Bayesian Estimation of Monetary DSGE Models and Testing for Indeterminacy

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Abstract

This thesis consists of three self-contained papers on U.S. monetary policy.

The first paper examines monetary policy in the early 2000s, a prolonged period of low interest rates for which the efficacy of policy is intensely debated. Through the lens of an estimated simple New Keynesian (NK) model, the paper finds that when measuring inflation using headline CPI, the Federal Reserve's response to inflation turns out to be passive, therefore implying indeterminacy. Only when measuring inflation using core PCE does monetary policy appear to have been sufficiently active to rule out indeterminacy. Faced with this dilemma, the paper finally estimates an extended model that distinguishes between core and headline inflation. Estimation results from this model decisively rule out indeterminacy and suggest that indeed the Fed has put more weight on core PCE.

The second paper contrasts interest rate rules featuring fixed versus time-varying inflation target. It finds that the rule embedding time variation in inflation target empirically fits better and that the Fed has been responding strongly to the inflation gap not only in the Great Moderation period but also in the Great Inflation era. Therefore, this finding rules out self-fulfilling inflation expectations as an explanation of the high inflation episode in the 1970s. The paper also documents that changes in monetary policy have dampened most of the fluctuations in the inflation gap and contributed to the decline in its persistence and predictability.

The third paper investigates the impact of commodity price fluctuations on monetary policy and estimates a NK model with an explicit role for commodity price fluctuations. It finds that the pre-Volcker period is characterized by a determinate version of the model featuring high degree of real wage rigidity In this environment, the commodity price shocks of the 1970s created a severe trade-off between inflation and output gap stabilization. Faced with this puzzle, the central bank chose to react aggressively to both inflation and output growth, but not to the output gap, thereby ruling out indeterminacy. The paper further documents that oil price shocks are no longer as inflationary as they used to be due to a decline in real wage rigidity, thereby explaining the resilience of the economy to sustained oil price hikes in the 2000s.

Declaration

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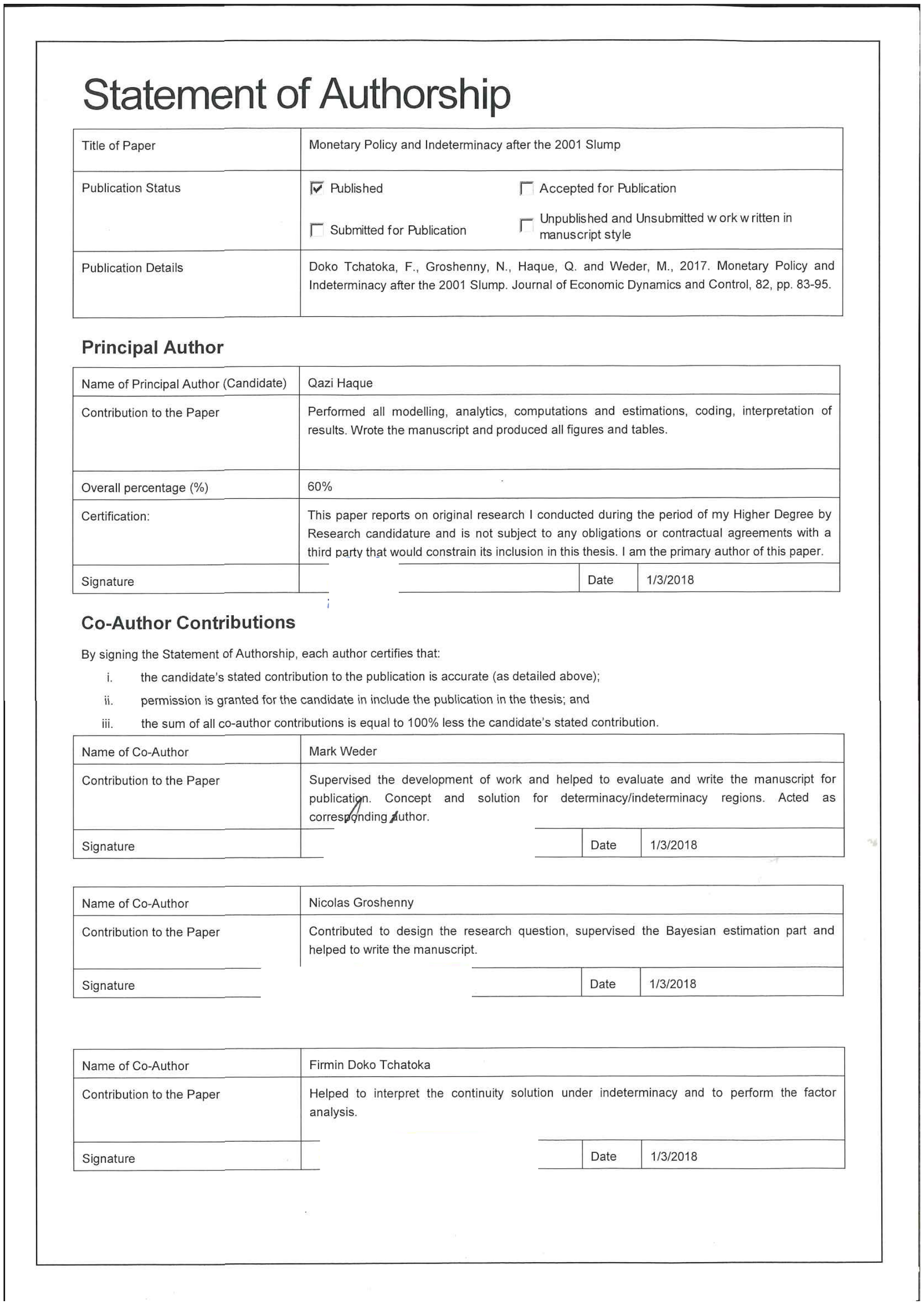
I Introduction

The aim of this thesis is to contribute to our understanding of monetary policy in the U.S. by investigating nominal interest rate setting through the lens of New Keynesian models. In this class of monetary dynamic stochastic general equilibrium (DSGE) models, it has become common practice to think of the central bank as following a kind of interest rate rule that features some feedback between economic variables and the nominal interest rate. However, it is well known that monetary policy can induce multiple equilibria, which is often referred to as indeterminacy. For instance, indeterminacy can arise if the monetary authority follows an interest rate rule that does not raise the interest rate aggressively enough in response to inflation. Such policy-generated indeterminacy can induce instability by opening the door to self-fulfilling inflation expectations or what is called sunspot fluctuations, thereby reducing economic welfare. In the following chapters, I apply Bayesian estimation techniques to evaluate the efficacy of monetary policy in the U.S. by assessing the quantitative relevance of equilibrium indeterminacy.

The first paper investigates the adequacy of monetary policy following the 2001 recession. The issue of loose monetary policy in the early 2000s is closely related to Stanford economist John Taylor who asserts that the Federal Reserve kept the policy rate too low for too long after the recession in 2001, thereby creating an environment that ultimately brought the economy close to a brink. Along these lines, the paper estimates a simple NK model of the U.S. economy over the period following the 2001 slump and prior to the onset of the global financial crisis (GFC). The paper finds that when inflation is measured using headline inflation, Fed's responsiveness to inflation turns out to be passive, implying indeterminacy and multiplicity of rational expectations equilibria and thereby lending some support to Taylor's assertion. However, when measuring inflation using core inflation as an observable, monetary policy appear to have been sufficiently responsiveness to inflation to rule out indeterminacy. This essentially poses a dilemma since the results are clearly dependent on the particular measure of inflation being used. The paper then relaxes the assumption that inflation in the model is measured by a single indicator and re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the empirical inflation series. However, indeterminacy can still be neither ruled in nor ruled out, thus leaving us essentially with the same dilemma. To resolve this ambiguity, the paper finally estimates a version of the model that distinguishes between core and headline inflation. Estimation results from this extended model suggest that the Federal Reserve has put more weight on core inflation in the conduct of its policy and has been strongly responsive to inflation. Thus the results corroborate the claims of Ben Bernanke who argues that the Fed has been focusing on core inflation during much this period.

The second paper examines the drivers of the high and volatile inflationary episodes in the 1970s as well as the decline in macroeconomic volatility and inflation gap predictability since the mid-1980s. In this paper, I model inflation dynamics using a NK framework with positive steady state (or trend inflation) while allowing for indeterminacy. First of all, allowing for trend inflation is important as it alters the New Keynesian Phillips Curve (NKPC) as well as the inflation dynamics and the determinacy properties of the model. For instance, higher trend inflation makes price-setting firms more forward-looking which flattens the NKPC and therefore the central bank needs to respond more strongly to inflation in order to guarantee determinacy. Next, on top of modeling the first moment of inflation, I compare the empirical fit of the model featuring fixed versus time-varying inflation target. In this line of argument, a fixed target is simply equal to steady state inflation in the model and stands for the Federal Reserve's long-run target compatible with its long-run goals such as inflation stability. In contrast, time-varying inflation target can be interpreted as short-run fluctuations around the long-run trend based on short-term goals pursued by the Fed conditional on economic situation. First of all, I find that when considering the model with fixed inflation target, indeterminacy cannot be ruled out in the 1970s while determinacy prevails in the Great Moderation period. This finding is in line with the empirical monetary policy literature. Yet, the upshot completely differs when allowing for time-varying target. This time the posterior density favors determinacy for both the Great Inflation and the Great Moderation period. Using posterior odds ratio to compare fixed versus time-varying target, I then find evidence in favor of time variation in the target inflation process. This result suggests that monetary policy, even in the 1970s, was sufficiently aggressive to inflation to rule out indeterminacy. Next, I find that diverse features of the inflation gap process provide mixed evidence regarding the decline in its persistence. Specifically, I document that it is actually the target-based inflation gap process, defined as the difference between inflation and the Fed's time-varying target, whose persistence has gone down, while mean-based gap, measured as the deviation of inflation from a constant steady-state, has continued to remain persistent. Through counterfactual experiments, I then show that the decline in inflation gap volatility and persistence is mainly driven by both a stronger response to the inflation gap as well as a better anchored inflation target. However, changes in monetary policy alone fail to explain the reduced variability of output growth which is explained by a reduction in the volatility of technology shocks in the model. Hence, in this sense, I find that both good policy and good luck are jointly required to explain the Great Moderation phenomenon.

The third chapter looks at the impact of commodity price fluctuations on monetary policy with a particular focus on the Great Inflation. Commodity price shocks in general and oil price shocks in particular were an important source of economic fluctuations in the U.S. during much of the 1970s. For instance, there were episodes of large increases in the price of oil triggered by the Yom Kippur war in 1973 and the Iranian revolution of 1979. Such adverse cost-push shocks arguably generated a trade-off between stabilizing inflation and stabilizing the output gap for the Federal Reserve. Existing empirical investigations on monetary policy in the 1970s find that policy failed to respond sufficiently strongly to inflation thereby opening the door to self-fulfilling inflation expectations. However, these studies abstract from modeling the role of commodity price fluctuations and the associated policy trade-off. Hence, the paper estimates a NK model with trend inflation and oil entering in both consumption and production while paying particular attention in identifying key features of the model through careful elucidation of observables. The paper finds that the Federal Reserve has been responding aggressively to inflation even in the 1970s to the extent that in completely rules out indeterminacy. This finding suggests that parameter estimates pertaining to the Taylor rule are biased when abstracting from modeling commodity price fluctuations and the associated trade-off. In fact, once this is taken into account, the empirical finding rules out self-fulfilling inflation expectations as an explanation of the high inflation episode in the 1970s thereby corroborating the findings of the second paper. Finally, the paper also documents that oil price shocks are no longer as inflationary as they used to be, allowing the central bank to respond less aggressively to an oil price shock, thereby explaining why the economy has remained remarkably resilient to sustained oil price hikes in the 2000s.



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II Monetary Policy and Indeterminacy after the 2001 Slump

This paper estimates a simple New Keynesian model of the U.S. economy, allowing for indeterminacy, over the period following the 2001 slump, an episode for which the adequacy of monetary policy is intensely debated. We find that only when measuring inflation with core PCE does monetary policy appear to have been sufficiently active to rule out indeterminacy. We then relax the assumption that inflation in the model is measured by a single indicator and re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the empirical inflation series. CPI and PCE provide better indicators of the latent concept while core PCE is less informative. Finally, we estimate an extended economy that distinguishes between core and headline inflation rates. This model comfortably rules out indeterminacy and confirms the view that the Federal Reserve put more weight on core PCE inflation when setting the policy rate during this period.

1 Introduction

It has become prevalent to think of monetary policy in terms of nominal interest rate feedback rules. In certain situations, for example, loose monetary policy, these rules may introduce indeterminacy and sunspot equilibria into otherwise stable economic environments. Lubik and Schorfheide (2004) and many others suggest that, empirically, such sunspots-based instability was confined to the seventies and that the post-Volcker years can ostensibly be characterized by determinacy. The current paper extends this analysis to more recent data leading up to the Great Recession.

The issue of loose monetary policy during the 2000s is closely related to Taylor (2007, 2012), who asserts that the Federal Reserve kept the policy rate too low for too long following the recession of 2001. While Taylor does not touch the issue of indeterminacy, he nevertheless argues that this loose policy created an environment that ultimately brought the economy close to the brink. To bolster his thesis of an extra easy monetary policy, Taylor constructs an artificial path for the Federal Funds rate that follows his proposed rule. He characterizes this counterfactual rate's loose fitting to the actual rate as

> "[...] the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007, 2]

His view is disputed by many. Amongst them, Bernanke (2010) argues that Taylor's use of the headline consumer price index (CPI) to measure inflation in the Federal Reserve's reaction function is misleading. In fact, the Federal Reserve switched the inflation measures that inform its monetary policy deliberations several times over the last two decades. In particular, it moved away from the CPI to the personal consumption expenditure deflator (PCE) in early 2000. In turn, PCE was abandoned midway through 2004 in favor of the core PCE deflator (which excludes food and energy prices).[1] Bernanke (2015) revisits Taylor's exercise and constructs his own counterfactual Federal Funds rate using core PCE. Bernanke's verdict of the Federal Reserve's policy during the 2000s is inimical to Taylor's and he says that

> "[...] the predictions of my updated Taylor rule and actual Fed policy are generally quite close over the past two decades. In particular, it is no longer the case that the actual funds rate falls below the predictions of the rule in 2003-2005." [Bernanke, 2015]

Our paper sheds further light on this debate. It takes as a point of departure Taylor's claim of an analogy between the 1970s and the 2000s as well as one of the key recommendations for monetary policy that has emanated from New Keynesian modelling: interest rates should react strongly to inflation movements to not destabilize the economy. Phrased alternatively, if the central bank's response to inflation is tuned too passively in a Taylor rule sense, multiplicity and endogenous instability may arise. In fact, the U.S. economy of the 1970s can be well represented by an indeterminate version of the New Keynesian model as was shown by Lubik and Schorfheide (2004). Along these lines, the current paper turns Taylor's *too low for too long* story into questioning whether the Federal Reserve operated on the indeterminacy side of the rule after the 2001 slump. Knowledge about the economy's

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[1] See Mehra and Sawhney (2010).

regime is important for policymakers because indeterminacy introduces sunspots and alters the propagation of fundamental shocks. Thus, for central banks to use models for policy analysis, a good understanding about the presence of (in-)determinacy is vital.

The empirical plausibility of a link between monetary policy and macroeconomic instability was first established by Clarida, Gali and Gertler (2000). They estimate variants of the Taylor rule and their research suggests that the Federal Reserve's policy may have steered the economy into an indeterminate equilibrium during the 1970s. Yet, they also find that the changes to policy which have taken place after 1980 – essentially a more aggressive response to inflation – brought about a stable and determinate environment. Lubik and Schorfheide (2004) reinforce this point but they refrain from using a single equation approach. They recognize that indeterminacy is a property of a rational expectations system and apply Bayesian estimation techniques to a general equilibrium model. Their results parallel the earlier findings that the U.S. economy veered from indeterminacy to determinacy around 1980 – largely as the result of a more aggressive response of monetary policy towards inflation.

Moreover, this monetary policy change had perhaps an even greater influence on the economy: the transformation from the Great Inflation of the 1970s to the Great Moderation is often conjoined to the conduct of monetary policy.[2] Yet, the Great Moderation came to an end sometime during the 2000s, and it was followed by enormous economic volatility. Our aim is to examine the possible connection between this transformation and an alteration in the Federal Reserve's monetary policy. In particular, we concentrate on the effects of a possibly too easy monetary policy after the 2001 slump. We frame our analysis from the perspective of (in-)determinacy and conduct it under the umbrella of the Bernanke versus Taylor dispute by considering the measures of inflation that repeatedly occur in the discussion: CPI, PCE and core PCE.

Accordingly, we estimate a small-scale New Keynesian model allowing for indeterminacy over the period between the 2001 slump and the onset of the Great Recession, thus, the NBER-dated 2002:I-2007:III window to be precise. To test for

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[2] See, for example, Benati and Surico (2009), Bernanke (2012), Coibion and Gorodnichenko (2011), Arias, Ascari, Branzoli and Castelnuovo (2014) and Hirose, Kurozumi and Van Zandweghe (2015).

indeterminacy, we employ the method of Lubik and Schorfheide (2004) to compute the posterior probabilities of determinacy and indeterminacy. We take as starting point the same basic New Keynesian model, priors and observables as Lubik and Schorfheide (2004). This strategy allows us to create a continuity between their and our results, which is important given the shortness of our period of interest.

We establish a number of new insights regarding recent U.S. monetary policy. For example, we can indeed expose a violation of the Taylor principle for most of the 2000s when using CPI to measure inflation. This finding supports the visual inspection checks based on single equations in Taylor (2012) who coined the phrase *Great Deviation* to refer to this period. Hence, the 2002:I to 2007:III period would appear to be best described by an indeterminate version of the New Keynesian model. Our upshot is different when basing the analysis on PCE data: we can neither rule in nor rule out indeterminacy. Finally, the evidence in favor of indeterminacy altogether vanishes when we use core PCE. Monetary policy then appears to have been quite appropriate. This conclusion parallels the insight from Bernanke's (2015) counterfactual Federal Funds rate. We thus establish that tests for indeterminacy are susceptible to the data used in the estimation.

We next consider whether our results are an artifact of the six year sample of data. To address this issue, we re-estimate the model on rolling windows of fixed length (23 quarters to match the length of the 2002:I-2007:III period) starting in the mid-1960s and focussing on the same inflation measure as Lubik and Schorfheide (2004) namely CPI inflation. The outcomes of the indeterminacy test performed on rolling windows are highly plausible. In particular, we identify only two broad periods (i.e. several consecutive windows) in which a passive policy has likely led to indeterminacy: the 1970s and the post-2001 period. The first period, which coincides with the span of the Burns and Miller chairmanships, exactly matches the indeterminacy duration, as well as the timing of the switch to determinacy in 1980, that Coibion and Gorodnichenko (2011) document. We take this analogy as a reassuring validation of our small sample approach, i.e. even though our period of interest is quite short, it is possible to infer meaningful information from it.[3]

We then attend the issue of how best to measure inflation in the New Keynesian

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[3] Judd and Rudebusch (1998) is another example of an evaluation of monetary policy over similarly short sample periods.

model. We tackle the ambiguity between the theoretical concept and the empirical inflation proxies by employing the DSGE-factor methodology proposed by Boivin and Giannoni (2006). Accordingly, we combine various measures of inflation in the measurement equation and re-estimate our model. CPI and PCE emerge as better indicators of the concept of inflation than core PCE and indeterminacy cannot be ruled out.

However, the finding that indeterminacy cannot be ruled out may hinge on the fact that the baseline three-equations New Keynesian model features a single concept of inflation. To address this question, we finally turn toward an artificial economy that distinguishes explicitly between core and headline inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active to comfortably rule out indeterminacy.

Perhaps most closely related to our work are Belongia and Ireland (2016) who, like us, evaluate monetary policy during the 2000s.[4] Belongia and Ireland (2016) estimate a time-varying VAR to track the evolution of the Federal Reserve's behavior throughout the 2000s. They find evidence of a change in the Federal Reserve's behavior away from stabilizing inflation towards stabilizing output and also of persistent deviations from the estimated policy rule. While similar in spirit to our results they do not address issues of indeterminacy.

Bianchi (2013) examines the Federal Reserve's policy post-WWII taking a Markov switching rational expectations approach with two monetary policy regimes (i.e. *Hawk* and *Dove*). Bianchi characterizes monetary policy in the early 2000s as Hawkish and identifies a switch to a Dove regime after 2005. His approach to deal with the issue of passive monetary policy is by requiring a linear representation of the Markov switching model to have a unique solution. Phrased alternatively, the regime transitions do not imply moving from determinacy to indeterminacy as both regimes are determinate. Hence, Bianchi's model cannot address questions involving sunspot equilibria as in our paper.

The remainder of the paper evolves as follows. The next section sketches the baseline model and its solution. Section 3 presents the econometric strategy and baseline results. Robustness checks are conducted in section 4. Section 5 relaxes the

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[4] See Fackler and McMillin (2015), Fitwi, Hein and Mercer (2015), Groshenny (2013) and Jung and Katayama (2014) for related exercises.

assumption that model inflation is properly measured by a single empirical indicator. In section 6 we consider an economy that features more than one inflation rate. Section 7 concludes.

2 Baseline model

The familiar three linearized equations summarize our basic New Keynesian model:

$$\begin{aligned} y_t &= E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \qquad \tau > 0 \qquad (1) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \qquad \kappa > 0,\ 0 < \beta < 1 \qquad (2) \\ R_t &= \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \le \rho_R < 1. \qquad (3) \end{aligned}$$

Here y_t stands for output, R_t denotes the nominal interest rate and π_t symbolizes inflation. E_t represents the expectations operator. Equation (1) is the dynamic IS relation reflecting an Euler equation. Equation (2) describes the expectational Phillips curve. Finally, equation (3) represents monetary policy, i.e. a Taylor-type rule in which $\psi_\pi > 0$ and $\psi_y > 0$ are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. The term $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. The other fundamental disturbances involve exogenous shifts of the Euler equation which are captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1$$

and

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1.$$

We denote by σ_g and σ_z the standard deviations of the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$. Finally, the term $\rho_{g,z}$ denotes the correlation between the demand and supply innovations. Then, the vector of model parameters entails

$$\theta \equiv \left[\psi_\pi, \psi_y, \rho_R, \beta, \kappa, \tau, \rho_g, \rho_z, \rho_{g,z}, \sigma_R, \sigma_g, \sigma_z\right]'.$$

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in our simple New Keynesian model, indeterminacy occurs when the central bank passively responds to inflation changes, i.e. when $\psi_\pi < 1 - \psi_y (1-\beta)/\kappa$.

To solve the model, we apply the method proposed by Lubik and Schorfheide (2003) in which case the full set of rational expectations solutions takes on the form

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{4}$$

where ϱ_t is a vector of model variables,

$$\varrho_t \equiv [y_t, R_t, \pi_t, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t]',$$

ε_t denotes a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[5] The coefficient matrices $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})$ and $\Phi_\zeta(\theta)$ are related to the structural parameters of the model. The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy can manifest itself in two ways: (i) through pure extrinsic non-fundamental shocks, ζ_t (a.k.a sunspots), disturbing the economy and (ii) through affecting the propagation mechanism of fundamental shocks via $\widetilde{\mathbf{M}}$.

3 Estimation and baseline results

3.1 Data and priors

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. The measurement equation relating the elements of ϱ_t to the three observables, x_t, is given by

$$x_t = \begin{bmatrix} 0 \\ r^* + \pi^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \varrho_t \tag{5}$$

where π^* and r^* are the annualized steady-state inflation and real interest rates respectively. Equation (4) and (5) provide a state-space representation of the linearized model that allows us to apply standard Bayesian estimation techniques. The technical appendix provides further details.

We use HP-filtered per capita real GDP and the Federal Funds Rate as our observable for output and the nominal interest rate. These choices follow Lubik and Schorfheide (2004) and make our baseline empirical analysis comparable to theirs

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[5] Under determinacy, the solution (4) boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

in all dimensions but the sample period. To draw up our analysis in the Bernanke versus Taylor debate, we consider in turn three different measures of inflation: CPI, PCE deflator and core PCE (all expressed in annualized percentage changes from the previous quarter). The data covers the period between the 2001 slump and the onset of the Great Recession, i.e. 2002:I to 2007:III.

Table 1 reports our baseline priors which are identical to the ones in Lubik and Schorfheide (2004) and imply a prior predictive probability of determinacy equal to 0.527. Following Lubik and Schorfheide (2004) we replace $\widetilde{\mathbf{M}}$ in equation (4) with $\mathbf{M}^*(\theta) + \mathbf{M}$ where $\mathbf{M} \equiv [M_{R\zeta}, M_{g\zeta}, M_{z\zeta}]'$. We select $\mathbf{M}^*(\theta)$ such that the responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and the indeterminacy regions. We set the prior mean for $\mathbf{M}$ equal to zero.

3.2 Testing for indeterminacy

For each measure of inflation, we estimate the model over the two different regions of the parameter space, i.e. determinacy and indeterminacy. To assess the quality of the model's fit to the data we present marginal data densities and posterior model probabilities for both parametric zones. We approximate the data densities using Geweke's (1999) modified harmonic mean estimator. Table 2 reports our results.

Following Lubik and Schorfheide (2004) and Taylor (2007, 2012), we begin by using headline CPI to measure inflation. In this case, the data favors the indeterminate model: the posterior probability of indeterminacy is 0.90. This result suggests that Taylor's characterization of monetary policy in the aftermath of the 2001 slump as *too low for too long* is in fact consistent with indeterminacy and the view that the Federal Reserve has potentially veered the economy into instability.

Yet, the upshot differs depending on which measure of inflation we employ in the estimation. Take Bernanke's (2015) suggestion that Taylor's counterfactual experiment should have been performed with core PCE. When making this choice, the posterior probability for our sample concentrates all of its mass in the determinacy region. This result flags that the Federal Reserve had not been responding passively to inflation during this period. However, the Humphrey-Hawkins reports to Congress document that the Federal Reserve based monetary policy deliberations on headline

Table 1: Priors and posteriors of DSGE parameters

		Priors		Posterior Mean [5th pct, 95th pct]	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Indeterminacy	Core PCE Determinacy
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	0.84 [0.61,0.98]	3.01 [1.97,4.17]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.19 [0.05,0.41]	0.28 [0.07,0.64]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.83 [0.74,0.90]	0.76 [0.64,0.85]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	3.28 [1.27,6.01]	1.99 [1.67,2.31]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.15 [0.47,2.01]	1.40 [0.84,2.01]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.20)	0.91 [0.51,1.41]	0.71 [0.31,1.19]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00 (0.50)	1.66 [1.00,2.49]	1.62 [0.95,2.48]
ρ_g	[0,1)	Beta	0.70 (0.10)	0.60 [0.45,0.73]	0.80 [0.72,0.87]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.80 [0.68,0.89]	0.61 [0.49,0.74]
ρ_{gz}	[-1,1]	Normal	0.00 (0.40)	−0.28 [−0.72,0.17]	0.86 [0.57,0.97]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−0.57 [−1.90,1.00]	
$M_{g\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−1.99 [−2.92,−1.05]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	0.41 [0.05,0.83]	
σ_R	$\mathbb{R}^+$	IG	0.31 (0.16)	0.16 [0.12,0.21]	0.16 [0.12,0.21]
σ_g	$\mathbb{R}^+$	IG	0.38 (0.20)	0.28 [0.18,0.40]	0.19 [0.14,0.25]
σ_z	$\mathbb{R}^+$	IG	1.00 (0.52)	0.74 [0.54,1.03]	0.62 [0.47,0.82]
σ_ζ	$\mathbb{R}^+$	IG	0.25 (0.13)	0.20 [0.12,0.30]	

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,\varsigma\right) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 4$ and ς equals $0.25, 0.3, 0.6$ and 0.2, respectively. The prior predictive probability of determinacy is 0.527.

Table 2: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
CPI	-95.48	-93.28	0.10	0.90
PCE	-85.42	-85.75	0.58	0.42
Core PCE	-64.60	-71.58	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.527.

PCE from the beginning of 2000 until mid-2004. Since Taylor is particularly critical of the monetary policy from 2002 to 2004, we next measure inflation using headline PCE data. We repeat the estimation and the finding is now ambiguous: the probability of determinacy is 0.58. Phrased alternatively, we cannot dismiss the possibility of indeterminacy.

Table 1 reports the posterior estimates of the parameters for the model specification favored under CPI and core PCE respectively.[6] The estimated policy rule's response to inflation, ψ_π, which essentially governs the indeterminacy, differs significantly depending on the way we measure inflation. In particular, when basing the estimation on CPI, the posterior mean equals 0.84 (with 90-percent interval $[0.61, 0.98]$). This result indicates that monetary policy violated the Taylor principle over the 2002-2007 period or in the words of Taylor:

> "[t]he responsiveness appears to be at least as low as in the late 1960s and 1970s." [Taylor, 2007, 8]

The opposite result ensues when using core PCE. In that case, the posterior mean of ψ_π is well above one at 3.01 (with 90-percent interval $[1.97, 4.17]$).

3.3 How important are sunspots and what drives the results?

Indeterminacy can manifest itself by affecting the propagation of fundamental shocks as well as introducing sunspot shocks. Given our above results, the question of how important sunspot fluctuations were during the 2000s comes up naturally. To

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[6] The appendix reports results for parameter estimates when using headline PCE inflation data.

answer this question, we study the propagation of shocks and the unconditional forecast error variance decomposition. A more detailed analysis can be found in the Appendix. Based on our estimation using CPI data, sunspots played only a marginal role with the most significant contribution being seven to eight percent in explaining the variances of the policy rate and inflation. However, indeterminacy qualitatively altered the propagation of demand shocks by changing the sign of the inflation response.

In sum, we find that indeterminacy outcomes are dependent on the inflation measure that is used. What is the intuition behind this result and which features of the data stand behind it? Headline inflation generally tends to be more volatile than core inflation that excludes the most volatile components, particularly in periods of persistent commodity price shocks. In fact, CPI and PCE are both more volatile than core PCE during our period of interest. This volatility feature of the data partly drives our findings through its influence on the estimates of the Taylor rule. With core PCE as the preferred measure of inflation, the monetary authority reacts to relatively small movements in inflation. In that case, any policy response to inflation has to be substantially larger for the estimation procedure to fit the Federal Funds rate data. In contrast, when measuring inflation with CPI, the estimated responsiveness to inflation turns out to be smaller due to the larger fluctuations of the inflation gap. As monetary policy fails to guarantee a unique rational expectations equilibrium whenever it is insufficiently active with respect to inflation, the posterior probability of indeterminacy is higher with headline than with core inflation.

Beyond the difference in the volatility of the inflation measures, another feature of the data that drives our (in)-determinacy results is a disconnect between core and headline inflation in face of persistent commodity price shocks. Our estimation based on CPI suggests that indeterminacy primarily affects the propagation of demand shocks. In particular, the parameter $M_{g\zeta}$ redirects the transmission of this disturbance, making it look similar to a cost-push shock. This mix of disturbances helps the model fit the joint behavior of headline inflation (especially CPI), real activity and monetary policy during the 2002-2007 episode.

4 Sensitivity analysis

We now investigate the sensitivity of our results in various directions. The robustness checks involve testing for indeterminacy on rolling windows and alternative measures of output as well as using real-time data.[7]

We conduct further robustness checks that involve (i) estimating the policy parameters only, (ii) alternative priors for ψ_π, (iii) alternative measure of inflation, (iv) serially correlated monetary policy shocks, and (v) trend inflation. For all these tests, our results remain unchanged.

4.1 Rolling windows

The size of our sample is undeniably short. So first and foremost, we want to assess the extent to which our results might be an artifact of the small sample. To do so, we re-estimate the model on rolling windows starting in the mid-1960's, and keeping the size of the windows fixed at 23 quarters to match the number of observations in our period of interest. Thus the first window is 1966:I-1971:III. We move the window forward one quarter at a time, and re-estimate all parameters each time.[8] Here we just consider CPI inflation as the Federal Reserve only began to base its monetary policy deliberations on PCE and core PCE in the 2000s. Moreover, doing so makes our results directly comparable to Lubik and Schorfheide (2004). Figure 1 presents the evolution of the posterior probability of determinacy for the U.S. economy from 1966:I to 2008:III. The end point is chosen to avoid obvious complications that emanate from hitting the zero lower bound. The graph suggests that the U.S. economy was likely in a state of indeterminacy during the 1970s. Thereafter, beginning with the Volcker disinflation policies, the economy shifted back to a determinate equilibrium. These findings are consistent with related studies such as Clarida, Gali and Gertler (2000), Lubik and Schorfheide (2004) and Coibion and

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[7] The Appendix conducts additional robustness checks that involve estimating the policy parameters only; alternative priors for ψ_π; an alternative measure of inflation; serially correlated monetary shocks; trend inflation. Our results are robust to all these extensions.

[8] This approach to estimate linear DSGE models was recently promoted by Canova (2009), Canova and Ferroni (2011a) and Castelnuovo (2012a,b). Rolling window estimation provides two benefits. It allows us to uncover time-varying patterns of the model's parameters, in particular, of the monetary policy coefficients. At the same time, the procedure permits us to remain within the realm of linear models and apply standard Bayesian methods.

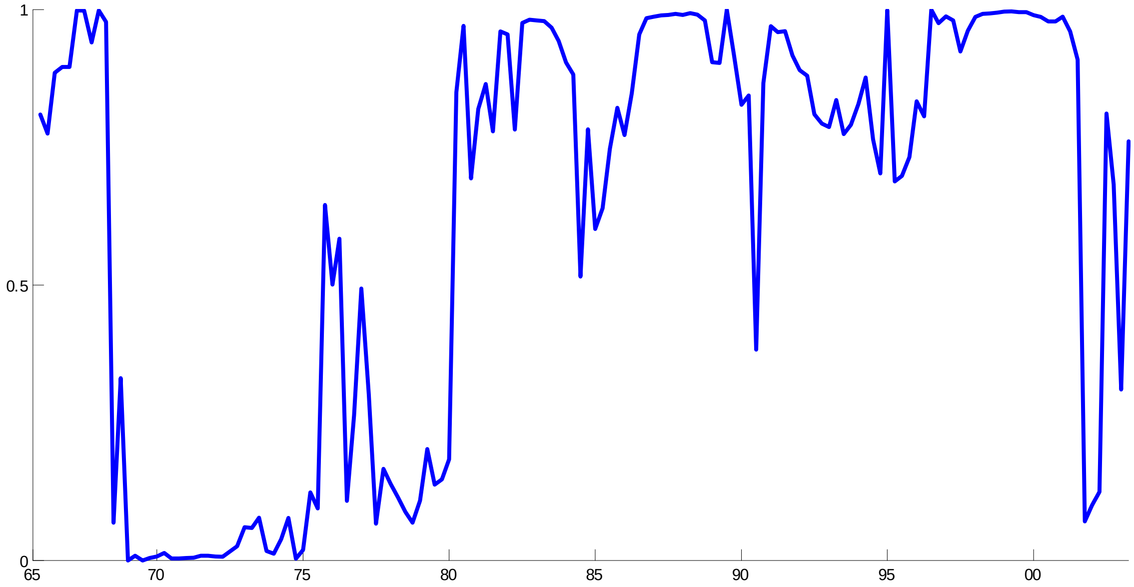

Figure 1: Probability of determinacy using rolling window estimation. The figure plots the probability at the first quarter of a window.

Gorodnichenko (2011).[9] We take this correspondence as a justification for estimating our model on a short window.[10] Our paper documents a second shift after the 2001 slump now from determinacy to indeterminacy.

4.2 Alternative measures of output

To make our baseline analysis comparable with Lubik and Schorfheide (2004), we used HP-filtered real GDP per capita to measure output fluctuations. However, as argued by Canova (1998), Gorodnichenko and Ng (2010), and Hamilton (2017) among others, HP-filtered data may induce spurious results. Accordingly, we now consider two alternative ways to gauge real economic activity. First, we replace the output trend extracted using the HP filter with the Congressional Budget Office's estimate of potential output as in Belongia and Ireland (2016) and others. Table 3 suggests that, again, our results remain robust. Second, we use output growth instead of an output gap measure. To this end, we assume that the artificial economy now

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[9] Figure 1 is comparable to Coibion and Gorodnichenko (2011, Figure 4). They report a moving average of the probability of determinacy which makes their series smoother than ours. Coibion and Gorodnichenko (2011) use a model with trend inflation. We explore such model in the Appendix.

[10] We furthermore experimented with the window length and the results appear to be robust.

Table 3: Determinacy versus Indeterminacy (Robustness)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	CBO output gap	-97.89	-95.85	0.12	0.88
	Output growth	-93.29	-89.58	0.02	0.98
PCE	CBO output gap	-88.08	-88.18	0.53	0.47
	Output growth	-82.89	-81.80	0.25	0.75
	Real-time data	-83.32	-83.06	0.44	0.56
Core PCE	CBO output gap	-68.53	-73.63	0.99	0.01
	Output growth	-62.54	-67.58	1	0
	Real-time data	-65.85	-70.24	0.99	0.01

features trend-stationary technology – it follows a deterministic trend as in Mattesini and Nisticò (2010) or Ascari, Castelnuovo and Rossi (2011).[11] Also, we no longer estimate the intertemporal rate of substitution, $1/\tau$, and instead set it equal to one to make the model consistent with balanced growth. Then, Table 3 shows that when using output growth, the case for indeterminacy becomes even stronger for CPI and PCE. Yet, it remains unchanged when measuring inflation via core PCE data.[12]

4.3 Real-time data

One important distinction between CPI and PCE price indices is that the former are not revised (except for seasonal adjustments), whereas the latter go through repeated rounds of revision as more information becomes available. In particular, the PCE-based measure of inflation in Bernanke's (2010) speech is a real-time measure, which, as he argues, may exhibit considerable differences relative to the revised PCE data. Hence, like Orphanides (2004), we now take into account that monetary policymakers make decisions based on contemporaneously available information. Therefore, our estimation uses real-time data on output, PCE and core PCE from the Real-Time Data Set for Macroeconomists provided by the Federal Reserve Bank of Philadelphia. Table 3 confirms that our findings remain robust: we can confidently rule out

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[11] The measurement equation now writes $\gamma_{yt}^{obs} = \gamma^* + \Delta\widehat{y}_t$ where γ_{yt}^{obs} is the observed growth rate of output, γ^* stands for the steady state growth rate and $\Delta\widehat{y}_t$ is the first-differenced logarithm of detrended model output. The prior distribution of γ^* is $\mathsf{N}(0.5, 0.1)$.

[12] Given the indicated issues with HP-filtered data and the essentially unchanged results when employing output growth, the remainder of this paper concentrates on output growth.

indeterminacy when basing our estimation on core PCE, while there is a possibility that indeterminacy might have prevailed under PCE.

4.4 Further tests of robustness

We conduct further robustness checks that involve (i) estimating the policy parameters only, (ii) alternative priors for ψ_π, (iii) alternative measure of inflation, (iv) serially correlated monetary policy shocks, and (v) trend inflation.[13] For all these tests, our main result so far that the basic New Keynesian model provides mixed evidence about indeterminacy is robust.

5 Which measure of inflation to choose?

Our baseline estimations have delivered mixed evidence regarding the probability of indeterminacy for the 2002:I to 2007:III period. The results are consistently dependent on the specific inflation measure used in estimation – only with core PCE series can we comfortably rule out indeterminacy. However, each inflation proxy may only provide an imperfect indicator of the model concept. Put differently, all three measures of inflation may contain relevant information. In this line of thinking, we will now depart from the assumption that model inflation is measured by a single series and draw on Boivin and Giannoni's (2006) dynamic factor analysis of DSGE models.[14] In a nutshell, we want to exploit the information from all the inflation series in the estimation to deliver more robust results. We treat the model concept of inflation as the unobservable common factor for which data series are imperfect proxies. More concretely, the estimation involves the transition equation (4)

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

or its determinacy equivalent

$$\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$$

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[13] In the model with trend inflation, it is no longer possible to analytically derive the indeterminacy conditions. Hence, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule (see also Justiniano and Primiceri, 2008).

[14] Canova and Ferroni (2011b) and Castelnuovo (2013) are recent applications.

and the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ FFR_t \\ \mathbf{X}_t \end{bmatrix} = \begin{bmatrix} \gamma^* \\ r^* + \pi^* \\ \underset{4\times 1}{\mathbf{0}} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_2 & \underset{2\times 4}{\mathbf{0}} \\ \underset{4\times 2}{\mathbf{0}} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta y_t \\ 4R_t \\ \boldsymbol{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{u}_t \end{bmatrix}. \tag{6}$$

Here ΔGDP_t stands for the growth rate of per-capita real GDP, FFR_t denotes the Federal Funds rate, $\mathbf{X}_t \equiv [\Delta CPI_t, \Delta PCE_t, \Delta corePCE_t, \Delta DEF_t]'$ is the vector of empirical inflation proxies,[15] $\mathbf{\Lambda} =$diag$(\lambda_{CPI}, \lambda_{PCE}, \lambda_{corePCE}, \lambda_{DEF})$ is a 4×4 diagonal matrix of factor loadings relating the latent model concept of inflation to the four indicators, $\boldsymbol{\pi}_t \equiv 4[\pi_t, \pi_t, \pi_t, \pi_t]'$ and $\mathbf{u}_t = [u_t^{CPI}, u_t^{PCE}, u_t^{corePCE}, u_t^{DEF}]' \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma})$ is a vector of serially and mutually uncorrelated indicator-specific measurement errors, with $\mathbf{\Sigma} =$diag$(\sigma^2_{CPI}, \sigma^2_{PCE}, \sigma^2_{corePCE}, \sigma^2_{DEF})$. We jointly estimate the parameters $(\mathbf{\Lambda}, \mathbf{\Sigma})$ of the measurement equation (6) along with the structural parameters θ. We calibrate π^* equal to 2.5 percent - a value roughly in line with the average of the sample means of the inflation series. We standardize the four indicators to have mean zero and unit variance. This standardization permits us to interpret the factor loadings, $\lambda_j s$, as correlations between the latent theoretical concept of inflation and the respective observables.[16] Our prior distribution for the loadings and measurement errors are $\lambda_j \sim Beta(0.50, 0.25)$ and $u_t^j \sim Inverse\ Gamma(0.10, 0.20)$ respectively. By employing a beta distribution, the support of the λ_j is restricted to the open interval $(0, 1)$ which is a necessary sign restriction.

Table 4 reports the resulting log-data densities which are -162.50 for determinacy and -161.83 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 34% versus 66%, hence, we cannot rule out indeterminacy.[17]

Table 5 reports the posterior estimates of the model parameters along with the factor loadings (i.e. the correlations between the latent factor and the proxies) as well as the standard deviations of the measurement errors. Conditional on both determinacy and indeterminacy the loadings on CPI and PCE are about three times as large as the loading on core PCE. Furthermore, there is evidence of substantial

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[15] DEF is the acronym for the GDP Deflator.

[16] See Geweke and Zhou (1996) and Forni, Hallin, Lippi and Reichlin (2000).

[17] We also replicated Lubik and Schorfheide (2004) with the DSGE factor model approach. The outcomes of the indeterminacy test for the pre-Volcker and post-1982 sample periods remain unaltered to this extension.

Table 4: Determinacy versus Indeterminacy (DSGE-Factor)

Log-data density		Probability	
Determinacy	Indeterminacy	Determinacy	Indeterminacy
-162.50	-161.83	0.34	0.66

Notes: The prior predictive probability of determinacy is 0.527.

indicator-specific component for core PCE as evident in the high standard deviation of its measurement error. These results imply that CPI and PCE provide better indicators of the latent concept of inflation, while core PCE, despite being promoted by Bernanke (2015), is less informative. In other words, while core PCE might better fit the Federal Reserve's behavior in isolation, the other inflation measures are more consistent with the New Keynesian model as a whole.

In sum, when taking the considered variants of the New Keynesian model, indeterminacy cannot be ruled out. What these model versions have in common though is that they all feature only one measure of inflation. In the next section we turn to an economy that explicitly differentiates between core and headline inflation rates.

6 An economy that distinguishes between core and headline inflation

Our baseline results on the issue of equilibrium determinacy were clearly dependent on the particular measure of inflation used in the estimation, thus leaving us with essentially the same dilemma that Taylor and Bernanke originally posed: should we measure inflation with CPI or Core PCE? In the previous section we have attempted to resolve this ambiguity by taking an econometric approach that draws on the DSGE-Factor analysis. Our estimation results there suggest that, for our period of interest, the concept of inflation in the basic New Keynesian model is more strongly correlated with broad indicators such as CPI and PCE than with narrower proxies such as core PCE. The immediate implication of this finding is that the indeterminate version of the model fits better than its determinate analogue.

However, the result that indeterminacy cannot be ruled out may hinge on the fact that the three-equation New Keynesian model features a single concept of inflation. Indeed, our DSGE-Factor approach forces the central bank to respond to the exact

Table 5: Parameter Estimation Results (DSGE-Factor)

	Determinacy		Indeterminacy	
	Mean	[5th pct, 95thpct]	Mean	[5th pct, 95th pct]
ψ_π	2.13	[1.29,3.13]	0.80	[0.61,0.98]
ψ_y	0.30	[0.07,0.65]	0.21	[0.05,0.45]
ρ_R	0.81	[0.72,0.88]	0.81	[0.73,0.88]
r^*	1.00	[0.45,1.67]	1.23	[0.57,2.00]
κ	0.74	[0.41,1.15]	1.00	[0.57,1.49]
γ^*	0.53	[0.45,0.62]	0.51	[0.44,0.58]
ρ_g	0.79	[0.68,0.87]	0.60	[0.45,0.74]
ρ_z	0.68	[0.50,0.85]	0.70	[0.54,0.84]
ρ_{gz}	0.14	[-0.33,0.70]	-0.31	[-0.74,0.15]
$M_{R\zeta}$			-0.31	[-1.53,1.17]
$M_{g\zeta}$			-1.77	[-2.59,-0.95]
$M_{z\zeta}$			0.30	[0.01,0.62]
σ_R	0.18	[0.13,0.25]	0.16	[0.12,0.21]
σ_g	0.19	[0.14,0.27]	0.28	[0.18,0.42]
σ_z	0.69	[0.50,0.94]	0.73	[0.53,1.00]
σ_ζ			0.18	[0.12,0.27]
λ_{CPI}	0.76	[0.55,0.93]	0.57	[0.37,0.79]
λ_{PCE}	0.79	[0.59,0.95]	0.59	[0.40,0.82]
$\lambda_{CorePCE}$	0.28	[0.07,0.52]	0.21	[0.06,0.40]
λ_{DEF}	0.53	[0.31,0.77]	0.41	[0.23,0.64]
σ_{CPI}	0.31	[0.20,0.43]	0.32	[0.22,0.43]
σ_{PCE}	0.18	[0.10,0.31]	0.18	[0.10,0.29]
$\sigma_{CorePCE}$	0.91	[0.72,1.14]	0.91	[0.72,1.14]
σ_{DEF}	0.71	[0.56,0.90]	0.70	[0.56,0.88]

Notes: The table reports posterior means and 90 percent probability intervals of the DSGE-Factor model parameters.

same measure of inflation (i.e. same combination of indicators) as the one that households consider in their consumption-spending decisions. But what (would be the consequences for equilibrium determinacy) if the Federal Reserve was actually focusing on core inflation in its conduct of monetary policy, as claimed by Bernanke (2015), while private-sector agents were looking at a different, broader, measure of inflation?

To address this question, we now turn toward a structural approach by employing an artificial economy that distinguishes explicitly between core and headline inflation, i.e. both inflation concepts simultaneously appear in the model.

6.1 Model

The artificial economy builds on Blanchard and Gali (2010) and Blanchard and Riggi (2013) who introduce imported oil into an otherwise standard New Keynesian model. We present the key aspects of the linearized model here and delegate the full description to the Appendix. Our exposition draws heavily on Blanchard and Gali (2010).

Oil is used by firms in production and by households in consumption. In particular, technology is given by a Cobb-Douglas production function that uses labor, n_t, and oil, m_t :

$$q_t = \alpha m_t + (1-\alpha) n_t \qquad 0 < \alpha < 1 \tag{7}$$

where q_t stands for gross output. Similarly, final consumption, c_t, is made up of domestically produced good, $c_{q,t}$, and imported oil, $c_{m,t}$:[18]

$$c_t = (1-\chi) c_{q,t} + \chi c_{m,t} \qquad 0 < \chi < 1. \tag{8}$$

Denoting the price of domestic output and the price of consumption by $p_{q,t}$ and $p_{c,t}$ respectively, and letting $p_{m,t}$ be the nominal price of oil, the following relationship arises between consumption-price inflation $\pi_{c,t}$ and domestic output-price inflation $\pi_{q,t}$:

$$\pi_{c,t} = \pi_{q,t} + \chi \Delta s_t \tag{9}$$

where s_t is the real price of oil, $s_t \equiv p_{m,t} - p_{q,t}$, which is exogenous. Following Aoki (2001) and Blanchard and Gali (2010), we interpret $\pi_{c,t}$ and $\pi_{q,t}$ as headline

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[18] If the shares α and χ are set to zero, the economy boils down to a simple three-equation New Keynesian model, similar to the one we have used in the previous sections.

and core inflation respectively. Utility maximization by the household yields the standard intertemporal optimality condition

$$c_t = E_t c_{t+1} + E_t z_{t+1} - R_t + E_t \pi_{c,t+1} + d_t - E_t d_{t+1} \tag{10}$$

and the intratemporal leisure-consumption trade-off

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t]. \tag{11}$$

Here R_t denotes the nominal interest rate, d_t is a discount-factor shock, z_t is a shock to the growth rate of technology, w_t denotes the nominal wage and φ stands for the inverse Frisch elasticity. The parameter $\gamma \in [0,1]$ captures the extent of real wage rigidity where larger values indicate higher degrees of rigidity. Notice in the household's Euler equation (10) that the model-consistent real interest rate that drives consumption dynamics involves headline consumption price inflation. Domestic firms are monopolistic competitors facing nominal rigidities à la Calvo. Firms' profit-maximizing pricing decisions result in the familiar aggregate New Keynesian Phillips curve which governs the dynamics of domestic-good sticky-price inflation (i.e. core inflation):

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \kappa \mu_t \tag{12}$$

where the slope coefficient $\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}$, ξ denotes the probability of not being able to reset prices, β represents the household's discount factor and μ_t is the price markup over nominal marginal costs. Cost minimization by firms gives rise to the following demand for oil:

$$m_t = q_t - \mu_t - s_t. \tag{13}$$

The requirement that trade be balanced (as oil is imported) delivers the following relationship between final consumption and domestic output:

$$c_t = q_t - \chi s_t + \eta \mu_t \tag{14}$$

where $\eta \equiv \frac{\alpha}{\mathcal{M}^P - \alpha}$ and $\mathcal{M}^P$ denotes the steady-state gross markup. Value added (i.e. GDP), denoted by y_t, is given by:

$$y_t = q_t + \frac{\alpha}{1-\alpha} s_t + \eta \mu_t. \tag{15}$$

Monetary policy follows a Taylor rule which reacts to inflation, deviations of GDP from the balanced-growth path and the growth rate of GDP, $gy_t \equiv y_t - y_{t-1} + z_t$:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi\{\omega\pi_{c,t} + (1-\omega)\pi_{q,t}\} + \psi_y y_t + \psi_{gy} gy_t] + \epsilon_{R,t}$$

where the monetary policy shock $\epsilon_{R,t}$ is i.i.d. $\mathsf{N}(0, \sigma_R^2)$. Notice that the central bank responds to a convex combination of headline and core inflation (with the parameter ω governing the relative weights; setting ω to zero implies that the central bank responds to core inflation only). As we have seen, the controversy between Taylor and Bernanke essentially boils down to the choice of the inflation measure in the monetary policy rule. By estimating ω, we will let the data speak as to whether the Federal Reserve was actually focusing on headline (Taylor, 2007) or core inflation (Bernanke, 2015). Lastly the structural disturbances s_t, z_t, and d_t are assumed to follow independent stationary $AR(1)$ processes:

$$s_t = \rho_s s_{t-1} + \varepsilon_{st} \qquad z_t = \rho_z z_{t-1} + \varepsilon_{zt} \qquad \text{and} \qquad d_t = \rho_d d_{t-1} + \varepsilon_{dt}.$$

We find that the *Taylor Principle* continues to hold in the Blanchard-Gali model.[19] In line with Carlstrom, Fuerst and Ghironi (2006), the indeterminacy condition is not dependent on any particular measure of inflation: as long as the central bank sets its response coefficient greater than unity to either headline or core inflation (or any convex combination of these two measures), such policy will ensure equilibrium determinacy.

6.2 Econometric strategy and results

To address typical identification issues, we calibrate a subset of the model parameters. We set the discount factor β to 0.99, the steady-state markup at ten percent, and the inverse of the labor-supply elasticity φ to one. Following the computations in Blanchard and Gali (2010) for their post-1984 sample period, we calibrate the shares of oil in production and consumption to $\alpha = 0.012$ and $\chi = 0.017$. Furthermore, we assume that shocks to the growth rate of technology are i.i.d., i.e. $\rho_z = 0$. We estimate the remaining parameters with Bayesian techniques. We use a loose Beta distribution centered at 0.5 to place an agnostic prior on both the wage-rigidity

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[19] Figure A5 in the Appendix shows the determinacy region for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity γ.

parameter, γ, and the weight on headline inflation in the monetary policy rule, ω. The other priors are similar to the ones we have used in the earlier sections and are reported in Table 6.

For our purpose, the main appeal of the Blanchard-Gali model is that it offers a micro-founded distinction between core and headline inflation which permits us to use both headline and core inflation data in the estimation. This approach will hopefully resolve some of the ambiguity that characterized our previous results.

At first, however, to maintain a continuity with our earlier findings, we estimate the new model using the exact same dataset with only three observables: the quarterly growth rate of real GDP per-capita, the Federal Funds rate and one of two alternative inflation rates, CPI or core PCE. Since we are initially using only one inflation series at a time, the weight ω in the Taylor rule is not well identified. Hence, when using CPI data, we calibrate this parameter to one, so that the central bank responds solely to headline inflation as in Taylor (2007). Similarly, when measuring inflation with core PCE, we set ω equal to zero, so that the monetary authority reacts to core inflation as Bernanke (2015) suggests. Table 6 reports the posterior estimates while Table 7 gives the log-data densities. In line with all our previous results, the estimation favors the indeterminate version of the model whenever we use CPI data, while it unambiguously selects determinacy under core PCE. Since we are using our original dataset, we can compare the marginal data densities of the augmented economy with the ones of the baseline model shown in Table 3 (the row labelled 'Output Growth'). The fact that these densities are of similar magnitude indicates that the additional micro-foundations of the Blanchard-Gali model are not rejected by the data.

We can now move on to our next exercise: treating simultaneously both headline and core inflation as observables. Hence, our dataset will now include four variables. This step enables us to properly identify the commodity-price shock as well as the weight ω in the policy rule. First, we measure headline and core inflation using PCE and core PCE data respectively. Then, we consider CPI as the proxy for headline inflation, while still using core PCE data to measure core inflation. Using CPI and core PCE data simultaneously to estimate the model helps us tackle the controversy between Taylor and Bernanke in a more direct way.[20] Table 7 (cf. the two rows

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[20] However, this combination of headline CPI and core PCE data is not ideal to measure the

Table 6: Priors and posteriors for DSGE parameters

		Posterior Mean [5th pct, 95th pct]					
		Three obs		Four obs		Five obs	
Name	Priors	CPI	CorePCE	PCE,CorePCE	CPI,CorePCE	PCE,CorePCE	CPI,CorePCE
ψ_π	G(1.1,0.5)	0.85 [0.63,0.98]	3.00 [2.01,4.14]	2.91 [1.94,4.03]	2.94 [1.99,4.03]	2.61 [1.57,3.86]	2.76 [1.69,4.03]
ψ_y	G(0.25,0.15)	0.22 [0.06,0.46]	0.28 [0.07,0.61]	0.30 [0.08,0.64]	0.30 [0.08,0.64]	0.11 [0.03,0.26]	0.07 [0.01,0.16]
ψ_{gy}	G(0.25,0.15)	0.47 [0.17,0.81]	0.28 [0.08,0.55]	0.29 [0.08,0.58]	0.30 [0.09,0.59]	0.62 [0.21,1.15]	0.69 [0.23,1.24]
ρ_R	B(0.7,0.1)	0.79 [0.70,0.86]	0.72 [0.61,0.81]	0.73 [0.62,0.82]	0.73 [0.63,0.82]	0.78 [0.66,0.88]	0.79 [0.70,0.87]
ω	B(0.5,0.2)	1	0	0.25 [0.08,0.47]	0.17 [0.06,0.32]	0.32 [0.10,0.59]	0.21 [0.06,0.41]
κ	G(0.5,0.1)	0.61 [0.45,0.80]	0.54 [0.39,0.72]	0.52 [0.38,0.70]	0.52 [0.37,0.69]	0.38 [0.25,0.53]	0.40 [0.25,0.57]
γ	B(0.5,0.2)	0.23 [0.07,0.46]	0.26 [0.07,0.50]	0.14 [0.04,0.28]	0.10 [0.03,0.19]	0.50 [0.30,0.68]	0.43 [0.24,0.60]
π^*	G(4,2)	2.92 [1.12,5.42]	1.96 [1.55,2.39]	1.99 [1.59,2.42]	2.02 [1.62,2.44]	1.95 [1.37,2.53]	1.99 [1.40,2.58]
r^*	G(2,1)	1.06 [0.43,1.85]	1.30 [0.72,1.99]	1.17 [0.64,1.75]	1.14 [0.60,1.75]	1.17 [0.59,1.84]	1.20 [0.59,1.87]
γ^*	N(0.5,0.1)	0.51 [0.39,0.64]	0.48 [0.38,0.60]	0.48 [0.37,0.59]	0.48 [0.38,0.59]	0.50 [0.37,0.64]	0.53 [0.39,0.66]
ρ_s	B(0.7,0.1)	0.70 [0.53,0.85]	0.70 [0.53,0.85]	0.88 [0.80,0.94]	0.90 [0.84,0.95]	0.88 [0.80,0.94]	0.91 [0.85,0.96]
ρ_d	B(0.7,0.1)	0.68 [0.52,0.81]	0.87 [0.79,0.93]	0.82 [0.72,0.91]	0.79 [0.68,0.89]	0.78 [0.66,0.88]	0.77 [0.64,0.87]
ρ_ν	B(0.7,0.1)	–	–	–	–	0.58 [0.39,0.81]	0.71 [0.50,0.90]
σ_z	IG(0.5,∞)	0.61 [0.46,0.80]	0.43 [0.34,0.55]	0.42 [0.33,0.54]	0.42 [0.33,0.54]	0.68 [0.52,0.89]	0.69 [0.53,0.89]
σ_R	IG(0.5,∞)	0.17 [0.12,0.24]	0.17 [0.12,0.23]	0.17 [0.12,0.24]	0.16 [0.12,0.23]	0.16 [0.11,0.24]	0.16 [0.11,0.22]
σ_s	IG(0.5,∞)	0.30 [0.15,0.59]	0.43 [0.16,1.00]	18.04 [14.1,22.9]	29.65 [23.2,37.9]	18.17 [14.08,23.36]	29.21 [22.77,37.40]
σ_d	IG(0.5,∞)	0.61 [0.26,1.08]	0.80 [0.53,1.21]	0.64 [0.42,0.99]	0.57 [0.39,0.84]	0.77 [0.48,1.17]	0.72 [0.45,1.07]
σ_ν	IG(0.5,∞)	–	–	–	–	0.62 [0.44,0.85]	0.80 [0.59,1.09]
σ_ζ	IG(0.5,∞)	0.20 [0.13,0.33]	–	–	–	–	–
$M_{z\zeta}$	N(0,1)	−0.36 [−0.63,−0.11]	–	–	–	–	–
$M_{R\zeta}$	N(0,1)	−0.17 [−1.12,0.90]	–	–	–	–	–
$M_{s\zeta}$	N(0,1)	0.01 [−0.71,0.76]	–	–	–	–	–
$M_{d\zeta}$	N(0,1)	−1.20 [−1.66,−0.87]	–	–	–	–	–

Notes: N stands for Normal, B Beta, G Gamma, and IG inverse gamma distribution. For each prior distribution, the parameters in parenthesis are the mean and standard deviation.

Table 7: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Det.	Indet.	Det.	Indet.
Three obs (CPI)	-93.98	-88.06	0	1
Three obs (CorePCE)	-61.14	-67.33	1	0
Four obs (PCE, CorePCE)	-111.55	-123.16	1	0
Four obs (CPI, CorePCE)	-126.01	-138.31	1	0
Five obs (PCE, CorePCE)	-156.30	-161.86	1	0
Five obs (CPI, CorePCE)	-174.66	-181.61	1	0

Notes: The prior predictive probability of determinacy is 0.51.

labelled "Four obs") shows that, no matter whether we measure headline inflation with PCE or CPI data, the whole posterior probability mass concentrates in the determinacy region. Looking at Table 6 (cf. the two columns labelled "Four obs"), the posterior mean of the weight on headline inflation in the policy rule, ω, is 0.25 with PCE data and 0.17 when we use CPI. Our estimation results therefore provide some empirical support for Bernanke's (2015) claim that the Federal Reserve was actively reacting to core inflation (as opposed to headline) during this period. Moreover, as anticipated, the parameters pertaining to the commodity-price shock are now better identified: the posterior mean estimates of ρ_s and σ_s are both significantly higher than the estimates we obtain when using only three observables.

A key parameter in the Blanchard and Gali (2010) model is the degree of real wage rigidity, γ. To sharpen the identification of this feature, we finally add real wage data, i.e. we ultimately employ five observables to estimate the model. We use observations on hourly compensation for the non-farm business sector for all persons as a measure of nominal wages. To get real wages, we then divide this proxy by, alternatively, the PCE or CPI price deflator (depending on how we measure headline inflation). To circumvent the issue of stochastic singularity, we add a labor supply

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theory's concepts of headline and core inflation: In the model, the core deflator is defined implicitly by excluding oil (the imported commodity) from the consumer's basket, without altering the weights of others goods. Yet, the CPI and PCE price index are assembled in different ways and attach different weights to different goods.

shock, ν_t.[21] As a result, the labor supply equation (11) becomes:

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t] + \nu_t. \tag{16}$$

Our main finding, that the data favors determinacy in this extended model, remains unchanged. The parameter estimate of γ becomes twice as large when we use real wage data, suggesting a substantial degree of real wage rigidity. This result contrasts with Blanchard and Riggi (2013) who find that real wages were highly flexible during the Great Moderation period. This divergence might be due to the different estimation strategy we employ. While Blanchard and Riggi (2013) adopt a limited-information approach that matches impulse responses to a commodity price shock in the DSGE model and in a structural VAR, we use a full-information Bayesian estimation with multiple shocks.

In summary, our estimation of the Blanchard-Gali model provides evidence that the Federal Reserve's monetary policy in the aftermath of the 2001 slump was responding mainly to core PCE and was sufficiently active to ensure equilibrium determinacy. These results line up with Bernanke's (2015) account.[22]

7 Concluding remarks

Using the Taylor rule as a benchmark for evaluating the Federal Reserve's interest-rate setting decisions, some commentators have argued that monetary policy was too accommodative during the 2002-2005 period. Along these lines, this paper starts by estimating a basic New Keynesian model of the U.S. economy for the time following the 2001 slump. Our assessment of the Federal Reserve's performance varies with the measure of inflation that is put into the model estimation. When measuring inflation with CPI or PCE, we find some support for the view that monetary policy during these years was extra easy and led to equilibrium indeterminacy. Instead, if the estimation involves core PCE, monetary policy comes out as active and the

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[21] As in Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), we normalize the labor supply shock such that it enters the household's intratemporal optimality condition with a unit coefficient. This procedure improves the identification of the standard deviation of the labor supply disturbance and facilitates the convergence of the MCMC algorithm.

[22] Likewise we have estimated the model with CPI and core CPI data. Furthermore, we have also used real-time data on per-capita real GDP growth rate, PCE and core PCE inflation. Our results remain robust and are reported in the Appendix.

evidence for indeterminacy dissipates. This divergence of results remains robust to several extensions. Our take is that each inflation series only provides an imperfect proxy for the model's concept of inflation. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the alternative empirical inflation series. Again, extra easy monetary policy as well as indeterminacy cannot be ruled out. This finding, however, may hinge on the fact that the model features a single concept of inflation. Thus, we finally move to an economy that explicitly distinguishes between headline and core inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active to comfortably rule out indeterminacy.

We chose to make these arguments while staying in relatively standard models. This choice enables to establish a bridge from existing research to our study which we believe is important given the short sample period that we consider. We specifically did not add asset markets to the model or in the estimation. Thus, in terms of possible extensions, it would be worthwhile to introduce housing into the model and in the econometric analysis. It is our intention to pursue these lines of research in the near future.

Acknowledgments

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Appendix A

This Appendix presents several extensions and robustness checks to our paper. Section A.1 describes the plain-vanilla New Keynesian model used in our baseline analysis, the solution method under indeterminacy as well as the data and estimation strategy. The section also discusses about the propagation of shocks, both fundamental and sunspots, and also the unconditional forecast error variance decomposition of shocks along with some extra results. Section A.2 presents various robustness checks. Finally, Section A.3 describes in details an artificial economy that distinguishes between core and headline inflation. The theoretical model in that section builds on Blanchard and Gali (2010) and Blanchard and Riggi (2013).

A.1 Framework of the structural analysis

Baseline New Keynesian Model

The artificial economy can be summarized in terms of the familiar linearized three equations of the plain-vanilla New Keynesian (NK) model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \tag{17}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \tag{18}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \varepsilon_{R,t.} \tag{19}$$

Here y_t stands for the output, R_t denotes the interest rate and π_t symbolizes the inflation rate. E_t represents the expectations operator. Equation (1) is the dynamic IS-relation reflecting an Euler equation in which τ can be interpreted as the intertemporal elasticity of substitution. Equation (2) describes the expectational Phillips curve where $0 < \beta < 1$ is the agents' discount factor. Finally, equation (3) describes monetary policy, i.e. a Taylor-type nominal interest rate rule in which $\psi_\pi$and ψ_y are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. $0 < \rho_R < 1$ is the usual smoothing term. $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. Fundamental disturbances involve exogenous shifts of the Euler equation captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1 \tag{20}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1. \tag{21}$$

The standard deviations for the demand and supply shocks are denoted by σ_g and σ_z. We allow for a non-zero correlation, $\rho_{g,z}$, between the demand and supply innovations.

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in the above New Keynesian model this can occur if the central bank only irresolutely responds to inflation changes. The precise analytical condition for indeterminacy corresponds to $\phi_\pi < 1 - \phi_y (1 - \beta) / \kappa$.

Rational-expectations solution under indeterminacy

Here we will outline the solution to this model which follows Lubik and Schorfheide (2003). Let us denote by η_t the vector of one-step ahead expectational errors. Moreover, define ϱ_t as the vector of endogenous variables and ε_t as vector of fundamental shocks. Then, the linear rational expectation system can be compactly written as

$$\Gamma_0(\theta)\varrho_t = \Gamma_1(\theta)\varrho_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t \tag{22}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$, and $\Pi(\theta)$ are appropriately defined coefficient matrices. We follow Sims' (2002) solution algorithm that was revisited by Lubik and Schorfheide (2003). This has the advantage of being general and explicit in dealing with expectation errors since it makes the solution suitable for solving and estimating models which feature multiple equilibria. In particular, under indeterminacy η_t will be a linear function of the fundamental shocks and the purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the LRE model entails

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{23}$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)$[23] are the coefficient matrices.[24] The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Accordingly, indeterminacy can manifest itself in one of two different ways: (i) pure extrinsic non-fundamental disturbances can affect model dynamics through endogenous expectation errors and (ii) the propagation of fundamental shocks cannot be uniquely pinned down and the multiplicity of equilibria affecting this propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following Lubik and Schorfheide (2004) we replace $\widetilde{M}$ with $M^*(\theta)+M$ and in the subsequent empirical analysis set the prior mean for M equal to zero. The particular solution employed in their paper selects $M^*(\theta)$ by using a least squares criterion to minimize the behavior of the model under determinacy and indeterminacy by assuming that it remains unchanged across the boundary. "Behavior" needs be described in some meaningful way and we follow them by choosing $M^*(\theta)$ such that the response of the endogenous variables to fundamental shocks, $\partial\varrho_t/\partial\varepsilon_t'$, are continuous at the boundary between the determinacy and the indeterminacy region.

Data

Figure 2 plots the three different measures of inflation, namely, CPI, PCE and core PCE. Headline inflation (both CPI and PCE) is more volatile than core inflation over the relevant period. In fact, headline inflation tends to be more volatile than core inflation measures that exclude or downweight the most volatile components, particularly in periods of persistent commodity price shocks.

Figure 3 plots the autocorrelation pattern (with five leads and lags) of the three different measures of inflation along with their cross-correlation with the growth rate of GDP and the Federal Funds rate. As seen in the figure, the cross-correlation patterns of headline inflation measures (CPI and PCE) on the one hand, and of core

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[23] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, they impose the normalization such that $M_\zeta = I$.

[24] Under determinacy, the solution boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

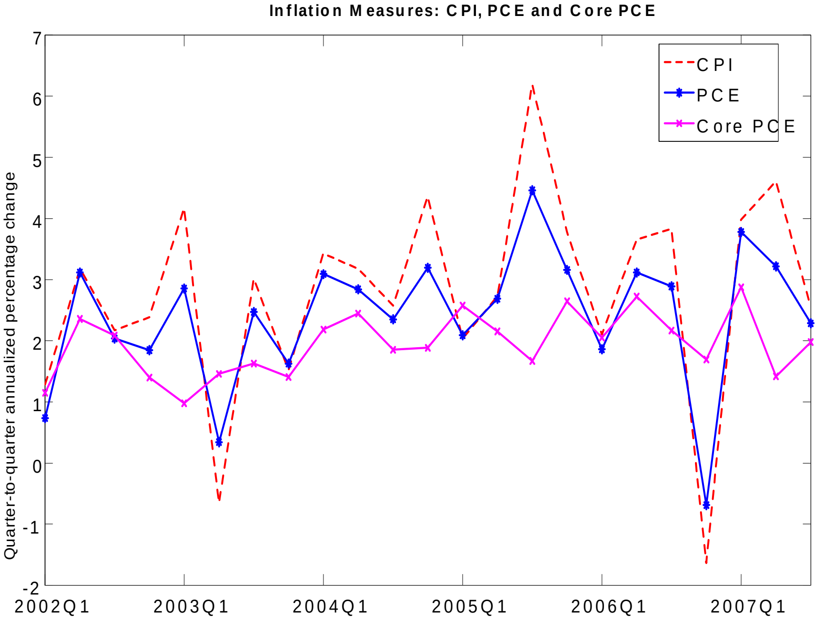

Figure 2: Inflation measures

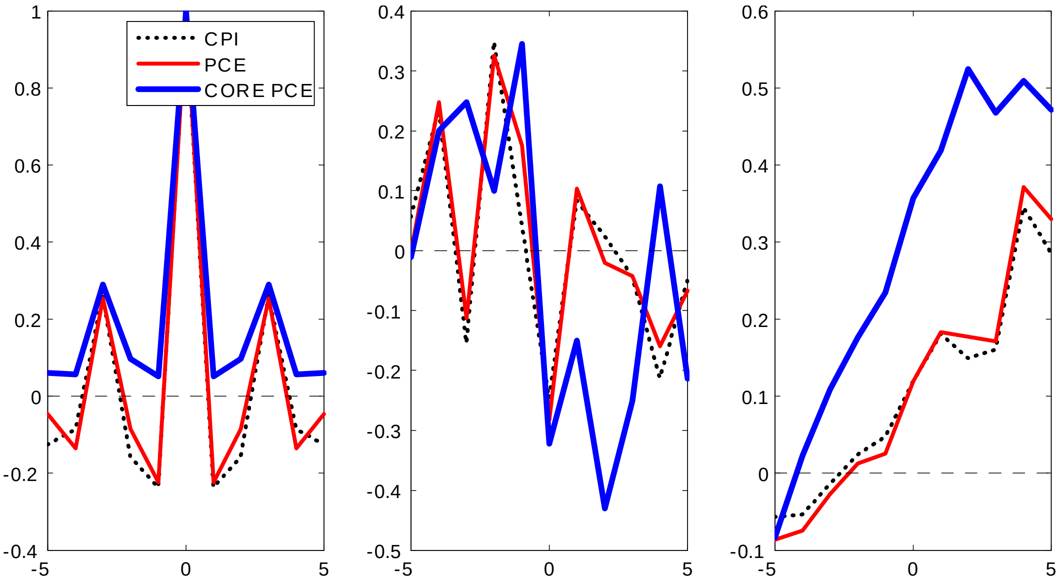

Figure 3: Correlation pattern of various inflation measures

PCE on the other hand, with the other two observables are notably different during our period of interest.

Estimation Strategy

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. In order to construct a likelihood function the DSGE model is turned into a Bayesian model. Toward that purpose we need to define a set of measurement equations that relate the elements of ϱ_t to a set of observables x_t which is given by

$$x_t = \begin{bmatrix} \gamma^* \\ r^* + \pi^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \varrho_t \tag{24}$$

where π^*, $r^*$and γ^* are annualized steady state inflation, annualized steady state real interest rates and quarterly steady state growth rate of real GDP per capita respectively.[25] Equations (7) and (8) provide a state-space representation for the linearized DSGE model that allows us to continue to apply standard Bayesian methodologies.

First priors are described by a density function of the form

$$p(\theta_S|S)$$

where $S \in \{D, I\}$ stands for a specific model, θ_S represents the parameter of the model S, $p(.)$ stands for probability density function. Next, the likelihood function describes the density of the observed data:

$$\mathcal{L}(\theta_S|X_T, S) \equiv p(X_T|\theta_S, S)$$

where X_T are the observations until period T. By using Bayes theorem we can combine the prior density and the likelihood function to get the posterior density:

$$p(\theta_S, X_T, S) = \frac{p(X_T|\theta_S, S)p(\theta_S|S)}{p(X_T, S)}$$

where $p(X_T|S)$ is the marginal marginal density of the data conditional on the model which is given by

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[25] When using HP-filtered data to measure real activity γ^* is set to zero.

$$p(X_T|S) = \int_{\theta_S} p(\theta_S; X_T) d\theta_S.$$

Finally, the posterior kernel corresponds to the numerator of the posterior density:

$$p(\theta_S|X_T, S) \propto p(X_T|\theta_S, S) p(\theta_S|S) \equiv \kappa(\theta_S|X_T, S).$$

We maximize the posterior kernel and find the posterior mode in the two regions of the parameter space using Sims' `csminwel`. The inverse Hessian is calculated at the posterior mode.[26] Next for each region of the parameter space we estimate the likelihood function with the help of the Kalman filter and generate 250,000 draws with a random-walk Metropolis Hastings algorithm. The algorithm is tuned to achieve 25 to 30 percent acceptance rate. Half of the parameter draws are discarded to ensure convergence and the remaining draws are used to generate our results. The marginal data densities for the two regions are computed with Geweke's (1999) modified harmonic mean estimator.

Propagation of Shocks

Here we study the propagation of sunspots as well as of fundamental shocks. Figure 4 depicts the impulse responses of output, inflation and the nominal interest rate under determinacy (the model being estimated using core PCE inflation) while Figure 5 graphs the responses under indeterminacy (using CPI inflation). Solid lines track the posterior means while the shaded areas cover the 90 percent probability intervals.

Let us begin with the model's reaction to sunspots. The bottom panels of Figure 4 display the reaction to an inflationary sunspot shock. The impulse responses show that the shock reduces the expected real return which subsequently increases current consumption and hence output. The Phillips curve then translates this into a rise of inflation thereby creating a self-fulfilling cycle: higher inflation expectations lead to higher actual inflation.

Fundamental shocks follow next. The first and second rows of Figures 4 and 5 plot the responses to monetary policy and cost-push shocks. The patterns of the key

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[26] For our rolling windown approach, if for a particular sample a region of the parameter space does not have a local mode, we use the inverse Hessian obtained from the nearest previous sample for that region.

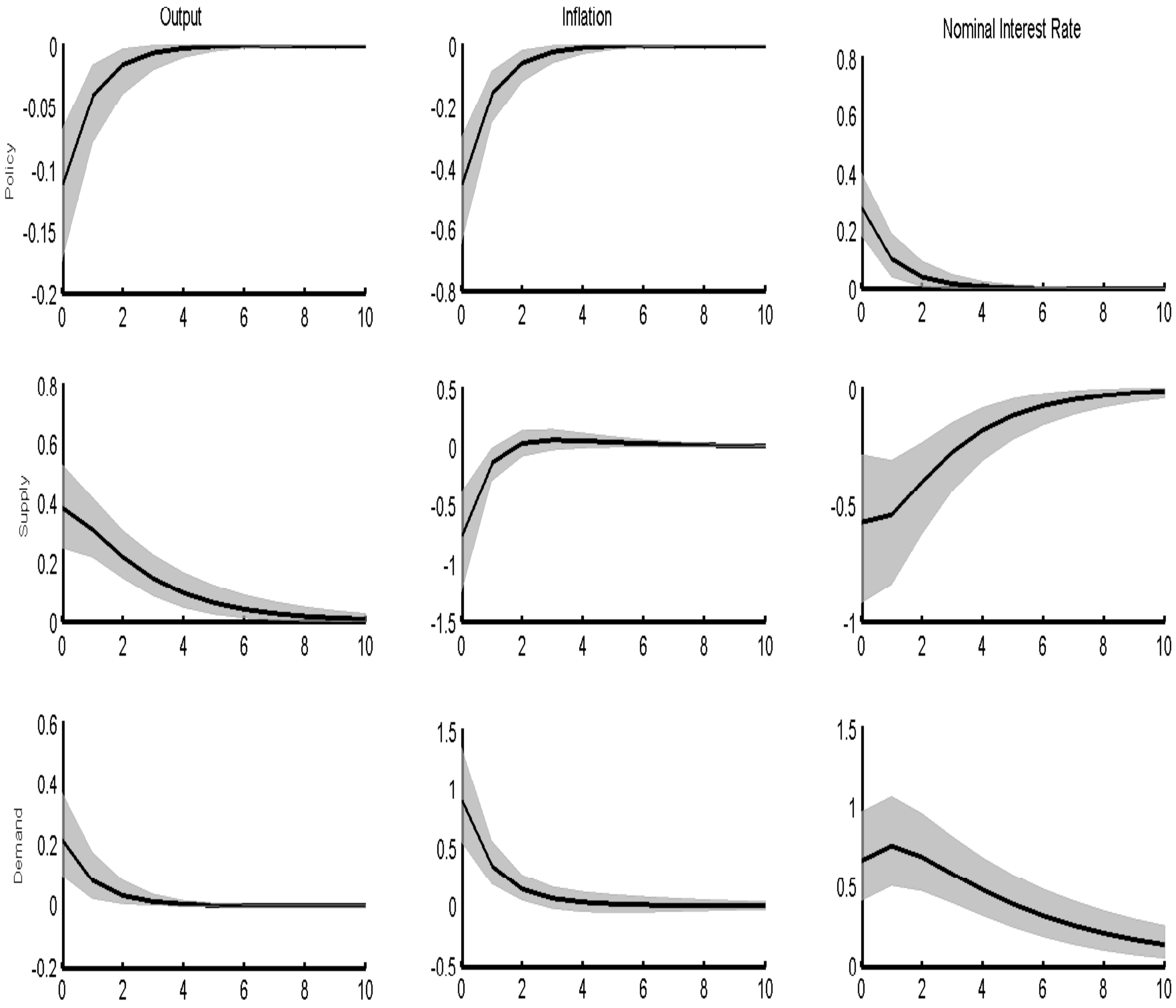

Figure 4: Impulse responses under determinacy from the model estimated over the period 2002:I - 2007:III using Core PCE inflation.

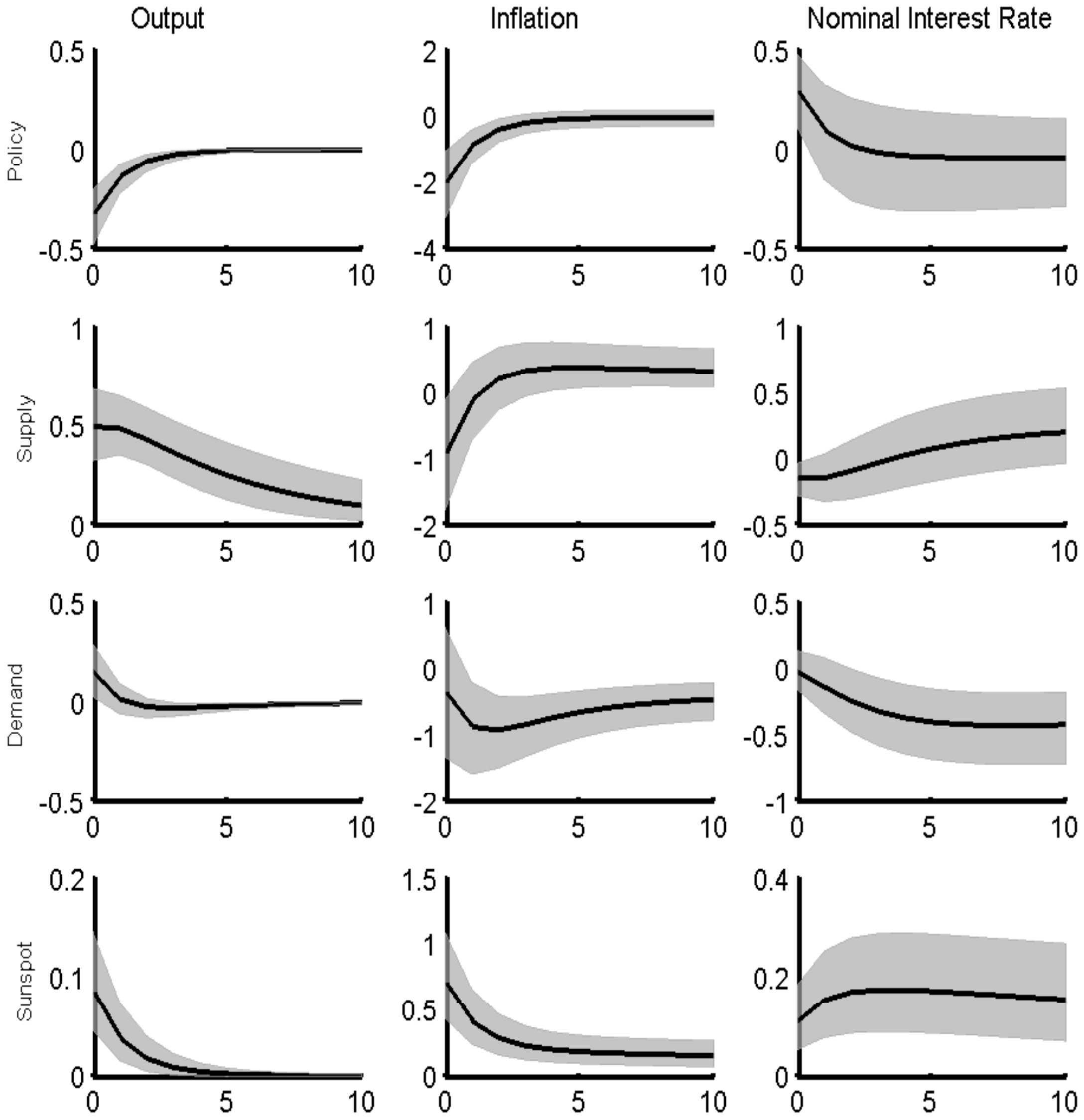

Figure 5: Impulse responses under indeterminacy from the model estimated over the period 2002:I - 2007:III using CPI inflation.

Table 8: Variance Decomposition

	Variables\Shocks	ε_R	ε_g	ε_z	ζ
CPI (Indet.)	y	9.44	7.47	82.37	0.71
	π	21.82	54.53	16.45	7.2
	R	1.29	74.28	16.24	8.20
Core PCE (Det.)	y	1.99	83.57	14.43	-
	π	39.25	31.03	29.72	-
	R	7.51	69.37	23.12	-

model variables look similar for both the indeterminate and the determinate versions of the model. This contrasts with the responses to aggregate demand shocks. While at impact we observe an increase of output in both regimes, the responses of inflation are quite different. The determinate model's response of inflation is conventional: it increases which is matched by the central bank tightening its policy – the nominal interest rate rises. However, inflation falls under indeterminacy which appears to reflect the alternative propagation of fundamental shocks in model versions that feature indeterminacy. These propagation dynamics are captured by the elements of the matrix $\mathbf{M}$. In particular, the posterior estimate of $M_{G\zeta}$ is far from zero at -1.99 and as such qualitatively alters the dynamics of a demand shock.

Variance Decomposition

The unconditional forecast error variance decomposition at the posterior mean for output (deviations from trend), inflation and interest rates are reported in Table 8. The ε_{gt} and ε_{zt} shocks are orthogonalized such that the cost-push shock only affects ε_{zt} and the demand shock affects both ε_{gt} and ε_{zt}. The rationale is that demand shocks will affect the labor supply decisions, hence, the firms' cost function.

The main message we take from this exercise is that in the indeterminacy regime, cost-push shocks cause over 80 percent of output fluctuations whereas in determinacy case aggregate demand disturbances are the main driver of aggregate fluctuations. Sunspot shocks play only a marginal role with the most significant contribution being eight percent in explaining the variance decomposition of the policy rate. This is in line with the results reported above. In conclusion, using different measures of inflation results in drastically different interpretations of the potential causes of

Table 9: Benchmark Model versus Determinate Model with Habit

Inflation measure	Specification	Log-data density Det.	 Indet.	Probability
CPI	Benchmark	-95.48	-93.28	0.87
	Habit	-95.18		0.13
PCE	Benchmark	-85.42	-85.75	0.26
	Habit	-84.70		0.74
Core PCE	Benchmark	-64.60	-71.58	0
	Habit	-62.73		1

output fluctuations.

Habit formation

It is well known that the determinate New Keynesian model features a poor internal propagation mechanism while the model potentially exhibits richer dynamics under indeterminacy. Accordingly, the posterior mass might be biased toward the indeterminacy region.[27]Hence, following Lubik and Schorfheide (2004), we extend the model by adding consumption habits. Log-data densities for the habit specification conditional on determinacy are reported in Table 9: the habit model fits better than the no-habit specification restricted to determinacy. The last column of Table 9 compares the respective posterior probabilities of the baseline model under indeterminacy and the habit model under determinacy. For example, when measuring inflation with CPI, the data favors the benchmark model under indeterminacy over the habit specification restricted to determinacy. Again, the results carry over from the benchmark exercise i.e. Table 2 in the paper.

Estimation Results under PCE

According to the semi-annual monetary policy reports to Congress (Humphrey-Hawkins reports), the Federal Reserve has also been looking at headline PCE inflation from 2000 to 2004. Hence, we employ PCE to measure inflation while estimating our model and the evidence is mixed at best, the probability of determinacy is 0.58. Phrased alternatively, we can neither exclude nor rule in indeterminacy. Table 10

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[27] See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007).

Table 10: Parameter Estimation Results

	PCE (Indeterminacy)		PCE (Determinacy)	
	Mean	90-percent interval	Mean	90-percent interval
ψ_π	0.82	[0.58,0.97]	2.13	[1.30,3.09]
ψ_y	0.21	[0.05, 0.45]	0.27	[0.06,0.59]
ρ_R	0.83	[0.74, 0.90]	0.85	[0.77,0.91]
π^*	3.36	[1.30, 6.21]	2.24	[1.63,2.84]
r^*	1.26	[0.55, 2.10]	1.17	[0.56,1.90]
κ	0.73	[0.40, 1.16]	0.75	[0.39,1.22]
τ^{-1}	1.69	[1.02 2.50]	1.83	[1.09,2.72]
ρ_g	0.60	[0.45, 0.73]	0.79	[0.70,0.86]
ρ_z	0.81	[0.70, 0.90]	0.62	[0.46,0.78]
ρ_{gz}	-0.27	[-0.72, 0.25]	0.64	[0.23,0.92]
$M_{R\zeta}$	-0.16	[-1.51, 1.40]		
$M_{g\zeta}$	-1.91	[-2.80, -1.01]		
$M_{z\zeta}$	0.43	[0.09, 0.81]		
σ_R	0.15	[0.12, 0.20]	0.16	[0.12,0.21]
σ_g	0.26	[0.17, 0.38]	0.19	[0.14,0.27]
σ_z	0.69	[0.50, 0.94]	0.70	[0.51,0.96]
σ_ζ	0.19	[0.12, 0.28]		

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters. The posterior summary statistics are calculated from the output of the Metropolis Hastings algorithm.

reports posterior estimates of the model parameters under both determinacy and indeterminacy.

A.2 Sensitivity analysis

We now investigate the sensitivity of our results in various directions. The robustness checks involve (i) estimating the policy parameters only, (ii) alternative priors for ψ_π, (iii) alternative measure of inflation, (iv) serially correlated monetary policy shocks, and (v) trend inflation.

Estimating the policy parameters only

As a further robustness check to address the small sample issue, we only estimate the policy parameters over the 2002-2007 period. More concretely, we exclusively estimate the three Taylor rule parameters along with the standard deviation of the monetary policy shock (as well as the sunspots related parameters, i.e. the **M**s and

σ_ζ, for the indeterminacy version of the model). As for the other parameters, all were calibrated at the posterior means obtained from estimating the determinate model over the period 1991:II to 2001:IV. The reason for beginning right after the 1990-91 recession is closely connected to Figure 1: it comfortably rules out indeterminacy even for "short" periods. Table 11 reports strong evidence for indeterminacy not only when we measure inflation with CPI but also with PCE. However, as before, the posterior probability puts all its weight on determinacy when inflation is measured using Core PCE.

Alternative priors

One possible drawback to using a small sample size is that the prior might *speak louder* than the data. To make our empirical analysis transparent, the priors we employ in our baseline estimation (Table 1) were set identical to the ones used by Lubik and Schorfheide (2004). Accordingly, our baseline specification implies a prior probability of determinacy equal to 0.53. To assess the sensitivity of our results to the priors, we alter the prior distribution for the key parameter that drives indeterminacy. Specifically, we change the prior mean of ψ_π from 1.1 to 1.3 and in doing so we ramp up the prior probability of determinacy from 0.53 to 0.7. Thus, the indeterminacy test will now find it harder to favor indeterminacy. Table 11 reports the posterior probabilities of (in-)determinacy under this alternative prior for each measure of inflation. The results remain largely unaltered. For example, the odds of indeterminacy versus determinacy are still five to one when estimating the model using CPI inflation. This finding provides some further support for our results.

GDP deflator

While not mentioned in the Humphrey-Hawkins reports to have informed Federal Reserve's policy deliberations during the 2000s, we lastly re-do the analysis with the GDP deflator as the inflation measure (as in Smets and Wouters, 2007). Then, the log-data densities are very close at -73.26 for determinacy and -74.16 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 71% versus 29% and again we cannot rule out indeterminacy.

Serially correlated monetary policy shocks

Our findings so far have lend some support to the conjecture that monetary

Table 11: Determinacy versus Indeterminacy (Robustness)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	Policy parameters only	-99.97	-95.50	0.01	0.99
	Alternative prior for ψ_π	-95.04	-93.58	0.19	0.81
	CBO output gap	-97.89	-95.85	0.12	0.88
	Output growth	-93.29	-89.58	0.02	0.98
	AR(1) policy shocks	-89.51	-85.68	0.02	0.98
	Trend Inflation with standard TR	-91.38	-87.13	0.02	0.98
	Trend Inflation with alternative TR	-85.16	-83.25	0.13	0.87
PCE	Policy parameters only	-99.36	-88.79	0.07	0.93
	Alternative prior for ψ_π	-85.04	-85.98	0.72	0.28
	CBO output gap	-88.08	-88.18	0.53	0.47
	Output growth	-82.89	-81.80	0.25	0.75
	Real-time data	-83.32	-83.06	0.44	0.56
	AR(1) policy shocks	-77.59	-77.25	0.42	0.58
	Trend Inflation with standard TR	-81.54	-82.01	0.62	0.38
	Trend Inflation with alternative TR	-75.79	-77.41	0.83	0.17
Core PCE	Policy parameters only	-63.49	-69.49	1	0
	Alternative prior for ψ_π	-64.47	-71.74	1	0
	CBO output gap	-68.53	-73.63	0.99	0.01
	Output growth	-62.54	-67.58	1	0
	Real-time data	-65.85	-70.24	0.99	0.01
	AR(1) policy shocks	-53.91	-62.09	1	0
	Trend Inflation with standard TR	-61.13	-64.53	0.97	0.03
	Trend Inflation with alternative TR	-56.68	-60.75	0.98	0.02

policy was extra easy following the 2001 recession. Our exercise interprets this view as a reduction in the Federal Reserve's systematic response to the inflation gap (thereby leading to indeterminacy of the rational expectations equilibrium). However, alternatively, extended periods of low interest rates could also arise due to discretionary deviations from the monetary policy rule (see also Rudebusch, 2002, Groshenny, 2013, and Belongia and Ireland, 2016). To assess the robustness of our interpretation, we next allow the monetary policy shocks to be serially correlated. Specifically, we assume that the policy shocks follow the AR(1) process

$$\epsilon_{R,t} = \rho_{\epsilon_R}\epsilon_{R,t-1} + \upsilon_t \qquad 0 \leq \rho_{\epsilon_R} < 1$$

where υ_t is $i.i.d.\mathsf{N}(0, \sigma_\upsilon^2)$ and jointly estimate the autocorrelation parameter, ρ_{ϵ_R}, and the standard deviation of the shock, σ_υ^2, along with the other parameters of the model.[28] Table 11 confirms that our results remain unaltered: we still cannot rule out passive responsiveness to inflation and thereby the possibility of indeterminacy.

Trend inflation

So far, our analysis had assumed that the U.S. economy is reasonably approximated by the standard New Keynesian model linearized around a zero inflation steady state. However, the Federal Reserve's implicit inflation target as well as the average inflation rate during the Great Moderation period was around two to three percent (depending on the chosen price index). Thus, we extend the baseline model to allow for positive trend inflation. This extension becomes meaningful for at least two further reasons as (i) positive trend inflation alters the determinacy properties of the model and (ii) as the determinate plain-vanilla New Keynesian model features a poor internal propagation mechanism, the posterior mass might be biased toward the indeterminacy region[29], however, trend inflation generates more endogenous persistence of inflation and output even in the determinacy case.

The estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian model (GNK). Unlike Ascari and Sbordone, we assume deterministic growth and we replace their labor supply disturbance by a discount factor shock, d_t, as our stand-in for demand shocks. Also, our Taylor rule involves responses to the

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[28] The AR(1) coefficient of the policy shock follows a beta prior with mean 0.5 and standard deviation 0.2.

[29] See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007).

output gap instead of log-deviations from the steady state. This then makes our setup similar to Hirose, Kurozumi and Van Zandweghe (2015).[30] The log-linearized (detrended) model consists of the Euler equation

$$y_t = E_t y_{t+1} - (R_t - E_t \pi_{t+1}) + d_t - d_{t+1}$$

where we have set the intertemporal rate of substitution equal to one to make the model compatible with balanced growth as well as the Taylor rule

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1$$

to capture the central bank's behavior. The supply side is no longer summarized by a single Phillips curve expression but rather it consists of the following three equations for inflation, an auxiliary variable, ψ_t, and price dispersion, s_t:

$$\begin{aligned}
\pi_t &= \varkappa E_t \pi_{t+1} + \vartheta \left[\varphi s_t + (1+\varphi) y_t - (1+\varphi) z_t\right] - \varpi E_t \psi_{t+1} + \varpi d_t \\
\psi_t &= (1 - \xi\beta\pi^{\varepsilon}) \left[\varphi s_t + (1+\varphi)(y_t - z_t) + d_t\right] + \xi\beta\pi^{\varepsilon} \left[E_t \psi_{t+1} + \varepsilon E_t \pi_{t+1}\right] \\
s_t &= \varepsilon \xi \pi^{\varepsilon} \left(1 - \frac{1 - \xi\pi^{\varepsilon}}{\pi - \xi\beta\pi^{\varepsilon}}\right) \pi_t + \xi \pi^{\varepsilon} s_{t-1}
\end{aligned}$$

where $\vartheta \equiv (1 - \xi\pi^{\varepsilon-1})(1 - \xi\beta\pi^{\varepsilon})/\xi\pi^{\varepsilon-1}$, $\varkappa \equiv \beta\left[1 + \varepsilon(\pi - 1)(1 - \xi\pi^{\varepsilon-1})\right]$, and $\varpi \equiv \beta(1-\pi)(1-\xi\pi^{\varepsilon-1})$. The term ξ denotes the Calvo-parameter and β stands in for the steady state discount factor. We set the Frisch elasticity of labor supply, φ, equal to one and calibrate the elasticity of substitution $\varepsilon = 11$ such that the steady state mark-up equals ten percent.

As mentioned above, the GNK model exhibits richer dynamics and the usual Taylor principle ($\psi_\pi > 1$) is no longer a sufficient condition for local determinacy of equilibrium. Due to the higher-order dynamics of the GNK model and our assumption of a unit Frisch elasticity of labor supply, it is not possible to analytically derive the indeterminacy conditions. To continue solving the model via Lubik and Schorfheide's (2004) continuity solution (where $\mathbf{M}^*(\theta)$ is selected such that the responses of the endogenous variables to the fundamental shocks are continuous at the boundary between the determinacy and indeterminacy region) one needs to resort to numerical methods. In particular, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule.[31]

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[30] They, however, assume firm-specific labor as well as stochastic growth.

[31] See also Justiniano and Primiceri (2008).

Table 12: Priors and posteriors for DSGE parameters (Trend Inflation)

				Posterior Mean [5th pct, 95th pct]			
				Standard TR		Alternative TR	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Ind.	CorePCE Det.	CPI Ind.	CorePCE Det.
ψ_π	$\mathbb{R}^+$	Gamma	1.40 (0.50)	0.93 [0.82,1.00]	2.65 [1.66,3.77]	0.94 [0.84,1.00]	2.53 [1.64,3.55]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.25 [0.07,0.54]	0.35 [0.09,0.72]	0.25 [0.07,0.52]	0.30 [0.07,0.65]
ψ_{gy}	$\mathbb{R}^+$	Gamma	0.25 (0.15)			0.33 [0.11,0.59]	0.35 [0.10,0.70]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.73 [0.63,0.82]	0.76 [0.64,0.85]		
ρ_{R1}	$\mathbb{R}$	Normal	1.00 (0.20)			1.09 [0.85,1.32]	1.13 [0.88,1.36]
ρ_{R2}	$\mathbb{R}$	Normal	0.00 (0.20)			-0.35 [-0.55,-0.14]	-0.35 [-0.57,-0.12]
π	$\mathbb{R}^+$	Gamma	2.50 (1.00)	2.21 [1.03,3.73]	1.89 [1.52,2.26]	2.12 [1.02,3.49]	1.93 [1.54,2.35]
r	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.01 [0.45,1.71]	1.27 [0.69,1.91]	0.93 [0.40,1.59]	1.22 [0.63,1.87]
γ	$\mathbb{R}$	Normal	0.50 (0.10)	0.49 [0.44,0.53]	0.55 [0.48,0.62]	0.49 [0.45,0.54]	0.55 [0.48,0.62]
ξ	[0,1)	Beta	0.70 (0.10)	0.26 [0.19,0.34]	0.64 [0.51,0.74]	0.31 [0.23,0.40]	0.65 [0.53,0.74]
ρ_d	[0,1)	Beta	0.70 (0.10)	0.71 [0.54,0.85]	0.82 [0.72,0.90]	0.73 [0.56,0.87]	0.82 [0.73,0.90]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.70 [0.58,0.82]	0.76 [0.61,0.89]	0.69 [0.55,0.81]	0.75 [0.59,0.88]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-0.74 [-1.82,0.42]		-0.84 [-1.97,0.39]	
$M_{d\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-1.47 [-2.57,0.13]		-1.15 [-2.62,0.61]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	1.98 [1.36,2.65]		1.68 [1.01,2.39]	
σ_R	$\mathbb{R}^+$	IG	0.50 (∞)	0.19 [0.13,0.26]	0.15 [0.11,0.21]	0.18 [0.12,0.26]	0.14 [0.10,0.20]
σ_d	$\mathbb{R}^+$	IG	0.50 (∞)	0.36 [0.15,0.82]	0.74 [0.51,1.09]	0.25 [0.13,0.47]	0.70 [0.46,1.04]
σ_z	$\mathbb{R}^+$	IG	0.50 (∞)	0.46 [0.35,0.59]	0.56 [0.37,0.85]	0.48 [0.36,0.63]	0.58 [0.48,0.62]
σ_ζ	$\mathbb{R}^+$	IG	0.50 (∞)	0.27 [0.15,0.47]		0.31 [0.16,0.56]	

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,\varsigma\right) \propto \sigma^{-\upsilon-1} e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 2$ and $\varsigma = 0.282$.

Table 13: Posteriors for DSGE parameters estimated using PCE (Trend Inflation)

	Posterior Mean [5th pct, 95th pct]			
Name	Stand. TR Indeterminacy	Stand. TR Determinacy	Alt. TR Indeterminacy	Alt. TR Determinacy
ψ_π	0.94 [0.83,1.00]	2.17 [1.46,3.03]	0.95 [0.83,1.04]	1.97 [1.31,2.83]
ψ_y	0.26 [0.07,0.55]	0.28 [0.07,0.59]	0.28 [0.07,0.60]	0.25 [0.06,0.55]
ψ_{gy}			0.36 [0.11,0.67]	0.42 [0.12,0.81]
ρ_R	0.73 [0.64,0.81]	0.79 [0.66,0.87]		
ρ_{R1}			1.11 [0.87,1.35]	1.20 [0.95,1.43]
ρ_{R2}			-0.37 [-0.58,-0.15]	-0.40 [-0.62,-0.17]
π^*	2.13 [1.01,3.49]	2.19 [1.58,2.81]	2.09 [1.02,3.43]	2.23 [1.57,2.91]
r^*	1.11 [0.51,1.79]	1.06 [0.51,1.70]	1.05 [0.49,1.69]	1.03 [0.49,1.66]
γ	0.49 [0.45,0.53]	0.55 [0.48,0.63]	0.51 [0.46,0.59]	0.55 [0.48,0.63]
θ	0.30 [0.22,0.39]	0.49 [0.36,0.61]	0.40 [0.26,0.62]	0.50 [0.38,0.62]
ρ_d	0.69 [0.53,0.84]	0.84 [0.76,0.92]	0.75 [0.58,0.89]	0.84 [0.75,0.91]
ρ_z	0.70 [0.59,0.81]	0.78 [0.64,0.89]	0.70 [0.56,0.83]	0.76 [0.61,0.88]
$M_{R\zeta}$	-0.29 [-1.55,1.06]		-0.43 [-1.90,1.02]	
$M_{d\zeta}$	-1.45 [-2.79,0.30]		-0.89 [-2.66,0.96]	
$M_{z\zeta}$	2.34 [1.62,3.14]		1.78 [-0.17,2.85]	
σ_R	0.17 [0.13,0.24]	0.18 [0.13,0.27]	0.17 [0.12,0.24]	0.17 [0.11,0.25]
σ_d	0.30 [0.14,0.59]	0.73 [0.48,1.08]	0.28 [0.14,0.54]	0.65 [0.40,0.99]
σ_z	0.46 [0.35,0.59]	0.58 [0.40,0.85]	0.55 [0.37,0.90]	0.61 [0.41,0.92]
σ_ζ	0.30 [0.16,0.53]		0.34 [0.16,0.65]	

As before we use the growth rate of GDP, the Federal Funds rate and the three measures of inflation sequentially. Table 11 provides the marginal data densities along with the posterior model probabilities while Table 12 reports the priors and the posterior estimates.[32] The emerging results parallel our earlier findings. When basing the estimation on CPI, the U.S. economy was very likely in an indeterminacy region, however, the opposite holds, again, under core PCE. Notably, as mentioned above, the posterior estimate of trend inflation under CPI is higher than under core PCE while the Calvo parameter is smaller implying more flexible prices under CPI.

Lastly, we investigate the sensitivity of our results to Coibion and Gorodnichenko's (2011) Taylor rule that allows for interest rate smoothing of order two, as well as a response to inflation, output growth, and the output gap. Coibion and Gorodnichenko document a shift in the Federal Reserve's response from output gap to output growth for the Great Moderation period and also show that the two lags of interest rate are required to remove the serial correlation in the monetary policy shocks. Thus, we re-estimate the GNK model by replacing the standard policy rule with the following formulation:

$$R_t = \rho_{R_1} R_{t-1} + \rho_{R_2} R_{t-2} + (1 - \rho_{R_1} - \rho_{R_2})(\psi_\pi \pi_t + \psi_y [y_t - z_t] + \psi_{gy} \Delta y_t) + \epsilon_{R,t}.$$

Even though the posterior probabilities of indeterminacy are now lower across the board, Table 11 shows that the only case in which we can confidently rule out the possibility of indeterminacy is when we use core PCE. Apart from the parameter estimates of the responsiveness to output growth, ψ_{gy}, and the interest rate lags, ρ_{R1} and ρ_{R2}, all other parameter estimates remain essentially unchanged.

Table 13 displays the estimation results under PCE for both the standard Taylor rule and the alternative rule following Coibion and Gorodnichenko (2011). Most of our parameter estimates are in line with the results from the previous table.

Estimation Results under Core CPI

Table 14 reports the marginal data densities for the benchmark specification as well as the various robustness exercises for the model estimated using core CPI. In line with the results of the model when estimated using core PCE, determinacy prevails in all cases.

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[32] The prior predictive probability is 0.539 for the standard rule and 0.503 for the alternative rule

Table 14: Determinacy versus Indeterminacy (Core CPI)

		Log-data density		Probability	
Inflation measure	Specification	Det.	Indet.	Det.	Indet.
Core CPI	Benchmark	-62.06	-69.64	1	0
	Policy parameters only	-63.64	-66.52	0.95	0.05
	Alternative prior for ψ_π	-61.61	-70.07	1	0
	CBO Output Gap	-65.60	-71.61	1	0
	Output Growth	-59.73	-64.68	0.99	0.01
	AR(1) policy shocks	-51.30	-58.78	1	0
	Trend Inflation with standard TR	-58.78	-62.40	0.97	0.03
	Trend Inflation with alternative TR	-53.88	-57.64	0.98	0.02

A.3 A micro-founded distinction between core and headline inflation

The artificial economy is a variant of Blanchard and Gali (2010) and Blanchard and Riggi (2013) and so our description of the model below draws heavily from their exposition. It is a New Keynesian economy with a commodity product which they interpret as oil. This model offers a micro-founded setup that naturally features various inflation rates. The economy consists of monopolistically competitive wholesale firms who produce differentiated goods using labor and oil. These goods are bought by perfectly competitive firms (retailers) that weld them together into the final good that can be consumed. People rent out their labor services on competitive markets. Firms and people are price takers on the market for oil.

People

The representative agent's preferences depend on consumption, C_t, and hours worked, H_t, and they are represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t u(C_t, N_t) \qquad 0 < \beta < 1$$

which the agent acts to maximize. Here, E_0 represents the expectations operator. The term d_t stands for a shock to the discount factor, β, which follows the stationary autoregressive process

$$\ln d_{t+1} = \rho_d \ln d_t + \epsilon_{d,t+1}$$

where $\epsilon_{d,t+1}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_d. The period utility is additively separable in consumption and hours worked and it takes on the functional form

$$u(C_t, N_t) = \ln C_t - \phi \nu_t \frac{N_t^{1+\varphi}}{1+\varphi} \qquad \phi > 0,\ \varphi \geq 0.$$

Logarithmic utility is the only additive-separable form consistent with balanced growth. The term φ is the inverse of the Frisch labor supply elasticity and ϕ governs the disutility of working in steady state. ν_t denotes a shock to the disutility of labor and it follows

$$\ln \nu_{t+1} = \rho_\nu \ln \nu_t + \epsilon_{\nu,t+1}$$

where $\epsilon_{\nu,t}$ is $N(0, \sigma_\nu^2)$. The overall consumption basket, C_t, is a Cobb-Douglas bundle of output of domestically produced goods, $C_{q,t}$, and the imported oil, $C_{m,t}$. In particular, we assume that

$$C_t = \chi^{-\chi}(1-\chi)^{-(1-\chi)} C_{m,t}^{\chi} C_{q,t}^{1-\chi} \qquad 0 < \chi < 1.$$

The parameter χ equals the share of energy in total consumption.

Retail firms combine the domestically-produced intermediate varieties $C_{q,t}(i)$, where $i \in [0,1]$, using a Dixit-Stiglitz aggregator to produce the consumption bundle $C_{q,t}$:

$$C_{q,t} = \left(\int_0^1 C_{q,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here, the term ε measures the elasticity of demand for each intermediate good.

The agent sells labor services to the wholesale firms at the nominal wage W_t and has access to a market for one-period riskless bonds, B_t, at the interest rate R_t. Any generated profits, Π_t, flow back to the representative household. Thus, the period budget is constrained by

$$W_t N_t + R_t B_{t-1} + \Pi_t \geq P_{q,t} C_{q,t} + P_{m,t} C_{m,t} + B_t$$

where $P_{q,t}$ denotes the domestic output-price index

$$P_{q,t} = \left(\int_0^1 P_{q,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

with $P_{q,t}(i)$ the price of intermediate good i.

The Euler equation is given by

$$\frac{d_t}{P_{c,t}C_t} = \beta E_t \frac{R_t d_{t+1}}{P_{c,t+1}C_{t+1}},$$

where $P_{c,t}$ is the price of the overall consumption basket.

The intratemporal optimality condition is described by

$$\frac{W_t}{P_{c,t}C_t} = \phi \nu_t N_t^{\varphi}.$$

In the optimal allocation, we have

$$P_{q,t}C_{q,t} = (1-\chi)P_{c,t}C_t$$

and

$$P_{m,t}C_{m,t} = \chi P_{c,t}C_t$$

where $P_{c,t} \equiv P_{m,t}^{\chi} P_{q,t}^{1-\chi}$ and $\mathrm{P}_{m,t}$ is the nominal price of oil. Also note that $P_{c,t} \equiv P_{q,t}S_t^{\chi}$, where $S_t \equiv \frac{P_{m,t}}{P_{q,t}}$ is the real price of oil.

Monopolistically competitive wholesale firms

Intermediate goods are produced using labor, $N_t(i)$, and oil, $M_t(i)$, both supplied on perfectly competitive factor markets. Each firm i produces output according to the production function

$$Q_t(i) = \left[A_t N_t(i)\right]^{1-\alpha} M_t(i)^{\alpha} \qquad\qquad 0 < \alpha < 1.$$

Here, α is the share of oil in production and A_t stands for labor augmenting technological progress whose growth rate, $z_t \equiv \frac{A_t}{A_{t-1}}$, follows an exogenous process

$$\ln z_t = \ln z + \epsilon_{z,t}$$

with $z > 1$ and $\epsilon_{z,t}$ is $N(0, \sigma_z^2)$. Each intermediate good-producing firm's nominal marginal costs are given by

$$\psi_t(i) = \frac{W_t}{(1-\alpha)Q_t(i)/N_t(i)} = \frac{P_{m,t}}{\alpha Q_t(i)/M_t(i)}$$

and the markup, $\mu_t(i)$, equals

$$\mu_t(i) = \frac{P_{q,t}(i)}{\psi_t(i)}.$$

The intermediate goods producers face a constant probability, $0 < 1 - \xi < 1$, of being able to adjust prices to a new optimal one, P_t^*, in order to maximize expected discounted profits

$$E_t \sum_{\tau=0}^{\infty} \xi^{\tau} \Lambda_{t,t+\tau} Q_{t+\tau|t} \left[P_t^* - \mathcal{M}^p \psi_{t+\tau|t}(i) \right] = 0,$$

where Q_t denotes gross output, $\Lambda_{t,t+\tau}$ denotes the household's stochastic discount factor and $\mathcal{M}^p \equiv \frac{\varepsilon}{\varepsilon - 1}$ is the desired gross markup.

The domestic price level evolves as

$$P_{q,t} = \left[\xi P_{q,t-1}^{1-\varepsilon} + (1-\xi) P_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Consumption and gross output are related as

$$P_{c,t} C_t = \left(1 - \frac{\alpha}{\mathcal{M}^p} \right) P_{q,t} Q_t$$

and the production function becomes

$$Q_t = \left(A_t N_t \right)^{1-\alpha} M_t^{\alpha}$$

Moreover,

$$M_t = \frac{\alpha}{\mathcal{M}^p} \frac{Q_t}{S_t}.$$

Value added (or GDP), Y_t, is given by

$$P_{y,t} Y_t = \left(1 - \frac{\alpha}{\mathcal{M}^p} \right) P_{q,t} Q_t$$

where $P_{y,t}$ is the GDP deflator defined via $P_{q,t} \equiv P_{y,t}^{1-\alpha} P_{m,t}^{\alpha}$.

Finally, the growth of the real price of oil follows an AR(1) process

$$\ln S_{t+1} = (1 - \rho_s) \ln S + \rho_s \ln S_t + \epsilon_{s,t+1.}$$

where the innovation $\epsilon_{s,t}$ is i.i.d. $N\left(0, \sigma_s^2\right).$

Log-linearized equations

Here we present the detailed log-linearized equations of the model. Lower case letters are proportional deviations from steady state.

Production is given by a Cobb-Douglas production function in labor and oil[33]:

$$q_t = \alpha m_t + (1-\alpha) n_t, \tag{25}$$

Consumption is given by a Cobb-Douglas consumption function in output and oil:

$$c_t = (1-\chi) c_{q,t} + \chi c_{m,t}, \tag{26}$$

The relationship between consumption price inflation and the domestic output price inflation is given by

$$\pi_{c,t} = \pi_{q,t} + \chi \Delta s_t, \tag{27}$$

where $\pi_{c,t} \equiv p_{c,t} - p_{c,t-1}$ is headline inflation and $\pi_{q,t} \equiv p_{q,t} - p_{q,t-1}$ is core inflation.

Note that if we set α and χ to zero, the Blanchard and Gali (2010) model boils down to a simple New Keynesian model similar to the one used in the previous sections of the paper.

The behavior of households is characterized by two equations. The first one is an inter-temporal Euler equation:

$$c_t = d_t - E_t d_{t+1} + E_t c_{t+1} + E_t z_{t+1} - \{R_t - E_t \pi_{c,t+1}\}, \tag{28}$$

where z_t is a shock to the growth rate of technology. Note that to be compatible with balanced growth we assume that the intertemporal elasticity of substitution is one.

The second condition characterizes labor supply and is given by[34]:

$$w_t - p_{c,t} = \gamma (w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t] + \nu_t, \tag{29}$$

where $\gamma \in [0,1]$ captures the extent of real wage rigidity. When $\gamma = 0$, the supply wage is equal to the marginal rate of substitution. The higher the value of γ, the higher the degree of real wage rigidity.

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[33] We assume that firms operate under constant returns to labor and oil. So, $1-\alpha$ is then the share of labor in output.

[34] As in Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), the labor supply shock is re-normalized such that it enters the labor supply equation with a coefficient of one as seen here. In this way, it is easier to choose a reasonable prior for its standard deviation denoted by σ_ν.

Domestic goods are imperfect substitutes in consumption, and firms are thus monopolistic competitors. Given the production function, cost minimization implies that the firms' demand for oil is given by:

$$m_t = -\mu_t - s_t + q_t. \tag{30}$$

Using this expression to eliminate m_t in the production function (9) gives a reduced-form production function:

$$q_t = n_t - \frac{\alpha}{1-\alpha} s_t - \frac{\alpha}{1-\alpha} \mu_t. \tag{31}$$

Combining the cost minimization conditions for oil and for labor with the aggregate production function yields the following factor price frontier:

$$(1-\alpha)(w_t - p_{c,t}) + (\alpha + (1-\alpha)\chi) s_t + \mu_t. \tag{32}$$

Firms are assumed to set prices à la Calvo (1983). The resulting inflation dynamics are described by the following expectational Phillips curve:

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \kappa \mu_t, \tag{33}$$

where $\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}$ is the slope of the Phillips curve.

Balanced trade gives us a relation between consumption and output:

$$c_t = q_t - \chi s_t + \eta \mu_t, \tag{34}$$

where $\eta \equiv \frac{\alpha}{\mathcal{M}^P - \alpha}$, with $\mathcal{M}^P$ denoting the steady state gross markup.

Combining the reduced form production function (15) with the above equation gives a relationship between consumption and employment:

$$c_t = n_t - (\frac{\alpha}{1-\alpha} + \chi) + (\eta - \frac{\alpha}{1-\alpha}) \mu_t. \tag{35}$$

The characterization of the equilibrium does not require us to introduce valued (or GDP). But it is needed to undertake the estimation of the model where we use GDP growth data. The definition of value added, combined with the demand for oil, yields the following relation between GDP and gross output:

$$y_t = q_t + \frac{\alpha}{1-\alpha} s_t + \eta \mu_t. \tag{36}$$

Table 15: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
5 obs (PCE, CorePCE)	-156.30	-161.86	1	0
5 obs (CPI, CorePCE)	-174.66	-181.61	1	0
5 obs (CPI, CoreCPI)	-177.12	-183.41	1	0
5 obs (Real-time data)	-173.66	-179.39	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.51.

The shocks s_t, ν_t and d_t are assumed to follow independent stationary AR(1) processes:

$$s_t = \rho_s s_{t-1} + \varepsilon_{st,} \qquad \nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t,} \qquad d_t = \rho_d d_{t-1} + \varepsilon.$$

Lastly, to close the model, the central bank's policy is described by a Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi\{\omega\pi_{c,t} + (1-\omega)\pi_{q,t}\} + \psi_y y_t + \psi_{gy} gy_t] + \epsilon_{R,t}, \qquad 0 \leq \rho_R < 1, \tag{37}$$

where $\epsilon_{R,t}$ is $N(0, \sigma_R^2)$ and $gy_t \equiv y_t - y_{t-1} + z_t$ stands for the growth rate of detrended output.[35] The central bank responds to a convex combination of headline and core inflation with the parameter ω governing the relative weights. For instance, setting ω to zero implies that the central bank responds to core inflation only.

Apart from estimating the model using PCE-core PCE and CPI-core PCE combinations as in the published paper (see Table 7), we have also estimated the model with CPI-core CPI data. Furthermore, we have also used real-time data on per-capita real GDP growth rate, PCE and core PCE inflation. Our results remain robust and are reported in Tables 15 and 16.

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[35] Unlike Blanchard and Gali (2010) and Blanchard and Riggi (2013), we assume that the central bank is perfectly credible. While these authors allow for a role of central bank credibility to explain the reduced impact of oil shocks in the 2000s, they restrict their attention to determinacy only. Our purpose in this present paper is to specifically test for indeterminacy due to passive monetary policy during our period of interest while allowing for a distinction between headline and core inflation. Hence, we assume that the central bank is perfectly credible.

Table 16: Priors and posteriors for DSGE parameters.

				Posterior Mean [5th pct, 95th pct]			
Name	Range	Density	Prior Mean (Std. Dev.)	5 obs PCE, CorePCE	5 obs CPI, CorePCE	5 obs CPI, CoreCPI	5 obs Real time
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	2.61 [1.57,3.86]	2.76 [1.69,4.03]	2.41 [1.47,3.62]	2.37 [1.31,3.54]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.11 [0.03,0.26]	0.07 [0.01,0.16]	0.09 [0.02,0.20]	0.05 [0.01,0.10]
ψ_{gy}	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.62 [0.21,1.15]	0.69 [0.23,1.24]	0.84 [0.31,1.44]	0.76 [0.20,1.44]
ρ_R	[0,1)	Beta	0.70 (0.10)	0.78 [0.66,0.88]	0.79 [0.70,0.87]	0.78 [0.68,0.86]	0.74 [0.63,0.84]
ω	[0,1)	Beta	0.50 (0.20)	0.32 [0.10,0.59]	0.21 [0.06,0.41]	0.22 [0.07,0.42]	0.32 [0.11,0.58]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.10)	0.38 [0.25,0.53]	0.40 [0.25,0.57]	0.36 [0.23,0.52]	0.44 [0.26,0.65]
γ	[0,1)	Beta	0.50 (0.20)	0.50 [0.30,0.68]	0.43 [0.24,0.60]	0.53 [0.35,0.69]	0.48 [0.18,0.77]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	1.95 [1.37,2.53]	1.99 [1.40,2.58]	2.05 [1.35,2.74]	1.86 [1.17,2.61]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.17 [0.59,1.84]	1.20 [0.59,1.87]	1.13 [0.54,1.79]	1.36 [0.71,2.12]
γ^*	$\mathbb{R}$	Normal	0.50 (0.10)	0.50 [0.37,0.64]	0.53 [0.39,0.66]	0.51 [0.38,0.65]	0.50 [0.37,0.63]
ρ_s	[0,1)	Beta	0.70 (0.10)	0.88 [0.80,0.94]	0.91 [0.85,0.96]	0.90 [0.83,0.95]	0.90 [0.83,0.95]
ρ_d	[0,1)	Beta	0.70 (0.10)	0.78 [0.66,0.88]	0.77 [0.64,0.87]	0.77 [0.65,0.88]	0.81 [0.70,0.90]
ρ_ν	[0,1)	Beta	0.70 (0.10)	0.58 [0.39,0.81]	0.71 [0.50,0.90]	0.62 [0.42,0.84]	0.78 [0.52,0.94]
σ_z	$\mathbb{R}^+$	IG	0.50 (∞)	0.68 [0.52,0.89]	0.69 [0.53,0.89]	0.75 [0.57,0.97]	0.79 [0.60,1.01]
σ_R	$\mathbb{R}^+$	IG	0.50 (∞)	0.16 [0.11,0.24]	0.16 [0.11,0.22]	0.16 [0.11,0.24]	0.19 [0.13,0.28]
σ_s	$\mathbb{R}^+$	IG	0.50 (∞)	18.17 [14.08,23.36]	29.21 [22.77,37.40]	28.47 [22.23,36.45]	20.91 [16.13,27.16]
σ_d	$\mathbb{R}^+$	IG	0.50 (∞)	0.77 [0.48,1.17]	0.72 [0.45,1.07]	0.74 [0.43,1.13]	0.70 [0.37,1.13]
σ_ν	$\mathbb{R}^+$	IG	0.50 (∞)	0.62 [0.44,0.85]	0.80 [0.59,1.09]	0.68 [0.49,0.94]	0.82 [0.49,1.23]

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,\varsigma\right) \infty \sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 2$ and $\varsigma = 0.282$. The prior predictive probability is 0.51.

Determinacy region

Figure 6 below shows the determinacy region for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity γ. As can be seen from the figure, the *Taylor Principle* continues to hold in this micro-founded model with a distinction between core and headline inflation. In line with the findings of Carlstrom, Fuerst and Ghironi (2006), equilibrium determinacy criterion does not imply a preference to any particular measure of inflation. As long as the central bank responds with a coefficient greater than unity to either headline inflation, core inflation or a combination of the two, then such policy will ensure equilibrium determinacy.

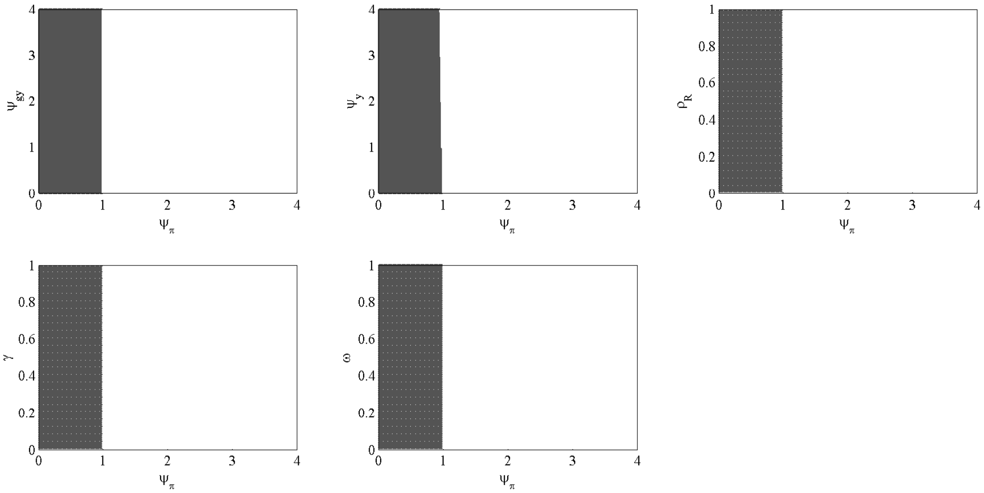

Figure 6: Determinacy region for the Blanchard and Gali (2009) model

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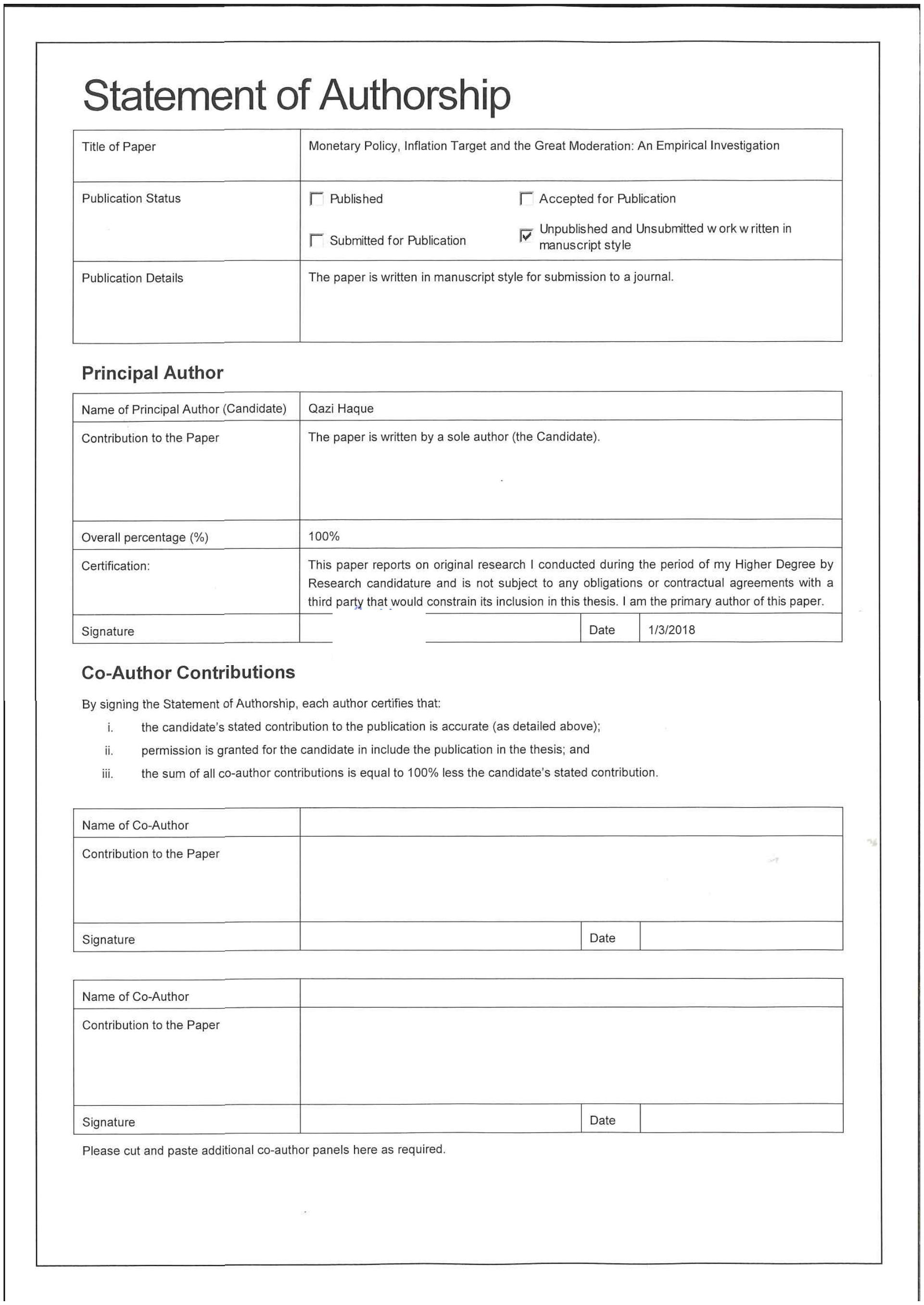

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By signing the Statement of Authorship, each author certifies that:

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III Monetary Policy, Inflation Target and the Great Moderation: An Empirical Investigation

This paper estimates a New Keynesian model with positive trend inflation and compares the empirical fit of the model featuring a Taylor rule with fixed versus time-varying inflation target while allowing for indeterminacy. The estimation is conducted over two different periods covering the Great Inflation and the Great Moderation. The rule embedding time variation in inflation target turns out to be empirically superior and determinacy prevails in both sample periods. This finding, therefore, rules out self-fulfilling inflation expectations as an explanation of the high inflation episode in the 1970s. Counterfactual simulations find that the decline in inflation-gap volatility and predictability is driven by better monetary policy. In contrast, the reduction in output growth variability is mainly explained by reduced volatility of technology shocks.

1 Introduction

Post-World War II U.S. economy is generally characterized by two particular eras: the Great Inflation and the Great Moderation. There is strong evidence that the former era is represented by highly volatile inflation and output growth while there has been a marked decline in macroeconomic volatility in the latter period (Blanchard and Simon, 2001; McConnell and Perez-Quiros, 2000; and Stock and Watson, 2002). The Great Moderation is also associated with changes in the predictability of inflation. For instance, Stock and Watson (2007) document that inflation has become *absolutely* easier, but *relatively* harder to forecast, in the Volcker-Greenspan era. They argue that forecasting inflation has become *absolutely* easier because of its reduced volatility while predicting inflation has become *relatively* harder due to its reduced persistence. What are the reasons behind this shift from the Great Inflation to the Great Moderation era?

One prominent explanation, put forth by Clarida, Gali and Gertler (2000) and further advocated by Lubik and Schorfheide (2004), suggests that the shift is attributable to changes in the behavior of the Federal Reserve. This literature argues that U.S. monetary policy in the 1970s failed to respond sufficiently strongly to inflation thereby generating indeterminacy.[1] Consequently, self-fulfilling inflation expectations is regarded as the driver of the high inflation episode in the 1970s. According to this view, a switch from a passive to an active response to inflation brought about a stable and determinate environment since the early 1980s.[2] In a conceptually related study, Boivin and Giannoni (2006) find that this switch has also been instrumental in reducing observed output and inflation volatility. Moreover, Benati and Surico (2008) show that by responding more strongly to inflation, monetary policy has contributed to the decline in persistence and predictability of inflation relative to a trend component.

While these studies only consider a constant zero inflation target (i.e. a zero inflation steady state), a different picture emerges from studies allowing for positive trend inflation. For instance, Coibion and Gorodnichenko (2011) and Hirose, Kurozumi and Van Zandweghe (2017) argue that a stronger response to inflation is not enough to explain the shift to determinacy after the Great Inflation. Instead, they document that a decline in trend inflation as well as a change in the policy response to the output gap and output growth have played a crucial role. Nonetheless, there is a large literature disputing the view of a fixed inflation target. Amongst them Kozicki and Tinsley (2005, 2009), Ireland (2007), Stock and Watson (2007), Cogley and Sbordone (2008) and Castelnuovo, Greco and Raggi (2014) find evidence in favor of time-varying inflation target. Furthermore, Cogley, Primiceri and Sargent (2010) argue that the decline in the variability of the Federal Reserve's inflation target is the single most important factor behind the reduction in inflation volatility and persistence.

Empirical investigations conducted so far have either looked at the plausibility of

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[1] Roughly speaking, *indeterminacy* refers to the multiplicity of rational expectations equilibria while an equilibrium that is locally isolated and uniquely determined by preferences and technologies is called *determinate*. See Farmer (1999) for a formal definition.

[2] A policy response to inflation is called *active* if it satisfies the Taylor Principle - an aspect of the Taylor rule that describes how, for each one percent increase in inflation, the central bank should raise the nominal interest rate by more than one percentage point to ensure determinacy. Otherwise, it is labelled as *passive*.

a switch from indeterminacy to determinacy through the lens of a model featuring fixed (either zero or positive) target or allowed for time-varying inflation target while restricting the model to determinacy alone.[3] Unfortunately, the assumption of a fixed versus time-varying inflation target is not innocuous for both the determinacy properties and the role of monetary policy in the Great Moderation. For instance, the parameter estimate of the Taylor rule's response to the inflation gap depends on whether the Federal Reserve is responding to deviations from a fixed target or time-varying target. This feature then affects the probability of being in a determinate or indeterminate regime.

This paper estimates a New Keynesian model with positive trend inflation and compares the empirical fit of the model featuring a Taylor rule with fixed versus time-varying inflation target while allowing for indeterminacy. The estimation is conducted over two different periods: a pre-Volcker sample from 1966:I - 1979:II and a post-1984 sample from 1984:I - 2008:II. In doing so, it makes two contributions. First, the paper shows that the rule embedding time variation in inflation target turns out to be empirically superior and determinacy prevails not only in the Great Moderation era but also in the pre-Volcker period. Therefore, unlike the literature's preponderant view, this finding rules out self-fulfilling inflation expectations, i.e. sunspots, as an explanation of the Great Inflation. Second, it shows that both good policy and good luck are jointly required to explain the Great Moderation.[4] Counterfactual exercises suggest that better monetary policy, both in terms of a stronger response to the inflation gap and smaller fluctuations of the inflation target process, has dampened most of the fluctuations in the inflation gap and contributed to the decline in its predictability. In contrast, changes in monetary policy alone fail to explain the reduced variability of output growth which is explained by a reduction

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[3] One exception is Coibion and Gorodnichenko (2011) who use a limited information single-equation approach to estimate a Taylor rule with time-varying coefficients which allow them to extract a measure of trend inflation and construct a time-series for the probability of determinacy for the U.S. economy. However, (in-)determinacy is a property of a rational expectations system that requires a full information estimation approach. Moreover, Coibion and Gorodnichenko (2011) estimate a constant term of the Taylor rule which contains trend inflation but also the equilibrium real interest rate and the Fed's targets for real GDP growth and the output gap. Consequently, the level of trend inflation is not separately indentified and hence they need to make additional assumptions.

[4] The good luck interpretation - a decline in the variance of the exogenous shocks hitting the economy - is supported by a number of authors including Stock and Watson (2002), Primiceri (2005), Sims and Zha (2006), Smets and Wouters (2007), and Justiniano and Primiceri (2008).

in the volatility of technology shocks.

In contrast to the existing literature, the current paper distinguishes between trend inflation and time-varying inflation target. Trend inflation, a term coined by Ascari (2004), stands for a strictly positive level of steady state inflation around which to approximate firms' first-order conditions in the derivation of the New Keynesian Phillips curve (henceforth NKPC). Allowing for positive trend inflation is crucial as it affects the determinacy properties of the model. Ascari and Ropele (2007, 2009) show that trend inflation makes price-setting firms more forward-looking which flattens the NKPC and widens the indeterminacy region. On the other hand, following Sargent (1999), Cogley and Sargent (2005), Primiceri (2006), and Sargent, Williams and Zha (2006) time-varying inflation target is interpreted as the short-term goal pursued by the Federal Reserve conditional on economic situation and its knowledge about the inflation-output volatility trade-off. In this line of argument, trend inflation stands for the Federal Reserve's long-run target compatible with its long-run goals such as inflation stability and sustainable economic growth. A fixed inflation target is simply equal to trend inflation in the model. In contrast, time-varying inflation target follows a persistent exogenous autoregressive process as in Cogley, Primiceri and Sargent (2010), but one whose unconditional mean is equal to positive trend inflation.[5]

The main findings can be summarized as follows. First, when considering the model with constant positive inflation target, indeterminacy can neither be ruled in nor ruled out before 1979 while determinacy prevails after 1984. This stands in contrast to Hirose, Kurozumi and Van Zandweghe (2017) who estimate a similar model allowing for positive constant trend inflation and find that the U.S. economy was explicitly in the indeterminacy region of the parameter space before 1979 and switched to determinacy afterwards. While these authors employ a model with firm-specific labour following Kurozumi and Van Zandweghe (2017), the current paper uses a model with homogenous labor in the benchmark specification following Ascari and Ropele (2009) and Ascari and Sbordone (2014). Indeed, when using firm-specific labor, this paper finds that the pre-Volcker period is unambiguously characterized by indeterminacy as well. Kurozumi and Van Zandweghe (2017) show that the model

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[5] For models in which inflation target evolves partly or fully endogenously, see Ireland (2007) and Eo an Lie (2017).

with firm-specific labor is more susceptible to indeterminacy induced by higher trend inflation than the model with homogeneous labor which explains the difference.

Yet, the upshot completely differs when allowing for time-varying inflation target. This time the posterior density favors determinacy for both the pre-1979 and post-1984 sub-samples. This result suggests that monetary policy, even during the pre-Volcker period, was sufficiently active to ensure determinacy. Using posterior odds ratio to compare the two specifications under the assumption of homogenous labor, the paper then reports evidence in favor of time variation in the inflation target process for both the Great Inflation and the Great Moderation period. Furthermore, when assuming firm-specific labor, the paper finds the fit of fixed versus time-varying inflation target to be comparable for both sample periods.

Perhaps most closely related to this paper are studies by Castelnuovo (2010), Cogley, Primiceri and Sargent (2010), Castelnuovo, Greco and Raggi (2014), and Hirose, Kurozumi and Van Zandweghe (2017). Both Castelnuovo (2010) and Cogley, Primiceri and Sargent (2010) estimate a New Keynesian model log-linearized around a zero inflation steady state and perform counterfactual simulations to assess the drivers of the Great Moderation. The current paper departs along the following dimensions. First, it estimates a model log-linearized around a positive steady state inflation. Ascari and Ropele (2009) and Ascari and Sbordone (2014) document that positive trend inflation substantially alters the NKPC relationship and therefore it changes the inflation dynamics and determinacy regions. Moreover, Ascari, Castelnuovo and Rossi (2011) and Hirose, Kurozumi and Van Zandweghe (2017) show that a model with positive steady state inflation fits better than its simple New Keynesian counterpart which is log-linearized around zero inflation steady state. Second, it compares the fit of fixed versus time-varying target while also allowing for indeterminacy. Finally, it employs the Sequential Monte Carlo (henceforth SMC) algorithm developed by Herbst and Schorfheide (2014, 2015) while both Castelnuovo (2010) and Cogley, Primiceri and Sargent (2010) employ Random-Walk Metropolis Hastings (henceforth RWMH) algorithm. Herbst and Schorfheide (2014, 2015) demonstrate that the SMC algorithm is better suited for multi-modal and irregular posterior distributions.[6]

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[6] See also Hirose, Kurozumi and Van Zandweghe (2017) who are the first ones to apply Bayesian estimation using the SMC algorithm to test for indeterminacy using Lubik and Schorfheide's (2003,

Castelnuovo, Greco and Raggi (2014) estimate a regime-switching policy rule featuring time-varying inflation target and compare it to a specification with fixed target. The authors find support in favor of time variation in inflation target as well. However, they employ a partial equilibrium single-equation approach with two monetary regimes, active and passive. They characterize monetary policy during much of the 1970s as passive and identify a switch to an active regime soon after Paul Volcker's appointment as Chairman of the Federal Reserve. First of all, using a partial equilibrium approach to characterize the likelihood of determinacy is not innocuous. As mentioned earlier, (in-)determinacy is a property of a rational expectations system that requires a full information estimation approach such that the parameter estimates of the Taylor rule account for the endogeneity of its targeted variables. Moreover, their approach to deal with the issue of passive monetary policy does not allow for multiplicity of equilibria. Phrased alternatively, the regime transitions do not imply moving from determinacy to indeterminacy as both regimes are determinate. Hence, their regime-switching policy rule cannot address questions involving self-fulfilling inflation expectations as in the current paper.

Finally, Hirose, Kurozumi and Van Zandweghe (2017) estimate a New Keynesian model with firm-specific labor and fixed inflation target (equal to positive steady state or trend inflation). They find that the pre-Volcker period is ostensibly characterized by indeterminacy while better systematic monetary policy as well as changes in the level of trend inflation resulted in a switch to determinacy after 1982.[7] In contrast, the current paper estimates a similar model with homogenous labor and allows for time variation in the inflation target process. The paper documents that time-varying inflation target empirically fits better (or at least no worse in the case of firm-specific labor) than a fixed target and determinacy prevails in both sample periods. Moreover, it conducts counterfactual exercises to uncover the driving forces of the Great Moderation. To the best of my knowledge, this paper is the first one to test for indeterminacy using a full-information structural approach while allowing

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2004) methodology.

[7] Arias, Ascari, Branzoli and Castelnuovo (2017) corroborate these findings as well as those in Coibion and Gorodnichenko (2011) by revisiting the relation between the sytematic component of monetary policy, trend inflation and determinacy within a medium-scale DSGE model. However, due to the complexities arising from the medium-scale nature of their model, they stop short by estimating the model over the period 1984:I - 2008:II focusing on determinacy alone.

for both positive trend inflation and time variation in the Federal Reserve's inflation target. The finding that the pre-Volcker period could possibly be characterized by a unique equilibrium is a novel result.[8]

2 Model

The estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian (henceforth GNK) model. The model economy consists of an intertemporal Euler equation obtained from the household's optimal choice of consumption and bond holdings, a discrete-time staggered price-setting model of Calvo (1983) that features a positive steady state trend inflation, and a Taylor rule that characterizes monetary policy. As discussed earlier, allowing for positive steady state inflation is important for the following reasons: (i) positive trend inflation makes price-setting firms more forward-looking which flattens the NKPC and makes the inflation rate less sensitive to current economic conditions; (ii) it alters the determinacy properties of the model; and (iii) trend inflation generates more endogenous persistence of inflation and output even in the determinacy case.[9] Unlike Ascari and Sbordone (2014), the paper assumes stochastic growth modelled as the technology level following a unit root process, replaces their labor supply disturbance by a discount factor shock as a stand-in for demand shocks and introduces external habit formation in consumption to generate output persistence. In light of the result of Cogley and Sbordone (2008) regarding the lack of empirical support for intrinsic inertia in the GNK Phillips curve, the model is estimated in the absence of rule-of-thumb price-setting. Finally, the Taylor rule involves responses to the output gap and output growth instead of log-deviations of output from the steady state. These assumptions

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[8] An exception is Orphanides (2004) who finds an active response to expected inflation in a Taylor-type rule estimated for the pre-1979 period, thereby claiming that self-fulfilling expectations cannot be a source of macroeconomic instability during the Great Inflation. However, Ascari and Ropele (2007, 2009) show that an active response to inflation does not guarante equilibrium determinacy when allowing for positive trend inflation. Moreover, Orphanides' (2004) finding is based on a single-equation framework. Instead, the current paper recognizes indeterminacy as the property of a system and hence uses full-information structural estimation.

[9] The plain-vanilla New Keynesian model features a poor internal propagation mechanism. As a result the posterior mass might be biased toward the indeterminacy region. See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007). However, trend inflation generates more endogenous persistence of inflation and output even under determinacy thus making the indeterminacy test less susceptible to bias.

then make the model similar to the one estimated by Hirose, Kurozumi and Van Zandweghe (2017). One important distinction is that the current paper allows for time variation in the Federal Reserve's inflation target.[10]

2.1 The log-linearized model

The log-linearized equilibrium conditions are given by the following equations[11]

$$\begin{aligned} y_t \;=\; & \left(\frac{h}{g+h}\right)[y_{t-1}-g_t]+\left(\frac{g}{g+h}\right)[E_t y_{t+1}+E_t g_{t+1}]-\left(\frac{g-h}{g+h}\right)[r_t-E_t\pi_{t+1}] \\ & +\left(\frac{g-h}{g+h}\right)[d_t-E_t d_{t+1}], \end{aligned} \tag{1}$$

$$\pi_t=\kappa E_t\pi_{t+1}+\vartheta\left[\varphi s_t+(1+\varphi)y_t\right]+\chi\left(\frac{h}{g-h}\right)[y_t-y_{t-1}+g_t]-\varpi E_t\psi_{t+1}+\varpi d_t, \tag{2}$$

$$\psi_t=(1-\xi\beta\pi^{\varepsilon})\left[\varphi s_t+(1+\varphi)y_t+d_t\right]+\xi\beta\pi^{\varepsilon}\left[E_t\psi_{t+1}+\varepsilon E_t\pi_{t+1}\right], \tag{3}$$

$$s_t=\varepsilon\xi\pi^{\varepsilon-1}\left(\frac{\pi-1}{1-\xi\pi^{\varepsilon-1}}\right)\pi_t+\xi\pi^{\varepsilon}s_{t-1}, \tag{4}$$

$$r_t=\rho_r r_{t-1}+(1-\rho_r)\left\{\psi_\pi\left(\pi_t-\pi_t^*\right)+\psi_x x_t+\psi_{\Delta y}\left(y_t-y_{t-1}+g_t\right)\right\}+\epsilon_{r,t}, \tag{5}$$

$$x_t=y_t-y_t^n, \tag{6}$$

$$y_t^n=\frac{h}{g(1+\varphi)-h\varphi}\left(y_{t-1}^n-g_t\right), \tag{7}$$

where $\kappa\equiv\beta\left[1+\varepsilon(\pi-1)(1-\xi\pi^{\varepsilon-1})\right]$, $\vartheta\equiv(1-\xi\pi^{\varepsilon-1})(1-\xi\beta\pi^{\varepsilon})/\xi\pi^{\varepsilon-1}$, $\chi\equiv(1-\xi\pi^{\varepsilon-1})(1-\xi\beta\pi^{\varepsilon-1})/\xi\pi^{\varepsilon-1}$ and $\varpi\equiv\beta(1-\pi)(1-\xi\pi^{\varepsilon-1})$. Lower case letters denote log-deviations from steady state. Here y_t and y_t^n stand for de-trended output and natural level of output respectively, x_t is the output gap, r_t denotes the nominal interest rate, π_t symbolizes inflation, π_t^* represents the Federal Reserve's time-varying

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[10] Moreoever, following Ascari and Sbordone (2014), the paper assumes homogenous labor whereas Hirose, Kurozumi and Van Zandweghe (2017) assume firm-specific labor.

[11] A full description of the model is delegated to the Appendix to conserve space.

inflation target, ψ_t is an endogenous auxiliary variable, s_t denotes the resource cost due to relative price dispersion and E_t represents the expectations operator. Eq. (1) is the dynamic IS relation reflecting an Euler equation where $h \in [0,1]$ represents the degree of habit persistence and g stands for the steady state gross rate of technological progress which is also equal to the steady state gross rate of balanced growth. Eq. (2) and (3) represent the GNK Phillips curve where $\beta \in (0,1)$ is the subjective discount factor, $\xi \in [0,1)$ is the fraction of firms whose prices remain unchanged from previous period, π is the steady state gross inflation rate or trend inflation, $\varepsilon > 1$ is the price elasticity of demand, and φ is the inverse elasticity of labour supply. Eq. (2) boils down to a standard NKPC when trend inflation is zero (i.e. $\pi = 1$) and this assumption also implies that $\psi_t = 0$. Eq. (4) is a recursive log-linearized expression for the price dispersion measure under Calvo pricing mechanism. Eq. (5) represents monetary policy, i.e. a Taylor-type rule in which $\psi_\pi, \psi_x, \psi_{\Delta y,}\rho_r$ are chosen by the central bank and echo its responsiveness to the inflation gap, output gap, output growth and the degree of inertia in interest rate setting respectively. The term $\epsilon_{r,t}$ is an exogenous transitory monetary policy shock whose standard deviation is given by σ_r. Eq. (6) is the definition of the output gap while the law of motion for the natural level of output is given by Eq. (7).

The remaining fundamental disturbances involve a preference shock d_t, a non-stationary technology shock g_t, and an inflation target shock π_t^*. Each of these three shocks follow $AR(1)$ processes:

$$d_t = \rho_d d_{t-1} + \epsilon_{d,t} \qquad 0 < \rho_d < 1,$$

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1,$$

and

$$\pi_t^* = (1 - \rho_{\pi^*})\,\pi + \rho_{\pi^*}\pi_{t-1}^* + \epsilon_{\pi^*,t} \qquad 0 < \rho_{\pi^*} < 1,$$

where the standard deviations of the innovations $\epsilon_{d,t}$, $\epsilon_{g,t}$ and $\epsilon_{\pi^*,t}$ are denoted by σ_d , σ_g and σ_{π^*} respectively.

Under a fixed inflation target, the paper assumes that the policy rules becomes

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left\{\psi_\pi \pi_t + \psi_x x_t + \psi_{\Delta y}(y_t - y_{t-1} + g_t)\right\} + \epsilon_{r,t,}$$

where the central bank's target is equal to steady-state inflation or trend inflation π.

2.2 Rational expectations solutions under indeterminacy

To solve the model, the paper applies the method proposed by Lubik and Schorfheide (2003). The linear rational expectations (henceforth LRE) system can be compactly written as

$$\Gamma_0(\theta)\varrho_t = \Gamma_1(\theta)\varrho_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t,$$

where ϱ_t, ε_t and η_t denote the vector of endogenous variables, fundamental shocks and one-step ahead expectation errors respectively and $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$ and $\Pi(\theta)$ are appropriately defined coefficient matrices. From a methodological perspective, the solution algorithm of Lubik and Schorfheide (2003) follows from that of Sims (2002). However, it has the added advantage of being general and explicit in dealing with expectation errors since it makes the solution suitable for solving and estimating models which feature multiple equilibria. In particular, under indeterminacy, η_t becomes a linear function of the fundamental shocks and purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the LRE model entails

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t, \tag{8}$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)$[12] are the coefficient matrices.[13] The sunspot shock satisfies $\zeta_t \sim i.i.d.\ \mathsf{N}(0, \sigma_\zeta^2)$. Accordingly, indeterminacy can manifest itself in one of two different ways: (i) purely extrinsic non-fundamental disturbances can affect the model dynamics through endogenous expectation errors; and (ii) the propagation of fundamental shocks cannot be uniquely pinned down and the multiplicity of equilibria affecting this propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following the methodology proposed by Lubik and Schorfheide (2004), $\widetilde{M}$ is replaced with $M^*(\theta)+M$ and the prior mean for M is set equal to zero. The particular solution employed selects $M^*(\theta)$ by using a least squares criterion to minimize the

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[12] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, the paper imposes their normalization such that $M_\zeta = I$.

[13] Under determinacy, the solution boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

distance between the impact response of the endogenous variables to fundamental shocks, $\partial\varrho_t/\partial\varepsilon'_t$, at the boundary between the determinacy and the indeterminacy region.[14] Analytical solution for the boundary in this model is unavailable and hence, following Justiniano and Primiceri (2008) and Hirose (2014), this paper resorts to a numerical procedure to find the boundary by perturbing the parameter ψ_π in the monetary policy rule.

2.3 Equilibrium determinacy and trend inflation

Before moving onto the empirical investigation, this subsection shows how allowing for trend inflation affects the determinacy properties of the model. Ascari and Ropele (2009) and Ascari and Sbordone (2014) argue that trend inflation makes price-setting firms more forward-looking thereby flattening the NKPC and widening the indeterminacy region. Figure 1 documents how trend inflation affects the determinacy region. Since analytical solution is infeasible unless one assumes indivisible labor, the determinacy results shown here are numerical.[15]

The determinacy region shrinks with trend inflation as documented by Ascari and Ropele (2009) and Ascari and Sbordone (2014).[16] In other words, a stronger response to the inflation gap together with a weaker response to the output gap is required to generate determinacy at higher levels of trend inflation. Therefore, monetary policy should respond more to the inflation gap and less to the output gap in order to stabilize inflation expectations. Moreover, in the case of positive trend inflation, Coibion and Gorodnichenko (2011) show that interest rate smoothing as well as stronger response to output growth, instead of the output gap, widen the determinacy region thereby making it easier for policy to guarantee determinacy.

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[14] This methodology has been used in previous studies, such as Benati and Surico (2009), Doko Tchatoka, Groshenny, Haque and Weder (2017) and Hirose (2007, 2008, 2013, 2014).

[15] The parameter values used in the numerical computation are: $\beta = 0.99$, $\varepsilon = 11$, $\xi = 0.75$, $h = 0$ implying no habit formation in consumption, and $g = 1.005$ such that the steady state growth rate of real per capita GDP is 2 per cent per year. The policy rule is a simple Taylor rule of the form $r_t = \psi_\pi \pi_t + \psi_x x_t$.

[16] The figure is the same as Figure 4 in Ascari and Ropele (2009) and Figure 11 in Ascari and Sbordone (2014).

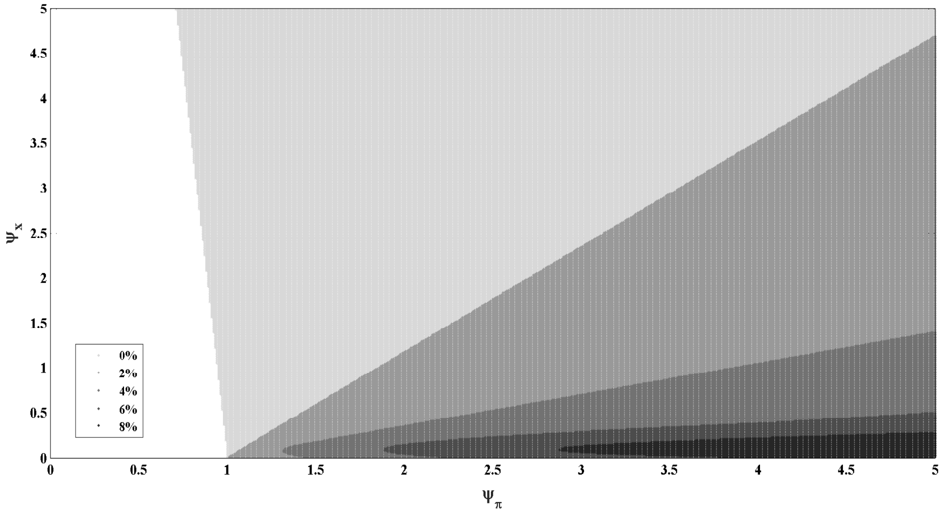

Figure 1: Determinacy region and trend inflation

3 Econometric strategy

3.1 Bayesian estimation with Sequential Monte Carlo (SMC) algorithm

The paper uses Bayesian techniques for estimating the parameters of the model and tests for indeterminacy using posterior model probabilities. It employs the SMC algorithm proposed by Herbst and Schorfheide (2014, 2015) which is particularly suitable for irregular and non-elliptical posterior distributions. Another practical advantage of using an importance sampling algorithm like SMC is that the process does not require one to find the mode of the posterior distribution, a task that can prove to be difficult particularly under indeterminacy.

First priors are described by a density function of the form

$$p(\theta_S|S),$$

where $S \in \{D, I\}$, D and I stand for determinacy and indeterminacy respectively, θ_S represents the parameters of the model S and $p(.)$ stands for the probability density function. Next, the likelihood function, $p(X_T|\theta_S, S)$, describes the density of the observed data where X_T are the observations through to period T. Following Bayes theorem, the posterior density is constructed as a combination of the prior density and the likelihood function:

$$p(\theta_S|X_T, S) = \frac{p(X_T|\theta_S, S)p(\theta_S|S)}{p(X_T|S)},$$

where $p(X_T|S)$ is the marginal data density conditional on the model which is given by

$$p(X_T|S) = \int_{\theta_S} p(X_T|\theta_S, S)p(\theta_S|S)d\theta_S.$$

Following Herbst and Schorfheide (2014, 2015), the paper builds a particle approximation of the posterior distribution through tempering the likelihood. A sequence of tempered posteriors is defined as

$$\pi_n(\theta_S) = \frac{[p(X_T|\theta_S, S)]^{\phi_n} p(\theta_S|S)}{\int_{\theta_S} [p(X_T|\theta_S, S)]^{\phi_n} p(\theta_S|S)d\theta_S},$$

where ϕ_n is the tempering schedule that slowly increases from zero to one.

The algorithm generates weighted draws from the sequence of posteriors $\{\pi_n(\theta_S)\}_{n=1}^{N_\phi}$, where N_ϕ is the number of stages. At any stage, the posterior distribution is represented by a swarm of particles $\left\{\theta_n^i, W_n^i\right\}_{i=1}^N$, where W_n^i is the weight associated with θ_n^i and N denotes the number of particles. The algorithm has three main steps. First, in the *correction* step, the particles are re-weighted to reflect the density in iteration n. Next, in the *selection* step, any particle degeneracy is eliminated by resampling the particles. Finally, in the *mutation* step, the particles are propagated forward using a Markov transition kernel to adapt to the current bridge density.

In the first stage, i.e. when $n = 1$, ϕ_1 is zero. Hence, the prior density serves as an efficient proposal density for $\pi_1(\theta_S)$. That is, the algorithm is initialized by drawing the initial particles from the prior. Likewise, the density of $\pi_n(\theta_S)$ is a good proposal density for $\pi_{n+1}(\theta_S)$.

Number of particles, Number of stages, Tempering schedule

The tempering schedule is a sequence that slowly increases from zero to one and is determined by $\phi_n = \left(\frac{n-1}{N_\phi-1}\right)^\tau$ where τ controls the shape of the schedule. The tuning parameters N, N_ϕ and τ are fixed ex ante. The estimation uses $N = 10,000$ particles and $N_\phi = 200$ stages. The parameter that controls the tempering schedule, τ, is set at 2 following Herbst and Schorfheide (2015).

Resampling

Resampling is necessary to avoid particle degeneracy. A rule-of-thumb measure of this degeneracy, proposed by Herbst and Schorfheide (2014, 2015), is given by the reciprocal of the uncentered variance of the particles and is called the effective sample size (ESS). Following them, the estimation employs systematic resampling whenever $ESS_n < \frac{N}{2}$.

Mutation

Finally, one step of a single-block RWMH algorithm is used to propagate the particles forward.

3.2 Data

The paper employs three U.S. quarterly time series: per capita real GDP growth rate $100\Delta \log Y_t$, quarterly growth rate of the GDP deflator $100 \log \Pi_t$ and the Federal Funds rate $100 \log R_t$. To compare the fit of fixed versus time-varying inflation target and to test for indeterminacy, it estimates the model over two sample periods. The first sample, 1966:I - 1979:II, corresponds to the Great Inflation period. The second one, 1984:I - 2008:II, corresponds to the Great Moderation period characterized by dramatically milder macroeconomic volatilities. The measurement equations relating the relevant elements of ϱ_t to the three observables are given by

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100 \log \Pi_t \\ 100 \log R_t \end{bmatrix} = \begin{bmatrix} g^* \\ \pi^* \\ r^* \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} + g_t \\ \pi_t \\ r_t \end{bmatrix}, \tag{9}$$

where $g^* = 100(g-1)$, $\pi^* = 100(\pi - 1)$ and $r^* = 100(r-1)$.

3.3 Calibrated parameters

The discount factor β is set to 0.99, the steady-state markup to ten percent (i.e. $\varepsilon = 11$), and the inverse of the labor-supply elasticity to one. Following Cogley, Primiceri and Sargent (2010), the autoregressive parameter of the inflation target shock is fixed at $\rho_{\pi^*} = 0.995$. Alternatively, one may follow Ireland (2007) by assuming that the inflation target shock follows a unit-root process. Instead, the paper follows Cogley, Primiceri and Sargent's (2010) calibration as they show that a unit-root inflation target process may counterfactually imply low inflation-gap predictability. The remaining parameters are estimated.

3.4 Prior distributions

Table 1 summarizes the specification of the prior distributions. The prior for the inflation coefficient ψ_π follows a gamma distribution centered at 1.10 with a standard deviation of 0.50 while the response coefficient to the output gap and output growth are centered at 0.125 with standard deviation 0.10. The paper uses Beta distributions with mean 0.50 for the smoothing coefficient ρ_r, the Calvo probability ξ, and habit persistence in consumption h, and 0.70 for the persistence of the discount factor shock. The autoregressive parameter of the TFP shock is centered at 0.40 since this process already includes a unit-root. The priors for the quarterly steady state rates of output growth, inflation and interest rate denoted by g^*, π^* and r^* respectively are distributed around their averages over the period 1966:I-2008:II.

For the shocks, the prior distributions for all but one follow an inverse-gamma distribution with mean 0.60 and standard deviation 0.20. The exception is the standard deviation of the innovation to the inflation target shock which is an important parameter in the analysis. Following Cogley, Primiceri and Sargent (2010), the paper adopts a weakly informative uniform prior on $(0, 0.15)$ for this parameter.

Finally, in line with Lubik and Schorfheide (2004), the coefficients M follow standard normal distributions. Hence, the prior is centered around the baseline solution of Lubik and Schorfheide (2004). The choice of the priors leads to a prior predictive probability of determinacy of 0.498, which is quite even and suggests no prior bias toward either determinacy or indeterminacy.

4 Estimation results

This section presents the findings in terms of model comparison, parameter estimates and forecast error variance decomposition.

Table 1: Prior distributions for parameters

Parameter	Density	Prior Mean [St Dev.]
ψ_π	Gamma	1.10 [0.50]
ψ_x	Gamma	0.125 [0.10]
$\psi_{\Delta y}$	Gamma	0.125 [0.10]
ρ_r	Beta	0.50 [0.20]
π^*	Normal	0.976 [0.50]
r^*	Gamma	1.612 [0.25]
g^*	Normal	0.50 [0.10]
h	Beta	0.50 [0.10]
ξ	Beta	0.50 [0.10]
ρ_d	Beta	0.70 [0.10]
ρ_g	Beta	0.40 [0.10]
σ_r	Inv-Gamma	0.60 [0.20]
σ_d	Inv-Gamma	0.60 [0.20]
σ_g	Inv-Gamma	0.60 [0.20]
σ_{π^*}	Uniform	0.075 [0.0433]
σ_ζ	Inv-Gamma	0.60 [0.20]
$M_{r,\zeta}$	Normal	0.00 [1.00]
$M_{d,\zeta}$	Normal	0.00 [1.00]
$M_{g,\zeta}$	Normal	0.00 [1.00]
$M_{\pi^*,\zeta}$	Normal	0.00 [1.00]

Note: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,\varsigma\right)\infty\sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$ where $\nu = 4$ and $\varsigma = 0.45$. The prior probability of determinacy is 0.498.

Table 2: Determinacy versus Indeterminacy

		Log-data density		Probability	
Sample	Inflation target	Det	Indet	Det	Indet
1966:I-1979:II	Fixed	-125.40	-125.94	0.60	0.40
	Time-varying	-122.48	-126.55	0.98	0.02
1984:I-2008:II	Fixed	-31.73	-42.08	1	0
	Time-varying	-28.64	-47.33	1	0

4.1 Model comparison

Table 2 collects the results for the empirical performance of the model with fixed versus time-varying inflation target. To assess the quality of the model's fit to the data, the paper uses log marginal data densities and posterior model probabilities for both parametric regions. The SMC algorithm-based approximation of the marginal data density is given by

$$p^{SMC}(X_T|S) = \prod_{n=1}^{N_\phi} \left(\frac{1}{N} \sum_{i=1}^{N} \widetilde{w}_n^i W_{n-1}^i \right),$$

where $\widetilde{w}_n^i$ is the incremental weight defined by

$$\widetilde{w}_n^i = [p(X|\theta_{n-1}^i, S)]^{\phi_n - \phi_{n-1}}.$$

In case of fixed inflation target, the evidence for (in-)determinacy for the pre-Volcker period is mixed while determinacy prevails after 1984. Phrased alternatively, the possibility of indeterminacy cannot be ruled out in the first sub-sample. Indeed, when assuming firm-specific labor instead of homogenous labor as in Hirose, Kurozumi and Van Zandweghe (2017), the pre-Volcker period is unambiguously characterized by indeterminacy (as shown in a later section).

However, when allowing for time variation in the inflation target pursued by the Federal Reserve, the results are drastically different. Both the pre-Volcker and post-1984 sample periods are now ostensibly characterized by determinacy as the posterior concentrates all of its mass in the determinacy region. This finding suggests that monetary policy, even during the Great Inflation period, was stabilizing and did not open the door for any sunspot fluctuations.

In terms of posterior odds ratio, the marginal likelihood points toward the empirical superiority of the specification featuring time variation in the inflation target. The Bayes factor involving fixed versus time-varying target reads about 20 for both the pre-Volcker and post-1984 sample periods. According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is "not worth more than a bare mention", between 3 and 20 suggests a "positive" evidence in favor of one of the two models, between 20 and 150 suggests a "strong" evidence against it, and larger than 150 "very strong" evidence. Hence, this result points toward a "positive" evidence in favor of the model where the Federal Reserve follows a time-varying inflation target.

4.2 Parameter estimates

Table 3 reports the posterior means and the standard deviations of the parameters under time-varying inflation target.[17] As seen in the table, the Taylor rule's response to the inflation gap was strongly active in the pre-1979 period. In fact, the point estimate is close to two which justifies why the posterior favors determinacy under time-varying target. Moving across the sample, the policy responses to the inflation gap and output growth more than doubled while trend inflation fell considerably by a third in line with the findings of Hirose, Kurozumi and Van Zandweghe (2017). Moreover, like Cogley, Primiceri and Sargent (2010), the innovation variance of the two shocks, $\epsilon_{\pi^*,t}$ and $\epsilon_{r,t}$, declined quite notably. According to the posterior mean estimates, the innovation variance fell from 0.07 to 0.04 for the inflation target shock, and from 0.42 to 0.21 for the policy-rate shock. However, unlike Cogley, Primiceri and Sargent (2010) who find a moderate increase in the responsiveness to the inflation gap, this paper finds quite a substantial increase across the two periods. This finding suggests that both the systematic response to the inflation gap and better anchoring of the inflation target might have played a key role in the decline in inflation-gap volatility and predictability.

Among the other parameters, habit remained unchanged while the degree of price stickiness increased slightly. As noted by Smets and Wouters (2007), the increase

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[17] Table 11 in the appendix reports parameter estimates under fixed target.

Table 3: Posterior estimates for DSGE parameters under time-varying target

	Pre-1979 period	Post-1984 period
Parameter	Mean [St Dev]	Mean [St Dev]
ψ_π	1.98 [0.41]	4.09 [0.61]
ψ_x	0.12 [0.09]	0.13 [0.10]
$\psi_{\Delta y}$	0.17 [0.09]	0.40 [0.18]
ρ_r	0.42 [0.13]	0.72 [0.06]
π^*	1.28 [0.23]	0.84 [0.21]
r^*	1.53 [0.22]	1.56 [0.20]
g^*	0.52 [0.09]	0.51 [0.07]
h	0.42 [0.06]	0.41 [0.06]
ξ	0.43 [0.11]	0.49 [0.08]
ρ_d	0.76 [0.07]	0.92 [0.02]
ρ_g	0.20 [0.06]	0.18 [0.04]
σ_r	0.42 [0.07]	0.21 [0.03]
σ_d	0.81 [0.30]	1.74 [0.35]
σ_g	1.34 [0.26]	0.71 [0.08]
σ_{π^*}	0.07 [0.03]	0.04 [0.01]

Note: Results are based on 10,000 particles from the final stage in the SMC algorithm in price stickiness is consistent with the hypothesis that low and stable inflation may reduce the cost of not adjusting prices and therefore lengthen the average price duration. In fact, Kurozumi (2016) shows that when the degree of price stickiness is endogenously determined in the Calvo model, the probability of price adjustment rises with trend inflation and this mitigates the effect of higher trend inflation on the likelihood of indeterminacy. However, following Ascari and Ropele (2007, 2009), Coibion and Gorodnichenko (2011) and Ascari and Sbordone (2014), the paper assumes that price stickiness is exogenously determined.

Among the non-policy shocks, there is an increase in the persistence and volatility of the discount factor shock, a finding shared with Hirose, Kurozumi and Van Zandweghe (2017). However, there is a decline in the volatility of technology shocks, which is in line with Smets and Wouters (2007) and Leduc and Sill (2007).

4.3 Forecast error variance decomposition

This section assesses the role of the various shocks by appealing to the forecast error variance decomposition (henceforth FEVD). The FEVDs are constructed by computing the contribution of each shock in explaining the forecast errors of the variables of interest. The computations, conditional on the estimated posterior means, refer to several horizons ranging from 1-step ahead up to ∞-step ahead to assess the contribution of each shock at various business cycle frequencies as well as the unconditional variances. Tables 4 and 5 report the results for the two sub-samples.

First of all, technology shocks play a dominant role in explaining the fluctuations in output growth for both sample periods accounting for over 95% of the fluctuations across all forecast horizons. This finding stands in contrast to Ireland (2004), who finds a secondary role for technology shocks and concludes that other shocks appear to be more important (or at least as important) than the technology shock in the New Keynesian model. One key difference is that the present paper log-linearizes the model around a positive steady state trend inflation while Ireland (2004) assumes zero inflation in the steady state. This modeling assumption is not innocuous as Ascari and Sbordone (2014) show that trend inflation substantially affects the propagation of technology shocks.

Yet technology shocks play a negligible role in explaining the fluctuations of the

Table 4: Forecast Error Variance Decompositions: Pre-1979 Sub-sample

Quarters Ahead	Policy	Preference	Technology	Inflation Target
Output Growth				
1	1.27	0.21	98.52	0.00
4	1.46	0.23	98.28	0.03
8	1.46	0.23	98.28	0.03
20	1.46	0.23	98.28	0.03
40	1.46	0.23	98.28	0.03
∞	1.46	0.23	98.28	0.03
Inflation Gap (Mean-based)				
1	41.65	27.52	6.72	24.10
4	25.85	24.33	6.40	43.43
8	18.80	18.70	4.65	57.84
20	10.88	10.90	2.69	75.53
40	6.81	6.82	1.69	84.68
∞	2.54	2.54	0.63	94.30
Inflation Gap (Target-based)				
1	49.18	32.50	7.93	10.39
4	37.01	34.83	9.16	19.00
8	31.98	31.81	7.91	28.29
20	24.03	24.07	5.95	45.96
40	17.78	17.81	4.40	60.00
∞	8.19	8.21	2.03	81.57
Interest Rate				
1	23.46	58.98	0.12	17.43
4	8.57	58.49	1.14	31.80
8	6.07	47.29	0.82	45.83
20	3.67	29.07	0.49	66.77
40	2.36	18.69	0.32	78.63
∞	0.91	7.17	0.12	91.80

Table 5: Forecast Error Variance Decompositions: Post-1984 Sub-sample

Quarters Ahead	Policy	Preference	Technology	Inflation Target
		Output Growth		
1	1.76	0.72	97.50	0.01
4	1.94	0.81	97.21	0.04
8	1.94	0.81	97.21	0.04
20	1.94	0.81	97.21	0.04
40	1.94	0.81	97.21	0.04
∞	1.94	0.81	97.21	0.04
		Inflation Gap (Mean-based)		
1	42.29	35.53	8.62	13.56
4	32.34	34.41	8.13	25.12
8	26.60	31.19	6.69	35.52
20	18.51	23.48	4.65	53.35
40	13.13	16.84	3.30	66.73
∞	5.71	7.32	1.44	85.54
		Inflation Gap (Target-based)		
1	47.32	39.74	9.65	3.29
4	41.34	43.97	10.39	4.30
8	39.04	45.76	9.81	5.39
20	36.34	46.09	9.14	8.43
40	34.49	44.22	8.67	12.62
∞	28.56	36.62	7.18	27.64
		Interest Rate		
1	18.91	75.08	0.10	5.91
4	4.73	86.41	0.38	8.48
8	2.94	85.79	0.24	11.03
20	2.04	79.09	0.17	18.71
40	1.72	69.08	0.14	29.05
∞	1.08	43.27	0.09	55.56

nominal variables. Here the paper focuses on both mean-based and target-based inflation gap. Mean-based inflation gap is defined as the difference between inflation and the central bank's long-run inflation target which is also the steady state inflation in the model; whereas target-based inflation gap is the difference between inflation and the central bank's time-varying short-run inflation objective. Importantly, the inflation target shock plays a considerable role as regards the inflation gap and policy rate, mainly at medium to low frequency. This result corroborates the findings in Castelnuovo (2010) who documents a similar role for inflation target shocks. As pointed out by Castelnuovo (2010), this finding is not necessarily a consequence of the calibration imposed on the autoregressive parameter for the inflation target ($\rho_{\pi^*} = 0.995$) since the volatility of the process, which is estimated, clearly matters as well. Moreover, while being relevant for the unconditional FEVDs of mean-based inflation gap (given its high persistence), the role of such a calibration is less obvious for the FEVDs of target-based gap even at lower frequencies.

As regards the policy-rate shock and the preference shock, the contribution is considerable in explaining the fluctuations in the inflation gap and policy rate at shorter horizons. For instance, the preference shock is most important in driving movements in the nominal interest rate at higher frequencies.

Finally, it is also interesting to compare the differences in the relevance of the shocks across sub-samples. As mentioned above, technology shock is the key driver of fluctuations in output growth in both sample periods. While in the Great Inflation era, inflation target shocks play a dominant role in explaining the fluctuations of target-based inflation gap and the policy rate, however, when moving to the Great Moderation sub-sample there are notable differences. The variance decompositions reveal that both preference and policy-rate shocks are important in explaining movements in target-based inflation gap even at longer horizons. Moreover, for policy-rate fluctuations, preference shocks play a key role at all horizons.

Overall, the variance decomposition exercise suggests that the decline in the innovation variance of inflation target shocks might have played a significant role with regard to the decline in inflation-gap volatility while technology shocks might have been more important for the decline in output growth volatility.

5 What explains the switch from indeterminacy to determinacy in the pre-Volcker period?

The finding that allowing for time-varying inflation target leads to determinacy for both sample periods might be surprising given that the literature has established the pre-Volcker period as characterized by indeterminacy. Yet, this finding relies on inflation dynamics which has been shown by Stock and Watson (2007) to be mostly driven by a permanent component during the Great Inflation. The question is: how can the model explain this phenomenon? Fujiwara and Hirose (2012) argue that a model under indeterminacy can generate richer persistent inflation dynamics compared to determinacy as fewer autoregressive roots in (8) are being suppressed. To highlight their argument, the paper presents a simple heuristic example that borrows from their illustration and also from Bianchi and Nicolò (2017).

Consider a classical monetary model characterized by the Fisher relation

$$r_t = E_t \pi_{t+1} + \nu_t, \tag{10}$$

and a simple Taylor rule

$$r_t = \psi_\pi \pi_t, \tag{11}$$

where r_t, π_t and ν_t denote the nominal interest rate, inflation rate and real interest rate respectively and E_t denotes the mathematical expectations operator. Following Bianchi and Nicolò (2017), the real interest rate follows a mean-zero Gaussian i.i.d. process. The rational expectations forecast error η_t is defined such that

$$\eta_t = \pi_t - E_{t-1}\pi_t. \tag{12}$$

The system is expressed as

$$E_t \pi_{t+1} = \psi_\pi \pi_t - \nu_t. \tag{13}$$

If $\psi_\pi > 1$, a unique non-explosive solution exists and is of the form

$$\pi_t = \frac{1}{\psi_\pi} E_t \pi_{t+1} + \frac{1}{\psi_\pi} \nu_t, \tag{14}$$

$$\pi_t = \frac{1}{\psi_\pi}\nu_t, \tag{15}$$

which implies that π_t follows an i.i.d. process and the last equality is obtained by recalling the assumption on ν_t.

In contrast, if $\psi_\pi \leq 1$, the solution to (13) is obtained by combining (13) with (12) and it takes the form

$$\pi_t = \psi_\pi \pi_{t-1} - \nu_{t-1} + \eta_t, \tag{16}$$

where the stability requirement imposes no restriction on the one-step ahead forecast error η_t.

For the present purpose, note that π_t in equation (16) exhibits richer dynamics than that in equation (15). As a result, the endogenous persistence implied by equation (16) suggests that indeterminacy cannot be ruled out under the assumption of a fixed inflation target.

On the other hand, this is not the case when allowing for time-varying inflation target. As documented by Cogley, Primiceri and Sargent (2010), inflation target shocks induce persistent responses to the inflation gap and capture the permanent component of inflation. According to the posterior estimates, inflation target was loosely anchored during the pre-Volcker period as evident from its higher innovation variance. This led to higher inflation-gap persistence due to a strengthening of the relative importance of this permanent component. As such, the model does not require the richer endogenous inflation dynamics that arises under indeterminacy. As a result, the posterior concentrates all of its mass in the determinacy region and the parameter estimate of the Taylor rule's response to the inflation gap turns out to be much stronger when the Federal Reserve is responding to deviations from a time-varying target.

6 Federal Reserve's inflation target

Before moving on to study the drivers of the Great Moderation, this section assesses the model-implied evolution of the Federal Reserve's inflation target. Here, the paper employs the Kalman smoother to obtain ex-post estimates of π_t^* based on the observations that are included in the construction of the likelihood function. As such,

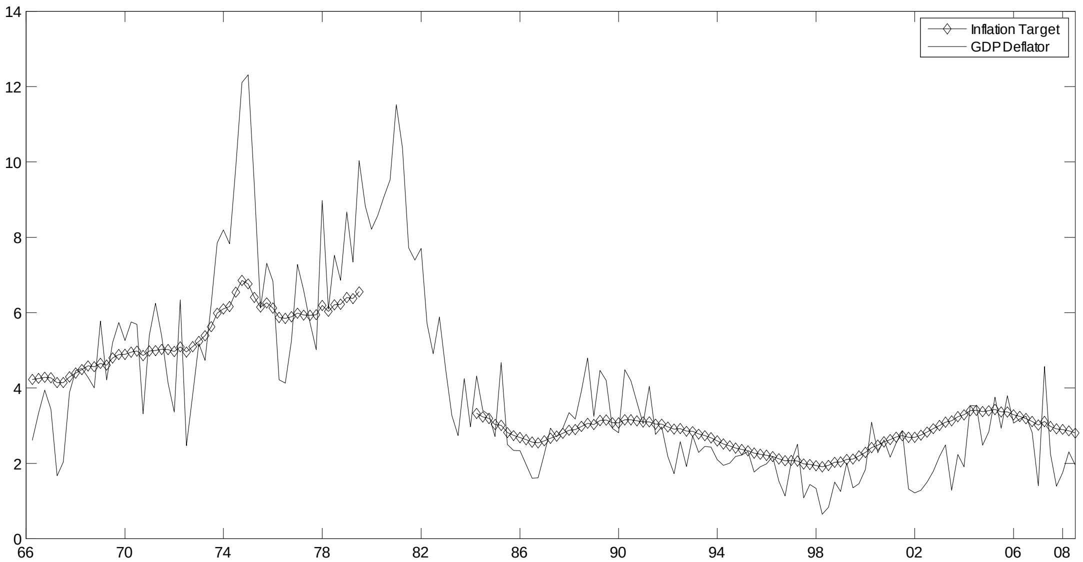

Figure 2: Federal Reserve's Inflation Target

this serves as an external validity check. Figure 2 plots the smoothed estimates of the (latent) inflation target process on top of actual annualized quarterly inflation of the GDP implicit price deflator. As seen in the figure, inflation target began rising in the mid-1960s and jumped above 6% in the aftermath of the 1973 oil crisis. Subsequently, it dropped significantly during the Volcker-disinflation period and somewhat settled around 2.5% since the mid-1980s.

How does the implicit inflation target compare with the evidence in the literature? Figure 3 compares the estimate with a selection of other proposed measures: Kozicki and Tinsley (2005), Ireland (2007), Leigh (2008), Cogley, Primiceri and Sargent (2010), Aruoba and Schorfheide (2011), and Castelnuovo, Greco and Raggi (2014).[18] Each panel plots GDP deflator inflation rate as well.

Several notable findings arise. First of all, there is a striking difference between the estimated target and that of Kozicki and Tinsley (2005). These authors estimate a VAR model allowing for shifts in the inflation target and imperfect policy credibility, defined by differences between the perceived and the actual inflation target. The disparity may be due to their imperfect credibility and learning mechanism whereby the private sector cannot perfectly distinguish between permanent target shocks and

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[18] Sources: Kozicki and Tinsley (2005), Ireland (2007), Leigh (2008), Cogley, Primiceri and Sargent (2010) and Castelnuovo, Greco and Raggi (2014) - original files provided by the authors; Aruoba and Schorfheide (2011) - *American Economic Review* (website).

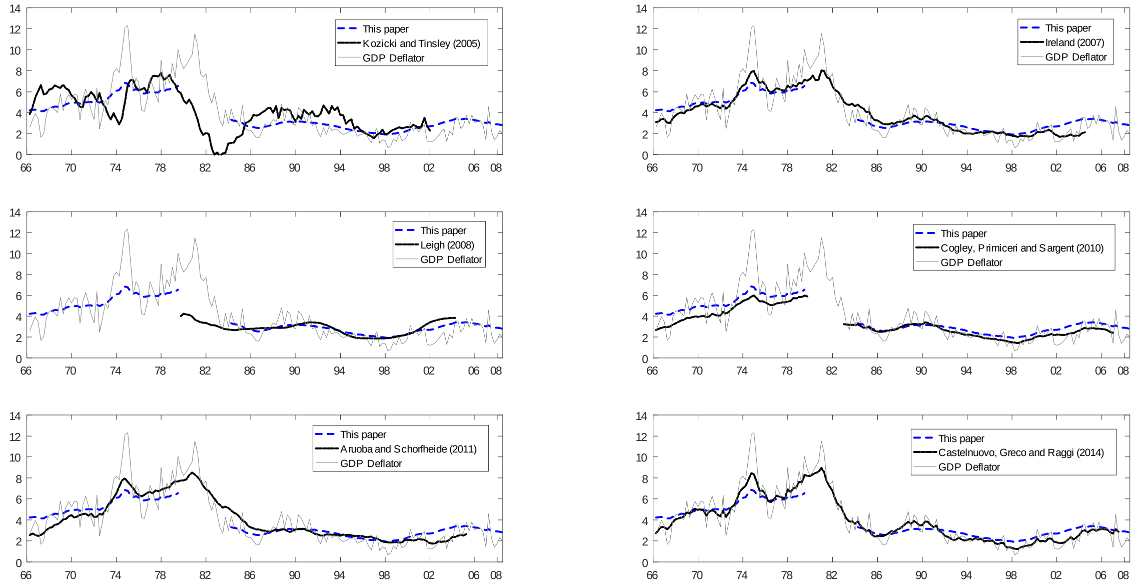

Figure 3: A comparison of inflation target estimates

transitory policy shocks.

As regards the estimates of Cogley, Primiceri and Sargent (2010), the co-movement between the two series is very similar: with a correlation of 0.98 and 0.87 for the pre-Volcker and post-1984 sub-sample respectively.[19] However, the fourth panel in Figure 3 documents clear evidence of a gap between the two inflation target series and points to the essence of trend inflation. While Cogley, Primiceri and Sargent (2010) leave the first moment of observed inflation unmodelled, the current paper overcomes this shortcoming by explicitly modelling inflation's long-run value (by log-linearizing around a positive steady state) on top of its dynamics.

The implicit inflation target is also close to that of Ireland (2007)[20], Aruoba and Schorfheide (2011) and Castelnuovo, Greco and Raggi (2014), particularly for the pre-Volcker period for which the correlation reads 0.99, 0.99 and 0.97 respectively. However, the estimated target turns out to be much smoother and somewhat different than theirs in the second sub-sample. In particular, since the early 2000s, there is a clear divergence. During this period, the estimate turns out to be higher than

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[19] The numbers are conditional on overlapping periods, i.e. 1966:I - 1979:II for the first sub-sample and 1984:I - 2006:IV for the second sub-sample.

[20] Ireland (2007) studies different inflation target processes, including some which allow for a systematic reaction to structural shocks hitting the economy. The second panel in Figure 3 plots the one labelled as "Federal Reserve's Target as Implied by the Constrained Model with an Exogenous Inflation Target" (see Figure 5, page 1869 in the published paper).

the alternative measures as well as actual inflation itself. This finding is intuitive and captures the fear of deflation among policymakers at that time which led to extra easy monetary policy and lowering of the Federal Funds rate.[21] As noted by Eggertsson and Woodford (2003), keeping interest rates low for an extended period of time is equivalent to a rise in the time-varying inflation target.

The estimated target is also similar to Leigh (2008) who uses a time-varying parameter Taylor rule and the Kalman filter focusing on the post-1980 sample period alone.[22] As in Leigh (2008, p. 2022-23), the time-varying implicit inflation target for the post-1984 sub-sample can be divided into separate chunks: (i) 'the opportunistic approach to disinflation' - a period covering from mid-1980s to mid-1990s - during which according to Orphanides and Wilcox (2002) the Fed did not take deliberate anti-inflation action but rather waited for external circumstances to deliver the desired reduction in inflation; (ii) 'the low-inflation equilibrium' in the late 1990s; and (iii) 'the deflation scare' in the early 2000s during which the inflation target rose above actual inflation.[23]

Finally, as a note of caution, one must be careful in drawing these comparisons. The differences could be due to differences in investigated samples, data transformation, structure imposed on the data and vintage of the data.

7 What explains the Great Moderation in the U.S.?

What are the reasons behind the decline in macroeconomic volatility and inflation gap predictability? To answer this question, the paper conducts counterfactual exercises following Castelnuovo (2010) and Cogley, Primiceri and Sargent (2010). The objective here is to disentangle the role played by good policy and good luck. In comparison to these studies, the exercises are still meaningful as the current paper estimates a model log-linearized around a positive steady state inflation rate. Ascari

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[21] See Bernanke (2002, 2010) and Bernanke and Reinhart (2004).

[22] Leigh (2008) focuses on estimating the implicit target based on both core PCE inflation and GDP/GNP implicit deflator inflation. The third panel in Figure 3 plots the one labelled as "Estimate of GDP/GNP deflator target (real-time forecasts)" (see Figure 5, page 2028 in the published paper).

[23] For alternative interpretation of monetary policy during the 2000s, see Groshenny (2013), Belongia and Ireland (2016) and Doko Tchatoka, Groshenny, Haque and Weder (2017).

Table 6: Implications of the model for volatility and predictability

		St. Dev	R_1^2	R_4^2	R_8^2
Output growth	1966:I-1979:II	1.10	-	-	-
	1984:I-2008:II	0.58	-	-	-
	Percent Change	-47	-	-	-
Mean-based Inflation Gap	1966:I-1979:II	1.46	0.94	0.90	0.87
	1984:I-2008:II	0.52	0.88	0.82	0.79
	Percent Change	-64	-6	-9	-9
Target-based Inflation Gap	1966:I-1979:II	0.81	0.84	0.78	0.74
	1984:I-2008:II	0.23	0.45	0.31	0.27
	Percent Change	-72	-46	-60	-64

and Ropele (2007, 2009) and Ascari and Sbordone (2014) show that this modelling assumption substantially alters the NKPC relationship and hence it changes the inflation dynamics. This assumption also facilitates analysis using both mean-based and target-based inflation gap.

Table 6 summarizes the model's implications for the volatility and predictability of the inflation gap and the volatility of output growth at the posterior mean of the model parameters. First and foremost, the estimated model is able to replicate the observed drop in output growth and inflation-gap volatility. The paper finds a fall of output growth variability of 47%, and a drop of mean-based and target-based inflation gap volatility of about 64% and 72% respectively.[24] The figures are similar to those reported in the literature. For instance, Justiniano and Primiceri (2008) report a fall of output growth variability of about 25% and a drop of inflation variability of about 75%. The numbers in Smets and Wouters (2007) read 35% and 58% respectively.

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[24] The data used in estimation implies a fall of the standard deviation of output growth of about 48% and that of inflation of about 57%.

This paper also focuses on the persistence of the inflation gap using the R_j^2 statistic proposed by Cogley, Primiceri and Sargent (2010).[25] To measure persistence at a given date t, these authors propose to calculate the fraction of the total variation in the inflation gap that is due to shocks inherited from the past relative to those that will occur in the future. They suggest that this is equivalent to one minus the fraction of the total variation due to future shocks. Since future shocks account for the forecast error, they express this as one minus the ratio of the conditional variance to the unconditional variance where j denotes the forecast horizon.

Table 6 reports R_j^2 statistic for inflation-gap predictability for forecast horizons of one, four and eight quarters. Similar to the findings reported in Cogley, Primiceri and Sargent (2010), there is a marked decline in the persistence of time-varying target-based gap at all three horizons. However, it is remarkably muted for mean-based inflation gap. This result shows that the persistence of these two series is considerably different, a finding in line with the autocorrelation of the two series based on pre and post-Volcker data reported in Ascari and Sbordone (2014). Moreover, it is also in line with Benati (2008) who fails to detect a change in raw inflation persistence in the U.S. around the time of the Volcker stabilization. Importantly, both mean-based inflation gap and raw inflation remained persistent as inflation target continued to drift after the Volcker disinflation. Instead, it is time-varying target-based inflation gap that has become less persistent. Hence, the results shed further light on the findings of Cogley and Sargent (2002), Cogley, Primiceri and Sargent (2010) on the one hand and Benati (2008) on the other.

7.1 Counterfactuals

Next the paper conducts counterfactual exercises designed to disentangle the role played by good policy and good luck in explaining the Great Moderation where it closely follows the counterfactual scenarios studied in Castelnuovo (2010) and Cogley, Primiceri and Sargent (2010). Following these authors, the paper divides the

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[25] Using this measure of persistence based on short- and medium-term predictability within a simple New Keynesian model, Benati and Surico (2008) show that a more aggressive poliy stance towards inflation causes a decline in inflation predictability. However, they estimate the model for the Great Moderation period only, thus stopping short of using the methodology of Lubik and Schorfheide (2003, 2004) to allow for indeterminacy and estimate the model during the Great Inflation period as well.

Table 7: Counterfactual standard deviations

Scenarios	Output growth		Mean-based inflation gap		Target-based inflation gap	
	St. Dev	% Change	St. Dev	% Change	St. Dev	% Change
Policy 2, Private 1	1.09	-1	0.58	-60	0.26	-68
ψ_π, ψ_x, $\psi_{\Delta y}$, ρ_r	1.09	-1	0.97	-34	0.39	-52
ψ_π	1.10	0	0.93	-36	0.28	-65
π^*	1.10	0	1.44	-1	0.79	-2
σ_{π^*}	1.10	0	0.90	-38	0.55	-32
Policy 1, Private 2	0.59	-46	1.49	+2	0.85	+5
σ_g	0.59	-46	1.45	-1	0.80	-1

experiment into two broad categories. First, it combines the parameters pertaining to the Taylor rule, i.e. ψ_π, ψ_x, $\psi_{\Delta y}$, ρ_r, π^*, σ_r, σ_{π^*}, of the post-1984 sub-sample with the private sector parameters of the pre-1979 period which is called 'Policy 2, Private 1'. This exercise is designed to capture the role of better monetary policy in reducing the volatility of the inflation gap (both mean-based and target-based) and output growth and the persistence of target-based inflation gap series. In the second category, it combines private sector parameters of the second sub-sample with the policy parameters of the first. This scenario, labelled 'Policy 1, Private 2', is designed to study the contribution of non-policy factors.

Table 7 reports the counterfactual results for the volatility of output growth and the two inflation gap series. The table reports the standard deviations and the percentage deviations with respect to the pre-Volcker scenario. First and foremost, the decline in inflation-gap volatility is driven by monetary policy (Policy 2, Private 1). However, changes in monetary policy alone cannot explain the decline in output growth variability, a finding shared with Leduc and Sill (2007) and Castelnuovo (2010). As in Leduc and Sill (2007), the decline in output growth variability is mainly explained by the reduction in the volatility of technology shocks. Hence, both good policy and good luck are jointly required to explain the reduction in output growth and inflation-gap volatility.

Digging further, the paper finds that both stronger response to the inflation gap (ψ_π) and better anchored inflation objective, i.e. a reduction in the volatility of inflation target shocks (σ_{π^*}), are key ingredients in the reduction of inflation-

Table 8: Counterfactual predictability

Scenarios	Target-based inflation gap					
	R_1^2	% Change	R_4^2	% Change	R_8^2	% Change
Policy 2, Private 1	0.36	-57	0.24	-69	0.23	-69
ψ_π, ψ_x, $\psi_{\Delta y}$, ρ_r	0.40	-52	0.32	-59	0.31	-58
ψ_π	0.69	-18	0.62	-21	0.59	-20
π^*	0.83	-1	0.77	-1	0.73	-1
σ_{π^*}	0.68	-19	0.58	-26	0.55	-26
Policy 1, Private 2	0.89	+6	0.81	+4	0.75	+1

gap variability. This outcome stands in contrast to Castelnuovo (2010) and Cogley, Primiceri and Sargent (2010) who both find that a stronger response to the inflation gap during the Great Moderation period only plays a minor role. Interestingly, the decline in the Federal Reserve's long-run inflation target (π^*) plays a negligible role. That a reduction in π^* is negligible for the reduced variability of target-based inflation gap is *a-priori* expected as π^* cancels out when looking at log-deviations of the inflation gap, $\pi_t - \pi_t^*$. However, that it is quantitatively unimportant for the variability of mean-based inflation gap as well is much less obvious given the qualitative result in Ascari and Sbordone (2014) that trend inflation affects the volatility of macroeconomic variables.

As regards the decline in inflation-gap persistence, the paper focuses on time-varying target-based inflation gap alone as the decline in the persistence of mean-based gap is rather muted. Table 8 reports the results. The main message from these experiments goes hand in hand with the counterfactuals related to volatility reduction. In particular, better monetary policy, mainly in terms of a stronger response to the inflation gap and a reduced variability of inflation target shocks, is the key driver of the decline in inflation-gap predictability. Moreover, the decline in the Federal Reserve's long-run inflation target, i.e. π^*, plays a quantitatively negligible role.

8 Further investigation

The robustness checks involve (i) testing for indeterminacy using a GNK model with firm-specific labor; and (ii) estimating the model over the entire region of the

Table 9: Determinacy versus Indeterminacy (firm-specific labor)

		Log-data density		Probability	
Sample	Inflation Target	Det	Indet	Det	Indet
1966:I-1979:II	Fixed	-132.27	-120.86	0	1
	Time-varying	-120.68	-123.41	0.94	0.06
1982:IV-2008:IV	Fixed	-46.96	-61.83	1	0
	Time-varying	-47.63	-70.96	1	0

parameter space, i.e. over both determinacy and indeterminacy.

8.1 Firm-specific labor

In contrasting fixed versus time-varying inflation target, the analysis so far has relied on a GNK model with homogenous labor following Ascari and Ropele (2009) and Ascari and Sbordone (2014). The paper finds that a model with time-varying inflation target empirically fits better than one featuring fixed inflation target and determinacy prevails in both the pre-Volcker as well as the post-1984 sample periods. However, Kurozumi and Van Zandweghe (2017) show that a similar model with firm-specific labor is more susceptible to indeterminacy induced by higher trend inflation than a model with homogenous labor. Hence, the paper conducts further investigation along this dimension and estimates the model of Hirose, Kurozumi and Van Zandweghe (2017) who employ firm-specific labor. In order to establish a valid comparison, it uses the exact same set of priors, observables and sample periods as they do.[26] However, to achieve identification between the inflation target process and the policy-rate shock, this paper assumes that the latter follows a transitory i.i.d. process while the former is a highly persistent AR(1) process as before. Table 9 collects the results for the marginal data densities and the posterior model probabilities.[27]

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[26] The pre-1979 period in Hirose, Kurozumi and Van Zandweghe (2017) is the same as in the current paper, i.e. 1966:I - 1979:II, while for the second sub-sample they use a slightly different period ranging from 1982:IV - 2008:IV. The choice of the second sub-sample is innocuous for the findings.

[27] Table 12 in the appendix reports parameter estimates.

In line with Hirose, Kurozumi and Van Zandweghe (2017), the pre-Volcker period is unambiguously characterized by indeterminacy while the post-1982 period is characterized by determinacy under the assumption of a fixed inflation target equal to trend inflation. However, when allowing for time-varying inflation target, determinacy prevails as before. In terms of the empirical fit of fixed versus time-varying target, it is comparable for both the pre-Volcker and the post-1982 period. Given that firm-specific labor makes the model more prone to indeterminacy due to higher trend inflation, this set of results somewhat mitigates, yet does not completely overturn, the main findings. The hypothesis that the Federal Reserve might have pursued a time-varying inflation target and as a consequence determinacy might have prevailed even in the pre-Volcker period is a possibility that cannot be empirically ruled out.

8.2 Estimation over the entire parameter spacer

A difficulty in the methodology of Lubik and Schorfheide (2003) is that the likelihood function of the model is possibly discontinuous at the boundary between the determinacy and indeterminacy region. In order to bridge the gap between the likelihood function and improve the test for indeterminacy, Lubik and Schorfheide (2004) pick $M^*(\theta)$ such that the impulse responses of the endogenous variables to fundamental shocks are continuous at the boundary. Yet, to test for indeterminacy, the authors estimate the model twice, first under determinacy, then under indeterminacy. Arguably, they do so because of the sampling technology available back then, i.e. the RWMH algorithm. However, it can get stuck near a local mode and fail to find the true posterior distribution. While, an importance sampling algorithm like SMC can use a single chain to explore the entire parameter space.

So far this paper has followed the conventional procedure of Lubik and Schorfheide (2004) by estimating separately under determinacy and indeterminacy. Instead, to take full advantage of the SMC algorithm, here the paper estimates the GNK model with homogenous labor over the entire parameter space just as Hirose, Kurozumi and Van Zandweghe (2017) do.[28] The likelihood function is now given by

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[28] For an alternative approach that allows estimation over the entire parameter space while using standard packages and estimation algorithms see Bianchi and Nicolò (2017).

Table 10: Posteriors distribution (Estimation over entire parameter space)

	Posterior Mean [Standard Deviation]			
	1966:I - 1979:II		1984:I - 2008:II	
Name	Fixed Target	Time-varying Target	Fixed Target	Time-varying Target
ψ_π	1.18 [0.18]	2.00 [0.42]	3.11 [0.45]	4.05 [0.57]
ψ_x	0.11 [0.09]	0.12 [0.09]	0.12 [0.10]	0.13 [0.11]
$\psi_{\Delta y}$	0.14 [0.09]	0.17 [0.09]	0.71 [0.19]	0.38 [0.18]
ρ_r	0.42 [0.11]	0.42 [0.13]	0.78 [0.03]	0.72 [0.06]
π^*	1.39 [0.14]	1.28 [0.22]	0.67 [0.05]	0.85 [0.21]
r^*	1.63 [0.15]	1.54 [0.21]	1.44 [0.14]	1.57 [0.20]
g^*	0.52 [0.09]	0.52 [0.09]	0.51 [0.07]	0.51 [0.07]
h	0.45 [0.07]	0.43 [0.07]	0.43 [0.06]	0.41 [0.06]
ξ	0.50 [0.09]	0.44 [0.11]	0.62 [0.06]	0.48 [0.08]
ρ_d	0.79 [0.06]	0.76 [0.07]	0.90 [0.02]	0.92 [0.02]
ρ_g	0.23 [0.06]	0.20 [0.07]	0.22 [0.06]	0.17 [0.04]
σ_r	0.37 [0.05]	0.42 [0.08]	0.20 [0.02]	0.21 [0.03]
σ_d	0.66 [0.21]	0.82 [0.32]	1.55 [0.25]	1.74 [0.36]
σ_g	1.45 [0.19]	1.34 [0.28]	0.76 [0.09]	0.71 [0.07]
σ_{π^*}	–	0.07 [0.03]	–	0.04 [0.01]
σ_ζ	0.57 [0.29]	0.58 [0.31]	0.54 [0.23]	0.55 [0.23]
$M_{r,\zeta}$	0.17 [1.04]	-0.02 [0.99]	-0.06 [1.01]	0.07 [0.99]
$M_{d,\zeta}$	0.03 [1.04]	0.00 [0.99]	0.05 [1.00]	-0.03 [1.00]
$M_{g,\zeta}$	-0.05 [0.98]	-0.02 [0.99]	0.00 [1.01]	0.01 [0.99]
$M_{\pi^*,\zeta}$	–	-0.03 [1.02]	–	-0.02 [0.98]
$\log p(X_T)$	-124.87	-122.36	-32.31	-28.76
$P\{\theta_S \in \Theta^D \mid X_T\}$	0.67	1.00	1.00	1.00

$\log p(X_T)$ represents the SMC-based approximation of the log marginal data density and $P\{\theta_S \in \Theta^D \mid X_T\}$ denotes the posterior probability of determinacy of equilibrium.

$$p(X_T|\theta_S, S) = 1\{\theta_S \in \Theta^D\}p^D(X_T|\theta_D, D) + 1\{\theta_S \in \Theta^I\}p^I(X_T|\theta_I, I),$$

where Θ^D, Θ^I are the determinacy and indeterminacy regions of the parameter space, $1\{\theta_S \in \Theta^S\}$ is the indicator function that equals 1 if $\theta_S \in \Theta^S$ and zero otherwise, and $p^D(X_T|\theta_D, D)$, $p^I(X_T|\theta_I, I)$ are the likelihood functions under determinacy and indeterminacy respectively.

Table 10 confirms that the findings are robust: indeterminacy can be confidently ruled out during the pre-Volcker period under time-varying inflation target, while there is still a possibility that it might have prevailed under a fixed target. Moreover, time-varying target fits better in both periods. The parameter estimates are also similar to the ones from the respectively favored models when estimated separately under determinacy and indeterminacy. Finally, the paper also estimates the model with firm-specific labor over the entire region. Once again, the results remain robust and are delegated to the appendix.[29]

9 Conclusion

This paper estimates a Generalized New Keynesian model with positive trend inflation. While allowing for indeterminacy, it assesses the empirical fit of fixed versus time-varying inflation target for the Great Inflation and the Great Moderation period. Several notable findings arise. First, when considering the model with fixed inflation target, the paper finds that indeterminacy cannot be ruled out in the pre-Volcker period while there is a switch to determinacy after the Volcker-disinflation. However, determinacy unambiguously prevails in both sample periods when the monetary authority follows a time-varying inflation target instead. The data support the model with time variation in the central bank's inflation objective as being empirically superior with respect to the standard constant-target model. To the best of my knowledge, this paper is the first one to test for indeterminacy using a full–information likelihood-based approach while comparing the fit of fixed versus time-varying target. The finding that even the pre-Volcker period could possibly be characterized by determinacy is a novel result. Furthermore, counterfactual simulations suggest that both good policy and good luck are jointly required to explain

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[29] See Table 13 in the appendix.

the Great Moderation. The decline in inflation-gap volatility and predictability is driven by better monetary policy, both in terms of a more active response to the inflation gap and a more anchored inflation target. In contrast, the reduction in output growth variability is mainly explained by reduced volatility of technology shocks.

The paper choose to make these arguments by assuming that trend inflation is positive but constant while the Federal Reserve pursues a time-varying inflation target. This choice helps to keep the analysis simple yet related to existing research. However, one could depart instead by log-linearizing the equilibrium conditions around a steady state characterized by drifting trend inflation which would result in a New Keynesian Phillips curve with drifting coefficients. DSGE models with time-varying coefficients and stochastic volatilities have been estimated by Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010). I wish to pursue these lines of research in the future.

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Appendix A

The artificial economy is a variant of the Generalized New Keynesian (GNK) model of Ascari and Sbordone (2014) and so the description of the model below draws heavily from their exposition. The model consists of a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. The behavior of these agents are described as follows.

A.1 Model

Households

The representative agent's preferences depend on consumption of final goods, $\widetilde{C}_t$, and labor,N_t, and they are represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t u(\widetilde{C}_t, N_t) \qquad 0 < \beta < 1$$

which the agent acts to maximize. Here, E_0 represents the expectations operator. The term d_t stands for a shock to the discount factor, β, which follows the stationary autoregressive process

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t}$$

where $\epsilon_{d,t}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_d. The period utility is additively separable in consumption and labor and it takes on the functional form

$$u(\widetilde{C}_t, N_t) = \ln\left(\widetilde{C}_t - hC_{t-1}\right) - d_n \frac{N_t^{1+\varphi}}{1+\varphi} \qquad\qquad d_n > 0,\, \varphi \geq 0.$$

Logarithmic utility is the only additive-separable form consistent with balanced growth. The term φ is the inverse of the Frisch labor supply elasticity and d_n governs the disutility of working in steady state.

The period by period budget constraint is given by

$$P_t\widetilde{C}_t + R_t^{-1}B_t = W_tN_t - T_t + D_t + B_{t-1}$$

where R_t is the gross nominal interest rate on bonds, B_t is one-period bond holdings, W_t is the nominal wage rate, T_t is lump sum taxes, and D_t is the profit income. The representative consumer's problem is to maximize the expected discount intertemporal utility subject to the budget constraint. The first-order conditions with respect to consumption, labor supply and bond holdings yield

$$\Xi_t = \frac{d_t}{C_t - hC_{t-1}}$$

$$\frac{W_t}{P_t} = \frac{d_n d_t N_t^{\varphi}}{\Xi_t}$$

$$1 = E_t \frac{\beta \Xi_{t+1}}{\Xi_t} \frac{R_t}{\pi_{t+1}}$$

where Ξ_t is the marginal utility of consumption, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate of the final-good price.

Firms

Firms come in two forms. Final-good firms produce output that can be consumed. This output is made from the range of differentiated goods that are supplied by intermediate-good firms who have market power.

Final-good firm

In each period t, a final good, Y_t, is produced by a perfectly competitive representative final-good firm, by combining a continuum of intermediate inputs, $Y_{i,t}$, $i \in [0,1]$, via the technology

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the elasticity of substitution among intermediate inputs. The first-order condition for profit maximization yields the final-good firm's demand for intermediate good i

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t.$$

The final-good market clearing condition is given by

$$Y_t = C_t.$$

Intermediate-good firms

Each intermediate-good firm i produces a differentiated good $Y_{i,t}$ under monopolistic competition using the production function

$$Y_{i,t} = A_t N_{i,t},$$

where A_t denotes the level of technology and follows the stochastic process

$$\ln A_t = \ln g + \ln A_{t-1} + g_t,$$

where g is the steady-state gross rate of technological progress which is also equal to the steady-state balanced growth rate, and g_t is a non-stationary technology shock which follows an AR(1) process

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t},$$

where $\epsilon_{g,t}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_g.

Unlike, Ascari and Sbordone (2014) I assume stochastic growth modelled as the technology level following a unit root process. The labor demand and the real marginal cost of firm i is therefore given by

$$N_{i,t}^d = \frac{Y_{i,t}}{A_t},$$

and

$$MC_{i,t} = \frac{W_t/P_t}{A_t}.$$

Due to the assumption of constant returns to scale and perfectly competitive labor markets, the real marginal cost of firm i, $MC_{i,t}$, depends only only aggregate variables and thus are the same across firms, i.e. $MC_{i,t} = MC_t$.

Firms' price-setting

The intermediate goods producers face a constant probability, $0 < \xi < 1$, of being able to adjust prices to a new optimal one, $P_{q,t}^*(i)$, in order to maximize expected discounted profits

$$E_t \sum_{j=0}^{\infty} \xi^j \beta^j \frac{\lambda_{t+j}}{\lambda_0} \left[\frac{P_{i,t}^* \left(\pi^{\omega j}\right)^{1-\mu} \left(\pi_{t-1|t+j-1}^{\omega}\right)^{\mu}}{P_{q,t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \frac{Y_{i,t+j}}{A_{t+j}} \right]$$

subject to the constraint

$$Y_{i,t+j} = \left[\frac{P_{i,t}^* \left(\pi^{\omega j}\right)^{1-\mu} \left(\pi_{t-1|t+j-1}^{\omega}\right)^{\mu}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j}$$

and

$$\begin{aligned} \pi_{t|t+j} &= \frac{P_{t+1}}{P_t} \times \frac{P_{t+2}}{P_{t+1}} \times ... \times \frac{P_{t+j}}{P_{t+j-1}} && \text{for } j \geqslant 1 \\ &= 1 && \text{for } j = 0 \end{aligned}$$

where π denotes the central bank's long-run inflation target and is equal to the level of trend inflation, $\Lambda_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_0}$ is the stochastic discount factor. This formulation is general as $\omega \in [0,1]$ allows for any degree of price indexation and $\mu \in [0,1]$ allows for any degree of geometric combination of the two types of indexation usually employed in the literature: to steady state inflation and to past inflation rates.

The first order condition for the optimized relative price $p_{i,t}^* (= \frac{P_{i,t}^*}{P_t})$ is given by

$$p_{i,t}^{*} = \frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{+j}} \left[\frac{Y_{t+j}}{A_{t+j}}\right] \left[\frac{\left(\pi^{\omega j}\right)^{1-\mu} \left(\pi_{t-1|t+j-1}^{\omega}\right)^{\mu}}{\pi_{t|t+j}}\right]^{-\varepsilon}}{E_t \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j} \left[\frac{(\pi^{\omega j})^{1-\mu} \left(\pi_{t-1|t+j-1}^{\omega}\right)^{\mu}}{\pi_{t|t+j}}\right]^{1-\varepsilon} Y_{t+j}}.$$

Moreover, the aggregate price level evolves according to:

$$\begin{aligned}
P_t &= \left[\int_0^1 {P_{i,t}}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\
1 &= \left[\xi \left(\pi^{1-\mu} \pi_{t-1}^{\mu}\right)^{\omega(1-\varepsilon)} \pi_t^{\varepsilon-1} + (1-\xi) \left(\frac{P_{i,t}^{*}}{P_t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \\
p_{i,t}^{*} &= \left[\frac{1-\xi\pi^{(1-\varepsilon)(1-\mu)\omega} \pi_{t-1}^{(1-\varepsilon)\mu\omega} \pi_t^{\varepsilon-1}}{1-\xi}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

Lastly, define price dispersion $S_t \equiv \int_0^1 (\frac{P_{i,t}}{P_t})^{-\varepsilon} di$. Under the Calvo price mechanism, the above expression can be written recursively as:

$$S_t = (1-\xi) {p_{i,t}^{*}}^{-\varepsilon} + \xi \pi^{-\varepsilon\omega(1-\mu)} \pi_{t-1}^{-\varepsilon\omega\mu} \pi_t^{\varepsilon} S_{t-1}$$

Recursive formulation of the optimal price-setting equation

The joint dynamics of the optimal reset price and inflation can be compactly described by rewriting the first-order condition for the optimal price in a recursive formulation as follows:

$$p_{i,t}^{*} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\psi_t}{\phi_t},$$

where ψ_t and ϕ_t are auxiliary variables that allow one to rewrite the infinite sums that appear in the numerator and denominator of the above equation in recursive formulation:

$$\psi_t = E_t \sum_{j=0}^{\infty} (\xi\beta)^j \pi_{t|t+j}^{\varepsilon} \frac{W_{t+j}}{P_{+j}} \left[\frac{Y_{t+j}}{A_{t+j}}\right] \Xi_{t+j} \pi^{-\varepsilon(1-\mu)\omega j} \pi_{t-1|t+j-1}^{-\varepsilon\mu\omega}$$

and

$$\phi_t = E_t \sum_{j=0}^{\infty} (\xi\beta)^j \pi_{t|t+j}^{\varepsilon-1} Y_{t+j} \Xi_{t+j} \pi^{(1-\mu)(1-\varepsilon)\omega j} \pi_{t-1|t+j-1}^{\mu\omega(1-\varepsilon)}.$$

Note that in defining these two auxiliary variables, we used the definition $\Xi = \frac{d_t}{C_t - hC_{t-1}} = \frac{d_t}{Y_t - hY_{t-1}}$.

Monetary Policy

Lastly, the central bank's policy is described by a Taylor rule

$$\log R_t = \rho_r \log R_{t-1} + (1-\rho_r) \left[\begin{array}{c} \log r + \psi_\pi (\log \pi_t - \log \pi_t^*) + \psi_x \log x_t + \\ \psi_{\Delta y} \left(\log \frac{Y_t}{Y_{t-1}} - \log g \right) \end{array} \right] + \epsilon_{r,t} \qquad 0 \leq \rho_r < 1.$$

where x_t is the output gap, $\epsilon_{r,t}$ is an i.i.d. monetary policy shock, $r \geq 1$ is the steady state gross policy rate. The parameters ψ_π, ψ_x and $\psi_{\Delta y}$ govern the central bank's responses to inflation, output gap and output growth respectively, and $\rho_r \in [0, 1]$ is the degree of policy rate smoothing . Here π_t^* denotes the central's banks time varying inflation target that is assumed to follow an exogenous process

$$\ln \pi_t^* = \rho_{\pi^*} \ln \pi_{t-1}^* + \epsilon_{\pi^*,t},$$

where $\epsilon_{\pi^*,t}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_{π^*}. Under fixed inflation target, I assume that the policy rules becomes

$$\log R_t = \rho_r \log R_{t-1} + (1-\rho_r) \left[\begin{array}{c} \log r + \psi_\pi (\log \pi_t - \log \pi) + \psi_x \log x_t + \\ \psi_{\Delta y} \left(\log \frac{Y_t}{Y_{t-1}} - \log g \right) \end{array} \right] + \epsilon_{r,t},$$

where the central bank's inflation target is equal to steady-state inflation or trend inflation π and the output gap is defined as

$$x_t = \frac{Y_t}{Y_t^n},$$

and Y_t^n is the natural rate of output. By considering flexible prices, the law of motion for Y_t^n is given by

$$\left(\frac{Y_t^n}{A_t} \right)^{1+\varphi} = \frac{\varepsilon - 1}{\varepsilon d_n} + h \left(\frac{Y_t^n}{A_t} \right)^{\varphi} \frac{Y_{t-1}^n}{A_t}.$$

Table 11: Fixed target and Homogenous labor

	Posterior Mean [Standard Deviation]		
	1966:I - 1979:II		1984:I - 2008:II
Name	Determinacy	Indeterminacy	Determinacy
ψ_π	1.25 [0.15]	1.00 [0.10]	3.17 [0.47]
ψ_x	0.10 [0.08]	0.14 [0.10]	0.12 [0.09]
$\psi_{\Delta y}$	0.14 [0.08]	0.12 [0.08]	0.73 [0.20]
ρ_r	0.42 [0.11]	0.44 [0.10]	0.78 [0.03]
π^*	1.38 [0.13]	1.39 [0.16]	0.67 [0.05]
r^*	1.62 [0.14]	1.62 [0.16]	1.43 [0.14]
g^*	0.51 [0.09]	0.51 [0.10]	0.51 [0.07]
h	0.45 [0.07]	0.47 [0.07]	0.43 [0.06]
ξ	0.48 [0.08]	0.53 [0.09]	0.62 [0.06]
ρ_d	0.80 [0.05]	0.78 [0.07]	0.91 [0.02]
ρ_g	0.23 [0.05]	0.25 [0.07]	0.23 [0.06]
σ_r	0.38 [0.05]	0.34 [0.04]	0.19 [0.02]
σ_d	0.69 [0.19]	0.57 [0.20]	1.61 [0.29]
σ_g	1.44 [0.18]	1.47 [0.22]	0.75 [0.09]
σ_{π^*}	–	–	–
σ_ζ	–	0.61 [0.36]	–
$M_{r,\zeta}$	–	0.54 [0.96]	–
$M_{d,\zeta}$	–	0.16 [1.06]	–
$M_{g,\zeta}$	–	−0.13 [1.00]	–
$M_{\pi^*,\zeta}$	–	–	–

Table 12: Firm-specific labor

	Posterior Mean [Standard Deviation]			
	1966:I - 1979:II		1982:IV - 2008:IV	
Name	Fixed Target Indeterminacy	Time-varying Target Determinacy	Fixed Target Determinacy	Time-varying Target Determinacy
ψ_π	0.98 [0.46]	2.54 [0.51]	2.66 [0.43]	3.11 [0.55]
ψ_x	0.25 [0.11]	0.18 [0.10]	0.09 [0.07]	0.11 [0.09]
$\psi_{\Delta y}$	0.14 [0.10]	0.20 [0.14]	0.86 [0.18]	0.75 [0.20]
ρ_r	0.77 [0.07]	0.75 [0.05]	0.83 [0.02]	0.82 [0.03]
π^*	1.48 [0.22]	1.35 [0.23]	0.71 [0.08]	1.04 [0.26]
r^*	1.66 [0.18]	1.59 [0.21]	1.43 [0.16]	1.66 [0.24]
g^*	0.45 [0.14]	0.49 [0.12]	0.43 [0.10]	0.42 [0.10]
h	0.56 [0.08]	0.59 [0.09]	0.56 [0.06]	0.58 [0.06]
ξ	0.53 [0.04]	0.51 [0.04]	0.48 [0.05]	0.49 [0.05]
ρ_d	0.50 [0.18]	0.46 [0.17]	0.92 [0.02]	0.90 [0.03]
ρ_g	0.64 [0.24]	0.53 [0.21]	0.25 [0.14]	0.31 [0.19]
σ_r	0.27 [0.03]	0.29 [0.03]	0.17 [0.01]	0.16 [0.01]
σ_d	1.71 [1.24]	2.09 [0.93]	2.37 [0.51]	2.22 [0.41]
σ_g	0.58 [0.24]	0.57 [0.22]	1.12 [0.19]	1.04 [0.27]
σ_{π^*}	–	0.10 [0.04]	–	0.04 [0.02]
σ_ζ	0.38 [0.06]	–	–	–
$M_{r,\zeta}$	-0.29 [0.43]	–	–	–
$M_{d,\zeta}$	0.00 [0.25]	–	–	–
$M_{g,\zeta}$	0.18 [0.42]	–	–	–
$M_{\pi^*,\zeta}$	–	–	–	–

Table 13: Firm-specific labor - estimation over entire parameter space

	Posterior Mean [Standard Deviation]			
	1966:I - 1979:II		1982:IV - 2008:IV	
Name	Fixed Target	Time-varying Target	Fixed Target	Time-varying Target
ψ_π	0.91 [0.40]	2.51 [0.56]	2.67 [0.43]	3.18 [0.58]
ψ_x	0.28 [0.13]	0.19 [0.11]	0.08 [0.06]	0.13 [0.10]
$\psi_{\Delta y}$	0.12 [0.09]	0.20 [0.12]	0.82 [0.16]	0.65 [0.18]
ρ_r	0.77 [0.07]	0.75 [0.05]	0.83 [0.02]	0.82 [0.03]
π^*	1.52 [0.22]	1.34 [0.22]	0.71 [0.07]	1.01 [0.28]
r^*	1.67 [0.18]	1.58 [0.21]	1.44 [0.16]	1.63 [0.26]
g^*	0.44 [0.14]	0.49 [0.13]	0.44 [0.10]	0.42 [0.10]
h	0.54 [0.08]	0.57 [0.09]	0.57 [0.06]	0.60 [0.06]
ξ	0.53 [0.04]	0.51 [0.04]	0.48 [0.04]	0.47 [0.05]
ρ_d	0.48 [0.18]	0.47 [0.17]	0.92 [0.02]	0.90 [0.03]
ρ_g	0.67 [0.23]	0.58 [0.19]	0.23 [0.12]	0.30 [0.19]
σ_r	0.27 [0.03]	0.29 [0.03]	0.17 [0.01]	0.16 [0.01]
σ_d	1.54 [1.08]	1.92 [0.89]	2.32 [0.45]	2.25 [0.41]
σ_g	0.57 [0.22]	0.58 [0.23]	1.16 [0.18]	1.06 [0.28]
σ_{π^*}	–	0.09 [0.04]	–	0.04 [0.02]
σ_ζ	0.38 [0.06]	0.64 [0.35]	0.60 [0.24]	0.56 [0.20]
$M_{r,\zeta}$	-0.26 [0.42]	-0.07 [1.01]	0.00 [0.98]	0.09 [1.01]
$M_{d,\zeta}$	-0.01 [0.25]	0.03 [1.00]	0.11 [0.98]	0.04 [0.98]
$M_{g,\zeta}$	0.25 [0.32]	-0.03 [0.99]	0.06 [1.00]	0.07 [1.00]
$M_{\pi^*,\zeta}$	–	0.07 [0.99]	–	0.11 [0.98]
$\log p(X_T)$	-120.23	-120.87	-48.42	-49.42
$P\{\theta_S \in \Theta^D \vert X_T\}$	0.00	0.98	1.00	1.00

Results are based on 10,000 particles from the final stage in the SMC algorithm. $\log p(X_T)$ represents the SMC-based approximation of the log marginal data density and $P\{\theta_S \in \Theta^D | X_T\}$ denotes the posterior probability of determinacy of equilibrium.

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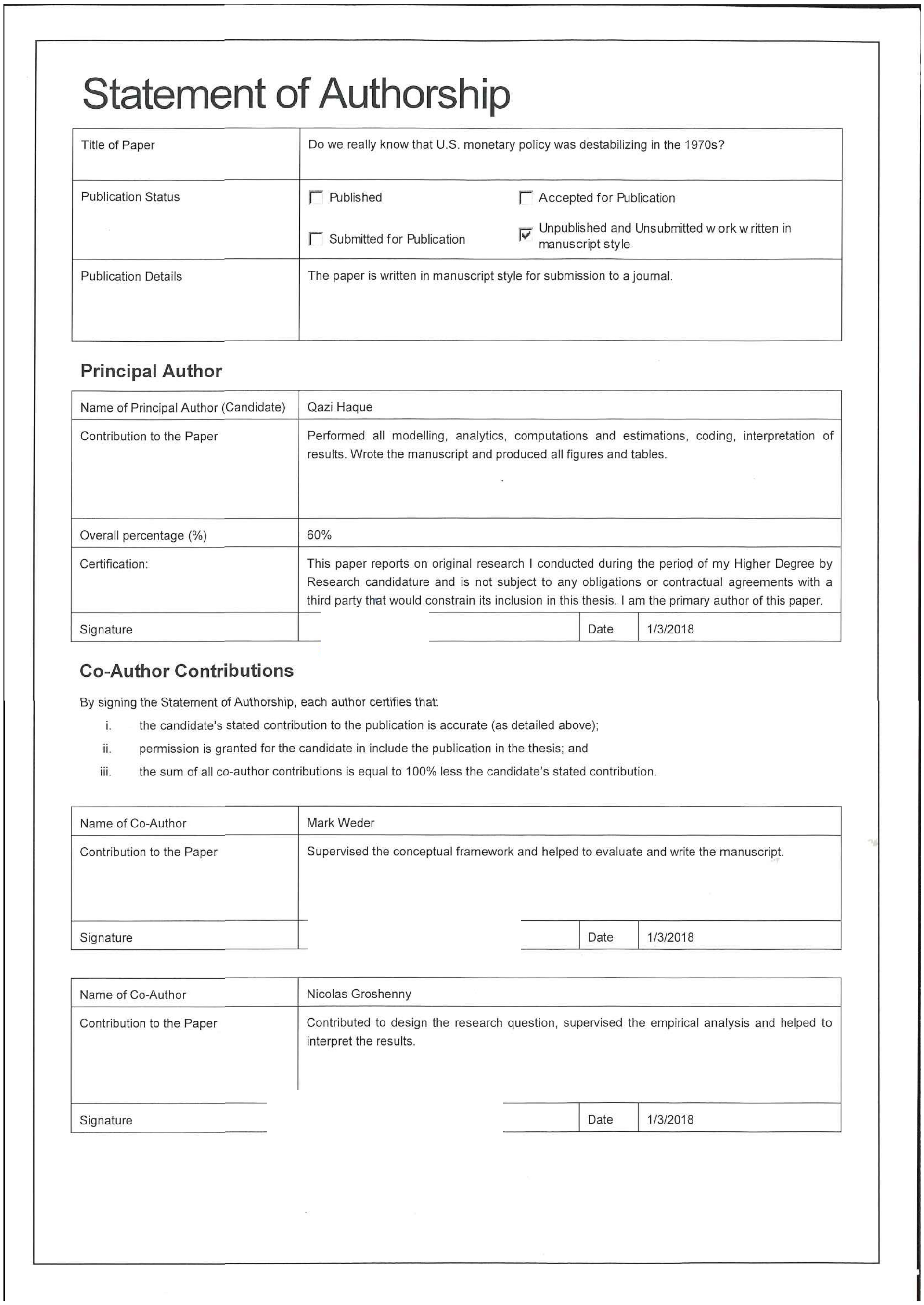

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Contribution to the Paper	Performed all modelling, analytics, computations and estimations, coding, interpretation of results. Wrote the manuscript and produced all figures and tables.		
Overall percentage (%)	60%		
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IV Do we really know that U.S. monetary policy was destabilizing in the 1970s?

This paper examines the role played by oil price shocks and monetary policy with a particular focus on the Great Inflation. Using Bayesian estimation techniques with a Sequential Monte Carlo algorithm while allowing for indeterminacy, we estimate a sticky price model with trend inflation and oil entering in both consumption and production. We find that the US economy during the pre-Volcker period is best described by a determinate version of the model that features a high degree of real wage rigidity. In this environment, the oil price shocks of the 1970s created an acute trade-off between inflation and output-gap stabilization. Faced with this dilemma, the Federal Reserve chose to react forcefully both to inflation and output growth, but not to the output gap, thereby preventing the appearance of multiple equilibria and sunspot shocks. We further document that oil price shocks are no longer as stagflationary as they used to be owing to lower real wage rigidity, thereby explaining the resilience of the U.S. economy to the sustained oil price increases in the 2000s.

1 Introduction

The Great Inflation episode is one of the defining macroeconomic events of the 20th century. From the late 1960s and throughout the 1970s, the U.S. economy not only went through rates of unemployment not seen since the 1930s but also at the same time experienced high and volatile inflation. This historical record was followed by a decline in macroeconomic volatility since the early 1980s, a phenomenon dubbed the Great Moderation. Since the seminal work of Clarida, Gali and Gertler (2000) and Lubik and Schorfheide (2004), the monetary policy literature has attributed the high inflation episode in the 1970s to self-fulfilling expectations-driven fluctuations arising due to "dovish" monetary policy. Clarida, Gali and Gertler (2000) were the first to argue that passive response to inflation in the pre-Volcker period resulted in equilibrium indeterminacy and opened the door to sunspot fluctuations which ultimately led to a higher level and volatility of inflation as well as overall macroeconomic instability. The switch to “hawkish” policy with the appointment of Paul Volcker as Federal Reserve Chairman stabilized inflation expectations, led to determinacy and removed sunspots as a source of economic instability. However, as pointed out by Bilbiie and Straub (2013), this ‘indeterminacy-based’ explanation of the Great Inflation has one obvious complication: sunspot shocks are demand-driven in nature as it increases both inflation and output whereas there were recurrent episodes of recession in the 1970s (hence the term Great Stagflation).

In contrast, an alternative hypothesis points to the role of commodity price shocks as an important source of economic fluctuations. For instance, Hamilton (1983) argues that most U.S. recessions were Granger caused by increases in the price of crude oil. A competing view suggested by Bernanke, Gertler and Watson (1997) is that the Great Stagflation is linked to the endogenous response of the Federal Reserve to exogenous oil price shocks. According to this view, policy-makers raised interest rates in response to the inflationary pressures caused by oil price shocks, thereby causing a deep recession that wouldn’t have occurred otherwise. Yet, this view is also disputed by many. Amongst them, Barsky and Kilian (2002) challenge the view that oil price changes were exogenous and provide evidence that the rise in oil prices in the 1970s was a response to macroeconomic forces, ultimately driven by shifts toward a less restrictive monetary policy regime following the breakdown of Bretton Woods. Nonetheless, such adverse cost-push shocks arguably generated a trade-off between stabilizing inflation and stabilizing the output gap for the Federal Reserve.

Existing empirical investigations that find passive response to inflation in the 1970s have largely disregarded the effect of commodity price fluctuations and the associated trade-off. Indeed with no trade-off between stabilizing inflation and the output gap, full price stability becomes optimal. The fact that inflation was highly volatile in the 1970s suggests that either policy was far from optimal or indeed there was a policy trade-off.

In this paper, we revisit the ‘indeterminacy-based’ explanation of the Great Inflation by adopting a framework which takes into account this trade-off faced by the central bank in the wake of adverse commodity price shocks. Toward this end, we extend the simple NK model with a role for oil in production and consumption and further allow for real wage rigidity as a mechanism that generates quantitatively meaningful trade-off faced by the central bank following Blanchard and Gali (2007, 2010). In this endeavor, what sets us apart from them is that we model inflation's long-run value on top of its dynamics by allowing for positive steady state or trend inflation. Recent theoretical work by Hornstein and Wolman (2005), Kiley (2007) and Ascari and Ropele (2009) has shown that the Taylor principle breaks down when trend inflation is positive. For instance, using a NK framework Ascari and Ropele (2009) and Ascari and Sbordone (2014) show that trend inflation makes price-setting firms more forward-looking which flattens the New Keynesian Phillips Curve (henceforth NKPC) and widens the indeterminacy region. To achieve a unique rational expectations equilibrium thus requires a stronger response to the inflation gap at higher levels of trend inflation. Moreover, the stronger the response to the output gap, the stronger the response to the inflation gap needs to be in order to guarantee determinacy. In a related study, Coibion and Gorodnichenko (2011) document that inertial policies also tend to stabilize inflation expectations as does a stronger response to output growth. Hence, to reassess the evidence of indeterminacy in the 1970s in the face of commodity price shocks, one must also take into account the level of trend inflation.

We estimate the model using Bayesian estimation techniques over the Great Inflation and the Great Moderation period. On top of reassessing the evidence of loose monetary policy in the 1970s, this further allows us to study changes in monetary policy as well as changes in the propagation of commodity price shocks over time. Our results read as follows. First, we find that when considering the model without any role for oil, indeterminacy prevails in the 1970s while determinacy gets favored in the Great Moderation period. This finding is in line with the empirical monetary policy literature, for instance, Clarida, Gali and Gertler (2000), Lubik and Schorfheide (2004) and Coibion and Gorodnichenko (2011). Second, once we turn on oil in the model, we find mixed evidence for indeterminacy in the 1970s, i.e. we can neither rule in nor rule out indeterminacy. However, important aspects of the analysis, such as commodity price shocks and the degree of real wage rigidity, remain not properly identified. Third, through careful elucidation of observables to sharpen the identification of these key features of the model, we then find that the Federal

Reserve has been responding aggressively to inflation even in the 1970s to the extent that it completely rules out indeterminacy as a possibility. These findings suggest that parameter estimates pertaining to the Taylor rule are biased when abstracting from modelling commodity price fluctuations and the associated trade-off. In fact, once we take this into account, our empirical finding rules out self-fulfilling inflation expectations as an explanation of the high inflation episode in the 1970s. Fourth, our results indicate that there have been important changes in the U.S. economy in terms of both the policy parameters as well as the stochastic environment, i.e. the shock processes, between the two sub-samples. Most notably, the policy response to inflation and output growth almost doubled while trend inflation fell considerably. We also find that the Federal Reserve moved its focus away from responding to headline inflation during the pre-1979 period toward core inflation during the post-1984 period. Finally, we document that oil price shocks are no longer as inflationary as they used to be, allowing the central bank to respond less aggressively to a given oil price shock, thanks to a shift toward more flexible wages in the second sub-sample. Therefore, this finding corroborates the claim of Blanchard and Gali (2010) that real wage rigidities have greatly reduced, thereby explaining why the economy has remained remarkably resilient to sustained oil price hikes in the 2000s.

Our paper is closely related to the empirical literature studying the link between monetary policy and macroeconomic stability, with a particular focus on the Great Inflation episode. Some recent contributions along this dimension include Coibion and Gorodnichenko (2011) and Hirose, Kurozumi and Van Zandweghe (2017). They find that the pre-Volcker period is characterized by indeterminacy while better systematic monetary policy as well as changes in the level of trend inflation resulted in a switch to determinacy in the early 1980s. In contrast, the current paper estimates a similar model while taking into account commodity price fluctuations and the trade-off faced by the central bank. The paper documents that once we take these key features into account, determinacy prevails not only in the Great Moderation period but also in the Great Inflation era. The outcome that pre-Volcker period is characterized by determinacy is in line with the findings of Orphanides (2004), Bilbiie and Straub (2013) and Haque (2017). Both Orphanides (2004) and Haque (2017) document strong anti-inflationary stance pursued by the Federal Reserve even in the 1970s. While Orphanides (2004) points toward the mismeasurement of output gap in real time, Haque (2017) suggests time variation in inflation target and its implications for the inflation gap as explanations for this finding. On the other hand, Bilbiie and Straub (2013) argue that limited asset market participation resulted in an inverted IS curve and inverted aggregate demand logic, i.e. interest rate increases becoming expansionary. Accordingly, they document passive monetary policy during the pre-Volcker period being consistent with equilibrium determinacy. However, to the best of our knowledge, the explanation proposed in this paper based on commodity price fluctuations and the associated trade-off is novel and may be seen as complementary to theirs.

The remainder of the paper evolves as follows. The next section sketches the model while the following sections present the solution and the estimation strategy. Section 5 presents the estimation results. Section 6 illustrates the trade-off between inflation and output gap stabilization and the role of real wage rigidity. In Section 7, we study the propagation of commodity price shock as well as how it has changed over time. Robustness checks are conducted in Section 8. Finally, Section 9 concludes.

2 Model

The artificial economy is a Generalized New Keynesian (GNK) economy with a commodity product which we interpret as oil. This model offers a micro-founded setup that naturally features various inflation rates and also accounts for positive trend inflation. The economy consists of monopolistically competitive wholesale firms that produce differentiated goods using labor and oil. These goods are bought by perfectly competitive firms (retailers) that weld them together into the final good that can be consumed. People rent out their labor services on competitive markets. Firms and households are price takers on the market for oil. The economy boils down to a variant of the model in Blanchard and Gali (2010) when approximated around a zero inflation steady state. Hence, the exposition below closely follows Blanchard and Gali (2010).

2.1 Households

The representative agent's preferences depend on consumption, C_t, and hours worked, N_t, and they are represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t u(C_t, N_t) \qquad 0 < \beta < 1 \quad ,$$

which the agent acts to maximize. Here, E_t represents the expectations operator. The term d_t stands for a shock to the discount factor β which follows the stationary autoregressive process

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} \quad ,$$

where $\epsilon_{d,t}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_d. The period utility is additively separable in consumption and hours worked and it takes on the functional form

$$u(C_t, N_t) = \ln\left(C_t - h\widetilde{C}_{t-1}\right) - \nu_t \frac{N_t^{1+\varphi}}{1+\varphi} \qquad \varphi \geq 0.$$

Logarithmic utility is the only additive-separable form consistent with balanced growth. The term φ is the inverse of the Frisch labor supply elasticity, $h \in [0,1]$ stands for the degree of (external) habit persistence in consumption, and ν_t denotes a shock to the disutility of labor which follows

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \quad ,$$

where $\epsilon_{\nu,t}$ is $N(0, \sigma_\nu^2)$. The overall consumption basket, C_t, is a Cobb-Douglas bundle of output of domestically produced goods , $C_{q,t}$, and imported oil, $C_{m,t}$. In particular, we assume that

$$C_t = \Theta_\chi C_{m,t}^{\chi} C_{q,t}^{1-\chi} \qquad 0 < \chi < 1,$$

where $\Theta_\chi \equiv \chi^{-\chi}(1-\chi)^{-(1-\chi)}$. The parameter χ equals the share of energy in total consumption and $C_{q,t}$ is an index of the domestic output described by

$$C_{q,t} = \left(\int_0^1 C_{q,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Here, the term ε measures the elasticity of demand for each intermediate good.

The agent sells labor services to the wholesale firms at the nominal wage W_t and has access to a market for one-period riskless bonds, B_t, at the interest rate R_t. Any generated profits, Π_t, flow back and the period budget is constrained by

$$W_t N_t + B_{t-1} + \Pi_t \geq P_{q,t} C_{q,t} + P_{m,t} C_{m,t} + \frac{B_t}{R_t} \quad ,$$

where $P_{q,t}$ denotes the domestic output price index.

The Euler equation is given by

$$\frac{d_t}{P_{c,t}\left(C_t - hC_{t-1}\right)} = \beta E_t \frac{R_t d_{t+1}}{P_{c,t+1}\left(C_{t+1} - hC_t\right)},$$

where $P_{c,t}$ is the price of the overall consumption basket.

The intra-temporal optimality condition is described by

$$\frac{W_t}{P_{c,t}} = \nu_t N_t^{\varphi}\left(C_t - hC_{t-1}\right) \equiv MRS_t.$$

Following Blanchard and Gali (2007, 2010) and Blanchard and Riggi (2013), we formalize real wage rigidities by modifying the previous equation as

$$\frac{W_t}{P_{c,t}} = \left\{\frac{W_{t-1}}{P_{c,t-1}}\right\}^{\gamma} \left\{MRS_t\right\}^{1-\gamma},$$

where γ is the degree of real wage rigidity.

In the optimal allocation, we have

$$P_{q,t}C_{q,t} = (1-\chi)P_{c,t}C_t$$

and

$$P_{m,t}C_{m,t} = \chi P_{c,t}C_t$$

where $P_{c,t} \equiv P_{m,t}^{\chi}P_{q,t}^{1-\chi}$ and $P_{m,t}$ is the nominal price of oil. Also note that $P_{c,t} \equiv P_{q,t}s_t^{\chi}$, where $s_t \equiv \frac{P_{m,t}}{P_{q,t}}$ is the real price of oil that follows an exogenous process given by

$$\ln s_t = \rho_s \ln s_{t-1} + \epsilon_{s,t}.$$

2.2 Firms

Final good firm

The representative final good firm produces homogenous good Q_t by choosing a combination of intermediate inputs $Q_t(i)$ to maximize profit. Specifically, the problem of the final good firm is to solve:

$$\max_{Q_t(i)} P_{q,t}Q_t - \int_0^1 P_{q,t}(i)Q_t(i)di \quad ,$$

subject to the CES production technology

$$Q_t = \left[\int_0^1 Q_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad ,$$

where $P_{q,t}(i)$ is the price of the intermediate good i and $\varepsilon > 1$ is the elasticity of substitution between intermediate goods.

Then the final good firm's demand for intermediate good i is given by

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\varepsilon} Q_t.$$

Substituting this demand for retail good i into the CES bundler function gives

$$P_{q,t} = \left[\int_0^1 P_{q,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} .$$

Intermediate good firm

Intermediate goods are produced using labor, $N_t(i)$, and oil, $M_t(i)$, both supplied on perfectly competitive factor markets. Each firm i produces according to the production function

$$Q_t(i) = \left[A_t N_t(i) \right]^{1-\alpha} M_t(i)^{\alpha} \qquad\qquad 0 < \alpha < 1,$$

where α is the share of oil in production and A_t denotes non-stationary labor-augmenting technology

$$\ln A_t = \ln \overline{g} + \ln A_{t-1} + z_t.$$

Here, $\overline{g}$ is the steady-state gross rate of technological change and z_t is a shock to the growth rate of technology following

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t},$$

where $\epsilon_{z,t}$ is $N(0, \sigma_z^2)$. Each intermediate good-producing firm's marginal cost is given by

$$\psi_t(i) = \frac{W_t}{(1-\alpha) Q_t(i) / N_t(i)} = \frac{P_{m,t}}{\alpha Q_t(i) / M_t(i)} \quad ,$$

and the markup, $\mathcal{M}_t^P(i)$, equals

$$\mathcal{M}_t^P(i) = \frac{P_{q,t}(i)}{\psi_t(i)}.$$

Given the production function, cost minimization implies that the firms' demand for oil is given by:

$$M_t(i) = \frac{\alpha}{\mathcal{M}_t^P(i)} \frac{Q_t(i)}{s_t} \frac{P_{q,t}(i)}{P_{q,t}}.$$

Letting Q_t also denote aggregate gross output and defining $\Delta_t \equiv \int_0^1 (\frac{P_{q,t}(i)}{P_{q,t}})^{-\varepsilon} di$ as the relative price dispersion measure, it follows that

$$M_t = \frac{\alpha}{\mathcal{M}_t^P} \frac{Q_t}{s_t} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}},$$

where we have used the demand schedule faced by intermediate good firm i and defined the average gross markup as $\mathcal{M}_t^P \equiv \int_0^1 \mathcal{M}_t^P(i) di$.

Next combining the cost minimization conditions for oil and for labor with the aggregate production function yields the following factor price frontier:

$$\left(\frac{W_t}{P_{c,t}}\right)^{1-\alpha} \mathcal{M}_t^P = \mathcal{C} A_t^{1-\alpha} s_t^{-\alpha-\chi(1-\alpha)} \Delta_t^{-\frac{1}{\varepsilon}},$$

where $\mathcal{C} \equiv \left[\frac{1}{(1-\chi)\Theta_\chi} \left(\frac{1-\chi}{\chi}\right)^\chi\right]^{\alpha-1} \alpha^\alpha (1-\alpha)^{1-\alpha}$.

Price setting

The intermediate goods producers face a constant probability, $0 < 1-\xi < 1$, of being able to adjust prices to a new optimal one, $P_{q,t}^*(i)$, in order to maximize expected discounted profits

$$E_t \sum_{j=0}^{\infty} \xi^j \beta^j \frac{\lambda_{t+j}}{\lambda_0} \left[\frac{P_{q,t}^*(i)}{P_{q,t+j}} Q_{t+j}(i) - \frac{W_{t+j}}{(1-\alpha) P_{q,t+j} A_{t+j}^{1-\alpha}} \left\{\frac{(1-\alpha) P_{m,t+j}}{\alpha W_{t+j}}\right\}^\alpha Q_{t+j}(i)\right],$$

subject to the constraint

$$Q_{t+j}(i) = \left[\frac{P_{q,t}^*(i)}{P_{q,t+j}}\right]^{-\varepsilon} Q_{t+j},$$

where

$$\lambda_{t+j} = \frac{d_{t+j}}{P_{c,t+j}\left(C_{t+j} - hC_{t+j-1}\right)}.$$

The first order condition for the optimized relative price $p^*_{q,t}(i) \equiv \frac{P^*_{q,t}(i)}{P_{q,t}}$ is given by

$$p^*_{q,t}(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{E_t \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{q,t+j} A_{t+j}^{1-\alpha}} \left[\frac{(1-\alpha)P_{m,t+j}}{\alpha W_{t+j}}\right]^{\alpha} \left[\frac{P_{q,t}}{P_{q,t+j}}\right]^{-\varepsilon} Q_{t+j}}{E_t \sum_{j=0}^{\infty} (\xi\beta)^j \lambda_{t+j} \left[\frac{P_{q,t}}{P_{q,t+j}}\right]^{1-\varepsilon} Q_{t+j}}.$$

The joint dynamics of the optimal reset price and inflation can be compactly described by rewriting the first-order condition for the optimal price in a recursive formulation as follows:

$$p^*_{q,t}(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{\kappa_t}{\phi_t},$$

where κ_t and ϕ_t are auxiliary variables that allow one to rewrite the infinite sums that appear in the numerator and denominator of the above equation in recursive formulation:

$$\kappa_t = \mathcal{C} \left(\frac{W_t}{P_{c,t}}\right)^{1-\alpha} s_t^{\chi(1-\alpha)+\alpha} A_t^{\alpha-1} Q_t \widetilde{\lambda}_t + \xi\beta \left[E_t \pi^{\varepsilon}_{q,t+1} \kappa_{t+1}\right],$$

and

$$\phi_t = Q_t \widetilde{\lambda}_t + \xi\beta \left[E_t \pi^{\varepsilon-1}_{q,t+1} \phi_{t+1}\right],$$

where we have used the definition $\widetilde{\lambda}_t = \lambda_t P_{c,t}$. Note that κ_t and ϕ_t can be interpreted as the present discounted value of marginal costs and marginal revenues respectively.

Moreover, the aggregate price level evolves according to:

$$\begin{aligned} P_{q,t} &= \left[\int_0^1 P_{q,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\ 1 &= \xi \pi^{\varepsilon-1}_{q,t} + (1-\xi) p^*_{q,t}(i)^{1-\varepsilon}, \\ p^*_{q,t}(i) &= \left[\frac{1-\xi\pi^{\varepsilon-1}_{q,t}}{1-\xi}\right]^{\frac{1}{1-\varepsilon}}. \end{aligned}$$

Gross output

Production function is characterized by the following:

$$Q_t \Delta_t = M_t^{\alpha} (A_t N_t)^{1-\alpha}.$$

Consumption

The condition that trade be balanced gives us a relation between consumption and gross output:

$$P_{c,t}C_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P}\Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t}Q_t.$$

GDP deflator

The GDP deflator $P_{y,t}$ is implicitly defined by

$$P_{q,t} \equiv (P_{y,t})^{1-\alpha}\,(P_{m,t})^{\alpha}\,.$$

GDP

Value added (or GDP) is then defined by

$$P_{y,t}Y_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P}\Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t}Q_t.$$

Price dispersion

Recall that price dispersion is defined as $\Delta_t \equiv \int_0^1 (\frac{P_{q,t}(i)}{P_{q,t}})^{-\varepsilon} di$. Under the Calvo price mechanism, the above expression can be written recursively as:

$$\Delta_t = (1-\xi)p_{q,t}^*(i)^{-\varepsilon} + \xi\pi_{q,t}^{\varepsilon}\Delta_{t-1}.$$

2.3 Monetary policy

Lastly, the model is closed by assuming that short-term nominal interest rate follows a feedback rule, of the type that has been found to provide a good description of actual monetary policy in the U.S. since Taylor (1993). Our specification of this policy rule features interest rate smoothing, a systematic response to deviations of inflation, output gap and output growth from their respective target values.

$$R_t = \widetilde{R}_t^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R\varepsilon_{R,t}\}, \qquad \widetilde{R}_t = \overline{R}\left\{\left(\frac{\pi_{c,t}}{\overline{\pi}}\right)^{\tau}\left(\frac{\pi_{q,t}}{\overline{\pi}}\right)^{1-\tau}\right\}^{\psi_\pi}\left\{\frac{x_t}{\overline{x}}\right\}^{\psi_x}\left\{\frac{Y_t/Y_{t-1}}{\overline{g}}\right\}^{\psi_{\Delta y}},$$

where $\overline{\pi}$ denotes the central bank's inflation target (and is equal to the gross level of trend inflation), $\overline{R}$ is the gross steady-state policy rate, $\overline{x}$ is the steady state output gap, $\overline{g}$ is the gross steady state growth rate of the economy and $\varepsilon_{R,t}$ is an i.i.d. monetary policy shock. The output gap x_t measures the deviation of the actual level of GDP Y_t from the efficient level of GDP, i.e. the counterfactual level of GDP that would arise in the absence of monopolistic competition, nominal price stickiness and real wage rigidity. The central bank responds to a convex combination of headline and core inflation (with the parameter τ governing the relative weights; setting τ to one implies that the central bank responds to headline inflation only). The coefficients ψ_π, ψ_x and ψ_g govern the central bank's responses to inflation, welfare-relevant output gap and output growth from their respective target values, and $\rho_R \in [0,1]$ is the degree of policy rate smoothing .

3 Solution under indeterminacy

To solve the rational expectations system, we follow the methodology of Lubik and Schorfheide (2003). This approach has the advantage of being general and explicit in dealing with expectation errors, thereby making the method suitable for solving models featuring multiple equilibria. Let us denote by η_t the vector of one-step ahead expectation errors. Moreover, define ϱ_t as the vector of endogenous variables and ε_t as vector of fundamental shocks. Then, the linear rational expectations system can be compactly written as

$$\Gamma_0(\theta)\varrho_t = \Gamma_1(\theta)\varrho_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t, \tag{1}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$ and $\Pi(\theta)$ are appropriately defined coefficient matrices. Under indeterminacy, η_t will be a linear function of the fundamental shocks and the purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the LRE model entails

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t, \tag{2}$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)$[1] are the coefficient matrices.[2] The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy alters the solution in two distinct ways.

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[1] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, we impose their normalization such that $M_\zeta = I$.

[2] Under determinacy, the solution boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

First and foremost, purely extrinsic non-fundamental disturbances, i.e. sunspots, affect model dynamics through endogenous formation of expectation errors. Second, the propagation of fundamental shocks are no longer uniquely pinned down and this multiplicity of equilibria affecting the propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following Lubik and Schorfheide (2004), we replace $\widetilde{M}$ with $M^*(\theta)+M$ and in the subsequent empirical analysis set the prior mean for M equal to zero. This strategy selects $M^*(\theta)$ by using a least squares criterion to minimize the distance between the impact response of the endogenous variables to fundamental shocks, i.e. $\partial\varrho_t/\partial\varepsilon_t'$, at the boundary between the determinacy and the indeterminacy region. Analytical solution for the boundary in this model is infeasible. Hence we follow Justiniano and Primiceri (2008) and Hirose (2014) and resort to a numerical procedure to find the boundary by perturbing the parameter ψ_π in the monetary policy rule.[3] In a later section, we also check the robustness of our results to an alternative perturbation for tracing the boundary.

4 Econometric strategy

This section sets up the estimation procedure, lists the data and discusses the calibration and priors.

4.1 Bayesian estimation with Sequential Monte Carlo (SMC) algorithm

We use Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. In our estimation, we employ the Sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2014, 2015) which is particularly suitable for irregular and non-elliptical posterior distributions. An added benefit of using an importance sampling algorithm like SMC is that the process does not require one to find the mode of the posterior distribution, a task that can prove to be difficult particularly under indeterminacy.

First the priors are described by a density function of the form

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[3] This methodology has been used in previous studies, such as Benati and Surico (2009), Doko Tchatoka, Groshenny, Haque and Weder (2017), Haque (2017), Hirose (2007, 2008, 2013, 2014) and Hirose, Kurozumi and Van Zandweghe (2017).

$$p(\theta_S|S).$$

Here $S \in \{D, I\}$ where D and I stand for determinacy and indeterminacy respectively, θ_S represents the parameter of the model S and $p(.)$ stands for probability density function. Next, the likelihood function

$$\mathcal{L}(\theta_S|X_T, S) \equiv p(X_T|\theta_S, S),$$

describes the density of the observed data and X_T denote observations through period T. By using Bayes theorem we can combine the prior density and the likelihood function to obtain the posterior density

$$p(\theta_S|X_T, S) = \frac{\mathcal{L}(\theta_S|X_T, S)p(\theta_S|S)}{p(X_T|S)},$$

in which $p(X_T|S)$ denotes the marginal density of the data conditional on the model which is given by

$$p(X_T|S) = \int_{\theta_S} \mathcal{L}(\theta_S|X_T, S)p(\theta_S|S)d\theta_S.$$

We employ the SMC algorithm of Herbst and Schorfheide (2014, 2015) to build a particle approximation of the posterior distribution through tempering the likelihood. A sequence of tempered posteriors is defined as

$$\pi_n(\theta_S) = \frac{[\mathcal{L}(\theta_S|X_T, S)]^{\phi_n} p(\theta_S|S)}{\int_{\theta_S}[\mathcal{L}(\theta_S|X_T, S)]^{\phi_n} p(\theta_S|S)d\theta_S},$$

where ϕ_n is the tempering schedule that slowly increases from zero to one and is determined by $\phi_n = \left(\frac{n-1}{N_\phi-1}\right)^\delta$ where δ controls the shape of the tempering schedule. The algorithm generates weighted draws from the sequence of posteriors $\{\pi_n(\theta)\}_{n=1}^{N_\phi}$, where N_ϕ is the number of stages. At any stage, the posterior distribution is represented by a swarm of particles $\left\{\theta_n^i, W_n^i\right\}_{i=1}^N$ where W_n^i is the weight associated with θ_n^i and N denotes the number of particles. The algorithm has three main steps. First, in the *correction* step, the particles are re-weighted to reflect the density in iteration n. Next, in the *selection* step, any particle degeneracy is eliminated by resampling the particles. A rule-of-thumb measure of this degeneracy, proposed by Herbst and Schorfheide (2014, 2015), is given by the reciprocal of the uncentered variance of the particles and is called the effective sample size (ESS) which is defined as:

$$\widehat{ESS_n} = \frac{N}{\frac{1}{N}\sum_{i=1}^{N}\left(\widetilde{W_i^n}\right)^2},$$

where $\widetilde{W_i^n}$ is the normalized particle weight. Following Herbst and Schorfheide (2014, 2015) we use systematic resampling whenever $\widehat{ESS_n} < \frac{N}{2}$. Finally, in the *mutation* step, the particles are propagated forward using a Markov transition kernel to adapt to the current bridge density. Here, we use one step of a single-block Random Walk Metropolis Hastings (RWMH) algorithm.

Note that in the first stage, i.e. when $n = 1$, ϕ_1 is zero. Hence, the prior density serves as an efficient proposal density for $\pi_1(\theta)$. That is, the algorithm is initialized by drawing the initial particles from the prior. Likewise, the idea is that the density of $\pi_n(\theta)$ may be a good proposal density for $\pi_{n+1}(\theta)$. In our estimation, the tuning parameters N, N_ϕ and δ are fixed ex ante. We use $N = 10,000$ particles and $N_\phi = 200$ stages and set δ at 2 following Herbst and Schorfheide (2015).

To assess the quality of the model's fit to the data we use log marginal data densities and posterior model probabilities for both parametric regions, i.e. determinacy and indeterminacy. The SMC algorithm-based approximation of the marginal data density is given by

$$p^{SMC}(X_T|S) = \prod_{n=1}^{N_\phi}\left(\frac{1}{N}\sum_{i=1}^{N}\widetilde{w}_n^i W_{n-1}^i\right),$$

where $\widetilde{w}_n^i$ is the incremental weight defined by

$$\widetilde{w}_n^i = [p(X|\theta_{n-1}^i, S)]^{\phi_n - \phi_{n-1}}.$$

4.2 Data

We define the set of observables, ϑ_t, which contains quarterly growth rate of real per-capita GDP, consumer price index (CPI), core consumer price index (Core CPI), real wage, and the Federal Funds rate. Wages come from the BLS (hourly compensation for the NFB sector for all persons). Hourly compensation is divided by the CPI in order to get the consumption real wage variable. The measurement equation is

$$\vartheta_t = \begin{bmatrix} g^* \\ \pi^* \\ \pi^* \\ g^* \\ R^* \end{bmatrix} + \begin{bmatrix} \widehat{g}_{y,t} \\ \widehat{\pi}_{c,t} \\ \widehat{\pi}_{q,t} \\ \widehat{g}_{w,t} \\ \widehat{R}_t \end{bmatrix}$$

where g^* is the quarterly steady state net output growth rate, π^* is the steady state net inflation rate, R^* stands for the steady state net interest rate, $\widehat{g}_{y,t}$ denotes the growth rate of output, $\widehat{\pi}_{c,t}$ is consumer price inflation, $\widehat{\pi}_{q,t}$ is core consumer price inflation, $\widehat{g}_{w,t}$ is the growth rate of real wages (deflated by the consumer price index), and $\widehat{R}_t$ denotes the nominal interest rate. Hatted variables stand for log deviations from the steady state. To test for indeterminacy and estimate the model parameters, we consider two sample periods in our benchmark analysis: 1966:I to 1979:II and 1984:I to 2008:II. We do not demean or detrend any series.

4.3 Calibrated parameters

We calibrate a subset of the model parameters. We set the discount factor β to 0.99, the steady-state markup at ten percent, i.e. $\varepsilon = 11$, and the inverse of the labor-supply elasticity to one. Following the computations in Blanchard and Gali (2010), we calibrate the shares of oil in production and consumption to $\alpha = 0.015$ and $\chi = 0.023$ for the first sample and $\alpha = 0.012$ and $\chi = 0.017$ for the second sample. Furthermore, we assume that shocks to the growth rate of technology are i.i.d., i.e. $\rho_z = 0$, since the process already includes a unit root. We also fix the autoregressive parameter of the commodity price shock at $\rho_s = 0.995$, in order to have the commodity price be very close to random walk yet be stationary. In our benchmark estimation, we abstract from price indexation. We estimate all the remaining parameters with Bayesian techniques.

4.4 Prior distributions

The specification of the prior distribution is summarized in Table 1. The prior for the parameter determining the central bank's responsiveness to inflation, ψ_π, follows a gamma distribution centred at 1.10 with a standard deviation of 0.50 while the response coefficient to output gap and output growth are centred at 0.125 with standard deviation 0.10. We use Beta distribution with mean 0.50 for the smoothing

Table 1: Prior distributions for parameters

Name	Density	Prior Mean	St. Dev
ψ_π	Gamma	1.10	0.50
ψ_x	Gamma	0.125	0.10
ψ_g	Gamma	0.125	0.10
ρ_R	Beta	0.50	0.20
τ	Beta	0.50	0.20
π^*	Normal	1.00	0.50
R^*	Gamma	1.50	0.25
g^*	Normal	0.50	0.10
ξ	Beta	0.50	0.05
γ	Beta	0.50	0.20
h	Beta	0.50	0.10
ρ_d	Beta	0.70	0.10
ρ_ν	Beta	0.70	0.10
σ_s	Inv-Gamma	5.00	2.00
σ_g	Inv-Gamma	0.50	0.20
σ_r	Inv-Gamma	0.50	0.20
σ_d	Inv-Gamma	0.50	0.20
σ_ν	Inv-Gamma	0.50	0.20
σ_ζ	Inv-Gamma	0.50	0.20
$M_{s,\zeta}$	Normal	0.00	1.00
$M_{g,\zeta}$	Normal	0.00	1.00
$M_{r,\zeta}$	Normal	0.00	1.00
$M_{d,\zeta}$	Normal	0.00	1.00
$M_{\nu,\zeta}$	Normal	0.00	1.00

Notes: The inverse gamma priors are of the form $p\left(\sigma|\upsilon,\varsigma\right)\infty\sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$ where $\nu = 4$ and $\varsigma = 0.38$ for all shocks but commodity prices while for commodity price shock $\varsigma = 3.81$. The prior probability of determinacy is 0.51.

coefficient ρ_R, the parameter governing the weight on headline inflation in the Taylor rule τ, the Calvo probability ξ, the real wage rigidity γ and habit persistence in consumption h. The prior distribution for the persistence of the discount factor shock and the labor supply shock is also a Beta with mean 0.70 and standard deviation 0.20.

For the standard deviations of the innovations, the priors for all but one follow an inverse-gamma distribution with mean 0.50 and standard deviation 0.20. The exception is the oil price shock for which we centre the prior at 5.00 with a standard deviation 2.00 to account for its higher volatility.

Finally, in line with Lubik and Schorfheide (2004), the coefficients M follow standard normal distributions. Hence, the prior is centered around the baseline solution of Lubik and Schorfheide (2004). The choice of the priors leads to a prior predictive probability of determinacy of 0.51, which is quite even and suggests no prior bias toward either determinacy or indeterminacy.

5 Estimation results

5.1 Model comparison

To assess the quality of the model's fit to the data, Table 2 presents marginal data densities and posterior model probabilities for both parametric zones. We find that determinacy unambiguously prevails in both the pre-Volcker and the post-84 sample periods. In other words, the posterior puts all its weight in the determinacy region. The finding that determinacy prevails in both the sample periods might be surprising given that the literature has established the high inflation episode of the 1970s as characterized by self-fulfilling inflation expectations. A natural question that arises is: what drives this result?

To shed light on our finding, we would like to start by bridging the gap between the current paper and the existing literature. As such, at first we shut down oil in the model by calibrating the oil share in consumption and production to zero. As a result, the model boils down to a simple Generalized New Keynesian (GNK) model with positive trend inflation ala Ascari and Ropele (2007, 2009) and Ascari and Sbordone (2014). To maintain continuity with the existing literature, we estimate this nested GNK model with only three observables: the quarterly growth rate of real per-capita

Table 2: Determinacy versus Indeterminacy

	Log-data density		Probability	
Model	Determinacy	Indeterminacy	Determinacy	Indeterminacy
1966:I-1979:II	-228.89	-241.06	1	0
1984:I-2008:II	-230.03	-251.05	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.51.

GDP, the Federal Funds rate and quarterly CPI inflation rate. Moreover, we set the weight τ in the Taylor rule to one as there is just a single concept of inflation in the simple GNK model with no distinction between headline and core. This then makes our set up similar to Hirose, Kurozumi and Van Zandweghe (2017). One exception is that the current paper employs a model with homogenous labor following Ascari and Ropele (2009) and Ascari and Sbordone (2014) while Hirose, Kurozumi and Van Zandweghe (2017) use a model with firm-specific labor following Kurozumi and Van Zandweghe (2017). Table 3 reports the log-data densities while Tables 4 and 5 give the posterior estimates. In line with the findings in the existing literature, the first row in the table confirms that the estimation favors the indeterminate version of the model in the pre-Volcker period.

Having bridged the gap with existing empirical studies, we now sequentially move on by adding one feature at a time. At first, we turn on oil in the model by resetting the values of α and χ to their benchmark calibration. This set up gives us a New Keynesian model with micro-founded cost-push shocks, a feature that is reminiscent of the environment in the 1970s, yet one that is missing in existing empirical investigation on (in)-determinacy. However, we continue to use three observables in our estimation. Furthermore, since we are still using one inflation series as an observable, τ is not identified. Hence, we calibrate this parameter to one such that the central bank responds solely to headline inflation. Once again, indeterminacy unambiguously prevails in the pre-Volcker period.

According to the posterior estimate of the innovation to oil-price shock σ_s, we find that the posterior is virtually indistinguishable from the prior suggesting possible identification issues. In fact, using only one inflation measure as an observable, i.e. CPI inflation alone in this case, does not provide sufficient information to pin down

Table 3: Determinacy versus Indeterminacy (1966:I - 1979:II)

	Log-data density		Probability	
	Det.	Indet.	Det.	Indet.
GNK $(\Delta y_t, R_{t,}\pi_{c,t})\,[\alpha, \chi = 0; \tau = 1]$	-121.14	-118.81	0.09	0.91
GNK with Oil $(\Delta y_t, R_{t,}\pi_{c,t})\,[\tau = 1]$	-123.01	-118.28	0.01	0.99
GNK with Oil $(\Delta y_t, R_{t,}\pi_{c,t}, \pi_{q,t})$	-157.93	-157.56	0.41	0.59
GNK with Oil $(\Delta y_t, R_{t,}\pi_{c,t}, \pi_{q,t}, \Delta w_t^1)$	-228.89	-241.06	1	0
GNK with Oil $(\Delta y_t, R_{t,}\pi_{c,t}, \pi_{q,t}, \Delta w_t^1, \Delta w_t^2)$	-279.02	-292.54	1	0

oil-price shocks. Hence, in our next exercise, we simultaneously treat both headline and core inflation as observables. Thus, our dataset now includes four variables. This step enables us to properly identify the oil-price shocks (or more generally commodity price shocks). Also, we estimate the weight τ in the Taylor rule which is now supposedly identified. Table 3 (third row) shows that the finding is now ambiguous: the probability of indeterminacy is 0.59. Phrased alternatively, we can neither rule in nor rule out indeterminacy. Moreover, as anticipated, the innovation to the oil-price shock σ_s is now better identified. Table 4 shows that the posterior mean estimate is significantly higher than the estimate we obtain when using only three observables.

A key parameter in this model is the degree of real wage rigidity γ. As Blanchard and Gali (2007, 2010) argue, the presence of real wage rigidity generates a trade-off between stabilizing inflation and stabilizing the output gap. Accordingly, higher real wage rigidity generates a more severe trade-off. To sharpen the identification of this feature, we next add real wage data, i.e. we employ five observables to estimate the model. We use observations on "hourly compensation for the non-farm business sector for all persons" as a measure of nominal wages. To get real wages, we then divide this proxy by the CPI price deflator. This then gives us our benchmark setup. The fourth row in Table 3 reproduces the log-data densities and posterior model probabilities from Table 2 for the pre-Volcker period. As argued above, the pre-Volcker period is then explicitly characterized by determinacy and a high degree of real wage rigidity.

Our argument can be summarized as follows. It is well known that commodity price shocks in general and oil price shocks in particular were an important source of economic fluctuations in the U.S. during much of the 1970s. For instance, there were episodes of large increases in the price of oil triggered by the Yom Kippur war in 1973 and the Iranian revolution of 1979. Such adverse cost-push shocks generated a trade-off between stabilizing inflation and stabilizing the output gap for the Federal Reserve. Existing empirical investigations on the efficacy of monetary policy in the 1970s find that policy failed to respond sufficiently strongly to inflation thereby generating indeterminacy. However, these studies abstract from modelling the role of commodity price fluctuations and the associated policy trade-off. Our first contribution is to employ a New Keynesian framework with positive trend inflation and an explicit role of oil in both consumption and production. In our framework, we also allow for a mechanism, i.e. the presence of real wage rigidity, which generates a quantitatively meaningful trade-off faced by the central bank following commodity price shocks. Our second contribution is to test for indeterminacy by estimating this model over the Great Inflation and the Great Moderation period. In this endeavor, what further sets us apart from existing empirical work is that we pay particular attention in identifying key features of the model through careful elucidation of observables. Our finding that determinacy prevails in the pre-Volcker period, therefore, rules out self-fulfilling inflation expectations or sunspots as an explanation of the high inflation episode in the 1970s.

As illustrated above, we follow Blanchard and Gali (2007, 2010) and Blanchard and Riggi (2013) by assuming real wage rigidities as a source of real imperfection which breaks down the divine coincidence with respect to commodity price shocks. In our empirical investigation, we find that real wage rigidity turns out to be significantly higher when we allow for wage data in the estimation and the parameter estimates of the Taylor rule turn out to be such that the data explicitly favors determinacy. However, as pointed out by Blanchard and Gali (2007, 2010), this way of modelling real wage rigidity is admittedly ad hoc but still a parsimonious way of capturing slow adjustment of real wages to labor market conditions arising due to some (unmodelled) labor market imperfection or friction. Nonetheless, the fact that we match a particular empirical wage inflation series to the latent concept of wage inflation in the model might have some bearing for the higher posterior estimate of the real wage rigidity parameter γ. In this line of thinking, we next depart from the assumption that wage inflation in the model is measured by a single series and draw on the methodology proposed by Boivin and Giannoni (2006) and recently adopted by Gali, Smets and Wouters (2011) and Justiniano, Primiceri and Tambalotti (2013). We match the wage inflation variable in the model with two data series. The first series is the same one as used in the estimations so far, i.e. "hourly compensation for the non-farm business sector for all employees". The second measure is the "average hourly earnings of production and non-supervisory employees". Following Justiniano, Primiceri and Tambalotti (2013), we further assume that both series represent an imperfect match to the concept of "wage" in the model and capture this mismatch through i.i.d. measurement errors. This assumption is important as Justiniano, Primiceri and Tambalotti (2013) find that most of the high frequency variation that characterizes the individual series on compensation is due to measurement error. More concretely, the estimation involves the following measurement equation for wage inflation

$$\left[\begin{array}{c} \Delta \log NHC_t \\ \Delta \log HE_t \end{array}\right] = \left[\begin{array}{c} 1 \\ 1 \end{array}\right] g^* + \left[\begin{array}{c} 1 \\ \lambda \end{array}\right] \widehat{g}_{w,t} + \left[\begin{array}{c} e_{1,t} \\ e_{2,t} \end{array}\right],$$

where $\Delta \log NHC_t$ and $\Delta \log HE_t$ denote the growth rate of the two measures of wages in the data (deflated using CPI), λ is a loading coefficient relating the second series to the latent concept of wage inflation in the model, and $e_{1,t}$ and $e_{2,t}$ are i.i.d. observation errors with distribution $N(0, \sigma_{e_1}^2)$ and $N(0, \sigma_{e_2}^2)$.[4] Our prior distributions for the loadings and measurement equations are $\lambda \sim N(1.00, 0.50)$ and $\sigma_{e_1}, \sigma_{e2} \sim IG(0.10, 0.20)$. Once again, the degree of real wage rigidity turns out to be substantially higher and as a corollary determinacy unambiguously prevails in the pre-Volcker period.

5.2 Parameter estimates

Tables 4 and 5 report the posterior mean and the standard deviation of the parameters under alternative specifications for the pre-1979 and the post-1984 sample periods respectively. First of all, we find that the estimated response to inflation

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[4] The other loading is normalized to 1 as standard in factor analysis.

in the Taylor rule is passive for the GNK model estimated using three observables and for the model with oil estimated using either three or four observables. This finding is in line with the literature's view that the policy response to inflation was passive during the Great Inflation period. However, once we allow for wage data in our estimation (either using just one wage series or using two series following Boivin and Giannoni's (2006) methodology), we find the degree of real wage rigidity to be significantly higher: the point estimate turns out to be around 0.9. As argued above, such a high degree of real wage rigidity worsens the trade-off faced by the central bank in the wake of commodity price shocks. Now the estimated response to inflation now turns out to be active during the pre-1979 period. Moreover, the response to output gap turns out to be substantially lower while the response to output growth and the degree of policy-rate smoothing turns out to be higher. This finding confirms our intuition that the parameter estimates of the Taylor rule during the pre-Volcker period might possibly be biased if the empirical investigation does not take into account the effect of commodity price shocks and the associated trade-offs faced by the central bank. Combined together, such changes in the parameter estimates of the Taylor rule push the posterior distribution toward the determinacy region of the parameter space.

Moving across the sample period while focusing on the parameter estimates of the GNK model with oil estimated using six observables (i.e. two wage series), we see that the policy response to inflation and output growth almost doubled while trend inflation fell considerably. The Federal Reserve also moved its focus away from responding to headline inflation during the pre-1979 period toward core inflation during the post-1984 period. Among the other structural parameters, habit persistence in consumption decreased slightly while the degree of price stickiness remained roughly unchanged. Furthermore, qualitatively in line with the findings of Blanchard and Riggi (2013), we find a substantial decline in real wage rigidity. However, our estimate still points toward the presence of moderate degree of rigidity while Blanchard and Riggi (2013) document perfect real wage flexibility. This divergence might be due to the different estimation strategies that we employ. While Blanchard and Riggi (2013) adopt a limited information approach that matches impulse responses to an oil price shock in the DSGE model and in a structural VAR, we use full-information Bayesian estimation with multiple shocks.

Table 4: Parameter Estimates (1966:I-1979:II)

	GNK (Indet) 3 obs	GNK-Oil (Indet) 3 obs	GNK-Oil (Indet) 4 obs	GNK-Oil (Det) 5 obs	GNK-Oil (Det) 6 obs
ψ_π	0.94 (0.11)	0.94 (0.11)	0.92 (0.12)	1.55 (0.19)	1.51 (0.17)
ψ_x	0.14 (0.11)	0.21 (0.14)	0.30 (0.11)	0.03 (0.02)	0.03 (0.03)
ψ_g	0.11 (0.07)	0.12 (0.07)	0.10 (0.05)	0.46 (0.16)	0.35 (0.14)
ρ_R	0.44 (0.08)	0.48 (0.08)	0.60 (0.10)	0.71 (0.05)	0.69 (0.05)
τ	1	1	0.35 (0.24)	0.57 (0.16)	0.58 (0.15)
π^*	1.42 (0.18)	1.34 (0.21)	1.37 (0.13)	1.36 (0.16)	1.36 (0.17)
R^*	1.56 (0.17)	1.51 (0.18)	1.58 (0.13)	1.52 (0.20)	1.51 (0.20)
g^*	0.48 (0.09)	0.51 (0.09)	0.50 (0.06)	0.47 (0.07)	0.45 (0.07)
ξ	0.50 (0.05)	0.54 (0.05)	0.66 (0.06)	0.62 (0.04)	0.60 (0.04)
γ	0.50 (0.24)	0.33 (0.17)	0.64 (0.25)	0.90 (0.03)	0.89 (0.04)
h	0.40 (0.07)	0.37 (0.07)	0.37 (0.05)	0.39 (0.07)	0.38 (0.07)
ρ_d	0.78 (0.08)	0.70 (0.10)	0.62 (0.09)	0.76 (0.06)	0.77 (0.07)
ρ_ν	$-$	0.69 (0.10)	0.67 (0.09)	0.80 (0.07)	0.86 (0.07)
σ_s	$-$	5.34 (2.23)	17.30 (1.38)	17.30 (1.62)	17.24 (1.62)
σ_g	1.59 (0.22)	1.51 (0.21)	0.80 (0.38)	0.63 (0.06)	0.48 (0.09)
σ_r	0.31 (0.04)	0.30 (0.03)	0.25 (0.04)	0.31 (0.04)	0.30 (0.03)
σ_d	0.54 (0.18)	0.39 (0.13)	1.57 (0.59)	1.94 (0.35)	1.86 (0.33)
σ_ν	$-$	0.36 (0.11)	0.62 (0.17)	0.41 (0.08)	0.38 (0.08)
σ_ζ	0.50 (0.27)	0.46 (0.20)	0.52 (0.20)	$-$	$-$
$M_{s,\zeta}$	$-$	-1.19 (0.58)	-0.07 (0.18)	$-$	$-$
$M_{g,\zeta}$	0.66 (0.86)	0.81 (0.70)	-0.14 (0.69)	$-$	$-$
$M_{r,\zeta}$	0.16 (0.97)	0.36 (1.00)	0.31 (0.74)	$-$	$-$
$M_{d,\zeta}$	0.13 (1.07)	-0.08 (1.02)	0.95 (1.09)	$-$	$-$
$M_{\nu,\zeta}$	$-$	-0.23 (1.01)	0.11 (0.93)	$-$	$-$
λ	$-$	$-$	$-$	$-$	1.07 (0.24)
σ_{e_1}	$-$	$-$	$-$	$-$	0.37 (0.10)
σ_{e_2}	$-$	$-$	$-$	$-$	0.46 (0.10)

Table 5: Parameter Estimates (1984:I-2008:II)

	GNK (Det) 3 obs	GNK-Oil (Det) 3 obs	GNK-Oil (Det) 4 obs	GNK-Oil (Det) 5 obs	GNK-Oil (Det) 6 obs
ψ_π	2.38 (0.34)	2.35 (0.32)	2.43 (0.24)	2.25 (0.30)	3.08 (0.36)
ψ_x	0.11 (0.09)	0.11 (0.09)	0.16 (0.13)	0.02 (0.01)	0.11 (0.06)
ψ_g	0.67 (0.20)	0.71 (0.21)	0.60 (0.14)	1.19 (0.20)	0.69 (0.15)
ρ_R	0.79 (0.03)	0.80 (0.03)	0.69 (0.04)	0.78 (0.03)	0.73 (0.04)
τ	1	1	0.22 (0.07)	0.23 (0.09)	0.14 (0.05)
π^*	0.83 (0.07)	0.84 (0.08)	0.84 (0.07)	1.01 (0.09)	0.95 (0.09)
R^*	1.39 (0.14)	1.40 (0.14)	1.43 (0.13)	1.53 (0.15)	1.44 (0.14)
g^*	0.51 (0.06)	0.51 (0.06)	0.50 (0.06)	0.38 (0.07)	0.16 (0.05)
ξ	0.47 (0.04)	0.49 (0.04)	0.65 (0.04)	0.80 (0.03)	0.62 (0.04)
γ	0.18 (0.10)	0.16 (0.09)	0.18 (0.11)	0.81 (0.07)	0.46 (0.12)
h	0.36 (0.06)	0.36 (0.06)	0.20 (0.04)	0.28 (0.06)	0.24 (0.05)
ρ_d	0.91 (0.02)	0.91 (0.02)	0.90 (0.04)	0.88 (0.03)	0.85 (0.04)
ρ_ν	–	0.71 (0.10)	0.72 (0.12)	0.94 (0.02)	0.99 (0.01)
σ_s	–	3.75 (1.10)	20.17 (1.40)	20.23 (1.42)	20.41 (1.50)
σ_g	0.76 (0.08)	0.76 (0.08)	0.66 (0.08)	0.83 (0.06)	0.44 (0.08)
σ_r	0.21 (0.02)	0.20 (0.02)	0.19 (0.02)	0.17 (0.02)	0.17 (0.02)
σ_d	1.50 (0.27)	1.50 (0.28)	1.33 (0.24)	1.40 (0.24)	1.23 (0.19)
σ_ν	–	0.35 (0.10)	0.36 (0.12)	0.55 (0.12)	0.75 (0.15)
λ	–	–	–	–	0.29 (0.08)
σ_{e_1}	–	–	–	–	0.66 (0.07)
σ_{e_2}	–	–	–	–	0.38 (0.04)

In terms of the standard deviations of the innovations, there is an increase in the volatility of commodity price shock and labor supply shock. As argued by Blanchard and Gali (2010), the increase in the size of commodity price shock is due to its limited variation before the 1973 crisis, despite the two large spikes in that year. On the other hand, the innovation variance of monetary policy shock and discount factor shock declined quite notably while the size of the technology shock remained fairly stable.

Finally, there is a substantial change in the estimate of the loading coefficient λ. In the pre-Volcker period, the estimate of λ is quite close to one implying a similarity in the two wage inflation series during that period. However, in the post-1984 period, it turns out to be much lower: the posterior mean estimate is 0.29. This further justifies the differences in some of the parameter estimates of the model for the post-1984 period depending on whether we employ the first empirical series alone as in our five observables case versus when we use both wage inflation series as in the six observables case.

5.3 Implications of the model for macroeconomic volatility

In this section, we assess the ability of the model to account for the Great Moderation, i.e. the marked decline in macroeconomic volatility in the second sub-sample. Table 6 summarizes the model's implications for the volatility of the inflation (both headline and core) and output growth at the posterior mean of the model parameters along with the data-based standard deviations over the indicated sample. The estimated model is able to replicate the observed drop in volatility.[5] We find a fall of output growth variability of 45% and a drop of headline and core inflation volatility of about 56% and 70% respectively. The figures are similar to those reported in the literature. For instance, Justiniano and Primiceri (2008) report a fall of output growth variability of about 25% and a drop of inflation variability of about 75%. The numbers in Smets and Wouters (2007) read 35% and 58% respectively. Despite the fact that our model is relatively small-scale in nature compared to the medium-scale models in these studies, we find it reassuring in terms of the empirical plausibility of our estimation results.

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[5] Although it overestimates the standard deviation, such mismatch is also present in medium-scale models as well. See Smets and Wouters (2007).

Table 6: The Great Moderation

	1966:I-1979:II		1984:I-2008:II		Percent Change	
	Data	Model	Data	Model	Data	Model
Headline Inflation	0.68	1.04	0.38	0.46	-44%	-56%
Core Inflation	0.60	0.89	0.28	0.27	-53%	-70%
Output Growth	1.01	1.14	0.53	0.63	-48%	-45%

6 Trade-off between inflation and output gap stabilization

In this section, we illustrate the importance of real wage rigidity in generating a quantitatively meaningful trade-off faced by the central bank in stabilizing inflation and output gap volatility in the wake of commodity price shocks. Figure 1 plots the impulse responses of headline inflation, core inflation, the welfare-relevant output gap and price dispersion to a one standard deviation commodity price shock under three alternative calibration of the real wage rigidity parameter. The structural parameters as well as the policy parameters are calibrated to their estimated posterior mean values for the pre-1979 period.

First of all, we see that in the absence of real wage rigidity, headline inflation increases while there is a decrease in core inflation, the output gap and price dispersion. The rise in headline inflation is somewhat obvious since part of the increase in oil prices is reflected mechanically in the oil component of the CPI. On the other hand, there is a reduction in core inflation owing to our assumption of real wage flexibility. With perfectly flexible real wages, an increase in the real price of oil reduces the consumption real wage and hence lowers the marginal cost. As a result, there is a fall in desired price as well as price dispersion. Moreover, the output gap goes down as well. To the extent that the central bank's objective is to stabilize both headline inflation as well as welfare-relevant output gap, it faces a trade-off even in the absence of real wage rigidity. However, divine coincidence holds when the central bank focuses on stabilizing core inflation instead as both output gap and core inflation goes down. In fact, one might argue that core inflation is a more natural reference point for

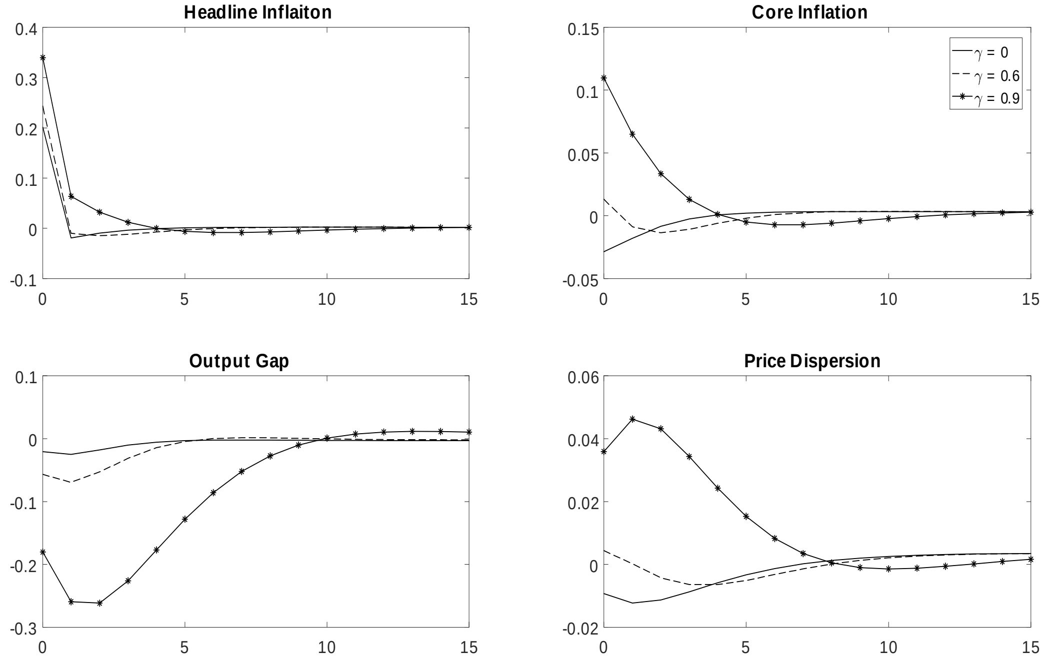

Figure 1: IRF to commodity price shock under alternative degree of real wage rigidity

monetary policy as policy can only affect the sticky price component. Hence, we qualify the results documented in Alves (2014) who argues that a non-zero steady state level of inflation makes it impossible for monetary policy to simultaneously stabilize inflation and output gap in response to preference and technology shocks. In any case, the response of the endogenous variables to a commodity price shock is quantitatively negligible when γ is set equal to zero.

In contrast, for high levels of real wage stickiness, policymakers face a quantitatively meaningful trade-off between output gap and inflation (either headline or core) stabilization. This trade-off arises from the fact that even in the equilibrium in which output gap is stabilized, desired prices are not constant in general. With real wages being rigid, an increase in the real price of oil will result in an increase in the firm's marginal cost, and hence in both desired price and core inflation. Due to fluctuations in desired prices, firms that reset their prices in different periods will charge different prices. This resulting increase in price dispersion will lead to instability in price inflation. Therefore, higher real wage rigidity generates a more severe trade-off faced by the central bank in the aftermath of commodity price shocks. A stable

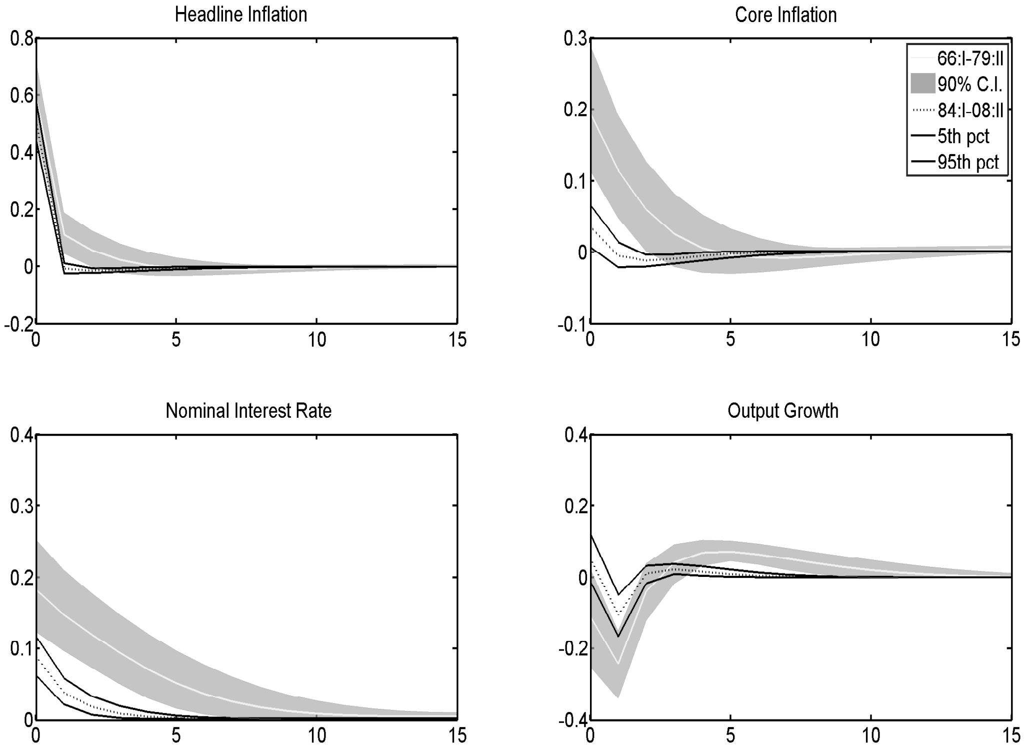

Figure 2: Estimated IRF to commodity price shock

welfare-relevant output gap is thus inconsistent with either stable headline and/or core inflation. As such, the parameter estimates of the Taylor rule during the 1970s might possibly be biased if the empirical investigation leaves out real wage rigidity and the associated trade-off faced by the Federal Reserve in the wake of commodity price shocks.

7 Propagation of commodity price shock

This section studies the propagation of commodity price shock as well as how it has changed over time. Figure 2 depicts the estimated mean impulse responses of headline inflation, core inflation, nominal interest rate and output growth for both sample periods along with the 90 percent probability interval. As evident from the figure, the effects of commodity price shocks have changed significantly over time. Our estimates point to much smaller effects on core inflation, real activity and interest rate in the second sub-sample despite the fact that the shocks are slightly larger in

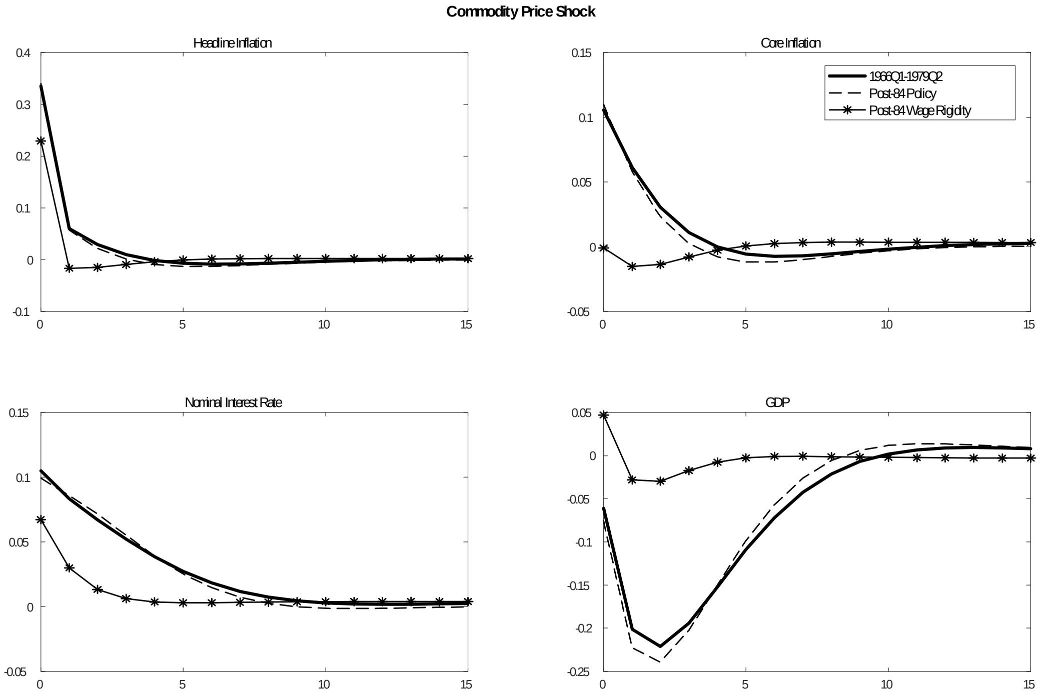

Figure 3: Counterfactual IRFs to commodity price shock

size. The only exception is the response of headline inflation, whose impact response is very similar, albeit with a reduced persistence. This is intuitive since, as argued above, part of the rise in oil prices is reflected automatically in the oil component of headline inflation. This finding is reassuring as it matches with the empirical VAR evidence put forth by Blanchard and Gali (2010) and Blanchard and Riggi (2013).

Next , we conduct counterfactual experiments to disentangle the driving force behind these changes over time. We divide the experiments into two categories. First, we combine the posterior mean estimates pertaining to the Taylor rule, i.e. $\psi_\pi, \psi_x, \psi_{\Delta y}, \rho_R, \pi^*, \tau$, of the post-1984 sub-sample with the remaining parameter estimates of the pre-1979 period which is called 'post-84 policy'. This exercise is designed to capture the role of monetary policy in the reducing the effect of a given change in commodity prices. In the second category, we combine the posterior mean estimates of the pre-1979 period (including the policy parameters) with the estimated (lower) real wage rigidity from the post-1984 period, labelled 'post-84 wage rigidity'. This scenario is designed to capture the role of the decline in real wage rigidity as a possible explanation.

Figure 3 depicts the impulse responses to a one standard deviation commodity price shock under the two alternative scenarios while calibrating the remaining parameters at the posterior mean estimates of the pre-1979 period. Looking at the figure, we can see that the decline in the effects of commodity price shocks is mainly explained by a reduction in real wage rigidity. As argued earlier, real wage rigidity generates a trade-off between inflation and output gap stabilization. A shift toward more flexible wages implies a reduction in this trade-off thereby explaining the smaller effects of the shocks in the more recent period. Thus, our finding corroborates one of the hypothesis put forth by Blanchard and Gali (2010) and is also in line with the empirical evidence documented in Blanchard and Riggi (2013).

8 Sensitivity analysis

We now conduct sensitivity of our results in various directions that involve (i) indexation to past inflation, (ii) alternative Taylor rule, (iii) alternative formulation of the boundary between the determinacy and indeterminacy region, (iv) flexible-price output gap, (v) estimation over the entire parameter space, and (vi) real oil price as an observable. For all these cases, the estimation is conducted using six observables, i.e. including both wage series ala Boivin and Giannoni (2006). Table 7 reports the log-data densities and the posterior probabilities while the parameter estimates are reported in Tables 8 and 9 in the appendix.

8.1 Indexation

In light of the result of Cogley and Sbordone (2008) regarding the lack of empirical support for intrinsic inertia in the generalized NKPC, the model is so far estimated by assuming absence of rule-of-thumb price-setting. Hence, we now estimate the model while allowing for indexation. To facilitate identification, we follow Ascari, Castelnuovo and Rossi (2011) by calibrating the relative degree of indexation μ to one and estimating the degree of indexation to past inflation ω in line with Benati (2009). While we find some support for moderate degree of indexation, our finding that the pre-Volcker period is characterized by determinacy remains robust.

Table 7: Determinacy versus Indeterminacy (Robustness)

		Log-data density		Probability	
		Det.	Indet.	Det.	Indet.
1966:I-1979:II	Indexation	-277.70	-291.52	1	0
	JPT Taylor rule	-286.71	-292.01	1	0
	Boundary	-279.02	-282.33	0.96	0.04
	Flex-price Output Gap	-276.25	-285.12	1	0
	Entire Parameter Space	-279.27		1	0
	Core CPI & Oil	-504.85	-515.42	1	0
1984:I-2008:II	Indexation	-287.87	-342.15	1	0
	JPT Taylor rule	-281.56	-317.89	1	0
	Boundary	-275.20	-361.36	1	0
	Flex-price Output Gap	-280.87	-312.90	1	0
	Entire Parameter Space	-275.71		1	0
	Core CPI & Oil	-619.62	-658.99	1	0

8.2 Alternative Taylor rule

Next we investigate the sensitivity of our findings with respect to an alternative formulation of the monetary policy rule. Following Justiniano, Primiceri and Tambalotti (2013), the specification of the rule now features a systematic response to deviations of annual inflation from a positive constant trend inflation (featuring weighted response to both headline and core inflation) and to deviations of observed annual GDP growth from its steady state level.[6] It also includes interest rate smoothing and response to welfare-relevant output gap as before. Thus, we re-estimate the model by replacing the standard policy rule with the following formulation:

$$R_t = \widetilde{R}_t^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \varepsilon_{R,t}\},$$

$$\widetilde{R}_t = \overline{R} \left\{ \left(\frac{(\Pi_{s=0}^{3} \pi_{c,t-s})^{1/4}}{\overline{\pi}} \right)^{\tau} \left(\frac{(\Pi_{s=0}^{3} \pi_{q,t-s})^{1/4}}{\overline{\pi}} \right)^{1-\tau} \right\}^{\psi_\pi} \left\{ \frac{x_t}{\overline{x}} \right\}^{\psi_x} \left\{ \frac{(Y_t/Y_{t-1})^{1/4}}{\overline{g}} \right\}^{\psi_{\Delta y}} .$$

We find a stronger response to output growth in both periods which is somewhat similar in magnitude to what Justiniano, Primiceri and Tambalotti (2013) reports. Other than this, the remaining results remain quite robust.

8.3 Boundary

As discussed earlier, the presence of positive trend inflation enriches the dynamics of the model and the usual Taylor principle ($\psi_\pi > 1$) is no longer a sufficient condition for local determinacy of equilibrium. Due to the higher-order dynamics, it is not feasible to analytically derive the indeterminacy conditions. To continue solving the model via Lubik and Schorfheide's (2004) continuity solution (where $M^*(\theta)$ is selected such that the responses of the endogenous variables to the fundamental shocks are continuous at the boundary between the determinacy and indeterminacy region) one needs to resort to numerical methods. In our applications so far, we follow Justiniano and Primiceri (2008) and Hirose (2014) by perturbing the response to inflation ψ_π in the monetary policy rule to numerically trace the boundary. However,

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[6] Strictly speaking, Justiniano, Primiceri and Tambalotti (2013) consider deviations of annual inflation from a time-varying inflation target.

due to the presence of trend inflation, the boundary becomes a complicated function of ψ_π along with other Taylor rule and structural parameters. As such, the (in-)determinacy test might be susceptible to how we trace the boundary. Hence, as an alternative, we now drag both the response to inflation ψ_π as well as the response to output gap ψ_x. This then possibly gets us to a different region of the boundary in the parameter space. Nonetheless, we still find that the data favors determinacy and the response to inflation is active even during the Great Inflation period.

8.4 Flexible-price output gap

We have argued earlier that allowing for wage data in the estimation helps us account for the higher real wage rigidity in the 1970s and generates a quantitatively meaningful trade-off faced by the central bank in the model economy. In the face of such trade-offs, our posterior estimates suggest an active response to inflation and a virtually negligible response to output gap during the pre-Volcker period which combined together push the posterior toward the determinacy region. In line with Blanchard and Riggi (2013), we have focused on welfare-relevant output gap, defined as the gap between actual and efficient output. Blanchard and Riggi (2013) justify their assumption by arguing that natural or potential level of output may move a lot with respect to oil price shock in a model with real wage rigidities whereas the efficient or welfare-relevant output moves much less, looks like a smooth time trend and appears to be what the Federal Reserve looks at. However, one could rightfully argue that natural or potential output is a better reference point for monetary policy as monetary policy is neutral in the long run and thus cannot offset fluctuations in the welfare-relevant output gap. As such, we replace the efficient output gap with the flexible-price output gap, defined as the gap between actual and potential output. We find that the estimate of the response to output gap during the pre-1979 period turns out to be somewhat higher this time. Yet, the findings that the pre-Volcker period is characterized by determinacy and active response to inflation remain unchanged.

8.5 Estimation over the entire parameter space

In our applications so far, follow Lubik and Schorfheide (2004) and estimate the model twice, first under determinacy, then under indeterminacy. While Lubik and Schorfheide (2004) possibly did so because of the sampling technology available back then which was Random Walk Metropolis Hastings (RWMH) algorithm, an importance sampling algorithm like SMC can use a single chain instead to explore the entire parameter space. Hence, to take full advantage of this algorithm, we now estimate the model simultaneously over both determinacy and indeterminacy region following Hirose, Kurozumi and Van Zandweghe (2017). The likelihood function is then given by

$$p(X_T|\theta_S, S) = 1\{\theta_S \in \Theta^D\}p^D(X_T|\theta_D, D) + 1\{\theta_S \in \Theta^I\}p^I(X_T|\theta_I, I),$$

where Θ^D, Θ^I are the determinacy and indeterminacy regions of the parameter space, $1\{\theta_S \in \Theta^S\}$ is the indicator function that equals 1 if $\theta_S \in \Theta^S$ and zero otherwise, and $p^D(X_T|\theta_D, D)$, $p^I(X_T|\theta_I, I)$ are the likelihood functions under determinacy and indeterminacy respectively. All our results, including the fit of the model and the parameter estimates, stay unaltered.

8.6 Oil as an observable

Lastly, we investigate the sensitivity of our results to directly using real oil price as an observable. In our effort to pin down the cost-push shocks, until now we have simultaneously employed both headline and core inflation measures as observables. This choice identifies the cost-push shocks as commodity price shocks in general (which includes the price of food and other commodities as well). To the extent that there were other driving forces of inflation in the 1970s other than oil price shocks, using both inflation measures simultaneously is a sound identification strategy. For instance, the two inflationary episodes in the 1970s also featured sizeable food-price hikes as documented by Blinder and Rudd (2012). Since food has a much larger weight in the price indexes than energy, ignoring them might constitute a key omission. Nonetheless, we also check the robustness of our results to directly using percentage change of the real price of oil as an observable to identify the episodes of oil price shocks in isolation. As such, we use the West Texas Intermediate oil price data from FRED II (2007).[7] We deflate the nominal oil price by the core consumer price index to be in line with the concept of real oil price in the model. The resulting series is then demeaned by it's sub-sample mean prior to estimation. We continue to use data on quarterly growth rate of GDP per capita, core CPI, the two (real) wage inflation series and the Federal Funds rate. Our results still remain robust.

9 Conclusion

This paper estimates a New Keynesian model with trend inflation and oil entering in both consumption and production. While allowing for indeterminacy, it examines the interaction between oil price shocks and monetary policy with a particular focus on the Great Inflation. First, when considering the model without any role for oil, we find that indeterminacy prevails in the pre-Volcker period while determinacy gets favoured in the post-1984 period. Next, when we introduce oil in the model, we find mixed evidence for indeterminacy in the 1970s. Yet, key features of the model, such as oil price shocks and the degree of real wage rigidity, are not properly identified. Hence, even after being hit with oil price shocks. there exist no quantitatively meaningful trade-off faced by the central bank between stabilizing inflation and the output gap. Therefore, to sharpen the identification of these important aspects, we then re-estimate the model using additional observables. We find that the pre-Volcker period in unambiguously characterized by a unique rational expectations equilibrium with a high degree of real wage rigidity. In this environment, the oil price shocks create an acute trade-off between inflation and output gap stabilization. Faced with this trade-off, we find that the Federal Reserve responded aggressively to inflation and output growth, but not to the output gap, thereby ruling out indeterminacy. Therefore, the finding that pre-Volcker period is characterized by a unique equilibrium has important implications for interpreting the Great Inflation and reassessing Federal Reserve's policy. We also estimate the model over the Great Moderation period and document that oil price shocks are no longer as inflationary as they used to be due to lower real wage rigidity, allowing the Federal Reserve to respond less aggressively to a given oil price shock. Therefore, this finding goes hand in hand with the hypoth-

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[7] Nakov and Pescatori (2010) use this same oil price series in their empirical exercise and find that oil played an important role in the Great Moderation.

esis of Blanchard and Gali (2010) that the decline in real wage rigidity partly helps explain the remarkable resilience of the economy to sustained oil price increases in the 2000s.

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Appendix A

A.1 Framework

The Non-linear Model

The non-linear model is described by the following equations.

$$\Xi_t = \frac{d_t}{C_t - hC_{t-1}} \tag{1m}$$

$$\frac{W_t}{P_{c,t}} = \nu_t N_t^{\varphi} \left(C_t - hC_{t-1}\right) \equiv MRS_t \tag{2m}$$

Following Blanchard and Gali (2007a,b) and Blanchard and Riggi (2013), we formalize real wage rigidities by modifying the previous equation as

$$\frac{W_t}{P_{c,t}} = \left\{\frac{W_{t-1}}{P_{c,t-1}}\right\}^{\gamma} \left\{MRS_t\right\}^{1-\gamma},$$

where γ is the degree of real wage rigidity.

$$\Xi_t = \beta E_t \left[\frac{R_t}{\pi_{c,t+1}}\right] \Xi_{t+1} \tag{3m}$$

$$P_{c,t}C_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t}Q_t \tag{4m}$$

$$Q_t \Delta_t = M_t^{\alpha} \left(A_t N_t\right)^{1-\alpha} \tag{5m}$$

$$M_t = \frac{\alpha}{\mathcal{M}_t^P} \frac{Q_t}{s_t} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}} \tag{6m}$$

$$\left(\frac{W_t}{P_{c,t}}\right)^{1-\alpha} \mathcal{M}_t^P = \mathcal{C} A_t^{1-\alpha} s_t^{-\alpha-\chi(1-\alpha)} \Delta_t^{\frac{-1}{\varepsilon}} \tag{7m}$$

$$p_{q,t}^*(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{\kappa_t}{\phi_t} \tag{8m}$$

$$\kappa_t = \mathcal{C}^{-1} \left(\frac{W_t}{P_{c,t}}\right)^{1-\alpha} s_t^{\chi(1-\alpha)+\alpha} A_t^{\alpha-1} Q_t \Xi_t + \xi\beta\overline{\pi}^{-\varepsilon\omega(1-\mu)} \pi_{q,t}^{-\mu\omega\varepsilon} E_t\left[\pi_{q,t+1}^{\varepsilon}\kappa_{t+1}\right] \tag{9m}$$

$$\phi_t = Q_t \Xi_t + \xi\beta\overline{\pi}^{(1-\mu)(1-\varepsilon)\omega} \pi_{q,t}^{\mu\omega(1-\varepsilon)} E_t\left[\pi_{q,t+1}^{\varepsilon-1}\phi_{t+1}\right] \tag{10m}$$

$$1 = \xi\overline{\pi}^{(1-\mu)(1-\varepsilon)\omega} \pi_{q,t-1}^{\mu\omega(1-\varepsilon)} \pi_{q,t}^{(\varepsilon-1)} + (1-\xi) p_{q,t}^*(i)^{1-\varepsilon} \tag{11m}$$

$$\Delta_t = (1-\xi) p_{q,t}^*(i)^{-\varepsilon} + \xi\overline{\pi}^{-\varepsilon\omega(1-\mu)} \pi_{q,t-1}^{-\mu\omega\varepsilon} \pi_{q,t}^{\varepsilon} \Delta_{t-1} \tag{12m}$$

$$P_{y,t} Y_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t} Q_t \tag{13m}$$

$$P_{c,t} \equiv P_{q,t} s_t^{\chi} \tag{14m}$$

$$R_t = \widetilde{R}_t^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \varepsilon_{R,t}\}, \qquad \widetilde{R}_t = (\overline{r\pi}) \left\{ \left(\frac{\pi_{c,t}}{\overline{\pi}}\right)^{\tau} \left(\frac{\pi_{q,t}}{\overline{\pi}}\right)^{1-\tau} \right\}^{\psi_\pi} \left\{\frac{x_t}{\overline{x}}\right\}^{\psi_x} \left\{\frac{Y_t}{\overline{g} Y_{t-1}}\right\}^{\psi_{\Delta y}} \tag{15m}$$

$$\ln s_t = \rho_s \ln s_{t-1} + \epsilon_{s,t} \tag{16m}$$

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t} \tag{17m}$$

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} \tag{18m}$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \tag{19m}$$

The Log-linearized Model

Following Ascari and Sbordone (2014), we take a log-linear approximation around a positive steady state trend inflation. Here hatted variables denote log-deviations from steady state or trend levels.

$$\widehat{\Xi}_t = \widehat{d}_t - \left(\frac{h}{g-h}\right)\widehat{g}_t - \left(\frac{g}{g-h}\right)\widehat{c}_t + \left(\frac{h}{g-h}\right)\widehat{c}_{t-1} \tag{1L}$$

$$\widehat{w}_t = \gamma\widehat{w}_t + (1-\gamma)\left\{\varphi\widehat{N}_t + \left(\frac{h}{g-h}\right)\widehat{g}_t + \left(\frac{g}{g-h}\right)\widehat{c}_t - \left(\frac{h}{g-h}\right)\widehat{c}_{t-1}\right\} + \widehat{\nu}_t \tag{2L}$$

Following Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), we normalize the labor supply shock such that it enters the labor supply equation with a coefficient of one. In this way, it is easier to choose a reasonable prior for the standard deviation of the shock.

$$\widehat{\Xi}_t = \widehat{R}_t - E_t\widehat{\pi}_{c,t+1} + E_t\widehat{\Xi}_{t+1} - E_t\widehat{g}_{t+1} \tag{3L}$$

$$\widehat{C}_t = \widehat{Q}_t - \chi\widehat{s}_t + \iota\widehat{\mu}_t - \iota\left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{4L}$$

$$\widehat{Q}_t = \alpha\widehat{M}_t + (1-\alpha)\widehat{N}_t - \widehat{\Delta}_t \tag{5L}$$

$$\widehat{M}_t = \widehat{Q}_t - \widehat{\mu}_t - \widehat{s}_t + \left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{6L}$$

$$(1-\alpha)\widehat{w}_t + \widehat{\mu}_t + \{\alpha + \chi(1-\alpha)\}\widehat{s}_t + \frac{1}{\varepsilon}\widehat{\Delta}_t = 0 \tag{7L}$$

$$\widehat{p^*_{q,t}}(i) = \widehat{\kappa}_t - \widehat{\phi}_t \tag{8L}$$

$$\begin{aligned}\widehat{\kappa}_t &= (1-\xi\beta\overline{\pi}^{\varepsilon(1-\omega)})\left[(1-\alpha)\widehat{w}_t + \{\chi(1-\alpha)+\alpha\}\widehat{s}_t + \widehat{Q}_t + \widehat{\Xi}_t\right] + \\ &\quad \xi\beta\overline{\pi}^{\varepsilon(1-\omega)}\left[\widehat{\kappa}_{t+1} + \varepsilon\widehat{\pi}_{q,t+1} - \mu\omega\varepsilon\widehat{\pi}_{q,t}\right]\end{aligned} \tag{9L}$$

$$\widehat{\phi}_t = (1-\xi\beta\overline{\pi}^{(\varepsilon-1)(1-\omega)})\left[\widehat{Q}_t + \widehat{\Xi}_t\right] + \xi\beta\overline{\pi}^{(\varepsilon-1)(1-\omega)}\left[\widehat{\phi}_{t+1} + (\varepsilon-1)\widehat{\pi}_{q,t+1} + \mu\omega(1-\varepsilon)\widehat{\pi}_{q,t}\right] \tag{10L}$$

$$\widehat{p^*_{q,t}}(i) = \frac{\xi\overline{\pi}^{(\varepsilon-1)(1-\omega)}}{1-\xi\overline{\pi}^{(\varepsilon-1)(1-\omega)}}\left[\widehat{\pi}_{q,t} - \mu\omega\widehat{\pi}_{q,t-1}\right] \tag{11L}$$

$$\widehat{\Delta}_t = \left[-\varepsilon(1-\xi\overline{\pi}^{\varepsilon(1-\omega)})\right]\widehat{p^*_{q,t}}(i) + \xi\overline{\pi}^{\varepsilon(1-\omega)}\left[-\mu\omega\varepsilon\widehat{\pi}_{q,t-1} + \varepsilon\widehat{\pi}_{q,t} + \widehat{\Delta}_{t-1}\right] \tag{12L}$$

$$\widehat{Y}_t = \widehat{Q}_t + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s}_t + \iota\widehat{\mu}_t - \iota\left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{13L}$$

$$\widehat{\pi}_{c,t} = \widehat{\pi}_{q,t} + \chi(\widehat{s}_t - \widehat{s}_{t-1}) \tag{14L}$$

$$\widehat{R}_t = \rho_R\widehat{R}_{t-1} + (1-\rho_R)\left[\psi_\pi\left\{\tau\widehat{\pi}_{c,t} + (1-\tau)\,\widehat{\pi}_{q,t}\right\} + \psi_x\widehat{x}_t + \psi_g(\widehat{Y}_t - \widehat{Y}_{t-1} + \widehat{g}_t)\right] + \varepsilon_{R,t} \tag{15L}$$

$$\widehat{s}_t = \rho_s\widehat{s}_{t-1} + \varepsilon_{s,t} \tag{16L}$$

$$\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \varepsilon_{g,t} \tag{17L}$$

$$\widehat{d}_t = \rho_d\widehat{d}_{t-1} + \varepsilon_{d,t} \tag{18L}$$

$$\widehat{\nu}_t = \rho_\nu\widehat{\nu}_{t-1} + \varepsilon_{\nu,t} \tag{19L}$$

A.2 Derivation of the Output Gap

Efficient Allocation (First Best)

We derive the efficient allocation by assuming perfect competition in goods and labor markets following Blanchard and Gali (2007) and Blanchard and Riggi (2013). From the firms' side we have

$$(1-\alpha)\,\widehat{w}_t = -\left(\alpha + (1-\alpha)\,\chi\right)\widehat{s}_t,$$

and from the consumer's side

$$\widehat{w}_t = \varphi \widehat{N}_t + \frac{h}{g-h}\widehat{g}_t + \frac{g}{g-h}\widehat{C}_t - \frac{h}{g-h}\widehat{C}_{t-1} + \widehat{\nu}_t.$$

At first, we substitute the aggregate resource constraint $\widehat{C}_t = \widehat{Q}_t - \chi\widehat{s}_t$ and combine both equations to get

$$(1-\alpha)\left\{\varphi \widehat{N}_t + \frac{h}{g-h}\widehat{g}_t + \frac{g}{g-h}\widehat{C}_t - \frac{h}{g-h}\widehat{C}_{t-1} + \widehat{\nu}_t\right\} = -\left(\alpha + (1-\alpha)\,\chi\right)\widehat{s}_t.$$

Using the reduced-form production function $\widehat{Q_t} = \widehat{N_t} - \frac{\alpha}{1-\alpha}\widehat{s}_t$ and after rearranging, we get the following expression for the first-best employment:

$$\begin{aligned}
\widehat{N_t^e} \;=\;& \left[\frac{h}{(g-h)\,\varphi + g}\right]\widehat{N_{t-1}^e} - \left[\frac{h}{(g-h)\,\varphi + g}\right]\widehat{g}_t \\
&+ \left[\frac{h\left(\alpha + (1-\alpha)\,\chi\right)}{(g-h)\,(1-\alpha)\,\varphi + g\,(1-\alpha)}\right]\left(\widehat{s}_t - \widehat{s_{t-1}}\right) - \left[\frac{g-h}{(g-h)\,\varphi + g}\right]\widehat{\nu}_t.
\end{aligned}$$

Given first-best employment, first-best output $\widehat{Y}_t^e$ can be written as

$$\begin{aligned}
\widehat{Y_t^e} &= \widehat{Q_t} + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s}_t \\
&= \widehat{N_t^e} - \left(\frac{\alpha}{1-\alpha}\right)\widehat{s}_t + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s}_t \\
&= \widehat{N_t^e}.
\end{aligned}$$

So,

$$\begin{aligned}
\widehat{Y_t^e} &= \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{Y_{t-1}^e} - \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{g}_t \\
&\quad + \left[\frac{h(\alpha+(1-\alpha)\chi)}{(g-h)(1-\alpha)\varphi+g(1-\alpha)}\right](\widehat{s}_t - \widehat{s_{t-1}}) - \left[\frac{g-h}{(g-h)\varphi+g}\right]\widehat{\nu}_t.
\end{aligned}$$

Therefore, we can write the welfare-relevant output gap defined as the difference between output and its first-best level

$$\begin{aligned}
\widehat{x_t^e} &= \widehat{Y}_t - \widehat{Y_t^e} \\
&= \widehat{Y}_t - \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{Y_{t-1}^e} + \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{g}_t \\
&\quad - \left[\frac{h(\alpha+(1-\alpha)\chi)}{(g-h)(1-\alpha)\varphi+g(1-\alpha)}\right](\widehat{s}_t - \widehat{s_{t-1}}) + \left[\frac{g-h}{(g-h)\varphi+g}\right]\widehat{\nu}_t.
\end{aligned}$$

Flexible Price Equilibrium (Second Best)

We derive the second-best level of employment and output by assuming that prices and wages are flexible. As before, from the firms' side we have

$$\begin{aligned}
(1-\alpha)\widehat{w}_t &= -(\alpha+(1-\alpha)\chi)\widehat{s}_t \\
\widehat{w}_t &= -\left(\frac{\alpha+(1-\alpha)\chi}{1-\alpha}\right)\widehat{s}_t
\end{aligned}$$

and from the consumer's side

$$\widehat{w}_t = \gamma\widehat{w_{t-1}} + (1-\gamma))\left[\varphi\widehat{N}_t + \frac{h}{g-h}\widehat{g}_t + \frac{g}{g-h}\widehat{C}_t - \frac{h}{g-h}\widehat{C}_{t-1}\right] + \widehat{\nu}_t,$$

where again we normalize the labor supply shock.

As before, we substitute the aggregate resource constraint $\widehat{C}_t = \widehat{Q}_t - \chi\widehat{s}_t$ and combine both equations to get

$$\begin{aligned}
-\left(\frac{\alpha+(1-\alpha)\chi}{1-\alpha}\right)\widehat{s}_t &= -\gamma\left(\frac{\alpha+(1-\alpha)\chi}{1-\alpha}\right)\widehat{s_{t-1}} \\
&\quad + (1-\gamma))\left[\varphi\widehat{N}_t + \frac{h}{g-h}\widehat{g}_t + \frac{g}{g-h}\widehat{C}_t - \frac{h}{g-h}\widehat{C}_{t-1}\right] + \widehat{\nu}_t,
\end{aligned}$$

Using the reduced-form production function $\widehat{Q_t} = \widehat{N_t} - \frac{\alpha}{1-\alpha}\widehat{s_t}$ and after rearranging, we get the following expression for the second-best employment:

$$\begin{aligned}
\widehat{N_t^f} &= \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{N_{t-1}^f} - \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{g_t} \\
&+ \left[\frac{(h-\gamma g)(\alpha+(1-\alpha)\chi)}{\varphi(1-\gamma)(1-\alpha)(g-h)+g(1-\gamma)(1-\alpha)}\right](\widehat{s_t}-\widehat{s_{t-1}}) \\
&- \left[\frac{g-h}{\varphi(1-\gamma)(g-h)+(1-\gamma)g}\right]\widehat{\nu}_t.
\end{aligned}$$

Given second-best employment, second-best output $\widehat{Y}_t^f$ can be written as

$$\begin{aligned}
\widehat{Y_t^f} &= \widehat{Q_t} + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s_t} \\
&= \widehat{N_t^f} - \left(\frac{\alpha}{1-\alpha}\right)\widehat{s_t} + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s_t} \\
&= \widehat{N_t^f}.
\end{aligned}$$

So,

$$\begin{aligned}
\widehat{Y_t^f} &= \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{Y_{t-1}^f} - \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{g_t} \\
&+ \left[\frac{(h-\gamma g)(\alpha+(1-\alpha)\chi)}{\varphi(1-\gamma)(1-\alpha)(g-h)+g(1-\gamma)(1-\alpha)}\right](\widehat{s_t}-\widehat{s_{t-1}}) \\
&- \left[\frac{g-h}{\varphi(1-\gamma)(g-h)+(1-\gamma)g}\right]\widehat{\nu}_t.
\end{aligned}$$

Therefore, we can write the welfare-relevant output gap defined as the difference between output and its first-best level

$$\begin{aligned}
\widehat{x_t^f} &= \widehat{Y}_t - \widehat{Y_t^f} \\
&= \widehat{Y}_t - \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{Y_{t-1}^f} + \left[\frac{h}{(g-h)\varphi+g}\right]\widehat{g_t} \\
&- \left[\frac{(h-\gamma g)(\alpha+(1-\alpha)\chi)}{\varphi(1-\gamma)(1-\alpha)(g-h)+g(1-\gamma)(1-\alpha)}\right](\widehat{s_t}-\widehat{s_{t-1}}) \\
&+ \left[\frac{g-h}{\varphi(1-\gamma)(g-h)+(1-\gamma)g}\right]\widehat{\nu}_t.
\end{aligned}$$

Table 8: Parameter Estimates, Robustness (1966:I-1979:II)

	Indexation	JPT rule	Boundary	Output Gap	Entire parm. space	CoreCPI-Oil
ψ_π	1.39 (0.17)	1.32 (0.14)	1.51 (0.17)	1.46 (0.14)	1.51 (0.17)	1.45 (0.15)
ψ_x	0.05 (0.04)	0.05 (0.05)	0.03 (0.03)	0.11 (0.08)	0.03 (0.03)	0.02 (0.02)
ψ_g	0.33 (0.15)	0.60 (0.18)	0.35 (0.14)	0.39 (0.14)	0.33 (0.13)	0.55 (0.17)
ρ_R	0.69 (0.06)	0.64 (0.06)	0.69 (0.05)	0.69 (0.05)	0.68 (0.06)	0.69 (0.05)
τ	0.59 (0.16)	0.78 (0.12)	0.58 (0.15)	0.48 (0.17)	0.58 (0.15)	0.39 (0.16)
π^*	1.35 (0.19)	1.37 (0.17)	1.36 (0.17)	1.39 (0.17)	1.37 (0.17)	1.44 (0.18)
R^*	1.52 (0.21)	1.56 (0.18)	1.51 (0.20)	1.53 (0.21)	1.53 (0.20)	1.60 (0.21)
g^*	0.45 (0.07)	0.46 (0.07)	0.45 (0.07)	0.44 (0.07)	0.45 (0.07)	0.41 (0.08)
ξ	0.60 (0.04)	0.62 (0.03)	0.60 (0.04)	0.60 (0.04)	0.60 (0.04)	0.65 (0.04)
γ	0.90 (0.03)	0.92 (0.02)	0.89 (0.04)	0.88 (0.04)	0.89 (0.03)	0.91 (0.03)
h	0.41 (0.07)	0.42 (0.07)	0.38 (0.07)	0.37 (0.07)	0.38 (0.07)	0.27 (0.06)
ω	0.44 (0.08)	–	–	–	–	–
ρ_d	0.76 (0.07)	0.69 (0.08)	0.77 (0.07)	0.77 (0.07)	0.76 (0.06)	0.86 (0.04)
ρ_ν	0.85 (0.06)	0.81 (0.07)	0.86 (0.07)	0.89 (0.05)	0.86 (0.07)	0.81 (0.07)
σ_s	17.23 (1.63)	17.07 (1.60)	17.24 (1.62)	17.30 (1.66)	17.31 (1.63)	17.61 (1.66)
σ_g	0.46 (0.08)	0.50 (0.09)	0.48 (0.09)	0.48 (0.08)	0.49 (0.09)	0.55 (0.10)
σ_r	0.29 (0.03)	0.28 (0.03)	0.30 (0.03)	0.30 (0.04)	0.30 (0.03)	0.34 (0.04)
σ_d	2.00 (0.37)	2.03 (0.33)	1.86 (0.33)	1.74 (0.31)	1.84 (0.32)	2.07 (0.43)
σ_ν	0.43 (0.09)	0.35 (0.07)	0.38 (0.08)	0.40 (0.08)	0.38 (0.08)	0.38 (0.08)
σ_ζ	–	–	–	–	0.44 (0.17)	–
$M_{s,\zeta}$	–	–	–	–	−0.01 (0.97)	–
$M_{g,\zeta}$	–	–	–	–	0.00 (0.98)	–
$M_{r,\zeta}$	–	–	–	–	0.01 (0.97)	–
$M_{d,\zeta}$	–	–	–	–	0.08 (0.98)	–
$M_{\nu,\zeta}$	–	–	–	–	0.01 (0.98)	–
λ	1.07 (0.22)	1.02 (0.25)	1.07 (0.24)	1.10 (0.23)	1.05 (0.23)	0.98 (0.32)
σ_{w_1}	0.38 (0.09)	0.34 (0.11)	0.37 (0.10)	0.39 (0.09)	0.36 (0.10)	0.31 (0.12)
σ_{w_2}	0.45 (0.10)	0.48 (0.10)	0.46 (0.10)	0.43 (0.11)	0.47 (0.09)	0.48 (0.12)

Table 9: Parameter Estimates, Robustness (1984:I-2008:II)

	Indexation	JPT rule	Boundary	Output Gap	Entire parm. space	CoreCPI-Oil
ψ_π	2.94 (0.31)	2.79 (0.32)	3.08 (0.36)	2.20 (0.22)	3.09 (0.35)	2.97 (0.37)
ψ_x	0.14 (0.07)	0.07 (0.04)	0.11 (0.06)	0.13 (0.09)	0.11 (0.05)	0.07 (0.03)
ψ_g	0.55 (0.13)	0.91 (0.16)	0.69 (0.15)	0.60 (0.14)	0.62 (0.13)	0.68 (0.15)
ρ_R	0.70 (0.05)	0.68 (0.04)	0.73 (0.04)	0.73 (0.04)	0.73 (0.04)	0.75 (0.04)
τ	0.13 (0.05)	0.21 (0.09)	0.14 (0.05)	0.21 (0.08)	0.14 (0.05)	0.21 (0.09)
π^*	0.93 (0.09)	1.03 (0.10)	0.95 (0.09)	0.95 (0.07)	0.97 (0.09)	0.98 (0.10)
R^*	1.44 (0.14)	1.47 (0.14)	1.44 (0.14)	1.44 (0.13)	1.46 (0.14)	1.47 (0.15)
g^*	0.17 (0.05)	0.15 (0.05)	0.16 (0.05)	0.13 (0.05)	0.17 (0.04)	0.15 (0.05)
ξ	0.52 (0.05)	0.69 (0.03)	0.62 (0.04)	0.68 (0.04)	0.61 (0.04)	0.64 (0.04)
γ	0.33 (0.11)	0.70 (0.07)	0.46 (0.12)	0.58 (0.10)	0.46 (0.11)	0.66 (0.10)
h	0.22 (0.05)	0.35 (0.06)	0.24 (0.05)	0.30 (0.06)	0.24 (0.05)	0.35 (0.06)
ω	0.34 (0.09)	–	–	–	–	–
ρ_d	0.85 (0.04)	0.80 (0.04)	0.85 (0.04)	0.85 (0.04)	0.84 (0.03)	0.83 (0.04)
ρ_ν	0.99 (0.01)	0.98 (0.01)	0.99 (0.01)	0.98 (0.01)	0.99 (0.01)	0.98 (0.01)
σ_s	20.43 (1.44)	14.94 (1.04)	20.41 (1.50)	20.20 (1.40)	20.14 (1.35)	12.90 (0.91)
σ_g	0.42 (0.08)	0.52 (0.09)	0.44 (0.08)	0.53 (0.09)	0.43 (0.07)	0.44 (0.08)
σ_r	0.18 (0.02)	0.15 (0.01)	0.17 (0.02)	0.17 (0.02)	0.17 (0.02)	0.17 (0.02)
σ_d	1.26 (0.29)	1.39 (0.17)	1.23 (0.19)	1.20 (0.16)	1.21 (0.18)	1.25 (0.19)
σ_ν	0.89 (0.15)	0.49 (0.08)	0.75 (0.19)	0.69 (0.12)	0.74 (0.13)	0.56 (0.13)
σ_ζ	–	–	–	–	0.47 (0.17)	–
$M_{s,\zeta}$	–	–	–	–	−0.10 (1.00)	–
$M_{g,\zeta}$	–	–	–	–	−0.11 (0.95)	–
$M_{r,\zeta}$	–	–	–	–	0.03 (0.96)	–
$M_{d,\zeta}$	–	–	–	–	0.06 (0.95)	–
$M_{\nu,\zeta}$	–	–	–	–	0.06 (0.97)	–
λ	0.28 (0.07)	0.29 (0.09)	0.29 (0.08)	0.31 (0.08)	0.30 (0.08)	0.38 (0.12)
σ_{w_1}	0.70 (0.07)	0.62 (0.07)	0.66 (0.07)	0.60 (0.08)	0.66 (0.07)	0.64 (0.07)
σ_{w_2}	0.38 (0.04)	0.38 (0.04)	0.38 (0.04)	0.36 (0.03)	0.38 (0.04)	0.37 (0.04)

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V Conclusion

This thesis contributes to our understanding of U.S. monetary policy over the Great Inflation and the Great Moderation, including the period of loose monetary policy in the early 2000s. The three self-contained papers apply Bayesian estimation techniques to investigate the adequacy of monetary policy by assessing the quantitative relevance of equilibrium indeterminacy through the lens of structural New Keynesian models. In doing so, this thesis furthers our comprehension of the drivers of the high and volatile inflationary episodes of the 1970s, the decline in macroeconomic volatility and inflation gap predictability since the mid-1980s, and the issue of loose monetary policy in the early 2000s.

The first paper examines monetary policy following the 2001 recession and its alleged link to the enormous macroeconomic instability of the Great Recession and establishes a number of new insights. It finds a violation of the Taylor principle for most of the 2000s when using CPI to measure inflation, thereby supporting the allegations made by Stanford economist John Taylor that Fed policy during the early 2000s was as loose as in the 1970s. In stark contrast, when measuring inflation using core PCE, monetary policy appear to have been quite appropriate and adhering to the Taylor principle, thus corroborating the claims of former Fed Chairman Ben Bernanke. However, these findings create a puzzle since the conclusions are so heavily dependent on the particular measure of inflation being used in the estimation. Moreover, the simple New Keynesian model features a single concept of inflation. Yet, it is possible that while households undeniably care about headline inflation, the central bank focuses on core inflation. To resolve the ambiguity, the paper accordingly turns toward an artificial economy that structurally distinguishes between core and headline inflation. Estimation results from this extended model indeed find that the Fed was responding mainly to core PCE and was sufficiently aggressive to inflation, thereby validating the assertions of Bernanke.

The second paper investigates the drivers of both the Great Inflation and the Great Moderation. The paper makes two contributions. First, it documents that the Federal Reserve has pursued a time-varying inflation target which captures the permanent component of inflation. As a corollary, policy has responded aggressively to the inflation gap not only during the Great Moderation as suggested by the literature but also during the Great Inflation. Second, the paper documents that both good policy, in terms of a stronger response to the inflation gap and a better anchored inflation target, and good luck, in terms of smaller aggregate technology shocks hitting the economy, are jointly required to explain the Great Moderation.

The final paper considers the impact of commodity price fluctuations on monetary policy with a particular focus on the oil price shocks of the 1970s. Arguably, those shocks generated a trade-off between stabilizing inflation and the output gap for the Federal Reserve and we model such a trade-off arising due to the presence of real wage rigidity. Indeed, the 1970s were times of strong labor unions. The paper finds that wage rigidity was, in fact, higher during the Great Inflation and in this environment the oil price shocks did create an acute trade-off. Faced with this dilemma, the paper shows that the Fed responded aggressively to both inflation and output growth, but not to the output gap, thereby ruling out self-fulfilling inflationary expectations or sunspots as an explanation of the Great Inflation. Finally, the paper shows that oil price shocks are no longer as inflationary as they used to be, allowing the Fed to be less aggressive and therefore explaining the absence of stagflationary outcomes in the 2000s.