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Abstract

In this paper, we take an analytical approach to examine possible adverse effects of the use of inflation targeting as a disinflation regime. The idea is that a strict interpretation of an inflation target may preserve inflationary distortions after price stability is attained. We show that such a policy not only creates a slump in output but may increase macroeconomic volatility substantially in a model in which wages are subject to a Taylor staggering structure.

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1 Introduction

Inflation targeting has become increasingly popular as a monetary policy regime. A distinctive feature of many emerging market economies among the inflation targeters regards the level of the inflation rate exhibited at the time of adoption of the policy. Industrial countries have adopted inflation targeting at inflation rates at least broadly consistent with price stability. In contrast, emerging market economies such as Mexico, Hungary or Poland exhibited initial inflation rates of around 10 percent or more.[1] This study is concerned with the macroeconomic effects of the explicit use of inflation targeting as a disinflation regime. The argument we make is that a strict inflation targeting policy employed for the purpose of disinflation may preserve inflationary distortions to an unusually large degree. In our model, these excess distortions not only cause slumps in real activity, but may additionally increase macroeconomic volatility.

An extensive literature has analyzed inflation targeting with regard to its properties as a monetary policy regime under price stability. Its advantages have been documented by, among others, Bernanke and Mishkin (1997), Svensson (1997) and Svensson and Woodford (2003). In an early contribution, Svensson (2000) differentiates between flexible and strict inflation targeting and shows that the former creates substantially less output variability than a strict interpretation of the policy, as it effectively targets inflation at a longer horizon. The present paper somewhat reinforces this result by showing that the suboptimality of strict inflation targeting may be even more severe in the context of a disinflation episode. Yun (2005) applies a similar line of reasoning as we do in this paper. He shows that the zero inflation optimality result (Woodford, 1999; Gali, 2000) must be refined in the presence of initial price dispersion.[2] The reason is that the pre-existing price dispersion adversely affects real activity in the economy and converges faster under alternative policies.[3]

The literature on the real effects of disinflations has concentrated on explaining stylized facts regarding the differential real effects of money- and exchange-rate-based disinflations. Among others, Ball (1994) and Ascari and Rankin

[1] See Landerretche (2001), Schmidt-Hebbel and Tapia (2002), Levin et al (2004), Roger and Stone (2005) and Batini et al (2006)

[2] Rotemberg and Woodford (1997) show that in a general class of sticky price dynamic general equilibrium (DSGE) models, consumer welfare can be well approximated by a quadratic loss function in inflation and real activity. It can be shown that welfare losses are then proportional to a discounted sum of squared deviations of the current inflation rate from a moving average of recent past inflation rates, rather than deviations from zero (Sheedy, 2005). Giannoni and Woodford (2005) conclude that inflation should not be reduced too abruptly if it has been allowed to exceed its optimal long-run level.

[3] The stabilization policy examined in Yun (2005) does not increase macroeconomic volatility. The reason is that the author uses Calvo (1983) price contracts. We elaborate on this below.

(2002) consider explanations for the finding that money-based disinflations typically cause slumps in output on impact. Exchange-rate-based policies, on the other hand, are frequently characterized by initial booms in real activity. Calvo and Vegh (1994) replicate this empirical finding by assuming that a collapse of the disinflation policy is rationally anticipated. Fender and Rankin (2008) explain the boom with the endogeneity of money supply and its ability to satisfy the excess demand for money balances following a disinflation.[4] The long run impact of disinflation policies in the framework of the New Keynesian model is discussed in Blanchard and Gali (2007) and Ascari and Merkl (2007). The present study is motivated by the idea that inflation targeting differs from other disinflation policies in important respects. In particular, a strict interpretation of an inflation target allows the policymaker to tolerate only minor deviations from target. But adjusting the policy instrument such that the inflation rate is reduced to a new target and defending this target rigorously must preserve inflationary distortions such as wage and price differentials to an exceptional degree.

In this paper, we use a Dynamic General Equilibrium Model with wage staggering of the type suggested by Taylor (1979a) to consider a rather extreme case of a disinflation exercise: an immediate and permanent reduction in the rate of CPI inflation to a newly set target. We interpret this policy as strict inflation targeting during a disinflation episode. The central bank sets the path of money supply such that the newly set inflation target is attained immediately and sustained throughout future periods. The particular nature of the policy requires us to solve the model in a rather unconventional way. We first impose the result of the disinflation policy, a reduction to a lower rate of CPI inflation, and then solve for the policy itself, i.e. the path that money supply has to follow in order to sustain the new inflation target throughout the future.

We consider both a closed and an open economy version of the model economy. As briefly mentioned above, we find that the disinflation policy we consider not only creates a slump in output on impact, but can additionally generate oscillatory behavior in both nominal and real variables along their post-disinflation adjustment path. The reason is that the immediate reduction in price inflation requires the real wage to fluctuate for some periods before it gradually converges to its new steady state. The oscillations can be permanent, such that the post-disinflation path of the economy is not saddlepath stable, when the economy is closed and the returns to labor in the production function are constant. From a modeling perspective, the presence of oscillations along the adjustment path is

[4]Kolver-Hernandez (2007) achieves a similar result by introducing elements of state-dependent pricing into the model economy.

a result that would not obtain in a model with a Calvo (1983) type staggering structure. We show that this is because price setters in such a model would always set prices equal to the prevailing price level. In the framework of our model, we distinguish two cases: one in which the economy is indeed characterized by oscillatory behavior along its post-disinflation path and one in which it reacts similarly to a conventional disinflation policy. Strikingly, the size of the initial slowdown in real activity is strongly positively related to the presence of oscillations along the adjustment path.

The presence and the magnitude of oscillations along the economy's adjustment path depend on the degree to which wages need to fluctuate in order to keep the inflation rate at the newly set target. Important determinants are thus the desired size of the reduction in the inflation rate as well as the returns to labor in the production function and the degree of openness of the economy. Greater returns to scale generate oscillations as they imply that wages are tied more closely to the behavior of prices. The degree of openness matters because the exchange rate acts as a stabilizer along the post-disinflation path of the economy by effectively relieving wages of part of the burden of reducing the inflation rate. The more open is the economy, the greater is the share of the burden it can successfully manage. In particular, we find that there are no oscillations at all along the post-disinflation path of the economy if the returns to labor and the degree of openness are sufficiently low. At the other extreme, in the case of constant returns to scale and a closed economy, the oscillations are large and permanent. The latter is thus the only case in which the economy does not gradually converge to a new steady state. When we investigate the policy's effects in the framework of the full nonlinear model instead of in a log-linear version of it, another particularity of this special case emerges. It turns out that the policymaker faces a surprisingly strict feasibility constraint that does not allow for the policy to be carried out in virtually any case of empirical relevance.

The rest of the paper is organized as follows. Section 2 examines the implications of our disinflation policy in a closed economy setting and highlights the degree of returns to scale as a decisive factor in determining the magnitude of the oscillations in the post-disinflation state. In Section 3, we extend the analysis to an open economy setting and discuss the role of the exchange rate as a stabilizer as well as the importance of the degree of openness of the economy. The section closes with an analysis of the disinflation policy's impact on the economy in the framework of the full nonlinear version of the model. Section 4 concludes.

2 Disinflation via Inflation Targeting in the Closed Economy

2.1 The Model Economy

In this section, we employ a closed economy Dynamic General Equilibrium Model with imperfect competition in the labor market and nominal wage rigidities of the type proposed by Taylor (1979). The structure of the model is kept simple which allows us to illustrate our main points in a clean way and to derive crucial results analytically. We omit derivations where they are standard in the literature.

The model economy is inhabited by a continuum of households $j\epsilon[0,1]$ and firms. The supply side of the economy produces a single consumption good using a technology in which labor is the only variable factor of production

$$Y_t = N_t^{\sigma} \tag{2.1}$$

where Y_t is output at time t, σ is the degree of returns to labor and $0 < \sigma \leq 1$. A typical firm demands a continuum of labor types $j\epsilon[0,1]$ to minimize the cost of achieving a particular composite labor input N_t, given by $N_t = [\int_0^1 L_{jt}^{(\varepsilon-1)/\varepsilon} dj]^{\varepsilon/(\varepsilon-1)}$. L_{jt} is the quantity of labor that household j supplies to the firm and $\varepsilon > 1$ is the elasticity of technical substitution across labor types. Solving the cost minimization problem yields the standard conditional demand for labor function

$$L_{jt} = N_t(\frac{W_t}{W_{jt}})^{\varepsilon} \tag{2.2}$$

where W_t is the wage index given by $W_t = [\int_0^1 W_{jt}^{1-\varepsilon} dj]^{1/(1-\varepsilon)}$. Goods markets are perfectly competitive. The firm's profit maximization problem thus yields the supply function

$$Y_t = N_t(\frac{W_t}{\sigma P_t})^{\sigma/(\sigma-1)} \tag{2.3}$$

We now move to the demand side of the economy. Household j supplies labor skill $j\epsilon[0,1]$ and sets its own wage W_{jt}. We assume that the economy consists of two sectors of households. Sector A comprises labor types $[0,0.5)$ and sector B comprises labor types $[0.5,1]$. Although households are monopolistic suppliers of their individual type of labor input, they are price takers in all other markets. We assume that they are completely symmetric in terms of their preference structure which implies that consumption must be equal across households in a given sector at any point in time. We also assume the existence of complete domestic asset markets. This implies that households can insure against any type of initial shock that might affect the two sectors differently due to the staggering structure to be defined below. Hence, $C_{jt} = C_{kt}$ must hold for any two given households j and k in sectors A and B. Finally, we define aggregate nominal consumption as $S_t = P_t C_t$ where P_t is the price of one unit of the composite consumption good.

Wages are set by each individual household subject to a staggering structure of the type proposed by Taylor (1979). In particular, we assume that households in sector A (B) set their wage in even (odd) periods and keep it fixed for the subsequent period. The wage newly set in period t is denoted by X_t independently of the sector in which it is set. Households are utility maximizers. A representative household j in sector A derives utility from consumption, liquidity holdings and leisure and maximizes her discounted lifetime utility U_j by choosing a pattern for personal consumption C_{jt}, bond holdings B_{jt}, wages W_{jt} and labor effort N_{jt} subject to a series of budget constraints, the conditional demand for labor and the wage setting constraint:

$$U_j = \sum_{t=0}^{\infty} \beta^t [\delta ln C_{jt} + (1-\delta) ln(M_{jt}/P_t) - \eta L_{jt}^{\zeta}] \tag{2.4}$$

subject to

$$M_{jt-1} + I_{t-1} B_{jt-1} + W_{jt} L_{jt} + \Pi_t + G_t = P_t C_{jt} + M_{jt} + B_{jt} \tag{2.5}$$

$$L_{jt} = N_t \left(\frac{W_t}{W_{jt}}\right)^{\varepsilon} \tag{2.6}$$

$$W_{jt} = W_{jt+1} = X_t, \quad t = 0, 2, 4, ... \tag{2.7}$$

where only the first two constraints must hold in all periods $t = 0, 1, 2, 3, ...$ and where $\beta < 1$, $\zeta \geq 1$, I_t is the domestic gross interest rate, M_t denotes money supply in period t, G_t is a lump-sum subsidy to households and Π_t denotes a share in firms' profits that is equal across households.[5] The optimization problem for household k in sector B is exactly equivalent except that the wage setting constraint holds in odd instead of even periods. The first order conditions of this optimization problem are given by the consumption Euler equation, the money demand optimality condition and the optimal wage setting condition

[5] A no-Ponzi-game condition ensures that individuals cannot borrow infinitely by repaying debt with further debt.

$$C_{jt+1} = \beta[I_t \frac{P_t}{P_{t+1}}]C_{jt} \quad (2.8)$$

$$\frac{M_{jt}}{P_t} = C_{jt}\frac{1-\delta}{\delta}\frac{I_t}{I_t - 1} \quad (2.9)$$

$$X_t = [\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}\frac{L_{jt}^{\zeta} + \beta L_{jt+1}^{\zeta}}{\frac{L_{jt}}{P_t C_{jt}} + \beta\frac{L_{jt+1}}{P_{t+1}C_{jt+1}}}] \quad (2.10)$$

The model economy further comprises a government that controls money supply. Its budget constraint is given by

$$G_t = M_t - M_{t-1} \quad (2.11)$$

The competitive equilibrium in the model economy is the sequence of prices $[X_{jt}, P_t]_{t=0}^{\infty}$ and allocations $[Y_t, C_{jt}, N_{jt}, L_{jt}, B_{jt}, M_{jt}]_{t=0}^{\infty}$ such that firms maximize profits, agents maximize utility and all markets clear.[6] We aggregate the equilibrium conditions across individuals and take a log-linear approximation around a reference steady state in which inflation is zero. Notice that in what follows we will distinguish between the reference zero inflation steady state (ZISS) around which the equilibrium conditions are linearized and a constant inflation steady state (CISS) in which the economy finds itself one period before the disinflation policy is applied. As the economy is closed, bond holdings are zero in both states.[7] The linearized equilibrium conditions are presented in Appendix A.[8]

2.2 Strict Inflation Targeting as a Disinflation Policy

We now proceed to assess the macroeconomic effects of a disinflation policy that uses strict inflation targeting to reduce the rate of CPI inflation from a positive to a non-negative value. We define strict inflation targeting following Svensson (2000). The definition implies that the policymaker only cares about stabilizing inflation at a given target. We assume that she has perfect control over the inflation rate and adjusts her policy instrument such that inflation is kept at target at any point in time. The economy is initially in a constant inflation steady state (CISS) in which real variables are constant and all nominal variables grow at some constant inflation rate μ_I. The policymaker decides

[6] Due to the staggering structure in wages the labor market does not clear in the Walrasian sense.

[7] While this must necessarily be the case in a closed economy setting, we will impose it upon the open economy by assumption in the next section

[8] The interested reader may refer to Fender and Rankin (2008) for the linearization of the wage setting condition.

to reduce inflation in period $t = 0$ from its initial rate μ_I to the lower but nonnegative value μ_D. This policy change is unexpected and credible. The policymaker takes action by announcing μ_D as the new target and adjusts the path of money supply such that the new target is attained immediately and sustained throughout future periods.[9]

There is no doubt that this disinflation policy is more rigid than policies applied in the real world. However, we present this extreme example as a benchmark case that allows for an analytical characterization of possible adverse effects of applying a too rigid inflation targeting policy for the purpose of disinflation. The definition of the policy requires us to solve the model in a rather unconventional way. We first impose the result of the disinflation policy, a reduction in the rate of inflation from μ_I to μ_D, and then solve for the path that the money supply needs to follow in order to achieve and sustain the newly set inflation target. Finally, we assess the macroeconomic effects of the disinflation policy.

2.2.1 The Initial Constant Inflation Steady State

Before we can investigate the macroeconomic impact of the disinflation policy, we need to solve for the equilibrium solution of the model in the initial CISS. We use the fact that the new wage x_t is homogeneous of degree one in nominal variables and, when normalized by money supply such that $v_t = x_t - m_t$, is constant over time at

$$v = -\frac{1}{2\gamma}\big[-\gamma\frac{1+\beta}{1-\beta} + \frac{1-\beta}{1+\beta}\big]\mu_I \tag{2.12}$$

with $\gamma = \frac{\zeta}{1+\varepsilon(\zeta-1)}$. The CISS value of nontradables output is then given by

$$y = \frac{\sigma(1-\beta)}{2(1+\beta)\gamma}\mu_I \tag{2.13}$$

We observe that nontradables output is affected positively by the inflation rate in the CISS. The effect is due to wage setters discounting future utility and is more closely discussed in Appendix C in the context of the full nonlinear model. In the CISS, all nominal variables grow at the rate μ_I. Defining m_{-1} as the level of money supply one period before the disinflation policy is applied, we can write the wage index in the same period as

$$w_{-1} = -\frac{1}{2}\big[\frac{-2\beta}{1-\beta} + \frac{1-\beta}{(1+\beta)\gamma}\big]\mu_I + m_{-1} \tag{2.14}$$

[9] There is a unique path of money supply, the exchange rate and the nominal interest rate that achieves this outcome. It is therefore not of importance, whether we think of the central bank as using one or the other as its policy instrument.

Equations (2.13) and (2.14) constitute the CISS solution of the model one period before disinflation.

The next step in the analysis is to understand, how changes in the rate of price inflation affect the rest of the economy and in particular the staggered variable, wages. The supply function (A.5) allows us to write the price index as a function of labor demand and the wage index

$$p_t = \frac{\sigma}{1-\sigma} y_t + w_t \tag{2.15}$$

The equation illustrates that a given change in the price level is accommodated partly by a reaction of wages and partly by firms adjusting production. The wage level's share in facilitating the price change depends positively on the degree of returns to labor. The reason is that a larger σ implies a less strongly upward sloping marginal cost curve and thus a weaker response of prices to changes in the level of production in the economy. When $\sigma = 1$, the marginal cost curve is constant, leaving no role for output in the determination of prices. This implies that the wage level must move one for one with the price index. In the context of our disinflation policy the latter case implies that a reduction in the rate of price inflation translates one for one into a reduction in the growth rate of the wage index. The subsequent section will show that this particularity allows for a purely analytic characterization of the disinflation policy's impact on the economy.

2.2.2 The Special Case: Constant Returns to Scale

We begin by solving for the impact of the disinflation policy in the special case of constant returns to scale. We do so because the distinctive implications of this assumption allow for a good understanding of the channels through which the policy affects the economy.

As a starting point, it is important to notice that there is a crucial implication arising from the assumption of constant returns to scale in the production function. In particular, equation (A.5) shows that this is the one and only case for which it is not possible to solve for output when the paths of both the wage index and the price level are known. The reason is that the aggregate supply curve, i.e. the relationship between output and the price level, is horizontal for a given wage. This implies that conventional solution techniques cannot be applied to determine the full post-disinflation solution of the model, a point that will perhaps become more obvious when we examine the general case of $\sigma < 1$ in the next section.

There is, however, another anomaly about the case of constant returns to scale. In the previous subsection we have discussed that $\sigma = 1$ implies that the wage index moves one for one with the price index and a policy of reducing CPI inflation effectively becomes one of reducing the growth rate of the wage index. Using this fact, it turns out to be possible to directly solve for the post-disinflation path of the real side of the economy by distinguishing the economy's law of motion in even periods from the law of motion in odd periods. To see this, remember that our disinflation policy reduces CPI inflation from its initial rate μ_I to a lower but non-negative rate μ_D. Given that the wage index moves one for one with prices, it is easy to see what such a policy implies for the behavior of sectoral wages. We take the wage index and its one period lag. We evaluate the difference at $t = 0$. This yields

$$w_0 - w_{-1} = \frac{1}{2}(x_0 - x_{-1}) + \frac{1}{2}(x_{-1} - x_{-2}) \tag{2.16}$$

We have assumed that the economy is in a CISS up until period $t = 0$. It follows that the second term on the RHS is predetermined and equal to $\frac{1}{2}\mu_I$. Reducing inflation to μ_D in period $t = 0$ then implies that the LHS of (2.16) is equal to μ_D. Hence,

$$x_0 - x_{-1} = 2\mu_D - \mu_I \tag{2.17}$$

Following the equivalent procedure for the subsequent periods, we have

$$x_t - x_{t-1} = 2\mu_D - \mu_I \tag{2.18}$$

for all $t = 0, 2, 4, \ldots, \infty$, and

$$x_t - x_{t-1} = \mu_I \tag{2.19}$$

for all $t = 1, 3, 5, \ldots, \infty$. In other words, wages grow at the constant rate $2\mu_D$ from one even period to the other and from one odd period to the other. But, since $\mu_D < \mu_I$ the new wage grows faster from even to odd periods than from odd to even periods. Three cases are possible. First, if $2\mu_D < \mu_I$, the new wage set in even periods is lower than the wage set in the previous odd period. Wages in sector A are thus smaller than wages in sector B throughout the post-disinflation state. Second, if $2\mu_D > \mu_I$, the new wage set in even periods is higher than the wage set in the previous odd period. Third, if $2\mu_D = \mu_I$, the new wage set in even periods is equal to the prevailing one from the previous odd period.

Figure 1 illustrates these three cases. The special case of $\mu_D = 0$ is represented by Case 1b. It represents a complete disinflation, i.e. a reduction in the rate of inflation from μ_I to zero. In particular, $\mu_D = 0$ implies that

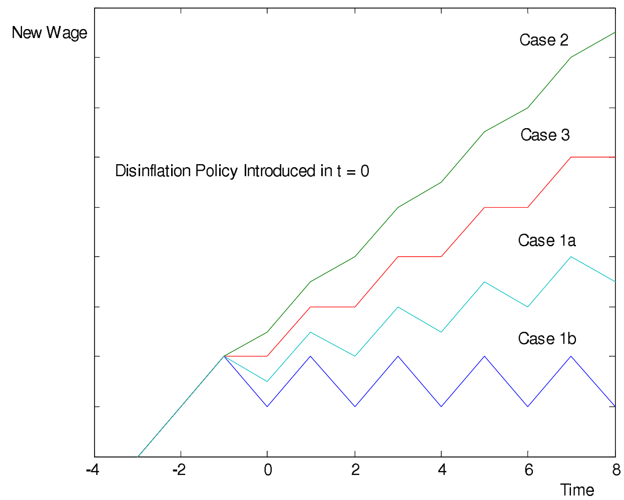

Figure 1: Post-Disinflation Path of the New Wage

$$x_{-2} = x_0 = x_2 = ... = w_{-1} - \frac{1}{2}\mu_I \tag{2.20}$$

$$x_{-1} = x_1 = x_3 = ... = w_{-1} + \frac{1}{2}\mu_I \tag{2.21}$$

That is to say that the disinflation policy produces a wage gap between the two sectors of households that is constant and permanent. The intuition is the following: we impose zero wage inflation from period zero onwards. The wage set in period $t = -2$ is smaller than the one set in period $t = -1$. This implies that the wage index requires downward pressure by the new wage set in period zero to ensure zero wage inflation. Due to the fact that the wage index is an equally weighted average of the two sectoral wages, the necessary amount of downward pressure is achieved if the wage in period $t = 0$ is set equal to wage set in period $t = -2$. The opposite reasoning applies in the subsequent period.

Let us get back to the more general case of a reduction in the rate of wage inflation from a constant value μ_I to a lower positive value μ_D. Having determined the wage setting behavior of households after the disinflation policy is introduced, we can now derive the entire post disinflation state of the economy. We substitute for both the wage index and employment in the wage setting condition (A.21) and obtain

$$x_t = \frac{1}{1+\beta}\left[s_t \frac{2\gamma}{1+\gamma} + x_{t-1}\frac{1-\gamma}{1+\gamma}\right] + \frac{\beta}{1+\beta}\left[s_{t+1}\frac{2\gamma}{1+\gamma} + x_{t+1}\frac{1-\gamma}{1+\gamma}\right] \tag{2.22}$$

As we are in effect imposing different wages for even and odd periods exogenously, we will have to look at every period separately. Using the wage setting condition (A.21) and substituting from equations (2.20) and (2.21), we have that

$$s_t + \frac{1}{\beta}s_{t-1} = \frac{1+\beta}{\beta}(w_{-1} + \frac{1}{2}\mu_I + t\mu_D) - \frac{1-\gamma}{2\gamma}(2\mu_D - \frac{1+\beta}{\beta}\mu_I) \quad (2.23)$$

$$s_t + \frac{1}{\beta}s_{t-1} = \frac{1+\beta}{\beta}(w_{-1} - \frac{1}{2}\mu_I + (t+1)\mu_D) - \frac{1-\gamma}{2\gamma}(\frac{1+\beta}{\beta}\mu_I - \frac{1}{\beta}2\mu_D) \quad (2.24)$$

in odd and even periods respectively. These relationships hold from period zero onwards. A similar relationship between s_0 and s_{-1} would be convenient as it would uniquely determine the post disinflation path of s_t. However, the equivalent procedure would give us a relationship between s_{-1} and the expectation of s_0 as of period $t = -1$. And this expectation is false as soon as the disinflation policy is introduced. The reason is that, in period $t = -1$, the wage setter does, by assumption, not know that the disinflation policy will be applied. Nominal consumption in the subsequent period then turns out to be different from his expectation as of period $t = -1$. But we can get around this problem by appealing to stability reasoning. From (2.23) and (2.24), we have that in odd and even periods respectively

$$s_{t+2} - \frac{1}{\beta^2}s_t = \mu_D\frac{\beta^2-1}{\beta^2}t + \frac{\beta^2-1}{\beta^2}w_{-1} - \mu_I(\frac{1+\beta}{\beta})^2\frac{1}{2\gamma} + \mu_D(3 + \frac{2\beta-\gamma}{\beta^2\gamma}) \quad (2.25)$$

$$s_{t+2} - \frac{1}{\beta^2}s_t = \mu_D\frac{\beta^2-1}{\beta^2}t + \frac{\beta^2-1}{\beta^2}w_{-1} + \mu_I(\frac{1+\beta}{\beta})^2\frac{1}{2\gamma} + \mu_D(3 - \frac{\gamma+1+\beta^2}{\beta^2\gamma}) \quad (2.26)$$

Equations (2.25) and (2.26) are the basic laws of motion of the economy in the post disinflation state. In order to arrive at a stationary solution, we deflate both difference equations by the wage level w_t. We create the new variable a_t defined as the ratio $a_t = s_t - w_t$. The resulting first order difference equations in a_t take the form

$$a_{t+2} = \frac{1}{\beta^2}a_t + constant \quad (2.27)$$

Since $\beta < 1$, the Eigenvalue of these equations is greater than one and unstable. This implies that there is a unique non divergent solution for each of these processes as Blanchard and Kahn (1980) show. This non divergent solution obtains when a_t is equal across odd and across even periods. The economy instantly jumps to its post disinflation state and remains there. Using the goods supply and demand functions, we find

$$y_{N_{odd}} = \frac{(1+\beta)}{2(1-\beta)\gamma}\mu_I - \frac{2\beta}{(1-\beta^2)\gamma}\mu_D \tag{2.28}$$

$$y_{N_{even}} = -\frac{(1+\beta)}{2(1-\beta)\gamma}\mu_I + \frac{(1+\beta^2)}{(1-\beta^2)\gamma}\mu_D \tag{2.29}$$

As is evident from equations (2.28) and (2.29), the disinflation policy results in output oscillations between even and odd periods. The amplitude of these oscillations is constant. Moreover, we can observe that the extent to which nominal and real variables fluctuate depends positively on the initial inflation rate and negatively on the post-disinflation rate. The reason is that the inflation rate determines the degree of wage dispersion present in the initial CISS. This suggests that a more gradual path of disinflation may attenuate the increase in output volatility resulting from the disinflation policy. The intuition for these findings becomes clear when the post disinflation path of money supply is derived. We have explained above that the policy instrument is endogenous in our solution procedure although it is exogenous in the interpretation of the policy experiment. Notice first that nominal consumption can be expressed as

$$s_t = y_t + w_t \tag{2.30}$$

Given the path of output documented in (2.28) and (2.29), and given that the wage index grows at the rate μ_D from period to period, we observe that post disinflation nominal consumption also grows at a lower rate in even periods than it does in odd periods. The post-disinflation path of money supply is given as a function of the path of nominal consumption

$$m_t = \frac{\beta}{1-\beta}(\frac{1}{\beta}s_t - s_{t+1}) \tag{2.31}$$

It is easy to see that money supply must follow a pattern that is qualitatively similar to the path of wages and output. The intuition is the following: the disinflation policy effectively imposes a path for the new wage that requires agents to set low wages in even periods and high wages in odd periods. In order to motivate this wage setting behavior, money supply must be set low in even and high in odd periods. This also explains the oscillatory behavior of output. Finally, we can see that average output in the post-disinflation state is lower than output in the initial CISS.[10] This is to say that the disinflation policy creates a slump in production.[11]

2.2.3 The General Case: Decreasing Returns to Scale

We now abstract from the assumption of constant returns to scale and return to the more general case of $\sigma < 1$. The analysis in the previous section has shown that the disinflation policy can result in oscillations in sectoral real wages that are permanent and constant over time. In this section, we show that this result is not robust to relaxing the assumption of $\sigma = 1$. In particular, we show that in the more general case the economy is indeed saddlepath stable and ultimately converges to a new steady state. The transition path may or may not be subject to oscillations, depending on the particular value of σ.

We investigate the effects of the disinflation policy in a different way than in the previous section. The reason is that it is now possible to derive the entire post-disinflation path of the economy by determining the response of the staggering variable wages to the reduction in price inflation. We again consider a reduction in the rate of price inflation from μ_I to μ_D in period zero. In particular, taking equation (2.22) and substituting for nominal consumption s_t using the definition (A.2) and the supply function (A.5), we can express the new wage in period t solely as a function of past and future wages as well as the price index

$$\begin{aligned} x_t = & \frac{1}{1+\beta}[\frac{2\gamma}{1+\gamma}(\frac{1}{1-\sigma}p_t - \frac{1}{2}\frac{\sigma}{1-\sigma}(x_t + x_{t-1})) + \frac{1-\gamma}{1+\gamma}x_{t-1}] \\ & + \frac{\beta}{1+\beta}[\frac{2\gamma}{1+\gamma}(\frac{1}{1-\sigma}p_{t+1} - \frac{1}{2}\frac{\sigma}{1-\sigma}(x_{t+1} + x_t)) + \frac{1-\gamma}{1+\gamma}x_{t+1}] \end{aligned} \quad (2.32)$$

In the post-disinflation state p_t and $p_{t+1} = p_t + \mu_D$ are exogenous and predetermined by definition of the disinflation policy. Therefore, the households wage setting condition can, from period $t = 0$ onwards, be expressed as

$$x_{t+1} - \frac{1+\beta}{\beta}\frac{1-\sigma+\gamma}{1-\sigma-\gamma}x_t + \frac{1}{\beta}x_{t-1} = -\frac{1+\beta}{\beta}\frac{2\gamma}{1-\sigma-\gamma}(p_t + \frac{\beta}{1+\beta}\mu_D) \quad (2.33)$$

[10]But output is greater or equal than in the ZISS. The reason is that the discounting effect of inflation on output is not only present in the initial CISS but also, albeit attenuated, in the post-disinflation state.

[11]It is typically found that money based disinflations cause a slump in output on impact, while exchange rate based disinflations can cause a boom. Explanations for these empirical findings can be found in Calvo and Vegh (1994), Rebelo and Vegh (1995), Fender and Rankin (2006) or Kolver Hernandez (2007).

This is the law of motion of the economy in the post-disinflation steady state. Notice that p_t is growing over time so that we need to stationarize the equation before we can solve it. We do so by defining $\phi_t = x_t - p_t$ and obtain

$$\phi_{t+1} - \frac{1+\beta}{\beta}\frac{1-\sigma+\gamma}{1-\sigma-\gamma}\phi_t + \frac{1}{\beta}\phi_{t-1} = \mu_D(\frac{1-\beta}{\beta} - \frac{2\gamma}{1-\sigma-\gamma}) \tag{2.34}$$

As we show in Appendix B.1, (2.34) is saddlepoint stable when $\sigma < 1$, i.e. one of its eigenvalues lies within and the other outside the unit circle. This result holds for any choice of parameter values within the defined limits.[12] We use the eigenvalue-eigenvector solution technique of Blanchard and Kahn (1980) to solve for the post-disinflation path of ϕ_t. As (2.34) is a scalar second order difference equation and its RHS is constant over time, its rational expectations solution is given by

$$\phi_t = \lambda_1\phi_{t-1} - \mu_D\frac{(1-\beta)/\beta - 2\gamma/(1-\alpha\sigma-\gamma)}{\lambda_2 - 1} \tag{2.35}$$

where λ_1 denotes the eigenvalue that is smaller in absolute value and λ_2 denotes the one that is bigger. Knowing both the path of the price index and the normalized sectoral wages ϕ_t, it is straightforward to derive the entire post-disinflation path of the economy. In reference to the previous section, notice that this would not have been possible in the special case of constant returns to scale in the production function. The reason is that the aggregate supply curve in this case is horizontal for a given wage level, implying that the post-disinflation path of the real economy could not be derived from knowledge about the path of prices and wages.

Whether the path of the economy is subject to oscillations as in the case of wage inflation targeting now crucially depends on the sign of the smaller eigenvalue. Appendix B.2 shows that the smaller eigenvalue is of negative sign, and thus induces oscillations, if and only if $1 - \sigma - \gamma < 0$ holds, where $\gamma = \frac{\zeta}{1+\varepsilon(\zeta-1)}$. This is to say that the post-disinflation path of the economy exhibits oscillations for a sufficiently large σ and sufficiently small ζ and ε.

We study the impact of the disinflation policy in both situations using the example of an initial inflation rate of two percent and a complete elimination in price inflation after the policy is applied. We first investigate the case in which $1 - \sigma - \gamma < 0$ holds.[13] Figure 2 illustrates the resulting path of the economy. As in the case of wage inflation targeting, we observe strong oscillations in both nominal and real variables after the disinflation policy is applied. The new wage

[12]When $\sigma = 1$, this is not the case and the economy is not saddlepath stable.

[13]We choose a rather extreme parametrization with $\sigma = 0.9$, $\zeta = 1.3$ and $\varepsilon = 4$.

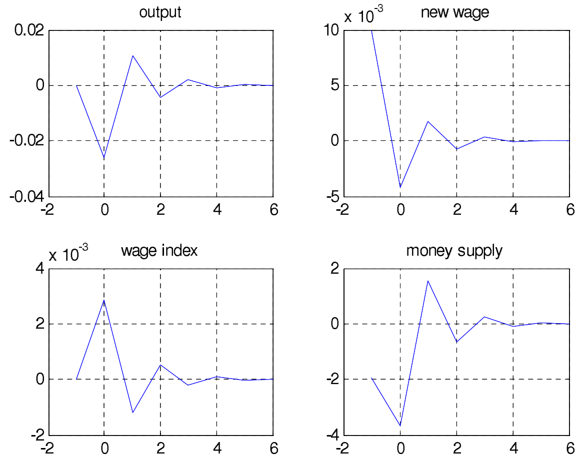

Figure 2: Price Inflation Targeting with Oscillations

falls on impact, oscillates for a few periods, and then gradually converges to its ZISS value. Money supply and output behave similarly. Moreover, average output from period zero onwards is substantially lower than prior to the application of the policy. The policy therefore not only increases macroeconomic volatility but also creates a fall in output. In the impact period of the policy, there is a drop of more than 2 percent in real activity.

These results are somewhat similar to the previously discussed special case of constant returns to scale in the production function. However, the crucial difference is that the economy is indeed saddlepath stable and converges over time after some periods of increased volatility. The reason is related to the assumption of decreasing returns to labor. As discussed previously, firms now face upward sloping marginal cost curves, which relieves wages of part of the burden of responding to desired changes in the price level or its growth rate. And the weaker is the response of wages in the impact period of the shock, the weaker will be the response in the subsequent period. Hence, the magnitude of the fluctuations in the economy decreases over time. The smaller is σ, the more rapidly this process takes place. The attenuating effect is strengthened the greater is the elasticity of substitution between labor types ε and the greater is the elasticity of the disutility of labor ζ. [14]

Let us now focus on the case of $1-\sigma-\gamma > 0$ in which σ is small enough and ε

[14] Regarding ε, the reason is that a higher substitutability of labor types implies that a given firm will employ more labor of the low wage type in period $t = 0$. This reduces the wage index and implies that the new wage set in period $t = 0$ does not have to be as low as would have been the case otherwise. A higher disutility per unit of work effort, on the other hand, scales wages upward and thus also scales the wage gap between one sector and the other.

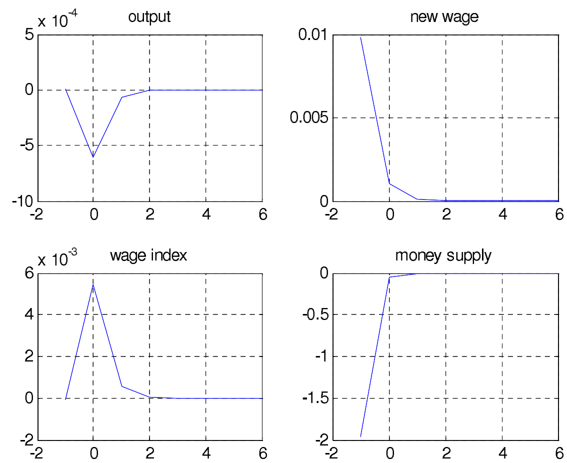

Figure 3: Price Inflation Targeting without Oscillations

and ζ are large enough such that there are no oscillations in the post-disinflation state. We choose $\sigma = 0.1$ and leave all remaining parameter values the same as in the previous case. Figure 3 shows that there are now indeed no oscillations in the post-disinflation path of the economy. All variables monotonically converge to their ZISS levels. In fact, the post-disinflation path of output reminds us of the one that is typically attained under a conventional disinflation policy in a model with a staggering structure in wages or prices.

To sum up, the analysis has shown that a strict inflation targeting policy employed to reduce CPI inflation will result in a slump in output on impact and may create oscillations of a substantial magnitude in both nominal and real variables. The presence of oscillations in the post-disinflation path of the economy crucially depends on the degree of returns to labor in the production function. For a large enough σ the disinflation policy will result in oscillatory behavior of both real and nominal variables. These oscillations die out more slowly over time the larger is σ and are permanent in the limiting case of $\sigma = 1$. Finally, notice that the slump in output following the disinflation policy is tiny when there are no post-disinflation oscillations in the economy and substantial if there are.

2.2.4 Digression: Calvo Staggering Structure

In this section we briefly show that the post-disinflation oscillations in response to our disinflation policy would not obtain in a model with a staggering structure as proposed by Calvo (1983). This is the reason why the policy experiment of Yun (2005) does not lead to an equivalent conclusion.

Following Yun and much of the literature, let us consider the case of price inflation targeting in an economy in which prices are subject to a Calvo-type staggering structure. We can write the price index in the absence of indexation as

$$P_t^{1-\varepsilon} = (1-\alpha)P_{t,t}^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon} \tag{2.36}$$

where P_t is the price index and $P_{t,t}$ is the price that is chosen by the fraction $(1-\alpha)$ of firms that get to adjust their price in period t. Let us assume a policy in which price inflation is reduced to zero once and for all in some period $t = 0$. Computing the ratio of P_t and P_{t-1} and imposing $\frac{P_t}{P_{t-1}} = 1$, we find that $P_{t,t} = P_{t-1}$. Thus, newly set prices in the post-disinflation state of the economy are always equal to the prevailing price level. The reason is that firms choose prices in a forward looking manner and face the identical problem in every period. Since prices gradually converge, there is no source for oscillatory behavior in the economy. The increase in macroeconomic volatility resulting from the disinflation policy applied in our model is thus a result that does not obtain in Calvo-type staggering models.

3 Disinflation via Inflation Targeting in the Open Economy

In this section, we investigate, how our conclusions from the previous section change in an open economy setting. In particular, we show that the exchange rate takes on a prominent role in alleviating wages from part of the burden of reducing the inflation rate. In addition to the returns to scale in the production function, the degree of openness of the economy is identified as a crucial factor determining the magnitude of the oscillations in the post-disinflation state of the economy. Moreover, we show that the nonlinear model delivers a set of additional results that cannot be detected in the framework of the linearized model.

3.1 Opening up the Model Economy

We now assume that the model economy is open in the sense that its agents trade goods and assets with a foreign country. The model economy's basic structure is the same as in the previous section. In opening up the economy we follow the formulation of Fender and Rankin (2008).

We assume that there are now two output sectors, one producing tradable goods and one producing non-tradable goods. Output in the tradables sector Y_{Tt}

is exogenous and normalized to one. The production function for non-tradable goods Y_{Nt} is equivalent to equation (2.1) in the closed economy specification. Markets for both types of goods are perfectly competitive. The foreign currency price of tradables is normalized to unity. Together with the assumption that the law of one price holds, this implies that $P_{Tt} = E_t$, where E_t is the nominal exchange rate, i.e. the domestic price of foreign currency. As regards financial markets, B_t is now an international bond traded between home and foreign agents. The currency of its denomination is immaterial since we assume that there are no initial outstanding bonds and there is no uncertainty after the disinflation policy is applied. A no-arbitrage condition implies interest rate parity

$$I_t = I_t^* \frac{E_{t+1}}{E_t} \tag{3.1}$$

where I_t^* is the foreign gross interest rate. The optimization problems of the individual agents are equivalent to the closed economy case except that households now consume both non-tradable and tradable goods. Their preference structure is revealed by the composite consumption index

$$C_{jt} = C_{Njt}^{\alpha} C_{Tjt}^{1-\alpha} \tag{3.2}$$

where $0 < \alpha < 1$ and C_{Njt} and C_{Tjt} denote household j's consumption of nontradables and tradables respectively. Utility from consumption is maximized subject to a given nominal spending constraint S_{jt} defined by $S_{jt} = P_{Nt}C_{Njt} + P_{Tt}C_{Tjt}$ such that

$$C_{Njt} = \alpha S_t / P_{Nt} \tag{3.3}$$

$$C_{Tjt} = (1-\alpha) S_t / P_{Tt} \tag{3.4}$$

where α is the degree of home bias in consumption. We denote $1 - \alpha$ the degree of openness of the economy. The consumer price index (CPI) is then a weighted average of the price of tradables and non-tradables. In particular,

$$P_t = \frac{P_{Nt}^{\alpha} P_{Tt}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{3.5}$$

The trade balance T_t can be expressed as $T_t = 1 - C_{Tt}$. In principle, the trade balance may be different from zero, indicating a trade surplus or deficit. Over time deficits must be balanced by surpluses and initial net foreign assets. A modified no-Ponzi game condition therefore reads

$$-I_{-1}B_{-1} = \sum_{t=0}^{\infty}[I_0 I_1 ... I_{t-1}]^{-1} P_{Tt} T_t \quad (3.6)$$

However, Fender and Rankin (2008) show that the assumptions of zero initial net foreign assets and the exogeneity of output in the tradables sector together imply that the trade balance is zero at any point in time. This result significantly simplifies the analysis as it allows to abstract from any dynamics introduced by the accumulation of net foreign assets. The log-linear definitions and equilibrium conditions of the open economy model can be found in Appendix A.

3.2 The Role of the Degree of Openness of the Economy

In opening up the model economy we left its basic structure unchanged. This implies that the initial constant inflation steady state (CISS) is the same as in the previous section. We can thus directly proceed to derive the post-disinflation path of the open economy. To fix language, we denote $1-\alpha$ the degree of openness of the economy. The disinflation policy is defined in the exact same way as before and is applied to reduce the rate of price inflation from its initial rate μ_I to the new target μ_D in period $t = 0$. We begin the analysis by noticing that price inflation in period t can be expressed as

$$p_t - p_{t-1} = (1 - \alpha\sigma)(e_t - e_{t-1}) + \alpha\sigma(w_t - w_{t-1}) \quad (3.7)$$

This implies that a reduction in CPI inflation can be the result of a reduction in wage inflation, an exchange rate appreciation or both.[15] It suggests that the post-disinflation path of wages may be subject to oscillations of a smaller magnitude than in the closed economy model if the exchange rate adjusts to alleviate them of part of the burden of reducing the inflation rate. In order to investigate this issue, we proceed as in the previous case and express the wage setting condition as

$$\begin{aligned} x_t = {} & \frac{1}{1+\beta}[\frac{2\gamma}{1+\gamma}(\frac{1}{1-\alpha\sigma}p_t - \frac{1}{2}\frac{\alpha\sigma}{1-\alpha\sigma}(x_t + x_{t-1})) + \frac{1-\gamma}{1+\gamma}x_{t-1}] \\ & + \frac{\beta}{1+\beta}[\frac{2\gamma}{1+\gamma}(\frac{1}{1-\alpha\sigma}p_{t+1} - \frac{1}{2}\frac{\alpha\sigma}{1-\alpha\sigma}(x_{t+1} + x_t)) + \frac{1-\gamma}{1+\gamma}x_{t+1}] \quad (3.8) \end{aligned}$$

Notice that setting $\alpha = 1$, we are back at equation (2.32), the post-disinflation law of motion in the closed economy setting. The subsequent steps are thus equivalent to the analysis in the previous section. We again define $\phi_t = x_t - p_t$.

[15] The reason is that the foreign price of tradables is assumed to be constant.

The law of motion of the open economy in the post-disinflation state is then given by

$$\phi_{t+1} - \frac{1+\beta}{\beta}\frac{1-\alpha\sigma+\gamma}{1-\alpha\sigma-\gamma}\phi_t + \frac{1}{\beta}\phi_{t-1} = \mu_D(\frac{1-\beta}{\beta} - \frac{2\gamma}{1-\alpha\sigma-\gamma}) \tag{3.9}$$

The stability proof in Appendix B.1 shows that (3.9) is saddlepoint stable as one of its eigenvalues lies within and the other outside the unit circle. This result holds for any choice of parameter values within the defined limits. We again use the eigenvalue-eigenvector solution technique of Blanchard and Kahn (1980) to solve the model. As (3.9) is a scalar second order difference equation and its RHS is constant over time, its rational expectations solution is given by

$$\phi_t = \lambda_1\phi_{t-1} - \mu_D\frac{(1-\beta)/\beta - 2\gamma/(1-\alpha\sigma-\gamma)}{\lambda_2 - 1} \tag{3.10}$$

where λ_1 denotes the eigenvalue that is smaller in absolute value and λ_2 denotes the one that is bigger. Appendix B.2 shows that the condition for the presence of oscillations in the post-disinflation state of the economy is now given by $1 - \alpha\sigma - \gamma < 0$. This suggests that an economy that is more closed is more likely to be subject to oscillatory behavior after the disinflation policy is applied. The reason is simply that the exchange rate takes a more significant share in the burden of reducing the rate of CPI inflation, the larger is the share of tradable goods in the consumption bundle and the price index.

Figures 4 and 5 illustrate these results. We have chosen the same parameterizations as in Figures 2 and 3 except that α now takes the value 0.9 in Figure 4 and the value 0.1 in Figure 5. A first look at the graphs shows that the exchange rate behaves in a qualitatively equivalent fashion as the new wage. In Figure 4 it initially appreciates and then follows an oscillatory path until it converges to its ZISS level. In Figure 5 the convergence process is monotonic. The exchange rate appreciates on impact and then gradually depreciates towards its ZISS value. Finally, notice that the oscillations in Figure 4 are of a slightly smaller magnitude than in Figure 2. This is in line with the intuition that the exchange rate contributes to a smoother post-disinflation path of the economy.

In sum, the exchange rate alleviates wages from part of the burden of reducing the inflation rate when the economy is open. The more open is the economy, the larger is the role played by the exchange rate and the smaller is the magnitude of the oscillations in the post-disinflation state of the economy. Even in the case of constant returns to scale in the production function, the economy always converges to a new steady state and oscillations are never permanent.

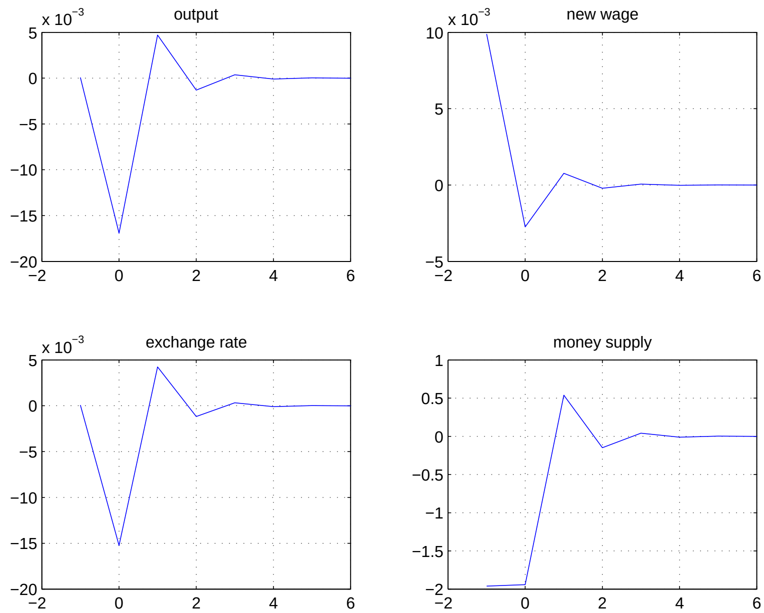

Figure 4: Price Inflation Targeting with Oscillations

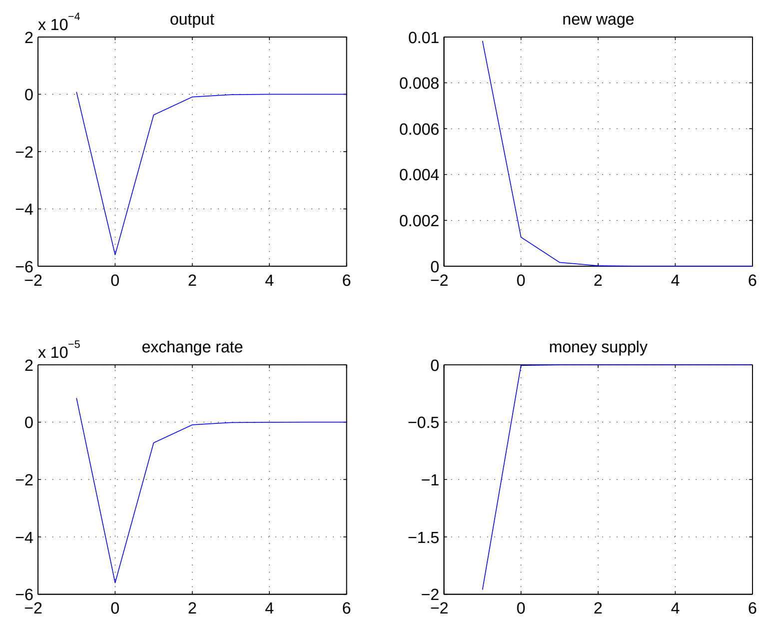

Figure 5: Price Inflation Targeting without Oscillations

3.3 The Nonlinear Open Economy

In this subsection, we investigate the impact of the disinflation policy on the open economy when the model is not linearized. We do so in order to illustrate that the log-linear model hides interesting findings by eliminating the model's non-linearities. In particular, we show that a disinflation from a constant inflation rate to zero cannot be implemented for any realistic parametrization of the model when the returns to scale in the production function are constant and the economy is closed. The reason is that the actions of the policymaker are in this case constrained by a liquidity trap. The case of a partial reduction in inflation is then even more problematic. If inflation is to be reduced to a positive new target, there is no perfect foresight solution to the model at all. However, in the subsequent subsection we show that these constraints are exclusive to the special case of of a closed economy and constant returns to scale.

3.3.1 The Initial Constant Inflation Steady State

As in the previous section, we assume that the economy is initially in a CISS in which real variables are constant and all nominal variables grow at the constant inflation rate $\mu_I = \frac{M_t}{M_{t-1}}$.[16] We again first determine the CISS solution of the model. In order to derive an expression for nontradables output in the CISS, we proceed as before and find that

$$Y_N = A[(\frac{(\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1})^{-\zeta}+\beta\mu_I^{\zeta}(\frac{1}{2}+\frac{1}{2}\mu_I^{1-\varepsilon})^{-\zeta}}{(\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1})^{-1}+\beta(\frac{1}{2}+\frac{1}{2}\mu_I^{1-\varepsilon})^{-1}})^{\frac{1}{\zeta}}(\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1})^{\frac{1}{1-\varepsilon}}]^{-\sigma} \quad (3.11)$$

where $A = (\frac{1}{\sigma\alpha})^{-\sigma}(\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1})^{-\frac{\sigma}{\zeta}} > 0$. The rate of money growth enters the expression for nontradables output in a more complex fashion than in the linearized version of the model. We now identify three channels through which the rate of money growth μ_I affects nontradables output in the CISS, namely the 'discounting channel', the 'productivity channel' and the 'disutility channel'. The intuition behind these channels is discussed in Appendix C.[17]

We move on to derive the CISS solution for the wage index one period before disinflation. Applying the same procedure as in the previous section, we have that

$$W_{-1} = M_{-1}V[\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1}]^{\frac{1}{1-\varepsilon}} \quad (3.12)$$

[16] Note that the definition of μ_I has changed. The definition of μ_D changes accordingly.

[17] Note that in the linearized version of the model, there is only one, namely the 'discounting channel'.

where V is given by

$$V = \frac{1}{Z}[\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}\frac{(\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1})^{-\zeta}+\beta\mu_I^{\zeta}(\frac{1}{2}+\frac{1}{2}\mu_I^{1-\varepsilon})^{-\zeta}}{(\frac{1}{2}+\frac{1}{2}\mu_I^{\varepsilon-1})^{-1}+\beta(\frac{1}{2}+\frac{1}{2}\mu_I^{1-\varepsilon})^{-1}}]^{\frac{1}{\zeta}} \tag{3.13}$$

where $Z_t = \frac{M_t}{S_t}$ and where, assuming that there is no foreign inflation, the money demand function and the interest rate parity condition imply that

$$Z = \frac{1-\delta}{\delta}\frac{\mu_I}{\mu_I-\beta} \tag{3.14}$$

Equations (3.11) and (3.12) determine the state of the economy one period before disinflation.[18]

3.3.2 The Special Case: Closed Economy with Constant Returns to Scale

We initially focus on the special case of a closed economy and constant returns to scale in the production function. The supply equation (2.3) shows that, as in the linearized version of the model, these assumptions imply that a policy of CPI inflation targeting is equivalent to one of wage inflation targeting. We concentrate on the particular case of $\mu_D = 1$ at first. In solving for the post-disinflation path of the economy, we follow the same procedure as in the case of the linearized model.

It is straightforward to determine what the disinflation policy implies for wages set from period $t = 0$ onwards. We use the wage index and impose that the rate of inflation is equal to μ_I until period $t = -1$ and to $\mu_D = 1$ thereafter. We find that in all odd periods starting with period one,

$$X_t = X_{-1} \tag{3.15}$$

and in all even periods starting with period zero,

$$X_t = X_{-2} = \frac{X_{-1}}{\mu_I} \tag{3.16}$$

The wage setting condition (2.10) can, in a complete markets equilibrium, be expressed as

$$X_t = [\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}\frac{W_t^{\varepsilon\zeta}N_t^{\zeta}+\beta W_{t+1}^{\varepsilon\zeta}N_{t+1}^{\zeta}}{W_t^{\varepsilon}N_t/S_t+\beta W_{t+1}^{\varepsilon}N_{t+1}/S_{t+1}}]^{\frac{1}{1+\varepsilon(\zeta-1)}} \tag{3.17}$$

[18] The disinflation policy is unexpected such that the expectation of $\frac{S_0}{S_{-1}}$ as of period $t = -1$ must be μ_I.

We substitute for the wage index and eliminate employment. This yields

$$X_t = [\frac{\varepsilon}{\varepsilon - 1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}(\frac{1}{2}X_t^{1-\varepsilon} + \frac{1}{2}X_{t-1}^{1-\varepsilon})^{1-\zeta}\frac{S_t^{\zeta} + \beta S_{t+1}^{\zeta}}{1+\beta}]^{\frac{1}{1+\varepsilon(\zeta-1)}} \tag{3.18}$$

This relationship holds for all $t = 0, 1, 2, \ldots, \infty$. Evaluating (3.18) at each particular time period and using (3.15) and (3.16), we find that

$$S_{t+2} = [[S_t^{\gamma} - (M_{-1}V)^{\zeta}\frac{1+\beta}{\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}(\frac{1}{2}+\frac{1}{2}B^{1-\varepsilon})^{1-\zeta}}(1-\beta B^{1+\varepsilon(\zeta-1)})]/\beta^2]^{\frac{1}{\zeta}} \tag{3.19}$$

$$S_{t+2} = [[S_t^{\gamma} - (M_{-1}V)^{\zeta}\frac{1+\beta}{\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}(\frac{1}{2}+\frac{1}{2}B^{1-\varepsilon})^{1-\zeta}}(B^{1+\varepsilon(\zeta-1)}-\beta)]/\beta^2]^{\frac{1}{\zeta}} \tag{3.20}$$

in odd and even periods respectively, where $B = (\frac{1}{2} + \frac{1}{2}\mu_I^{\varepsilon-1})^{\frac{1}{1-\varepsilon}}/(\frac{1}{2} + \frac{1}{2}\mu_I^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$.

Equations (3.19) and (3.20) represent the fundamental laws of motion of the economy in the post-disinflation state. In order to learn about the stability properties of the two equations, we differentiate both and evaluate them at their respective steady states. The resulting expressions turn out not to be analytically tractable. We are thus left with the option to establish stability results for given parameter values. We therefore compute the slope of each of the two equations at their respective steady states for various combinations of parameter values within the defined limits. We find a robust result across all parameterizations, namely that both equations exhibit a slope greater than one at their respective steady states and are thus locally unstable. The unique non divergent solution of (3.19) and (3.20) must therefore be given by their respective steady states. We proceed to rule out unstable solutions as in the previous section. In the post-disinflation state, nominal consumption is thus given by

$$S_t = [(M_{-1}V)^{\zeta}\frac{1+\beta}{\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}(\frac{1}{2}+\frac{1}{2}B^{1-\varepsilon})^{1-\zeta}}(1-\beta B^{1+\varepsilon(\zeta-1)})/(1-\beta^2)]^{1/\zeta} \tag{3.21}$$

$$S_t = [(M_{-1}V)^{\zeta}\frac{1+\beta}{\frac{\varepsilon}{\varepsilon-1}\frac{\eta\zeta}{\delta}(\sigma\alpha)^{\zeta-1}(\frac{1}{2}+\frac{1}{2}B^{1-\varepsilon})^{1-\zeta}}(B^{1+\varepsilon(\zeta-1)}-\beta)/(1-\beta^2)]^{1/\zeta} \tag{3.22}$$

in odd and even periods respectively. The results show that, as in the linearized version of the model, nominal consumption fluctuates between even and odd periods throughout future periods in the special case of $\mu_D = 1$. The same is true for money supply which is given as a function of nominal consumption by

$$M_t = \frac{1-\delta}{\delta} \frac{S_t}{1 - \beta S_t / S_{t+1}} \tag{3.23}$$

However, taking a closer look at (3.22), we observe that a steady state solution for nominal consumption does not exist in even periods for sufficiently large values of μ_I. We know that the initial rate of money growth μ_I is greater than one. Thus, $B < 1$. In fact, B decreases with μ_I. For sufficiently large values of μ_I, the term $B^{1+\varepsilon(\zeta-1)} - \beta$ is negative. This implies that the steady state solution for nominal consumption in even periods does not exist. The reason is the following: nominal consumption is high in odd periods in which wage setters set high wages. It is low in even periods in which wage setters set low wages. A large μ_I implies a high degree of wage dispersion and thus strong oscillations in macro variables. For sufficiently large values of μ_I, the wage set in even periods becomes arbitrarily small and so does nominal consumption. Eventually, there is no positive level of nominal consumption in even periods that is consistent with the wage setting behavior of households and the disinflation policy becomes infeasible.[19]

The constraint $B^{1+\varepsilon(\zeta-1)} - \beta \geq 0$ does not, however, turn out to be binding in this model. The reason is that there is a second constraint for the policy to be feasible which takes the form of a liquidity trap. The set of combinations of parameter values that do not violate the second constraint is a strict subset of those that do not violate the first. This implies that there are cases in which the economy hits the liquidity trap although a solution for even periods' nominal consumption exists. In order to fully understand why the economy hits a liquidity trap in this framework, notice that the consumption Euler equation (2.8) is given by

$$I_t = \frac{1}{\beta} \frac{S_{t+1}}{S_t} \tag{3.24}$$

The optimal intertemporal choice of consumption requires the nominal interest rate to be low in odd periods in order to induce a fall in nominal consumption

[19] The maximum size of the initial inflation rate μ_I for which the disinflation policy is still feasible in terms of this constraint depends on the choice of parameter values. Unsurprisingly, it hinges crucially on the parameters β, ε and ζ. The reason is that these parameters determine the wage setting behavior of households and thereby the amplitude of the oscillations in macro variables. The greater is the magnitude of each of these parameters, the stronger incentives must be set to induce the required wage setting behavior on the part of the households.

Table 1: Maximum Value of μ_I Allowing the Policymaker to Avoid the Liquidity Trap

β	ε	ζ	μ_I
0.90	1.4	1.2	1.0052
0.95	3.5	1.4	1.0007
0.95	4.5	1.9	1.0004
0.97	6.0	2.5	1.0001

in the subsequent even period. Similarly, a rise in nominal consumption in odd periods requires a high nominal interest rate in the previous even period. The money demand function in equation (2.9) shows that the interest rate increase in even periods causes a fall in even periods' money demand, while the decrease in odd periods causes a rise in odd periods' money demand. The Euler equation and the money demand function taken together imply the following: the greater are the oscillations in nominal consumption that the disinflation policy produces in the post disinflation state, i.e. the smaller is $\frac{S_{t+1}}{S_t}$ in odd periods, the smaller is the nominal interest rate in even periods. As $\frac{S_{t+1}}{S_t}$ approaches β, the nominal interest rate approaches its lower bound $I_t = 1$ and money demand approaches infinity. For $\frac{S_{t+1}}{S_t} < \beta$, the nominal interest rate becomes (notionally) smaller than one and the demand for money flicks from infinity to negative infinity. Mathematically, the reason is that the demand for money is a hyperbola as equation (2.9) shows. Intuitively, the key insight is that the strict disinflation policy hits the lower bound of the nominal interest rate. For a sufficiently strong reduction in inflation, there is no odd periods' nominal interest rate $I_t > 1$ that could induce an upward jump in money demand of the magnitude that is required for the policy to work. The monetary authority is thus constrained by a liquidity trap.

Table 1 lists the maximum feasible initial inflation rates that do not violate this feasibility constraint for given parameterizations. It is immediately obvious that the constraint is too strict for the policy to be of any practical relevance in this case.

The above results imply that for sufficiently high initial inflation rates it may not be possible for the monetary authority to set the future path of its policy instrument such that the inflation rate is immediately reduced to zero and the zero inflation rate is sustained throughout future periods. The magnitude of the feasible reductions in the inflation rate is surprisingly small. If one considers the last two cases in Table 1 to be realistic calibrations, the analysis implies that the maximum feasible rate of wage inflation which can be immediately and permanently reduced to zero amounts to less than 0.05 percent. The disinflation policy admittedly is very strict. But the results suggest that a complete inflation reduction is infeasible for any practical purposes. An analysis of the effects of the use of strict inflation targeting to reduce the rate of inflation from a positive to a lower positive value therefore suggests itself.

In the framework of the linearized model, we found that the nature of the disinflation policy's impact on the economy in the case of a partial reduction of inflation is equivalent to the particular case of eliminating inflation alltogether. In the nonlinear model, however, it turns out that there is no equilibrium of the model that could support such a policy. In particular, the inflation rate cannot be kept at the new target after disinflation throughout all future periods. Due to the nonlinearity in the wage index formula, the new wage is subject to fluctuations between even and odd periods which grow over time. The new wage set in one of the two sectors grows, while the other falls and eventually hits its zero lower bound. Hence, an equilibrium does not exist. Surprisingly, this is not the case if the inflation rate μ_I is reduced to a new but negative inflation target $0 < \mu_D < 1$. Intuitive explanations as well as mathematical proofs for these findings are given in Appendix D.

3.3.3 The General Case: Open Economy and Decreasing Returns to Scale

The purpose of the present section is to show that both a complete and a partial disinflation of an empirically relevant size are possible in the nonlinear model if one abstracts from the special case of a closed economy and constant returns to scale. The post-disinflation path of the economy moreover looks strikingly similar to the equivalent case in the log-linear model.

As before we solve for the post-disinflation path of the economy by focusing on its law of motion, i.e. the wage setting condition, which can be expressed as

$$X_t = [\frac{\varepsilon}{\varepsilon - 1}\frac{\eta\zeta}{\delta}\frac{W_t^{\varepsilon\zeta}Y_{Nt}^{\zeta/\sigma} + \beta W_{t+1}^{\varepsilon\zeta}Y_{Nt+1}^{\zeta/\sigma}}{W_t^{\varepsilon}Y_{Nt}^{1/\sigma}/S_t + \beta W_{t+1}^{\varepsilon}Y_{Nt+1}^{1/\sigma}/S_{t+1}}]^{\frac{1}{1+\varepsilon(\zeta-1)}} \tag{3.25}$$

As in the case of the linearized model, we need to normalize all nominal variables by the price level before we can solve the model. For each nominal variable Q_t we define $\tilde{Q}_t = \frac{Q_t}{P_t}$. Using this definition as well as the fact that $P_{t+1} = P_t\mu_D$ in the post-disinflation state, it is straightforward to show that the following system of equations determines the post-disinflation path of the economy:

$$\tilde{X}_t = [\frac{\varepsilon}{\varepsilon - 1}\frac{\eta\zeta}{\delta}\frac{\tilde{W}_t^{\varepsilon\zeta}Y_{Nt}^{\zeta/\sigma} + \beta(\tilde{W}_{t+1}\mu_D)^{\varepsilon\zeta}Y_{Nt+1}^{\zeta/\sigma}}{\tilde{W}_t^{\varepsilon}Y_{Nt}^{1/\sigma}/\tilde{S}_t + \beta(\tilde{W}_{t+1}\mu_D)^{\varepsilon}Y_{Nt+1}^{1/\sigma}/(\tilde{S}_{t+1}\mu_D)}]^{\frac{1}{1+\varepsilon(\zeta-1)}} \quad (3.26)$$

$$\alpha^{\alpha} = \tilde{P}_{Nt}^{\alpha}\tilde{S}_t^{1-\alpha} \quad (3.27)$$

$$\tilde{W}_t = [0.5\tilde{X}_t^{1-\varepsilon} + 0.5(\frac{\tilde{X}_{t-1}}{\mu_D})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (3.28)$$

$$Y_{Nt} = (\frac{\sigma\alpha\tilde{S}_t}{\tilde{W}_t})^{\sigma} \quad (3.29)$$

$$\tilde{P}_{Nt} = (\alpha\tilde{S}_t)^{1-\sigma}(\tilde{W}_t/\sigma)^{\sigma} \quad (3.30)$$

We need to resort to numerical methods to solve this system of equations. A set of initial conditions can be derived based on equations (3.11) and (3.12). The latter define the state of the economy one period before disinflation. As a solution procedure, we use the Newton-type algorithm first proposed by Laffargue (1990).[20]

Figures 6 and 7 consider a disinflation from two percent down to price stability. The solid lines represent the post-disinflation path of the economy in the log-linear version of the model using the same parameterizations as in Figures 2 and 3.[21] The dashed lines represent the corresponding path of the economy in the full nonlinear model. It is immediately obvious that a disinflation of a realistic size is indeed possible in the nonlinear version of the model. Moreover, not only is the post-disinflation path of the model economy qualitatively equivalent to the case of the log-linear version of the model, it is even quantitatively very similar. While the magnitude of the fluctuations in nominal variables is greater in the nonlinear model, the movements in output are almost precisely of the same magnitude. This implies that we can draw an equivalent set of conclusions from these results as we did in the previous section for the log-linear version of the model.

Figures 8 and 9 present the results from reducing the rate of price inflation from 5 percent to 3 percent. The nominal variables are shown as normalized by the price index. It is immediately obvious that, contrary to the special case of a closed economy and constant returns to scale, not only disinflations of an empirically relevant size but also gradual disinflations are now possible. As in the log-linear version of the model, the qualitative conclusions arising from

[20] The algorithm is employed in Dynare for the solution of deterministic models.

[21] We converted the values from log deviations back to level terms.

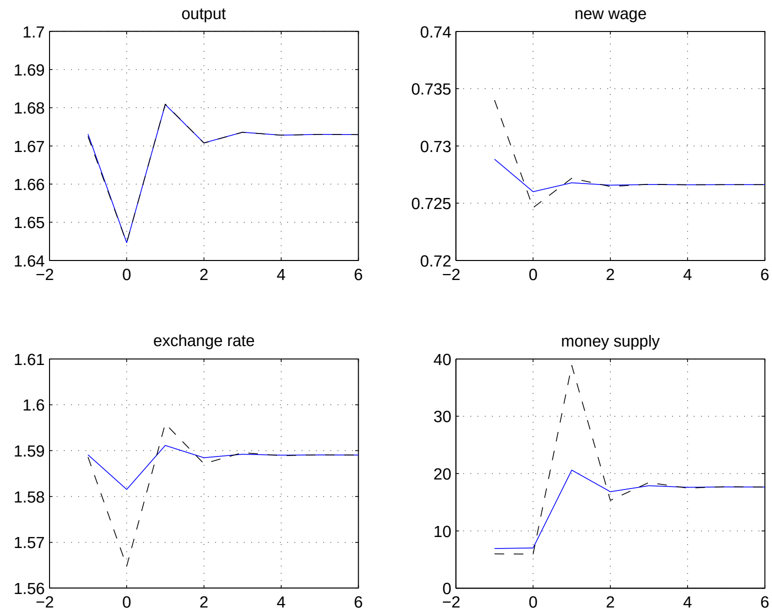

Figure 6: Price Inflation with Oscillations: Comparison of Linear and Nonlinear Model

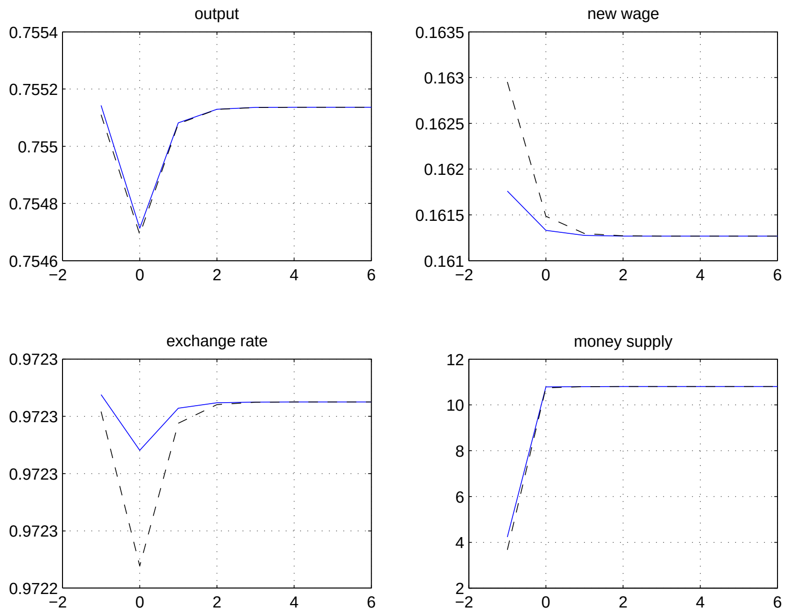

Figure 7: Price Inflation without Oscillations: Comparison of Linear and Nonlinear Model

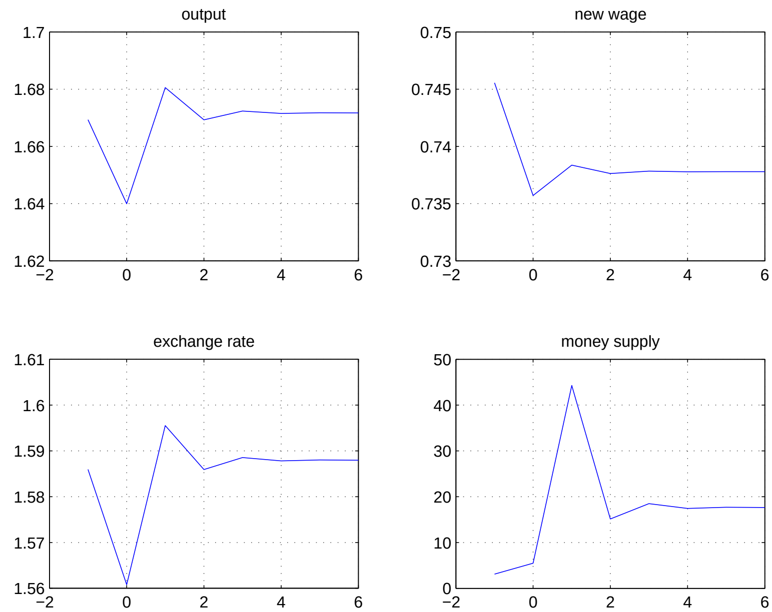

Figure 8: Partial Disinflation with Oscillations

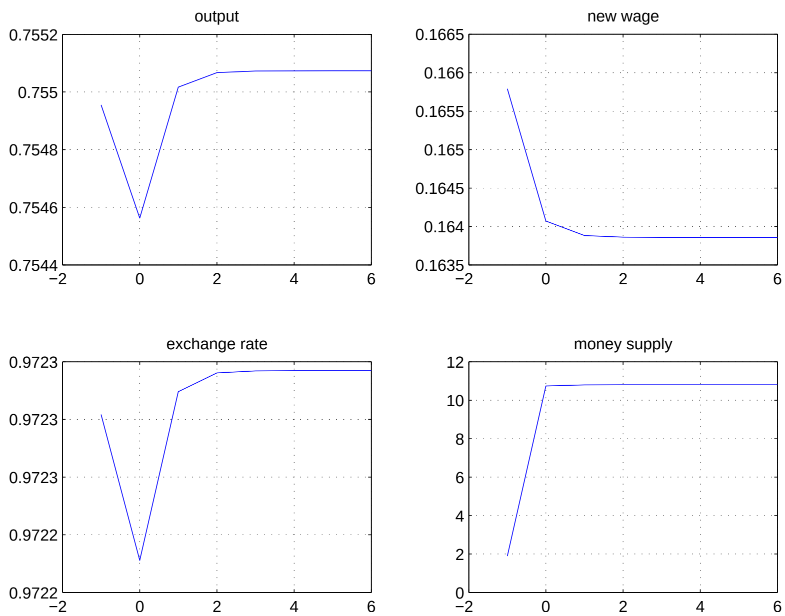

Figure 9: Partial Disinflation without Oscillations

partial disinflations are the same as in the case of a complete disinflation to price stability.

4 Discussion

The present study was motivated by the idea that inflation targeting differs from other disinflation policies in important respects. In particular, a strict interpretation of an inflation target allows the policymaker to tolerate no deviations from target. But adjusting the policy instrument such that the inflation target is attained and defending the new target rigorously may preserve inflationary distortions such as wage and price differentials to an exceptional degree. The implications of these distortions might be severe for both real activity and volatility in the economy. In order to formalize this idea, we have used an open economy Dynamic General Equilibrium Model with wage staggering of the type suggested by Taylor (1979a) to consider a rather extreme case of a disinflation policy: an immediate and permanent reduction in the rate of CPI inflation to a newly set target. We found that the disinflation policy not only creates a slump in output on impact, but can additionally generate oscillatory behavior in both nominal and real variables along their post-disinflation adjustment path. These oscillations can be permanent when the economy is closed and the returns to labor in the production function are constant. This is also the only case in which the economy does not gradually converge to a new steady state. Moreover, we showed that the size of the initial slowdown in real activity and the magnitude of the oscillations are positively related.

From a modeling perspective, the analysis has shown that the presence of oscillations along the adjustment path would not occur in a model with a Calvo (1983) type staggering structure as in Yun (2005). In the framework of the particular model we employed, we found that the presence of oscillations along the post-disinflation path of the economy as well as their magnitude strongly depend on the desired size of the reduction in the inflation rate as well as the returns to labor in the production function and the degree of openness of the economy. In particular, we illustrated that there are no oscillations at all along the post-disinflation path of the economy if the returns to labor are sufficiently low and the economy is rather open. At the other extreme, in the case of constant returns to scale and and a closed economy, the oscillations are large and permanent. The reason is that, in the latter case, CPI inflation targeting becomes equivalent to a policy of wage inflation targeting and the exchange rate looses its role as a stabilizer.

The present study is in line with the analysis of Yun (2005) in that it identifies the slow convergence of prices as the major source of inefficiency resulting from an unexpected, immediate and permanent reduction in the inflation rate. Moreover, the analysis has shown that the negative consequences of strict inflation targeting identified by Svensson (2000) may be exacerbated when the policy is used as a disinflation regime. We interpret this as an explanation for the finding of Roger and Stone (2005) that target misses are particularly common for disinflating inflation targeters despite the fact that monetary authorities in these economies should be particularly eager to avoid credibility losses.[22] In sum, there are good reasons to be cautious when adopting inflation targeting as a disinflation regime. This may be particularly true for emerging market economies which have a limited experience with an independent monetary authority.

[22]The authors provide evidence that suggests that central banks rather shift up the planned trajectory for the inflation rate than tightening policy in order not to deviate from it. This behavior complies with the concept of opportunistic disinflation proposed by Clifton (1999).

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Appendix

A Log-linear Definitions and Equations

In the following equations lower-case symbols denote log deviations of variables from their reference steady state values, i.e. $v_t = log\frac{V_t}{V_R}$:

A.1 The Closed Economy

$$\mu_{t+1} = m_{t+1} - m_t \tag{A.1}$$

$$s_t = p_t + c_t \tag{A.2}$$

$$m_t - p_t = c_t - \frac{\beta}{1-\beta} i_t \tag{A.3}$$

$$s_{t+1} - s_t = i_t \tag{A.4}$$

$$y_t = \frac{\sigma}{1-\sigma}(p_t - w_t) \tag{A.5}$$

$$\begin{aligned} x_t = \frac{1}{1+\varepsilon(\zeta-1)}[&\frac{1}{1+\beta}[s_t + \varepsilon(\zeta-1)w_t + (\zeta-1)n_t] \\ &+\frac{\beta}{1+\beta}[s_{t+1} + \varepsilon(\zeta-1)w_{t+1} + (\zeta-1)n_{t+1}]] \end{aligned} \tag{A.6}$$

$$w_t = \frac{1}{2}x_t + \frac{1}{2}x_{t-1} \tag{A.7}$$

$$n_t = \frac{1}{\sigma} y_t \tag{A.8}$$

A.2 The Open Economy

$$\mu_{t+1} = m_{t+1} - mu \tag{A.9}$$

$$s_t = p_t + c_t \tag{A.10}$$

$$m_t - p_t = c_t - \frac{\beta}{1-\beta} i_t \tag{A.11}$$

$$s_{t+1} - s_t = i_t \tag{A.12}$$

$$e_{t+1} - e_t = i_t \tag{A.13}$$

$$y_{Nt} = \frac{\sigma}{1-\sigma}(p_{Nt} - w_t) \tag{A.14}$$

$$y_{Tt} = 0 \tag{A.15}$$

$$c_{Nt} = s_t - p_{Nt} \tag{A.16}$$

$$c_{Tt} = s_t - e_t \tag{A.17}$$

$$p_t = \alpha p_{Nt} + (1-\alpha) e_t \tag{A.18}$$

$$\frac{1}{\alpha} y_t = y_{Nt} \tag{A.19}$$

$$\tau_t = -c_{Tt} \tag{A.20}$$

$$\begin{aligned} x_t = \frac{1}{1+\varepsilon(\zeta-1)}[&\frac{1}{1+\beta}[s_t + \varepsilon(\zeta-1)w_t + (\zeta-1)n_t] \\ &+ \frac{\beta}{1+\beta}[s_{t+1} + \varepsilon(\zeta-1)w_{t+1} + (\zeta-1)n_{t+1}]] \end{aligned} \tag{A.21}$$

$$w_t = \frac{1}{2} x_t + \frac{1}{2} x_{t-1} \tag{A.22}$$

$$n_t = \frac{1}{\sigma} y_{Nt} \tag{A.23}$$

B Price Inflation Targeting

In the following, we show in a first step that there is a unique perfect foresight solution for the law of motion of the economy in the post-disinflation state when either $\sigma < 1$ or $\alpha < 1$. In a second step, we show that the sign of the smaller eigenvalue depends on the degree of returns to labor σ and the degree of openness of the economy $1 - \alpha$ as well as the parameters ζ and ε. In the special case of $\alpha = 1$ and $\sigma = 1$ a saddlepath equilibrium does not exist. The proofs are outlined for the open-economy setting but hold equivalently in the closed economy setting if α is set to unity.

B.1 Stability

As long as $\alpha < 1$ or $\sigma < 1$, the system of equations determining the law of motion of the economy in the post-disinflation steady state has exactly one stable eigenvalue such that there is a unique perfect foresight solution. To see this, notice that the law of motion of the economy is given by

$$\phi_{t+1} - \frac{1+\beta}{\beta}\frac{1-\alpha\sigma+\gamma}{1-\alpha\sigma-\gamma}\phi_t + \frac{1}{\beta}\phi_{t-1} = \mu_D(\frac{1-\beta}{\beta} - \frac{2\gamma}{1-\alpha\sigma-\gamma}) \tag{B.1}$$

The characteristic equation of this 2nd order difference equation is given by

$$\omega^2 + b\omega + c = 0 \tag{B.2}$$

where $b = -\frac{1+\beta}{\beta}\frac{1-\alpha\sigma+\gamma}{1-\alpha\sigma-\gamma}$ and $c = \frac{1}{\beta}$. Notice that in the special case of $\alpha = 1$ and $\sigma = 1$, the two eigenvalues of this difference equation are readily derived as $\lambda_1 = -1$ and $\lambda_2 = -\frac{1}{\beta}$, implying that one eigenvalue lies on the unit circle and the economy will never converge. In order to prove stability for all other cases, we apply standard results from the continuous time case. In particular, we define the variable z such that $z = \frac{\omega-1}{\omega+1}$ and $\omega = \frac{1+z}{1-z}$.[23] We therefore transform the equation to

$$(1-b+c)z^2 + 2(1-c)z + (1+b+c) = 0 \tag{B.3}$$

Applying standard results for a continuous time characteristic equation, the following condition must hold for the characteristic equation to have exactly one stable eigenvalue.

$$\frac{1+b+c}{1-b+c} < 0 \tag{B.4}$$

[23] A stable eigenvalue in the continuous time case has a negative real part. It can be shown that ω lies in the unit circle if and only if z has a negative real part.

After some manipulations we have

$$\frac{1+b+c}{1-b+c} = -\frac{\gamma}{1-\alpha\sigma} \tag{B.5}$$

And since $\alpha, \sigma < 1$, it is clear that this expression is strictly negative. We can conclude that for any parameterization within the restrictions given, there will be exactly one stable eigenvalue. This implies that there is a unique perfect foresight solution to the model and the economy ultimately converges to a new steady state. In the special case of $\alpha = 1$ and $\sigma = 1$ a saddlepath equilibrium does not exist.

B.2 The Sign of the Smaller Eigenvalue

We call $p(\omega)$ the 2nd order characteristic polynomial function in ω, i.e. the LHS of the characteristic equation. We have shown that there is one and only one stable eigenvalue when $\alpha < 1$ or $\sigma < 1$. This implies that the characteristic function cuts the horizontal axis precisely once within the range $(-1, 1)$. Now, suppose that the stable eigenvalue is of a positive sign. This implies that $p(0)$ and $p(1)$ must be of opposite sign. Therefore, for the unique stable eigenvalue to be of a positive sign, the following condition must hold:

$$\frac{c}{1+b+c} < 0 \tag{B.6}$$

After some manipulations

$$1 - \frac{1-\alpha\sigma+\gamma}{1-\alpha\sigma-\gamma} < 0 \tag{B.7}$$

Noticing that multiplying by the denominator of the fraction might involve multiplying by a negative number, we find that the condition is fulfilled if and only if $1 - \alpha\sigma - \gamma > 0$, where $\gamma = \frac{\zeta}{1+\varepsilon(\zeta-1)}$. This implies that the unique and stable eigenvalue will be negative for sufficiently high α and σ and sufficiently small ζ and ε.

C The Effect of the Rate of Money Growth on Non-Tradables Output

In this section, we discuss the three channels through which the rate of money growth affects non-tradables output in the constant inflation steady state (CISS) in the framework of the full non-linear model.

(1) *Discounting Channel*: Under wage staggering, the wage that households set in period t has to lie between the ideal wage for period t and the projected ideal wage for period t+1. Under positive inflation, the ideal wage for period t must be lower than the one for period t+1. And as individuals discount future utility according to the discounting parameter $\beta < 1$, the wage set in period t will be set closer to the ideal current wage than to the projected ideal wage for the subsequent period. This, in turn, allows firms to employ more labor at the same cost and increases output. The magnitude of the effect increases the greater is the dispersion between the two ideal wages, i.e. the greater is μ_I. In sum, μ_I has a positive effect on equilibrium output due to its depressing effect on real wages through the discounting parameter β.[24]

(2) *Productivity Channel*: In the presence of inflation, the optimal wage in period t is smaller than the optimal wage in period t+1. Wage staggering then implies that in an arbitrary period t, wages differ between the sector that has just adjusted its wage and the sector whose wage is prevailing from the previous period. This implies that inflation creates wage dispersion between households with the two sectors alternating in setting the higher wage.[25] To be able to infer what wage dispersion implies for equilibrium output, notice that labor skills are imperfect substitutes in the production function. This implies that, ceteris paribus, using equal amounts of each labor type yields a higher average productivity of labor than using unequal amounts. In the presence of wage dispersion, profit maximizing firms substitute labor from the low wage sector for labor from the high wage sector. This choice is optimal under wage dispersion but is inefficient in comparison with conditions under which households in different sectors set the same wages and supply the same amounts of labor and thus attain a higher level of average productivity per unit of labor.[26] In sum, inflation creates wage dispersion which leads to labor substitution and a reduction in productivity. The higher is ε, the less severe is this effect. However, the less severe the effect is, the more do firms engage into labor substitution and the lower is the level of output attained in equilibrium.

(3) *Disutility Channel*: We have elaborated on the fact that wage dispersion induces firms to substitute labor from the low wage sector for labor from the high wage sector. As long as $\zeta > 1$, the resulting intertemporal fluctuation

[24]The smaller is β, the lower will households set their wages.

[25]The exact same degree of wage dispersion that is present in the CISS will be preserved in the post-disinflation state due to the particular nature of the disinflation policy.

[26]These conditions are for instance satisfied in the ZISS around which we have linearized the equilibrium conditions in the previous section.

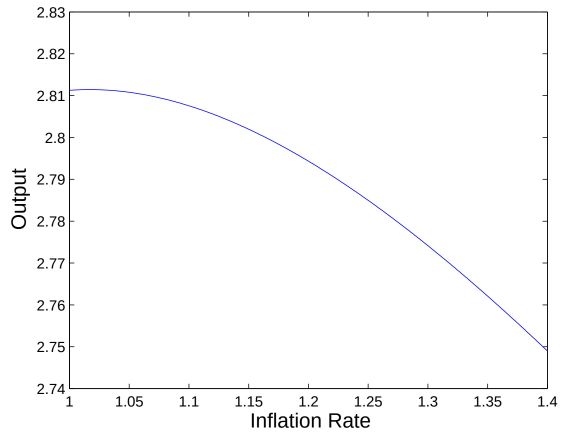

Figure 10: Effect of the Inflation Rate on Nontradables Output

in the demand for a given household's labor supply increases the disutility the household derives from providing it. This increase in disutility induces households to demand compensation payments in the form of wage increases. The higher the degree of wage dispersion, the greater the increase in the wage level and hence the greater the reduction in output. In sum, μ_I has a negative effect on equilibrium output in the CISS as it increases the overall disutility of work effort. This effect is stronger the greater is ζ.

Figure 10 illustrates the effect of μ_I on CISS output. The impact of a marginal increase in inflation varies across different levels of the inflation rate. An increase in the inflation rate affects output positively if the inflation rate is rather low. The reason is that, at low rates of inflation, the 'discounting channel' dominates. The greater is μ_I, the stronger is the degree of wage dispersion and the more do the 'productivity channel' and the 'disutility channel' gain in relative importance. For sufficiently large values of μ_I, the net effect of a marginal increase in μ_I on output is negative. This implies that the distortion resulting from an additional percentage point of inflation worsens with the size of the rate of inflation. The smaller the magnitude of β, ζ and ε, the stronger is the discounting channel relative to the productivity channel and the disutility channel.[27]

[27] Ascari (1998) and Graham and Snower (2004) derive the effect of money growth on output in similar frameworks and reach the same qualitative conclusion.

D Impossibility of a Partial Reduction in the Rate of Wage Inflation

We show in a first step that $\frac{X_0}{X_{-1}} = \frac{X_2}{X_1}$ holds only in the special case of $\mu_D = 1$. This implies that in all other cases the economy is not in the post-disinflation state characterized by perpetual oscillations immediately after the policy is applied. The second step is to show that there is no convergence process of the ratio $\frac{X_t}{X_{t-1}} = \frac{X_{t+2}}{X_{t+1}}$ to its steady state $\frac{X_t}{X_{t-1}} = \mu_D$ thereafter if $1 < \mu_D < \mu_I$. The steady state is unstable and the disinflation policy is infeasible.

Proposition D.1. *The equality* $\frac{X_0}{X_{-1}} = \frac{X_2}{X_1}$ *holds in the post-disinflation state only in the special case of* $\mu_D = 1$*.*

Proof. The ratio of the wage index in period t and its first lag is given by

$$\frac{W_t}{W_{t-1}} = \frac{[\frac{1}{2}X_t^{1-\varepsilon} + \frac{1}{2}X_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}}{[\frac{1}{2}X_{t-1}^{1-\varepsilon} + \frac{1}{2}X_{t-2}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}} \tag{D.1}$$

Setting $\frac{W_t}{W_{t-1}} = \mu_D$ and manipulating yields

$$\frac{X_t}{X_{t-1}} = (\mu_D^{1-\varepsilon}(1 + (\frac{X_{t-2}}{X_{t-1}})^{1-\varepsilon}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.2}$$

This implies that we have

$$\frac{X_0}{X_{-1}} = (\mu_D^{1-\varepsilon}(1 + \mu_I^{\varepsilon-1}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.3}$$

$$\frac{X_1}{X_0} = (\mu_D^{1-\varepsilon}(1 + (\frac{X_{-1}}{X_0})^{1-\varepsilon}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.4}$$

$$\frac{X_2}{X_1} = (\mu_D^{1-\varepsilon}(1 + (\frac{X_0}{X_1})^{1-\varepsilon}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.5}$$

etc. I will now show that for a given ε the equality $\frac{X_0}{X_{-1}} = \frac{X_2}{X_1}$ only holds if $\mu_D = 1$ and/or if $\mu_D = \mu_I$. Plugging (D.4) into (D.5), we have

$$\frac{X_2}{X_1} = (\mu_D^{1-\varepsilon}(1 + (\mu_D^{1-\varepsilon}(1 + (\frac{X_{-1}}{X_0})^{1-\varepsilon}) - 1)^{-1}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.6}$$

Now using (D.3),

$$\frac{X_2}{X_1} = (\mu_D^{1-\varepsilon}(1 + (\mu_D^{1-\varepsilon}(1 + (\mu_D^{1-\varepsilon}(1 + \mu_I^{\varepsilon-1}) - 1)^{-1}) - 1)^{-1}) - 1)^{\frac{1}{1-\varepsilon}} \tag{D.7}$$

Now we set the RHS of (D.7) equal to the RHS of (D.3). We obtain

$$\mu_D^{1-\varepsilon}(1+\mu_I^{\varepsilon-1}) = \mu_D^{1-\varepsilon}(1+(\mu_D^{1-\varepsilon}(1+(\mu_D^{1-\varepsilon}(1+\mu_I^{\varepsilon-1})-1)^{-1})-1)^{-1}) \quad \text{(D.8)}$$

and finally after some simplification

$$\mu_I^{1-\varepsilon}+1 = \mu_D^{1-\varepsilon}+(1+\mu_I^{\varepsilon-1}-\mu_D^{\varepsilon-1})^{-1} \quad \text{(D.9)}$$

For a given ε, there are only two positive real valued solutions to this equation. These are $\mu_D = 1$ and $\mu_D = \mu_I$. This proves the above proposition.

□

Proposition D.2. *If $1 < \mu_D < \mu_I$, then the steady state $\frac{X_t}{X_{t-1}} = \mu_D$ is locally unstable and it is impossible to carry out the disinflation policy. If $0 < \mu_D < 1$, then the steady state $\frac{X_t}{X_{t-1}} = \mu_D$ is locally stable and the disinflation policy can be carried out.*

Proof. From equation (D.2) in Proposition D.1 we have that

$$(\frac{X_t}{X_{t-1}})^{1-\varepsilon} = \mu_D^{1-\varepsilon}(1+(\frac{X_{t-1}}{X_{t-2}})^{\varepsilon-1})-1 \quad \text{(D.10)}$$

We define $(\frac{X_t}{X_{t-1}})^{1-\varepsilon} = r_t$. Then we have the first order non-linear difference equation

$$r_t = \mu_D^{1-\varepsilon} - 1 + \frac{\mu_D^{1-\varepsilon}}{r_{t-1}} \quad \text{(D.11)}$$

This difference equation has a steady state at $r = \mu_D^{1-\varepsilon}$ or $r = -1$. A negative value of r is economically meaningless. Hence, the only economically meaningful steady state is $r = \mu_D^{1-\varepsilon}$, which implies that $\frac{X_t}{X_{t-1}} = \mu_D$. Furthermore, notice that differentiating r_t with respect to its one period lag and evaluating the derivative at the steady state yields a value of $\frac{1}{\mu_D^{1-\varepsilon}}$, the slope of the phase line at the steady state.

Case 1: $1 < \mu_D < \mu_I$ This case corresponds to a partial disinflation policy. It is illustrated in figure 11. We begin by noticing that there is a natural initial condition for r_t, namely r_0, which is predetermined in this setting. Given this initial value for r_t, the time path of r_t is divergent as the steady state is not locally stable. The reason is that the slope of the phase line given by equation (D.11) is smaller than -1 at the SS. Its horizontal asymptote occurs at a negative value for r_t. This means that eventually $r_t < 0$ which implies that the disinflation policy cannot be carried out. However, notice that our discussion here only covers local as opposed to global stability. As the economy

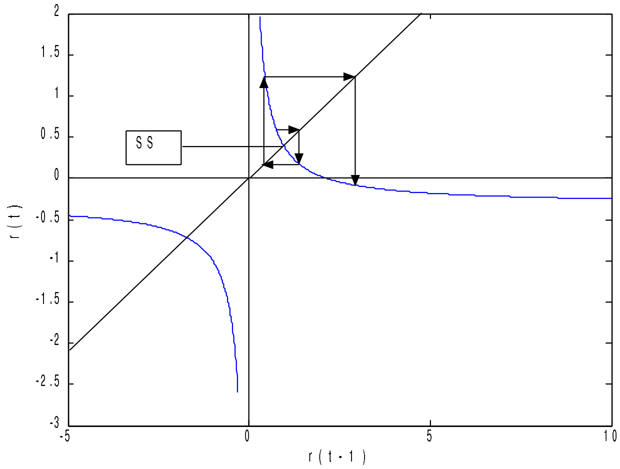

Figure 11: Case 1: Unstable Pattern

moves farther away from the steady state, and hence nonlinearities become more important, it is possible that the economy behaves in an unexpected way. It is for instance possible that it converges on a so-called 2-cycle, i.e. a path on which fluctuations continue forever at the same amplitude. An analysis of global stability, however, is not within the scope of this paper.

Case 2: $0 < \mu_D < 1$ This case corresponds to disinflation policy to a negative inflation rate. It is illustrated in figure 12. Notice that r_t is locally stable. The reason is that the slope of the phase line given by equation (D.11) is greater than -1 at the SS. Its horizontal asymptote occurs at a positive value for r_t. In contrast to Case 1, this implies that the time path of r_t will follow a stable pattern. The disinflation policy can thus be carried out.[28].

□

Intuitively, the driving factor behind these results is the non(log)linearity of the wage index W_t in X_t and X_{t-1}.[29] This non(log)linearity is due to the fact that labor types are not perfectly complementary but at least partially substitutable in the production function. Simply put, if households in one sector set a lower wage than households in the other, the resulting wage index is not given by the simple average of the two wages - as in the linearized version of

[28]As discussed above, notice that we only consider local stability.

[29]If the production function were of Cobb-Douglas type, the wage index would be loglinear and labor types would be less easily substituted than in the present case.

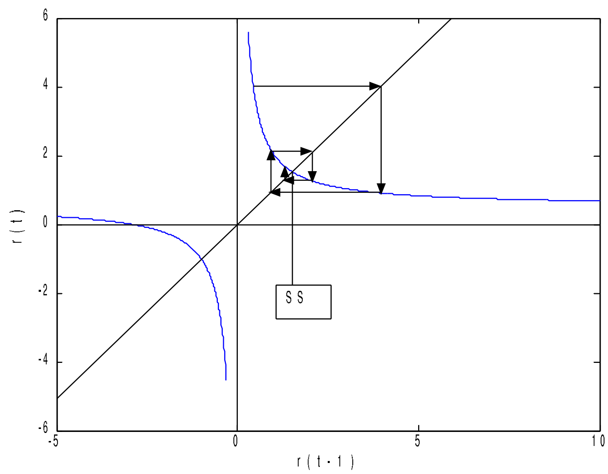

Figure 12: Case 2: Stable Pattern

the model - but by some value smaller than it. The reason is that firms will substitute labor types from the low wage sector for labor types from the high wage sector. Furthermore, firms will find it optimal to engage more strongly into labor substitution the greater is the wage gap between the two sectors and the greater is the elasticity of technical substitution between labor types, ε. This labor substituting behavior of firms is the reason, why it is not possible to keep wage growth at target throughout future periods after disinflation to positive rate of inflation. In the following, we will contrast wage setting after (a) a complete disinflation, (b) a disinflation to a positive rate of wage inflation and (c) a disinflation towards a negative rate of wage inflation.

$\mu_D = 1$ is the case of a complete disinflation. We have shown that a complete disinflation implies that wages set in the post-disinflation state are constant through time for each sector of households. Furthermore, these wages are equal to the respective wage of each sector one period before disinflation. Therefore, this is the one case in which the policy does not introduce dynamics into the behavior of sectoral wages.

Let us now consider the case when $1 < \mu_D < \mu_I$. Before starting the discussion, it is helpful to take a quick look at Figure 1 in the main text which illustrates the post-disinflation path of the new wage in the framework of the linearized version of the model. The question is then, why we do not observe equivalent patterns when investigating disinflations to positive new inflation targets in the framework of the nonlinear model.

Let us assume that we are investigating a disinflation that implies a strong enough reduction in the rate of wage inflation for the new wage in period $t = 0$ to be forced to fall below the prevailing new wage that was set in period $t = -1$.[30] In the framework of the linearized model, we would be looking at case 1a in which the new wage set in period zero is lower than in the previous period.

We have chosen the case in which the new inflation target is attained if X_0 takes a value somewhere between X_{-1} and X_{-2}. Now, for a given X_{-1} and X_{-2}, let X_{0_A} be the new wage that would attain the new inflation target in period $t = 0$ in the absence of labor substitution. Notice that $X_{0_A} > X_{-2}$ implies that sectoral wages are less far apart in period $t = 0$ than they were in $t = -1$. Consequently, if we do allow for labor substitution in period $t = 0$, firms substitute less labor away from the high wage sector in period $t = 0$ than they did in period $t = -1$. This implies that the higher wage gains weight in the wage index in period $t = 0$ relative to period $t = -1$ and thereby exerts additional upward pressure on it. This 'labor substitution effect' implies that X_0 must lie below X_{0_A}, the value it would have taken in the absence of labor substitution. Therefore, in terms of log deviations from the reference steady state, the new wage in period zero must be low relative to its counterpart in the framework of the linearized model.

Moving on to period $t = 1$, the smaller is X_0, the greater must agents choose X_1 in order to keep wage growth constant. Furthermore, the higher is X_1, i.e. the greater is the wage gap, the more do firms engage into labor substitution away from the higher wage and the higher does X_1 have to be chosen in order to attain the new inflation target. This implies that the 'labor substitution effect' that put downward pressure on X_0 now exerts upward pressure on X_1. And it exerts more upward pressure on X_1, the more downward pressure it exerted on X_0. The fluctuations in the new wage are therefore self-enforcing with the result that the ratio $\frac{X_t}{X_{t-1}}$ falls over time throughout even periods and increases over time throughout odd periods until the new wage hits its zero lower bound in even periods.

$0 < \mu_D < 1$ is the rather unrealistic case of a reduction in wage inflation towards a negative inflation rate. Here, the economy is saddlepath stable. The ratio $\frac{X_t}{X_{t-1}}$ converges towards its post-disinflation steady state $\frac{X_t}{X_{t-1}} = \mu_D$. Notice that this result follows from the above argument as the magnitude of the reduction in the inflation rate is so great that firms engage more strongly into labor substitution in period $t = 0$ than they did in period $t = -1$.

[30] The reasoning is similar for the opposite case.