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Abstract: This paper identifies the knowledge that emerges from collaborative situations, in an initial teachers' education with integration of digital technology for teaching of function, in a blended and collaborative modality course, aiming to emerge Technological, Pedagogical and Content Knowledge (TPACK) in pre-service teachers. The method comprised the design of a teachers' collaborative training experiment. The experiment had five phases, from a theoretical discussion to plan an Instrumental Orchestration to teach function with digital resources. Collaboratively, students worked in groups. The participants were students from a Mathematics teachers' education. The experiment took place in the Teaching Methodology of Mathematics discipline at a public university in Brazil. This article focuses on the analysis of students' knowledge in the planning stage, carried out collaboratively. The data analysis pointed out the three types of Technological, Pedagogical and Content knowledge were identified in different phases of the experiment, sometimes collectively, sometimes individually. In addition, we also identified that some intersections of TPACK emerged from the interactions; despite being in the initial stage of training, the students show pedagogical knowledge linked to knowledge of the content they had at the time of the training experiment.

Keywords: TPACK, Instrumental orchestration, Mathematical function, Collaborative learning, Pre-service teachers' education.

Introduction

Nowadays, the presence of technology in people's lives is a reality in the vast majority of sectors of society. However, some resistance is found mainly in education. Schools still prioritize teaching with traditional resources, timidly using digital technology. Teachers often use digital resources to reproduce traditional teaching models (Lins, 2010). Cantini *et al*. (2006) point out that teachers need to integrate technologies into their practices, starting from their insertion into society. However, there is a lack of teachers' training that contemplates the use of these

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technologies for teaching (Nogueira *et al.*, 2013; Dantas, 2005). Thus, it is necessary to propose, even in pre-service teachers' education, situations for challenging preservice teachers to integrate technologies into classroom practice, exploring the potential of digital technologies for teaching.

In Brazil, the mathematics teachers' initial education curricular matrix at some institutions in the northeast focuses on digital technologic training in terms of algorithms and programming language (Araújo Filho, 2019). In general, pre-service teachers have contact with digital technology in the specific didactic-pedagogical disciplines of the area.

Research studies (Borba, & Penteado, 2010, Bairral, 2007, Charles, & Gitirana, 2014, Nishio, & Hora, 2016) points to the need to integrate technology in initial teachers' education. Also, the National Curriculum Guidelines (DCN) (Brasil, 2001) reinforce this need. Araújo Filho (2019) identified some gaps in the literature regarding teachers' knowledge in initial education. In our literature review, we mapped the research that dealt with the pre-service teachers' knowledge. However, at the national level, he identified that the research addresses in-service teachers' knowledge, who are at in-service education programs.

From this gap, Araújo Filho (2019) proposes an initial teachers' education program, with a diversity of digital technological artefacts, to promote teaching knowledge with technology to emerge in mathematics pre-service teachers. Thus, his object is part of the theoretical framework of TPACK, which is based on the research of Shulman (1986), Mishra and Khoeler (2006, 2008). These authors discuss the integration of content and pedagogical knowledge (Shulman, 1986) and technological, pedagogical and content knowledge (Mishra, & Khoeler, 2006). According to them, the integration of this knowledge and its intersections constitute essential knowledge for teaching practice. The dissociation of technological, pedagogical knowledge and content, does not contribute or contributes incompletely, in the constitution of the teacher's professional profile (Mishra, & Khoeler, 2006).

In this paper, we aim to identify knowledge that emerged from collaborative situations, while subjects elaborate lesson objectives and plan its execution based on the group discussions. Thus, we focus on the analysis analysing the fourth stage of Araújo Filho (2019) experiment and its results.

In the theoretical framework, we discussed the knowledge involved in TPACK and its intersections; computer-supported collaborative learning (CSCL) and its main aspects and collaborative teacher education. Then, we describe the teachers' education experiment method and its respective phases and, finally, we discuss the results and final considerations.

Technological, Pedagogical and Content Knowledge

According to Mishra and Khoeler (2008), knowledge is seen as propositions and skills that someone builds and exercises (Powell, 2014). Shulman (1986) points out that content and pedagogical knowledge are part of this set of essential knowledge for pre-service teacher education. The teachers' education must help them integrate specific content knowledge with didactic and pedagogic ones. The technology development led to the technological knowledge inclusion, introduced by Mishra and Khoeler (2006) to the framework proposed by Shulman, structuring the TPACK triad.

The content knowledge (CK)

According to Shulman (1986), content knowledge is the specific component which should be learned and taught in teaching practice. It is the knowledge that the teacher needs to master. As it needs to be taught, it needs to understand the epistemological nature of the content. Teachers should not only understand “what” or “which”, but also “why”. He/she must ensure as few obstacles as possible to the content knowledge itself in its intersection with the pedagogical knowledge. In doing so, teachers can understand the reason for teaching and learning this content.

The pedagogical knowledge (PK)

Pedagogical knowledge concerns the didactic issues of teaching practice, the teacher-student relationship and the student's social knowledge, teaching and learning methodologies and other didactic elements that constitute teacher training (Shulman, 1986). Some teacher education programs treated content knowledge and pedagogical knowledge mutually exclusive; emphasizing one type in detriment of the other.

Mishra and Khoeler (2006) point out that it is necessary to transform content knowledge into knowledge teaching. It means that teachers have to adapt content knowledge for the teaching and learning process. It is possible only by looking at its intersection with pedagogical knowledge. However, along with technological evolution, other knowledge became necessary in teachers' education programs: technological knowledge, the third element of the triad that forms TPACK.

The technological knowledge (TK)

Technological knowledge refers to the knowledge about non-digital technologies, such as whiteboards, pencils, books, and digital technologies. In the educational context, knowing it means the teacher develops the ability to use the teaching resources, such as books, e-mail, calculators, spreadsheets, files, documents, etc. (Mishra, & Khoeler, 2006). Nowadays, digital technologies are in people's lives; however, their insertions in education are rare.

In the scenario of the new coronavirus pandemic, we realize that many teachers are focused on training programs that deal with the use of digital technologies. Moreover, the need to use technology leads many of them to autonomous training by research on the web.

Despite the emerging need for technological knowledge, there are still factors that need to be added to the reality of remote education, as the potential of digital technologies is not always used to transform teaching. It is necessary to discuss technological knowledge combined with content knowledge and pedagogical.

Technological Pedagogical Content Knowledge (TPACK)

TPACK emerges from integrating TK with CK and PK. Mishra and Khoeler (2006) introduce TK to Shulman's model (1986), which combined only CK and PK. Next, we show the proposed model for the integration of the three types of knowledge.

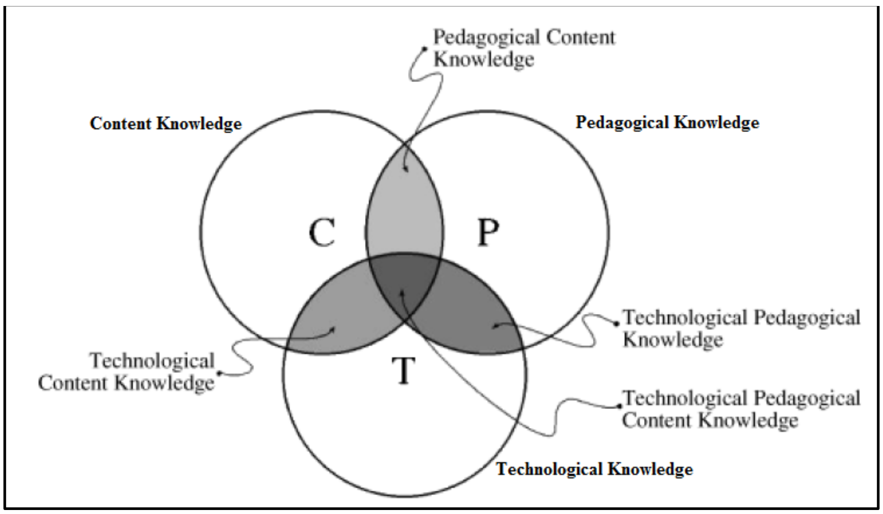


Figure 1: TPACK diagram (Mishra, & Khoeler, 2006, p. 1026)

According to Mishra and Khoeler (2006):

> Technological content knowledge (TCK) is knowledge about the manner in which technology and content are reciprocally related. Although technology constrains the kinds of representations possible, newer technologies often afford newer and more varied representations and greater flexibility in navigating across these representations.
> Pedagogical and Content Knowledge (PCK): The idea of pedagogical content knowledge is consistent with, and similar to, Shulman's idea of knowledge of pedagogy that is applicable to the teaching of specific content. This knowledge includes knowing what teaching approaches fit the content, and likewise, knowing how elements of the content can be arranged for better teaching.
> Technological and Pedagogical Knowledge (TPK): is knowledge of the existence, components, and capabilities of various technologies as they are used in teaching and learning settings [...] This might include an understanding that a range of tools exists for a particular task, the ability to choose a tool based on its fitness, strategies for using the tool's affordances, and knowledge of pedagogical strategies.
> Technological, Pedagogical and Content Knowledge (TPACK): is an emergent form of knowledge that goes beyond all three components [...] This knowledge is different from

> knowledge of a disciplinary or technology expert and also from the general pedagogical knowledge shared by teachers across disciplines. TPCK is the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (Mishra, & Koehler, 2006, p. 1027-1028).

According to Powel (2014) and Mishra and Khoeler (2006), it is necessary to observe dyads TCK, TPK and PCK to find them in TPACK. The authors claim that TPACK can start with a teaching situation exploration when teachers realize that technology needs didactic or content support and when content needs didactic or technological support. They also emphasize that it is necessary to clarify that technological knowledge does not arise from the simple use of a digital or non-digital resource but how teachers use them for teaching (Mishra, & Khoeler, 2006).

To analyze those types of knowledge in teaching situations, Powell (2014) suggests looking at them separately, or even at the mentioned dyads. It is very complex to observe TPACK in a global situation, searching to show the interception of those pieces of knowledge from an entire situation or a class. In this paper, we bring a synthesis of the analyzes of teachers' initial training, analyzing emerging knowledge from Powell's perspective.

In the next section, we introduce Computer-Supported Collaborative Learning that helped us structuring our experiment in terms of design and interactions.

Computer Supported Collaborative Learning (CSCL)

Computer-Supported Collaborative Learning (CSCL), according to Stahl *et al.* (2006), is a theoretical framework to investigate the constraints and potentialities of synchronous and asynchronous interactions between people who seek to solve problems or proposed situations in a computational environment.

There is no consensus among researchers about the concept of collaboration. It is confronted and confused with cooperation, which differs from collaboration in some aspects. Stahl et al. (2006) distinguish them as follows:

> In cooperation, learning is carried out by individuals who contribute to their individual results and present their aggregation as the product of the group. [...] in collaboration, individuals are involved with group members, but the activities in which they are engaged are not individual learning activities, but in group interactions, such as negotiation and sharing. Participants do not isolate themselves to perform activities individually, but remain engaged in a shared task that is built and maintained by and for the group as such. (Stahl et al., 2006, p. 3).

Dillenbourg (2002) affirms that collaborative learning to be effective depends on multiple conditions, from the characteristics of subjects (such as age and gender) to the instruments that promoted the communication. Therefore, in a CSCL environment for mathematics teaching, we should think about which software allows collaboration. The assembled environment should favour various audio, video, and writing forms, especially if it is synchronous. After the communication, we must be concerned with the software needed to solve the problem or situation proposed; if the situation requires an algebraic language, one should think of some platform that offers interaction through this representation support.

After deciding the resources, the session needs a script (Dillenbourg, 2002), defined as "a more detailed and more explicit didactic contract between the teacher and the group of students regarding their model of collaboration and how they should solve the problem" (p. 62). The script should describe the actions that participants must follow and the necessary resources to the moment of the interaction. Dillenbourg presents some script templates. In this research, we chose to use the instruction script. The instruction script is a model in which the mediator describes, in instruction form, everything that participants should do, describing step by step what should be done either by audio or in writing.

Another important concept when talking about CSCL is group learning. The interaction of the participants does not occur spontaneously, without motivation for arguing about the proposed problem. Online argumentation has been investigated with actual results for collaboration. Stahl (2012) argues that collaborative argumentation in online environments develops a greater capacity for individual argumentation.

The mediator of the collaborative session has a crucial role in the learning process. It can influence the process, similar to the structure influences, promoting more productive interactions (Dillenbourg, 2002). At CSCL, the mediator leads the discussion and seeks to eliminate arising obstacles during the process. We agree with Dillenbourg (2002) when he argues that the mediator should take a position when necessary in the discussion, without interfering in the groups' dynamics.

Gonçalves (2013) used Bakers' analysis model to analyse the collaborative interactions. It structures the analysis of the collaboration in three dimensions: symmetry, agreement and alignment. Those dimensions classified the participants' positions in the sessions. Thus, when someone argues, we see whether s/he agrees or not; and whether there is or not collaboration among participants.

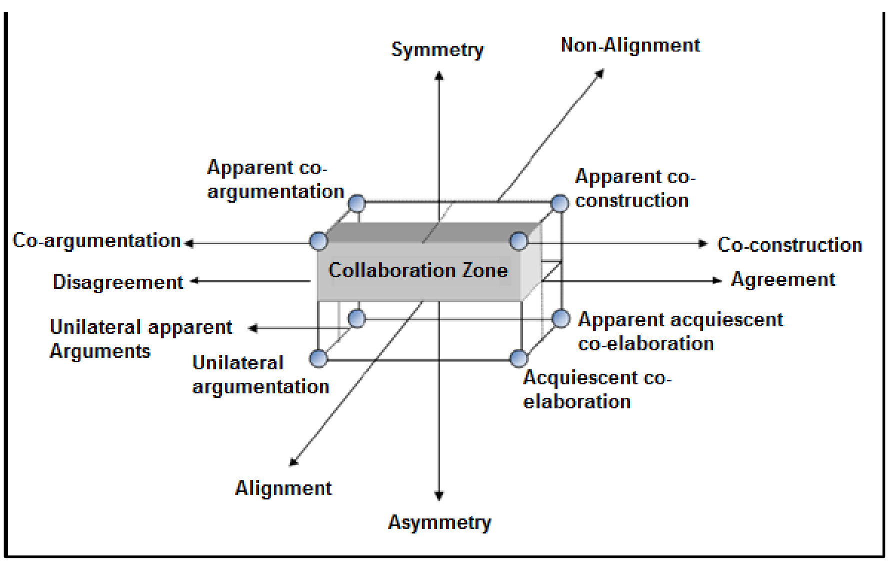


Figure 2: Baker's analysis model (Gonçalves, 2013, adapted from Baker, 2002, p. 592).

For analyzing the symmetry dimension, we characterize the role of each participant in the session as a proposer, who argues, or receiver, who responds to the argument placed. The agreement is a dimension that defines the positioning of the receiver. S/he can assume four positions in the face of the agreement: implicit-positive, explicit-positive, implicit-negative and explicit-negative. The implicit-positive position means that the participant partially agrees with the proposer's argument, while the explicit-positive when s/he explicitly agrees with the proposer. Similarly, the implicit-negative occurs when the receiver partially denies the proposer's argument and the explicit-negative, when he effectively denies the arguments.

Alignment defines the user's participation in the session. More specifically, the participant may be present at the session but not be collaborating, and s/he may just be watching the interaction. On the other hand, s/he may also be collaborating but not performing the same activity as the other participants. Thus, the alignment is subdivided into four categories: in phase, out of phase, in grounding, and out of grounding.

The participant is *in phase* when they perform the same activity as the others, for example, they are all discussing in a chat. Otherwise, we say it is *out of phase*. When he is *in reasoning*, he is collaborating and on the same semantic basis as the other participants. Otherwise, we say that it is out of grounding.

The three dimensions described guide the session structure and, combined with the epistemological knowledge treated in the research, form categories to define and help us analyze the interaction. In Figure 2, we have a schema that brings together the three dimensions of collaboration and the subsequent forms of collaboration. Following forms of collaboration are defined based on the three dimensions, following the research by Gonçalves (2013), collaboration occurs when the problem is appropriated. The author considers that appropriation occurs in situations where there is co-construction or co-argumentation.

According to Baker (2002), co-construction happens if the interaction is aligned, symmetrical, and the students agree. On the other hand, co-argumentation occurs if the interaction is aligned, symmetrical and the students are at odds (usually, they are arguments in which they propose solutions and alternatives to resolve the situation).

We used Baker's model to analyze the interactions between teachers in training, and, in parallel, we map whether or not there was collaborative learning from identified knowledge.

Collaborative Teachers' Education

The teacher education process from a collaborative perspective appears in the literature, focusing on teachers' continuing education. Here, we will discuss some research results from the perspective of collaborative training.

Powell (2014) discusses the results of a collaborative geometric situation undertaken with in-service teachers' education by discussing the problem through VMTChat. The collaborative interactions allowed in this environment use a Geogebra platform that allows users to share control of actions in the software and include a chat. In addition, the research aimed to map the teachers' knowledge based on TPACK and, from the collective interactions, it was clear that collaborative interactions contributed to users reaching the intersection between TPACK kinds of knowledge.

On the other hand, in pre-service teachers' education, Araújo Filho (2015) carried out an experiment that analyzed teachers' learning in a collaborative situation of planning a lesson to teach functions. The research project based its method on Stahl et al. (2006) and Dillenbourg (2002) to build the experimental model. The model included an instruction script with an environmental design that added different software for sharing information. The results showed that the groups that interacted better collaboratively were the small ones, reaching collaborative learning, according to Bakers' analysis model.

Method

Our study used an experimental method for the pre-service Mathematics teachers' education lesson to plan a class to teach functions collaboratively.

We chose as the locus of the research, a discipline of Methodology of Teaching Mathematics, from a pre-service undergraduation for mathematics teachers at a Brazilian state university. The discipline aims to discuss theoretical aspects linked to teacher practice involving mathematical concepts. Thus, in the research context, teaching and learning functions are objects of the discipline. This course had the purpose of inserting the undergraduates in a context of integration of digital technologies to the theoretical-methodological discussions included in the syllabus. In addition, we had access and consent from the lecturer responsible for the class and the students.

Here, we discuss all stages of the training experiment proposed in the main study. It is necessary to discuss all the stages to understand the context of the 4th stage of training, which is the object of analysis and discussion in this study. We chose the fourth stage since the students discussed the class planning in this stage, starting from the aims and concluding with the activity proposals and the use of software for the teaching of Mathematics.

Subjects

Twenty-one undergraduate students participated in the research. To identify the subjects, we will use the following nomenclature: S5, S6, S7 and S8 which correspond to: student 5, student 6, student 7 and student 8. Next, we will discuss a little about the etape of the experiment.

Stages of the experiment

The training experiment had five stages, as follows:

- **1st Stage** - Theoretical training on semiotic representations and their relevance in teaching functions.
- **2nd Stage** - Technological training on Geogebra
- **3rd Stage** - Constitution of the collaborative groups
- **4th Stage** - Collaborative sessions - Planning a lesson to teach functions with Geogebra
- **5th Stage** - Presentation of the lesson in the Methodology classroom

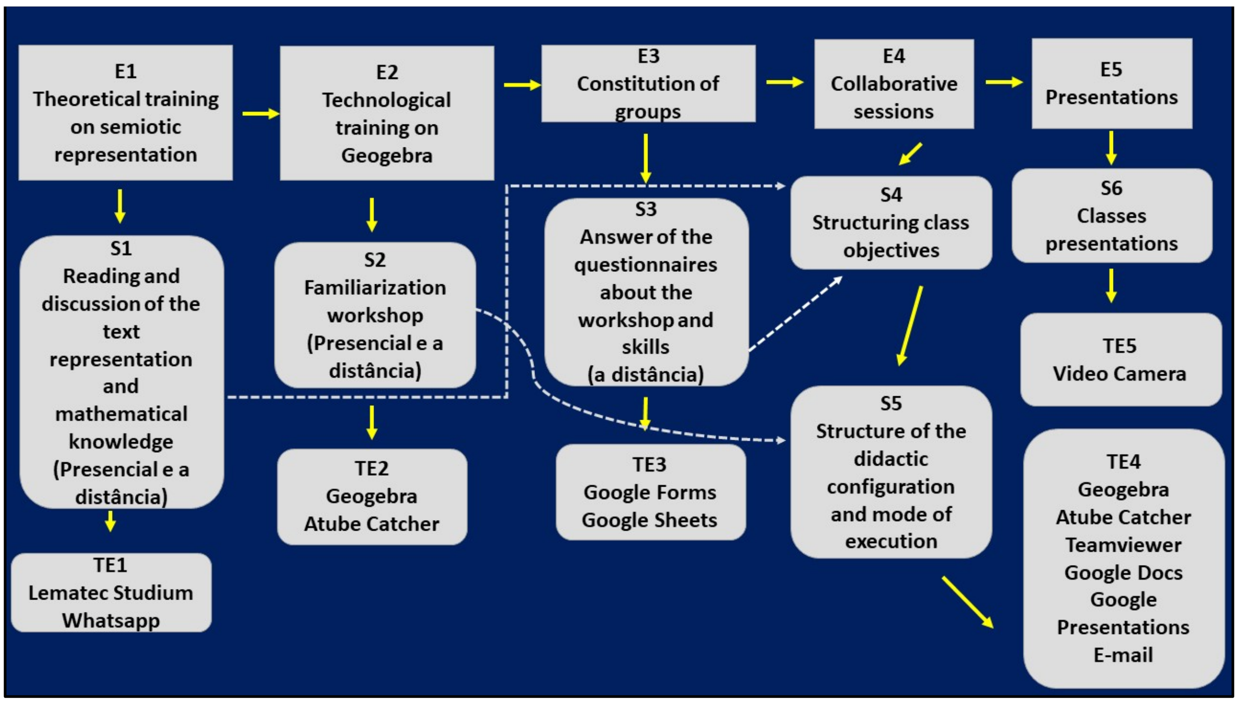


Figure 3: General outline of the training experiment (translated from Araújo Filho, 2019, p. 69)

In Figure 3, we show each stage (E), its situation (S) and the technologies (T) used. The continuous lines connect the proposed situations and the technologies used within them. On the other hand, the dotted lines interconnect the situations of different stages, which directly influence the performance of the other stages. At each stage (E1, ..., E5), we have situations (S1, ..., S6) proposed to undergraduates intending to support the collaborative elaboration of an instrumental orchestration using different technologies (TE1, ..., TE5). In stage E1, situation S1 was essential for the students to discuss and structure the S4 class objectives, just as S2 helped directly in S5 in executing the class proposal.

1st Stage

As the proposed experiment is an online collaborative training, we realized the students' need for theoretical appropriation. In this sense, in the first stage, we planned a theoretical discussion on the Theory of Semiotic Representation Records (TSRR) (Duval, 2011), focusing on functions representations, given their relevance for the teaching and learning of this mathematical object.

Therefore, the first stage was organized as follows:

A. Each student should previously read the text "Representation and mathematical knowledge" (Lucena et al., 2016) and watch the video "Representations and learning of mathematics" (Gitirana, 2014). These resources served as a theoretical framework for the students to reflect on the representations to learn functions. The authors introduce the firsts notions of TRRS and exemplify, through some situations, the semiotic transformations, treatment and conversion, regarding functions.

B. Then, the students had a week to elaborate a question and send it to the group and respond to at least one of the questions made by their colleagues.

At this stage, the students were expected to articulate theoretical discussions - pedagogical knowledge - with questions involving the mathematical object - Function - as content knowledge.

2nd Stage

In the second stage of the training, we aimed to familiarize the students with Geogebra to plan the class. Therefore, we scheduled a face-to-face meeting to build a simulation in the Geogebra, making available different technological devices: smartphones, tablets and laptops.

This stage was carried out during a discipline meet and lasted approximately one hour and thirty minutes.

During the face to face meeting, some difficulties related to the devices used and the time scheduled for the training were identified. With that, it was necessary to expand this stage to the distance modality so that it was not necessary to interfere directly in the teaching time of the undergraduates. We readapted what was planned so that they discussed in groups at a distance, constituted by each one's choice about the construction started in the classroom.

With the change in our planning, we let available a video with the beginning of the simulation construction made in the classroom. The simulation dealt with the construction of a polygon homothety and a graph relating one of its sides to its area. As the polygon enlarged, the point representing the relationship moved in the cartesian system, and its trail was marked, forming the graph. We wished to lead the students to perceive the Geogebra aid to teach functions and discuss their representations based on previous theoretical discussions.

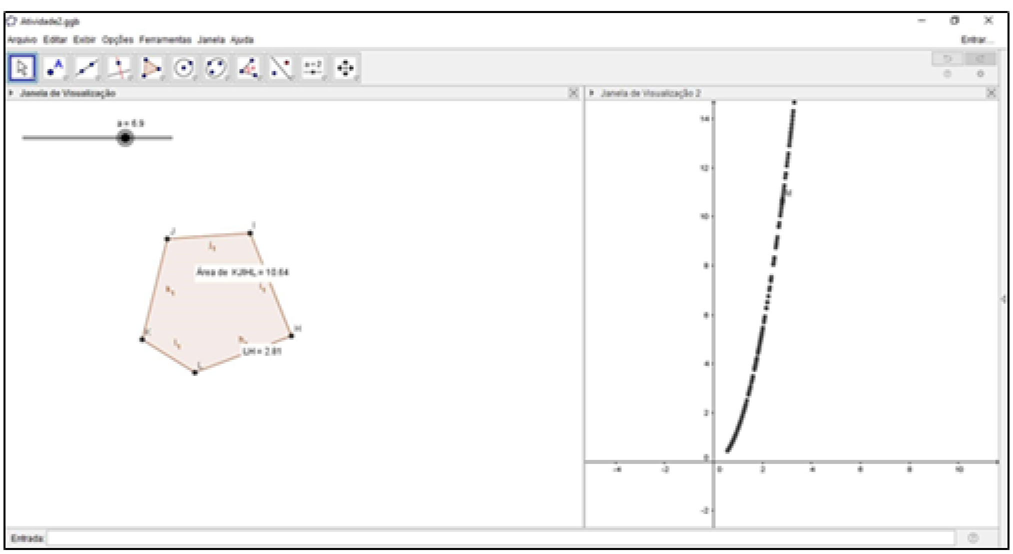


Figure 4: Simulation of the Homothetic (Araújo Filho, 2019, p. 73)

After watching the video, the students had to finish the simulation construction, record the screen with the process, identify the function's graph formed on the screen, and model the algebraic form from the graph. The videos recorded at this stage were sent by e-mail to the researcher and the teachers' educator.

3rd Stage

In the third stage, we organize the students into groups for the collective work. Therefore, we made available an online formulary so that they could self-evaluate their knowledge. We used google forms to build it, using a Linkert scale from 1 to 4, with one for being less skilful and four being very skilful.

TPACK knowledge was our theoretical base in the forms as well; we named them mathematical (content), didactical (pedagogical), and technological knowledge. According to Stahl et al. (2006), for better interaction between subjects in solving a collaborative problem, it is necessary to trace the abilities of group members. So, we grouped the students to have a more skilled member in each group who mapped TPACK knowledge. From there, we proceeded to the stages of class planning.

4th Stage

In the fourth stage, we provided an environment designed to support students' collaboration and class planning. Thus, there were two meetings for each group: the first aimed at elaborating the class objectives, supporting material sharing and discussion among the students; the second, at class planning based on the Instrumental Orchestration (Trouche, 2005). The following is a schematic of the design of the environments for the two meetings:

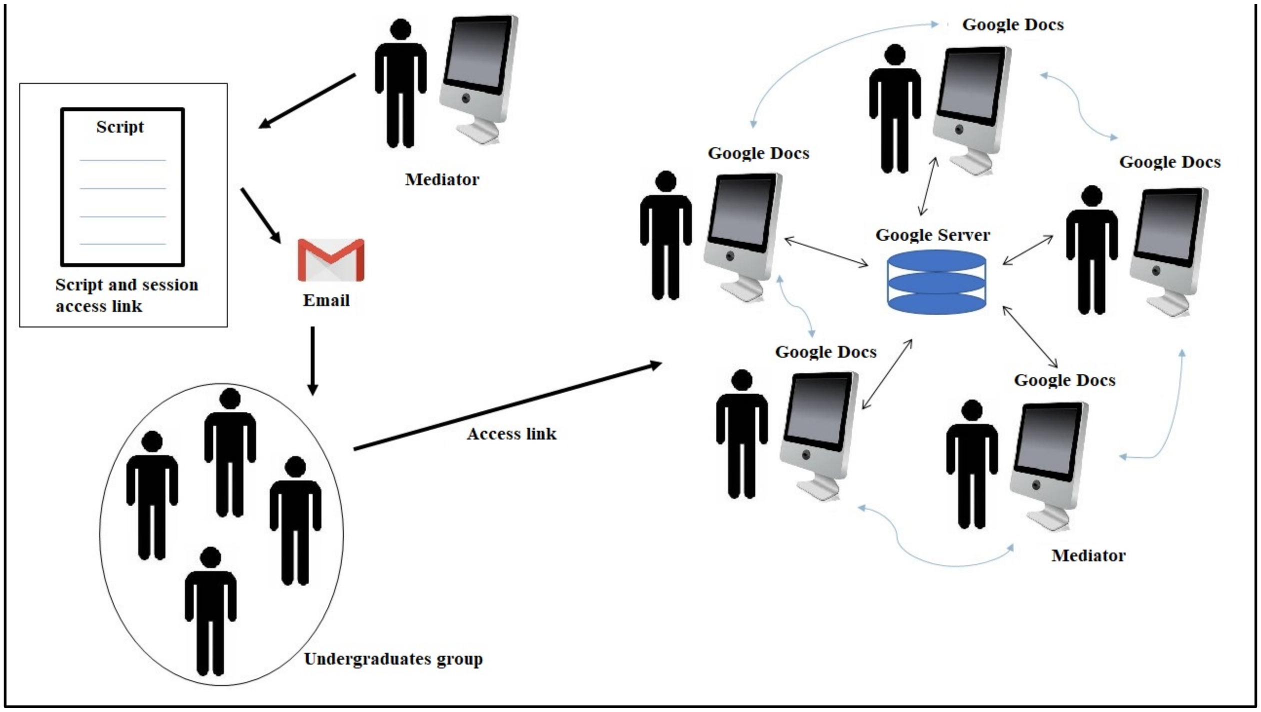


Figure 5: First meeting scheme (translated from Araújo Filho, 2019, p. 78)

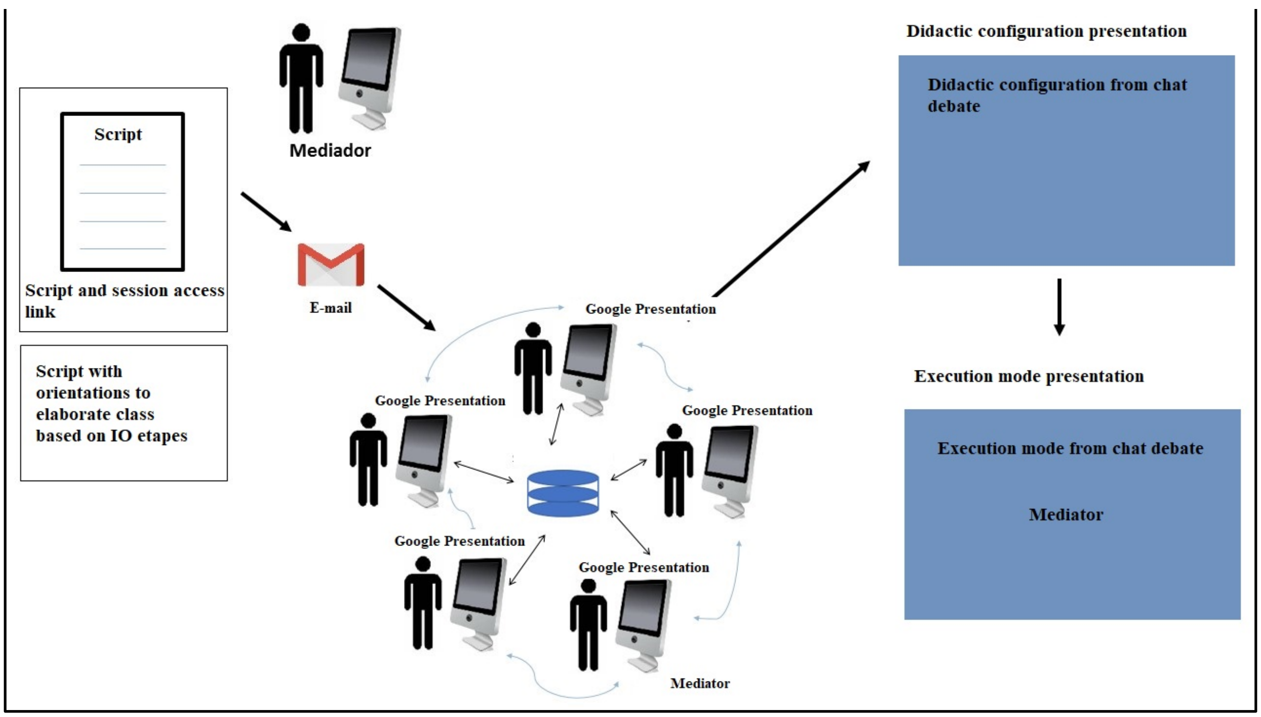


Figure 6: Second meeting scheme (translated from Araújo Filho, 2019, p.79)

When logging into the environment, the researcher briefly reviewed the script sent by email. Then, at the first meeting, they should carry out some tasks:

A. To search in the literature (articles) about difficulties in learning functions.
B. To search the teaching orientation regarding function in the Curricular proposals at the national and the state level: the Common National Curricular Base (BNCC) and the Curriculum Parameters of Pernambuco.
C. To seek the material exploring functions available from the GeogebraTube platform.
D. To see the approaches of functions in the textbooks from the National Textbook Program (PNLD, 2014).

The tasks requested from the students, at this stage, seek to resume their knowledge about the theoretical framework. In item a, we aim to discuss content knowledge and pedagogical content knowledge with current approaches to functions. In item b, we seek to promote pedagogical and content knowledge since it discusses the Brazilian official orientations. In item c, we aim to promote technological knowledge. Furthermore, we search for what is available in terms of digital resources to help students build the ideas and arguments in the class planning. Finally, in item (d), we aim to raise the pedagogical content knowledge developed in the classroom by in-service teachers.

In the second moment, that is, the moment of construction of the activities, we make available another environment of Google Drive, the Google Presentations, or simply the slides, so that the undergraduates could work collaboratively. Then, after defining the class objectives, they should present a roadmap for the development of these classes, based on the components of Instrumental Orchestration (IO) (Trouche, 2005).

> An instrumental orchestration is exactly the systematic arrangement by an intentional agent of the elements (artefacts and humans) of an environment in order to implement a given situation and, more generally, to guide learners in the instrumental genesis and in the evolution and balancing of their instrumental systems (Trouche, 2005, p. 126, our translation).

As IO proposes to assist and investigate the teaching practice in a technology-rich environment, we sought to use two of three essential components of the model - didactic configuration and exploration mode - so that students could plan their classes.

5th Training Stage

In the fifth stage, we intended to socialize the actions carried out in the distance mode to plan the class. Thus, it happened face-to-face, recording all groups' presentations. Students should describe the outlined objectives and the planned IO, its didactic configuration and its exploration mode.

Data Collection

Data collection was carried out considering specificities and the objectives of each stage. We used various techniques and instruments.

Data Collect					
Actions	**1st Stage**	**2nd Stage**	**3rd Stage**	**4th Stage**	**5th Stage**
Data Collet Instruments	Whatsapp, Lematec Studium.	Email, Atube Catcher	Online form, shared spreadsheet, email.	Atube Catcher, Shared document, Google Document chat.	Researchers' camcorder.
Method	Observation.	Footage	Interview	Footage, researcher notes.	Footage, researcher notes.
Data Produced	Answers and questions by text messages.	Computer screen capture video	Response of subjects to chat skills, distribution of subjects to build groups.	Screen capture video, text document, slides presentation, session script.	Presentations videos, documents from subjects slides.
Participants	Researcher	Pre-service teachers			

Figure 7: System of data collection

Our focus in this study will be to investigate the students' interactions during class planning, that is, in the 4th stage. Next section, we detail the data collection process in this stage.

Data collection in the 4th stage

As discussed in the section on stages of the experiment, the 4th stage took place in two distinct moments: in the first moment, which we will treat here as the first session, the students discussed themes provided by the mediator in order to elaborate the objectives of the lesson; in the second session, based on what they had already read about IO, they built the didactic configuration and the exploration mode.

Thus, in the 4th stage, we had a screen capture of their discussion and protocol data. The screen used to capture was the mediator's one. Regarding the protocols, they were from the records left by the students in collaborative google documents and slides, both in the first and the second session. Figure 8 shows the data collected schema.

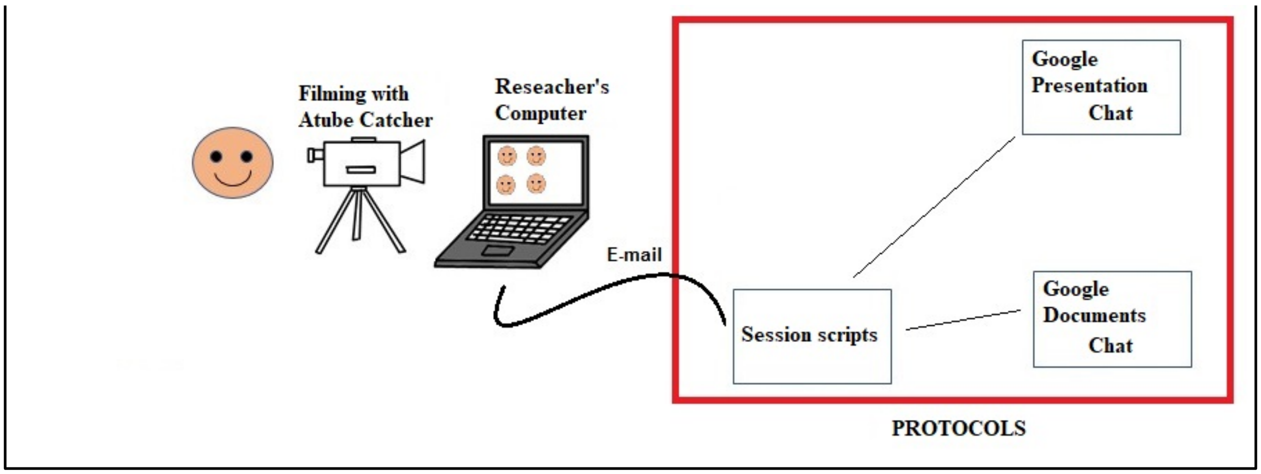


Figure 8: Data collected in 4th stage.

We collected approximately 3 hours of video, considering the two sessions in which we subdivided this stage of the experiment. So that is approximately one hour and thirty minutes in each session.

Data analysis method

As we showed in the previous section, one of the main collection methods used in the 4th stage was screen capture video. In addition, we used microgenetic analysis of the data. According to Meira (1994), it consists of videography, a detailed investigation of the events identified by the researcher. These events are also called episodes. We selected the events from the video records or even from the researcher protocols. The videos must be watched multiple times to identify the events correlated to our research questions. The chat was also the source of the events. We transcribed videos so that we could better analyze the chosen events.

Next section, we will discuss the analysis of the fifth stage, the choice of the objectives and the planning of the stage of the IO.

Data Analysis

This paper will focus on analysing the 4th stage, composed of two interrelated sessions, namely, defining the class objectives and planning the IO. We chose this stage due to the quality of that. In the fourth stage, we identified from collaborative interactions the knowledge that goes toward the intersection of TPACK.

Class Objectives

We selected a group to analyse here based on their self-assessment form. Thus, the group had four students; each one considers him/herself more potent abilities in one of the knowledge dimensions, in such a way as to form a heterogeneous group: S5 (Student 5) - in Mathematics; S6 (Student 6) - in Didactics; S7 (Student 7) - in Mathematics and S8 (Student 8) - in Technologies. To choose the class objectives, they started presenting the role of each one, written in the script. Below, we present the event in which the students justify their attributions:

> **Event 1 - Roles distribution**
> ***L17*** *S8: I was in charge of the technological part and interactions with software because I had made a computer technician.*
> *[...]*
> ***L34*** *S7: I was responsible for the topic of difficulties in learning functions since his division placed me in the mathematical category, and, starting from that point, I seek to understand this problem.*
> ***L35*** *S6: As in the distribution (of roles), I kept the didactic part, I will be student 4.*
> ***L36*** *S5: I kept the textbook approach part because my best skill is math.*
> (Araújo Filho, 2019, p. 141)

This event reveals both the students' self-declared ability and those they attributed to the researcher's distribution. Those two allow us to compare the choices with the actions developed collectively and the knowledge that we will identify in each one.

Then, their discussions changed to address teaching, learning and mathematical content. The discussion emerged from S7's searches in official state government documents, which provides guidance on the teaching of functions, a mathematical object to be addressed by them in class planning. Let us see the event from the conversation:

> **Event 2 - Discussion on curricular proposals for teaching functions**
> ***L57*** *S7: There is little in the curriculum.*
> ***L58*** *S5: Joined the chat*
> ***L59*** *S8: Because it seems to be incomplete. The final version has not come out yet.*
> ***L60*** *S5: Left the chat*
> ***L61*** *S7: But in the Pernambuco Parameters, they talk about the relationship between quantities; I will bring it here:*
> *The study of functions is essential at this stage of schooling, mainly because of its role as a mathematical model for studying variations between quantities in phenomena in the natural or social world. While exploring functions, the most important thing is that the students understand, in addition to modelling real phenomena, aspects related to the growth and decrement of each studied function, which allows them to develop functional thinking. The means removing the emphasis in general attributed to algebraic-symbolic manipulation, ordinarily privileged in high school, shifting the focus from the relationship between quantities to the study of equations and inequalities and calculations with logarithms...*
> ***L62*** *S6: So, do students' difficulties happen because teachers do not teach like that?*
> ***L63*** *S7: I think they confuse many equations with function. You always have to find some x.*
> ***L64*** *S8: It's true… and any graph for them is a function.*
> ***L65*** *S7: In the papers, I observed that teachers work a lot with the algebraic part.*
> ***L66*** *S8: So I think we have to work more with problems, like ENEM*
> ***L67*** *S6: Yes!*
> ***L68*** *S7: I think so too.*
>
> (Araújo Filho, 2019, p. 146)

The conversation brings a discussion from S7's findings. It shows that S6 and S8 engage, connection with the theoretical perspective on the representations, the object of the first stage of the teachers' education experiment. In lines 62 and 63, it is evident that students were concerned with algebrization in the teaching of functions and how teachers approach this in the classroom. In addition, in lines 66 to 68, they go to their proposals to class planning, as possible "solution" to the identified problems.

According to Mishra and Khoeler (2006), pedagogical knowledge (PK) encompasses theoretical issues involving a particular concept. In our case, the students point out the need to investigate how the articulation of semiotic registers of representation is relevant to teaching and learning functions. Furthermore, it is noticeable that the content knowledge is also involved in the discussion and, with this, the interaction between the subjects makes the pedagogical and content knowledge (PCK) emerge.

The following event shows the moment when the students define the objective of the class:

> **Event 3 - Class objective**
> ***L77*** *S8: The objective is basically to promote student understanding of a function, not only called f(x) but also as a quantity between dependent and independent variables.*
> ***L78*** *S7: This is the general objective; it has specific objectives.*
> *[...]*
> ***L85*** *S8: I will put some more things on my part, with my words.*
> *[...]*
> ***L90*** *S6 and S7 enter the specific objectives in the document.*
> ***L91*** *S8 clears screenshots.*
> ***L92*** *S7: What will be the teaching methodology and our resources?*
> ***L93*** *R (Researcher): This will be decided in another session.*
> (Araújo Filho, 2019, p. 148-149)

The conversation in Event 3 shows a break in students' alignment because the objective brought (in L77) by the S8 was individual. Nonetheless, there is agreement among the other students, despite the lack of discussion about proposals for the objectives. In this case, PK is evidenced, in lines 77 and 92, because of the reference to didactic issues of the class. In the table below, we summarize the class objectives and contents from the students' productions.

<table>
<tr><th colspan="2">Objectives</th><th rowspan="2">Content</th></tr>
<tr><th>General</th><th>Specific</th></tr>
<tr><td>The objective is basically to make the student understand what is a function, not only called f(x) but also as a quantity between dependent and independent variables with the use of affine functions.</td><td>To determine domain, image and root of the affine function.

To solve problems involving first degree equations.

To build the proportionality between quantities and associate it with a graph of the linear function.

To solve and elaborate problems from the information in a graph or a table.

To identify its coefficients and what is a consequence generated from them.

To identify the growth and decay of the function.</td><td>● Domain and image
● Constant function
● Linear function
● Affine function
● Increasing and descending function
● First degree Inequation
● Graph</td></tr>
</table>

Figure 9: Objectives and contents defined by the students (Araújo Filho, 2019, p. 153).

We brought in this section some events analysis from the first session of the 4th stage. The selected events show the moments of their discussion that led them to reach the class objective. Nonetheless, in total, we had seven events analyzed. Figure 10 synthesizes the knowledge that emerged from the interactions between the students in each session.

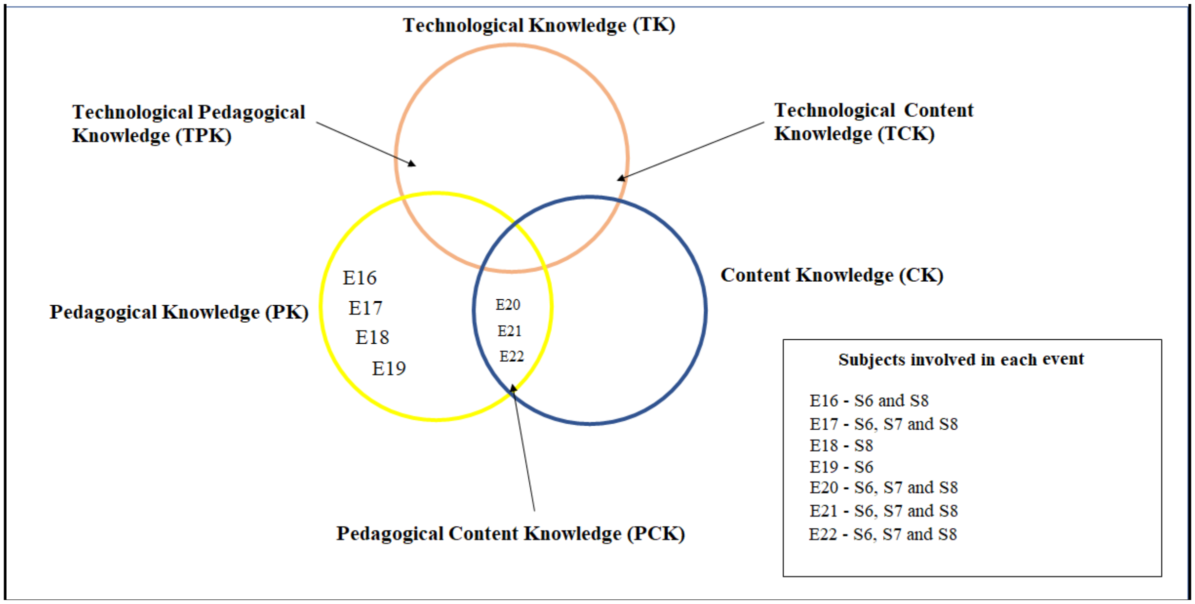


Figure 10: Knowledge identified in the 1st event of the 4th stage (Araújo Filho, 2019, p. 154, our translation).

Figure 10 shows that all the students stood out regarding pedagogical knowledge (PK) or pedagogical and content knowledge (PCK). However, by contrasting these results with their selfies-declared knowledge from the third stage, only S6 reflects self-declared good abilities regarding didactic knowledge. Despite feeling more skilled in other types of knowledge, the others reveal that they stand out from PCK or PK.

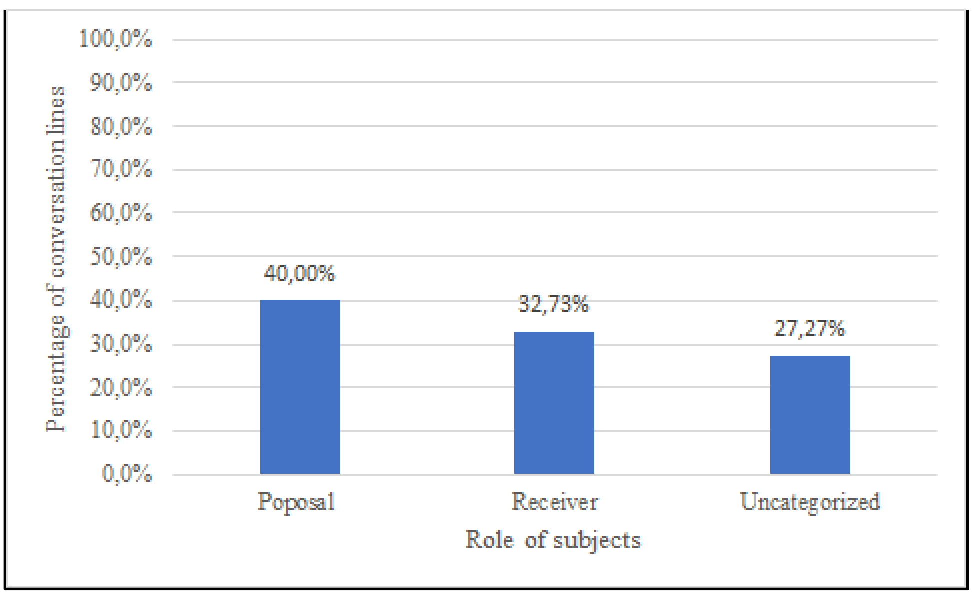
Collaborative interactions in the first session: dimensions of Baker's model

Analyzing interactions from the perspective of collaboration according to Baker's model, we identified the three dimensions in the events from the fourth stage. First, for the symmetry and agreement analysis, we categorized the lines of the conversation event, with each student's speech, as follows:

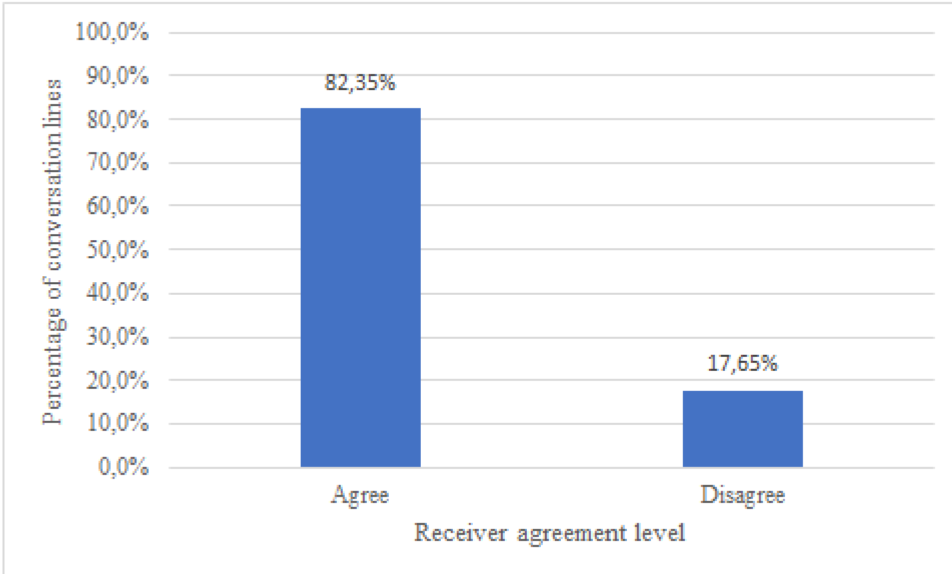
Dimension	Category
Symmetry	Proposer
	Receiver
Agreement	Agree
	Disagree

Figure 11: Collaborative dimensions and their categories. Source: Author's own.

After categorizing the chatting, we analyzed the percentage identified in each category to help us locate the group in the model. Observing the graphs below, we can see no exchange of roles; thus, we cannot characterize symmetry. These results of symmetry with the high level of agreement leave the students in the collaboration zone. In the graphs below, we can see the distribution of the percentage of categorized chatting in each dimension:



Graph 1: Percentage distribution of the symmetry dimension (translated from Araújo Filho, 2019, p. 152).



Graph 2: Percentage distribution of the agreement dimension (translated from Araújo Filho, 2019, p. 153)

Regarding the alignment dimension, we do not observe the students' speeches individually but the conversation and their engagement in the discussion.

The collective choices, as well as all members participating in the discussion, characterize their group engagement. There was no discrepancy in the topics discussed among the members. Thus, in Baker's model, they fit in the collaboration zone and are co-construction; that is, there was a collaboration in the interactions.

Planning of IO components

The planning of the IO components was part of the fourth stage, such as the second session of interactions between them. Therefore, we performed the analysis in this section following the format of the previous one, highlighting the events from the conversation and identifying the emerging knowledge of the collective work.

The first component of the planned IO was the Didactic Configuration. According to Trouche and Bellemain (2016), some actions of the didactic configuration include analysis of the curriculum, elaboration of the mathematical situation, organization of the student's participation, the definition of their roles, including the teacher's role, and the choice of techniques to approach mathematics concepts integrating the digital technologies.

In this paper, we selected some of the events in which they discuss the components of IO and which evidenced their TPACK knowledge. The event below shows their discussion about didactic configuration:

> **Event 4 – Discussion about didactic configuration**
> ***L84*** *S8: Two people would perform the task algebraically, and the other 2 would plot the graph in Geogebra. That was my idea.*
> ***L85*** *S6: I thought similarly.*
> ***L86*** *S5: So, do I.*
> ***L87*** *S8: It's similar to the puppet made in a classroom.*
> ***L88*** *S6: Huuuum...*
> ***L89*** *S8: There is no mt (SIC) (to much) to innovate.*
> ***L90*** *S6: And this class would have to address all objectives.*
> *[...]*
> ***L93*** *S5: We could pose a problem.*
> ***L94*** *S6: But that's it.*
> ***L95*** *S8: A problem/exercise.*
> ***L96*** *S5: Solved by math duo.*
> *[...]*
> ***L98*** *S8: But what's up.*
> ***L99*** *S8: Is this idea that we are going to use?*
> *[...]*
> ***L102*** *S5: For example, we could have students build the graph of the problem in Geogebra and then ask them to find the domain and image of the function.*
> ***L103*** *S6: That...*
> ***L104*** *S8: It's a good*
> ***L105*** *S6: Valuing student participation.*
> ***L106*** *S8: We have to think about the problem.*
> ***L107*** *S5: For that, they would have to calculate outside the software, and that's where the teacher comes in.*
> ***L108*** *S8: it would have to be a problem in which students will have to assemble the function equation.*
> ***L109*** *S8: to later put in geogebra.*
> ***L110*** *S6: this... It could be an everyday situation.*
> *[...]*
> ***L116*** *S7: we could tell them to create a line and analyze the consequences according to the changes made, like making a slider for coefficient a and another for b*
>
> (Araújo Filho, 2019, p. 161)

In Event 4, the students discuss how to organize the classroom and what activity would be relevant considering the way mentioned during the previous session. We realized that pedagogical knowledge (PK) permeates the conversation, taking into consideration Mishra and Khoeler (2006) points that planning and teaching methodology make up this type of knowledge (PCK). It appears when they involve the issues of the concept of Function: domain, image, graphics, to structure the teaching situation. In L116, S7 makes technological knowledge emerge individually, highlighting the use of the software to explore elements of the concept.

On the other hand, there are evident issues in L102 and L103, which reaffirm what Lins (2010) points out: using technology to reinforce traditional practices. For example, pupils must perform procedures outside the software for the pre-service teachers to have a teacher's intervention. In this case, they explored the potential of technology for teaching (TPK).

The second stage of the IO planned by the undergraduates was the Exploration Mode. According to Trouche and Bellemain (2016), it comprises the way of using the chosen artefacts and the decisions on how the task will be introduced and worked on (Drijvers et al., 2010).

Event 5 - Exploration mode
L121 *S5: Activity 1 graphs the affine function*
L122 *S8: Will it be more than one activity?*
L123 *S5: Yes*
L124 *S8: Do you already have these activities in mind?*
L125 *S5: I don't know how to download, but it would be basic*
L126 *S5: and the first thing that appears in Geogebra*
L127 *S5: graph construction*
L128 *S5: We could only build the graph and analyze*
L129 *S5: A graph of an affine function is given to the group that is 2 has to represent it in Geogebra*
[...]
L131 *S7: In producing parts, a factory has a fixed cost of R$16.00 plus a variable cost of R$1.50 per unit produced. If x number of unit pieces produced, determine:*
a) The law of function that provides the cost of producing x pieces
b) Calculate the production cost of 400 pieces
[...]

(Araújo Filho, 2019, p. 164)

The previous event shows the interaction of undergraduates in planning the exploration mode. In addition, the conversation shows interactions between two members, S5 and S8, which characterize the group as not aligned in the collaboration perspective. However, S7 and S6 were engaged in elaborating the text that describes the group's decisions. So, they are engaged in the discussion and, therefore, aligned.

In the same way as didactic configuration, the group presents only the pedagogical content knowledge (PCK) in the exploration mode. Pedagogical knowledge emerges from the discussion on how to approach function through a problem situation, proposed by S7 and discussing the most appropriate methodology to explore the mathematical object. We also identified content knowledge linked to pedagogical knowledge. Figure 12 shows their IO planning in the second session:

Instrumental orchestration	
Didactic configuration	**Exploration mode**
The students will be organized into groups of four; Each group will have to split into two pairs, the first will solve the problem algebraically, and the second pair will solve the problem without software. In the classroom, they will use: ● The brain and its cognitive connections. ● Their knowledge from the class about affine functions. ● Electronics devices as tablets and smartphones, with Geogebra installed. ● Paper and pencil.	It must operate at maximum capacity. It brings a methodology that promotes students' participation in their development of knowledge, highlighting the importance of the teacher in mediating learning. This methodology is contrary to the traditional one. Before the activities, there will be a brief explanation about affine functions, and from there, the students should try to solve the exercises. Two activities will be proposed to each group in the class, to be solved according to the instructions.

Figure 12: IO components planned by the students (Araújo Filho, 2019, p.168).

From the analysis of the events from the conversation in the planning, we were able to classify the students in the following diagram:

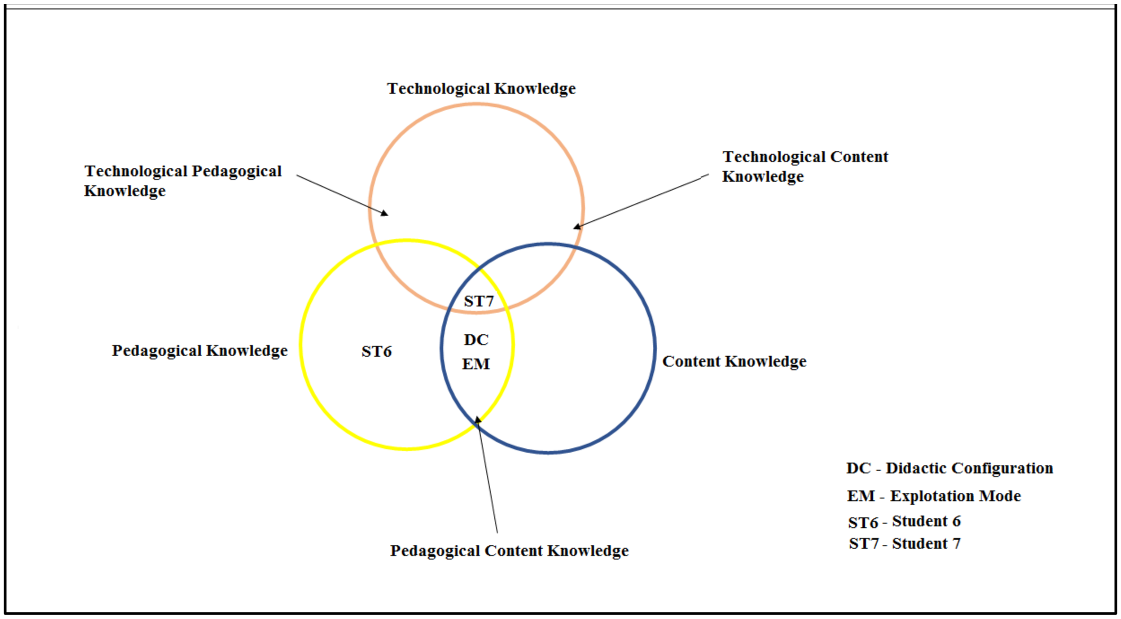
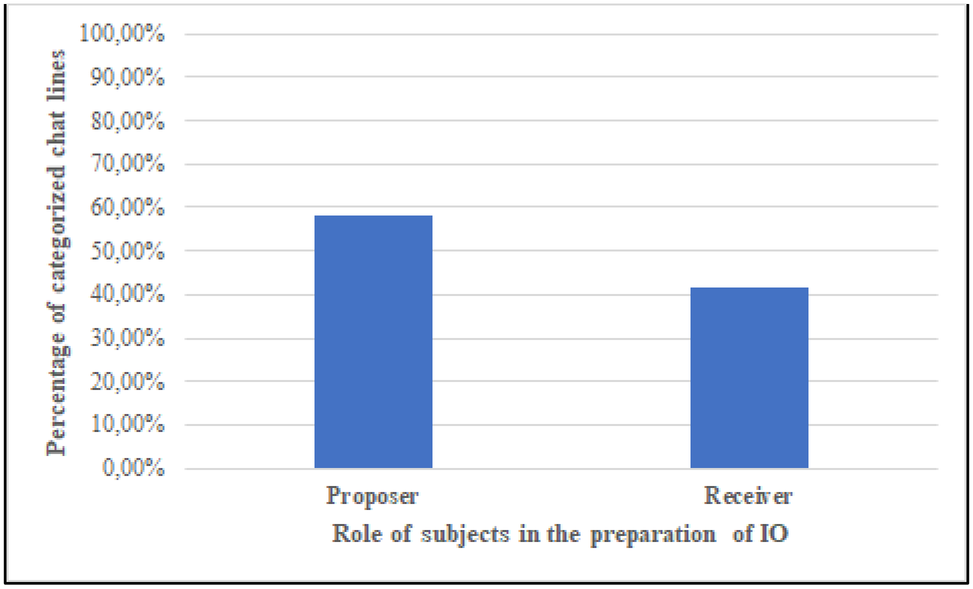


Figure 13: Knowledge in planning the IO components (Araújo Filho, 2019, p.168)

Figure 13 shows that S7 is inserted at the intersection of the three types of knowledge, evidenced by his individual actions in the interactions. On the other hand, S6 actions reaffirm his self-declared knowledge at this stage. The IO components inserted in the intersection representing PCK show that in this group, the collective production fits into the connection between two types of knowledge, which diverges from personal knowledge.

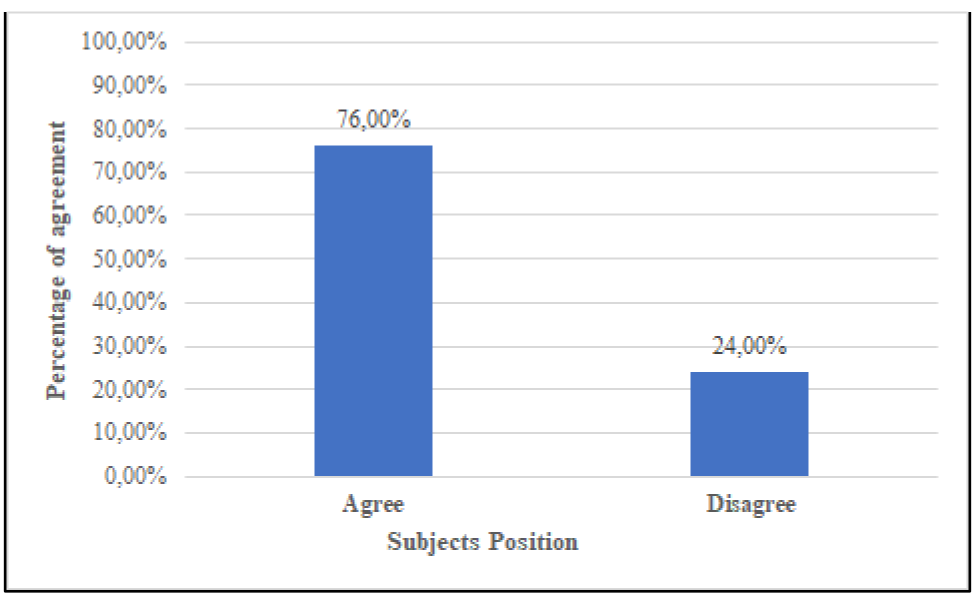
Collaborative interactions in the second session: dimensions of Baker's model

The analysis of the students' collaborative activity in the second session took place in the same way as in the first. Therefore, we will describe what the data represents, from the symmetry and agreement graphs, according to Baker's model.



Graph 3: Symmetry dimension in IO planning (translated from Araújo Filho, 2019, p. 167)

Graph 3 shows that the percentage of talks between proposers and reactors is close. This proximity characterizes symmetry; that is, the reversal of roles demonstrates that the students are symmetrical. The percentage difference in the graph, which is approximately 15%, allows us to conclude that they were symmetrical in the second session.



Graph 4: Agreement dimension in IO planning (Araújo Filho, 2019, p. 167, our translation)

In the agreement dimension, according to Baker's model, the students are in agreement when the percentage of speeches that agree exceeds the percentage of speeches that disagree. In our case, in Graph 3, we can see that the percentage of agreement between the students' is approximately 80%, characterizing that they were in agreement in the second session.

The group was aligned both while elaborating the didactic configuration and in the exploration mode. In addition, the other dimensions of collaboration showed that they were symmetrical and in agreement, based on the number of speeches analyzed and classified concerning the roles - proposer and receiver - and to the position - agree or disagree. Thus, based on Baker's model, they are classified into co-construction, confirming their classification in the first session. This result characterizes collaboration in the interactions between the students.

Concluding Remarks

In this paper, we present a part of a doctoral thesis that aims to analyze pre-service teachers' knowledge in the context of an experimental model of collaborative teachers' education with the integration of digital resources. We focused here on investigating the knowledge that emerged within the TPACK framework. Thus, we brought an excerpt from the data analysis of one of the stages, which mapped this knowledge and allowed us to trace the individual and collective knowledge that emerged in this process.

The types of knowledge most stood out in the class planning stage were the pedagogical and pedagogical-content knowledge (PK and PCK). They emerged in all group's collective interactions. In addition, individually, pedagogical knowledge (PK) and technological, pedagogical and content knowledge (TPACK) also appeared.

We emphasize that the analyzed group were pre-service teachers, enrolled in the second semester of the degree in Mathematics Licensors, bringing with them elements of teaching practice experienced by them as students (Fiorentini, 2005). Thus, the knowledge that emerged collectively may reflect the little experience in contexts of teachers' education. However, they stood out the results they had throughout the five stages of training proposed in our general study. We believe that this study points to an emerging need for technological integration in initial teachers' education, as the demand for technological knowledge on the part of the future teacher is also a demand of today's society, as pointed out (Brasil, 2001; Dantas, 2005).

The experimental training model proposed in the main study was designed according to the research needs, aiming to make teachers' TPAC knowledge emerge in the students during the training. The training model started from a theoretical perspective, in the discussion of records of semiotic representation, through digital resources instrumentalization, raising reflections and questions in the students. In the 4th stage, focused in this paper, the students considered the theoretical aspects discussed initially to build their class objectives and structure the software activities. These questions show that the design of the experiment and the collaborative interaction provided the students with different facets of teaching practice. It points out collaborative planning and theoretical training as a path to future teachers' education.

Despite the study limitations, such as the lack of validation of the planned OI, it contributes with a new training model for initial teachers' education to integrate digital technologies in their practices. In addition, we also believe that we have contributed with an online collaborative training model, which uses different digital technological resources, which can bring perspectives for future research in the creation or improvement of other models of initial training.

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