N. M. NEWMARK

~~FIRST PROGRESS REPORT~~

HIGHWAY BRIDGE IMPACT ~~INVESTIGATION~~ INVEST.

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MAY 1951

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

FIRST PROGRESS REPORT

of the

HIGHWAY BRIDGE IMPACT INVESTIGATION

Conducted by

THE DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY OF ILLINOIS

In Cooperation With

THE DIVISION OF HIGHWAYS
STATE OF ILLINOIS

and

THE BUREAU OF PUBLIC ROADS

U. S. DEPARTMENT OF COMMERCE

MAY 1951

HIGHWAY BRIDGE IMPACT INVESTIGATION

CONTENTS

SYNOPSIS

This progress report presents the results of the first years' investigation of the effects of moving loads on highway bridges. The report is not a complete description of the work of the investigators because some time was spent in checking past work in the field and some was expended on unsuccessful methods of analysis before a workable scheme was devised.

During the period covered by this report, effort has been concentrated on the development of a method for the analysis of smoothly running mass loads on highway bridges. This effort has been successful in the main, although much remains to be done to reduce the amount of time and labor involved and possibly also to increase the accuracy. The chief advantages of the procedure lie in its ability to take full account of the inertia of the several axle loads which can act on the span as well as the inertia and elasticity of the bridge. It is also capable of extension to include the influence of damping and of truck springing.

Readers whose time is limited are advised to turn directly to Chapter IV in which some results of the first analytical predictions of the investigation are described. This material has been limited to simple span bridges, thus far, in order to reduce the length of time required for the analysis. The numerical values given are intended to be illustrative of the general character of the results obtainable, rather than as an exhaustive study of the influence of any one factor on highway bridge impact stresses.

The first chapter of this report is a brief history of past investigations. The second summarizes pertinent structural properties of certain types of highway bridges. The third chapter contains a description of the process of analysis employed, the fourth some typical results, and the last a discussion of the problems which seem most important, suitable and ready for analysis in the immediate future.

A definite effort has been made to write the material in a direct, readily understandable manner, making allowance for the difficulty of some of the points in question. The investigators will appreciate having their attention directed to any obscurities which may inadvertently have found their way into the report.

PERSONNEL.

The investigation, during the period covered by this report, has been under the general supervision of Professors N. M. Newmark, C. P. Siess, and L. E. Goodman of the Department of Civil Engineering, University of Illinois.

Detailed conduct of the work and the preparation of this report have been in the hands of Dr. T. P. Tung, Mr. W. H. Milley and Professor Goodman. To the first two of these are due most of the new results and techniques described herein.

HIGHWAY BRIDGE IMPACT INVESTIGATION.

CHAPTER I.

SUMMARY OF PAST INVESTIGATIONS OF THE EFFECTS OF MOVING LOADS ON BRIDGES.

The question of the effects of moving loads on bridges appears to have first attracted attention in 1847 when a British Royal Commission was appointed to investigate the "application of iron in structures subject to violent concussions and vibrations". Experiments were conducted to examine the effect of the velocity of a train in increasing or decreasing the tendency to failure of a girder bridge over which the train was passing. These experiments, in which a smoothly running load of considerable weight passed over a light, flexible, simply supported bridge at various velocities showed that the deflection of the bridge increased with the velocity of the moving load, up to a certain point, and that in certain cases this increase in deflection ammounted to two or three times the central statical deflection which would be produced by the load at rest on the span. It seemed highly desirable to investigate the problem analytically but it was realized that the exact calculation of the motion would be extremely difficult because the forces acting on the load and on any element of the bridge depend upon the positions and motions, or rather the accelerations, both of the load and of the bridge. Professor Willis, a member of the commission, derived a differential equation for the deflection of the bridge at the point over which the moving load was passing, on the assumption that the weight of the bridge was negligible compared with the weight of the load. This differential equation was solved for Professor Willis by Sir G. G. Stokes (12).* Stokes presented many tables based on his solution of the equation which made it possible actually to apply the analysis. It should be pointed out that the Stokes solution is only applicable when the ratio of the weight of load to the weight of bridge is large. As a consequence it cannot be applied in the case of highway bridge loadings where this weight-ratio is of the order of 0.2. The same problem was later investigated by H. Zimmermann (6). A brief discussion of the solution by S. Timoshenko is presented in his book "Vibration Problems in Engineering." Timoshenko approximated the solution of Willis' differential equation by assuming the trajectory of the moving load to be the same as it would be if the load moved very slowly. By "trajectory" of the load is meant the path of the point of contact of the load and the beam.

The case of a very small ratio of moving load to bridge weight was first studied by A. N. Kryloff (66) in 1905. He imagined the load to have so little mass that its inertia could be neglected and it could be thought of as exerting a constant force. The constant force case was later completely solved by S. Timoshenko (68) and by Sir. C. E. Inglis (21). One of the contributions of the present investigation has been to show that this analysis is unuseable for the mass ratios encountered in highway bridge design. The solution for this case expresses the deflection of the load point in terms of forced and free oscillation components determined by the speed of the load and

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* In the text, numbers in parentheses refer to the Bibliography.

the natural vibration characteristics of the bridge respectively. Since the forced oscillation component of deflection can be shown to be equivalent to the "crawl deflection", or the deflection which the bridge would undergo if the load moved very slowly across it, the only dynamic effect in this case is produced by the free oscillation deflection component which in the case of a highway bridge is insignificant. Influence lines for dynamic bending moment in a simple span are symmetrical with regard to the center of the bridge as found by this method while it is known that they are not so in reality. Although deflections as found by the constant force solution agree with experimental results more closely than do the stresses, the constant force solution cannot be regarded as giving a true picture of the dynamic response of a bridge to a moving load. The inertia of the load cannot be neglected.

The general problem of a moving load where both the mass of the load and of the bridge are considered was first investigated by H. H. Jeffcott (67) in 1929. Jeffcott solved the differential equation for this case by a series of successive approximations, first neglecting the inertia terms and obtaining a solution, then using this solution and considering the inertia terms so as to obtain a second solution and so on. Jeffcott's iteration method has been shown not to converge in certain cases.

Another analysis of the general case is that due to A. Schallenkamp (39), published in 1937. The Schallenkamp method involves writing the trajectory in the form of a series and noting that the expression for the load on the beam then contains a constant force and an infinite number of oscillating components which are determined by the weight of the load and its speed. The generalized coordinates for this type of disturbance are known, hence a new equation can be written for the trajectory which when expanded in a Fourier series leads, in principle, to a solution of the problem. Schallenkamp obtained good agreement between theory and certain experiments.

An investigation into the dynamic influence of moving loads on bridges is being conducted at present by A. Hillerborg at the Royal Institute of Technology, Stockholm, Sweden. It comprises both theoretical and experimental work. The simplest cases are being thoroughly studied to begin with. Eventually it is hoped to combine the various effects one by one to determine their separate and cumulative actions. Hillerborg has employed, with ingenious modifications, an assumption due to Inglis as to the form of the deflection curve. Expressions have been derived for the "dynamic increment" in deflection and bending moment in terms of dimensionless quantities. These dynamic increments which are the fractional increases in deflection and moment caused by the moving load are presented in graphical form making it practical to apply the analysis to simple experiments. Hillerborg's results apply only to a single smoothly running load.

CHAPTER II.

STRUCTURAL PROPERTIES OF HIGHWAY BRIDGES.

Before starting an analytical study of the effect of moving loads on highway bridges it was felt necessary to investigate and compare certain physical characteristics of the common types of highway bridges in Illinois. Although the actual properties of these bridges were not required in the study of impact at the outset, since the problem can be expressed in general terms, they were necessary to give meaning to the final results.

Seven continuous three-span I-beam bridges were studied initially, each having been designed for H-15-44 loading. The outside spans of these bridges varied in length from 26.25 ft. to 88.50 ft. while the center spans ranged from 33 ft. to 113 ft., as shown in Table 1B. The ratio of the outside span length to the center span length was found always to lie within the narrow limits of 0.777 to 0.782. The bridges consisted of a 6 1/2-in. or 7-in. reinforced concrete slab deck 24 ft. to 30.5 ft. in width supported on 5 WF steel girders whose depth varied from 21 in. to 36 in. and which were spaced at equal intervals of from 6 ft.-0 in. to 6 ft.-10 in., as shown in Table 1A. The roadway width was either 22 ft. or 24 ft. with curbs 8 in. high varying in width from 6 in. to 33 in. The handrail in every bridge except one was made of structural steel and in this one case it was made of concrete. The bearings of each bridge consisted of roller supports at both abutments and either a roller or a keyed rocker at the two intermediate piers. Three of the bridges were level longitudinally, three had grades of 0.33 per cent each way from the center-line and only one had a significant grade of 3.87 per cent throughout.

The stiffness of each bridge given in Table 1C was calculated on the assumption that the steel girders and concrete slab behave as a composite beam. The ratio E_s/E_c was taken as 7 with $E_c = 4.5 \times 10^6$ psi. The assumption of a composite beam is good enough for the small deflections which would be produced in the free vibration of the bridge but for larger deflections slippage may occur between the girders and the slab. Neglecting the handrail, but including the curbs in the calculations, the transverse stiffness computed on the foregoing assumptions ranged from 5.29×10^9 ft.^{2} lb. to 32.81×10^9 ft.^{2} lb. corresponding to the center spans of 49.3 ft. and 113.0 ft. shown in Table 1B. The heights of the neutral axis of this transverse section ranged from 90 per cent to 113 per cent of the girder height measured above the bottom of the outer beam and are shown in Table 1C.

In order to find the three lowest frequencies of vibration of these unloaded three-span continuous bridges of varying span length we proceed as follows: Taking the origin of coordinates at the left end of each span the equation for the deflection of each span during free vibration is:

$$\mu = a_r (\cos kx - \cosh kx) + c_r \sin kx + d_r \sinh kx.$$

Since for the continuous bridge the slope of the beam and the bending moment of the beam are continuous over any interior support, we may write the first and second derivatives of the above equation with proper boundary conditions to get three equations in three unknowns, a_r, c_r, and d_r. Solving for these three constants and letting:

$$\phi_r = \coth kL_r - \cot kL_r$$

$$\psi_r = \operatorname{cosech} kL_r - \operatorname{cosec} kL_r$$

we can then write the slope equation at each interior support in terms of ϕ and ψ, so that for a bridge of "i" spans we will have (i - 1) equations of the type:

$$-a_2 (\phi_1 + \phi_2) + a_3 \psi_2 = 0$$

$$a_2 \psi_2 - a_3 (\phi_2 + \phi_3) + a_4 \psi_3 = 0$$

$$a_{i-1} \psi_{i-1} - a_i (\phi_{i-1} + \phi_i) = 0$$

By equating to zero the determinant of these equations the frequency equation for the vibration of the continuous bridge is obtained and for a three-span bridge where the two outside spans are equal we get:

$$\phi_1 + \phi_2 = \pm \psi_2$$

where $kL_1 = 0.795\, kL_2$.

To solve these equations it is convenient to draw a graph of the functions $(\phi_1 + \phi_2)$, ψ_2, $-\psi_2$. The three lowest values of "kL" where these graphs intersect represent the three lowest frequencies of vibration of the bridge and these frequencies were calculated from the formula:

$$f = \frac{(kL)^2}{2\pi L^2} \sqrt{\frac{EI}{m}}.$$

The three mode shapes of vibration corresponding to the three natural frequencies found are shown in Fig. 1.

Later in the investigation a similar set of properties was compiled for four simple span I-beam bridges designed for H-15-44 loading since the analytical development of the impact problem had reached the stage where a practical application of the methods seemed desirable. Each of the four bridges was designed and built by a separate State Highway Department so that they should represent a good cross-section of bridge design practice in the United States. A bridge having similar properties to those shown in Table 2 was chosen for the analysis described in Chapter IV, the dimensions being changed only very slightly for convenience in the analysis. The properties listed in Table 2 were found in the same way as those for the three-span bridges except that the natural frequencies of vibration in the simple span case are merely n^2 times the fundamental frequency, where "n" is the order of the mode in question.

TABLE 1A.

GENERAL DESCRIPTION OF THREE-SPAN I-BEAM BRIDGES.
STATE OF ILLINOIS.

NAME	TOTAL WIDTH (Outside of Curbs)	ROADWAY WIDTH	GIRDERS	CURB		SLAB THICK-NESS	CROWN
				Width	Height		
DOYLES BRANCH	30'-6"	24'-0"	5-21" WF-59# 6'-10" o.c.	24"	8"	$6\frac{1}{2}$"	1-3/8"
LAWS CREEK	26'-0"	24'-0"	5-24" WF-87# 6'-$2\frac{1}{2}$" o.c.	12"	8"	$6\frac{1}{2}$"	1-3/8"
LITTLE MUDDY RIVER	26'-0"	24'-0"	5-36" WF-150# 6'-1" o.c.	12"	8"	7"	1-3/8"
PECATONICA RIVER	29'-6"	24'-0"	5-36" WF-94# 6'-$7\frac{1}{2}$" o.c.	33"	8"	7"	1-3/8"
BIG CREEK	24'-0"	22'-0"	5-24" WF-84# 5'-$8\frac{1}{2}$" o.c.	12"	8"	$6\frac{1}{2}$"	1-3/8"
RANGE CREEK	25'-0"	24'-0"	5-21" WF-73# 6'-0" o.c.	6"	8"	$6\frac{1}{2}$"	1-3/8"
JOHNSON COUNTY	24'-0"	22'-0"	5-30" WF-124# 5'-7" o.c.	12"	8"	7"	1-3/8"

TABLE 1B.

PROPERTIES OF THREE-SPAN I-BEAM BRIDGES.

STATE OF ILLINOIS.

NAME OF BRIDGE	LOCATION	DESIGN LOADING	TOTAL LENGTH, (ft.)	WEIGHT per ft. (lb.)	SPAN LENGTH Outside $L_1 = L_3$	SPAN LENGTH Center L_2	$\frac{L_1}{L_2}$
DOYLES BRANCH FA Proj. 531-C	Clark County S.A. Rt. 8, Sec. 26-B	H-15	85.50	4166	26.25	33.00	0.795
LAWS CREEK	Clay County S.A. Rt. 5-A, Sec. 10-B	H-15	155.50	2950	47.42	60.67	0.782
LITTLE MUDDY RIVER Proj. S-151 (1)	Jackson County F.A.S. Rt. 869, Sec. 11-B, F	H-15-44	266.50	3632	81.17	104.17	0.779
PECATONICA RIVER Proj. S-134 (2)	Stephenson County F.A.S. Rt. 57, Sec. 27-B S.A. Rt. 10	H-15	290.00	4748	88.50	113.00	0.783
BIG CREEK	Fulton County S.A. Rt. 14, Sec. 32 B-1- MFT.	H-15	160.00	2943	48.75	62.50	0.780
RANGE CREEK	Cumberland County S.A. Rt. 4, Sec. 21-B -15 d	H-15	126.00	2808	38.33	49.33	0.777
JOHNSON COUNTY Proj. S-152 (1)	Johnson County F.A.S. Rt. 928, Sec.24-B	H-15-44	220.00	3237	67.00	86.00	0.779

TABLE 1c.

PROPERTIES OF THREE-SPAN I-BEAM BRIDGES. (Con'd).

STATE OF ILLINOIS

NAME OF BRIDGE	EI of SECTION (ft.^{2}lb.) x 10^9	*HEIGHT OF NEUTRAL AXIS (in.)	NATURAL FREQUENCIES (c.p.s.)			GRADE Per Cent	HANDRAIL
			f_1	f_2	f_3		
DOYLES BRANCH	5.90	23.82	12.4	19.0	23.3	3.87	Concrete
LAWS CREEK	8.07	24.45	5.2	8.1	9.8	0.33 each way from center	Steel
LITTLE MUDDY RIVER	23.95	32.37	3.8	4.3	5.2	0.33 each way from center	Steel
PECATONICA RIVER	32.81	33.40	2.4	3.7	4.5	0	Steel (with concrete posts)
BIG CREEK	7.82	24.34	4.8	7.5	9.1	0	Steel
RANGE CREEK	5.29	21.82	6.5	10.2	12.4	0	Steel
JOHNSON COUNTY	15.30	28.23	3.4	5.3	6.5	0.33 each way from center	Steel

*Above bottom of outside girder

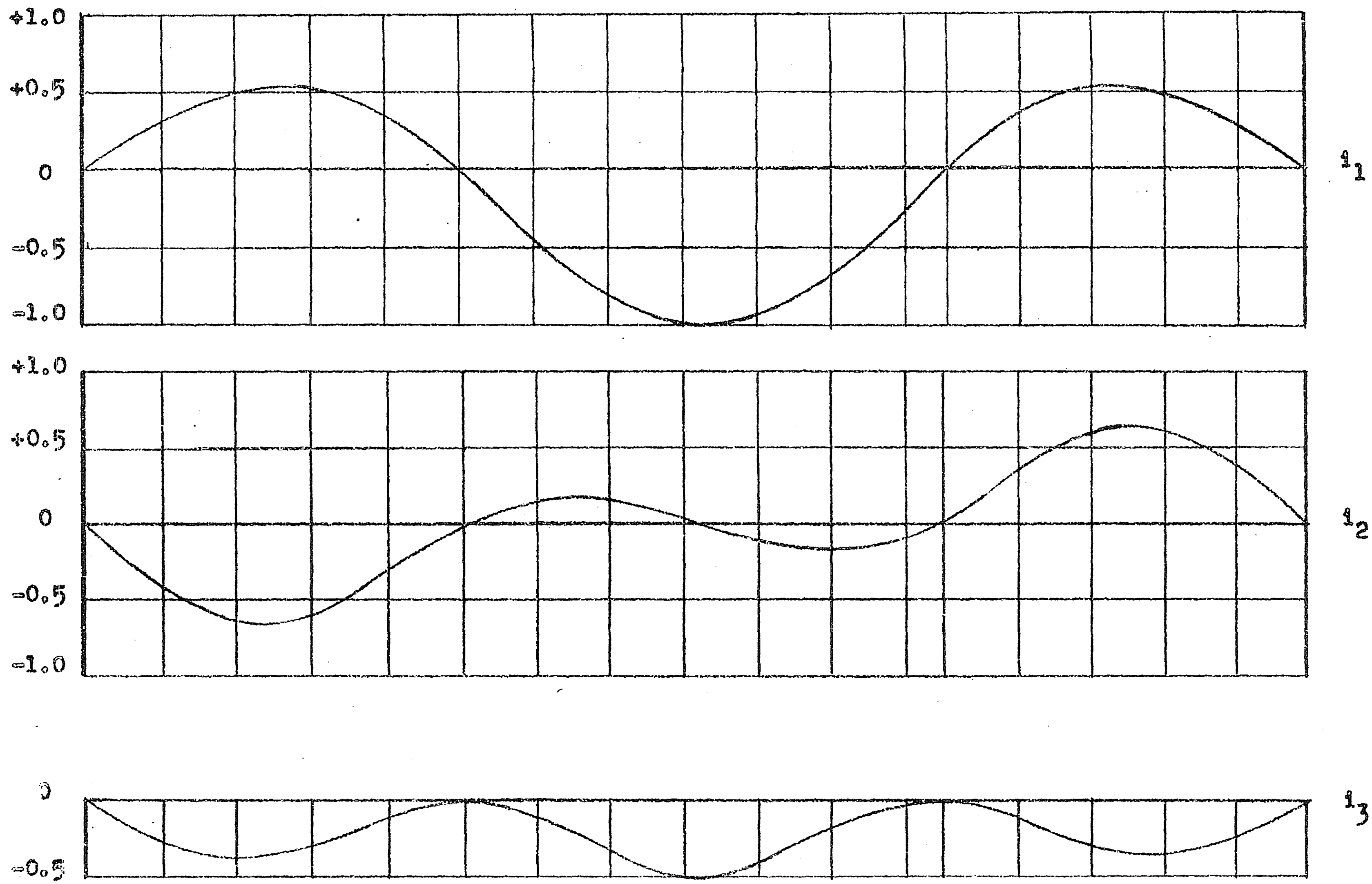
TABLE 2.

SIMPLE-SPAN I-BEAM BRIDGE PROPERTIES.

(All Designed for H-15 Loading).

BRIDGE	SPAN (ft.)	WEIGHT per ft. (lb.)	EI of SECTION ft.^{2}lb. x 10^9	NATURAL FREQUENCIES (c.p.s.)			HANDRAIL	ROADWAY WIDTH (ft.)
				f_1	f_2	f_3		
OREGON 1. Vancouver Ave. Bridge Columbia Slough, Multnomah County	75.00	4640	31.67	4.1	11.6	37.2	Concrete	44*
WISCONSIN 1. Bridge No. 582 Town of Weston, Dunn County	68.92	3670	32.68	5.6	22.5	50.5	Steel	24
MAINE 3. Knightly Bridge at Waterford, Oxford County	55.00	2860	11.85	6.0	24.1	54.2	Concrete	24
UTAH 4. Skull Valley Drain Bridge. Low-Timple Tocele County	60.00	2710	20.85	6.9	27.5	61.9	Concrete	44*

*Properties converted to 24 ft. width.



TYPICAL MODE SHAPES

THREE-SPAN CONTINUOUS BRIDGES.

FIG. 1.

CHAPTER III.

METHOD OF ANALYSIS.

I. Simplifications Employed.

The analysis which has been developed for the purposes of the investigation is explained in detail in this section. The method has been devised with a view toward including all of the important factors which influence the stresses and deflections of highway bridges. At the same time simplifications have been introduced which permit the question to be investigated without the use of an elaborate mathematical treatment. In practice the analysis is a repeated sequence of numerical operations - additions, subtractions and multiplications. The simplifications or assumptions which make this possible are first discussed.

To begin with, the bridge structure with its distributed mass and elasticity is replaced by a structure having concentrated mass and elasticity as indicated schematically in Fig. 1.

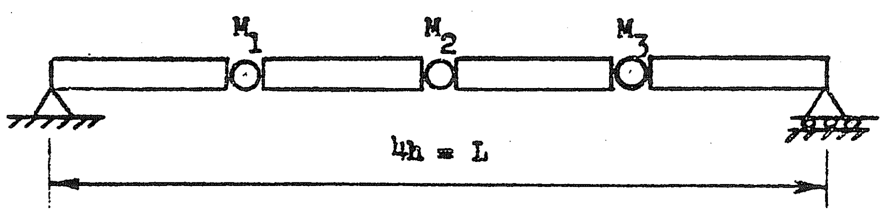


FIG. 1.

IDEALIZED SPAN.

The bridge span is divided into n equal segments. At each joint is placed a mass equal to 1/nth of the total mass of the structure. At each joint a hinge or spiral spring is considered to resist angle change at the joint. The stiffness of these hinges are such that the deflection of any joint for a static load on the bridge is exactly the same as the deflection of the original bridge structure with its distributed mass.

In practice it is necessary to set a limit on the value of n employed in the computation. An actual bridge can deflect in a very complicated manner. In fact, strictly speaking, it is necessary to know the deflection of each point in order to specify the deflection configuration completely. This ideal can be approached by using a large number of divisions, making n very large. On the other hand, the method, while unchanged in principle, becomes more tedious in actual computation when this is done, so that some compromise based on engineering judgment is required. During the past year most work has been based on a four-division scheme, and the remaining portions of this discussion will deal with this case. Extension to any number of divisions is straightforward.

To get a clearer idea of what is being neglected by the assumption of concentrated mass and elasticity we recall that any deflection shape of a simple beam can be expressed as the sum of a series of sine waves. In the first of these, the beam deflects to a half sine wave shape. This shape can be closely approximated by four straight-line segments. The second mode distorts the beam into a full sine wave shape which can still be well represented by four straight-line segments. Even the third mode is accounted for. But the fourth and higher modes are too sinuous in shape to be modeled by so simple a structure. The use of a "model" having only n - 1 degrees of freedom implies that the influence of modes of vibration higher than the n-1st is to be sacrificed.

On the other hand, there is good experimental evidence that the effects of damping are much more pronounced in the higher modes than in the lower. Certainly it is difficult to excite the higher modes of actual highway spans in the field by means of mechanical oscillators. These field observations provide some basis for believing that the quantities neglected in the analysis are precisely those which ought to be neglected on physical - as contrasted with purely mathematical - grounds. In the final analysis, however, this point is one which can only be resolved by experiment.

There is a second kind of error introduced by replacement of the continuous span with four segments. The traveling mass representing a truck wheel is, as a consequence of this simplification, taken to have a deflection equal to that of the mode point just passed plus a fraction of the difference between the deflections of the end points of the segment over which it is moving. In reality, the flexibility of the span between mode points would modify this straight-line relation somewhat.

The analysis proceeds by dividing the total time required for the mass, traveling with uniform velocity v, to traverse the beam into short intervals of duration τ seconds. Within each of these intervals the vertical acceleration of all points on the span is assumed to vary linearly. So also is the acceleration of the traveling mass in contact with the span. If we call this deflection y, then

$$\text{acceleration} = a = \frac{d^2 y}{dt^2} = a_o + ct \qquad (1)$$

where a_o is the deflection at the beginning of a time interval of length τ . Integrating once

$$\text{velocity} = v = \frac{dy}{dt} = v_o + a_o t + \frac{1}{2} ct^2 \tag{2a}$$

and using Equation (1) to eliminate ct,

$$v = v_o + \frac{1}{2} t\,(a_o + a). \tag{2b}$$

Integrating Equation (2a),

$$y = y_o + v_o t + \frac{1}{2} a_o t^2 + \frac{1}{6} ct^3 \tag{3a}$$

$$y = y_o + v_o t + \frac{1}{3} a_o t^2 + \frac{1}{6} a t^2 \tag{3b}$$

If quantities pertaining to the end of the time interval are denoted by double accents (e.g. v''), and those pertaining to the beginning of the time interval by single accents (e.g. v'), then from Equations (2b) and (3b) we have directly:

$$v'' = v' + \frac{1}{2} \tau\,(a' + a'') \tag{4a}$$

$$y'' = y' + v'\tau + \frac{1}{3} a' \tau^2 + \frac{1}{6} a'' \tau^2 \tag{4b}$$

II. Equations of Motion.

Having developed the simplifications and assumptions of the procedure, equations of motion for the joints can be derived. To begin with, the moving load is in the left-hand panel. Let us choose the time interval τ such that the panel, of length [illegible], is traversed in, say, twenty intervals.

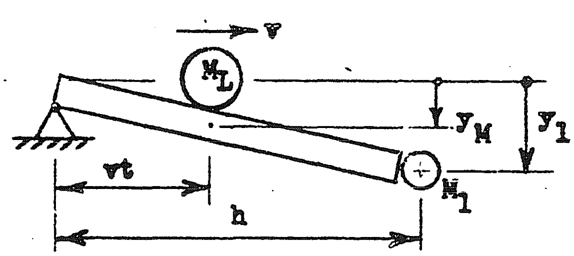


FIG. 2.
FIRST PANEL

The mass, at the end of n time intervals, is distant from the left support an amount

$$vt = v n \tau \qquad (5)$$

It has passed over a fraction, α, of the panel length, where

$$\alpha = \frac{vt}{h} = \frac{n}{20} \qquad (6)$$

and consequently

$$y_M = \alpha\, y_1 = \frac{n}{20} y_1 \qquad (7)$$

The forces which act on the concentrated mass M_1 are shown in Fig. 3. The force exerted by the moving load, $\alpha M_L (g - \ddot{y}_M)$.[1]

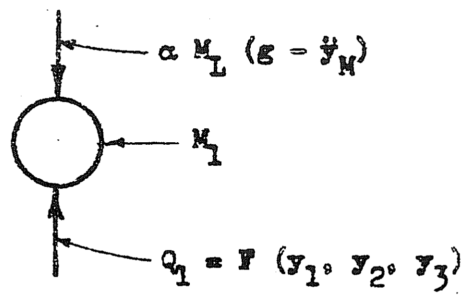


FIG. 3
FORCES ACTING ON M_1.

depends not only on the weight of the truck axle, $M_L g$, but also on the downward acceleration of the point of contact, y_M. If, for example, the point of contact were moving downward with an acceleration of 32.2 feet per second per second, there would be no instantaneous pressure exerted by the moving load on the span. It is this interaction between force and deflection that makes it impossible to utilize solutions in which the moving mass is replaced by a constant force or even by a force varying in some simple manner.

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(1) Dotted superscripts indicate differentiation with respect to time, for example, $\dot{y} = dy/dt$ and $\ddot{y} = d^2y/dt^2$.

The other force exerted on the mass M_1 is that due to the elasticity of the span. When the span deflects to positions y_1, y_2, y_3, internal forces Q_1, Q_2, Q_3 are developed at each node point to resist this deflection. The deflections are related to the forces by the expressions

$$\left.\begin{aligned} y_1 &= 9\,Q_1 + 11\,Q_2 + 7\,Q_3 \\ y_2 &= 11\,Q_1 + 16\,Q_2 + 11\,Q_3 \\ y_3 &= 7\,Q_1 + 11\,Q_2 + 9\,Q_3 \end{aligned}\right| \times \frac{L^3}{EI}\ \frac{1}{768} \qquad (8)$$

These expressions are just the deflections of the quarter points of a simple span due to loads Q_1, Q_2, Q_3. For example, the deflection of the center point, y_2, due to a concentrated force Q_2 is

$$y_2 = 16\,\frac{L^3}{EI}\,\frac{1}{768}\,Q_2 = \frac{Q_2 L^3}{48\,EI},$$

a well-known relationship. Inverting the above Equations (8) we have

$$\left.\begin{aligned} Q_1 &= 630.85706\,y_1 - 603.42841\,y_2 + 246.85703\,y_3 \\ Q_2 &= -603.42841\,y_1 + 877.71410\,y_2 - 603.42841\,y_3 \\ Q_3 &= 246.85703\,y_1 - 603.42841\,y_2 + 630.85706\,y_3. \end{aligned}\right| \times \frac{EI}{L^3} \qquad (9)$$

The forces Q are simple linear functions of the deflections.

Newton's equation of motion, F = ma, can now be written for the three node points.

$$\begin{aligned} M_1\,\ddot{y}_1 &= \alpha\,M_L\,(g - \ddot{y}_M) - Q_1 \\ M_2\,\ddot{y}_2 &= -Q_2 \\ M_3\,\ddot{y}_3 &= -Q_3 \end{aligned} \qquad (10)$$

These expressions are valid so long as the truck axle is in the first panel. When it enters the second panel the term $\alpha M_L (g - \ddot{y}_M)$ must be removed from the first of Equations (10). While the truck is in the second panel, moving load terms appear in the first two of Equations (10), while it is in the third panel they appear in the last two equations, and while it is in the last panel the moving load term appears only in the last equation. The treatment of these cases is not discussed explicitly in this report but it follows the first-panel case in a straightforward manner.

III. Solution of the Equations of Motion.

The first step in the solution of Equations (10) is the elimination of the term $\ddot{y}_M$. In view of Equation (7)

$$y_M = \alpha y_1$$

whence

$$\dot{y}_M = \dot{\alpha} y_1 + \dot{y}_1 \alpha$$

where, in view of Equation (6) $\dot{\alpha} = v/h =$ constant.

$$\ddot{y}_M = \frac{2v}{h} \dot{y}_1 + \alpha \ddot{y}_1 \tag{11}$$

Using Equation (11), Equations (10) become

$$\begin{aligned} \ddot{y}_1 \, (1 + 4\alpha^2 R) &= 4\alpha v g - 8\alpha R \frac{v}{h} \dot{y}_1 - Q_1/M \\ \ddot{y}_2 \qquad\qquad &= - Q_2/M \\ \ddot{y}_3 \qquad\qquad &= - Q_3/M \end{aligned} \tag{12}$$

where

$$R = \frac{M_L}{4M} = \frac{\text{traveling weight}}{\text{weight of bridge}} \tag{13}$$

The second step in the solution is the removal of the velocity term, $\dot{y}_1$, from the right-hand side of the first of Equations (12). This is done by means of Equation (4a) which relates the velocity at any time to the velocity at a previous instant and to the accelerations at the present and previous instants. Denoting the values of quantities at a previous instant by an additional zero subscript (e.g. y_{10}) is the deflection y_1 at an instant previous to the one under consideration, we have from (4a),

$$\dot{y}_1 = \dot{y}_{10} + \frac{1}{2}\tau\,(\ddot{y}_{10} + \ddot{y}_1) \qquad (15)$$

Substituting this in the first of Equations (12),

$$\begin{aligned} \ddot{y}_1\left(1 + \alpha^2 R - \frac{4\alpha R v\tau}{h}\right) &= \alpha R\left[4g - \frac{8v}{h}\left(\dot{y}_{10} - \frac{\tau}{2}\ddot{y}_{10}\right)\right] - Q_1/M \\ \ddot{y}_2 &= -Q_2/M \qquad (16) \\ \ddot{y}_3 &= -Q_3/M \end{aligned}$$

At this point there are two ways in which it is possible to proceed. Both yield the same result and the choice between them is largely one of convenience. The first method described is due to N. M. Newmark. It is an iterative procedure.

A part of the solution is assumed, the remainder is then computed, and finally the computed part is used to obtain a new value for the quantities originally assumed. If these values agree with the original assumptions the process is complete, otherwise it must be repeated using the new values as a starting assumption:

Method A:

Suppose that at the beginning of some time interval the displacements, velocities, and accelerations of the node points are known. Since the properties of the bridge and the load are also known, the only unknowns which appear in Equations (16) are the accelerations at the end of the time interval ($\ddot{y}_1$, $\ddot{y}_2$, $\ddot{y}_3$) and the displacements at the end of the time interval (y_1, y_2, y_3). These last appear implicitly Q_1, Q_2 and Q_3 by virtue of Equations (9).

1. Assume values for the "final" accelerations $\ddot{y}_1$, $\ddot{y}_2$, $\ddot{y}_3$.

2. Compute the corresponding values of the final displacements and velocities using Equations (4b) and (4a).

3. With these values of the displacements and velocities at the end of the interval, solve Equations (16) for new values of the accelerations. These should agree with the values assumed in Step 1. If they do not agree, to the accuracy desired, the computation is repeated, using the derived values of the accelerations as a new starting point.

4. It may be noted that the method depends on knowing the displacements, velocities, and accelerations at the beginning of some time interval. These quantities are known for the first time interval; all successive values follow, step by step.

Method B:

As in A, it is assumed that at the beginning of some time interval, say the first, the displacements, velocities and accelerations of the joints are known.

1. To begin, the accelerations $\ddot{y}_1$, $\ddot{y}_2$, $\ddot{y}_3$ are removed from Equations (16) by direct substitution from Equation (4b). The resulting expressions are

$$Q_1/M + \frac{6}{\tau^2}\left(1 + 4\alpha^2 R + \frac{4\alpha R v\tau}{h}\right)y_1 = \frac{6}{\tau^2}\left(1 + 4\alpha^2 R + \frac{4\alpha R v\tau}{h}\right)\left(y_{10} + \dot{y}_{10}\tau + \frac{1}{3}\tau^2 \ddot{y}_{10}\right)$$
$$+ \alpha R \left[4g - \frac{8v}{h}\left(\dot{y}_{10} - \frac{\tau}{2}\ddot{y}_{10}\right)\right]$$

$$Q_2/M + \frac{6}{\tau^2} y_2 = \frac{6}{\tau^2}\left(y_{20} + \dot{y}_{20}\tau + \frac{1}{3}\tau^2 \ddot{y}_{20}\right) \qquad (17)$$

$$Q_3/M + \frac{6}{\tau^2} y_3 = \frac{h}{\tau^2}\left(y_{30} + \dot{y}_{30}\tau + \frac{1}{5}\tau^2 \ddot{y}_{30}\right)$$

2. These expressions are a set of three linear algebraic equations in the three unknowns y_1, y_2, y_3. (It should be remembered that Q_1, Q_2, Q_3 are simply linear functions of the y's, as defined by Equations (9).) The quantities appearing on the right-hand sides of Equations (17) depend only on previous values of the acceleration, velocity and displacement. They are known constants as far as the solution of the equations is concerned.

3. While the Equations (17) can, in principle, be solved directly, it is preferable to reduce them to a dimensionless form. Let the time of passage of the load over the span be divided into, say, 80 parts, 20 per panel, as in Section II of this Chapter. Let n be the number of the time interval under consideration, also as in Section II of this Chapter. Then

$$\alpha = n/20 = vt/h = v\ n\ \tau/h; \quad v\ \tau/h = 0.05 \tag{18}$$

Instead of measuring deflections y in conventional units, for the purposes of the calculation, it is desirable to introduce the dimensionless quantity $\bar{y}$ defined by

$$y = \bar{y}\ g\ \tau^2 \tag{19}$$

Time is to be measured by means of the dimensionless quantity t^* where

$$t = t^*\ \tau \tag{20}$$

Derivatives taken with respect to t^* will be indicated by asterisks

$$\text{e.g. } \frac{dy}{dt^*} = \overset{*}{y}$$

It should be noted that

$$\frac{dy}{dt} = \frac{dy}{dt^*}\ \frac{dt^*}{dt} = \overset{*}{y}\ \frac{1}{\tau}$$

Finally it is necessary to choose the length of the time interval τ. The proper choice of time interval is discussed in some detail in Section IV of this chapter. For purposes of illustration let us consider a time interval one thirty-second of the fundamental period, i.e.

$$\tau = T/32. \tag{21}$$

where

$$T = \frac{4}{\pi}\sqrt{\frac{ML^3}{EI}} \tag{22}$$

On using the relationships (18) to (22) and replacing Q_1, Q_2, and Q_3 by their values in terms of y_1, y_2 and y_3 as given by Equations (9), the equations of motion take the form:

$$[630.85706\, y_1 - 603.42841\, y_2 + 246.85703\, y_3]\,\frac{1}{64\,\pi^2}$$

$$+ 6\,\bar{y}_1\left(1 + \frac{n\,R(1+n)}{100}\right) = 6\left(1 + \frac{n\,R(1+n)}{100}\right)\left(\bar{y}_{10} + \overset{*}{\bar{y}}_{10} + \overset{**}{\bar{y}}_{10}\right)$$

$$+ \frac{n\,R}{20}\left[4 - \frac{4}{10}\left(\overset{*}{\bar{y}}_{10} - \frac{1}{2}\overset{**}{\bar{y}}_{10}\right)\right]$$

$$[-603.42841\,\bar{y}_1 + 877.71410\,\bar{y}_2 - 603.42841\,\bar{y}_3]\,\frac{1}{64\,\pi^2} + 6\,\bar{y}_2$$

$$= 6\left(\bar{y}_{20} + \overset{*}{\bar{y}}_{20} + \frac{1}{3}\,\overset{**}{\bar{y}}_{20}\right) \tag{23}$$

$$[246.85703\,\bar{y}_1 - 603.42841\,\bar{y}_2 + 630.85706\,\bar{y}_3]\,\frac{1}{64\,\pi^2} + 6\,\bar{y}_3$$

$$= 6\left(\bar{y}_{30} + \overset{*}{\bar{y}}_{30} + \frac{1}{3}\overset{**}{\bar{y}}_{30}\right)$$

4. Since the initial values of y, $\dot{y}$ and $\ddot{y}$ are all zero, the Equations (23) are easily written down for the first time interval, $n = 1$. Suppose, for example, that, as in one of the cases studied, the mass ratio $R = 3.5$. Then for $n = 1$ Equations (23) take the form:

$$7.418735\,\bar{y}_1 - 0.955312\,\bar{y}_2 + 0.390809\,\bar{y}_3 = 0.7$$

$$-0.955312\,\bar{y}_1 + 7.389544\,\bar{y}_2 - 0.955312\,\bar{y}_3 = 0$$

$$0.390809\,\bar{y}_1 - 0.955312\,\bar{y}_2 + 6.998735\,\bar{y}_3 = 0$$

of which the solution is

$$\bar{y}_1 = 0.096090$$

$$\bar{y}_2 = 0.011940$$

$$\bar{y}_3 = -0.003736$$

From these values we may at once compute

$$\overset{**}{\bar{y}}_1 = 6\,\bar{y}_1 - 6\,\bar{y}_{10} - 6\,\overset{**}{\bar{y}}_{10} - 2\,\overset{**}{\bar{y}}_{10} = 6 \times 0.096090 = 0.576540$$

and

$$\overset{*}{\bar{y}}_1 = \overset{*}{\bar{y}}_{10} + 0.5\,(\overset{**}{\bar{y}}_1 + \overset{**}{\bar{y}}_{10}) = 0 + 0.5 \times 0.576540 = 0.288270$$

$$\overset{**}{\bar{y}}_2 = 0.071640 \qquad \overset{*}{\bar{y}}_2 = 0.035820$$

$$\overset{**}{\bar{y}}_3 = -0.022416 \qquad \overset{*}{\bar{y}}_3 = -0.011208$$

The procedure may now be repeated for the second time interval.

5. A valuable check is obtained by computing

$$P = \alpha\, M_L\,(g - \ddot{y}_M) = \frac{n}{20} M_L\, g\,(1 - 0.1\,\overset{*}{\bar{y}}_1 - 0.05\,n\,\overset{**}{\bar{y}}_1) \tag{24}$$

$$Q_1 = [0.998735\,\bar{y}_1 - 0.955312\,\bar{y}_2 + 0.390809\,\bar{y}_3]\, g\, M \tag{25}$$

then

$$\overset{**}{\bar{y}}_1 = \frac{P - Q_1}{Mg} \tag{26}$$

In the example of Step 4,

$$P = \frac{n}{20} M_L\, g\,(1 - 0.028827 - 0.028827) = 0.047117\, M_L\, g$$

$$Q = 0.083102\, Mg$$

and since $M_L/M = 4 \times 3.5 = 14$,

$$\overset{**}{\bar{y}}_1 = 14 \times 0.047117 - 0.083102 = 0.576540$$

which checks the result of the previous step.

Similar checks can be performed for $\ddot{\bar{y}}_2$ and $\ddot{\bar{y}}_3$. They are simpler because P vanishes for these joints when the load is in the first panel.

6. Finally, bending moments at the quarter points are computed.

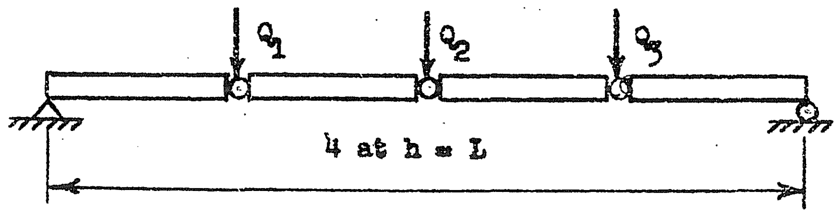


FIG. 4.

BENDING MOMENT COMPUTATION

$$M_{1/4} = h\left(\frac{1}{4}Q_3 + \frac{1}{2}Q_2 + \frac{3}{4}Q_1\right) Mg$$

$$M_{1/2} = 2h\left(\frac{1}{4}Q_3 + \frac{1}{2}Q_2 + \frac{1}{4}Q_1\right) Mg \qquad (27)$$

$$M_{3/4} = h\left(\frac{3}{4}Q_3 + \frac{1}{2}Q_2 + \frac{1}{4}Q_1\right) Mg$$

7. It is convenient to arrange the computations in a tabular form, as shown in Table 1. Here are indicated the computations for the first three time intervals.

IV. Proper Choice of the Time Interval.

It is clear that the iterative method of solution (Method A) is successful only if the newly computed values of acceleration are closer to the true accelerations than were the original assumed values. This will be the case provided that the time interval during which the acceleration is assumed to be constant is smaller than a certain limiting value. This limiting length of the time interval, τ, is related to the shortest period of the substitute structure. Newmark has shown that to secure convergence in ordinary vibration problems it is necessary to have $$\tau < \frac{T_{min}}{\pi} \tag{28}$$

For the case of a uniform bridge span

$$T_{max} = 1.2732\sqrt{\frac{ML^3}{EI}} \qquad T_3 = 0.1415\sqrt{\frac{ML^3}{EI}} \tag{29}$$

and for the substitute structure having concentrated mass and elasticity

$$T_{max} = 1.2736\sqrt{\frac{ML^3}{EI}} \qquad T_{min} = 0.1510\sqrt{\frac{ML^3}{EI}} \tag{30}$$

where M is, as before, one-quarter of the total mass of the structure. The figures are cited to show how closely the substitute structure with its concentrated mass and elasticity corresponds to the "original" structure with distributed mass and elasticity.

Designating the fundamental or maximum period simply as T, we have

$T_{min} = 0.1185\ T$ and therefore we must have

$$\tau < 0.0377\ T \tag{31}$$

In the illustrative problem of the pervious section, τ was chosen as

$$\tau = T/32 = 0.0312\ T \tag{32}$$

which falls within the allowable range.

At first glance it might appear as though Method B would be free from this limitation on the allowable length of the tine interval. Actually this is not the case. If the time interval is too large, the assumption of a linear fariation of acceleration will be seriously in error. Accuracy and convergence are closely allied and for this reason use of a time interval larger than that given by Equation (28) seems inadvisable. In the present investigation the time intervals have been checked by recomputing, using finer time divisions. The criterion presented in this section has proven to be entirely adequate.

TABLE 1.

TYPICAL COMPUTATION SHEET.

n	1	2	3	4
$\bar{\bar{y}}_1$	+ 0.096090	+ 0.684764	+ 1.880921	+ 3.560016
$\bar{\bar{y}}_1^{*}$	+ 0.288270	+ 0.901212	+ 1.461375	+ 1.879044
$\bar{\bar{y}}_1^{**}$	+ 0.576540	+ 0.649344	+ 0.470982	+ 0.364356
P_1	+ 0.659642	+ 1.182922	+ 1.644752	+ 2.069828
Q_1	+ 0.083102	+ 0.533574	+ 1.173772	+ 1.705474
$\frac{P_1 - Q_1}{Mg}$	+ 0.576540	+ 0.649348	+ 0.470980	+ 0.364354
$\bar{\bar{y}}_2$	+ 0.011940	+ 0.141769	+ 0.684247	+ 1.888349
$\bar{\bar{y}}_2^{*}$	+ 0.035820	+ 0.282027	+ 0.852993	+ 1.545741
$\bar{\bar{y}}_3^{**}$	+ 0.071640	+ 0.420774	+ 0.721158	+ 0.664338
P_2	0	0	0	0
Q_2	− 0.071636	− 0.420768	− 0.721157	− 0.664340
$\frac{P_2 - Q_2}{Mg}$	+ 0.071636	+ 0.420768	+ 0.721157	+ 0.664340
$\bar{\bar{y}}_3$	− 0.003736	− 0.038104	− 0.130762	− 0.117910
$\bar{\bar{y}}_3^{*}$	− 0.011208	− 0.069480	− 0.091950	+ 0.197862
$\bar{\bar{y}}_3^{**}$	− 0.022416	− 0.094128	+ 0.049188	+ 0.530436
P_3	0	0	0	0
Q_3	+ 0.022415	+ 0.094123	− 0.049185	− 0.530437
$\frac{P_3 - Q_3}{Mg}$	− 0.022415	− 0.094123	+ 0.049185	+ 0.530437
M(1/4)	+ 0.0321 Mgh	+ 0.2133 Mgh	+ 0.5075 Mgh	+ 0.8143 Mgh
M(1/2)	− 0.0189	− 0.1070	− 0.1589	− 0.0768
M(3/4)	+ 0.0018	− 0.0064	− 0.1040	− 0.3036

CHAPTER IV.

IMPACT LOADS ON SIMPLE SPANS.

Essentially two types of problem have been studied to date. Both deal with smoothly running loads on simple-span bridges. The first type of investigation has been aimed at establishing the practical usefulness of the method of analysis and does not deal with a specific bridge under a specific truck loading. The second investigation, on the other hand, concerns itself with the magnitude of the impact effect which would be produced by a common type of truck passing smoothly but at rather high velocity over a typical highway span.

The first of these studies, although admittedly academic, has brought into focus a number of important points, so that it is worth citing in some detail. The load in this case consisted of a single axle carrying a mass one-tenth the mass of the bridge, a more or less representative value for highway spans.

$$v = R = \frac{\text{mass of truck axle}}{\text{mass of bridge}} = 0.1 \qquad (1)$$

The only other parameter which need be specified is the quantity

$$\alpha = \frac{v\,T}{2\,L} \text{ taken equal to } 0.2 \qquad (2)$$

In this expression v is the velocity of the moving load, T the fundamental period of the structure, and L the span length.

The fact that α and R are the only quantities that need be specified for the analysis was first noticed by Inglis (see reference). The expression α can be written in a form which brings out its physical significance in a more striking way.

$$\alpha = \frac{\text{1/2 Fundamental period of the bridge}}{\text{Time required for the load to traverse the span}} \qquad (3)$$

The speed of the moving load is of importance, but only as compared with the fundamental period of the structure. In the investigation in question this quantity was taken to be two-tenths, a value which would represent, for example, a bridge having a span of 44 feet, and a fundamental period of 0.2 seconds, acted on by a single axle traveling at 60 m.p.h.

In Fig. 1 the deflection of the midpoint of the span is plotted vertically and the position of the moving load horizontally. When, for example, the load is three-tenths of the way across the span, the step-by-step or dynamic deflection of the midpoint is $2.80 \text{ g } T^2 \times 10^{-3}$. The three curves which appear in the figure are:

1. The static deflection. This is a parabolic curve symmetrical about the midspan. It is the deflection which the load would produce if moved very slowly. For this reason it is known as the "crawl" deflection.

2. The dynamic deflection obtained by the step-by-step analysis developed in this investigation. This is the true dynamic deflection of the midpoint. It lags behind the "crawl" deflection at first because of the inertia of the structure. Its maximum value is $0.569 \text{ g } T^2 \times 10^{-3}$ as compared with $0.512 \text{ g } T^2 \times 10^{-3}$ for the static case. The dynamic amplification factor for deflection is

$$\frac{y_{max. dynamic}}{y_{max. static}} = \frac{0.569}{0.512} = 1.11 \qquad (4)$$

We notice that the bridge swings back and its deflection becomes negative (upward) just before the load leaves the span.

3. The "constant force" solution. This is the dynamic deflection which would be produced by a constant force, as contrasted with a constant mass, traversing the span. The distinction lies in the fact that the force exerted by a mass on the bridge depends on the weight of the mass and also on the acceleration of the point over which the mass is passing. If that point, for example, is moving downward with an acceleration of 32.2 ft./sec.^{2} then the mass exerts no force on the span. It is precisely this interaction of mass and span which complicates the analysis. Actual truck loadings correspond to masses crossing the structure, at least if we neglect for the moment the springing of the truck. The statement is sometimes made that if the mass ratio R is small, the traveling mass may be replaced by a constant force. The solution shows that this is not so even for mass ratios as small as one-tenth.

Actually, bending moments are of greater engineering interest than deflections. Since stresses are directly proportional to bending moments, it is these which usually control design and which provide an index of the safety of the structure. In Figs. 2, 3 and 4 the bending moments at the quarter points of the bridge are plotted against the position of the load. The heavy curve of static bending moment represents the bending moment which the axle would produce if it moved slowly across the bridge. It is proportional to the influence line for bending moment at the point in question. On the other hand, the dashed curve represents the actual dynamic bending moment as obtained from the step-by-step analysis.

Referring to Fig. 2, the dynamic bending moment lags behind the static due to the inertia of the span. The dynamic amplification factor for bending moment or stress is 0.99, very nearly equal to unity.

At the third quarter-point, as shown in Fig. 4, the dynamic amplification factor is:

$$\frac{\text{max. dynamic moment}}{\text{max. static moment}} = \frac{0.405}{0.300} = 1.35 \qquad (5)$$

There is a thirty-five per cent increase in bending moment due to the dynamic effect.

The peak stress occurs when the traveling load is near the point in question. The explanation of the relatively high amplification factor at the third quarter-point, as compared with the first, lies in the fact that the moving load passes the third quarter-point after the response of the bridge has been fully developed. On the other hand, it passes the first quarter-point before the structure has had time to respond. Since vehicles are, presumably, as likely to travel in one direction as the other, the "third" quarter-point may be at either end of the span.

Figure 3 gives the bending moment at midspan. Although the amplification factor is smaller than at the third quarter-point, being in fact

$$\frac{\text{max. dynamic bending moment at midspan}}{\text{max. static bending moment at midspan}} = 1.12 \qquad (6)$$

the maximum static bending moment is larger by a factor of 4/3, so that the peak dynamic stresses are nearly equal at midspan and third quarter-point.

The "constant force" solution is shown for academic purposes only. It does not resemble the correct dynamic solution and would, in fact yield peak stresses on the unsafe side.

The second type of question investigated was intended to correspond closely to an actual highway loading. The vehicle in this case was taken to be a two-axle truck, shown schematically at the top of Fig. 5. In the Illinois State Traffic Survey of 1945 trucks of this type were found to account for roughly one-third of all commercial traffic. The total weight, 34,000 lb., is about as high as is commonly carried on a two-axle vehicle. The axle spacing of 16 feet and the division of load between axles (84 per cent to the rear and 16 per cent to the front axle) were chosen very close to the mean values for these vehicles disclosed in the traffic survey. The speed of the vehicle was taken to be 60 m.p.h., a high but not unreasonable value.

As mentioned in Chapter II, the bridge analyzed was intended to be more or less typical of State Highway Department practices for structures of the slab and I-beam type. Plans from a number of states were studied. The pertinent results of the study are given in Chapter II. The bridge actually analyzed had the following properties:

Span length $L = 64$ ft.

Weight of bridge $W = 218,000$ lb.

Fundamental period $T = 0.146$ seconds

Stiffness $= EI = 33.8 \times 10^9$ ft.^{2}lb.

It should be noted that in this case the quantity α of expressions (2) and (3) has the value 0.10. The bridges in question had been designed for H-15-44 loading so that the vehicle represents very nearly the load for which the structure was actually designed. It should be emphasized that the properties of the structure analyzed are an average of the properties of bridges of the same common type designed for the same loading.

The dynamic bending moments, as computed by the step-by-step method of analysis discussed in Chapter III, are shown in Figs. 5, 6 and 7. In each case the horizontal scale gives the position of the front wheel. The curves extend beyond the end of the span because the rear wheel is on the span even after the fron wheel leaves it. As in previous figures, the static or "crawl" bending moment is given by solid lines and the dynamic bending moment by dashed lines.

Referring to Fig. 5, the first point to be noted is that the dynamic amplification factor for moment at the first quarter-point is very nearly equal to unity. Both dynamic and static maximum bending moments are

$$\text{Maximum Moment} = 0.448 \frac{WL}{16} \qquad (7)$$

The dynamic response lags behind the static, reaching its maximum value 0.545 seconds after the rear wheel has passed.

At the midpoint, as shown in Fig. 6, the maximum dynamic bending moment materially exceeds the static.

$$\text{Max. static bending moment} = 0.577 \frac{WL}{16}$$

$$\text{Max. dynamic bending moment} = 0.625 \frac{WL}{16} \qquad (8)$$

$$\text{Amplification factor} = \frac{0.625}{0.577} = 1.08$$

It is interesting to note that in this case the dynamic curve oscillates about the static curve with a period nearly equal to the fundamental period of the span.

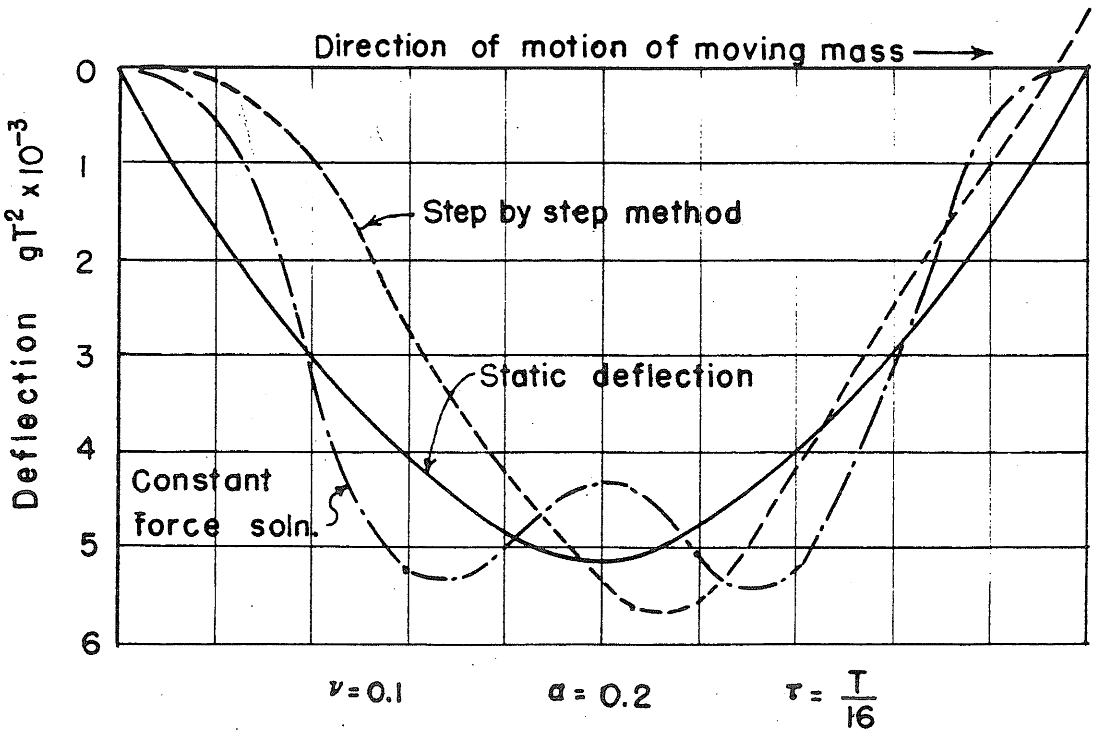
At the third quarter-point, as shown in Fig. 7, the dynamic amplification factor is larger than at the midpoint.

$$\text{Max. static bending moment} = 0.395 \frac{WL}{16} \qquad (9)$$

$$\text{Max. dynamic bending moment} = 0.445 \frac{WL}{16}$$

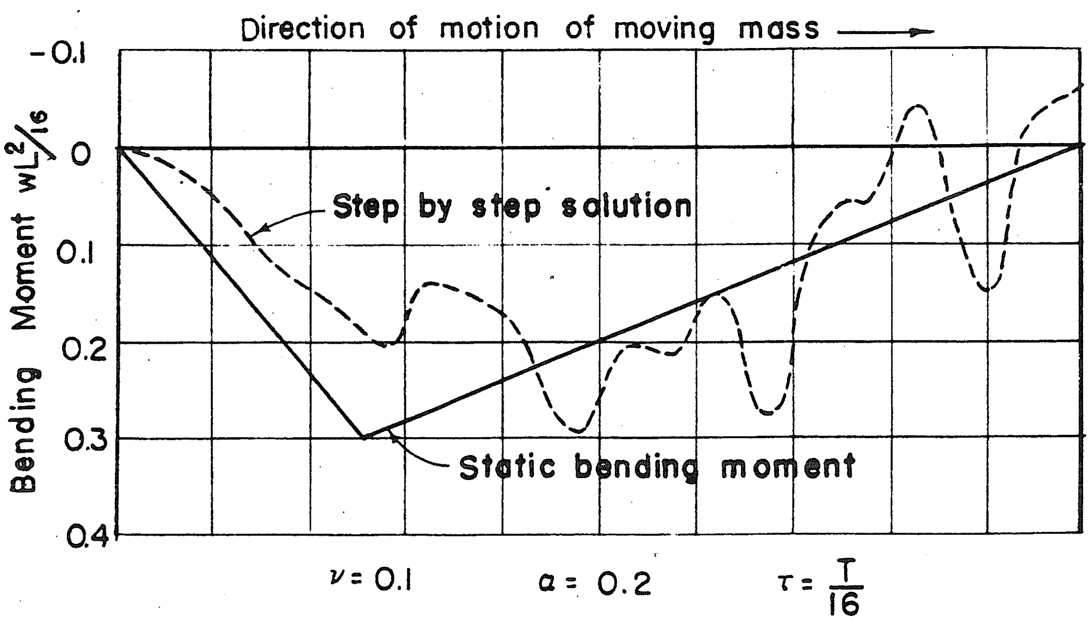
$$\text{Amplification factor} = \frac{0.445}{0.395} = 1.13$$

The higher amplification factor at this point is entirely reasonable, the explanation being the same as that given in connection with a similar observation in the case of the first problem of this Chapter. Indeed the reasoning in the two cases is substantially the same. The front axle, being light, has only a secondary influence on the dynamic response.



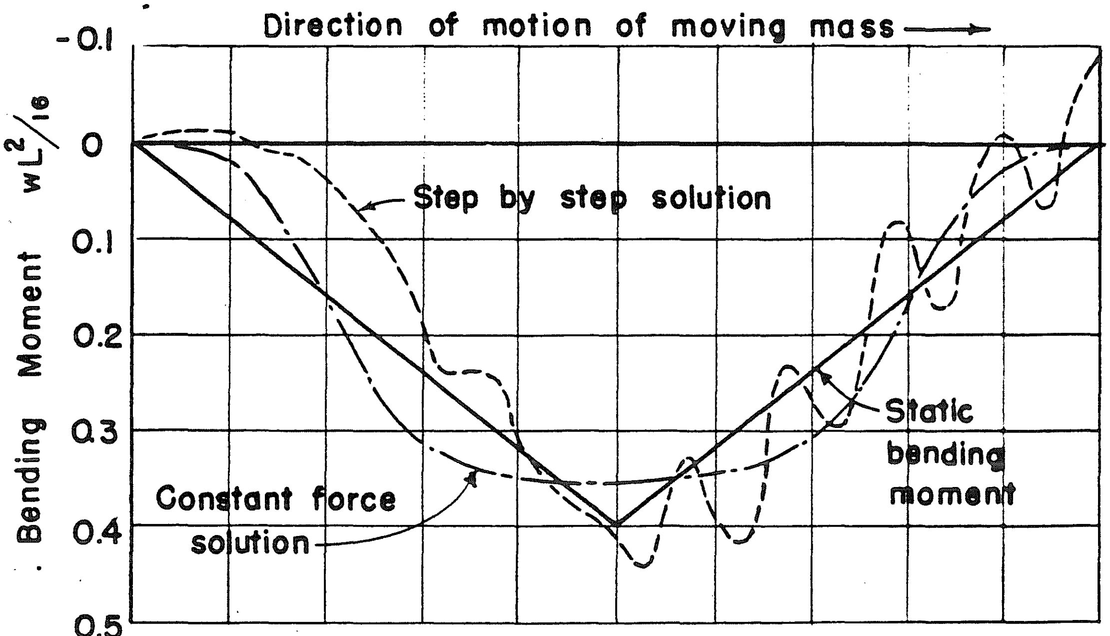
DYNAMIC DEFLECTION OF MID-POINT OF A SIMPLE BEAM DUE TO A MOVING MASS LOAD

FIG. 1



DYNAMIC BENDING MOMENT AT FIRST QUARTER POINT IN A SIMPLE BEAM DUE TO A MOVING MASS LOAD

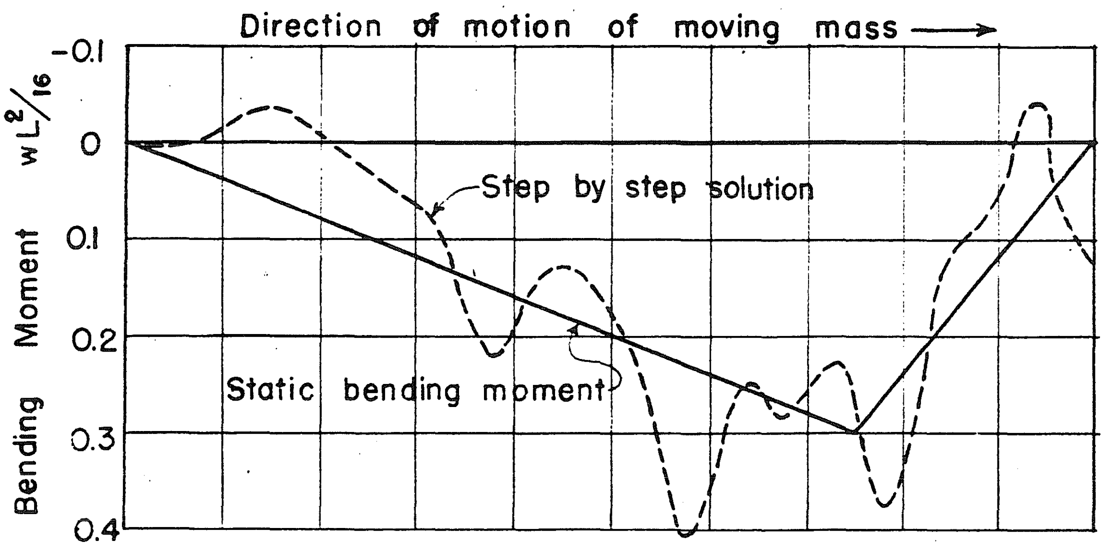
FIG. 2



$\nu = 0.1$ $\quad \alpha = 0.2$ $\quad \tau = \frac{T}{16}$

DYNAMIC BENDING MOMENT AT MID-POINT OF A SIMPLE BEAM DUE TO A MOVING MASS LOAD

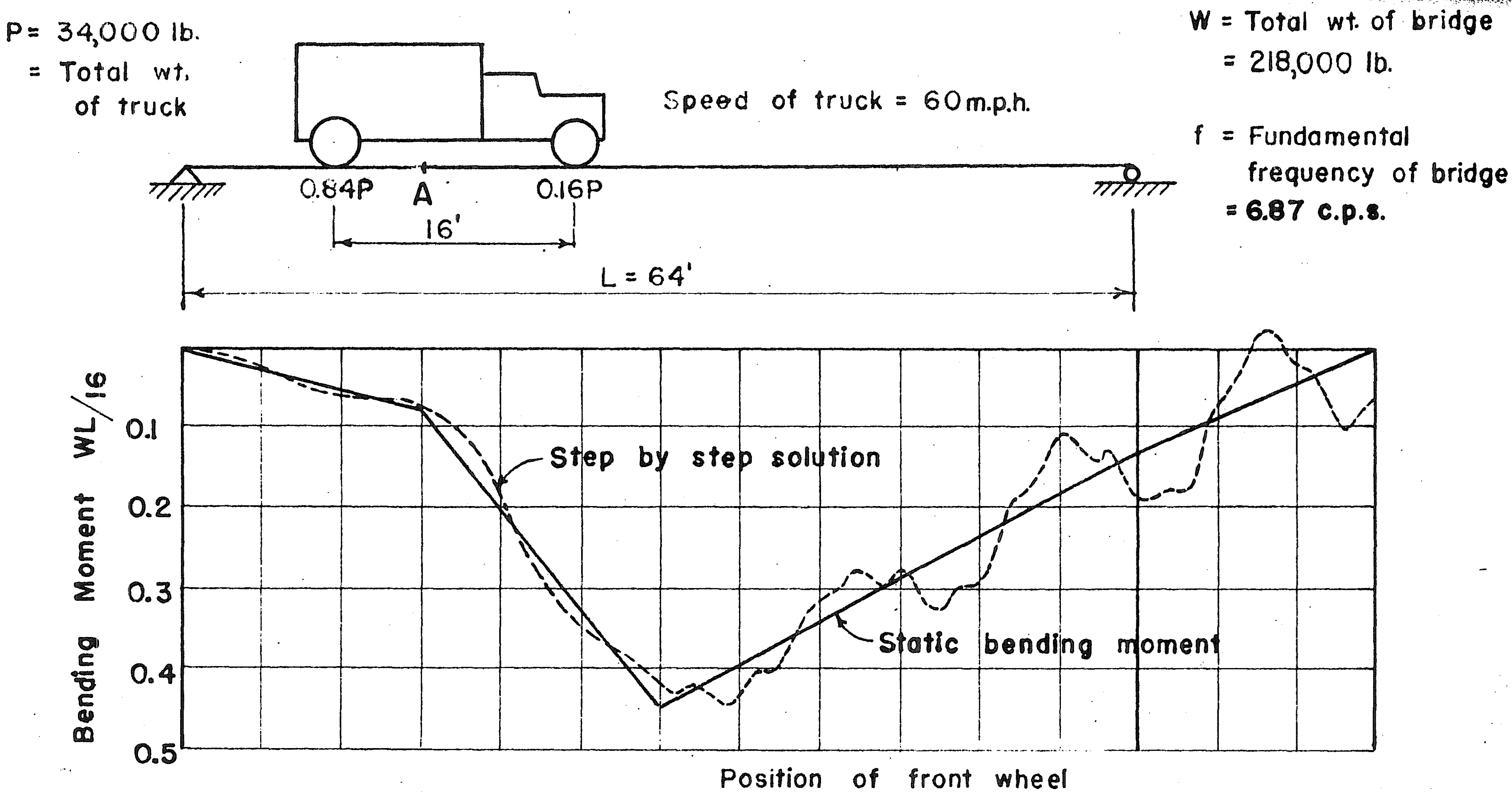
FIG. 3



$\nu = 0.1$ $\quad \alpha = 0.2$ $\quad \tau = \frac{T}{16}$

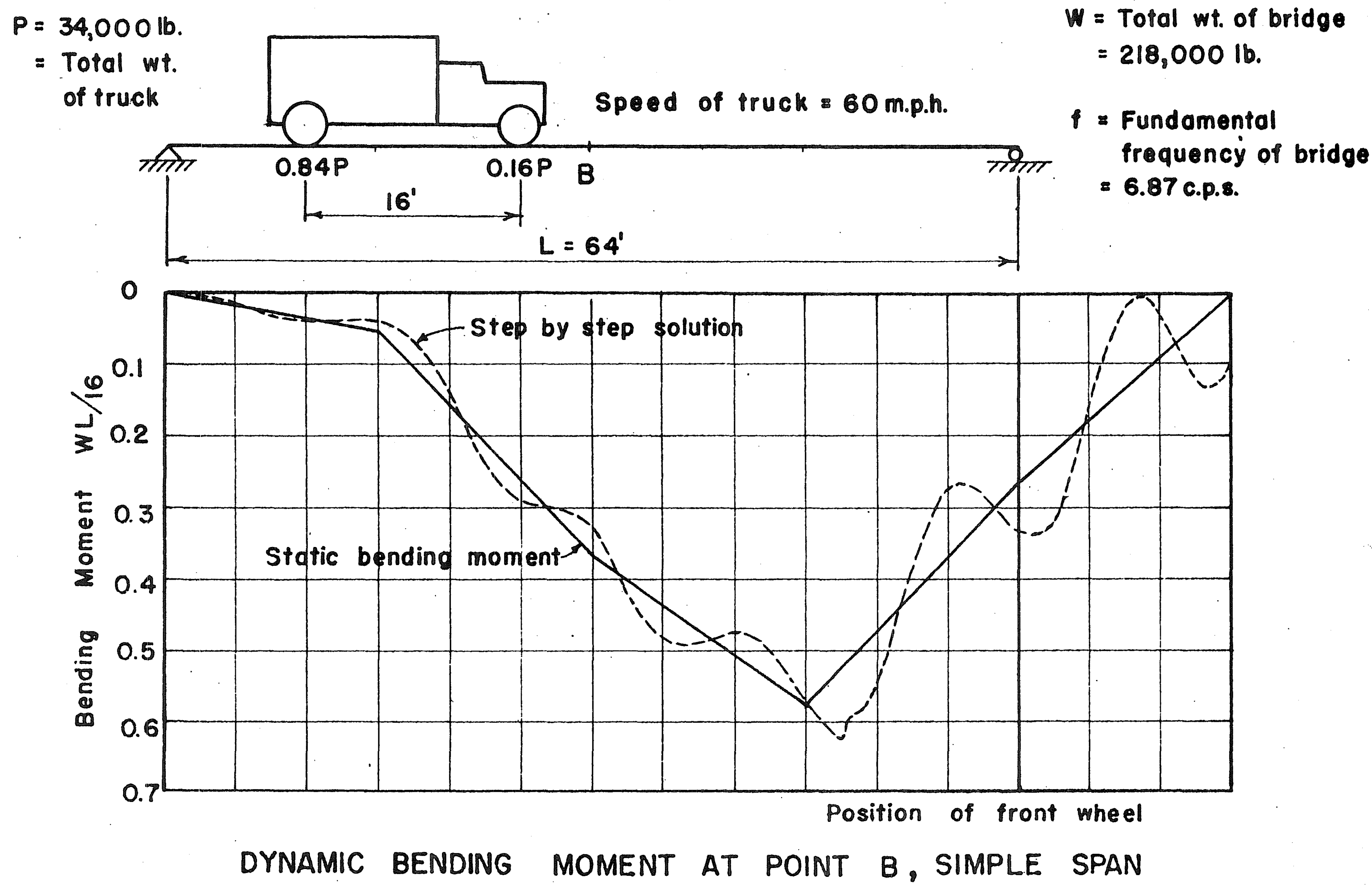
DYNAMIC BENDING MOMENT AT THIRD QUARTER POINT OF A SIMPLE BEAM DUE TO A MOVING MASS LOAD

FIG 4



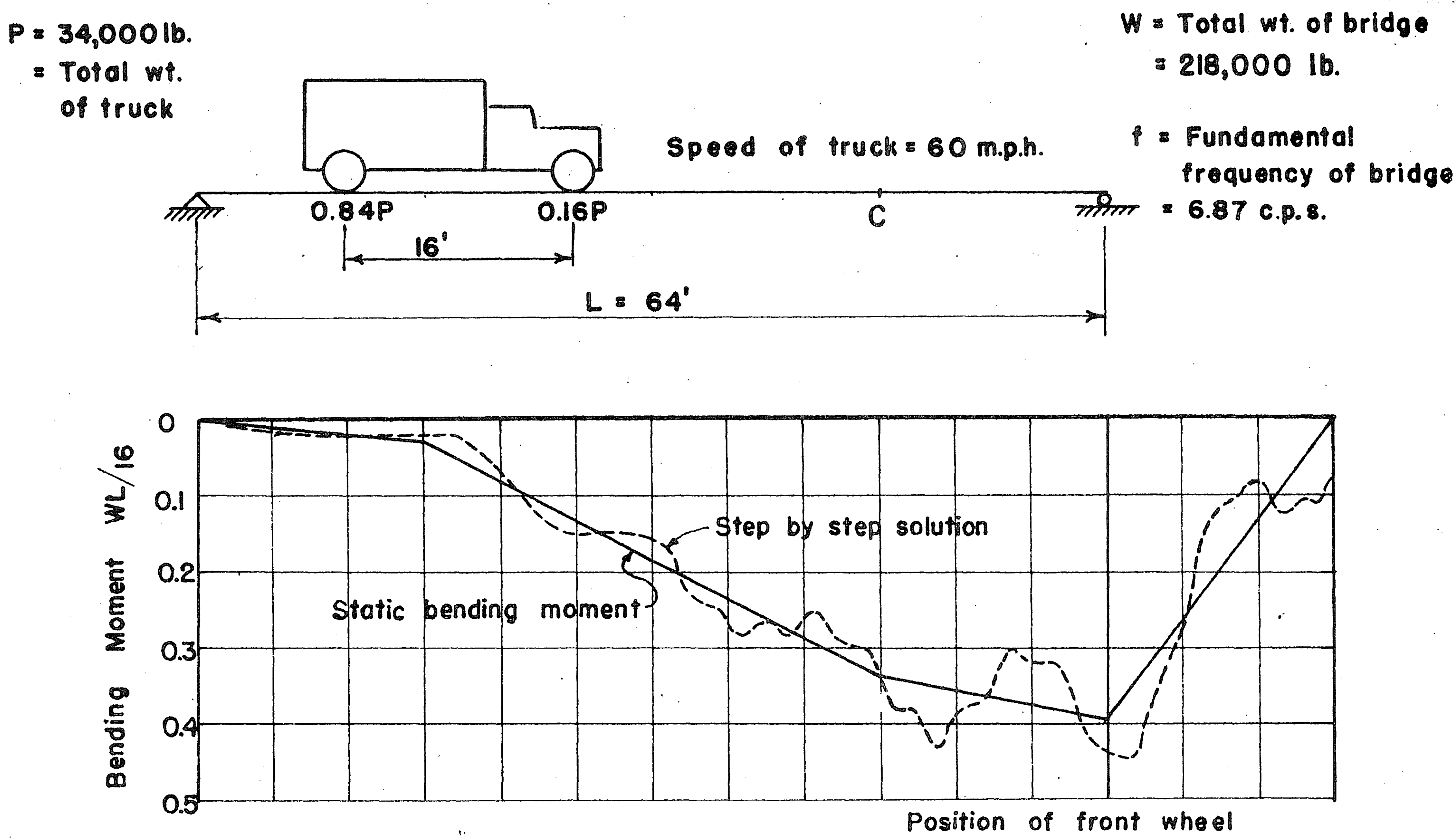
DYNAMIC BENDING MOMENT AT POINT "A", SIMPLE SPAN

FIG. 5



P = 34,000 lb.
= Total wt. of truck
W = Total wt. of bridge
= 218,000 lb.
Speed of truck = 60 m.p.h.
f = Fundamental frequency of bridge
= 6.87 c.p.s.
0.84P
0.16P
B
16'
L = 64'
Step by step solution
Static bending moment
Bending Moment WL/16
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Position of front wheel
DYNAMIC BENDING MOMENT AT POINT B, SIMPLE SPAN

FIG. 6



DYNAMIC BENDING MOMENT AT POINT C, SIMPLE SPAN

FIG. 7

CHAPTER V.

SUMMARY AND SUGGESTED FUTURE WORK.

During the past year the investigation has accomplished the following objectives:

1. Completed a study of the literature in the field of moving loads on bridges.
2. Tabulated significant structural properties of typical slab-and-girder highway bridges.
3. Developed a workable method of analysis for smoothly running mass loads.
4. Completed several analyses of representative truck loadings traveling at high speed over a typical simple-span slab-and-girder bridge.

In connection with item 3 above, certain drawbacks in the method of analysis should be noted. The primary difficulty is that the computation is tedious if conducted by hand with only the aid of a desk calculator. The results presented in Chapter IV of this report required about five weeks (each) of such computation. On the other hand, the advantages of the method are so great that it does not seem advisable to place any major effort on the development of a substitute at this time. Later, perhaps, when a large number of cases have been worked out, it may be desirable to develop a simplified method of predicting peak stresses which will give results in good agreement with the lenthier analysis. For the immediate future, it is suggested that priority be given to the coding of the analysis so that the computations can be carried out rapidly on I.B.M. punched card equipment. These machines are available at the Statistical Bureau of the University.

The results presented in Chapter IV are merely an indication of the potentialities of analysis. They should be supplemented by a systematic study of

1. The "speed" effect.
2. The weight effect.
3. The importance of the truck chassis springs.
4. The effect of a jolt or series of jolts as contrasted with the effect of a smoothly running load.

In all of these cases what is desired primarily is a value of the dynamic amplification factor for stress. In other words, what is the ratio of the peak dynamic stress to the peak static stress at any point on the bridge? Since the static stress is the one with which the designer works, the "dynamic amplification factor for stress" is a genuine impact factor.

In order to avoid any possibility of misunderstanding, it should be stated plainly that deflection measurements and computations, while of some value in themselves, do not provide any safe guide to the state of stress in the bridge structure. The midpoint of a bridge may deflect only 10 per cent more under a moving truck than it would under the same truck standing still, yet the dynamic stress may well be much more than 10 per cent in excess of the static stress. Bending moment and stress depend on curvature. To appreciate the importance of this circumstance it is only necessary to visualise two identical simple beams having the same maximum deflection. One of these, however, is loaded so as to deflect to a half-sine-wave shape, the other to a parabola. The two deflection shapes will be so nearly identical as to be almost indistinguishable. But the bending moment or stress variations will be entirely different. In the parabolic case bending moment would be constant along the beam, while in the sinusoidal case it would vary as a half-sine-wave. For this reason, theories which predict only deflections, or which are justified only by an agreement between observed and computed deflections, should be viewed with the greatest reserve.

It is suggested that the work of the investigation now be directed toward the accumulation of stress amplification factors for simple and three-span bridges for the variables 1 ....4 enumerated in the previous paragraph.

BIBLIOGRAPHY

The following bibliography represents only a partial compilation of material on impact loadings. Future progress reports will contain further references.

1. Great Britain, Report of the Commissioners Appointed to Inquire Into the Application of Iron and Railway Bridges, 1849, London.
This report was reprinted in the "Treatise on the Strength of Timber, Cast and malleable Iron by P. Parlow, 1851, London.

2. S. W. Robinson, Vibration of Bridges, ASCE Trans. 1887, V. 16, p. 42.

3. Souleyre, Action Dynamique des charges roulante sur les Poutres Rigide qui ne travaillent qu'a la flexion.
Annals des Ponts et Chaussées, 1889, 2nd Semester, p. 341.

4. Deslandres, Action des Chocs Rythmes sur les traves métalliques. Note sur les épreuves par charge roulante et l'action des Chocs."
Annals des Ponts et Chaussées 1892.

5. Rabut, L'Etude Expérimental des Ponts Métalliques"
Annals des Ponts et Chaussées 1896.

6. Zimmermann, Die Schwingungen eines Trager mit bewegter Last.
Zentralblatt der Bauverwaltung 1896, 16, p. 249, 257, 264.

7. Lebert, Étude des Mouvements Vibratoires dans les Ponts a Poutres droites a une travée et dans les Ponts suspendus a tablier continu simplement appuyés aux culées.
Annals des Ponts et Chaussées, 1899, p. 215.

8. F. E. Turneaure, Some Experiments on Bridges under Moving Train Loads. ASCE Trans. 41, 1899, 410.

9. Rabut, Conférence sur l'expérimentation des Ponts"
Annals des Pont et Chaussées, 1901, p. 123.

10. Impact, AREA Proc. 1905, 6, 255.

11. Report of Subcommittee on Impact, AREA Proc. 1911, 12, pt. 3, 13 or AREA Bull. 125.

12. G. G. Stokes, Discussions of a Differential Equation Relating to the Breaking of Railway Bridge.
Trans. Cambridge Phil. Soc. 1869.

13. G. Lindenthal, A New Impact Formula.
Engineering News, 1912, 68, p. 216.

14. C. T. Brady, Turneaure's Impact Formula too Complex.
Eng. News 1916, 75, 1056.

15. Impact Tests on Norfolk and Western Railway made during the Summer 1916.
AREA Proc. 1917, 18, p. 1226 or Bull. 195.

16. J. A. L. Waddell, New Impact Formulas Needed in Designing Bridges of Various Types.
Eng. News Record, 1918, 81, 924

17. Impact Test on the Electrified Section of the Chicago-Milwaukee and St. Paul Railway.
AREA Proc. 1918, 19, 781.

18. J. A. L. Waddell, Railway Bridge Impact Allowance
Eng. News Record, 1919, 82, 786.

19. F. W. Cardiner, Notes on Impact
ASCE Trans. 1921, 84, 437.

20. First and Second Interim Reports of Bridge Sub-committee 1925. on "Impact and Revision of Bridge Rules," Tech. paper no. 247 Railway Board. Government of India Calcutta, 1925.
See also five reports of the Indian Railway Bridge Committee, Tech. papers nos. 187, 198, 211, 224, 225, 228, 233.

21. C. E. Inglis, Theory of Transverse Oscillations in Girders and its Relation to Live Load and Impact Allowances.
Minutes of Proc. Inst. of Civil Eng. 1923, 218, p. 225.
Impact of Railway Bridges ditto, 234, p. 358.

22. Report of the Bridge Stress Committee, 1928, London, 1928, His Majesty's Stationery Office.

23. S. Timoshenko, Vibration of Bridges,
ASME Trans. 1928, 53, Railway, p. 53.

24. C. Gribble, Impact on Railway Bridges with particular references to the Report of the Bridge Stress Committee.
Minutes of Proc. Inst. Civil Engr. 1928, 228, 46.

25. W. Hort, Stoss Beanspruchungen und Schwingungen der Haupttrager Statisch bestimmter Eisenbahnbrucken. Die Bautechnik, 1928, V. 6, 37-50.

26. A. Buhler, Die Entwicklung der Messung dynamischer Wirkungen bei Brucken, Die Bautechnik, 1929, 7. 421-448.

27. F. E. Turneaure, Effect of Dead Load on Impact
AREA Proc. 1929, 30, 1017.

28. F. E. Turneaure, Abstract of Report of the Bridge Stress Committee of Britain.
AREA, Proc. 1930, 31, 975.

29. J. B. Hunley, Impact on Railway Bridges
AREA Proc. 1930, 31, 989.

30. The International Railway Congress Madrid.
Eng. (London) 1930, 129, 754.

31. Impact in Highway Bridges
Final Report of the Special Committee
ASCE Trans. 1931, 95, 1089.

32. W. E. Gelson, Moving Load Stresses in Short Span Railway Bridges,
Minutes of Proc. Inst. Civil Engrs. 1933, 237, 314

33. R. W. Forler, E. H. Crest, Hammer Blows Impact on Main Girders of Railway Bridges.
Minutes of Proc. Inst. of Civil Engrs. 1933, 237, 283.

34. R. Bernhard, Beitrag zur Berechnung der dynamischen Beanspruchung der Hauptträger Stahleisen Brücken infolge Befoleneus durch Dampflocomotive. Der Stahlbau 1934, 7, 123 and 139.

35. R. R. Leffler, The New Impact Formula of Committee IV
AREA Bull. 366, 1934

36. Kaufmann, Die Beurteilung der Resonance-fähigkeit von Stahl und Fachwerken auf Grund von Modellversuchen. Der Stahlbau 1936, 9, 177.

37. F. Bleich, Beitrag zur Dynamik der Brückenträgwerke
Der Stahlbau 1936, 9, 81.

38. J. B. Hunley, Impact in Steel Railway Bridges of Simple Span
AREA Proc. 1936, 37, 747.

39. A. Schallenkamp, Schwingungen von Trägern bei bewegten Lasten
Ing. Arch. 1937.

40. F. H. Frauland, European Development in the Study of Impact and Fatigue in Structures.
Civil Engring. (ASCE) 1940, 10, 208.

41. Report on Vibration Measurements on Road Bridges with Continuous Plate Girders under Moving Traffic Loads.
Strasse, 1941, 8 (23/24), 324-5 (Forschungsgesellschaft für des Strassenwesen)
Test on 6-7 spans. Deflection reduced by the effect of the neighbor span.

42. Description and Analysis of the Bridge Impact Test made by the Penn-R.R.
AREA Proc. 1941, 42, 397.

43. R. K. Bernhard, Dynamic Stress Analysis of Railway Bridges
ASCE Trans. 1942, V. 107, 327.

44. E.E.R. Fratman, Impact Test on Railway Bridges.
E. N. R. Dec. 16, 1943.

45. Description and Analysis of Bridge Impact Test made on Short Steel Bridges under Diesel Electric Locomotives
AREA Proc. 1943, 29 - 625.06/AME

46. C.T.G. Looney, Impact on Railway Bridge
Univ. Ill. Bulletin No. 19. 42, 1944.

47. L. E. Vandergrift, Vibration Studies of Continuous Span Bridges.
Ohio State Univ. Eng. Exp. Stat. Bull. No. 119, 1944.

48. A. L. Hillerborg, A study of Dynamic Influences of Moving Loads on Girders.
Int. Asso. for Bridge and Structural Engg. Congress 1948 at Liege. Preliminary Publication.

49. M. Casse, Dynamic Research of Bridges of the S.N.C.F. (In French)
Int. Asso. for Bridge and Structural Engg. 3rd Congress, Liege, Sept. 1948, Preliminary Publication.

50. E. Forslind, Effect of Dynamic Forces on Structures.
Int. Asso. for Bridge and Stru. Engg. Sept, 1948. Preliminary Publication.

51. V. Kolousek, Damped Oscillation of Frame Girders (In French)
Int. Asso. for Bridge and Structural Engg. Third Congress Liege, Sept, 1948. Preliminary Publication.

52. D. L. Thornton, Impact Loading of Structures Engineering 165, V. 4292. Apr. 30, 1948, p. 409-12.

53. S.T.A. Ödman, Differential Equation for Calculation of Vibrations Produced in Load-bearing Structures by Moving Loads.
Preliminary Publication, Int. Asso. for Bridge and Str. Engg. Third Congress, Liege Sept. 1948.

54. R. G. Norman, Vibrations of A Highway Bridge, New Zealand Engg. 5, 3, p. 239, Mar. 15, 1950.
Test on a two-span continuous highway bridge. 2-80.
Test speed - 10 m.p.h. - 50 m.p.h.

55. R. K. Bernhard, Dynamic Relations Between Moving Loads and Structures, Mechanical Engg. 60, 9, 697.

56. O.J. Horger, C.W. Nelson, Vibration of Locomotive Driving Wheels Caused by Unbalance.
J. app. Mech. 6, 4, December 1939.

57. G. Bautlinger, Dynamische Untersuchungen von Brücken u. Hochbauten.
Publ. Int. Asso. Bridge and Str. Engg. Vol. 1. 387- 1932.

58. C. E. Inglis, Math. Investigation of the Vertical Path of a Wheel Moving Along a Straight Railway Track.
Fifth, Inter. Cong. App. Mech. Cambridge, Mass, 1938.

59. G. R. Mitchell, Problems of Impact and Fatigue and their Effect on Permissible Stresses in Cast Iron Girder Bridges.
Publ. Int. Asso. Bridge and Struc. Engg. 1949. 9, 61.

60. O. Berg, Biegungsschwingungen eines in beiden Enden unterstützten
ZAMM Bd. 24. Nr. 1, 1944.

61. Description and Analysis of Bridge Impact Tests made on Short Span Open and Ballast Deck Steel Bridges under Diesel and Steam Locomotive.
AREA Proc. 1945, 46, 19). 625.06 / AME

62. R.K. Bernhard, W. Spaeth, Dynamic Method of Investigating Stresses in Buildings, Der Stahlbau, No. 6, 1926.

63. R. Desprets, Math. Studies of Vibrations of Girders in Particular Cases. International Railroad Asso. 1930 (Madrid)
International Bridge Asso. 1930 (Luettich)

64. R. K. Bernhard, Dynamic Tests by Means of Induced Vibrations
Proc. ASTM 37, Pt. II, 634-649.

65. E. Homann, R. K. Bernhard, Mechanical Vibrations of Bridges.
Verkehrstechnische Lehrmittel, gesellschaft, Berlin, 1933.

66. A. N. Kryloff, "Über der erzwungenen Schwingungen von gleichförmigen elastischen Stäben", Math. Annalen V. 61, 1905

67. H. H. Jeffcott, "On the Vibration of Beams under the action of Moving Loads" Phil. Magazine V. 8, 1929.

68. S. Timoshenko, "Vibration Problems in Engineering".
D. Van Nostrand, 2nd Ed.