



Kaonic hydrogen and deuterium in Hamiltonian effective field theory

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ABSTRACT

The anti-kaon nucleon scattering lengths resulting from a Hamiltonian effective field theory analysis of experimental data and lattice QCD studies are presented. The same Hamiltonian is then used to compute the scattering length for the K^-d system, taking careful account of the effects of recoil on the energy at which the $\bar{K}N$ T-matrices are evaluated. These results are then used to estimate the shift and width of the 1S levels of anti-kaonic hydrogen and deuterium. The K^-p result is in excellent agreement with the SIDDHARTA measurement. In the K^-d case the imaginary part of the scattering length and consequently the width of the 1S state are considerably larger than found in earlier work. This is a consequence of the effect of recoil on the energy of the $\bar{K}N$ energy, which enhances the role of the $\Lambda(1405)$ resonance.

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1. Introduction

The kaon has played an important role in advancing particle physics since it was discovered in cosmic rays. The related θ - τ puzzle resulted in the proposal of parity violation. A bound state of the kaon and nucleon was predicted and the candidate $\Lambda(1405)$ was discovered in experiment, even before the birth of the quark model [1,2]. The study of kaonic nuclei and atoms constitutes an important source of information as we seek to understand the non-perturbative properties of QCD and many experiments have been designed to investigate them (see, for example, Ref. [3] for a recent review).

In a kaonic atom an electron is replaced by a K^- , which is still bound primarily by the attractive electromagnetic interaction. Since the kaon is three orders of magnitude heavier than the electron, the size of the kaonic orbit in a light atom is only ~ 100 fm, while the binding energy is \sim keV. If the kaon is captured by the nucleus, it can be annihilated in the formation of a hyperon. The behavior of these exotic atoms and nuclei allows us to explore new aspects of hadronic interactions. For example, the $\Lambda(1405)$ lies close below threshold in the $J^P = 1/2^- \bar{K}N$ system, while the lowest-lying $J^P = 1/2^-$ strangeness-zero baryon, the $N(1535)$, is far from the πN threshold. This difference has led to a great deal

of speculation regarding the adequacy of the conventional quark model [4–15].

Since the atomic size is beyond the range of the strong interaction, the kaonic atom can be studied pretty well within QED. However, for the 1S state, the wave function $\psi_{1S}^{QED}(\vec{r}=0)$ is non-zero and the strong interaction plays a role. Therefore, the experimental energy, E_{1S} , is different from the QED prediction, E_{1S}^{QED} , because of the strong interaction, $\Delta E_{1S} \equiv E_{1S} - E_{1S}^{QED}$. Moreover, the $\bar{K}N$ system can decay to $\pi\Sigma$ and $\pi\Lambda$ and thus the 1S level has a width Γ_{1S} , in addition to the energy shift ϵ_{1S} , $\Delta E_{1S} \equiv E_{1S} - E_{1S}^{QED} = \epsilon_{1S} - \frac{1}{2}\Gamma_{1S}$. The energy distributions of X-rays emitted from the excited kaonic atom may be measured to obtain ΔE_{1S} in experiment [16–21]. The difference between the measured and QED-predicted $nP - 1S$ transition energy is essentially the same as ΔE_{1S} , since the nP level is scarcely affected by the strong interaction. The SIDDHARTA experiment gave the energy shift and width for 1S kaonic hydrogen as

$$\begin{aligned} \epsilon_{1S}^p &= 283 \pm 36(\text{stat}) \pm 6(\text{sys}) \text{ eV}, \\ \Gamma_{1S}^p &= 541 \pm 89(\text{stat}) \pm 22(\text{sys}) \text{ eV}, \end{aligned} \quad (1)$$

which helps us to constrain the K^-p interaction at low energy [16,17]. The reduced mass of the anti-kaon and the deuteron is a little larger than that of $\bar{K}N$ and thus kaonic deuterium would be easier to form under the pure electromagnetic interaction. There has been a proposal to search for kaonic deuterium in SIDDHARTA-2 and the J-PARC E57 experiment [18–21]. However, technically it

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is far more difficult to measure the kaonic deuteron than kaonic hydrogen.

The study of kaonic deuterium may be expected to benefit from a comparison with earlier studies of pionic deuterium. The latter was proposed and observed in experiment very early – see Ref. [22] for a review. The energy shifts and widths of the 1S states of pionic hydrogen and deuterium were measured with precision at the Paul Scherrer Institute (PSI) at Villigen [23].

Kaonic deuterium has already been investigated by many authors [24–34]. In recent studies, after the resummation of contributions from kaon multiple scatterings between the two nucleons in the deuteron, solving the full three-body Hamiltonian or Faddeev equations, the energy shift for kaonic deuterium has been found to be of order 700~900 eV, while the width has been reported to be in the range 800~1200 eV [28–33].

The nucleons in the deuteron have been assumed to be static in most studies of kaonic deuterium, perhaps because this approximation has been shown to be quite accurate for pionic deuterium. However, the situation merits further consideration for the following reason. In the pionic case the closest resonance, the $\Delta(1232)$, has angular momentum one and is still 150 MeV away from the πN threshold. Even allowing for its width, it is far away. On the other hand, the difference between the $\Lambda(1405)$ resonance and the $\bar{K}N$ threshold is less than 30 MeV and, moreover, its 50 MeV width means they overlap to some extent. In such a case, even if the recoiling nucleon shifts the center-of-mass energy of $\bar{K}N$ a relatively small distance from the threshold, the scattering amplitude may be very different from that at threshold, which the static approximation uses. These considerations led us to examine the effect of recoil carefully in the $\bar{K}d$ system.

Some earlier studies of the effect of nucleon recoil for $\bar{K}d$ scattering was undertaken within effective field theory in Refs. [35,36]. There non-local effects were taken into account perturbatively by using the effective-range expansion of the $\bar{K}N$ amplitude. The recoil effect for the double scattering contribution was found to be the order of 10-15%, as compared to the static term [35]. The effect on the single scattering process was not discussed there, while it is a key issue in this study.

In this work, we focus particularly on the recoil effects for the 1S level shifts of anti-kaonic deuterium with Hamiltonian Effective Field Theory (HEFT). HEFT was developed to study the low-lying resonances based on a combined analysis of both lattice QCD data and experimental scattering data, while preserving the constraints of chiral perturbation theory where appropriate. It has led to important insights into the properties of the $N(1535)$, $N(1440)$, $\Lambda(1405)$, and so on [12,37–44]. In Ref. [12], we studied both the cross sections for K^-p scattering and the relevant lattice QCD data and reproduced the two-pole structure of the $\Lambda(1405)$. If not explicitly pointed out, references of the $\Lambda(1405)$ refer to the pole close to the experimental mass.

We first briefly review the HEFT for $\bar{K}N$ in Sec. 2. Then we use it to compute the K^-p scattering lengths and the corresponding energy shift ΔE_{1S}^p , with no further adjustment of the parameters determined in that previous work [12]. The scattering length of the K^-d system and the energy shift ΔE_{1S}^d for kaonic deuterium, with and without the recoil correction, are presented in Secs. 3 and 4. We conclude with a discussion and summary in Sec. 5.

2. Kaonic hydrogen and HEFT

The T matrix for the two-body scattering process $\alpha \rightarrow \beta$ is related to the corresponding S matrix and cross section by

$$S_{\beta\alpha} = \delta_{\alpha\beta} - 2i\sqrt{\rho_\alpha\rho_\beta}T_{\beta\alpha}, \quad (2)$$

$$\sigma_{\beta\alpha} = \frac{4\pi^3 k_\beta^{\text{cm}} \omega_{\alpha M}^{\text{cm}} \omega_{\alpha B}^{\text{cm}} \omega_{\beta M}^{\text{cm}} \omega_{\beta B}^{\text{cm}}}{k_\alpha^{\text{cm}} (E_{\text{cm}})^2} T_{\beta\alpha} T_{\beta\alpha}^*, \quad (3)$$

where

$$\rho_\alpha = \pi \frac{\omega_{\alpha M}^{\text{cm}} \omega_{\alpha B}^{\text{cm}}}{E_{\text{cm}}} k_\alpha^{\text{cm}}. \quad (4)$$

In this work, the channel label, α or β , refers to $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, and $\eta\Lambda$ but, of course, it could be extended to more general cases. The labels α_M and α_B refer to the meson and baryon in channel α , respectively. $\omega_{\alpha X}^{\text{cm}}$ and k_α^{cm} denote the energy and spatial momentum of the particle in the center-of-mass frame, and E_{cm} is the total energy in the center-of-mass frame.

For the process $\alpha \rightarrow \alpha$, the cross section $\sigma_{\alpha\alpha}^{\text{thr}}$ at threshold can be related to the scattering length a_α

$$\sigma_{\alpha\alpha}^{\text{thr}} = 4\pi |a_\alpha|^2, \quad (5)$$

where the scattering length is defined as

$$a_\alpha \equiv - \lim_{k_\alpha^{\text{cm}} \rightarrow 0} \pi \frac{\omega_{\alpha M}^{\text{cm}} \omega_{\alpha B}^{\text{cm}}}{E_{\text{cm}}} T_{\alpha\alpha} = -\pi \mu_\alpha T_{\alpha\alpha}^{\text{thr}} \quad (6)$$

and μ_α is the reduced mass for the channel α .

We used HEFT to study both the cross sections of $K^-p \rightarrow K^-p/\bar{K}^0n/\pi^- \Sigma^+/\pi^0 \Sigma^0/\pi^+ \Sigma^-/\pi^0 \Lambda$ and the corresponding energy levels in a finite volume (for comparison with lattice QCD simulations). The results were consistent with both the experimental scattering data and the spectra from lattice QCD simulations [12]. Here we want to further study the scattering length, a_{K^-p} , within this framework. Although there are two different scenarios which give different behavior at larger pion masses in Ref. [12,45], we have checked that both yield the same threshold observables with negligible numerical differences. Thus we can choose the simpler scenario for this work. In our framework, the T matrix can be obtained by solving the Bethe-Salpeter equation

$$T_{\alpha\beta}^I(k, k'; E) = V_{\alpha\beta}^I(k, k') + \sum_\gamma \int q^2 dq V_{\alpha\gamma}^I(k, q) \frac{1}{E - \omega_\gamma(q) + i\epsilon} T_{\gamma\beta}^I(q, k'; E), \quad (7)$$

with the Weinberg-Tomozawa potentials

$$V_{\alpha\beta}^I(k, k') = g_{\alpha,\beta}^I \frac{[\omega_{\alpha M}(k) + \omega_{\beta M}(k')] u(k) u(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha M}(k)} \sqrt{2\omega_{\beta M}(k')}}}, \quad (8)$$

where f is the decay constant of the pion, the isospin I can be 0 or 1. k and k' are the center-of-mass spatial momenta in the final and initial states, respectively. The form factor $u(k) = (1 + k^2/\Lambda^2)^{-2}$ is used, with $\Lambda = 1$ GeV [12].

To fix the nonzero couplings, $g_{\alpha,\beta}^I$, in Ref. [12] we fitted the cross sections for $K^-p \rightarrow K^-p/\bar{K}^0n/\pi^- \Sigma^+/\pi^0 \Sigma^0/\pi^+ \Sigma^-/\pi^0 \Lambda$ with $\chi_{d.o.f} = 1.5$ for laboratory energies of the kaon up to 250 MeV. There are two poles for $\Lambda(1405)$ at $1428 - 23i$ and $1338 - 89i$ MeV with this set of parameters. We also compared our energy levels with lattice QCD results in Ref. [12].

We have not fine tuned the coupling constants, $g_{\alpha,\beta}^I$, in this work to exactly fit the scattering length. Instead, we use the values of $g_{\alpha,\beta}^I$ found in Ref. [12] to predict a_{K^-p} and further check the reliability of HEFT. The scattering lengths for $\bar{K}N$ in the isospin $I=0$ and $I=1$ channels in the HEFT, using Eq. (6), are

$$a_{I=0} = -1.77 + 1.08i \text{ fm}, \quad a_{I=1} = 0.27 + 0.52i \text{ fm}. \quad (9)$$

With the isospin relation, one obtains

$$a_{K^-p} = -0.75 + 0.80i \text{ fm}, \quad (10)$$

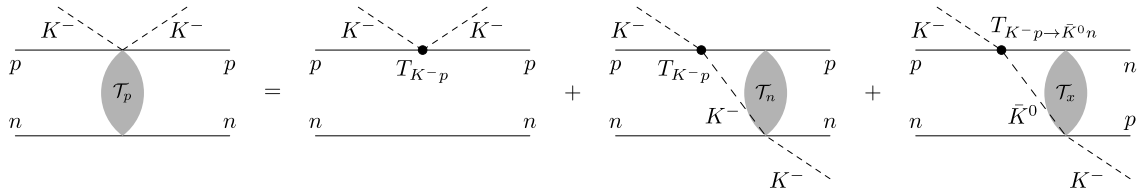


Fig. 1. Illustration of the Faddeev equation for \mathcal{T}_p in kaon-deuteron scattering. The $T_{\bar{K}N}$ can be obtained from the study of kaon nucleon scattering, which contains the coupled-channel effect from $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, and $\eta\Lambda$. There are two other similar diagrams for equations on \mathcal{T}_n and \mathcal{T}_x which can be found in Ref. [31].

and

$$\begin{aligned} a_{K^-n} &= 0.27 + 0.52i \text{ fm}, & a_{\bar{K}^0n} &= -0.75 + 0.80i \text{ fm}, \\ a_{\bar{K}^0n \rightarrow K^-p} &= 1.02 - 0.28i \text{ fm}. \end{aligned} \quad (11)$$

The relation between the energy shift ϵ_{1S}^p and width Γ_{1S}^p of kaonic hydrogen and a_{K^-p} can be given by the improved Deser formula [46], but the logarithmic contribution at higher orders is important for kaonic deuterium [32] and thus throughout this work we use the “double-improved” Deser formula in which the logarithmic terms are summed to all orders [35]

$$\epsilon_{1S}^p - \frac{i}{2}\Gamma_{1S}^p = \frac{-2\alpha_e^3 \mu_{K^-p}^2 a_{K^-p}}{1 + 2\alpha_e \mu_{K^-p} (\ln \alpha_e - 1) a_{K^-p}}, \quad (12)$$

where α_e is the electromagnetic fine-structure constant. With this resummed formula we obtain

$$\epsilon_{1S}^p = 307 \text{ eV}, \quad \Gamma_{1S}^p = 533 \text{ eV}. \quad (13)$$

These results are consistent with the result of the SIDDHARTA experiment given in Eq. (1).

3. Results for kaonic deuterium with the static nucleon approximation

The scattering amplitude $T_{\bar{K}d}$ can be divided into two parts

$$T_{\bar{K}d} = \int d^3\vec{q} \psi_d(\vec{q}) \int d^3\vec{q}' \psi_d^*(\vec{q}') (\mathcal{T}_p + \mathcal{T}_n), \quad (14)$$

where $\psi_d(\vec{q})$ is the wave function of the nucleon in the deuteron in momentum space and \vec{q} (\vec{q}') is the relative momentum between the two nucleons in the initial (final) state. The $\bar{K}NN$ scattering amplitude \mathcal{T} can be obtained by solving the Faddeev equations [47] illustrated in Fig. 1

$$\mathcal{T}_p \sim T_{K^-p} \delta^3(\vec{q} - \vec{q}') + \int_{\delta} T_{K^-p} G_0 \mathcal{T}_n + \int_{\delta} T_{K^-p \rightarrow \bar{K}^0n} G_0 \mathcal{T}_x,$$

$$\mathcal{T}_n \sim T_{K^-n} \delta^3(\vec{q} - \vec{q}') + \int_{\delta} T_{K^-n} G_0 \mathcal{T}_p,$$

$$\mathcal{T}_x \sim T_{\bar{K}^0n \rightarrow K^-p} \delta^3(\vec{q} - \vec{q}') + \int_{\delta} T_{\bar{K}^0n} G_0 \mathcal{T}_x + \int_{\delta} T_{\bar{K}^0n \rightarrow K^-p} G_0 \mathcal{T}_n. \quad (15)$$

It corresponds to the scattering length calculation, where the kaon momentum is zero and hence the momentum of the spectator nucleon (to the kaon-nucleon scattering) is its momentum in the deuteron center of mass. G_0 is the propagator

$$G_0 = (\epsilon_d - \frac{\vec{p}_K^2}{2m_K} - \frac{\vec{p}_{N_1}^2}{2m_N} - \frac{\vec{p}_{N_2}^2}{2m_N} + i0^+)^{-1}, \quad (16)$$

where \vec{p}_K , \vec{p}_{N_1} , and \vec{p}_{N_2} are the momenta of the intermediate particles, and ϵ_d is the binding energy of deuteron.

The first term in Eq. (15) corresponds to the single scattering (SS) of the kaon with only one nucleon, which contains the effects of the coupling of $\bar{K}N$ to the channels $\pi\Sigma$, $\pi\Lambda$, and $\eta\Lambda$. The remainder is generated by the multiple scattering (MS) of the kaon between the two nucleons in the deuteron.

If one uses the static nucleon approximation, usually on the basis that the momenta are relatively small, the momentum-dependent amplitude $T_{\bar{K}N}$ can be approximated by the value at threshold, $T_{\bar{K}N}^{\text{thr}}$. With this approximation, the multiple scattering series integration can be solved analytically and the scattering length, a_{K^-d} , can be simplified as [31]

$$a_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3\vec{r} |\psi_d(\vec{r})|^2 \hat{A}_{K^-d}(r), \quad (17)$$

where $\psi_d(\vec{r})$ is the wave function for nucleons in the deuteron, and

$$\hat{A}_{K^-d}(r) = \frac{\tilde{a}_{K^-p} + \tilde{a}_{K^-n} + (2\tilde{a}_{K^-p}\tilde{a}_{K^-n} - b_x^2)/r - 2b_x^2\tilde{a}_{K^-n}/r^2}{1 - \tilde{a}_{K^-p}\tilde{a}_{K^-n}/r^2 + b_x^2\tilde{a}_{K^-n}/r^3}, \quad (18)$$

with $\tilde{a}_{\bar{K}N} = a_{\bar{K}N}(1 + m_K/m_N)$, and

$b_x = \tilde{a}_{\bar{K}^0n \rightarrow K^-p} / \sqrt{1 + \tilde{a}_{\bar{K}^0n}/r}$. This corresponds to the commonly used fixed center approximation (e.g. Ref. [48]) in which excited nucleon intermediate states are neglected.

Using the $\bar{K}N$ scattering lengths given in Eq. (11) and the deuteron wave function obtained from a one-boson-exchange model [49], the numerical result with the static nucleon approximation is

$$a_{K^-d}|_{\text{StaticApprox}} = -1.55 + 1.57i \text{ fm}. \quad (19)$$

With the resummed formula (12), the energy shift and width of the 1S level of kaonic deuterium is

$$\epsilon_{1S}^d|_{\text{StaticApprox}} = 855 \text{ eV}, \quad \Gamma_{1S}^d|_{\text{StaticApprox}} = 1127 \text{ eV}. \quad (20)$$

4. Recoil effects for kaonic deuterium

The root-mean-square radius of the deuteron is about 2 fm, which means the momentum q of the nucleons in the deuteron should be of order $1/(2 \text{ fm}) \sim 100 \text{ MeV}$ on average. The variation of momentum is not very large, but the resonance $\Lambda(1405)$ is around the $\bar{K}N$ threshold and thus the effect from the nucleon recoiling in the deuteron should be carefully examined.

Firstly, we study the single scattering term of K^-d , which can be represented as the second diagram in Fig. 1. Denoting the scattering amplitude of a static kaon and a nucleon with momentum \vec{q} in the deuteron as $T_{\bar{K}N}(\vec{q})$, the integrated amplitude is

$$\langle T_{\bar{K}N}^d \rangle \equiv \int d^3\vec{q} |\psi_d(\vec{q})|^2 T_{\bar{K}N}(\vec{q}). \quad (21)$$

Obviously, the amplitude of K^-d from the single scattering contribution is exactly $T_{K^-d}^{SS} = \langle T_{K^-p}^d \rangle + \langle T_{K^-n}^d \rangle$.

To calculate $T_{\bar{K}N}(\vec{q})$, we need to recognize that if the spectator nucleon has momentum \vec{q} , the $\bar{K}N$ center of mass energy

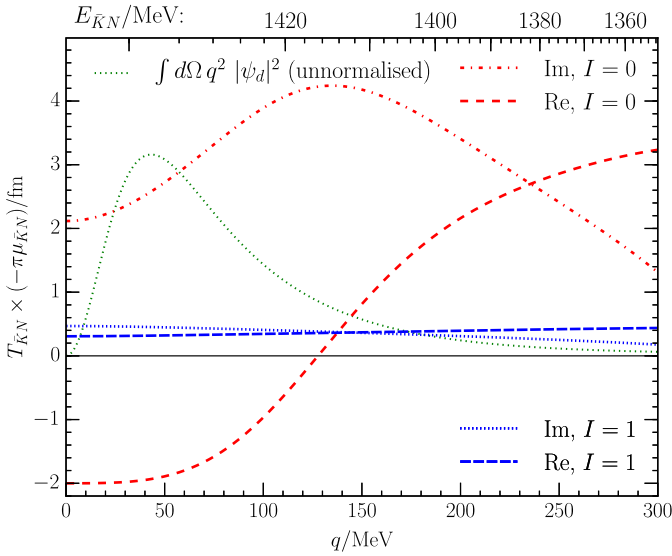


Fig. 2. The scattering amplitude $T_{\bar{K}N}(\vec{q})$ for a static kaon and a nucleon of momentum q in the deuteron. The lower x-axis is the spatial momentum q of a nucleon, while the upper x-axis is the center-of-mass energy $E_{\bar{K}N}$ of $\bar{K}N$, and they are related by $E_{\bar{K}N} = (m_d/2 + m_K) - q^2/2\mu_3$ with $\mu_3 = m_N(m_N + m_K)/(2m_N + m_K)$. The deuteron wave function is also shown as the green dotted lines.

at which the two-body T-matrix must be evaluated is effectively $E_{\bar{K}N} = m_d + m_K - m_N - q^2/2\mu_3$ within Faddeev formalism where $\mu_3 = m_N(m_N + m_K)/(2m_N + m_K)$. We use the following relation to obtain $T_{\bar{K}N}(\vec{q})$

$$T_{\bar{K}N}(\vec{q}) = T_{\bar{K}N, \bar{K}N}(k, k; E_{\bar{K}N}), \quad (22)$$

where $k = qm_K/(m_N + m_K)$.

For the multiple scattering terms, we use the approximation $T_{\bar{K}N} \rightarrow \langle T_{\bar{K}N}^d \rangle$ to simplify the calculation. Finally, we can evaluate a_{K-d} . By replacing $a_{\bar{K}N}$ with $\langle a_{\bar{K}N}^d \rangle \equiv -\pi \mu_{\bar{K}N} \langle T_{\bar{K}N}^d \rangle$ in the formula given in Eq. (17), the K^-d scattering length, including the recoil correction, may be obtained.

5. Numerical results and discussion

More than 96% of the contribution to $\langle T_{\bar{K}N}^d \rangle$ in Eq. (21) comes from $q < 300$ MeV, which corresponds to $k \lesssim 110$ MeV and $\Delta E_{\bar{K}N} \lesssim 80$ MeV, a region in which our fit with HEFT is very reliable [12]. The wave function $\psi_d(\vec{q})$ is extracted from the Bonn potential [50]. The wave function of the deuteron is well constrained both by experiments and theory, and thus we directly cite two different results from Refs. [49] and [50] for $\psi_d(\vec{r})$ and $\psi_d(\vec{q})$ in the coordinate and momentum spaces respectively. We do not transform between them and simply neglect the modest differences between the models.

Before providing the average $\langle T_{\bar{K}N}^d \rangle$, we show the scattering amplitude $T_{\bar{K}N}(\vec{q})$ in Fig. 2. By comparing the two x-axes, one can see that even if the momentum q increases from 0 to 200 MeV where the wave function ψ_d is extremely close to 0, the center-of-mass energy $E_{\bar{K}N}$ of the $\bar{K}N$ system varies over a relatively small range, roughly 1430 to 1400 MeV. Usually such a small change of $E_{\bar{K}N}$ would not be expected to lead to large variations in related amplitudes. For example, the blue dashed lines for the amplitudes $T_{\bar{K}N}(I=1)$ are relatively flat for isospin 1. However, the issue is very different when there is a resonance or bound state in a channel below and close to the threshold.

In our previous work [12] we searched for two poles for the $\Lambda(1405)$ in the $I=0$ channel within HEFT, finding one close to the $\bar{K}N$ threshold with width about 50 MeV. As $E_{\bar{K}N}$ passes by

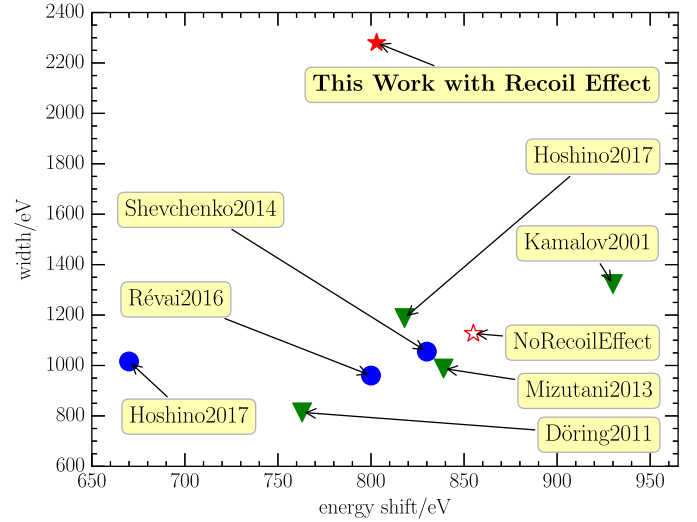


Fig. 3. Comparison of the energy shifts and widths for the 1S state of kaonic deuterium. The data with green triangles are from Refs. [29–32] and ΔE_{1S}^d are extracted from the scattering lengths of K^-d using the resummed formula (12). The data with blue circles are from Refs. [28,32,33] where the dynamical equations directly provide the energy level of kaonic deuterium without the help of K^-d scattering length. For example, there are two points labeled with “Hoshino2017” which are from the same paper [32], but the left blue circle is directly obtained by solving the Schrödinger equation while the right green triangle is translated from the double-improved Deser formula (12) from the K^-d scattering length. Our result (labeled with hollow star) without the recoiling effect is close to other works, but the result after considering the recoiling effect (labeled with a filled star) has a much larger width.

the pole, the scattering amplitude $T_{\bar{K}N}(I=0)$ changes rapidly, as shown by the red dashed and dot-dashed lines in Fig. 2. The phase shift usually crosses 90° when a resonance appears, which is equivalently exhibited by the real part of the amplitude changing its sign for $E_{\bar{K}N}$ around 1410 MeV (red dashed). The imaginary part of $T_{\bar{K}N}(I=0)$ is related to the experimental $\pi\Sigma$ invariant mass distribution and thus the red dot-dashed line shows a bump [51].

From this analysis, we would expect that the average value of $\langle T_{\bar{K}N}^d \rangle$ may deviate significantly from the threshold value in the $I=0$ channel. Equivalently speaking, $\langle a_{I=0}^d \rangle$ is very different from the scattering length $a_{I=0}$, while $\langle a_{I=1}^d \rangle$ should be still close to $a_{I=1}$. We find

$$\langle a_{I=0}^d \rangle = -1.07 + 3.01 i \text{ fm}, \quad \langle a_{I=1}^d \rangle = 0.32 + 0.40 i \text{ fm}, \quad (23)$$

which in comparison with the threshold scattering lengths in Eq. (9) shows that the average imaginary part in the channel with $I=0$ increases by more than 1 fm. This will, of course, increase the width of the 1S K^-d energy level significantly.

We show the K^-d scattering length after taking the recoiling effect into consideration

$$a_{K-d} = -0.59 + 2.70 i \text{ fm}. \quad (24)$$

With the resummed formula of Eq. (12), the energy shift and width of 1S kaon deuterium are

$$\epsilon_{1S}^d = 803 \text{ eV}, \quad \Gamma_{1S}^d = 2280 \text{ eV}. \quad (25)$$

By comparing these results with those given in Eq. (20) of Sec. 3, we can clearly see that the recoil effect plays a very important role for the K^-d system.

We compare the scattering lengths a_{K-d} in this work with some other studies in Table 1 and Fig. 3. Without including the effect of recoil our results in the fifth column are close to those of earlier

Table 1

Comparison of scattering lengths a_{K-d} in units of fm. The first four columns are from Refs. [29–32]. The last four columns are the results in this work with (✓) and without (×) the multiple scattering contributions or the recoil effects, respectively.

a_{K-d}	Kamalov [31]	Doring [29]	Mizutani [30]	Hoshino [32]	(MS ✓, recoil ×)	(MS ×, recoil ✓)	(MS ×, recoil ×)	(MS ✓, recoil ✓)
Re	−1.62	−1.46	−1.58	−1.42	−1.55	−0.06	−0.58	−0.59
Im	1.91	1.08	1.37	1.60	1.57	2.55	1.59	2.70

work. That is, our result would be close to others if the momentum of the nucleon in the deuteron is neglected. However, our corrected result has a much larger width.

The contribution to a_{K-d} from the single scattering term is exactly

$$a_{K-d}^{SS} = \frac{\mu_{\bar{K}d}}{\mu_{\bar{K}N}} \left(\frac{1}{2} \langle a_{I=0}^d \rangle + \frac{3}{2} \langle a_{I=1}^d \rangle \right). \quad (26)$$

We divide a_{K-d} into a_{K-d}^{SS} and a_{K-d}^{MS} , which are from the single and multiple scattering contributions, respectively

$$a_{K-d} = a_{K-d}^{SS} + a_{K-d}^{MS} = (-0.06 + 2.55i) + (-0.53 + 0.15i) \text{ fm}. \quad (27)$$

By comparing this with

$$a_{K-d}|_{\text{StaticApprox}} = (-0.58 + 1.59i) + (-0.97 - 0.02i) \text{ fm}, \quad (28)$$

the main difference in the imaginary part clearly arises from the single scattering term. This gives us confidence in the conclusion that the ground state of kaonic deuterium is broad. Our final result for the K^-d scattering length is

$$a_{K-d} = -0.59 + 2.70i \text{ fm}. \quad (29)$$

In summary, we first established that our earlier Hamiltonian effective field theory study of the $\bar{K}p$ system did indeed reproduce the empirical K^-p scattering length, as well as producing an energy shift and width for the ground state of kaonic hydrogen which is consistent with the SIDDHARTA experiment. The calculation was then extended to the K^-d system, where it was found that the effect of recoil in the energy argument of the $\bar{K}p$ T-matrix within the Faddeev formalism is very significant, making the 1S level of kaonic deuterium considerably more broad and short lived.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] R.H. Dalitz, S.F. Tuan, A possible resonant state in pion-hyperon scattering, *Phys. Rev. Lett.* **2** (1959) 425–428.
- [2] M.H. Alston, et al., Study of resonances of the sigma-pi system, *Phys. Rev. Lett.* **6** (1961) 698–702.
- [3] A. Gal, E.V. Hungerford, D.J. Millener, Strangeness in nuclear physics, *Rev. Mod. Phys.* **88** (3) (2016) 035004, arXiv:1605.00557.
- [4] E.A. Veit, B.K. Jennings, A.W. Thomas, R.C. Barrett, S wave meson - nucleon scattering in an SU(3) cloudy bag model, *Phys. Rev. D* **31** (1985) 1033.
- [5] E.A. Veit, B.K. Jennings, R.C. Barrett, A.W. Thomas, Kaon - nucleon scattering in an extended cloudy bag model, *Phys. Lett. B* **137** (1984) 415–418.
- [6] Y.S. Zhong, A.W. Thomas, B.K. Jennings, R.C. Barrett, Radiative decays of the K^-p atom, *Phys. Rev. D* **38** (1988) 837–844.
- [7] N. Kaiser, T. Waas, W. Weise, SU(3) chiral dynamics with coupled channels: eta and kaon photoproduction, *Nucl. Phys. A* **612** (1997) 297–320, arXiv:hep-ph/9607459.
- [8] E. Oset, A. Ramos, Nonperturbative chiral approach to s wave anti-K N interactions, *Nucl. Phys. A* **635** (1998) 99–120, arXiv:nucl-th/9711022.
- [9] L. Roca, E. Oset, Isospin 0 and 1 resonances from $\pi\Sigma$ photoproduction data, *Phys. Rev. C* **88** (5) (2013) 055206, arXiv:1307.5752.
- [10] Y. Ikeda, T. Hyodo, W. Weise, Improved constraints on chiral SU(3) dynamics from kaonic hydrogen, *Phys. Lett. B* **706** (2011) 63–67, arXiv:1109.3005.
- [11] Z.-H. Guo, J.A. Oller, Meson-baryon reactions with strangeness -1 within a chiral framework, *Phys. Rev. C* **87** (3) (2013) 035202, arXiv:1210.3485.
- [12] Z.-W. Liu, J.M.M. Hall, D.B. Leinweber, A.W. Thomas, J.-J. Wu, Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory, *Phys. Rev. D* **95** (1) (2017) 014506, arXiv:1607.05856.
- [13] J.-J. Xie, B.-C. Liu, C.-S. An, Coupling constant for $\Lambda(1405)\bar{K}N$, *Phys. Rev. C* **88** (1) (2013) 015203, arXiv:1307.3707.
- [14] M. Mai, U.-G. Meißner, Constraints on the chiral unitary $\bar{K}N$ amplitude from $\pi\Sigma K^+$ photoproduction data, *Eur. Phys. J. A* **51** (3) (2015) 30, arXiv:1411.7884.
- [15] R. Molina, M. Döring, Pole structure of the $\Lambda(1405)$ in a recent QCD simulation, *Phys. Rev. D* **94** (5) (2016) 056010, arXiv:1512.05831; *Phys. Rev. D* **94** (7) (2016) 079901 (Addendum).
- [16] M. Bazzi, et al., SIDDHARTA Collaboration, A new measurement of kaonic hydrogen X-rays, *Phys. Lett. B* **704** (2011) 113–117, arXiv:1105.3090.
- [17] M. Bazzi, et al., Kaonic hydrogen X-ray measurement in SIDDHARTA, *Nucl. Phys. A* **881** (2012) 88–97, arXiv:1201.4635.
- [18] C. Curceanu, et al., Unlocking the secrets of the kaon-nucleon/nuclei interactions at low-energies: the SIDDHARTA(-2) and the AMADEUS experiments at the DAΦNE collider, *Nucl. Phys. A* **914** (2013) 251–259.
- [19] J. Zmeskal, et al., Probing strong interaction with SIDDHARTA-2, *JPS Conf. Proc.* **26** (2019) 023012.
- [20] J. Zmeskal, et al., Measurement of the strong interaction induced shift and width of the 1s state of kaonic deuterium at J-PARC, *Acta Phys. Pol. B* **46** (1) (2015) 101–112, arXiv:1501.05548.
- [21] C. Curceanu, et al., The modern era of light kaonic atom experiments, *Rev. Mod. Phys.* **91** (2) (2019) 025006.
- [22] A.W. Thomas, R.H. Landau, Pion - deuteron and pion - nucleus scattering: a review, *Phys. Rep.* **58** (1980) 121.
- [23] P. Hauser, et al., New precision measurement of the pionic deuterium S wave strong interaction parameters, *Phys. Rev. C* **58** (1998) R1869–R1872.
- [24] U.-G. Meißner, U. Raha, A. Rusetsky, Kaon-nucleon scattering lengths from kaonic deuterium experiments, *Eur. Phys. J. C* **47** (2006) 473–480, arXiv:nucl-th/0603029.
- [25] R.C. Barrett, A. Deloff, Strong interaction effects in kaonic deuterium, *Phys. Rev. C* **60** (1999) 025201.
- [26] P. Doleschall, J. Révai, N.V. Shevchenko, Three-body calculation of the 1s level shift in kaonic deuterium, *Phys. Lett. B* **744** (2015) 105–108, arXiv:1411.2411.
- [27] A.N. Ivanov, et al., On kaonic deuterium: quantum field theoretic and relativistic covariant approach, *Eur. Phys. J. A* **23** (2005) 79–111, arXiv:nucl-th/0406053.
- [28] N.V. Shevchenko, J. Révai, Faddeev calculations of the $\bar{K}NN$ system with chirally-motivated $\bar{K}N$ interaction. I. Low-energy K^-d scattering and antikaonic deuterium, *Phys. Rev. C* **90** (3) (2014) 034003, arXiv:1402.3935.
- [29] M. Döring, U.-G. Meißner, Kaon-nucleon scattering lengths from kaonic deuterium experiments revisited, *Phys. Lett. B* **704** (2011) 663–666, arXiv:1108.5912.
- [30] T. Mizutani, C. Fayard, B. Saghai, K. Tsushima, Faddeev-chiral unitary approach to the K^-d scattering length, *Phys. Rev. C* **87** (3) (2013) 035201, arXiv:1211.5824.
- [31] S.S. Kamalov, E. Oset, A. Ramos, Chiral unitary approach to the K^- deuteron scattering length, *Nucl. Phys. A* **690** (2001) 494–508, arXiv:nucl-th/00110054.
- [32] T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, Constraining the $\bar{K}N$ interaction from the 1S level shift of kaonic deuterium, *Phys. Rev. C* **96** (4) (2017) 045204, arXiv:1705.06857.
- [33] J. Révai, Three-body calculation of the 1s level shift in kaonic deuterium with realistic $\bar{K}N$ potentials, *Phys. Rev. C* **94** (5) (2016) 054001, arXiv:1608.01802.
- [34] A. Sibirtsev, J. Haidenbauer, S. Krewald, U.-G. Meißner, Kaon-deuteron scattering at low energies, *J. Phys. G* **32** (2006) R395–R416, arXiv:nucl-th/0608028.

- [35] V. Baru, E. Epelbaum, A. Rusetsky, The role of nucleon recoil in low-energy antikaon-deuteron scattering, *Eur. Phys. J. A* 42 (2009) 111–120, arXiv:0905.4249.
- [36] M. Mai, V. Baru, E. Epelbaum, A. Rusetsky, Recoil corrections in antikaon-deuteron scattering, *Phys. Rev. D* 91 (5) (2015) 054016, arXiv:1411.4881.
- [37] J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W. Thomas, R.D. Young, Finite-volume matrix Hamiltonian model for a $\Delta \rightarrow N\pi$ system, *Phys. Rev. D* 87 (9) (2013) 094510, arXiv:1303.4157.
- [38] J.M.M. Hall, et al., Lattice QCD evidence that the $\Lambda(1405)$ resonance is an antikaon-nucleon molecule, *Phys. Rev. Lett.* 114 (13) (2015) 132002, arXiv:1411.3402.
- [39] Z.-W. Liu, et al., Hamiltonian effective field theory study of the $N^*(1440)$ resonance in lattice QCD, *Phys. Rev. D* 95 (3) (2017) 034034, arXiv:1607.04536.
- [40] Z.-W. Liu, et al., Hamiltonian effective field theory study of the $N^*(1535)$ resonance in lattice QCD, *Phys. Rev. Lett.* 116 (8) (2016) 082004, arXiv:1512.00140.
- [41] J.-j. Wu, D.B. Leinweber, Z.-w. Liu, A.W. Thomas, Structure of the roper resonance from lattice QCD constraints, *Phys. Rev. D* 97 (9) (2018) 094509, arXiv:1703.10715.
- [42] J.-j. Wu, T.-S.H. Lee, A.W. Thomas, R.D. Young, Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD, *Phys. Rev. C* 90 (5) (2014) 055206, arXiv:1402.4868.
- [43] D. Leinweber, et al., N^* spectroscopy from lattice QCD: the roper explained, *JPS Conf. Proc.* 10 (2016) 010011, arXiv:1511.09146.
- [44] Z.-W. Liu, et al., Study of low-lying baryons with Hamiltonian effective field theory, *PoS INPC2016* (2017) 288, arXiv:1701.08582.
- [45] B.J. Menadue, W. Kamleh, D.B. Leinweber, M.S. Mahbub, Isolating the $\Lambda(1405)$ in lattice QCD, *Phys. Rev. Lett.* 108 (2012) 112001, arXiv:1109.6716.
- [46] U.G. Meissner, U. Raha, A. Rusetsky, Spectrum and decays of kaonic hydrogen, *Eur. Phys. J. C* 35 (2004) 349–357, arXiv:hep-ph/0402261.
- [47] R. Aaron, et al., in: A.W. Thomas (Ed.), *Modern Three-Hadron Physics*, Springer-Verlag, Berlin, Heidelberg, 1977, Chap. 1.
- [48] L. Roca, E. Oset, A description of the $f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ resonances as multi- $\rho(770)$ states, *Phys. Rev. D* 82 (2010) 054013, arXiv:1005.0283.
- [49] R. Chen, A. Hosaka, X. Liu, Prediction of triple-charm molecular pentaquarks, *Phys. Rev. D* 96 (11) (2017) 114030, arXiv:1711.09579.
- [50] R. Machleidt, in: J.W. Negele, E. Vogt (Eds.), *Advances in Nuclear Physics*, vol. 19, Plenum, New York, 1989, Chap. 2.
- [51] R.J. Hemingway, Production of $\Lambda(1405)$ in K^-p reactions at 4.2-GeV/c, *Nucl. Phys. B* 253 (1985) 742–752.