# INVESTIGATION OF NON-LINEAR MARITIME SIGNAL ESTIMATION SCHEME FOR PASSIVE ACOUSTIC AND ELECTROMAGNETIC UNDERWATER TRACKING AND UNDERWATER SURVEILLANCE 

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#### Abstract

Objectives: Modified gain extended Kalman filter (MGEKF) created by Song and Speyer was turned out to be an appropriate calculation for points just detached target following applications in air.

Methods: As of late, roughly altered increases are displayed, which are numerically steady and exact. In this paper, this enhanced MGEKF calculation is investigated for submerged applications with a few changes.

Results: In submerged, the commotion in the estimations is high, turning rate of the stages is low, and speed of the stages is likewise low when contrasted and the rockets in air. These attributes of the stage are concentrated on in detail, and the calculation is adjusted appropriately to track applications in submerged.

Conclusions: Monte-Carlo analysis comes about for two run of the mill situations are introduced with the end goal of clarification. From the outcomes, it is watched that this calculation is, especially reasonable for this nonlinear edges just detached target following.


Keywords: Estimation, Sonar, Kalman filter, Simulation, Modified gain, Angles-only target tracking.

## INTRODUCTION

In the sea environment, three-dimensional (3D) edges just target movement examinations is by and large utilized. A spectator screens uproarious sonar orientation and heights from a transmitting target, which is thought to go in a consistent course with uniform speed. The estimations are removed from a solitary eyewitness and the spectator forms these estimations to discover target movement parameters such as go, course, bearing, rise, and speed of the objective. Here, the estimations are nonlinear; making the entire procedure nonlinear Nonetheless, the changed increase broadened kalman channel (modified gain extended Kalman filter [MGEKF]) created by Song and Speyer [1-6], is the fruitful commitments in this field. MGEKF performs superior to anything EKF and also pseudo estimation channel. However, the adjusted pickup capacities was determined in light of the pseudo estimations. As of late, the adjusted pick up capacities are enhanced and exhibited by Longbin et al. [2-15], in which recipe of the bearing estimation is the same as in [1] and that of height estimation is more exact than the first.

So far, the angles in azimuth alone are considered. Now elevation angles are also considered. A point P, as shown in Fig. 1 whose elements are $\mathrm{x}, \mathrm{y}, \mathrm{z}$.

A line from P is drawn onto xy plane. This line is parallel to Z axis. Let this line touch xy plane at $P^{\prime}(x, y)$. Let the angle of elevation, $\phi$, be defined as the angle between +ve Z axis (or z up) and the line OP. Let the azimuth angle be the angle between True North and the line OP'. In Fig. 2a OPP' is denoted as $\phi$ We can write the following equations.

$$
\begin{equation*}
\mathrm{r}_{\mathrm{z}}=\mathrm{r} \cos \phi \quad \mathrm{r}_{\mathrm{xy}}=\mathrm{r} \sin \phi \tag{1}
\end{equation*}
$$

Where, $r$ is the distance from point 0 to $P$ (in 3D space), $r_{x y}$ is the distance from point 0 to $P^{\prime}$ (in two-dimensional space).

Also $\frac{r_{y}}{r_{x y}}=\cos B$ and $\frac{r_{x}}{r_{x y}}=\sin B$
$r_{y}=r_{x y} \cos B \quad r_{x}=r_{x y} \sin B$
Substituting Equations (1) in (2),
$r_{x}=r \sin \phi \sin B \quad r_{y}=r \sin \phi \cos B$
and $r_{z}=r \cos \phi$.
Where, $r=\sqrt{r_{x}^{2}+r_{y}^{2}+r_{z}^{2}}=\sqrt{r_{x y}^{2}+r_{z}^{2}}$
$r_{x y}=\sqrt{r_{x}^{2}+r_{y}^{2}}$
and $\frac{r_{x}}{r_{y}}=\frac{\sin B}{\cos B}=\tan B$
$\therefore$ Bearing, $\mathrm{B}=\tan ^{-1} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{y}}}$
$\tan \phi=\frac{\mathrm{r}_{\mathrm{xy}} / \mathrm{r}}{\mathrm{r}_{\mathrm{z}} / \mathrm{r}}=\frac{\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}_{\mathrm{z}}}\{$ From (1) $\}$
$\phi=\tan ^{-1} \frac{r_{x y}}{r_{z}}$
Let the measurement be $\mathrm{Z}=\left[\begin{array}{l}\mathrm{B}_{\mathrm{m}} \\ \phi_{\mathrm{m}}\end{array}\right]=\left[\begin{array}{l}\tan ^{-1} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{y}}}+\sigma_{\mathrm{B}} \\ \tan ^{-1} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{z}}}+\sigma_{\phi}\end{array}\right]$
Considering the state vector as $X_{s}=\left[\begin{array}{llllll}\dot{\mathrm{x}} & \dot{\mathrm{y}} & \dot{z} & \mathrm{r}_{\mathrm{x}} & \mathrm{r}_{\mathrm{y}} & \mathrm{r}_{\mathrm{z}}\end{array}\right]^{\mathrm{T}}$

Then
$H=\frac{\partial \mathrm{h}\left(\mathrm{X}_{\mathrm{s}}\right)}{\partial \mathrm{X}_{\mathrm{s}}}=\left[\begin{array}{cccccc}\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{x}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{y}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{z}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{x}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{y}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{z}}} \\ \frac{\partial \mathrm{h}(\varphi)}{\partial \dot{\mathrm{x}}} & \frac{\partial \mathrm{h}(\varphi)}{\partial \dot{\mathrm{y}}} & \frac{\partial \mathrm{h}(\varphi)}{\partial \dot{\mathrm{z}}} & \frac{\partial \mathrm{h}(\varphi)}{\partial \mathrm{r}_{\mathrm{x}}} & \frac{\partial \mathrm{h}(\varphi)}{\partial \mathrm{r}_{\mathrm{y}}} & \frac{\partial \mathrm{h}(\varphi)}{\partial \mathrm{r}_{\mathrm{z}}}\end{array}\right]$
$\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{x}}}=\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{y}}}=\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \dot{\mathrm{z}}}=0 \quad \mathrm{III}^{\mathrm{ly}} \quad \frac{\partial \mathrm{h}(\phi)}{\partial \dot{\mathrm{x}}}=\frac{\partial \mathrm{h}(\phi)}{\partial \dot{y}}=\frac{\partial \mathrm{h}(\phi)}{\partial \dot{\mathrm{z}}}=0$
$\frac{\partial h(B)}{\partial r_{x}}=\left[\frac{1}{1+r_{x}{ }^{2} / r_{y}{ }^{2}}\left\{\frac{1}{r_{y}}\right\}\right]=\frac{r_{y}{ }^{2}}{r_{x y}^{2}} \cdot \frac{1}{r_{y}}=\frac{r_{y}}{r_{x y}^{2}}=\frac{r_{y}}{r_{x y}} \cdot \frac{1}{r_{x y}}=\frac{\cos \hat{B}}{\hat{r}_{x y}}$


Fig. 1: Target-observer geometry
$\frac{\partial h(B)}{\partial r_{y}}=\frac{r_{y}^{2}}{r_{x y}^{2}} \cdot r_{x}\left(-\frac{1}{r_{y}^{2}}\right)=\frac{-r_{x}}{r_{x y}^{2}}=\frac{-r_{x}}{r_{x y}} \frac{1}{r_{x y}}=\frac{-\sin \hat{B}}{\hat{r}_{x y}}$
$\frac{\partial h(B)}{\partial r_{z}}=0$
$\frac{\partial \mathrm{h}(\varphi)}{\partial \mathrm{r}_{\mathrm{x}}}=\left[\frac{1}{1+\mathrm{r}_{\mathrm{xy}}{ }^{2} / \mathrm{r}_{\mathrm{z}}^{2}}\left\{\frac{1}{\mathrm{r}_{\mathrm{z}}} \frac{\partial \mathrm{r}_{\mathrm{xy}}}{\partial \mathrm{x}}\right\}\right]=\frac{\mathrm{r}_{\mathrm{z}}^{2}}{\mathrm{r}^{2}} \cdot \frac{1}{\mathrm{r}^{2}} \frac{\partial \sqrt{\mathrm{r}_{\mathrm{x}}^{2}+\mathrm{r}_{\mathrm{y}}^{2}}}{\partial \mathrm{r}_{\mathrm{z}}}$

$$
=\frac{r_{z}}{\mathrm{r}^{2}} \cdot \frac{2 \mathrm{r}_{\mathrm{x}}}{2 \mathrm{r}_{\mathrm{xy}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{xy}}}=\frac{\cos \hat{\phi} \sin \hat{B}}{\hat{\mathrm{r}}}
$$

$$
\begin{align*}
& \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{r}_{\mathrm{y}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \cdot \frac{1}{\mathrm{r}_{\mathrm{z}}} \cdot \frac{2 \mathrm{r}_{\mathrm{y}}}{2 \mathrm{r}_{\mathrm{xy}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \cdot \frac{\mathrm{r}_{\mathrm{y}}}{\mathrm{r}_{\mathrm{xy}}}=\frac{\cos \hat{\phi} \cos \hat{B}}{\hat{\mathrm{r}}} \\
& \frac{\partial \mathrm{~h}(\phi)}{\partial \mathrm{r}_{\mathrm{z}}}=\frac{\mathrm{r}_{\mathrm{z}}^{2}}{\mathrm{r}^{2}} \mathrm{r}_{\mathrm{xy}}\left(-\frac{1}{\mathrm{r}_{\mathrm{z}}^{2}}\right) \cdot=\frac{-\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}^{2}}=\frac{-\mathrm{r} \sin \phi}{\mathrm{r}^{2}}=\frac{-\sin \hat{\phi}}{\hat{\mathrm{r}}} \\
& \therefore \mathrm{H}=\left[\begin{array}{lllllc}
0 & 0 & 0 & \frac{\cos \hat{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \hat{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0 \\
0 & 0 & 0 & \frac{\cos \hat{\varphi} \sin \hat{B}}{\hat{\mathrm{r}}} & \frac{\cos \hat{\varphi} \cos \hat{B}}{\hat{\mathrm{r}}} & -\frac{\sin \hat{\varphi}}{\hat{r}}
\end{array}\right] \tag{10}
\end{align*}
$$

## Horizontal plane and bearing measurements

If the range in horizontal plane is $\sqrt{\mathrm{r}_{\mathrm{x}}^{2}+\mathrm{r}_{\mathrm{y}}^{2}}$, then the estimated range be:

$$
\begin{equation*}
\hat{\mathrm{r}}_{\mathrm{xy}}=\sqrt{\mathrm{r}_{\mathrm{x}}^{2}+\mathrm{r}_{\mathrm{y}}^{2}} \tag{11}
\end{equation*}
$$



Fig. 2: (a-c) Errors in target motion parameters for given scenario

As $\quad \mathrm{r}_{\mathrm{x}}=\mathrm{r}_{\mathrm{xy}} \sin \mathrm{B} \quad \hat{\mathrm{r}}_{\mathrm{x}}=\hat{\mathrm{r}}_{\mathrm{xy}} \sin \hat{\mathrm{B}}$

$$
\begin{equation*}
r_{y}=r_{x y} \cos B \quad \hat{r}_{y}=\hat{r}_{x y} \cos \hat{B} \tag{12}
\end{equation*}
$$

$r_{x} \sin B+r_{y} \cos B=r_{x y} \sin ^{2} B+r_{x y} \cos ^{2} B=r_{x y}$
$\hat{r}_{x} \sin B+\hat{r}_{y} \cos B=\hat{r}_{x y}$
By adding $\mathrm{r}_{\mathrm{xy}}+\hat{\mathrm{r}}_{\mathrm{xy}}$,
$r_{x y}+\hat{r}_{x y}=r_{x} \sin B+r_{y} \cos B+\hat{r}_{x} \sin \hat{B}+\hat{r}_{y} \sin \hat{B}$
Adding both sides $-r_{x} \sin \hat{B}-\hat{r}_{y} \cos \hat{B}-\hat{r}_{y} \cos B+r_{y} \cos \hat{B}$ to the above equation.
$r_{x y}+\hat{r}_{x y}-r_{x} \sin \hat{B}-\hat{r}_{x} \sin B+r_{y} \cos B+r_{y} \cos \hat{B}=\sin B\left(r_{x}-\hat{r}_{x}\right)$
$+\cos B\left(r_{y}-\hat{r}_{y}\right)-\sin \hat{B}\left(r_{x}-\hat{r}_{x}\right)-\cos \hat{B}\left(r_{y}-\hat{r}_{y}\right)$
$=\left(r_{x}-\hat{r}_{x}\right)(\sin B-\sin B)+\left(r_{y}-\hat{r}_{y}\right)(\cos B-\cos \hat{B})$
Substituting for $r_{x^{\prime}} \hat{r}_{x}$ and $r_{y^{\prime}}, \hat{r}_{y}$ on LHS of (14)
$r_{x y}+\hat{r}_{x y}-r_{x y} \sin B \sin \hat{B}-\hat{r}_{x y} \sin \hat{B}$
$\sin B-\hat{r}_{x y} \cos \hat{B} \cos B-\hat{r}_{x y} \cos B \cos \hat{B}=$ RHS of (14)
$\left(r_{x y}+\hat{r}_{x y}\right)(1-\sin B \sin \hat{B}-\cos B \cos \hat{B})=$ RHS of $(14)$
$\left(r_{x y}+\hat{r}_{x y}\right)(1-\cos (B-\hat{B}))=$ RHS of $(14)$
$r_{x y}+\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B-\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B-\cos \hat{B})}{1-\cos (B-\hat{B})}$

By subtracting $\mathrm{r}_{\mathrm{xy}}$ from $\hat{\mathrm{r}}_{\mathrm{xy}}$,
$r_{x y}-\hat{r}_{x y}=-\hat{r}_{x} \sin \hat{B}-\hat{r}_{y} \sin \hat{B}+r_{x} \sin B+r_{y} \cos \hat{B}$

Adding both sides $-\hat{r}_{x} \sin \hat{B}-\hat{r}_{x} \sin B-\hat{r}_{y} \cos B-\hat{r}_{y} \cos \hat{B}$ to (16) and continuing the previous procedure,
$r_{x y}-\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B+\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B+\cos \hat{B})}{1+\cos (B-\hat{B})}$
Using (15) and (17),

$$
\begin{align*}
2 \hat{r}_{x y}= & \left(r_{x}-\hat{r}_{x}\right)\left[\frac{\sin B-\sin \hat{B}}{1-\cos (B-\hat{B})}-\frac{\sin B+\sin \hat{B}}{1+\cos (B-\hat{B})}\right] \\
& +\left(r_{y}-\hat{r}_{y}\right)\left[\frac{\cos B-\cos \hat{B}}{1-\cos (B-\hat{B})}+\frac{\cos B+\cos \hat{B}}{1+\cos (B-\hat{B})}\right] \tag{18}
\end{align*}
$$

(18) can be simplified as:

$$
\begin{align*}
& \frac{\sin B-\sin \hat{B}}{1-\cos (B-\hat{B})}-\frac{\sin B+\sin \hat{B}}{1+\cos (B-\hat{B})} \\
& =\frac{1+\cos (B-\hat{B})(\sin B-\sin \hat{B})-(1-\cos (B-\hat{B})(\sin B+\sin \hat{B})}{1-\cos ^{2}(B-\hat{B})} \\
& =\frac{2 \sin B \cos (B-\hat{B})-2 \sin \hat{B}}{\sin ^{2}(B-\hat{B})} \\
& =\frac{2 \cos B \sin (B-\hat{B})}{\sin ^{2}(B-\hat{B})}=\frac{2 \cos B}{B \sin (B-\hat{B})} \tag{19}
\end{align*}
$$

III ${ }^{\text {ly }}\left(r_{y}-\hat{r}_{y}\right)$ coefficient is simplified to $\frac{2 \cos (B-\hat{B}) \cos B-2 \cos \hat{B}}{\sin ^{2}(B-\hat{B})}$
$=-\frac{2 \sin B}{\sin (B-\hat{B})}$
$\therefore 2 \hat{r}_{x y}=\frac{2 \cos B\left(r_{x}-\hat{r}_{x}\right)}{\sin (B-\hat{B})}-\frac{2 \sin B\left(r_{y}-\hat{r}_{y}\right)}{\sin (B-\hat{B})}$
(21) is rewritten as,
$\sin (B-\hat{B})=\frac{\cos B\left(r_{x}-\hat{r}_{x}\right)-\sin B\left(r_{y}-\hat{r}_{y}\right)}{\hat{r}_{x y}}$

## Angle measurement

$\tan ^{-1} \frac{r_{x}}{r_{y}}=B$ generates $\sin (B-\hat{B})=\frac{\cos B\left(r_{x}-\hat{r}_{x}\right)-\sin B\left(r_{y}-\hat{r}_{y}\right)}{\hat{r}_{x y}}$
$\tan ^{-1} \frac{\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}_{\mathrm{z}}}=\phi$ generates $\sin (\phi-\hat{\phi})=\frac{\cos \phi\left(\mathrm{r}_{\mathrm{xy}}-\hat{r}_{\mathrm{xy}}\right)-\sin \phi\left(\mathrm{r}_{\mathrm{z}}-\hat{r}_{\mathrm{z}}\right)}{\mathrm{r}}$
Where, $\mathrm{r}_{\mathrm{x}} \rightarrow \mathrm{r}_{\mathrm{xy}} \mathrm{r}_{\mathrm{y}} \rightarrow \mathrm{r}_{\mathrm{z}^{\prime}}, \mathrm{B} \rightarrow \phi \quad \mathrm{r}_{\mathrm{xy}} \rightarrow \mathrm{r}_{\mathrm{xyz}}(=\mathrm{r})$
$\mathrm{r}_{\mathrm{xy}}-\hat{\mathrm{r}}_{\mathrm{xy}}$ is given by (17) as follows,
$r_{x y}-\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B+\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B+\cos \hat{B})}{1+\cos (B-\hat{B})}$
It is known that $\cos (p+q)+\cos (p-q)=2 \cos p \cos q$,
Here $\mathrm{p}+\mathrm{q}=\mathrm{B} \mathrm{p}-\mathrm{q}=\hat{\mathrm{B}}$ giving $\quad \frac{\mathrm{B}+\hat{\mathrm{B}}}{}$ and $\mathrm{p}=\frac{\mathrm{B}-\hat{\mathrm{B}}}{2}$
So $\cos B+\cos \hat{B}=2 \cos \frac{B+\hat{B}}{2} \cos \frac{B-\hat{B}}{2}$
$\mathrm{III}^{\mathrm{ly}} \sin (\mathrm{p}+\mathrm{q})+\sin (\mathrm{p}-\mathrm{q})=2 \sin \mathrm{p} \sin \mathrm{q}$,
$\sin \mathrm{B}+\sin \hat{\mathrm{B}}=2 \sin \frac{\mathrm{~B}+\hat{\mathrm{B}}}{2} \sin \frac{B-\hat{B}}{2}$
III ${ }^{\text {ly }} 1+\cos 2 \alpha=\frac{2 \cos ^{2} \alpha}{2}=\cos ^{2} \alpha$,
so $1+\cos (B-\hat{B})=2 \cos ^{2} \frac{B-\hat{B}}{2}$
Using (24), (25) and (26), equation (17) becomes,

$$
\begin{array}{r}
\left(r_{x}-\hat{r}_{x}\right) 2 \sin \frac{(B+\hat{B})}{2} \cos \frac{(B-\hat{B})}{2} \\
r_{x y}-\hat{r}_{x y}=\frac{+\left(r_{y}-\hat{r}_{y}\right) 2 \cos \frac{(B+\hat{B})}{2} \cos \frac{(B-\hat{B})}{2}}{2 \cos ^{2} \frac{(B-\hat{B})}{2}} \\
=\frac{\left(r_{x}-\hat{r}_{x}\right) \sin \frac{(B+\hat{B})}{2}+\left(r_{y}-\hat{r}_{y}\right) \cos \frac{(B+\hat{B})}{2}}{\cos \frac{(B-\hat{B})}{2}} \tag{27}
\end{array}
$$

Substituting (27) in (23),
$\sin (\varphi-\hat{\varphi})=\frac{\cos \varphi}{\hat{r}}\left[\frac{\left(r_{x}-\hat{r}_{x}\right) \sin \frac{(B+\hat{B})}{2}+\left(r_{y}-\hat{r}_{y}\right) \cos \frac{(B+\hat{B})}{2}}{\cos \frac{(B-\hat{B})}{2}}\right]-\frac{\sin \varphi}{\hat{r}}\left(r_{z}-\hat{r}_{z}\right)$

As $\phi-\hat{\phi}$ tend to zero and $\sin (\phi-\hat{\phi}) \rightarrow(\phi-\hat{\phi})$ and $\sin (B-\hat{B}) \rightarrow(B-\hat{B})$. Equations (22) and (28) can be written in matrix form as,
$\left[\begin{array}{c}(\mathrm{B}-\hat{\mathrm{B}}) \\ (\varphi-\hat{\varphi})\end{array}\right]=\left[\begin{array}{ccc}\frac{\cos \mathrm{B}}{\hat{r}_{\mathrm{xy}}} & -\frac{\sin \mathrm{B}}{\hat{r}_{\mathrm{xy}}} & 0 \\ \frac{\sin \left(\frac{\mathrm{~B}+\hat{\mathrm{B}}}{2}\right) \cos \varphi}{\cos \left(\frac{\mathrm{B}+\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{\cos \left(\frac{\mathrm{B}+\hat{\mathrm{B}}}{2}\right) \cos \varphi}{\cos \left(\frac{\mathrm{B}-\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{-\sin \varphi}{\hat{r}}\end{array}\right]\left[\begin{array}{l}\mathrm{r}_{\mathrm{x}}-\hat{\mathrm{r}}_{\mathrm{x}} \\ \mathrm{r}_{\mathrm{y}}-\hat{\mathrm{r}}_{\mathrm{y}} \\ \mathrm{r}_{\mathrm{z}}-\hat{\mathrm{r}}_{\mathrm{z}}\end{array}\right]$

True bearing is not available, if it is replaced by measured bearing,

$$
\begin{aligned}
& {\left[\begin{array}{c}
(B-\hat{B}) \\
(\varphi-\hat{\varphi})
\end{array}\right]=\left[\begin{array}{ccc}
\frac{\cos B_{m}}{\hat{r}_{x y}} & -\frac{\sin B_{m}}{\hat{r}_{x y}} & 0 \\
\frac{\sin \left(\frac{B_{m}+\hat{B}}{2}\right) \cos \varphi}{\cos \left(\frac{B_{m}+\hat{B}}{2}\right) \hat{r}} & \frac{\cos \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{B}}{2}\right) \cos \varphi}{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{-\sin \varphi}{\hat{\mathrm{r}}}
\end{array}\right]} \\
& {\left[\begin{array}{l}
\mathrm{r}_{\mathrm{x}}-\hat{\mathrm{r}}_{\mathrm{x}} \\
\mathrm{r}_{\mathrm{y}}-\hat{\mathrm{r}}_{\mathrm{y}} \\
\mathrm{r}_{\mathrm{z}}-\hat{\mathrm{r}}_{\mathrm{z}}
\end{array}\right]} \\
& =g\left[\begin{array}{c}
r_{x}-\hat{r}_{x} \\
r_{y}-\hat{r}_{y} \\
r_{z}-\hat{r}_{z}
\end{array}\right]
\end{aligned}
$$

Where g is given by,
$\mathrm{g}=\left[\begin{array}{ccc}\frac{\cos \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0 \\ \frac{\sin \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \cos \varphi}{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \cos \varphi}{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{-\sin \varphi}{\hat{\mathrm{r}}}\end{array}\right]$
Considering $\dot{x}, \dot{\mathrm{y}}$ and $\dot{\mathrm{z}}$ also g is given by,
$\mathrm{g}=\left[\begin{array}{cccccc}0 & 0 & 0 & \frac{\cos \mathrm{~B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}=\hat{\mathrm{r}}_{\text {sin } \hat{\varphi}}} & \frac{-\sin \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}=\hat{\mathrm{r}}_{\text {sin } \hat{\varphi}}} & 0 \\ 0 & 0 & 0 & \frac{\cos \varphi \sin \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right)}{\hat{\mathrm{r}} \cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{B}}{2}\right)} & \frac{\cos \varphi \cos \left(\frac{\mathrm{B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right)}{\hat{\mathrm{r}} \cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right)} & -\frac{\sin \varphi}{\hat{\mathrm{r}}}\end{array}\right]$
$\mathrm{H}=\left[\begin{array}{cccccc}0 & 0 & 0 & \frac{\cos \hat{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \hat{\mathrm{B}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0 \\ 0 & 0 & 0 & \frac{\cos \hat{\varphi} \sin \hat{B}}{\hat{r}} & \frac{\cos \hat{\varphi} \cos \hat{B}}{\hat{r}} & -\frac{\sin \hat{\varphi}}{\hat{r}}\end{array}\right]$
Implementation of Kalman filter
It is assumed that the target is not changing depth.
Let $X_{S}=\left[\begin{array}{lllll}\dot{x} & \dot{y} & r_{x} & r_{y} & r_{z}\end{array}\right]^{T}$
Where, $\dot{x} \dot{y}$ are target (absolute) velocity components,

Let $\mathrm{X}(0 \mid 0)$ be $\mathrm{X}_{\mathrm{f}}(0 \mid 0)$
$\mathrm{X}_{\mathrm{f}}[0 \mid 0]=\left[\begin{array}{llll}10 & 10 & 10 & 15000 \\ \sin \mathrm{~B}_{\mathrm{m}} \sin \varphi_{\mathrm{m}}\end{array}\right.$

$$
\begin{equation*}
\left.15000 \cos \mathrm{~B}_{\mathrm{m}} \sin \varphi_{\mathrm{m}} 15000 \cos \varphi_{\mathrm{m}}\right]^{\mathrm{T}} \tag{35}
\end{equation*}
$$

$P(0 \mid 0)=I$
$\mathrm{G}(\mathrm{k}+1)=\mathrm{P}(\mathrm{k}+1) \mathrm{H}^{\mathrm{T}}(\mathrm{k}+1)\left[\mathrm{H}(\mathrm{k}+1) \mathrm{P}(\mathrm{k}+1 \mid \mathrm{k}) \mathrm{H}^{\mathrm{T}}(\mathrm{k}+1)+\mathrm{r}(\mathrm{k}+1)\right]$
Where, $\mathrm{r}(\mathrm{k}+1)=\left[\begin{array}{cc}\sigma_{\mathrm{B}}^{2}(\mathrm{k}+1) & 0 \\ 0 & \sigma_{\varphi}^{2}(\mathrm{k}+1)\end{array}\right]$
Where, $\sigma_{\mathrm{B}}^{2}$ and $\sigma_{\phi}^{2}$ are input error bearing and elevation measurement covariances respectively.
$\hat{\mathrm{X}}(\mathrm{k}+1 \mid \mathrm{k}+1)=\hat{\mathrm{X}}(\mathrm{k}+1 \mid \mathrm{k})+\mathrm{k}(\mathrm{k}+1)[\mathrm{Z}(\mathrm{k}+1)-\mathrm{h}\{\hat{\mathrm{x}}(\mathrm{k}+1 \mid \mathrm{k})\}]$
Where, $\mathrm{Z}(\mathrm{k}+1)=\left[\begin{array}{l}\mathrm{B}_{\mathrm{m}} \\ \varphi_{\mathrm{m}}\end{array}\right]$
$P(k+1 \mid k+1)=(I-G g) P(I-G g) P(I-G g)^{T}+G r G^{T}$
$=(I-G(k+1) g) P(k+1 \mid k)\left(I-G(k+1) g^{T}\right)+G(k+1) r(k+1) G^{T}(k+1)$
For next cycle $\hat{x}(k \mid k)=\hat{x}(k+1 \mid k+1)$
and $P(k \mid k)=P(k+1 \mid k+1)$
$\hat{\mathrm{x}}(\mathrm{k}+1 \mid \mathrm{k})=\phi(\mathrm{k}+1 \mid \mathrm{k}) \hat{\mathrm{x}}(\mathrm{k} \mid \mathrm{k})+\mathrm{B}$
Where, $B=\left[\begin{array}{c}0 \\ 0 \\ 0 \\ -\left\{\mathrm{x}_{0}(\mathrm{k}+1)-\mathrm{x}_{0}(\mathrm{k})\right\} \\ -\left\{\mathrm{y}_{0}(\mathrm{k}+1)-\mathrm{y}_{0}(\mathrm{k})\right\} \\ -\left\{\mathrm{z}_{0}(\mathrm{k}+1)-\mathrm{z}_{0}(\mathrm{k})\right\}\end{array}\right]$
$P(k+1 \mid k)=\phi(k+1 \mid k) P(k \mid k) \phi^{T}(k+1 \mid k)+Q(k+1)$
Where, Q is plant covariance matrix.
A maneuvering target and tracking using bearing and elevation measurements
$\dot{x}(\mathrm{k}+1)=\dot{\mathrm{x}}(\mathrm{k})+\mathrm{t} \ddot{\mathrm{x}}(\mathrm{k})$
$\int \dot{x}(k+1)=\int_{0}^{t} \dot{x}(k) d z+\int_{0}^{t} \ddot{x}(k) d z$
$x(k+1)-x(k)=\dot{x}(k) \cdot t+\frac{t^{2}}{2} \ddot{x}(k)$
$x(k+1)=x(k)+t \dot{x}(k)+\frac{t^{2}}{2} \ddot{x}(k)$
This can be easily remembered as,
$X\left(t_{n}+\varsigma\right)=X\left(t_{n}\right)+\varsigma \dot{x}\left(t_{n}\right)+\frac{\varsigma^{2}}{2} \ddot{x}\left(t_{n}\right)$
$x(K t+t)=x(K t)+t \dot{x}(k t)+\frac{t^{2}}{2} \ddot{x}(k t)$
or simply,
$x(k+1)=x(k)+t \dot{x}(k)+\frac{t^{2}}{2} \ddot{x}(k)$

## IMPLEMENTATION OF THE ALGORITHM FOR UNDERWATER APPLICATION

The above mentioned improved MGEKF algorithm is implemented for underwater passive target tracking as follows. In underwater, the variance of the noise in the measurements is very high, and so the measurements are preprocessed (averaging the measurements over some duration, say 20 seconds) to reduce the variance of the noise in the measurements. Hence, although the measurements are available every one second, the update of the solution is presented at every 20 seconds. This does not hamper the results as the vehicles move in water at very low speeds when compared with that of in air. The underlying target state vector is picked as takes after. As just bearing and height estimations are accessible, and there is no real way to figure the speed parts of the objective, these segments are each thought to be $10 \mathrm{~m} / \mathrm{s}$ which is near the sensible speed of the vehicles in submerged. The scope of the day, say $15,000 \mathrm{~m}$, is used in the computation of beginning position gauge of the objective is as:
$X(0 \mid 0)=\left[\begin{array}{llllll}\mathrm{x} & \mathrm{y} & \mathrm{z} & \mathrm{x} & \mathrm{y} & \mathrm{z}\end{array}\right]^{\mathrm{T}}=$
$\left[\begin{array}{llll}10 & 10 & 10 & 15000 *\end{array} \sin _{B_{m}}(0) * \sin _{m}(0)\right.$
$\left.15000 * \sin _{\mathrm{m}}(0) * \cos \mathrm{~B}_{\mathrm{m}}(0) 15000 * \cos _{\mathrm{m}}(0)\right]^{\mathrm{T}}$

Where, $\mathrm{B}_{\mathrm{m}}(0)$ and ${ }_{\mathrm{m}}(0)$ are initial bearing and elevation measurements. The initial covariance matrix is chosen according the standard procedure [3].

## SIMULATION RESULTS

PC calculation is produced and tried with recreated information to outline the execution of this estimator. Every single crude bearing and height estimations are adulterated by added substance zero mean Gaussian commotion with a r.m.s level of $1^{\circ}$ and 0.3 separately and after that preprocessed over a time of 20 seconds. Relating to a strategic situation in which the objective is at the underlying scope of 19,000 yards ( 17373.6 m ) at beginning bearing and height of $0^{\circ}$ and $45^{\circ}$ individually, the mistakes in assessments are plotted in Figs. 1, and 2(a-c) (on seawaters, as a rule range is communicated in yards and speed in tangles). The objective is thought to move at a consistent course of $140^{\circ}$ at a speed of 25 bunches ( $12.875 \mathrm{~m} / \mathrm{s}$ ). Spectator is accepted to moving at a consistent speed of 7 bunches ( $3.605 \mathrm{~m} / \mathrm{s}$ ) with a pitch point of $45^{\circ}$. At last the outcomes have been troupe found the middle value of more than a few Monte Carlo runs. When all is said in done
the blunder permitted in the evaluated target movement parameters in submerged is $8 \%$ in range assess, $3^{\circ}$ in course gauge and three meters/ sec in speed appraise. It is watched this required exactness is acquired from 240 seconds onwards thus this calculation is by all accounts particularly valuable for submerged aloof target following.

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