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Verônica Gitirana

Rosilângela Lucena

Rogério Ignácio

Roberto Araújo Filho

José Wilson Pereira

*See next page for additional authors*

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## The Man Creates Instruments that Transform Himself: An Overview of GERE Research within Mathematics Education

### Authors

Verônica Gitirana, Rosilângela Lucena, Rogério Ignácio, Roberto Araújo Filho, José Wilson Pereira, and César Thiago da Silva

**The Man Creates Instruments that Transform Himself:  
An Overview of GERE Research within Mathematics Education**

Verônica Gitirana<sup>1</sup>

Universidade Federal de Pernambuco-CAA, Brazil

Rosilângela Lucena

Universidade Federal de Pernambuco-CAA, Brazil

Rogério Ignácio

Universidade Federal de Pernambuco-CAp, Brazil

Roberto Araújo Filho

Universidade do Estado do Rio Grande do Norte, Brazil

José Wilson Pereira

Universidade Federal de Pernambuco-EDUMATEC, Brazil

César Thiago da Silva

Universidade Federal de Pernambuco-EDUMATEC, Brazil

**Abstract:** This paper discusses GERE (Study Group on Resources for Education) members' research trajectory in three different lines: Teaching practice and practice management, Teacher education and identity, and Support for learning and resource generation. It discusses how researchers seek to understand technology integration in the practice of distance education in Brazil, with the instrumental orchestrations lens, revealing the changes made in the didactic configurations, from the multiplicity of teachers responsible for each discipline. Teachers' documentation is discussed in the process of resource elaboration and use and guided by design, experimentation, and reflection of the generated instrument, such as digital textbooks and mathematical games. The notion of integrating artefacts is discussed both from the perspective of collaborative learning and of instrumental meta-orchestration, a teacher education model about instrumental orchestration. Finally, thinking about the artefact as a support for learning also made us work on generating devices aimed at specific concepts such as covariation in learning functions.

**Keywords:** resource, instrumental orchestration, teacher as designer, instrument, documentational approach to didactic.

## Introduction

The man builds instruments to improve the ability to solve problems within situations and expand the spectrum of his abilities to perform tasks. Artefacts and knowledge gather to create these instruments that,

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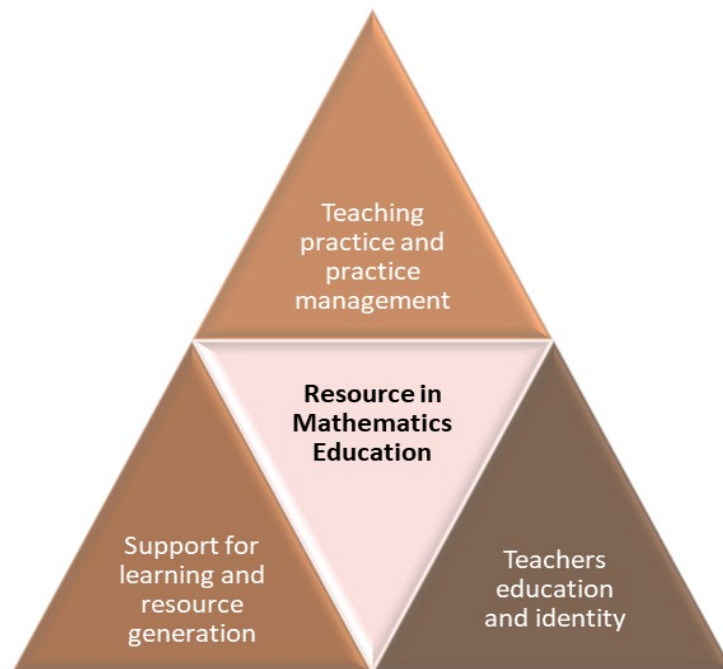
<sup>1</sup> veronica.gitirana@gmail.com

in turn, change ways of carrying out an activity and dealing with new challenges. This process allows inferences, new knowledge, new strategies, and new instruments to emerge.

The instrument developed for one type of situation can be just an artefact in another, which gradually gains new attributes through the emergence of new knowledge, becoming a new instrument for a new type of activity, thus modifying the subject himself. It establishes a whole evolutionary process of the subject's actions mediated by the artefacts.

Investigating this process and its different nuances related to resources in education has been of interest to many researchers (Gueudet & Trouche, 2009; Drijvers, Gitirana et al., 2019). GERE (Study Group on Resources for Education), created in May 2018, shares this interest, particularly for mathematics education.

This paper seeks to bring to the public a discussion about the resources in mathematics education from the research developed within the GERE research group, considering their respective lines of research (**figure 1**): Teaching practice and practice management; Teacher education and identity; Support for learning and resource generation.



**Figure 1:** GERE research lines schema.

In the following sections, we discuss some of the developed and underdevelopment research in each of the three GERE research lines, revealing their specific thematic and how they articulate themselves

with the common theme - resources in mathematics education. We also present a brief overview of our main theoretical framework.

## Teaching Practice and Practice Management

We will start by discussing resources as a support for teaching practice from the perspective of integrating resources and practice management, particularly in distance and online education. In this context, the group framework is firmly based in the instrumental orchestration (IO) model as developed by Trouche (2005) and complemented by Drijvers & Trouche (2008).

They also use reflective methodology (Pepin, Gueudet, & Trouche, 2013) to undertake his study. They developed reflective methodology as a methodology for the documentary approach to didactics. This approach is the second theoretical framework most used in GERE research.

## **Instrumental orchestration: Its Concepts and Framework**

The instrumental orchestration has its base in the concept of the situation (Vergnaud, 2009) and the instrumental approach (Rabardel, 1995). Vergnaud (2009) affirms that cognitive development is essentially an adaptation process. Nonetheless, to think on adaptation demands to reflect about: “what is it that adapts itself, and to what?” To him,

[...] what adapts are the forms of organization of activity, the schemes, and they adapt to situations. Therefore, the pair scheme/situation is conceptually more interesting and more powerful than the pair response/stimulus, and it is also more viable to describe and analyse behaviour and representation using the pair scheme/situation than the pair subject/object. (Vergnaud, 2009, p.85)

It is through the experience with and resolution of situations that someone develops their schemes. Vergnaud (2009, p. 88) semantically defines the scheme as “the invariant organization of activity for a certain class of situations.” He also introduces the analytical definition of the scheme, which has allowed many pieces of research to go further, in the study of schemes. Analytically, the scheme is defined as formed by goals; rules of action, rules of controls and information gathering; invariants (concepts in action and theorems in action) and possible inferences. To face new situations, the subject checks his existing schemes, defines goals and anticipation, mobilises rules of action, or gathers information, which, in turn, involves concepts and theorems in action, and, when necessary, revises and produces inferences

by building new invariants or new rules of action, of control or information gathering. Schemes evolve, therefore, the subject's knowledge, as part of the schemes about the situations with which he deals, also evolves.

When studying human activity, Rabardel (1995) develops the notion of instrumental genesis, a process by which man develops scheme of use and scheme of instrumented action for an artefact, transforming it into an instrument to face contingencies arising when dealing with its activity. According to him, instrumental genesis has two distinct but inseparable processes: instrumentalisation and instrumentation.

Based on the instrumental approach, and the notion of situation, Trouche (2005), inspired by the metaphor of an orchestra, develops the instrumental orchestration. According to him, it aims to allow the teacher, based on a situation, to develop a well-structured arrangement to promote the instrumental genesis. The authors define instrumental orchestration as:

a systematic and intentional arrangement, by an agent, of the elements (artefacts and humans) of an environment to implement a given situation and, more generally, to guide learners in the instrumental genesis and the evolution and balancing of their systems of instruments. (Trouche, 2005, p. 126, our translation)

Therefore, Trouche introduces two crucial elements to think about an *instrumental orchestration*, in addition to the *situation*: the *didactic configuration* and the *exploitation mode*. When reflecting on the metaphor and considering the need to think about unexpected situations, common to the classroom, Drijvers et al. (2010) introduce the *didactic performance*, among which teachers' *ad hoc* decisions is one of the focus.

Lucena (2015) and Pereira (2017) used Instrumental Orchestration as a model to analyse systematic arrangements and teaching practices in distance education at UAB - Brazil.

## **The Documentational Approach to didactics: the documentational genesis**

The documentational approach to didactics (DAD) (Gueudet, & Trouche, 2009a) presents itself as a theoretical scope aimed at studying the activity of teachers with a focus on the resources they use and develop. It is an extension of the instrumental approach (Rabardel 1995), which aims at the work of the

teacher in the face of the diversity of resources, mainly digital, and the need to articulate studies focused on the teaching work with these resources. Therefore, we consider that, among others, it inherits the notion that all human activity presupposes the use and development of schemes and artefacts.

As regards the notion of resource used in DAD, the researchers rely on Adler's (2000) conception of resource as a verb (besides a noun). Resource as a verb is all that can renew teachers' activity, being re-signified by him/her as a means of achieving his didactic goals.

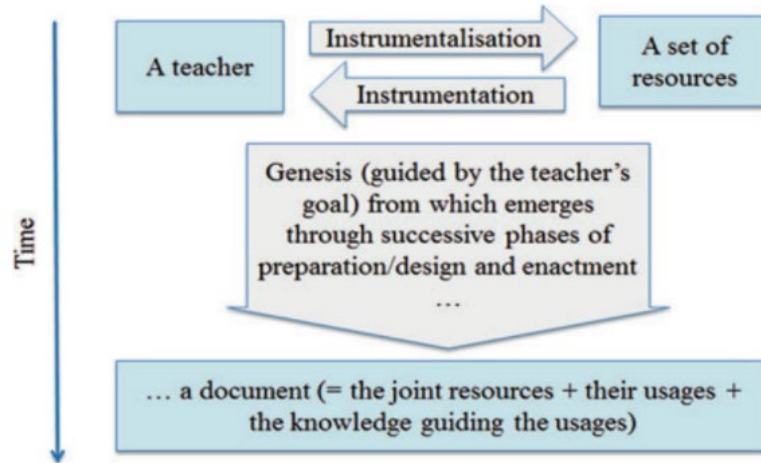
The notion of schemes adopted in DAD follows, in a more general way, the analytical definition elaborated by Vergnaud (2009). More specifically, as it refers to the mediation of action by resources, it considers the notion of the scheme of utilisation (Sh.U.) developed by Rabardel (1995) and that, the Sh.U., comprises two other schemes as components distinguishable by their goals:

- The scheme of use focuses on the actions related to the handling of the artefact.
- The scheme of instrumented action focuses on promoting the transformations intended by the subject with the use.

The concept of document and documentational genesis appears in DAD in a similar way to the concepts of instrument (as a mixed entity containing two poles, artefact and scheme of utilisation) and instrumental genesis (the process of creating an instrument from transformations aimed at the artefact, instrumentalisation, and focused on the subject, instrumentation).

In the context of DAD, the term document is retained "for designing 'something bearing an intention', and dedicated to a given usage in a given context" (Trouche, Gueudet, & Pepin, 2018, p.2). This concept is the result of a process called documentational genesis (**figure 2**), according to which the teacher, faced with situations related to his professional life, develops forms of organisation to use a set of resources, aiming to deal with such situations.





**Figura 2:** Representation of documentational genesis (Trouche, Gueudet, & Pepin, 2018, p.4).

The duality (resource pole and scheme pole) of this notion, the document, creates know-how-with which is integrated into the teacher's knowledge base, forming a document system. In other words, documents are not isolated but integrated into a functional whole aimed at the subject's handling with the situations (grouped into classes and these into families of activities of the teacher). Gueudet and Trouche (2009b, p. 11, our translation) distinguish three activity families:

- *organise own teaching*: adapt mathematical organisations, design teaching organisations, evaluate and revise teaching;
- *participate in the educational organisation through institutions management to different school areas*: build school reports, participate in class council at school, participate in parent-teacher meetings;
- *develop a reflection on their practice* (Vasquez-Bronfman 2000): participate in teachers' education, question oneself, cultivate oneself, "enter" into a software program or a site to appropriate them.

To understand such structures, Gueudet and Trouche (2012) developed the reflective investigation methodology that allows the analysis of the teacher's documentation both as a product and as a process. Such research methodology has four principles, namely: long-term follow-up, to observe genesis processes, which occur over long periods; monitoring inside and outside the class, which takes into account the teaching work in the classroom, but also at school, at home, in teachers' in-service education; an extensive collection of material resources used and produced in the documentation work, during the monitoring; reflective monitoring of the documentation work to intimately involve the teacher in data collection and generate a reflective attitude.

## Tutors' Didactic Reconfiguration

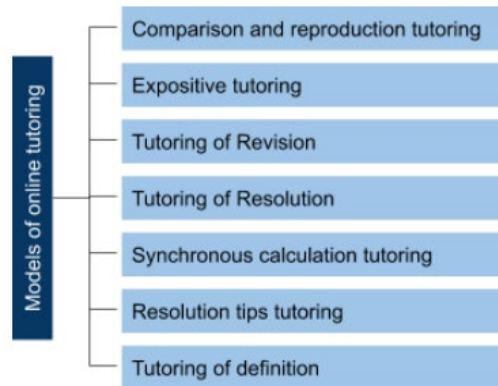
In Brazilian distance education, several teachers are responsible for the execution of teaching enhanced by didactic and technological artefacts. For each activity, there is a specific person whose action influences the didactic configuration: The content teacher, responsible for the content, that produces the textbook, virtually distributed to the students; the executing teacher who plans and systematises the whole discipline and decide what will be done in class; and the distant tutor - teacher who does didactic mediation; the face-to-face tutor, responsible for welcoming and supporting students in the teaching centres.

Lucena (2015) analyses tutors' mediation through the *chat*, in two classes of analytical geometry courses of distance undergraduation. For this purpose, she used the two components of IO defined in Trouche (2004; 2005) (the didactic configuration and the exploitation mode) served as an analysis tool for the tutors' practice of mediation. She detected that the tutors modify didactic configurations defined by teachers who plan the course and, thus, introduces the notion of didactic reconfigurations.

In the investigated reality, the *chat* is the interface that the tutors and the students must interact to discuss the specific knowledge addressed during the modules of the course. This kind of mediation faces many difficulties, mainly due to the limitations of mathematical representations allowed in the chat, particularly on algebraic and Cartesian representations.

The mathematical knowledge that is the focus of the investigation, analytical geometry, focuses on the articulation between two representations. Thus, Lucena (2015) takes the theory of registers of semiotic representation - TRSR (Duval, 2003; 2009) as a theoretical basis to look at the situations discussed according to treatment or conversion needed. Associated with the two theoretical bases, the theory of cognitive mediation and didactic mediation - TCMDM (Lenoir, 1996) supported the tutors' mediations analyses.

Based on the theoretical triad, she characterises seven types of online mediation (**figure 3**), according to the didactic mediation strategies developed by the tutor centred on the chat.



**Figure 3:** Online Tutoring Models (Translated from Lucena 2015, p. 126).

In the Comparison and Reproduction Tutoring, the tutor asks the student first to compare the situation he has doubts with a similar situation already resolved and made available in the virtual learning environment (VLE) or the chat session. As for the Expositive Tutoring, the tutor presents definitions, concepts, properties, and theorems, concluding with examples. Revision Tutoring is when the tutor does a general revision of the course content for the evaluation test. Tutoring of Resolution happens when the tutor resolves for the students the activities that they have difficulties in resolving. In the Synchronous Calculation Tutoring, the tutor accompanies and guides the students synchronously while he solves the mathematical situation, using a draft. Such a draft notebook is part of the student's physical space and cannot be seen by the tutor, who follows the student's reasoning through his chat messages. As for the Resolution Tips Tutoring, the tutor gives tips on how the student can solve the questions presented. Finally, in Tutoring of Definition, the tutor presents concepts, definitions, but there is no demonstration or resolution of activities to exemplify what is being tutored.

As for the types of instrumental orchestration developed by tutors in didactic mediations, Lucena (2015) notes that tutors perform a didactic reconfiguration. From the intersections between the strategies and resource groups of didactic mediations and the types of semiotic transformations of the mathematical situations presented, she characterised four types of instrumental orchestration of online tutoring, namely: Online Calculation Reconfiguration, Resource Function Reconfiguration, Confirmation of Didactic Configuration and Indication of Didactic Reconfiguration.

The most used was the Resource Function Reconfiguration. In these, during the didactic reconfiguration, the tutor chooses a (re)source, but, due to the need for didactic mediation, he uses the resource for interaction, that is, he establishes a new function for it.

She also found that the instrumental orchestration model of online tutoring promotes the simultaneous use of two instruments, chat and sketch, by the tutor and the student based on the need to convert between algebraic and Cartesian representation (Duval, 2003). In the context of online tutoring, they performed the calculation in real-time through sketching and chat, which favours the instrumental genesis of both the student and the tutor, considering that both develop instrumented actions. They use the chat to discuss instructions and the draft to develop algebraic calculations and graphs that they cannot do in the chat.

In summary, Lucena (2015) identifies changes made in the didactic configurations of the teacher who plans from the needs of mediation by the tutor, the didactic reconfigurations. The tutor exchanges or adds artefacts and alters functions assigned to the use of these resources by the teacher. It thus promotes transformations in the instruments provided by the teacher, creating a new instrument from a defined artefact or incorporating artefacts.

### **Didactic Configuration: the path from institution to the distant tutor**

Pereira (2017), in continuity with Lucena (2015), studied the didactic configurations of executing teachers when planning the course. He identified several stages of configurations of a course at the distance education. Developed in the scenario of distance education (DE), in the model of UAB (Open University of Brazil), his study focused on the choices of the executing teachers, regarding the management of resources made available by the institution and the schemes of use planned in his didactic configuration for the analytical geometry courses at Mathematics graduation. This management comprises choosing of teaching situations, mapping of resources and planning schemes of use.

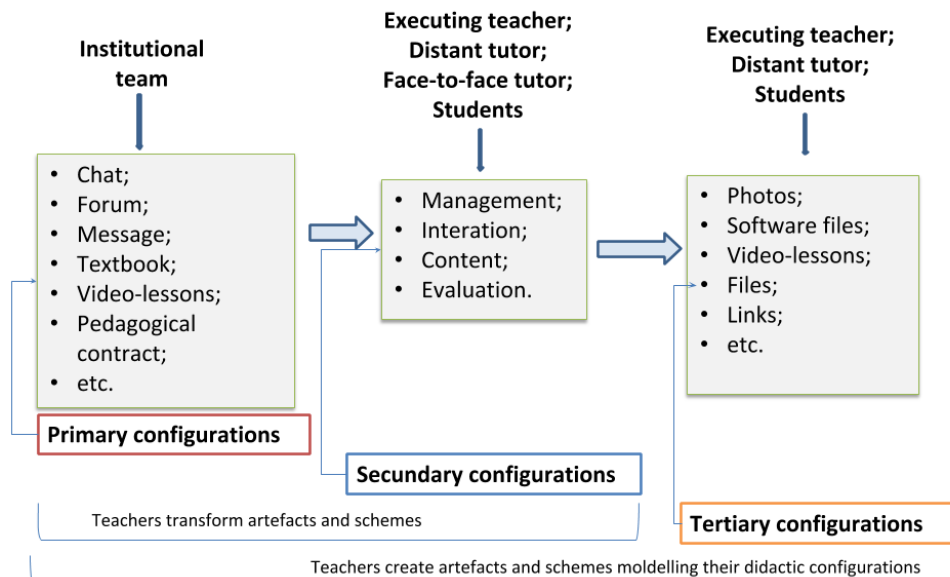
The theoretical framework comprises the instrumental orchestration model (Trouche, 2005) concerning didactic configuration, and the reflective methodology adopted in the documentary approach (Pepin, Gueudet, & Trouche, 2013).

By considering the complexity of the actors responsible for teaching practice at DE-UAB, Pereira (2017) describes how complex this teaching system is. He also claims that it must be carried out harmoniously during a course to be successful. All the actors influence the didactic configurations and the exploitation mode planned by the executing teacher.

Pereira (2017) points out the changes in the didactic configurations of the executing teachers. It comprises a path which begins with the didactic configuration proposed by the institutional team, which influences the executing teacher's plans for the interactions between the students and the tutors in the VLE. These changes occur due to the need to adapt the content presented in the digital textbook, made available by the institution, and produced by the content teacher.

The VLE configuration starts with the institutional team by providing artefacts and idealising goals and rules of use, which we will name strategies. With the VLE configured, the executing teacher chooses some of the artefacts and traces their schemes of use, considering the strategies indicated by the institutional team. Besides, he creates and inserts other artefacts and plans schemes of use that articulate with the instruments of the VLE.

In **figure 4**, the diagram represents the didactic configurations that take place in the virtual classroom.



**Figure 4:** Stages of didactic configuration in DE-UAB (adapted from Pereira, 2017, p. 107).

The schemes of use attributed to artefacts by executing teachers also influence the different stages of didactic configurations. Pereira (2017) points to an evolution in the didactic configurations (**figure 4**):

- a primary configuration made by the institutional team;
- a secondary didactic configuration elaborated by the executing teacher, adding or modifying artefacts and their schemes of use;
- a tertiary didactic configuration happened in the moments of interaction with students.

In the Secondary stage, the executing teacher has the support of the distant and the face-to-face tutors. In the latter, the distant tutors play an essential role, as they are closer to the students.

One of the teachers participating in the research points out, in his speech, artefacts, and evidence of the didactic configurations: “[...] Primary resources are these here: like... pedagogical contract, textbook, tasks, you place them there when you organise the classroom, and the secondary ones are placed there latter, by the [distant] tutor” — interview with one of the executing teacher (Pereira, 2017, p. 42).

These stages of didactic configurations revealed barriers in the initial planning and the need for dialogue among the different actors that make up this teaching modality, as shown by the speech of one of the interviewed teachers:

[...] For planning, I have to have availability, discussion with other authors, think and build collectively, and this is almost impossible to do. If I could argue with the guy who writes the textbook, the teachers who already touch the geometries [courses], or to have a contribution in parallel, [with] the teachers who will be there that are the tutors, the distant tutors, to know early difficulties of the class, it would be the ideal. [...] It takes much work, but it would be ideal — interview with the executing teacher. (Pereira, 2017, p. 51)

The lack of dialogue, mentioned by the executing teacher, reveals that the tertiary configuration is the moment when the teacher, with the contribution of the distant tutor, performs the most significant transformations in the didactic configurations. It is the time when students meet artefacts and teachers’ schemes of use that the significant evolution of the didactic configurations becomes evident.

These artefacts undergo transformations, which will depend on the scheme of use planned by the institution team or by the executing professor, who also creates artefacts, assigns strategies of use and provides them to students. The exploitation of artefacts provides a rich moment when they are interconnected and connected with external artefacts.

Teachers develop didactic configurations that are similar in the sense of transformations and the evolution of resources and their schemes of use, but each one enriches their configurations differently. One relies on instruments that serve as a source of content, video, internal and external to the VLE. Another focuses on mediation using the forum, enriching it with software, videos, videoconferencing, followed by chat. Furthermore, the third teacher enriches his didactic configurations by sending artefacts that serve as a source of content and as an evaluation instrument.

### **Some considerations: the resource in teaching practice and practice management**

Instrumental orchestrations were used as a research tool by GERE team to understand DE-UAB teaching practice, which is an environment necessarily rich in technology. In this context, where many teachers are responsible for defining the didactic configuration, the results show that there are different stages of didactic configurations for each teacher who works. They feel the need to rearrange the configuration, redefine instruments and even change artefacts.

## Teacher Education and identity, and the curricular resources

Teacher education and their identity, concerning the use of resources, especially those of digital technologies, is the second focus of GERE. In this research line, the spectrum of theories expands in the group. By assuming the importance for teachers' practice of the knowledge about IO, Lucena (2018) extends the model to instrumental meta-orchestration, a model for theoretic-practical and reflective teacher education about IO. In a documental approach, Ignácio (2018) uses the DAD (Gueudet, & Trouche, 2009a) to understand the birth of the teaching documentation, even in the stage of pre-service education, with the development of digital teaching resources. Pre-service teacher education based on didactic resource design projects is also explored by Gitirana (2018) when studying the documentation of prospective mathematics teachers, joining DAD, activity theory (Leontiev, 1978; Engeström, 2001) and curricular ergonomics (Choppin et al., 2018). Computer-supported collaborative learning (CSCL) (Dillenbourg, 1999) associated with the constructs of teaching knowledge for the use of technologies (TPACK) (Koehler, & Mishra, 2009) and IO comprises the theoretical framework of Araújo Filho (2019) to investigate a pre-service collaborative education of mathematics teacher to integrate technology in practice.

### **Instrumental Meta-Orchestration: a model for teacher education about IO**

Instrumental orchestration (Trouche, 2005) is assumed by Lucena (2018) to be essential knowledge for the organisation of teaching practice to integrate artefacts. Thus, she develops, tries out and validates a teacher education model about IO and based on it, the instrumental meta-orchestration (IMO).

The conception of instrumental meta-orchestration highlighted the need to use a methodological model that promotes an organisation of a teacher education design about IO. Thus, she integrated the design experiments (Brown, 1992; Collins, 1992; Cobb et al., 2003) into the theoretical and methodological framework of the research. The design experiments assumptions guide the use of theories to conceive designs, develop and try it out iteratively, in the perspective of improving and validating them.



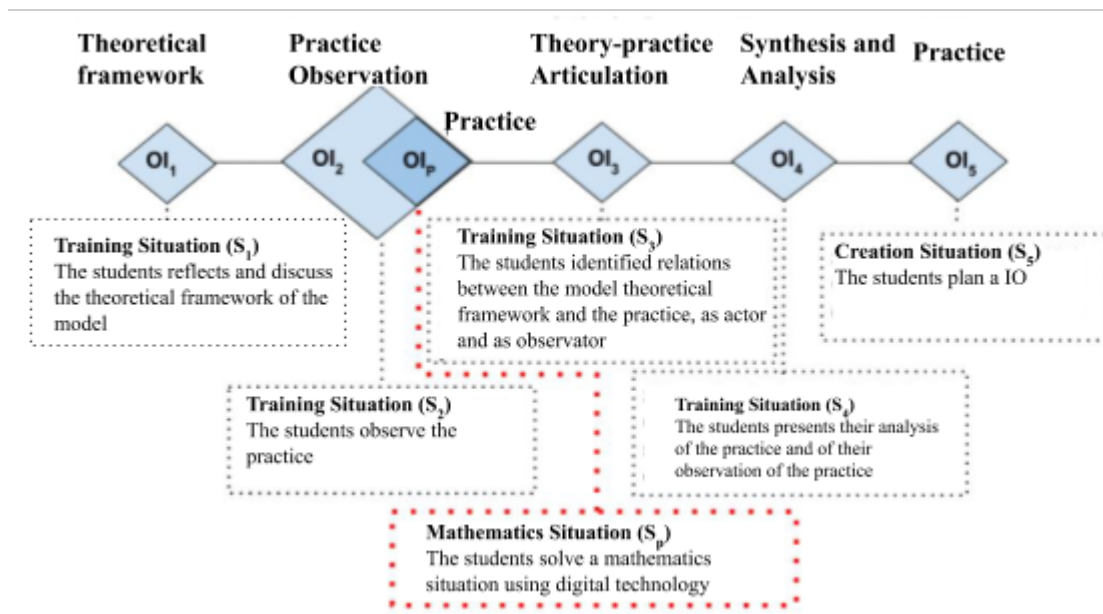
Lucena (2018) defines IMO as the systematic and intentional management, by an agent (trainer(s)), of artefacts and subjects (teachers and future teachers) confronted with a meta-situation, with the goal of appropriating the instrumental orchestration model. Meta-situation is a composition of situations with their nature and difficulties.

An IMO has eight characteristics, three of which already prescribed by Trouche (2005) to IO: systematisation, intentionality, and richness of technologies. The others emerged from the refinement process in the preliminary and main studies of the IMO: composition, articulation, flexibility, adaptation, and interaction promotion.

Didactic meta-configuration, Execution meta-method and didactic meta-performance are principles of IMO, coined by Lucena (2018) based on the principles of IO (Trouche, 2005; Drijvers et al., 2010). As Lucena (2018) states:

- *Didactic meta-configuration*: it is an architecture of the subjects (students), artefacts, didactic choices, and situations, defined by the conductor (teacher/trainer). This architecture presupposes, in addition to the set of instrumental orchestrations internal to each orchestration, the organisation, articulation and management of the instrumental orchestrations that will compose the IMO.
- *Meta-exploitation mode*: it consists of different ways of exploiting the architecture of each instrumental orchestration of the didactic meta-configuration. To develop an IMO, someone must prescribe at least one way to exploit the didactic meta-configuration.
- *Didactic meta-performance*: it consists of the viability of the architecture created for teachers' or pre-service teachers' appropriation of the IO model.

So, she offers a model and structure for teacher education design. **Figure 5** shows the Design of IMO didactic meta-configuration for the main study, a (Re)Design.



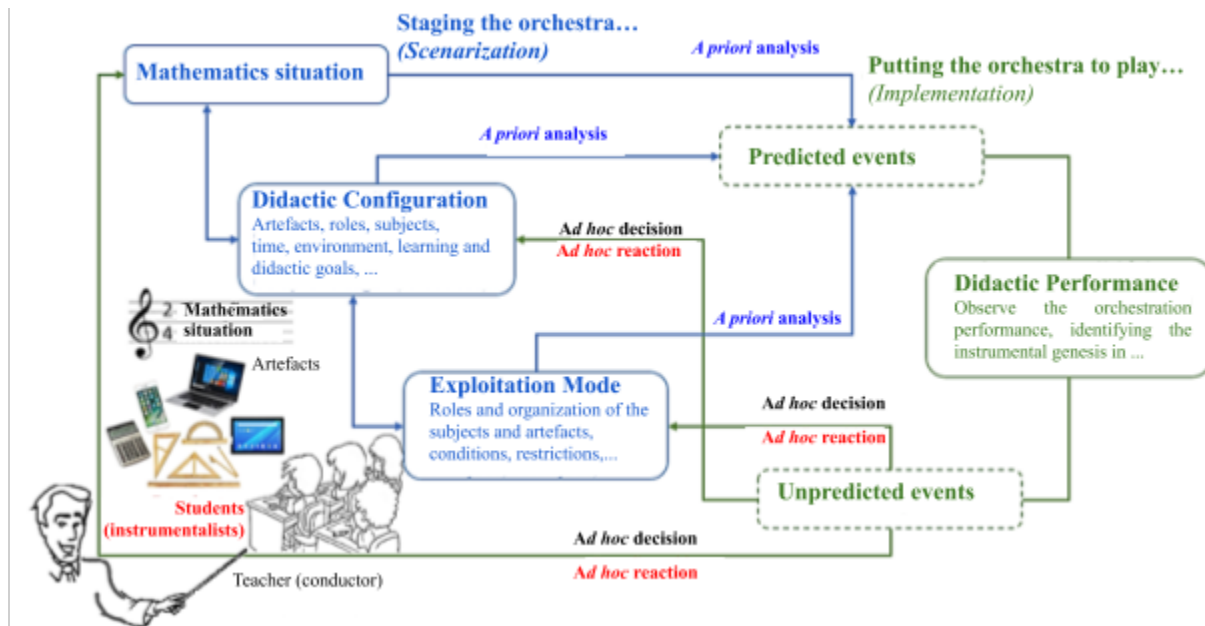
**Figure 5:** (Re)Design of IMO didactic meta-configuration (Lucena, 2018, p.135, our translation).

In **figure 5**, IMO is shown as a structure aiming at the exploitation of a meta-situation, composed of six situations of different nature and complexities. Thus, the design consists of six instrumental orchestrations, one for each situation.

The six IO intend to support the proposed situations and aim at: theoretical foundation, observation of an IO experience practice of experiencing an IO, execution of an OI, the articulation between studied theory and IO exploitation practice, analysis and synthesis of data produced in the light of IO and, finally, creation of an IO. All situations seek teachers or pre-service teachers' appropriation of the IO model. They are all formative situation, although the one that proposes the experience of an IO, aims, on the one hand, at learning mathematics with the integration of digital technologies; and, on the other hand, aims to serve as a resource to the composition of  $IO_2$ , to observe students' instrumental genesis with digital resource integration.

She undertook two iterated experiments of IMO. The first was in a master's class with 26 master's students and two trainers, which served as the basis for the origin of this research, a preliminary study. The principal study was carried out in a pre-service mathematics teachers' course at a state institution of higher education. Twenty-three pre-service teachers and two trainers participated in the experience.

In the first test of the model, Lucena (2018) advances on the OI model, presenting the elements of an IO in two acts (**figure 6**): the first - Scenarisation - the moment of IO planning, the didactic configuration and the exploitation mode (Trouche, 2004), to support the realisation of the situation; the second - Implementation - denotes the moment when the orchestra performs, that is, how teacher performs the IO, how it is favouring, or not, the students' instrumental genesis regarding the situation - the didactic performance (Drijvers et al., 2010).



**Figure 6:** (Re)Organisation of the IO model according to Lucena (2018, p.34).

The first act (**figure 6**), Scenarisation, is orchestrated by the teacher who determines the conditions, such as the situation to be solved, chooses artefacts to be made available, defines the roles, functions of each subject, the duration. He predicts at least an exploitation mode and undertakes theoretical analysis of scenarisation to predict the events. The second act, Implementation, is the performance of the first act, by the teacher and his/her students. In this act, the predictions are confirmed, or not. At this moment, one can identify teachers' *ad hoc* decisions (Drijvers et al., 2010) and students' *ad hoc* reactions. A term coined by Lucena (2018) to refer to the reactions of students to unforeseen events. In performing the IO, considering conditions of the situations and restrictions imposed on those who will solve them, unpredicted events emerge, not only from the teacher's viewpoint but also from the student's point of view. Thus, students react to resolve the unforeseen - *ad hoc* reaction, a definition that extends that given

by Drijvers et al. (2010), in the sense of looking at it from the point of view of the student's action that alters the IO.

The complex composition of orchestrations, in sequence and overlapping, raised significant challenges in terms of analysing the students' evolution during IMO. In this sense, she developed analysis instruments to facilitate the classification of events that give evidence of schemes or parts of a scheme of the participants. Thus, it enables her to follow participants' trajectories, individual or collective, to ascertain their performance regarding either the mathematical situation at IOp or the appropriation of concepts related to IO at IMO.

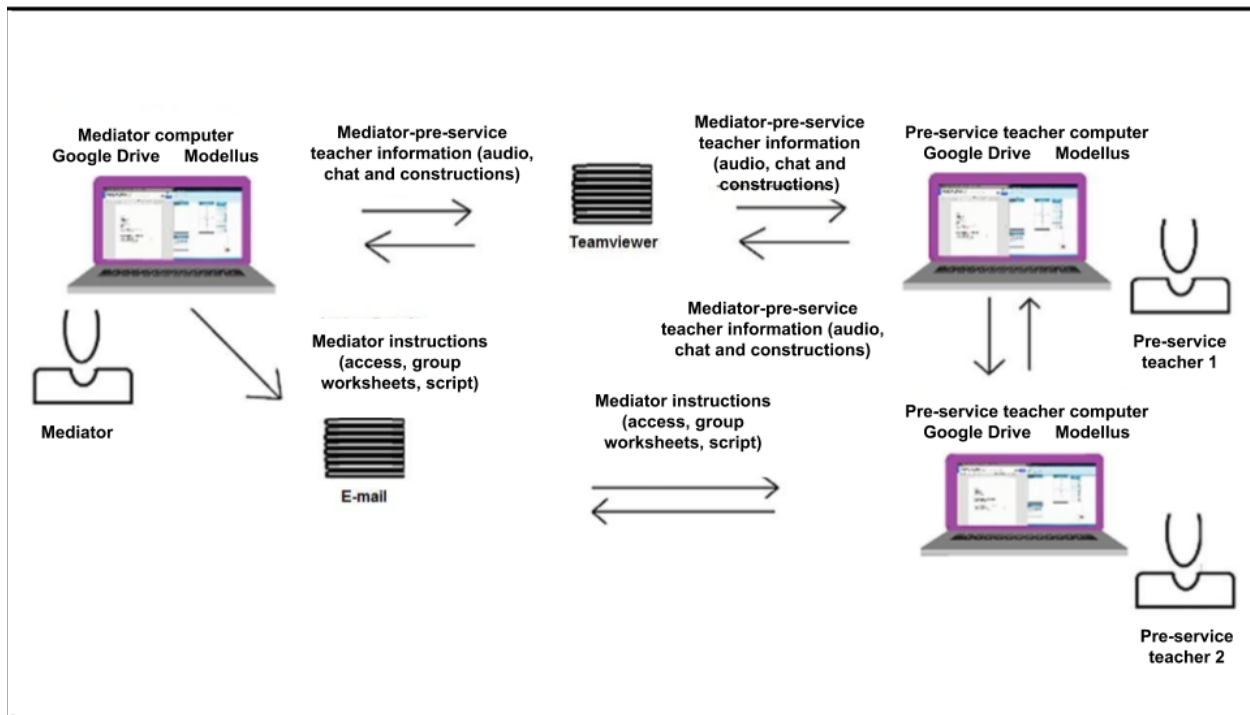
The evaluation of didactic documents based on the web (webdocs) (Bellemain, & Trouche, 2016) is another relevant aspect to be discussed. The formative situation of IO<sub>1</sub> aimed at introducing and discussing the IO model theoretically, in a forum discussion, after reading a Webdoc with examples. To conceive and analyse the webdoc, the authors considered the "Forms of Destination" (Remillard, 2010) as crucial elements. She also adds to them the media to be integrated. Remillard's (2010) modes of engagement are the focus of analysis; that is, how the participants interact with the forms of destination of the webdoc while solving the situation.

Of all the artefacts available at IMO, the webdoc was the only used by participants in all orchestrations both to solve the proposed situation and to generate new artefacts (protocols) to be used in other IO. There were also different ways of engaging participants with these artefacts, most of them combined with other artefacts and integrated in the instrumental orchestration.

## **Collaboration and instrumental orchestration in pre-service teacher education**

In parallel, Araujo Filho (2015, 2019) brings up a perspective of collaboration in the integration of digital artefacts for distant teacher education. The CSCL scenario (Dillenbourg, 1999) is rethought with the design integrating platforms (**figure 7**) for pre-service teacher education, working collaboratively, seeking to remedy teachers' education lack regarding the integration of digital artefacts.

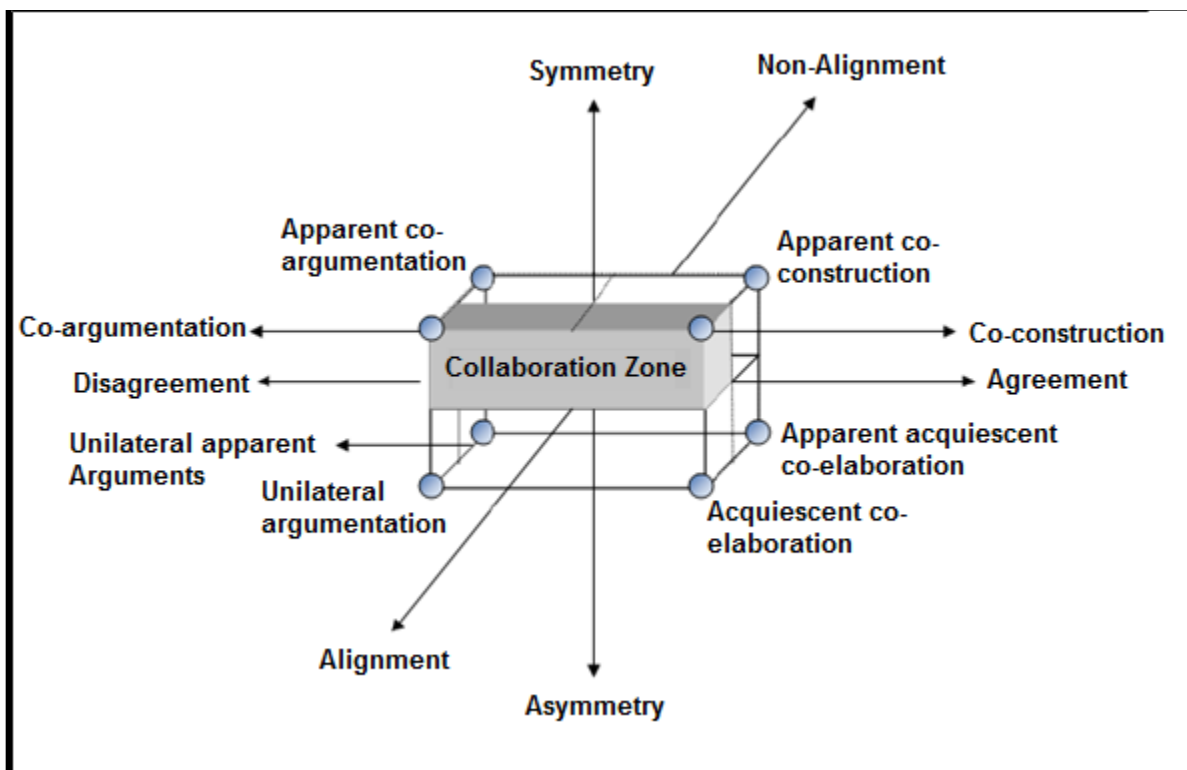
Araújo Filho (2015) proposes an experiment for the class planning, synchronously and collaboratively, with pre-service mathematics teachers. The perspective of online collaboration, he addressed some aspects of CSCL, such as the construction of scripts, design of the collaborative environment (Dillenbourg, 1999) and Baker’s method of analysis for collaboration (Baker, 2002). The collaborative script is a session script to guide the participants’ interactions. The design of the environment, according to Dillenbourg (1999), must integrate digital resources that support collective work and communication between participants.



**Figure 7:** Schema of the created collaborative environment (Araujo Filho, 2015, p.52).

**Figure 7** illustrates the design of the environment. The mediator, in the sessions, plays the role of keeping the students engaged in the discussions and of promoting interactions, when needed. The mediator’s computer worked as a server that shared the collective information, to be viewed and edited by the pre-service teachers. For that, he used the Teamviewer software, a remote access software. A videoconference with chat and audio centred on the mediator’s computer. The mediator also used email to communicate with the students, sending access instructions and the script of the sessions.

He analysed the interactions among pre-service teachers, grouped in teams of five students, based on CSCL criteria. Baker's model (Baker, 2002) (**figure 8**) supported the analysis of the interactions, identifying symmetry through the roles assumed by the participants, agreement or disagreement, alignment of the discussions, what led to classify the level of collaboration or cooperation among them. Symmetry concerns the constant inversion of proposer-receiver roles, indicating a high level of symmetry among the subjects. The agreement maps the level of agreement between the members. The alignment has a more general look at the interactions and aims to identify the engagement of the subjects throughout the session. Thus, symmetry and alignment are crucial for collaboration to occur since the agreement is part of the process and the agreement and disagreement between the subjects are beneficial to the interaction process.

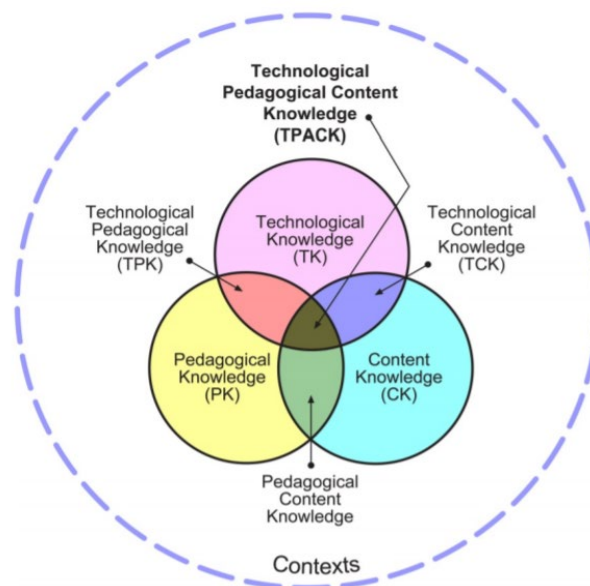


**Figure 8:** Bakers' analysis of interaction model (adapted from Baker, 2002).

Based on Baker's model, Araújo Filho (2015) claims that small groups collaborated better than the larger groups. He identified collaborative emergence of students' knowledge in these groups.

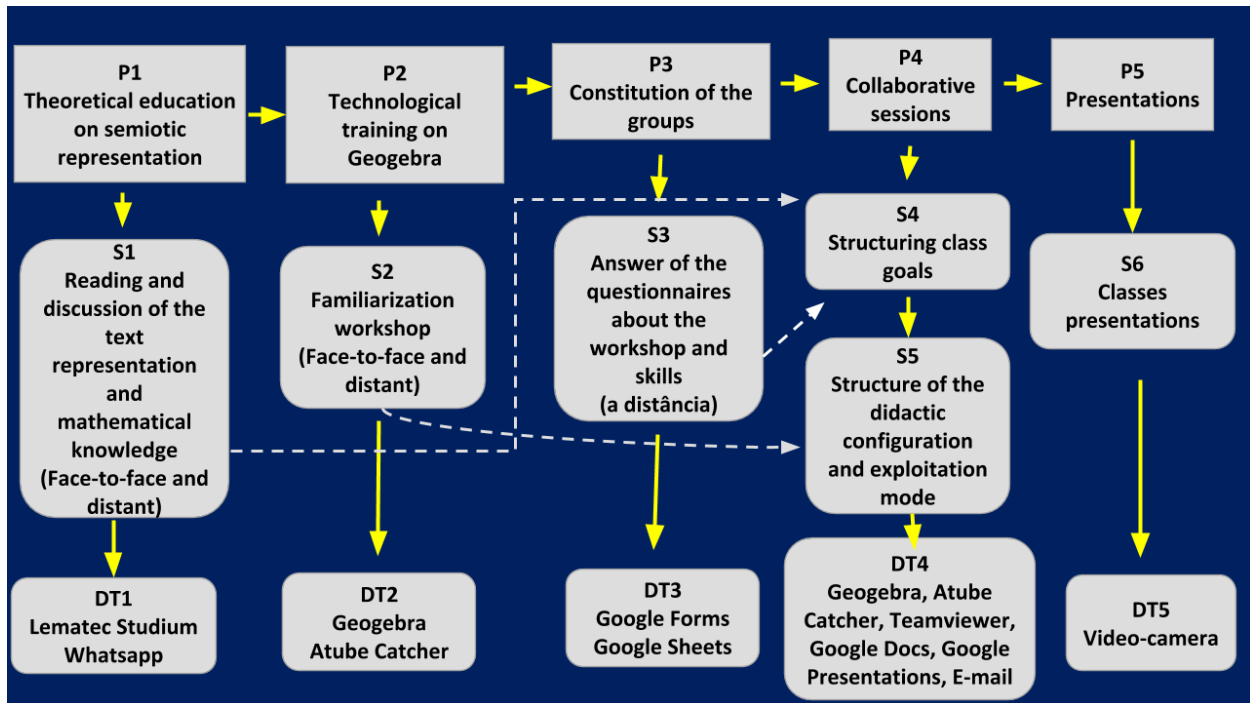
In this perspective, latter, Araujo Filho (2019) reinforces the need for the integration of digital technologies for teacher education. He builds up an online collaborative model for mathematics pre-service education, with the integration of digital technologies. He also highlights the scarcity of research that deals with the teaching knowledge of teachers in training, requiring more investigation to understand the nature of teaching knowledge that emerges from a collaborative workgroup with digital technologies integration.

To identify the teaching knowledge emerging from the built model, the author assumed the theoretical framework of the technological, pedagogical and content knowledge - TPACK (Khoeler, & Mishra, 2009). They integrate technological knowledge to the Shulman (1986) model, constituting the TPACK (figure 9).



**Figure 9:** The TPACK framework and its knowledge components (Khoeler, & Mishra, 2009, p.63).

In his model, Araujo Filho (2019) integrated IO (Trouche, 2005), as propellants for collaboration and, consequently, a determining factor for the emergence of TPACK knowledge. In the group, students were challenged to build an IO that integrates Geogebra for teaching functions. His model had five Phases (figure 10), ranging from Situations of theoretical discussion to instrumentation to the development of the IO, with didactic configuration and exploitation mode. All them involving a set of (DT) digital technologies.

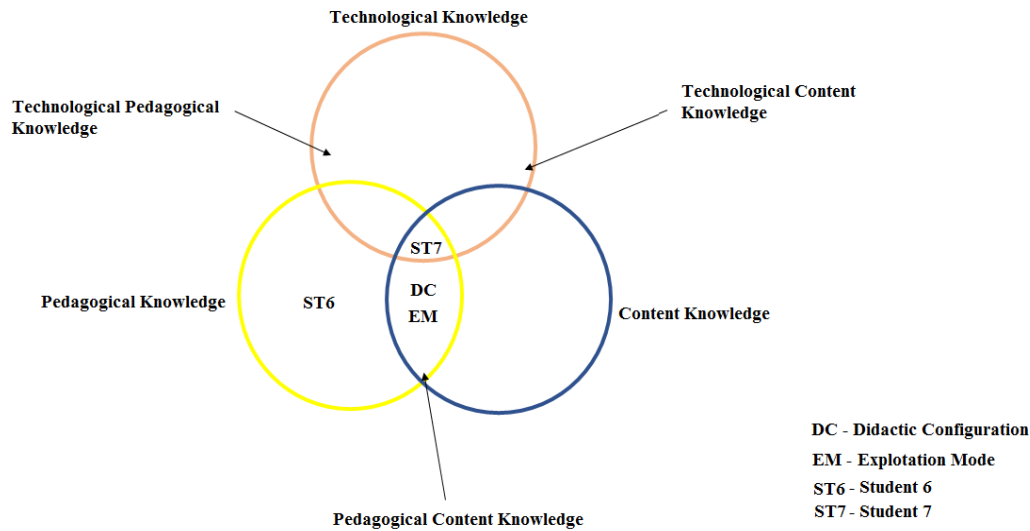


**Figure 10:** Schema of the teacher education model (Araujo Filho, 2019, p. ????, our translation)

In figure 10, the dotted arrows interconnect one situation that directly influences the performance of another situation of a different phase. S1 is essential to prepare them to discuss and structure the lesson goals of S4, just as S2 directly assists S5, that is, in the IO scenarisation.

The hypothesis that the situation, collaboratively involving scenarisation of IO, could raise integrated knowledge was confirmed. In **figure 11**, we can see that, in the analysis of the knowledge identified in the elaboration of the IO components, the integration of the three types of knowledge arises, individually with student 7 (ST7), but from the group conversations. Regarding the knowledge of this group, while planning the didactic configuration and predicting an exploitation mode, their knowledge characterises as pedagogical content knowledge.





**Figure 10:** Synthesis of types of knowledge identified (Araujo Filho, 2019, p.168)

The students' collaborative interaction contributed effectively to raise TPACK triad knowledge. Also, the integration of digital technologies as part of the experimental training model favoured collaborative activity. Communication between the subjects, mediator and trainers took place without significant obstacles since the digital resources available favoured collective and individual activity. Another result was that the students did not use all the available artefacts, but in some situations, they act autonomously. They sought for external resources as a complement to what was available.

Ignácio (2018) also studied pre-service teachers' education, but differently from Araujo Filho (2019), he used the documental approach to didactics.

### **The pre-service teachers' education: The birth of teachers' documentation**

Based on the DAD, Ignácio (2018) studied the birth of teachers' documentation in the locus of the pre-service mathematics teachers' education. He focused on following the moment reserved for the pre-service teacher first encounter with teaching practice, in supervised internships. The study of the documental genesis of the student in mathematics project for the elaboration and improvement of a chapter on mathematics digital textbooks (DT) reveals aspects of the origin, the birth, of the teacher's documentation. In this study, Ignácio identifies the creation of a documentation structure around the

support material (textbook) through adaptations of previous schemes originated in the mathematics teachers' education course and the development of new schemes related to professional activities.

When analysing the pre-service teacher documentation genesis, he traces the evolution of teachers' pre-service documentation system in a reflective education within the project that happened in the pre-service teacher's curricular Teaching Practice.

The textbook is a resource widely distributed in Brazil. Between 2018 and 2020, the Brazilian Federal Government acquired more than 450 million textbook volumes to distribute among students within the PNLD (National Textbook Program). In its version of 2014 (Brazil 2012), PNLD introduced a set of digital objects as the appendix of each volume, which pointed to the possibility to combine the current quality of textbooks with a digital environment in which the teacher and students could expand their performance. Regarding the mathematics teacher, a diverse range of digital objects can be integrated with conventional textbooks (games, simulators, microworlds, plotters, among others). The emergence of several platforms for developing and sharing teaching materials, such as the Geogebra book, these possibilities can go beyond acting from users to authors of their digital textbooks. In this scenario, Ignácio problematised mathematics pre-service teacher education to deal with digital resource development in the form of a book.

In inferring the subject's documentary system, Ignácio (2018) considered that DAD recommends that "[...] the analysis of the evolutions of the documentary system can base the analysis of teachers' professional development" (Gueudet, & Trouche, 2008, p.16, our translation). He also chose to follow the principles of the reflective investigation methodology (Ibid) to support his study.

For two semesters, Ignácio (2018) recorded the activities of a graduate student in the teaching practice field (a basic education school), accompanied by two supervising teachers, one each semester. During the practice, the pre-service teacher elaborated, experimented, reflected and recreated a chapter in the Geogebra book to teach functions in the 9th grade of elementary school. The data comprised the recordings of

- all the meetings for planning and formatting the activities of the Geogebra book (trainee and supervisor),
- the trainee's rules using the material created, and
- the subsequent meeting for reflections/replanning between the supervisor and the trainee.

The trainee's previous work in the disciplines of the mathematics course, especially in Methodology of Mathematics Teaching, was also recovered through his resource map, interviews, and document collection.

He analysed the data to trace the evolution of schemes of use, documentary evolution. The results show an evolution of the intern student in the sense of assuming a part of the responsibility with his training path, structuring a documentary system based on the resources previously developed and integrated with his proposal for a digital textbook chapter.

Ignácio (2018) showed that the student's involvement with the design of a digital textbook chapter, a teaching resource, served as an element of unbalance, demanding new ways of dealing with teaching resources. There was an emphasis on the fact that the subject's organisation to deal with the challenges faced allowed the development of a documentation structure around the support material (textbook) in terms of curriculum, collective work organisation and resignification of resources for integration with the proposal.

Taking a similar approach regarding the involvement in a design curricular resource project, Gitirana (2018) traces the evolution of a student resource system in a pre-service teacher education.

### **Curricular Resource Design: a study of a pre-service teacher resource system**

In the perspective of the evolution of teaching documentation, still in pre-service education, Gitirana (2018) discusses the teacher as a designer, in the teacher education approach that she named authoring. For that, her research unites activity theory (Leont'ev, 1978; Engeström, 2001), DAD (Gueudet & Trouche, 2009a; Pepin, Gueudet, & Trouche, 2013) and curricular ergonomics (Choppin et al. 2018). Collectively, students developed the design of a didactic game for teaching mathematics, from the

prototype to the analysis of use in the class. Pepin et al. (2019) show how documentation and engagement evolve during the various stages of design.

In a long-term teacher education project (3 semesters) for the development of mathematical didactic games, she analysed how a game design process in group-work of pre-service teachers affects their formation and the origin of their documentation. The project was developed in mathematics teaching methodology courses (II, III and IV), using the principles listed in figure 11



**Figure 11:** Principles of the teacher education.

During the two years of the experiment, 15 students participated in all the activities and worked on the design of four mathematical didactic games. The study showed that the students, collaboratively, work in the design of a mathematical game as a curricular resource. The students designed a mathematical game, writing its description. Moreover, to try out the developed game, they used it in different levels of practice, ranging from testing with specialists (colleagues in the discipline) to use in school classes, reflecting on this use. Thus, they wrote an article about the investigation and presented it in class. Some students continued to develop their essay investigating the use of their game (Silva, 2019; Silva Neto, 2018; Pontes, 2018; Mendonça, 2018; Souza, 2018; Melo, 2018).

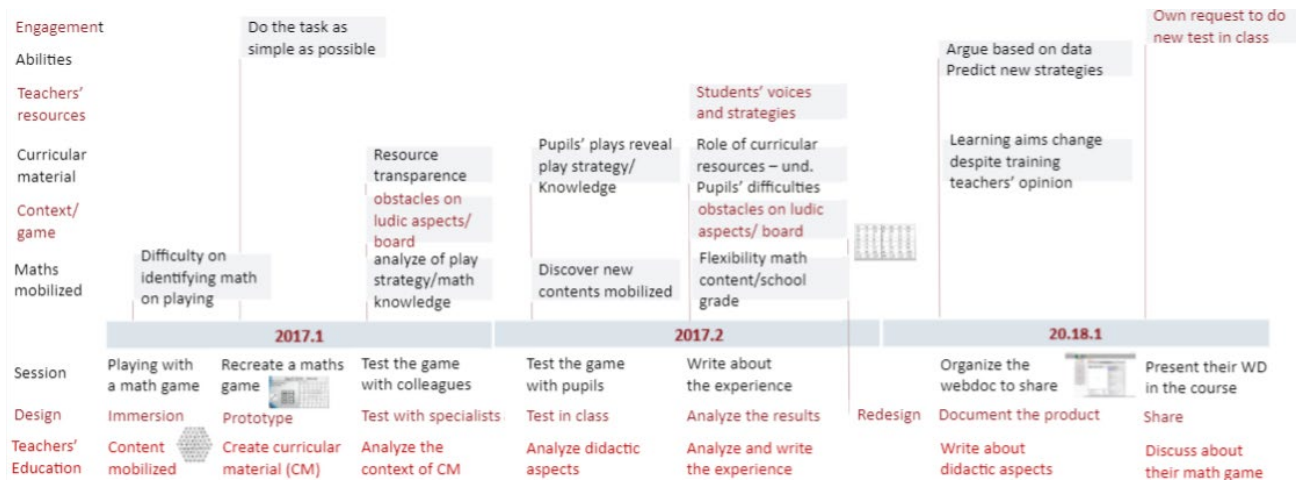
Based on DAD (Gueudet, & Trouche, 2009a), Gitirana (2018) studied students' resources, mainly, on how the game served as a resource for their formation in the evolution of their documentation, as shown in the timeline of **figure 12**. In addition to the resources of the pre-service teacher, the timeline shows the analysis of one of the students focusing his teaching skills, engagement with the task (of creating a game), his invariants about curriculum resources, games in the educational context, and mathematics. The timeline shows the development in three semesters, indicating session, design activities and teacher education aims. In the first session, the students played a mathematics game called "Calculus" and were asked to identify the mathematical knowledge mobilised, which served as an immersive moment in terms of design. At that moment, the students' difficulty in identifying the mathematics worked in the game was revealed. They were able only to identify the most superficial aspects of the content, such as the four operations and the mental calculation. The properties involved in the plays, for example, were not identified.

In a second moment, the students had to modify a popular game, to explore mathematics, recreating curriculum material (**figure 12**) for teaching mathematics. So, they designed a prototype of a game that they should use in practice and publish. Many of the groups did the activity to satisfy the teaching demand as simply as possible.

As a first test of the game, students are asked to describe the game. Then a test with specialists was undertaken with their colleagues. They tried out the game and the description as well.

In the first semester of the project, the student already starts to discuss the transparency of the didactic material (Adler, 2000). According to the student group discussed here, the game needed to bring mathematics and the game itself in a balanced way, without overlapping the other. In terms of game context, they begin to identify some obstacles in the board design they made. They began to anticipate students' play strategies as a tool to think about the game.

In a second semester, they tried the game in a class of elementary school and, when authorised, filmed, or recorded the experimentation. After that, they analysed and presented the experience in class.



**Figure 12:** Trajectory of a pre-service teacher documentation

At this semester, the student being analysed in his group demonstrated to know the game as curricular material. He began to discuss how pupils' game moves reveal their mathematics knowledge. So, they argued that they do not need an exam to find out what pupils knew or not. As for mathematical knowledge, they go deeper into the identified knowledge. At the end of the semester, in the text and presentation, the students already established that pupils' feedback was an important resource for class decisions. They used the game to identify pupils' difficulties. They started to analyse the obstacles of the game board for its playfulness. They also argued that one game could explore different contents, depending on the level of education.

In the third semester (**figure 12**), students are asked to build a webdoc, in terms of the game itself, as well as its didactic aspects. Students' presentation of the webdoc revealed that they were able to predict pupils' strategies as a tool to discuss the didactic possibilities of the game. The game objective changes regardless of the ongoing education process.

At the end with the redesign of the board from the reflection on the obstacles to playfulness, the group requested to redo the entire application of the game in class using the new board, and only then deliver the webdoc. It represented a significant transformation in engagement that moved from making the task as simple as possible, to redo all the experimentation without being asked, with the improvement of their product, the game.

### **Some considerations: teachers' education and identity and curricular resources**

GERE research works showed us the importance of creating and designing instruments, both by instrumental genesis and by designing and redesigning of artefacts, in order to advance in learning from practice. The importance of thinking about artefacts and instrumental genesis in teacher education led to the design of instrumental meta-orchestration, to offer teachers and students a theoretical-practical education about the IO. We also developed a teachers' education model to integrate digital technology that allows the students to act and think collaboratively generating different teaching knowledge: of the content, technological and pedagogical.

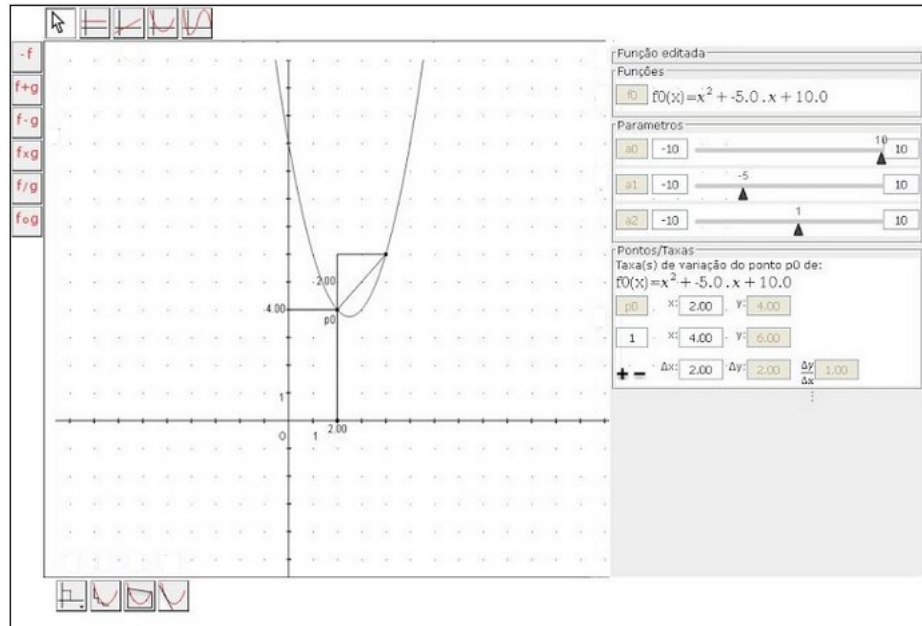
### **Support for learning and resource generation**

In GERE, resources are also studied in terms of supporting learning and resource generation, studies that, in turn, serve to elicit requirements for the engineering of digital artefacts (Silva, 2016). Silva studied the design and use of computational artefacts in teaching and learning functions, focusing on the covariation perspective.

Generating and integrating digital instruments in the teaching and learning of mathematics requires attention to specific aspects related to mathematics teaching and learning, the nature of mathematical knowledge and mathematics representation in the digital environment.

Silva (2016) analysed the design and validation process of a digital artefact to address functions, focusing the rate of change and joint variation between variables. This emphasis is essential for the development of covariational reasoning, defined by Carlson et al. (2002) as being the “[...] cognitive

activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other.” (p.354)



**Figure 13:** *Function Studium* Screen

Guided by a software development model (Bellemain & Tibúrcio, 2017; Santos, 2016), Silva (2016) pointed out some cognitive, didactic, epistemological and computer aspects related to the rate of variation. These aspects articulate and permeate the process of design and implementation of the artefact, called *Function Studium* (**figure 13**).

The software engineering model that guided the conception and validation of *Function Studium* comprises six phases: (i) delimitation of the content field; (ii) preliminary analysis; (iii) requirements analysis; (iv) a priori analysis and prototyping; (v) experimental phase and (vi) a posteriori analysis and validation (Santos, 2016, p.53).

In the preliminary analysis, Silva (2016) articulated computational aspects applied to the function (Kaput, 1992) and aspects of teaching, learning and mathematical epistemology of the rate of change in the covariation perspective. This analysis served to feed the requirements analysis phase, in which the characteristics were defined in terms of the computational programming of the artefact, such as programming language, aspects of the interface, menus and windows functionalities. The requirements



analysis phase is, in itself, the translation of aspects of the mathematical concept in terms of language and computational representation.

The prototyping phase is when the team materialises the artefact. It was operationalised through development, discussion, and analysis cycles of the adequacy of the prototype, in collaboration with the engineer-programmer. At this moment, a priori analysis, inspired by didactic engineering (Artigue, 1996), allowed not only to anticipate possible interactions between the artefact and students in the experimental phase but also to use these anticipations to implement adjustments and improvements in the artefact programming.

In the experimental phase, Function Studium was used by a pair of undergraduates in mathematics teaching as a support to solve two activities on the rate of variation of polynomial functions. Silva (2016) analysed the experiment in terms of how the artefact features supported, or not, the students to mobilise covariational reasoning. Aspects such as the possibility of dynamic variation of variables and the dynamic and simultaneous articulation between representation records enabled this support. Function Studium software validation resulted from (1) an analysis of the experiment results and (2) an analysis of how each phase of the development process contributed to these results.

The articulation between aspects of software engineering and theoretical-educational aspects in the design of digital artefacts is a necessary process to think about the generation of resources for mathematics education. Disregarding the importance of this process, it can result in artefacts without a significant contribution to learning.

## Final Considerations

The GERE studies developed show results that can contribute to mathematics education that go towards the integration between software engineering processes and theoretical and methodological aspects of mathematics education and their contribution to think about the design of digital resources for significant support for teaching and learning (Silva, 2016). The design of curricular artefacts also appears as an essential part of teacher training (Gitirana 2018, Ignácio 2018), the authorship process transforms

the subject and its documentation, not just the instrument. In the context of teacher training, the research also contributed with training models for integrating digital resources (Lucena, 2018, Araújo Filho, 2019), with instrumental meta-orchestration, which focuses on IO learning and a collaborative training model with the focus on teaching knowledge. The studies also bring the IO model as an instrument for analysing the teaching practice of EAD-UAB. Pereira (2017) and Lucena (2015) show the complexity of the didactic configuration process imposed in the construction of courses and mediations.

Artefacts and resources in the field of education are multifaceted. In the research of the GERE group, a resource may be the theory that allows the interpretation of the teacher's work in the virtual classroom or that which underlies the construction of a training model about himself. It can also be an artefact as a platform that allows the integration of other platforms to promote collaborative work between different users in real-time. A resource can be a text, which through a documentation process becomes a new document for the subject. Moreover, through an authoring process, an existing resource can be transformed, becoming another resource. Resources sometimes serve as a support for teaching and sometimes for learning. There are those used to plan, others to evaluate. It has built features, while others support creation. There are so many possibilities waiting for human hands and minds that will give them meaning, meanings and different uses. Investigating resources for education, in particular for mathematical education, is more than analysing uses or evaluating reflexes and results. The relationship between human and resources is intertwined interdependent. Man shapes his instruments to carry out his practices, but they also shape him.

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