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An Engineering Design STEM Project: T-Shirt Launcher

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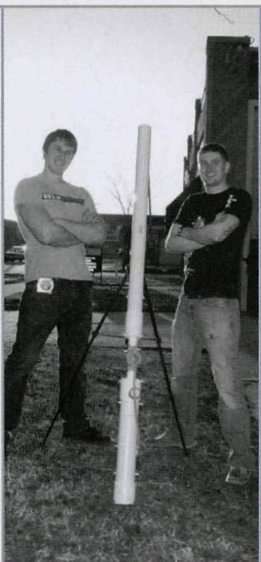

AN ENGINEERING DESIGN STEM PROJECT t-shirt LAUNCHER

BY TODD D. FANTZ AND MELVA R. GRANT

“Students become enthusiastic about applying science and mathematics for solving interesting and complex problems.

Photo Credit: Francis Hauris and Robert Stuart

INTRODUCTION

Technology education has the potential to be the glue for integrating science, technology, engineering, and mathematics (STEM) education through the use of the design process. This should result in increased student interest in science and math, resulting in increased standardized science and math scores (Silk, C. Schunn, & Strand, 2009). In order for this to happen, students need to integrate their grade-level mathematics and science content knowledge in their technology and/or engineering design (Tran & Nathan, 2010). Hopefully, this can be accomplished without losing student interest generated by hands-on, kinesthetic learning. This article provides one example of getting technology education students interested in science and math through the use of a design project. This T-shirt design project was designed for junior and senior level high school students who have completed or are currently taking physics and precalculus.

It has been the authors' experience that students become enthusiastic about applying science and mathematics for solving interesting and complex problems, such as an optimum design to launch a T-shirt across a stadium. The project requires students to use the following science principles and mathematics concepts in their designs:

Physics Principles	Mathematics Concepts
Projectile motion	Trigonometric functions of angles
Pascal's principle	Trigonometric identities
Newton's second law	Solving literal equations
Kinematic motion	Solving systems of equations

This project offers opportunities for students to use educational technology as well. For example, students can use spreadsheet software such as Microsoft Excel to perform repetitive calculations that will later assist them in selecting optimal design parameters. They can also use drafting software to finalize and communicate the design to others.

DESIGN BRIEF FOR T-SHIRT LAUNCHER

The task for this project involves designing an air-powered device to distribute rolled-up T-shirts to people sitting in a large venue, such as a stadium or auditorium. For this paper, the project was situated in a sports arena, and the design teams were instructed to design a T-shirt launcher with the following constraints on the launch device:

- T-shirts must reach fans seated at the upper level, 40 ft. horizontal and 58 ft. vertical distances from the launch device.
- Nozzle must be locked at a fixed angle.

- Launcher must be portable and must not damage floors (e.g., hardwood, carpet, etc.).
- Launcher must operate free from the air compressor during launch.
- T-shirts will be launched with a maximum pressure of 50 psi.
- Launcher can be built within a budget of $170, maximum.

THE ANALYTICAL DESIGN

Students use their understanding of the aforementioned mathematics concepts and science principles to determine the launch velocity, nozzle length, and launch angle needed to project T-shirts to reach the fans seated in the stands of a sports arena (Figure 1) and meet the other design constraints. In this paper, we offer one detailed explanation for solving this problem for teachers' use in understanding our process, but not for guiding students. Our intent is to offer adequate explanation to teachers so they may provide sufficient, but minimal support to maximize students' opportunities to think and design independently in meaningful ways. Keep in mind that this design project was meant to engage students in applying mathematics concepts and science principles in real-world contexts; if all challenges and decisions are removed during classroom implementation, the project is reduced to little more than an exercise.

We started our design process by examining projectile motion given the situation (Figure 1). Students who have taken or are currently taking physics and precalculus will likely be ready to begin thinking about the project and working with little or no guidance. Teachers will need to consider what level and type of support other students may need in order to maximize their ability to work with peers independently and still be challenged.

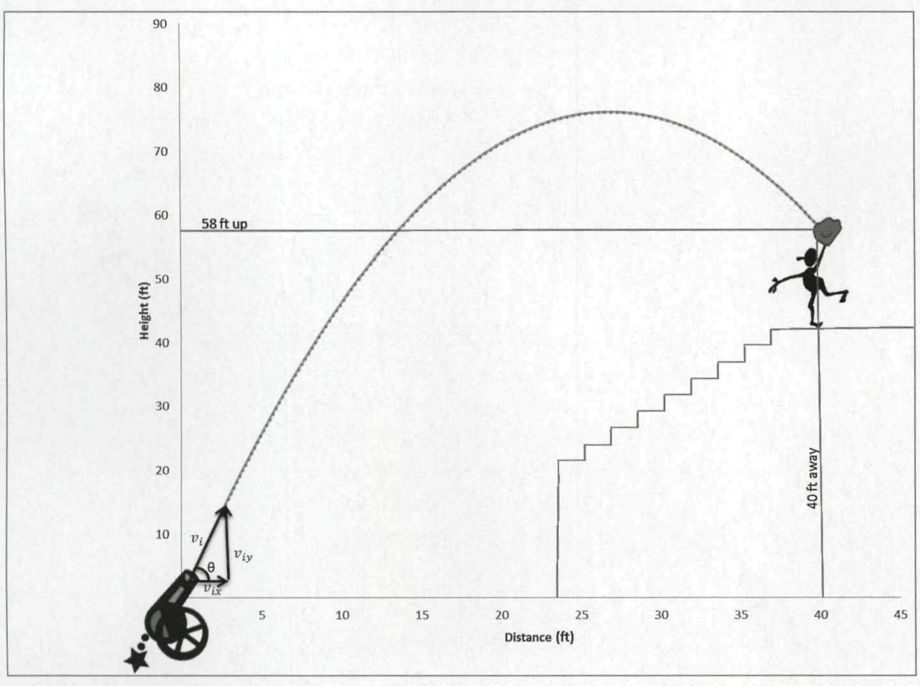

Figure 1. The T-Shirt Launcher's Projectile Motion

Examining the diagram of the T-shirt launcher (Figure 1), we can use trigonometric relationships to write equations for the initial velocity vector, vi, in terms of the launch angle, θ, and velocity vectors, v_{ix} and v_{iy}, for each direction x (horizontal) and y (vertical), respectively as:

$$v_{ix} = v_i \cos\theta \quad (1)$$

$$v_{iy} = v_i \sin\theta \quad (2)$$

The equations of Kinematic motion can be used to relate distance, velocity, acceleration, and time. For this specific situation, we can use the Kinematic equation for uniformly accelerated linear motion in one direction at a time. That is, we can write two equations, one to express the horizontal motion in terms of x, and another to express the vertical motion in terms of y. The Kinematic motion equation we use is given by:

$$s = ut + \frac{1}{2}at^2 \quad (3)$$

where

s = the distance between the start and end positions of the T-shirt

u = initial velocity in a specific direction (x, horizontal and y, vertical)

a = acceleration, a constant for this situation

t = the time taken for the motion

The only acceleration force involved with the T-shirt flight is due to gravity, a constant (-32.2 ft/s^2). Gravity only applies to the vertical direction, y. The acceleration for the horizontal direction, x, is 0 ft/s^2. Using equation (3), we can write two equations, one for *x* and one for *y*. For the horizontal motion of x, substitute 0 ft/s^2 for acceleration, and for the vertical motion of y, substitute *G* for acceleration, and we get the following two equations:

$$s_x = v_{ix}t \quad (4)$$

$$s_y = v_{iy}t + \frac{1}{2}Gt^2 \quad (5)$$

We can make the equations easier to read by replacing s_x with x and s_y with y. Additionally, to reduce the number of variables within the two equations, we can substitute the initial velocities from equations (1) and (2) into equations (4) and (5) for each direction, respectively.

$$x = v_i t \cos\theta \quad (6)$$

$$y = v_i t \sin\theta + \tfrac{1}{2} Gt^2 \quad (7)$$

(Note, by convention the *t* was written before the trigonometric function in each equation after the substitution was made.)

This is a good point to offer students who require guidance an opportunity to work independently. Equations (6) and (7) are a system of equations that need to be solved. Give students an opportunity to consider options for proceeding. Perhaps they will contemplate using substitution as a method for combining the system into a single equation with one unknown; allot sufficient time for them to contemplate options. There are many variables within the system; however, the design constraints provide numerical values for some variables, and perhaps students will recognize this to help them as they contemplate a solution.

In our design approach, we solved literal equations; equation (6) was solved for initial velocity, v_i, as shown in equation (8). Then, we used substitution to solve the system of equations. We substituted v_i from equation (8) into equation (7) to get equation (9).

$$\mathbf{v_i} = \frac{x}{t\cos\theta} \quad (8)$$

$$y = \frac{x}{t\cos\theta}\, t\sin\theta + \tfrac{1}{2} Gt^2 \quad (9)$$

Then we solved the equation for *t*, substituted the known and given numerical values, and simplified the equation. For this example, we know our T-shirt launcher must launch shirts to the top of the arena, which is x = 40 ft. and y = 58 ft, (refer to Figure 1) and gravity, G = -32.2 ft/s^2. We can further simplify the equation by using the trigonometric identity, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Thus, using these numerical values and trigonometric identity, solving for *t*, and simplifying gives us:

$$t = \sqrt{\frac{58\ ft - 40\ ft\ (\tan\theta)}{-16.1\ ft/s^2}} \quad (10)$$

$$v_i = \frac{40}{t\cos\theta} \quad (11)$$

Summarizing, we have a system of two equations (10) and (11) that we can use to find the T-shirt flight time for a fixed nozzle angle, and an initial velocity. Equation (10) is an equation in two variables, *t* (shirt flight time) and θ (nozzle launch angle); if we pick one value, we can find the other. Then, using equation (11), we can find the initial velocity, v_i for each launch angle, θ and the corresponding time, *t*.

Theoretically speaking, there exists a minimum launch angle and a maximum launch angle that will get the T-shirt to the top of the stands in the sports arena, and between these angles there are an infinite number of launch angles. For example, if we select a nozzle launch angle of 60 degrees, substituting that value into equation (10) gives a flight time, *t*, of 0.837s. Then, substituting $t = 0.837$s and $\theta = 60$ into equation (11) gives an initial velocity of 95.57 ft/s or 65.16 mph. Repeating the process for a launch angle of 70 degrees and 80 degrees results in initial velocities of 65.14 ft/s and 71.13 ft/s respectively. Each launch angle will have a different trajectory, as shown in Figure 2.

Determining an optimal launch angle and velocity can be done using different educational technology approaches, such as graphing calculators, spreadsheets, etc. This is an optimal decision point for students to make, so allow them the opportunity to decide how to proceed.

For this article, we describe our use of a spreadsheet to inform our design decision. Comparing launch angles and corresponding velocities can be easily shown using a spreadsheet. In Column 1, labeled Theta (deg), we entered values ranging from 50 degrees to 89 degrees, increasing by 1 degree increments. In Column 2, labeled Time (s), we found the time by entering

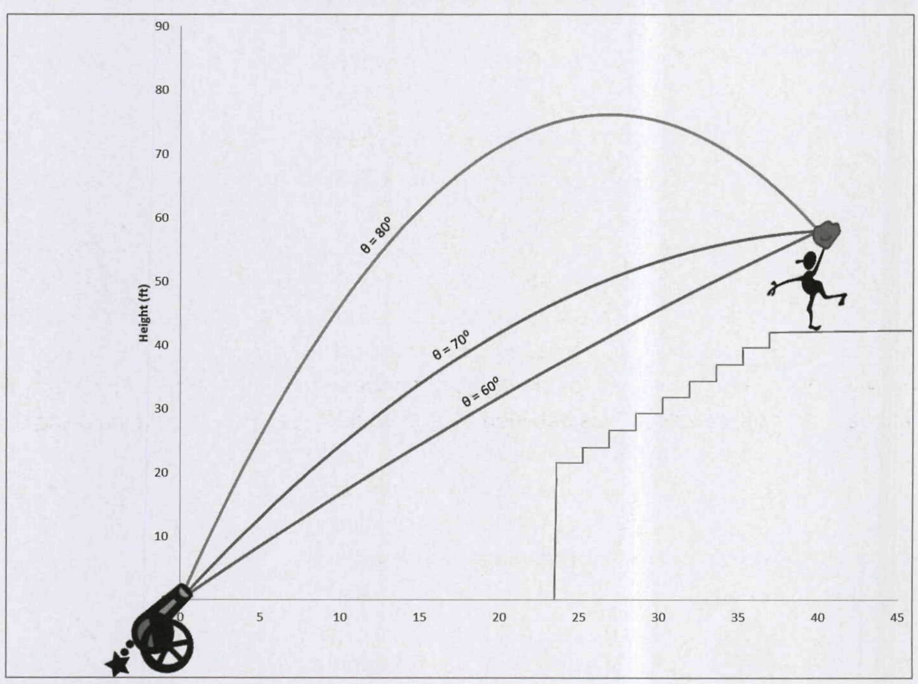

Figure 2. T-Shirt launch trajectories for nozzle angles 60°, 70°, and 80°.

equation (10) and using Column 1 values for the launch angle within the equation (formula), thus computing the shirt flight time for each corresponding angle in Column 1. In Column 3, labeled Initial Vel. (ft/s), we used equation (11) to find the initial velocity that corresponded to values for each launch angle (Column 1) and time (Column 2). In Column 4, labeled Initial Vel. (mph), we converted each initial velocity value from ft/s (Column 3) to mph. We chose to convert initial velocities into mph because our students have a better conceptual understanding of velocities expressed in those units. Selected rows from our spreadsheet are shown in Table 1:

Table 1: Selected Rows From Spreadsheet Calculations

Theta (deg)	Time (s)	Initial Vel. (ft/s)	Initial Vel. (mph)
50	#NUM!	#NUM!	#NUM!
51	#NUM!	#NUM!	#NUM!
55	#NUM!	#NUM!	#NUM!
56	0.284424	251.4968936	171.4751547
57	0.472511	155.4314746	105.9760054
58	0.611148	123.510493	84.21169977
70	1.795424	65.13903866	44.41298091
71	1.900775	64.63791239	44.0713039
72	2.010954	64.36882539	43.88783549
73	2.126936	64.32357832	43.85698522
74	2.249866	64.50080996	43.97782497
75	2.381111	64.90588089	44.2540097
76	2.522338	65.55133534	44.69409228
77	2.675621	66.45802133	45.31228727
87	6.618464	115.4788935	78.73560919
88	8.218484	139.4598168	95.08623874
89	11.77849	194.5875543	132.6733325

Using this spreadsheet we were able to draw several conclusions and make design decisions based upon relationships between the calculated values for the range of nozzle angle values. **Note**: For nozzle angles between 50 and 55 degrees, initial velocities are undefined. A quick analysis of equation (10) shows that until $\theta > 55$ degrees, the value inside the radical is negative, a direct consequence of the design constraints placed on the horizontal (40 ft.) and vertical (58 ft.) distances that must be traveled to reach the stands (refer to Figure 1). This suggests that the T-shirts are unable to reach the stands when the nozzle angle is less than 55 degrees, and this is true independent of the initial velocity.

While the minimum angle needed to reach the stands is 56 degrees, the T-shirt would need to be launched at speeds over 171 mph! Surely, 171 mph is an unsafe speed for launching T-shirts at fans in an arena. In our analysis, we decided to pick the optimal launch angle of 73 degrees because it corresponded with the lowest initial velocity; perhaps the best choice to ensure the safety of the fans. Thus, for the remainder of our design, we used a launch angle of 73 degrees with initial velocity of 64.3 ft/s.

NOZZLE LENGTH

Neglecting friction, Newton's second law, F = m*a, informs us that the T-shirt accelerates as long as it is in the nozzle. Therefore, we need to determine how long to make the nozzle in order to accelerate to the velocity of 64.3 ft/s required to deliver the T-shirt in the stands. Other factors determining the length of the nozzle are the diameter of the nozzle and the air pressure. The diameter is determined by the size of a rolled up T-shirt and commonly available material sizes. We assumed that the holding tank for the air is large enough that the pressure drop as the T-shirt moves through the nozzle is negligible.

The nozzle design begins by applying Pascal's principle that allows us to determine the force on the T-shirt while it is accelerating through the nozzle. The relationship between the air pressure (P), nozzle cross-sectional area (A), and force (F) is given by:

$$F = PA \qquad (12)$$

The cross-sectional area of the nozzle was found using the equation for the area of a circle as given in Equation 13. Area is given in terms of D (diameter) in lieu of the more common circular area formula in terms of r (radius) because PVC pipe is typically sold in terms of diameter and length. If the students question the formula, allow them to use whatever formula they prefer.

$$A = \frac{\pi D^2}{4} \qquad (13)$$

For our example, we will assume a 3" diameter nozzle and a T-shirt weighing 0.4375 lbs. Using Equation 13, the cross-sectional area is 7.07 in^2. Then, using Equation 12 and using P = 50 psi from the design constraints and the cross-sectional area, the force on the T-shirt is 353 lbs. Next solve Newton's second law for acceleration, *a*, to get Equation 14.

$$a = \frac{F}{m} \qquad (14)$$

Using Equation 14 we can find the T-shirt's acceleration from the T-shirt's force from the air pressure (353 lbs.) and mass

(0.4375 lbs.) to get 806.9 ft/s[2]. Now that the acceleration and the velocity needed at the end of the nozzle are known, we can find how far the T-shirt must travel with this acceleration to reach our selected launch velocity (64.3 ft/s), or the length of the nozzle. Revisiting Kinematic motion, the relationship we need is one between final and initial velocities and constant acceleration. The equation is:

$$v_f^2 = v_i^2 + 2a\Delta s \qquad (15)$$

Where

v_f = final velocity at the end of the nozzle, 64.3 ft/s for our example

v_i = initial velocity of the T-shirt, 0 for our example

a = acceleration of the T-shirt, 806.9 ft/s2 for our example

Δs = distance the T-shirt accelerates, unknown length of nozzle

Solving literal equations, we can rewrite Equation 15 to find the length of the nozzle, Δs:

$$\Delta s = \frac{v_f^2 - v_i^2}{2a} \qquad (16)$$

For our design, and with respect to the nozzle, the final velocity, v_f, in Equation 16 is the speed at which the T-shirt will leave the launcher; however, up until now we have referred to this value as our initial velocity. Thus, v_f = 64.3 ft/s, which we selected from the initial velocities calculated in our spreadsheet in Column 3 (refer to Table 1). The initial velocity, vi = 0 is used for Equation 16 because when the T-shirt is placed in the nozzle, it will not be moving, and a = 806.9 ft/s2, which we calculated using Equation 14. Thus, making the appropriate substitutions into Equation 16, the nozzle length, Δs, is 2.56 ft.

In summary, to reach a spectator in the stands 40 feet horizontally and 58 feet vertically with 50 psi pressure and a launch angle of 73 degrees, the T-shirt launcher must have a nozzle length of 2.56 feet (or 2 ft. 6 ¾ in.). Depending on the level of science and mathematics understanding of the students in your class, certain elements of this design can be shown, while some elements are left to students to design using their own decisions and analysis to determine their desired launch angle, nozzle diameter, and T-shirt weight. Then design approaches may be compared using different criteria.

Photo Credit: Francis Hauris and Robert Stuart

THE PHYSICAL DESIGN

The basic design for a T-shirt launcher contains three parts: an air storage chamber, valve, and nozzle. The analytical design has given us the design length of the nozzle. The air storage tank's volume should be large enough so that changes in pressure during launch can be neglected (at least twice the volume of the nozzle). The valve can be a ball valve or another type of fluid valve such as a sprinkler valve. Sprinkler valves are efficient and work well, but they must be modified to work. Numerous examples with various complexities and features are found commercially and on the Internet as well as in journals (Gurstelle, 2011). Students should begin the physical design process by researching what others have done and how applicable it will be to meet their particular design criteria, especially considering the budgetary constraints.

Ideally, the engineering design process will be used by the students to create several possible designs of the T-shirt launcher. While all designs should accomplish the goal of getting the T-shirt to travel the specified distance, other criteria such as aesthetics, weight, cost, and ease of use can be used to choose an optimal design.

NATIONAL STEM STANDARDS

This project spans the science, technology, and mathematics curriculum through the use of design. Standards are addressed in each respective subject area as shown in Table 2 (page 20).

EXTENSIONS

Some students will want to explore methods to produce a T-shirt launcher that has greater capabilities than one size shirt shooting at one set distance with a set air pressure. A possible extension is for students to design multiple nozzles based on launch distances needed. The physical design could allow for screwing different nozzles on/off or a rotating system similar to a Gatling gun. A similar system could be used for T-shirts with different weights and/or sizes. Another option is for students to develop a table relating air pressure and distance the T-shirt will travel. The physical design should have a way to adjust and gauge the air pressure before each launch. This can be done using a secondary air chamber with a bleed-off valve and pressure gauge. For students seeking an additional analytical challenge and who are familiar with calculus, a more accurate length of nozzle can be found by developing an equation for the change of volume as a function of time as the T-shirt travels up the nozzle. The change in volume relates to the change in pressure as a function of time through Boyle's gas law ($p_1V_1 = p_2V_2$). From there, students can develop equations for the nonconstant force and acceleration of the T-shirt. Another possible extension is to examine how different projectile shapes affect the flight trajectory. An example of how this can be done is found in a previous *Technology and Engineering Teacher* article (Lammi & Greenhalgh, 2011).

SUMMARY

Student motivation to use science and mathematics can be generated from the excitement of an authentic engineering design project. The challenge is getting students to use grade-appropriate science and math before deciding on a final design solution. To accomplish this challenge, the criteria and constraints must contain grade-appropriate requirements for a successful design. The authors have found that the T-shirt launcher design project has generated enough excitement that students are willing to tackle difficult scientific and mathematic concepts within their designs. Hopefully you will too.

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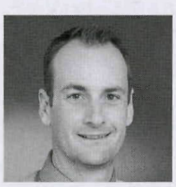
Table 2: Connections to National Technology, Science, and Mathematics Standards

NATIONAL STEM STANDARDS		
Content Area	**Standard**	**Description**
Standards for Technological Literacy (ITEA/ITEEA, 2000/2002/2007)	Standard 3	Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.
	Standard 8	Students will develop an understanding of the attributes of design.
	Standard 9	Students will develop an understanding of engineering design.
	Standard 10	Students will develop an understanding of the role of troubleshooting, research and development, invention and innovation, and experimentation in problem solving.
	Standard 11	Students will develop the abilities to apply the design process.
Benchmarks for Science Literacy (AAAS, 1993)	3A/H2	Mathematics, creativity, logic, and originality are all needed to improve technology.
	3A/H4	Engineers use knowledge of science and technology, together with strategies of design, to solve practical problems. Scientific knowledge provides a means of estimating what the behavior of things will be even before they are made. Moreover, science often suggests new kinds of behavior that had not even been imagined before, and so leads to new technologies.
	3B/H1	In designing a device or process, thought should be given to how it will be manufactured, operated, maintained, replaced, and disposed of and who will sell, operate, and take care of it. The costs associated with these functions may introduce yet more constraints on the design.
	4F/H1	The change in motion (direction or speed) of an object is proportional to the applied force and inversely proportional to the mass.
	4F/H7	In most familiar situations, frictional forces complicate the description of motion, although the basic principles still apply.
Common Core State Standards for Mathematics (2011)	MP.1	Make sense of problems and persevere in solving them.
	MP.4	Model with mathematics.
	A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
	A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
	G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.



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