Sovereign Debt Default: The Impact of Creditor Composition

Amrita Dhillon, Javier García-Fronti and Lei Zhang

No 901

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

THE UNIVERSITY OF
WARWICK

Sovereign Debt Default: The Impact of Creditor Composition *

Amrita Dhillon, Javier García-Fronti and Lei Zhang

University of Warwick

March, 2009

Abstract

The main motivation of this paper is to study the impact of the composition of creditors on the probability of default and the risk premium on sovereign bonds, when there is debtor moral hazard. In the absence of any legal enforcement, relational contracts work only when there are creditors who have a repeated relationship with the borrower. We show that ownership structures with a larger fraction of long term lenders are associated with a lower default probability and lower risk premia. Moreover, competitive markets structures lead to loss in efficiency as well when there is moral hazard, in contrast to the case with perfect enforceability and information.

JEL: F34, G34, H63, D82, D86

Keywords: Sovereign default, Institutions, Reputation, Uncertainty.

1 Introduction

In recent years countries have turned increasingly from bank loans to bond issues to raise capital. As a result, the international capital markets are more diversified and function more efficiently. Specifically, there is a broader investor base available to provide financing for emerging market sovereigns, which has helped diversify risk. But there is a serious downside if a country faces unsustainable debt. Private creditors have become increasingly numerous, anonymous and difficult to coordinate. (IMF, 2003)

During the 1980s, lending in sovereign debt markets was very different from what it is now: there were syndicated bank loans and a small number of banks which operated on a common set of assumptions that tended to avoid legal action. In contrast, Brady bonds and subsequent new debt issues in the 90s were purchased by thousands of new investors, including institutional hedge funds – see Figure 1 Wright (2005). This greater diversity among creditors meant that they were less likely to be constrained by tacit understandings about a shared collective interest.

This stylized fact has great relevance for sovereign debt markets. As Wright (2005) suggests the changing nature of such markets might be good in terms of increased competition for lending from the point of view of the debtor country when debt is enforceable, but the case of sovereign debt is vastly different. When debt cannot be contractually enforced in the debtor country but not the creditor countries, then creditors who can co-ordinate punishments may achieve much better outcomes in terms of default incentives than un-coordinated creditors. So competition in such markets is not necessarily always good.

Co-ordination problems can arise at various levels: e.g. at the level of the contracts that are signed: how favourable they are to creditors, at the level of punishment in case of default and at the level of the re-negotiation of debt in case of partial default.

For example, let us examine the events after the Argentine default of 2001. The ex-post default evidence on bond issues has been studied by e.g. Dhillon, García-Fronti, Ghosal, and Miller (2006) and Sgard (2005). It reveals an interesting pattern of creditor composition post default: a large number of small creditors (more than 1/4 million) and a few very big lenders. This creditor heterogeneity has been cited as one of the causes for delay in the post default renegotiation. The coordination problem was resolved just before the 2005 swap: a significant fraction of the small lenders sold their bonds cheap to big lenders allowing them to start bargaining with Argentina. In general, besides re-negotiation of debt, large creditors can have co-ordinating effects on the market for sovereign debt. This is what we try to capture in our model.

In this paper we study the question of how the composition of debt, in particular, the presence of large creditors who have a repeated relationship with the debtor affects (ex-ante) the probability of default and the risk premium on bonds.

Our formal model is a stochastic dynamic game where there is a fraction of "large" lenders and a fraction of "small" lenders all of whom buy one period bonds from the borrower country. The only

difference between the two types of lenders is that large borrowers internalize the effects of their own actions on the rate of interest and on the probability of default, and have access to a commitment technology for delivering on promises next period while small lenders do not. Small lenders always offer to buy bonds at any rate of interest higher than the risk free rate. Large lenders however have bargaining power in setting the rate of interest while small lenders take it as given and only decide whether to buy bonds or not. The borrower country chooses the level of effort (or investment): the higher the effort the higher the probability of a good outcome. In case there is a bad outcome, the country must default on it's debt. In this setting, the optimal contract for the borrower country must take into account two incentive problems: moral hazard and repudiation of debt servicing in the good state and the participation constraint of the country as well. Large lenders choose their profit maximizing rate of interest subject to the presence of small creditors, and the incentive compatibility conditions. Hence small lenders exert a negative externality on large lenders through the lowering of the rate of interest as well as through the free riding on providing incentives to repay. There is an indirect effect on incentives by small lenders: lowering the rate of interest encourages repayment. The direct effect on incentives is however negative since they free ride on rewarding the borrower for repayment or punishing for default. We show that the effect of small creditors on the probability of default is positive: the higher the fraction of bonds held by small creditors, the higher is the probability of default and the higher are the risk premia. This result has a similar flavor to a phenomenon pointed out by Hellman and Stiglitz (2000) that banks have tendencies to gamble on investments much more the higher is the competition they face in the market.

1.1   Related Literature

One of the first questions to understand in the absence of well developed enforcement mechanisms in a sovereign debt market is: why do countries repay debt?[1] One possible explanation is that they repay because they are worried about sanctions in case of default. There is a literature (Bulow and Rogoff (1989) Fernandez and Rosenthal (1990)) focusing on direct punishment in the form of sanctions to the defaulter country. The problem with this interpretation is that due to the nature of the sovereign debt market, there is no clear evidence of effective sanctions.

A second interpretation argues that countries repay because they are worried about the impact of a default on their reputation (Eaton and Gersovitz (1981)). These papers argue that a country's commitment to repay arises through the threat of no future credit in the market. Following a similar argument, Eaton (1996) describes a model in which there are two types of borrowers. The bad one always defaults if it is optimal and the good one always try to repay if they can. In any case, at low levels of debt, bad borrowers have a reputational incentive to repay in order to mimic the good types. Kletzer and Wright (2000) describe how lending can be sustained in equilibrium, allowing for

---

[1]For a detailed description of the issue, please see Sturzenegger and Zettelmeyer (2006)

partial market exclusion in case of default with the creditor extracting all future gains from the debtor. However, in equilibrium there is no default because in the hypothetical case of default, the equilibrium in the subgame is just as painful as autarky. The main result is that they have a model where, without insurance, without sanctions and without international institutions; sovereign debt markets still exist. The common feature of the reputation models is that the only way that creditors could punish a country is by denial of future credit. An important assumption of the reputational models is that creditors are able to fully coordinate on punishments. Wright (2002) describes how the market could be sustained if lenders collude in the lending condition and punishing in case of default. Finally, Wright (2005) analyses how the developing country sovereign debt market has became progressively more competitive and how this harms coordination on punishment among creditors. He describes how this affects welfare and concludes that reduced competition in the market is welfare improving when there are enforceability problems . However, while Wright (2005) is concerned more about self enforcement of contracts, there are no incentive problems stemming from moral hazard in the investment decision. Moreover, he is concerned with efficiency of the investment decision while we focus on the probability of default and risk premia. In terms of efficiency, our results are similar. We show that while competition might be welfare improving in the absence of enforceability and moral hazard constraints, this does not remain the case when these constraints become important.

A related paper by Kovrijnykh and Szentes (2007) analyzes the effects of default (debt overhang) on market structure. They show that when debt overhang occurs, then seniority rules imply that only the incumbent lender is willing to lend to the country and so he exploits his monopoly power. After a series of good shocks however the lender finds it optimal to allow access to other lenders again. Our paper, in contrast, looks at how market structure affects the probability of default.

Evidence that supports our results on the importance of creditor coordination is provided in a paper by Drelichman and Voth (2008) that looks at defaults in the age of Philip II, in the 16th century. Philip II of Spain accumulated huge debts and defaulted four times but yet was able to get access to funds. They show that in fact the lenders were a highly coordinated group with the ability to cut off Philip II's access to smoothing services. Lending morataria were sustained by a "cheat the cheater" strategy as in Kletzer and Wright (2000). We show that when syndicated bank loans give way to bonds and there is a reduced co-ordination among lenders it makes it more difficult to sustain lending morataria: this in turn leads to higher defaults and market break-downs.

The paper is organised in two sections. The next section introduces the setup of the model. Section three analyses Sequential equilibria of the repeated game. Finally, we draw some conclusions.

2   The Model

The model investigates the link between the composition of bond ownership, the probability of default and the risk spread on the debt. What we have in mind here is that there are some creditors in the

market who are more "powerful" in the sense that (a) they internalize the effect of their actions on the repayment rate and have access to a commitment technology to deliver future loans and (b) they can influence the interest rate of the bonds even though they are issued by the borrower country. Formally, we assume that a fraction $1 - \alpha$ of creditors are "large" creditors who do influence the rate of interest and who act strategically. All large creditors are assumed to be identical and own α_l (exogenously given) shares in the total bond issue while small creditors are identical and own $\alpha_s < \alpha_l$ (exogenously given) shares of the total bond issue each. Let n_l, n_s be the numbers of large and small creditors who own bonds respectively: then $\alpha = n_s \alpha_s$ and $1 - \alpha = n_l \alpha_l$.[2] Small creditors are assumed to be non-strategic: they are willing to buy bonds as long as the bonds pay in expected terms at least the risk free rate , normalized to 1.

Large creditors use bargaining with the borrower to set the rate of interest. To represent the outcome of this bargaining game we simply assume a very reduced form where the rate chosen depends on a weighted sum of the offer by small creditors which we assume to be the risk free rate and the offer by large creditors R_l. Hence $R = \alpha R_s + (1 - \alpha) R_l$. Large creditors choose R_l to maximize joint utility. Moreover small creditors have a reservation rate of return equal to 1 (the risk free rate), and Bertrand competition between the small creditors drives R_s to the point where $R_s = 1$. Hence $R = (1 - \alpha) R_l + \alpha$, so that $R_l = \frac{1}{1-\alpha}(R - \alpha)$. Large creditors therefore get a gross return of $R_L = (1 - \alpha) R$ so that $R_L = (1 - \alpha)^2 R_l + \alpha(1 - \alpha)$. R_l is the choice variable for the large lenders.

The sovereign bond market consists of one borrower, large creditors and small creditors. There are two goods. The borrower country has no endowment of good 1. The sovereign borrows 1 unit of good 1 in every period to invest in a risky technology which yields the end of period payoffs of Y_H of good 2 with probability P and $Y_L = 0$ with $1 - P$. The probability is assumed to be an increasing and concave function of the level of effort (or investment measured in units of good 2), e. All agents are risk neutral. Good 2 is measured in terms of good 1 units.

The time line is given in Figure 2 below. First the debt contract is signed with the borrower receiving one unit of lending (in good 1) and promising to pay R (in good 2) a period later. Given this contract and expected future payoffs, the borrower then determines his optimal effort (investment). Finally, outcomes of the borrower's investment are realized. Output is not observed by creditors, so contracts are not state contingent. If high output occurs, the borrower could pay off the debt and engage in a new round of borrowing; or could default. If the bad state is realized we assume that $Y_H > R > 0$, so that when output is low there is no option but to default and pay nothing when there is no possibility of legal enforcement. In this case the borrower is excluded from the market forever[3].

---

[2]We can justify this assumption with the following interpretation: large creditors are able to perfectly co-ordinate their actions: it is worthwhile for them to do so only if they can cover the fixed costs of coordination by owning a large enough fraction of bonds.
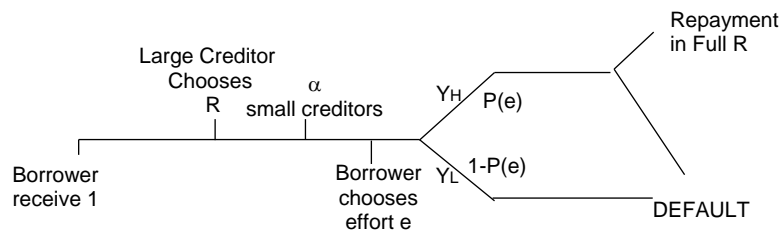
[3]We assume this for the moment. As discussed before, lack of coordination can show up either in commitment to rewards or commitment to punishments but it is not clear that in the case of risk neutrality it makes much difference to the analysis. We simplify by allowing commitment to play a role only in the commitment to rewards rather than

If the game is a one shot game then defaulting may be deterred if there is a sufficiently high and enforceable output penalty imposed by the courts for default . This implies that in the absence of enforcement the one shot game leads to sure default and anticipating this, no lenders will be willing to lend. In order to get an equilibrium with positive lending, when there are no other reward or punishment menchanisms, we therefore need at least some creditors who have a repeated interaction (commitment to lend in future) with the borrower country. We assume that large creditors have access to a commitment technology, while small creditors do not. This is captured by introducing a participation constraint only for small creditors (which is already taken care of earlier in the computation of R_L).

The extensive form game (2) is the stage game of an infinite horizon repeated game with imperfect monitoring and we investigate the properties of the sequential equilibria of this game. The effort is unobservable to creditors, and is costly so the debtor is subject to moral hazard. Large creditors interact with the borrower repeatedly while small creditors only interact for one period. Large creditors are able to condition the rewards (through future values of R_L) for not defaulting on the full history of repayment. A summary of the variables is provided in Table 2.

R	Full repayment	$R > 1$
e	Borrower's effort	$e \geq 0$
P	Probability of good state	$P'(e) > 0, P''(e) < 0, P'''(e) < 0$
		$P(0) = 0, P'(0) \to \infty$
Y_H	Output in the good state	$Y_H > R$
α	Small creditors' bonds share	$0 \leq \alpha \leq 1$
R_F	Repayment at risk free rate	
β	Discount factor for large creditors	

punishments.

Large Creditor
Chooses
α
R   small creditors

Borrower
receive 1

Borrower
chooses
effort e

Y_H P(e)

Y_L 1-P(e)

Repayment
in Full R

DEFAULT

In this set up there are two driving forces that are important determinants of the probability of default: the larger the fraction of small creditors the bigger the externality imposed through the determination of R (i.e. the lower is the monopoly power of large lenders in setting price) and also the larger the per capita cost of ensuring repayment and effort incentives: since by definition small creditors get the returns R per unit, without incurring any of the costs of ensuring repayment. Small creditors therefore free ride: if the actual R is bigger than 1 (the risk free rate) then they gain by getting a higher repayment while the presence of large creditors ensure a lower probability of default since they choose R_l (which affects R) anticipating the effect on the incentive to repay. What is the combined effect of these two forces? The direct competitive effect on R is to lower it, thus improving incentives to repay. The indirect effect of high α is to increase the net per capita cost to large lenders of ensuring repayment by large creditors since they get only $(1-\alpha)R$ of the total repayment but they provide all the promised rewards in terms of lower R_l for repayment. The competitive effect leads to falling risk premia but the externalities imposed on incentives leads to increasing risk premia to compensate large lenders. This is what we show in the next section.

3   Sequential equilibria of the repeated game

This section analyses the sequential equilibrium resulting from repeated interactions between the borrower, the large creditor and a fringe of small creditors. Given that the borrower's output is not observable by the creditor, we look for a non-state-contingent optimal relational contract. By observing the past payment history of the borrower, the large creditor decides R_l on the current loan. We first describe the sequential problem facing the large lenders. The state of the economy at time t is indexed by a state $\omega \in \{G, B\}$ where G stands for the good state and B for the bad state. The state is only observed by the borrower.Notice that we restrict attention to maximum punishment contracts, so once there is default the game ends. Clearly, the modelling should take account of free riding on punishment- hence allowing re-negotiation in the model and letting small creditors free ride on large creditors would have been the best modelling strategy. This would imply that the country can get back into credit markets even after default, albeit with a higher punishment. The problem however might become intractable given that we have asymmetric information on effort as well as enforcement problems. Also both borrower and creditors are risk neutral so using varying rewards for repayment rather than varying punishment for default might well yield qualitatively similar results. We also assume that the borrower always defaults in the bad state. The publicly observable history is the sequence of repayments if there has been no default till time t. (Note that we do not need to formulate the decision of the borrower once a bad state has occurred). The optimal dynamic contract is a history dependent sequence of repayments $\{R(h_t)\}_{t=1}^{\infty}$ which maximizes the large lenders' payoffs. In the equilibrium we must have the following:

(1) The borrower's choose e^4 in period t to maximize:

$$U_t = max_e\{P(e)[Y_H - R(h_t) + \beta U_{t+1}] - e\} \tag{1}$$

having observed the contract $R(h_t)$. Observe that the price setting process implies that $R_L = (1-\alpha)^2 R_l + \alpha(1-\alpha)$. (2) Large lenders then choose the contract to maximize their payoff:

$$V_t = \max_{\{R_l(h_t)\}} \{P(e)[(1-\alpha)^2 R_l(h_t) + \beta V_{t+1}] - (1-\alpha)^2\}, \tag{2}$$

subject to the following constraints:

(i) Borrower's participation constraint:

$$U_t(e^*(h_t)) \geq 0 \tag{3}$$

where the autarky payoff is normalized to 0.

(ii) No Default Constraint (or Self Enforcement constraint) of Borrower in the good state:

$$Y_H - R(h_t) + \beta U_{t+1}(R(h_{t+1})) \geq Y_H \tag{4}$$

(iii) Small creditors' participation constraint:

$$P_t(e^*(h_t))R(h_t) = 1 \tag{5}$$

where in any period, the relation $R = (1-\alpha)R_l + \alpha$ must hold as well (dropping the time subscript for notational convenience).

This problem has a recursive formulation, using lifetime utility of the borrower as a state variable, following Spear and Srivastava (1987). This variable is enough to summarize information about a borrowers' default history. We consider the space of contracts where incentive problems can be partially overcome using memory and future promises. Contracts are restricted to depend only on publicly observable outcomes, which in this case is just whether there is default or not. We follow the previous literature (Spear and Srivastava (1987), Thomas and Worrall (1988), Abreu, Pearce and Stachetti (1990), Phelan and Townsend(1991) among others) in formulating the contracting promise recursively using a "promised value". The contracts specify moreover that upon default, the borrower will be permanently excluded from the future credit market. The contract design problem of the above setup needs to take into account two basic elements: one is the lack of commitment mechanism on the part of the borrower i.e. that debt can be repudiated, the other is the private information concerning the actual realised states. One-sided commitment problem in a similar context has been studied by Kocherlakota (1996) who looks at self-sustaining insurance contract in a village economy where villagers face idiosyncratic endowment shocks. There, full insurance is not possible as the optimal contract has to take into account the lack of commitment on the part of villagers even if their

---

[4]There is a one to one mapping between effort and the disutility from effort.

endowments are public information. Instead, the optimal relational contract derived exhibits history dependence, where history summarizes all past endowments. Such history dependence is dealt with in a recursive manner by using a "promised value".[5] A similar problem with asymmetric information has been investigated by Thomas and Worrall (1988).[6]

To induce the borrower to repay the loan in the good state,[7] the large creditor has to promise more favorable terms in future loan contracts. Notice that these large creditors are like the "swiss banks" in the model of Cole and Kehoe (1995). How does this "promised value" capture history dependence in our model? Let h_t track borrower's past output realizations up to time t. As the loan contract is not state contingent, the borrower will default in the bad state. So h_t simply counts the number of times that the borrower has made full repayments (or a string of realizations of good state).

Let δ_t be the "promised value" (present value of lifetime utility in t) made by the large creditor in the period $t-1$ for the delivery in period t. Given δ_t, the current period interest rate is determined, $R_t(\delta_t)$. Conditioned on this interest rate and the future promise δ_{t+1}, the borrower decides on the optimal effort, $e^*(R_t, \delta_{t+1})$. The future promise also affects whether the borrower would repay the loan in the good state. So when period $t+1$ arrives, δ_{t+1} would be delivered only if there was a good state in period t. This implies that δ_{t+1} depends on δ_t and a realisation of Y_H at t, $\delta_{t+1} = f(\delta_t, Y_H)$. Iterating this relationship forward from the initial δ implies that δ_{t+1} depends on h_t. In what follows, we denote δ the promise made by the large creditor in the last period, and δ' the promise made in the current period. In the optimisation faced by the large creditor, δ serves as a state variable.

We want to focus on the trade off between the competitive effect of having more creditors in the game (captured by a higher fraction of small creditors) and the free riding on the provision of reputational incentives for repayment to the borrower. We solve now for the dynamic optimal contract among those with maximum punishment for this problem with one sided commitment. Before that we analyse the effect of market structure on efficiency.

Observe that as in Wright (2005), absent any enforcement and information problems, a competitive market structure leads to higher efficiency (i.e. higher e^*) than a collusive or co-ordinated market structure. In our model, competition increases as α increases. Hence, we first show (after introducing the maximization problem for the large creditors below) that in the absence of the incentive constraints and full enforceability and observability of effort, higher α leads to higher effort.

Let us first focus on the borrowers optimization problem. Let $x = \beta\delta' - R$. Given the contract (R, δ') the borrower chooses optimal effort to maximize:

$$u(e; R, \delta') \equiv P(e)(Y_H + x) - e \tag{6}$$

where we assume that the discount factor of the borrower, β is identical to that of the large creditor,

---

[5] For other applications using "promised value" approach, see Ljungqvist and Sargent (2000, Chapters 15 and 16).

[6] Atkeson (1991) looked at a problem of lending to a sovereign in the absence of complete information and enforcement, where distribution of future states depends on investment.

[7] Note that our setup implies that the borrower always defaults in the bad state.

and the probability of the good state, $P(e)$, is increasing and concave in effort, e. In addition, we assume that $P'''(e) < 0$, $P(0) = 0$, $P'(0) \to \infty$, $P(\infty) = 1$ and $P'(\infty) = 0$. This yields the following first order condition of the borrower:

$$P'(e) = \frac{1}{Y_H + x} \tag{7}$$

We define e^* as the optimal effort:

$$e^*(x) = arg \max_e u(e; x). \tag{8}$$

Hence the participation constraint of the borrower becomes:

$$u(e^*; x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq 0. \tag{9}$$

In addition, for the borrower to have incentive to make full repayment in the good state, he needs to be "rewarded" when honouring the current contract. This is reflected in the following "no-default" constraint

$$Y_H + x \geq Y_H. \tag{10}$$

This constraint says that conditional on the good state, the borrower will prefer to honour the contract than to default. Clearly it is equivalent to asking $\beta \delta' - R \geq 0$: this is asking that the continuation value of the future relationship is large enough that the borrower prefers to pay whenever the good state is realized rather than default and terminate the relationship. Notice that the participation constraint for the borrower must be satisfied in every period so that we have $\delta' \geq 0$.

In the following we assume first that only one type of bond contract can be written for all creditors. The bonds are one period contracts. Large creditors are players who are committed to re-lending in every subsequent period conditional on no default. We also assume that large creditors are fully coordinated: they decide both R_l and δ' as a group. In the absence of this assumption, there would be free riding even among large creditors leading to suboptimal contracts. Solving now for the optimal contract of large lenders: Let $V(\delta)$ be the maximum expected present value to the large creditor conditional on the state δ. Then $V(\delta)$ at any given time must satisfy the following Bellman equation

$$V(\delta) = \max_{\{R_l, \delta'\}} \{P[(1-\alpha)^2 R_l + \beta V(\delta')] - (1-\alpha)^2\}, \tag{11}$$

where $0 < \beta < 1$ is the large creditor's discount factor, δ' denote the current period interest rate and promise respectively, and P is the probability of good state. Equation (11) simply specifies $V(\delta)$ as the expected payoffs under the optimal contract. If the good state is realized, the large creditor receives $(1-\alpha)$ fraction of the full repayment and the discounted continuation value associated with the future relationship. In the bad state, the borrower defaults, creditors obtain nothing and the relationship is terminated. Since the promise made by the large creditor is on the total amount of lending, while the return is only on $(1-\alpha)$, small creditors will free ride on borrower's full repayment in good state. Moreover, notice that the presence of small lenders depresses the repayment R as well.

11

As δ reflect the "transfer" from the large creditor to the borrower, we must have $V'(\delta) < 0$. We show this later.

Given that the large creditor is engaged in current period lending, the assumption that creditors are committed to paying their "promised value" made in the previous period, δ, implies the following constraint:

$$u(e^*; x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq \delta, \tag{12}$$

where Y_H is output realised in the good state respectively, and e^* is the optimal effort chosen by the borrower. Equation (12) is the so-called "promise-keeping" constraint on the part of the large creditor. It is clear that δ is measured in terms of borrower's utility. The promise keeping constraint reflects the set of (R, δ') that are consistent with the borrowers participation constraint, given that lenders are committed to honoring their promise. If this is violated the borrower prefers not to accept the contract and then get its reservation value δ.

This maximization is done subject to the No repudiation constraint of the borrower (10) and the participation constraint (12).

Finally the contract must also satisfy the participation constraints of large and small creditors. Note that the rationality condition of the borrower requires $\delta \geq 0$. The rationality condition of the large creditor, $V(\delta) \geq 0$, implies $\delta \leq \delta_{max}$ as $V(\delta)$ is decreasing in δ. So the domain of the value function we consider will be in $\delta \in [0, \delta_{max}]$. The participation of small creditors is satisfied by assumption in the definition of R. Generally speaking, the optimal contract of this repeated game may possess non-stationary equilibrium.

The Appendix shows that the all equilibria in this game are stationary, except for the first period. Existence of a stationary equilibrium is quite intuitive since the game itself has a stationary structure: maximum punishment ensures that whenever there is default, all lending is stopped. Since the only observable variable is the number of defaults, punishment is fixed, and there is no re-negotiation, the only way that history could matter in through the number of repayments in the past . However, the initial δ captures history so that the predicted pattern of default would depend on history. Now we look at the properties of the stationary equilibria.

Proposition 1 *Let $V(\delta)$ be continuously differentiable, and Y_H sufficiently large. Then (a) there exists a unique solution, $V(\cdot)$ to the Bellman equation (11) subject to constraints (10) and (12); (b) $V(\delta)$ is decreasing and concave; and (c) $V(\cdot) \geq 0$.*

PROOF: Appendix 1.

Proposition 2 *Let Y_H be sufficiently large. Then there exists a unique optimal contract which is always stationary after the first period.*

PROOF: Appendix 2.

Proposition 3 *In the stationary optimal contract, $\frac{\partial R}{\partial \alpha} > 0, \frac{\partial P}{\partial \alpha} < 0$ and $\frac{\partial e^*}{\partial \alpha} < 0$.*

PROOF: Appendix 3.

The intuition for Proposition 3 is quite simple. Note that from the Bellman equation (11), the contract has two different effects on the value function. On the one hand, given the probability P, the large creditors would like to choose the largest R and smallest δ' so as to increase its payoff in the good state. We term this as the collusive (the extreme case is when $\alpha = 0$) effect. This means that creditors will have incentives to choose the smallest possible x. On the other, the large creditors also have the incentive to have high P. We term this as the probability increasing effect. This requires high x. The optimal contract depends which of these two effects dominates. The probability increasing effect depends on α since the higher is α the greater is the free riding by small creditors on large so that the participation constraint of large creditors is affected.

When α is large, the presence of the free-riding small creditors will decrease the payoff to the large creditor in the good state. So the large creditors will have more incentive to decrease x, leading to this collusive effect dominating the probability increasing effect. In this case, the participation constraint of the borrower,(9) is binding. From Proposition 3, it is also clear that as α increases, optimal x decreases. This implies that large fraction of small creditors increase the probability of default but lead to higher risk premia.

In terms of efficiency Proposition 3 shows that as α increases, the effort decreases. So competition has a bad effect on incentives to repay when there are moral hazard and enforceability problems. This conclusion is consistent with Wright (2005).

4   Conclusions

In this paper we presented a stylized model to analyze the effects of creditor composition on the probability of default by sovereign governments. In the model we assumed that small creditors own a fixed proportion α of the total bond issue. Small creditors have a direct effect on the price of bonds through increasing the competition in the market, however they free ride on large creditors by not taking the impact of their chosen R on the incentives of the borrower to decrease the probability of default. The net effect according to our model is that increasing competition in the market is bad. Indeed, an increasing share of small creditors in the bond market increases the probability of default and increases the risk premium. Our model can explain why a shift from syndicated loans to bonds might lead to more volatility in the market than before.

Obviously, a lot remains to do. We are interested in endogenizing the entry of large and small creditors: if there are secondary markets in bonds then small creditors might have incentives to sell to large creditors so that the ownership structure in the end may be no different from syndicated loans! Second, we would like to relax the maximum punishment rule we imposed: it would be interesting

to analyze the case where instead of only rewards to the borrower the lenders can use reductions in access to the market as punishment. The result of negotiations after default depend on the ownership structure and that in itself would alter the default rates. We might also consider allowing repayments to some creditors and not others: an endogenous seniority rule that emerges in response to reputational concerns.

References

BULOW, J., AND K. ROGOFF (1989): "A Constant Recontracting Model of Sovereign Debt," Journal of Political Economy, 97(1), 155–178.

COLE, H. L., AND P. J. KEHOE (1995): "The role of institutions in reputational models of sovereign Debt ," Journal of Monetary Economics, 35(1), 45–64.

DHILLON, A., J. GARCÍA-FRONTI, S. GHOSAL, AND M. MILLER (2006): "Debt Restructuring and Economic Recovery: Analysing the Argentine Swap," The World Economy, 29(4), 377.

DRELICHMAN, M., AND H.-J. VOTH (2008): "Lending to the Borrower from Hell: Default in the age of Philip II, 1556-1598," Manuscript, University of British Columbia.

EATON, J. (1996): "Sovereign Debt, Reputation and Credit Terms," International Journal of Finance and Economics, 1, 25–35.

EATON, J., AND M. GERSOVITZ (1981): "Debt with Potential Repudiation: Theory and Estimation," Review of Economic Studies, 48(2), 289–309.

FERNANDEZ, R., AND R. ROSENTHAL (1990): "Strategic Models of Sovereign-Debt Renegotiations," Review of Economic Studies, 57(3), 331–349.

HELLMAN, THOMAS.F.AND MURDOCK, K., AND J. STIGLITZ (2000): "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?," American Economic Review, 90(1), 147–165.

KLETZER, K., AND B. WRIGHT (2000): "Sovereign Debt as Intertemporal Barter," American Economic Review, 90(3), 621–39.

KOVRIJNYKH, N., AND B. SZENTES (2007): "Equilibrium Default Cycles," Journal of Political Economy, 115(3), 403–46.

SGARD, J. (2005): "La Dette Argentine et le Déclin du FMI," Lettre du CEPI.

STURZENEGGER, F., AND J. ZETTELMEYER (2006): "Has the Legal Threat to Sovereign Debt Restructuring Become Real?," Manuscript, Universidad Torcuato Di Tella.

WRIGHT, M. (2002): "Reputations and Sovereign Debt," Manuscript, Stanford University.

WRIGHT, M. (2005): "Coordinating Creditors," American Economic Review, 95(2), 388–392.

A Proofs

Appendix 1: Proposition 1

Proof.

We first illustrate that large creditors' value function and its promised value are bounded from above so a metric can be defined, we then show there exists a unique value function to the Bellman equation for some δ subject to constraints (10) and (12). Finally, we show that the value function is decreasing and concave.

(a) Value function and promised value are bounded from above

Suppose that we can implement some contract to generate the Pareto frontier to the problem, the combined benefits to all players are given by the following value function

$$W = \max_e \{P(e)Y_H - e - 1 + P(e)\beta W\} \tag{13}$$

where W defines the Pareto frontier. Given $Y_H < \infty$, then

$$W = \max_e [P(e)Y_H - e - 1]/[1 - P(e)\beta] \equiv \bar{W} < \infty. \tag{14}$$

For any allocation of this benefits between creditors and the borrower, we must have $V \leq \bar{W}$. So for any bounded pair of V, we can define a metric. For the same reason, the borrower's utility must also be bounded from above by $\bar{W}$. This implies that the promised value by the large creditor must be bounded from above. Denote the highest promised value by $\bar{\delta}$, then

$$\delta, \delta' \leq \bar{\delta}. \tag{15}$$

(a) We use Blackwells sufficient condition to show that there exists a unique solution to the Bellman equation (11).

(1) Monotonicity (for a given δ')

Define the right hand side of (11) as a function of x and V:

$$T(V(\delta')) = \max_{R_l, \delta'} \{P(e)(1-\alpha)^2 R_l + P(e)\beta V(\delta')\} - (1-\alpha)^2 \tag{16}$$

subject to constraints (10) and (12), where T is an operator in a metric space. For monotonicity, we have to show that $\tilde{V} \geq V$ implies $T(\tilde{V}) \geq T(V)$.

Construct the Lagrangean

$$L = \max_{R_l, \delta'} \{P[(1-\alpha)^2 R_l + \beta V(\delta')] - (1-\alpha)^2 + \lambda[P(e^*)(Y_H + x) - e^*(x) - \delta] + \mu x + \phi(P[(1-\alpha)R_l + \alpha] - 1)\} \tag{17}$$

where λ, μ are Langrange multipliers. The FOCs to (17) determines R_l and δ'. Using the envelope theorem

$$\partial L/\partial V = \partial T/\partial V = \beta P(e) > 0 \tag{18}$$

16

So $T(V)$ is monotonic.

(2) Discounting

We have to show that $T(V + c) \leq T(V) + \beta c$ for $\beta \in (0, 1)$. It is straightforward to show that

$$T(V + c) = \max_x \{P(x)(1 - \alpha)^2 R_l + P(x)\beta[V(\delta') + c]\} - (1 - \alpha)^2 \qquad (19)$$

$$= T(V) + P\beta c \leq T(V) + \beta c$$

Hence the operator T satisfies monotonicity and discounting, so by Blackwell's sufficient condition, V exists and is unique.

(b): Now we prove that the value function is decreasing and concave. Using the envelope theorem (see (17)) we have: $V'(\delta) = \partial L / \partial \delta = -\lambda$, so $V(\delta)$ is decreasing.

Let the iteration of the value function be given by

$$V_{i+1} \equiv T(V_i) = \max_{R_l, \delta'} \{P[(1 - \alpha)\alpha + (1 - \alpha)^2 R_l - 1 + \alpha + \beta V_i(\delta')] \qquad (20)$$

$$+ \lambda[P(e^*)(Y_H + x) - e^*(x) - \delta] + \mu x + \phi(P[(1 - \alpha)R_l + \alpha] - 1)\}$$

Assume that V_0 is the initial value function and w.l.o.g assume it to be concave. Since $P[(1 - \alpha)^2 R_l + \beta V_i(\delta')]$ is concave in R_l and δ', constraints (10), (12) form a convex set of R_l and δ', so V_1 is a concave function of δ. Through this consecutive iteration process, $\lim_{i \to \infty} V_i$ must converge to a concave function.

Clearly, $V(\delta)$ is a decreasing and concave function in the restricted domain $\delta \in [0, \delta_{max}]$.

(3) $V(\cdot) \geq 0$

To show that $V(\cdot) \geq 0$, we only have to show that the feasible set used in optimising the RHS of (17) is not empty. This is sufficient for there to exist a $V(\cdot)$. If there is a solution, then $PR_l \geq 1$, so period utility and the value function of the large lender is non-negative.

Note first that the small creditors participation constraint, $P(e^*(x))R = 1$ generates a decreasing convex function in (R, x) space, i.e., $S = \{(R, x) : P(e^*(x))R \geq 1\}$ is a convex set. Observe that the LHS of the borrower's participation constraint (9) is strictly increasing in x, so that the constraint is equivalent to $x \geq \underline{x}$. So the intersection of these two sets is a non-empty convex set.

It remains to check this intersection is non-empty when we add the constraint (15). The binding constraint (15) translates into a downward sloping straight line in (R, x) space. The set associated with constraint (15) is simply a triangle formed by the two axes and the downward sloping line mentioned above. This set is clearly convex. When Y_H increases, this set expands. Therefore, there exists a Y_H such that the feasible set is convex and non-empty.

∎

Appendix 2: Proposition 2

Proof.

In what follows, we first show the existence of a stationary equilibrium when Y_H is sufficiently large. Then we look at the properties of the stationary equilibrium.

Given δ, the optimal choice of δ' is determined by the following FOC:

$$V'(\delta') = -(1-\alpha)^2. \tag{21}$$

If such a stationary equilibrium, δ', exists, then it must be reached in just one period.

Note that without imposing $\delta \geq 0$ and $V(\delta) \geq 0$, Proposition 1 ensures that there always exists some δ' such that (21) can be satisfied. However, from Proposition 1, the rationality conditions $\delta \geq 0$ and $V(\delta) \geq 0$ imply that the value function is restricted to some domain $\delta \in [0, \delta_{max}]$ where $V(\delta_{max}) = 0$. To show the existence of the stationary equilibrium δ', we only need to show that $\delta' \in [0, \delta_{max}]$.

From Proposition 1, if Y_H is large enough, $V(\cdot) \geq 0$. Consider the case where $\delta = 0$. Note that the borrower's period utility $u(e^*; x) = P(e^*)(Y_H + x) - e^*(x)$ is increasing in Y_H, so for large enough Y_H, $u(e^*; x) > 0$. In this case constraint (10) is binding and (12) is not (for $\delta = 0$). Using the envelope theorem, it is clear that $\partial V(0)/\partial \delta = 0$.

Now, we look at the local behaviour of the value function $V(\delta)$ near δ_{max}. Since $V(0) > 0$, so $V(\delta_{max}) = 0$ only if $V'(\delta_{max}) < 0$. From the envelope theorem, this is the case where $\lambda > 0$, so constraint (12) is binding, i.e., $P(e^*)(Y_H + x) - e^*(x) = \delta_{max}$. Differentiating both sides of the constraint with respect to δ and incorporating the FOC (7) yields

$$P(e^*)\frac{\partial x^*}{\partial \delta} = 1. \tag{22}$$

Using the Bellman equation (11) at δ_{max},

$$V(\delta_{max}) = P(e^*)[(1-\alpha)R^* + \beta V(\delta')] - (1-\alpha). \tag{23}$$

one can differentiate both sides with respect to δ to obtain

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P'(e^*)\frac{\partial e^*}{\partial x}\frac{\partial x}{\partial \delta}[(1-\alpha)R + \beta V(\delta')] + P(e^*)(1-\alpha)\frac{\partial R}{\partial \delta} + P(e^*)\beta V'(\delta')\frac{\partial \delta'}{\partial \delta}. \tag{24}$$

Using $x = \beta\delta' - R$ we have $\frac{\partial R}{\partial \delta} = \beta\frac{\partial \delta'}{\partial delta} - \frac{\partial x}{\partial \delta}$. Hence equation (24) can be re-written as:

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P'(e^*)\frac{\partial e^*}{\partial x}\frac{\partial x}{\partial \delta}[(1-\alpha)R + \beta V(\delta')] + P(e^*)(1-\alpha)[\beta\frac{\partial \delta'}{\partial \delta} - \frac{\partial x}{\partial \delta}] + \beta\frac{\partial \delta'}{\partial \delta}V'(\delta') \tag{25}$$

or

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P'(e^*)\frac{\partial e^*}{\partial x}\frac{\partial x}{\partial \delta}[(1-\alpha)R + \beta V(\delta')] + P(e^*)\beta\frac{\partial \delta'}{\partial \delta}[(1-\alpha) - (1-\alpha)^2] - (1-\alpha)P(e^*)\frac{\partial x}{\partial \delta} \tag{26}$$

Noting that $P(e^*)\frac{\partial x}{\partial \delta} = 1$ we get

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P'(e^*)\frac{\partial e^*}{\partial x}\frac{\partial x}{\partial \delta}[(1-\alpha)R + \beta V(\delta')] + P(e^*)\beta\frac{\partial \delta'}{\partial \delta}\alpha(1-\alpha) - (1-\alpha) \tag{27}$$

Assume that the future equilibrium δ' exists and is stationary (which will be validated later), and let Y_H be very large so $P'(e^*) \to 0$, then

$$\frac{\partial V(\delta_{max})}{\partial \delta} = -(1 - \alpha) < V'(\delta') = -(1 - \alpha)^2. \tag{28}$$

Since the value function is continuously differentiable and concave, there must be an interior solution such that $V'(\delta') = -(1 - \alpha)^2$ which is stationary. ∎

Appendix 3: Proposition 3

Before proving this Proposition we need the following lemma:

Lemma 1 *Suppose there exists an optimal contract which is stationary. Then in equilibrium, the participation constraint of borrowers (9) is always binding.*

Proof.

By Proposition 2, in a stationary equilibrium we have $V'(\delta') = -(1 - \alpha)^2$. Moreover by the Lagrangean equation (17), $V'(\delta) = \lambda$. By Stationarity, $\delta' = \delta$ so $V'(\delta) \neq 0$ implies that $\lambda \neq 0$. Hence the constraint (9) is always binding in equilibrium.

∎

Proof. First notice that at equilibrium we have

$$V'(\delta') = -(1 - \alpha)^2. \tag{29}$$

Hence, by the implicit function theorem we have $\frac{d\delta'}{d\alpha} = \frac{2(1-\alpha)}{V''(\delta')} < 0$ since $V'' < 0$.

From Lemma (1), we know that the participation constraint of the borrower, (9) is binding, so:

$$P(e^*)(Y_H + x) - e^*(x) = \delta \tag{30}$$

By the implicit function theorem:

$$P'(e^*)\frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha}(Y_H + x) + P(e^*)\frac{\partial x}{\partial \alpha} - \frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha} = \frac{\partial \delta}{\partial \alpha} \tag{31}$$

From the first order condition of the borrower, (7), we know that $P'(e^*) = \frac{1}{Y_H + x}$. Substituting this in equation (31) above, we get:

$$P(e^*)\frac{\partial x}{\partial \alpha} = \frac{\partial \delta}{\partial \alpha} \tag{32}$$

Now, $x = \beta\delta - R$, in the stationary equilibrium, so again, using the Implicit Function theorem:

$$\frac{\partial R}{\partial \alpha} = \beta\frac{\partial \delta}{\partial \alpha} - \frac{\partial x}{\partial \alpha} \tag{33}$$

Using equation (32) above, we get:

$$\frac{\partial R}{\partial \alpha} = \beta\frac{\partial \delta}{\partial \alpha} - \frac{1}{P}\frac{\partial \delta}{\partial \alpha} = \frac{\beta P - 1}{P}\frac{\partial \delta}{\partial \alpha} > 0. \tag{34}$$

Consider the first order condition for the borrower equation (7): By the Implicit Function theorem, we have that

$$\frac{\partial e}{\partial x} = -\frac{1}{(Y_H + x)^2}\frac{1}{P''} > 0 \qquad (35)$$

Also by assumption $P' > 0$ so $\frac{\partial P}{\partial \alpha} = P'\frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha} < 0$. Hence, when the constraint (9) is binding then as α increases, R increases and P decreases.

Finally, $\frac{\partial e^*}{\partial \alpha} = \frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha} < 0$.

∎