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# In-Medium $K^+$ Electromagnetic Form Factor with a Symmetric Vertex in a Light Front Approach

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Using the light-front kaon wave function based on a Bethe-Salpeter amplitude model for the quark-antiquark bound state, we study the Electromagnetic Form Factor (EMFF) of the kaon in nuclear medium within the framework of light-front field theory. The kaon model we adopt is well constrained by previous and recent studies to explain its properties in vacuum. The in-medium kaon EMFF is evaluated for the + component of the electromagnetic current, $J^+$, in the Breit frame. In order to consistently incorporate the constituent up and antistrange quarks of the kaon immersed in symmetric nuclear matter, we use the Quark-Meson Coupling (QMC) model, which has been widely applied to various hadronic and nuclear phenomena in a nuclear medium with success. We predict the in-medium modification of the kaon EMFF in symmetric nuclear matter. It is found that, after a fine tuning of the regulator mass, i.e. $m_R = 0.600$ GeV, the model is suitable to fit the available experimental data in vaccum within the theoretical uncertainties, and based on this we predict the in-medium modification of the EMFF.

PACS numbers:

## I. INTRODUCTION

The main purpose of this work is to investigate the in-medium modifications of kaon EMFF in symmetric nuclear matter combined with the QMC model [1], where the kaon model [2] is adjusted so as to provide the best description of the EMFF data in vacuum. The study of the lighter pseudoscalar mesons plays an important role to understand the low energy QCD. Their static and dynamical properties have also been investigated theoretically and experimentally [3–12]. With respect to the description of bound states on the light cone, a detailed review of hadronic wave functions in QCD-based models can be found in Ref. [13]. Additional important knowledge about the meson's internal structure can be inferred from their valence-quark parton distribution functions. The theoretical framework we adopt is the light-front field theory [13, 14], more specifically, we use a symmetric vertex model for $|q\bar{q}\rangle$ kaon bound-state in the light-front approach for the Bethe-Salpeter amplitude. The light-front component $J^+$ of the electromagnetic current has been successfully used to calculate elastic form factors. For the symmetric $K - q\bar{q}$ vertex model [15], the components of the current are conveniently obtained in the Drell-Yan frame, where the light-front bound state wave functions are defined on the hypersurface $x^0 + x^3 = 0$ and are covariant under kinematical boosts due to the stability of Fock-state decomposition [13, 16]. In this work, we consider the symmetric vertex function with the intention to optimize and unify the parameter set to simultaneously reproduce the electromagnetic form factor. Our numerical results are compared with experimental data in vacuum up to $\approx 0.10$ GeV$^2$ to explore the validity of the model $Q^2 = -q^2 > 0$, with $q$ being the four-momentum transfer.

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## II. THE MODEL

The electromagnetic current for a two-fermion bound state system with spin zero and intrinsic negative parity, $0^-$, a $K^+$ meson ($|q\bar{q}\rangle$ bound state), is calculated in one-loop approximation (triangle diagram shown in Fig. 1), modelling the Bethe-Salpeter amplitude through a symmetric vertex function in momentum space with a pseudoscalar coupling between $K^+$ meson and quark. This coupling is given by the effective Lagrangian [2, 15, 17]:

$$\mathcal{L}_{eff} = -\imath \frac{\hat{m}}{f_{K^+}} \bar{q}(\lambda_4 + \imath\lambda_5)\gamma^5 q(\phi_4 - \imath\phi_5)\Lambda^*, \tag{1}$$

here, $q = (u, d, s)^T$ and $K^+ = (\phi_4 - \imath\phi_5)$, $\hat{m}$ is given by the $\frac{m_u^* + m_{\bar{s}}}{2}$ with $m_u^* = m_u + V_s$ , $\Lambda^*$ the symmetric vertex function in nuclear medium and $f_{K^+}$ the $K^+$ meson decay constant. In this study we approximate to use $f_{K^+}$ value in vacuum. In the Hartree mean field approximation the modifications enter as the shift of the quark momentum via $P^\mu \to P^\mu + V^\mu = P^\mu + \delta_0^\mu V^0$ due to the vector potential, and in the Lorentz-scalar part through the Lorentz-scalar potential $V_s$ as $m_u \to m_u + V_s$ [1, 18] and $m_{\bar{s}} \to m_{\bar{s}}^* = m_{\bar{s}}$ based on the QMC model [1]. The QMC model has been applied to many nuclear and hadronic phenomena in a nuclear medium with success, and the input quantities in vacuum as well as quantities calculated in-medium shown in Table 1 were adopted in the model to describe the effects of nuclear medium. The electromagnetic current for $K^+$ meson with the plus component, is obtained from the covariant expression Eq. (2) corresponding to the triangle diagram in figure 1:

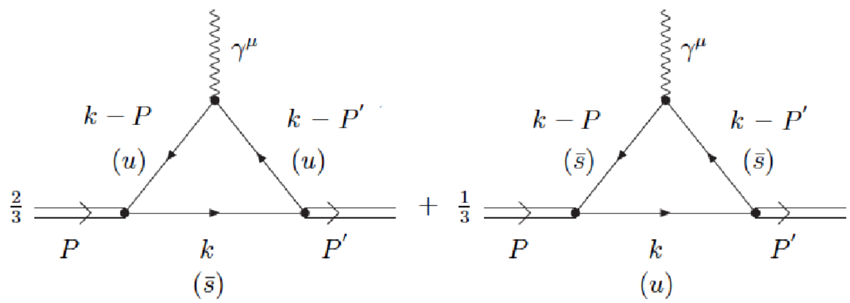


FIG. 1: Light-Front Feymnan diagram for $K$ meson exchange momentum with a photon.

$$J^+(q^2) = -ie\frac{\hat{m}^2}{f_{K^+}^2} N_C \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{2}{3} Tr[S(k, m_{\bar{s}})\gamma^5 S(k^* - P', m_u^*)\gamma^+ S(k^* - P, m_u^*)\gamma^5] + \frac{1}{3} Tr[S(k^*, m_u^*)\gamma^5 S(k - P', m_{\bar{s}})\gamma^+ S(k - P, m_{\bar{s}})\gamma^5] \right\} \Lambda^*(k + V, P)\ \Lambda^*(k + V, P'). \tag{2}$$

where $(k^*)^\mu = k^\mu + \delta_0^\mu V^0$, $\Lambda^*(k + V, P')$ and $\Lambda^*(k + V, P)$ are the symmetric vertex function in nuclear medium, where the symmetric vertex function in nuclear medium is given by [15]

$$\Lambda^*(k + V, P) = \frac{C}{(k + V)^2 - m_R^{*2} + i\epsilon)} + \frac{C}{((P - k - V)^2 - m_R^{*2} + i\epsilon)}, \tag{3}$$

with $N_C = 3$ being the number of colors in QCD, and

$$S(k^* - P, m_u^*) = \frac{1}{(\not{k}^* - \not{P} - m_u^* + i\epsilon)} \text{ and } S(k - P, m_{\bar{s}}) = \frac{1}{(\not{k} - \not{P} - m_{\bar{s}} + i\epsilon)}, \tag{4}$$

are corresponding to the up and antistrange quark propagators, respectively, in symmetric nuclear matter.

We summarize here the light-front model for the symmetric vertex function $\Lambda^*$ (described above) for the pseudoscalar bound states. Also, we work in the Breit frame and using light-front variables, $k^+ = k^0 + k^3$, $k^- = k^0 - k^3$ and $k^\perp = (k^1, k^2)$, and one has

$$q^+ = -q^- = \sqrt{-q^2}\sin\alpha,\ q_x = \sqrt{-q^2}\cos\alpha,\ q_y = 0 \text{ and } q^2 = q^+q^- - (\vec{q}_\perp)^2, \tag{5}$$

where the Drell-Yan condition $q^+ = 0$ is recovered with $\alpha = 0^o$ [6, 15, 19]. As is well known the $K^+$ meson form factor can be extracted from the covariant expression below:

$$F_{K^+}^*(q^2) = \frac{1}{e(P + P')^\mu} < P'|J^\mu|P > . \tag{6}$$

If covariance and current conservation are fulfilled, one can obviously use any frame and any nonvanishing component of the current to calculate the electromagnetic form factor. In the light-front approach, however, besides the valence component of the electromagnetic current, we can have the nonvalence contribuition or zero modes; thus in the light-front, this two contributions enter in the full electromagnetic form factor:

$$F^{*}_{K^+}(q^2) = F^{*(I)}_{K^+}(q^2,\alpha) + F^{*(II)}_{K^+}(q^2,\alpha), \tag{7}$$

where $\alpha = 0^o$, $F^{*(I)}_{K^+}(q^2,\alpha)$ has the loop integration on $k^-$ constrained by $0 \le k^+ < P^+$ (see the light-front time-ordered diagram in Fig. 2 (a)) the valence $(u\bar{s})$, and $F^{*(II)}_{K^+}(q^2,\alpha)$ has the loop integration on $k^-$ in the interval $P^+ \le k^+ \le P'^+$ (see Fig. 2 (b)) the nonvalence $|u\bar{s}\rangle$, pair production contributions with $q^+ > 0$.

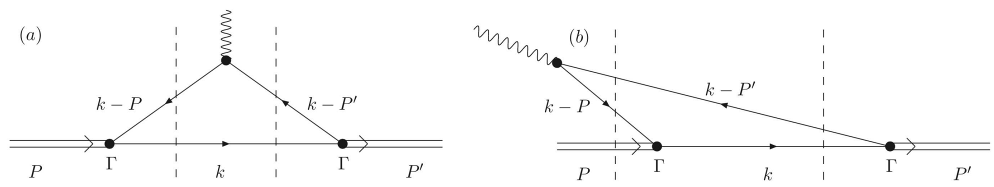


FIG. 2: Light-Front Feymnan diagrams: (a) valence contribution and (b) non-valence contributions.

Moreover, after $k^-$ integration from $J^+$ current Eq. (2) using the Cauchy's Teorem, a light-front wave function emerges for a symmetric $|u\bar{s}\rangle$ vertex function with the change of variable $x^*P^{*+} = xP^+ + V^+$ where $x = \frac{k^+}{P^+}$. The kaon light-front wave function is defined as:

$$\Phi(x,\vec{k}_\perp) = \frac{P^+}{m^{*2}_{K^+} - \mathcal{M}_0^2}\left[\frac{\mathcal{N}}{(1-x)(m^{*2}_{K^+} - \mathcal{M}_0^2)} + \frac{\mathcal{N}}{x(m^{*2}_{K^+} - \mathcal{M}_R^2))}\right] + [u \leftrightarrow \bar{s}], \tag{8}$$

where $\mathcal{N}$ is the normalization factor, $\mathcal{M}_0^2$ is a mass operator and $\mathcal{M}_R^2$ is a regulator mass function given by

$$\mathcal{M}_0^2 = \frac{k_\perp^2 + m_u^{*2}}{x^*} + \frac{(P-k)_\perp^2 + m_{\bar{s}}^2}{(1-x^*)} - p_\perp^2 \text{ and } \mathcal{M}_R^2 = \frac{k_\perp^2 + m_u^{*2}}{x^*} + \frac{(P-k)_\perp^2 + m_R^2}{(1-x^*)} - p_\perp^2, \tag{9}$$

and $[u \leftrightarrow \bar{s}]$.

| $\rho/\rho_0$ | 0.00 | 0.25 | 0.50 | 0.75 |
|---|---|---|---|---|
| $m_K [GeV]$ | 0.494 | 0.472 | 0.453 | 0.435 |
| $m_u\ [GeV]$ | 0.220 | 0.180 | 0.143 | 0.110 |
| $V\ \ [GeV]$ | 0.000 | 0.029 | 0.058 | 0.088 |
| $m_{\bar{s}}\ [GeV]$ | 0.440 | | | |
| $m_R\ [GeV]$ | 0.600 | | | |

TABLE I: Parameters used to compute the electromagnetic form factor for $K^+$ meson in medium, where is comes from QMC model, to details see Ref. [1, 18].

## III. NUMERICAL RESULTS

We have three model parameters: the regulator mass, $m_R$, the quark masses, $m_u$ and $m_{\bar{s}}$ to compute EMFF. Our main aims of this work is to jointly analyze the $K^+$ meson's elastic form factors to determine more accurately the model's quark masses in view of future applications and to test whether a single mass scale, $m_R$, can satisfactorily describe experimental data for $K^+$ meson [10, 11], as well as to study the in-medium $K^+$ EMFF. We use the quantitites described in Table 1 for vacuum, and in-medium inputs computed by the QMC model [1, 18]. Our main results of this study are shown in Fig. 3. As nuclear matter density increases $K^+$ EMFF decreases faster than that in vacuum, and thus this implies $K^+$ charge radius increases in-medium.

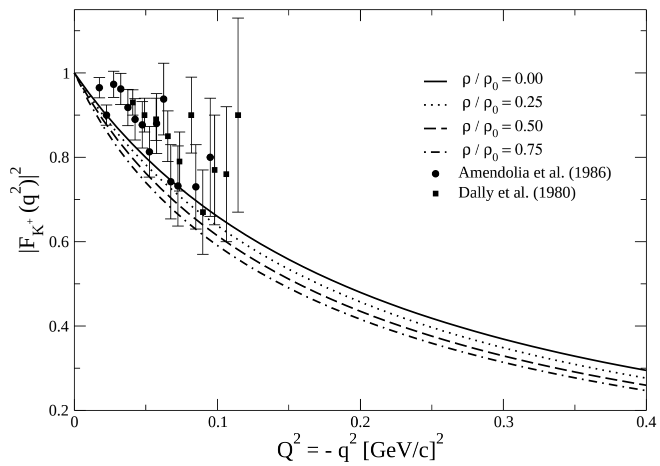


FIG. 3: Results for $K^+$ meson form factor squared with different density using parameters from table 1. Experimental data form vaccum given by references [10, 11].

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