

# Wavelet Methods and System Identification

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*Thesis submitted for the degree of*

*Doctor of Philosophy*

*in*

*the School of Mathematical Sciences*

*at*

*The University of Adelaide*

*(Faculty of Engineering, Computer and Mathematical Sciences)*

School of Mathematical Sciences



August 2016

# Acknowledgments

I would like express my appreciation and acknowledgment to various people who have help and support me in the process of completing this Ph.D. thesis.

First of all to my supervisors, Dr Andrew Metcalfe and Dr David Green, who keep supervising me through this adventurous and tough journey. I only could arrived at this point through their excellent mentoring, where they spend countless of hours reading my reports, and give some feedback and ideas on solving my thesis problems.

Most importantly my wife, Noratiqah, who has been the backbone, supporting me mentally and emotionally through this journey. Also to my loving parents, Abu Bakar and Zaiton, who keep reminding me to finish the thesis. To my loving son, Muhammad Lutfi, who being the motivation for me to finally complete this thesis.

I would like to express my gratitude to Universiti Kebangsaan Malaysia and Ministry of Higher Education, Malaysia for the scholarship which make it possible for me to do my Ph.D. in The University of Adelaide.

Also to the School of Mathematical Sciences, The University of Adelaide, on accepting me as their postgraduate student and for their supports and services during the whole time I spend there.

# Abstract

I begin with a brief introduction to dynamic systems, the identification of system parameters from records of input and output, and also wave energy converters which provide case studies to motivate the research. The dynamic systems discussed are categorized as linear or nonlinear dynamic systems. I present brief reviews of strategies for identification of dynamic systems which cover the history and also the areas of applications. The discretization of differential equations for dynamic systems is a recurrent theme and I consider forward, backward and central differences in detail for linear systems. The estimation techniques discussed are the principle of least squares, the Kalman filter and spectral analysis. Several system identification techniques for nonlinear dynamic systems in the time domain and in the frequency domain are presented and compared.

The main focus of the thesis is estimation methods based on wavelets. I present some introduction to the wavelet transforms, which cover both continuous and discrete wavelet transforms. Wavelet methods for system identification of linear and nonlinear dynamic systems are discussed.

Throughout this research, I have published four research articles guided by my supervisors. The first article discusses the wavelet based technique for linear system, and the technique was compared to the spectral analysis technique. The second article compare two types of wave energy converters, where the heaving buoy wave energy converter (HBWEC) is modelled as a linear system and the oscillating flap wave energy converter (OFWEC) as a nonlinear system. The frequency domain technique for system identification of nonlinear dynamic systems have been applied on the OFWEC model. Unscented Kalman filter have been discussed in the third article where the nonlinear OFWEC system have been used as the case study. A

wavelet approach for nonlinear system identification has been discussed in the fourth article together with the probing technique. The probing technique was used to find the generalized frequency response functions of the nonlinear dynamic systems based on the nonlinear autoregressive with exogenous input (ARX) model. Both techniques were compared for two weakly nonlinear oscillators, the Duffing and the Van der Pol. Once again, we selected the OFWEC system as a case study.

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# Statement of Originality

I, Mohd Aftar Abu Bakar, certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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## Published Works

M.A.A. Bakar, D.A. Green, and A.V. Metcalfe. Comparison of spectral and wavelet estimators of transfer function for linear systems. *East Asian Journal on Applied Mathematics*, 2(3):214-237, 2012.

M.A.A. Bakar, D.A. Green, A.V. Metcalfe, and G. Najafian. Comparison of heaving buoy and oscillating flap wave energy converters. In *AIP Conference Proceedings: Proceedings of the 20th National Symposium on Mathematical Sciences*, 1522: 86-101, 2013.

M.A.A. Bakar, D.A. Green, A.V. Metcalfe, and N.M. Ariff. Unscented Kalman filtering for wave energy converters system identification. In *AIP Conference Proceedings: Proceedings of the 3rd International Conference on Mathematical Sciences*, 1602: 304-310, 2014.

M.A.A. Bakar, N.M. Ariff, D.A. Green and A.V. Metcalfe. Comparison of autoregressive spectral and wavelet characterizations of nonlinear oscillators. Submitted to *East Asian Journal on Applied Mathematics*, 2016.

# Preamble

This thesis has been submitted to the University of Adelaide for the degree of Doctor of Philosophy. According to the University's Specification for Thesis, a Doctoral thesis may comprise,

*a combination of conventional written narrative presented as typescript and publications that have been published and/or submitted for publication and/or text in manuscripts,*

and this thesis takes this form.

The thesis has been divided into eight chapters. The first chapter is a brief introduction to: dynamic systems; the identification of system parameters from records of input and output; and also wave energy converters which provide case studies to motivate the research.

In the second chapter I discuss dynamic systems, which can be divided into linear and nonlinear dynamic systems. I also present brief reviews of strategies for identification of dynamic systems which cover the history and also the areas of applications.

In the third chapter, I consider linear systems. The discretization of differential equations for dynamic systems is a recurrent theme and I consider forward, backward and central differences in detail for linear systems. Then I consider estimation techniques including: least squares, the Kalman filter and spectral analysis.

The fourth and fifth chapter discussed several system identification techniques for nonlinear dynamic systems in time domain and frequency domain, respectively.

The sixth chapter starts with an introduction on the wavelet transforms. This cover both continuous and discrete wavelet transforms. Several wavelet methods for

system identification of dynamic systems are discussed here.

The seventh chapter presents four published papers from this research which form the main component of this thesis. The outline of the paper is given for each paper. All the papers are presented in the format they were printed.

In the last chapter, I discuss the conclusions from this research. Together with the conclusions, I also suggest possible potential future research following this thesis and also on system identification generally.