# On the need for unbiasing azimuthal asymmetry in signals measured by surface detector arrays 

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A surface detector array samples the lateral distribution of an extensive air-shower (EAS) at the ground, i.e. the density of particles as a function of the distance from the axis of the shower. The azimuthal symmetry of this measured lateral distribution is broken for EAS with a nonzero zenith angle. The resulting asymmetry, caused by atmospheric attenuation and geometrical effects, increases with the inclination of the shower and introduces a bias in the reconstruction of the shower parameters. Using simulated sets of air-showers, we present a model to correct the azimuthal asymmetry in signals measured by water-Cherenkov detectors and exemplified using the geometry and detector response of the Pierre Auger Observatory. Testing showers initiated by proton and iron primaries using Epos-LHC and QGSJetII-04 as hadronic models, we developed a fine-tuned model of the amplitude of the asymmetry as a function of the zenith angle, shower size and distance of a detector from the shower axis. The improvements resulting from the application of the correction are quantified in terms of the biases and resolutions in the impact-point and arrival direction.

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Figure 1: Left: Schematic view of the shower geometry. The label "MC" refers to simulated values while "rec" are for reconstructed ones. $\zeta$ is the projected polar angle into the shower plane. Right: 2D distribution of $\Delta x=x_{\mathrm{rec}}-x_{\mathrm{MC}}$ and $\Delta y=y_{\mathrm{rec}}-y_{\mathrm{MC}}$. The color scale of the z -axis represents the number of entries. These coordinates have been computed in the reference frame of the simulated shower.

## 1. Introduction

Observations of ultra-high energy cosmic-rays are only possible from the ground where a large surface of detection can be built. Entering into the atmosphere and interacting with the air molecules, a cosmic ray initiates the production of a cascade of secondary particles, the Extensive Air Shower (EAS). These secondary particles are then travelling from their point of emission to the ground and can be detected by one of the numerous detectors constituting an array such as the surface detectors of the Pierre Auger Observatory or the Telescope Array. This sampling of the lateral distribution of density of particles of an EAS at the ground is then modelled with a Lateral Distribution Function (LDF) from which the characteristics of the primary cosmic ray are derived.

For vertical air showers an azimuthally symmetric LDF can be applied, where the term $a z$ imuthal refers to the polar angle $\zeta$ of a detector, at distance $r$ from the shower axis, in a plane perpendicular to the shower axis as illustrated in Fig. 1 (left). According to this definition of the polar angle, two regions are delimited: the upstream region where $|\zeta|<\pi / 2$ and the downstream region where $|\zeta|>\pi / 2$. For inclined air showers, the equal-density contours in the detector plane are no longer circular but elliptic. Moreover, due to the inclination of the shower, the travel length of particles - and thus the attenuation - of the downstream region is higher than the one in the upstream region. These geometrical and attenuation effects shift the position of the core into the upstream direction.

In this study we focus on the simulation of the detector response of water-Cherenkov detectors (WCD) of the surface detector of the Pierre Auger Observatory, using the $\overline{\text { Offline }}$ simulation and reconstruction software [1]. In Ref. Fig. 1 (right), the differences between the coordinates of the injected and reconstructed cores are illustrated. Each point represents a simulated event, injected with a zenith angle $\theta_{\mathrm{MC}}$ and a impact point $\left(x_{\mathrm{MC}}, y_{\mathrm{MC}}\right)$ and reconstructed with $\theta_{\mathrm{rec}}$ at a position ( $x_{\text {rec }}, y_{\text {rec }}$ ) as developed in [2]. An mean bias of $\sim 40 \mathrm{~m}$ in the position of the core of the shower is observed in the upstream direction. For more inclined showers ( $\sim 45^{\circ}$ ), this bias increases to $\sim 70$ to 80 m which is comparable to the core resolution reported by the Pierre Auger Observatory [2]. A model describing the azimuthal asymmetry in signals is thus needed to improve the reconstruction


Figure 2: Amplitude of the asymmetry illustrated at different distances from the shower axis. From left to right, and top to bottom, the distances are of $158 \mathrm{~m}, 1000 \mathrm{~m}, 1685 \mathrm{~m}$, and 2285 m . The total non-saturated signal is represented with black circles while the non-saturated sub-components signals are shown with red triangles and blue squares for the electromagnetic and muonic components respectively. These distributions use showers initiated by a proton primary with $\lg (S(1000)) \approx 100 \mathrm{VEM}, \theta \approx 40^{\circ}$ and EPOS-LHC as hadronic model.
of the position of the core. In this model, the signals measured at the ground by one WCD are expressed as

$$
\begin{equation*}
S(r, \zeta)=S\left(r_{\mathrm{opt}}\right) f_{\mathrm{LDF}}(r)\left[1+\alpha\left(r, \theta, S\left(r_{\mathrm{opt}}\right)\right) \cos \zeta\right] \tag{1}
\end{equation*}
$$

where $S\left(r_{\mathrm{opt}}\right)$ is the estimator of the shower size, with $r_{\mathrm{opt}}=1000 \mathrm{~m}$. In the following sections, we will simply denote it as $S(1000) . f_{\mathrm{LDF}}(r)$ is the normalized LDF $\left(f_{\mathrm{LDF}}\left(r_{\mathrm{opt}}\right) \equiv 1\right)$ which, in the case of the framework used, is a modified-NKG parametrized in Ref. [2]. $\alpha\left(r, \theta, S\left(r_{\mathrm{opt}}\right)\right)$ is the amplitude of the azimuthal asymmetry modeled for different inclination of the shower, energy of the primary cosmic-ray and different distances.

## 2. Asymmetry in simulated events

To study the azimuthal asymmetry in the signal measured at the ground by WCD, such as those constituting the Pierre Auger Observatory, different data-sets of simulations have been created. The mass composition of the cosmic ray and the development of Extensive Air Showers (EAS) in the atmosphere are still under deep investigations. Thus, to avoid any misleading results, the
detector response of 3000 Corsika showers [3] have been simulated, with an energy from $10^{18.5} \mathrm{eV}$ to $10^{20} \mathrm{eV}$ and a flat distribution in $\sin ^{2} \theta$ from $0^{\circ}$ to $60^{\circ}$, for both iron and proton primaries and for Epos-LHC [4] and QGSJet-II. 04 [5] as hadronic models. In each of these four data-sets of simulations, two arrays have been considered. A regular array mimicking the array of the surface detector of the Pierre Auger Observatory and a dense array constituting of 20 dense rings of 24 detectors at fixed distances from the shower axis. Below 1500 m , the distances of the rings have been selected following a natural logarithmic function while above 1500 m , the rings are distant from 100 m . With these selected distances a reliable study of the signals has been performed avoiding side effect caused by large fluctuations of the lateral distribution close to the core or at large distances. In the following parts the results are shown using detectors of the dense array only. Cross-checks have been performed using detectors of the regular array.

### 2.1 Amplitude of the asymmetry

The distribution of $S(r, \zeta) /\langle S(r, \zeta)\rangle$, where $\langle S(r, \zeta)\rangle$ is the mean signal in a particular dense ring, is computed in each dense ring and binned in $\sin ^{2} \theta$ and in $\lg (S(1000))$. Examples of these distributions are shown in Fig. 2 from where the amplitude of the asymmetry $\alpha(r, \theta, S(1000), \zeta)$ is extracted. As a function of $\sin ^{2} \theta$, the amplitude of the asymmetry $\alpha(r, \theta, S(1000))$ for proton primary is increasing linearly from 0 to $\sim 0.25$ ( $\sim 0.2$ for iron) before decreasing to $\sim 0.15$ ( $\sim 0.1$ for iron) for $\theta \gtrsim 40^{\circ}$. It is quite interesting to note, that despite their differences or disagreements, the hadronic models Epos-LHC and QGSJet-II. 04 give similar results, as it will be illustrated in Fig. 4. As a function of the distance, the amplitude $\alpha(r, \theta, S(1000)$ ) is increasing from 0 to $\sim 500 \mathrm{~m}$, before reaching a plateau whose amplitude depends on the zenith angle. A slow decrease is then observed from $\sim 1500$ to 2000 m . The higher is the energy of the cosmic ray, the larger is the distance at which the decrease begins. A description will be given in the next section with the formulation of the model derived to correct the azimuthal asymmetry.

The use of the regular detectors gives the same results. Moreover, introducing a random bias ${ }^{1}$ in the position of the core increases the uncertainties of the amplitude of the asymmetry but does not modify them.

### 2.2 An equilibrium between the shower components

The total amplitude of the azimuthal asymmetry results in the combination of the amplitude of its sub-components. Using simulations, it is possible to track back the history of particles reaching the ground. Let us define the electromagnetic component (EM) as the sum of the signals produced in WCD by (a) electrons, (b) photons and (c) muons produced through photoproduction, and the muonic component (Muon) as the signals attributed to (a) muons or (b) electrons and photons produced by the decay of muons. Close to the shower axis (top-right panel in Fig. 2), the electromagnetic component is dominating and thus driving the amplitude of the azimuthal asymmetry. Above 1000 m , the ratio between the electromagnetic and muonic signals is closer to 1. The flat azimuthal distribution of muons is thus flattening the total amplitude. This effect can be seen in Fig. 3 (left) where the amplitude of the asymmetry of the electromagnetic component is increasing with the distance, while the amplitude for the muonic component is almost null ( $\leq 0.05$ ).

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Figure 3: Left: Amplitude of the asymmetry as a function of distances from the shower axis for the electromagnetic (triangles) and muonic (squares) component and for four zenith angles: $15^{\circ}$ (green), $30^{\circ}$ (red), $45^{\circ}$ (blue) and $55^{\circ}$ (black). Showers initiated by a proton primary with $\lg S(1000) \sim 100$ VEM, using Epos-LHC as hadronic model have been used to produce this figure. Right: Schematic view explaining the reversal of the amplitude of the asymmetry.

In Fig. 3 (left) and Fig. 2 (bottom, right), the muon component exhibits a negative amplitude of the asymmetry. As introduced in Section 1, the observed asymmetry at the ground is a combination of geometrical and attenuation effects. The latter can be expressed as $f_{\text {att }}(d(\theta))=\exp (-d(\theta) / \lambda)$ where $d(\theta)$ represents the distance of the detector to the shower maximum and $\lambda$ is the attenuation length depending on the particles considered. The geometrical effects can be modelled considering the number of particles hitting a detector as seen from the apex of the shower, with a solid angle $\Delta \Omega \propto 1 / d^{2}$ and considering the Angular Distribution Function (ADF) of the emission of particles, formulated as $\operatorname{ADF}(\delta) \propto\left(\delta / \delta_{0}\right)^{-\gamma}$. An expansion (see Ref. [6] for a detailed derivation) of the signal measured at the ground with these assumptions leads to a derivation of the amplitude of the asymmetry as $\alpha \propto 2-\gamma+d(\theta) / \lambda$. We see that, at large distances, the value of the exponent $\gamma$ of the ADF can be large enough to compensate the attenuation and produce a negative amplitude of the asymmetry.

## 3. Correction of the asymmetry

### 3.1 Parametrization of the amplitude

To correct the bias introduced by the azimuthal asymmetry, a parametrization of the amplitude $\alpha\left(r, \theta, S\left(r_{\mathrm{opt}}\right)\right)$ has been derived from simulated showers and is best fit with

$$
\alpha\left(r, \theta, S\left(r_{\mathrm{opt}}\right)\right)=a\left(\theta, S\left(r_{\mathrm{opt}}\right)\right) \operatorname{erf}\left(\frac{r}{r_{0}\left(\theta, S\left(r_{\mathrm{opt}}\right)\right)}\right)
$$

where:

$$
a\left(\theta, S\left(r_{\mathrm{opt}}\right)\right)=\frac{a_{0}+a_{1} \sin ^{2} \theta}{\left(1+\exp \left(-\frac{\sin ^{2} \theta-a_{2}\left(S\left(r_{\mathrm{opt}}\right)\right)}{a_{3}\left(S\left(r_{\mathrm{opt}}\right)\right)}\right)\right)}
$$

and:

$$
r_{0}\left(\theta, S\left(r_{\mathrm{opt}}\right)\right)=r_{1}\left(S\left(r_{\mathrm{opt}}\right)\right)+r_{2}\left(S\left(r_{\mathrm{opt}}\right)\right) \sin ^{4} \theta
$$



Figure 4: Top: Mean bias of the position of the core in the shower plane, as a function of shower size $\lg (S(1000) /$ VEM $)$ (left) and as a function of $\sin ^{2} \theta$ (right). Bottom: Resolution of the core as a function of the shower size (left) and as a function of $\sin ^{2} \theta$ (right).

The parameters $a_{2}, a_{3}, r_{1}$ and $r_{2}$ depend on energy following a second-order polynomial in $\lg S(1000)$. The derivation of the parameters, through a minimization using Minuit framework, has been performed using dense rings from 0 to 1500 m . For larger distances, the amplitude of the asymmetry is assumed to be constant and equal to $\alpha(1500 \mathrm{~m})$. The model is then applied to the reconstruction of the simulated data-sets following Eq. (1).

### 3.2 Effect on the shower geometry

## Position of the core

One of the main objectives of this study is the correction of the bias in the core position introduced by the azimuthal asymmetry in the measured signal. This bias is computed as a function of shower size and zenith angle as the mean value in the upstream-downstream direction (y-axis) of the 2D-distribution illustrated in Fig. 1 (right).

In Fig. 4 (top left), the mean bias in the upstream-downstream direction is, as a function of shower size, constant around -40 m . The application of the developed model reduces this bias to $\pm 5 \mathrm{~m}$. This improvement is more remarkable as a function of zenith angle, Fig. 4 (top right), where the dependency of the mean bias with the inclination of the shower is removed to a residual bias $<10 \mathrm{~m}$.


Figure 5: Left: Relative uncertainty $\sigma(S(1000)) / S(1000)$ of the shower size reconstruction as a function of shower size $\lg (S(1000) / V E M)$. Right: Angular resolution of the arrival direction as a function of $\sin ^{2} \theta$.

Note that to be complete, the mean bias in the perpendicular direction (x-axis) is zero irrespective of applying the correction of the asymmetry or not.

The resolution of the reconstruction of the position of the core, in the shower plane, is extracted computing the distance at which $68.3 \%$ of the cumulative distribution of the distances between the position of the simulated core and the reconstructed one, is reached. The results are reported in Fig. 4 (bottom) as a function of shower size and zenith angle. Applying the correction of the asymmetry is removing the observed bias in the position of the core. The resolution of the position of the core is, after the application of the correction, of $\sim 80 \mathrm{~m}$ at the lowest values of shower size and decrease to $\sim 40$ at the highest values. by $\sim 20$, improving the resolution by to $10-30 \mathrm{~m}$. As a function of zenith angle, Fig. 4 (bottom) right, the core resolution is, after the correction, constant at $\sim 40 \mathrm{~m}$ to an inclination of the shower of $45^{\circ}$.

The model derived does not take into account the decrease of the amplitude of the asymmetry at large distances. This assumption seems to be validated by the results shown in Fig. 4. Indeed, the reconstruction of the position of the core of the shower is driven by the closest detectors ( $r<1500 \mathrm{~m}$ ). However, one could imagine to improve the model developed in this paper taking into account effects of the muonic component in the amplitude of asymmetry.

## Angular resolution and uncertainties in shower size

A precise knowledge of the position of the core is essential to obtain an accurate estimator of the energy, $S(1000)$. The uncertainties in the shower size are evaluated comparing the reconstructed $S(1000)_{\text {rec }}$ and the "simulated" one $S(1000)_{\text {MC }}$. The lack of knowledge of the true shape of the LDF is preventing us from knowing the true value of $S(1000)$. Instead, in this study, $S(1000)_{\text {MC }}$ is computed averaging the signal measured in the 24 detectors constituting the dense ring with a radius of exactly 1000 m in the shower plane. The standard deviation of the distribution of $S(1000)_{\text {rec }} / S(1000)_{\text {MC }}$ is thus, an estimator of the accuracy of the reconstruction of $S(1000)$. Despite the suppression of the bias in the position of the core and the improvements of its resolution, no improvements of the uncertainties in $S(1000)$ are observed after correcting the asymmetry, as reported in Fig. 5. The uncertainties are decreasing with the energy of the shower from $12 \%$ to between 7 and $8 \%$. The plateau observed at the highest energies is the consequence of the saturation
of the closest detectors to the shower axis.
The angular resolution is defined as the angle at which the cumulative distribution of the opening angle $\eta$ reaches $68.3 \%$, where $\sin \eta=\left|\hat{a}_{\mathrm{MC}} \times \hat{a}_{\text {rec }}\right|$ and $\eta$ is the opening-angle between the simulated arrival direction $\hat{a}_{\mathrm{MC}}$ and the reconstructed one $\hat{a}_{\text {rec }}$. As for the uncertainties in $S(1000)$, the angular resolution is not affected by the application of the correction of the azimuthal asymmetry and is decreasing from $1^{\circ}$ to $0.4^{\circ}$ for the most inclined showers considered.

## 4. Conclusion

The azimuthal asymmetry in signals measured by surface detector arrays is still a subject deserving our attention. In the case of an array similar to the surface detector of the Pierre Auger Observatory, not considering asymmetry introduces a bias in the position of the core around 40 m in the upstream direction and worsens its resolution by 20 m on average. With a model describing the amplitude of this asymmetry as a function of the distance of the shower axis, the shower size and the zenith angle, these effects are suppressed without affecting the resolution of the reconstruction of the arrival direction and shower size.

However recent studies [8] have shown that the simulations are suffering from a deficit of produced muons. Knowing that the muonic component is decreasing the amplitude of the azimuthal asymmetry, a comparison with observed data is necessary to avoid an overestimation of the amplitude and overcompensating the position of the core too much in the downstream direction. The comparison with measured data could also provide another method to test the hadronic interactions, similar to the one performed in Ref. [7]

This study has been performed considering WCDs which emphasize the contribution of muonic component to the total signal. For surface detectors using scintillator detectors, which are more sensitive to the electromagnetic component, the mean bias introduced by the azimuthal asymmetry is expected to be larger. It would be interesting for surface detectors such as the Telescope Array or the upgrade of the Pierre Auger Observatory, to continue the development of a model correcting the azimuthal asymmetry in signals.

## References

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[^0]:    $37^{\text {th }}$ International Cosmic Ray Conference (ICRC 2021)
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[^2]:    ${ }^{1} \mathrm{~A}$ bias $>100 \mathrm{~m}$ has to be introduced to see an effect on the amplitude of the asymmetry.

