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Abstract**:** In the first months of life, babies develop visual perception. The notions of space evolve in the everyday of experiences, the recognition of the self through your body, and relationships with others. The topological notions developed by babies correspond to closeness, proximity, continuity and separation. As babies grow, their skills are developed both in the projective space and in the geometric space. These even influence the baby's development in an integral way. This article intends to present results of the topological notions of closure, proximity separation and projections in the baby's space. This qualitative research is developed under a descriptive perspective with interdisciplinary contributions. Data collection was made from cartography, photographic and filmic records of babies in different cities in Brazil and Colombia. The reflections developed point to the development of perception from the offer of multiple experiences since the first months. In addition, it is evident that the understanding of important mathematical concepts happens since the beginning of life, from everyday experiences of exploration and relationship with spaces and regardless of the formal school learning of geometry and its concepts

*Keywords***:** Space, Topological notions, Projective space, Child education, Baby studies.

Introduction

Babies develop visual perception skills from birth. That is, the spatial sense begins to develop from the baby's daily life. From the moment babies start to know their body, they identify the part and the relationship with the whole (of their body) from their senses and visual associations. As babies interact with their surroundings and with the environment through their movements, they appropriate a space (their universe), and experiences of perception of (another) increasingly larger space appears. For example, a simple rotation inside the cradle, attempts to walk, or even crawling – among other exploratory movements in the world –, will enable the development of skills for coordinating movements, walking, and perceiving the world. Likewise, the notions of space conquered since the babies' first

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experiences and all learning and hypotheses built by them in this period without school systematization help consolidate a conceptual basis for learning geometry at a later stage (Santos & Almouloud, 2014; Alsina & Giralt, 2017; Rodríguez et al., 2019; Vilotta et al., 2019).

When talking about the perception of space, the matter is not necessarily limited to knowledge governed by a specific school curriculum. It is assumed that the construction of knowledge develops from a very early age, participating in multiple *experience*s *(re) organizing* recognition of the world, which are provided by the environment, such as, for example, coming into contact with a toy, with an adult or even crawling on the floor to reach a specific point. Visual perception develops through the skills of (flat) representation (of three-dimensional or mental objects), transformation (translation, rotation, reflection, etc.), generalization (about the actions of the object), communication (seen from the argument and proof about representations), documentation, and reflection on visual information (Hershkowitz, 1990). That is, visualization is conceived from the different flat representations of three-dimensional objects, and the actions on these representations can be determined by transformations that lead to generalization. Thus, the experiences, which relate the interaction to the environment, will help improve cognitive development and facilitate, from the use of well-structured abstract concepts throughout their lives, the visualization and interpretation of reality in their daily activities or in research.

However, it cannot be deduced that this learning occurs only when a teacher monitors a school class; on the contrary, learning is developed in the experiences to which the baby is exposed since the first years, that is, learning is consolidated based on these notions (Acevedo-Rincón, 2010). These notions of space and the subsequent development of concepts of spatial thinking will allow the children, in the future, to have an easier time learning the forms and their relationships, in addition to locating, in space and trajectory traveled, articulating axes of spatial thinking. Spatial thinking as part of geometric systems and defined as the set of cognitive processes through which the mental representations of spatial objects, the relationships between them, their transformations, and their various translations into material representations are constructed and manipulated (MEN, 1998) . It is in this sense that different theoretical

references (Bishop, 1983; Wheatley, 1990; Shea et al., 2001; Quinn, 2004) consider that learning forms and relationships, compared to the reference space, relates to visual perception (Wheatley, 1990). Therefore, according to the investigations, two important processes intervene in the development of the spatial sense. In this order of ideas, when the baby has contact with a wider universe, he/she develops several skills that are opportunities to learn new notions (Cando, 2011).

Early childhood research in mathematics education focuses, above all, on children's school learning. This means that it does not consider that learning comes from years ago, when babies just start to recognize themselves in the world. The results of the childhood mathematics education report (National Research Council of the National Academies, 2014) emphasize that research on babies in early education and with babies focuses on learning counting and natural numbers. Aspects such as representations of fractional quantities and geometric and proportion relations, and shapes and measures categories were analyzed. However, the importance of researching on babies and children in the studies is considered, taking into account that these are bases for investigations on mathematics learning at higher levels, such as the case of symbolic language.

In this sense, this research presents results that contribute to the development of mathematical education in the early years, with baby studies, since the recognition of babies' experiences in the first years of life offers multiple possibilities of posterior learning. Likewise, it is stated in the literature that the development of research focused on spatial thinking could contribute to mathematical concepts at higher levels, such as quantities and measures, part-whole relationships, and the understanding of Euclidean geometry (Skolnick, Langbort & Day, 1982; Ansari et al., 2003), in addition to more global learning, as relationships between verbal and mathematical skills (Clements et al., 2007; Stewart, Leeson & Wright, 1997; Wheatley, 1990).

Based on this, this article presents the importance not only of learning about space from early childhood education, but of being able to live and experience that space. In this regard, the type of

experiences babies are offered/allowed from the first months is important. Subsequently, some conceptual elements about the baby's topology and perceptual space are presented, in order to consider the four categories proposed for the study: enclosure, proximity, continuity, and division. Next, the methodology describes the research type, the data collection method and the interinstitutional research participants. Finally, multiple scenes are presented as a result of observation of babies' interactions, in day care centers and family homes, and then conclusions about this type of research and contributions to mathematical education.

Space and experiences in early childhood education

The education of babies and young children is understood as a partnership between families and day care center. In Brazil, educational legislation states that early childhood education and care should contribute to the integral development of babies and children from zero to five years of age (BRASIL, 1996, 2009, 2018). During this period, the objective is not to offer formal school content, but to provide situations that enable the child to share and build knowledge based mainly on interactions and games.

Early childhood education in Brazil is understood as all children's right and the first stage of basic education, as established by article 29 of the National Education Guidelines and Framework Law (Brazil, 1996). In Colombia, initial education becomes a comprehensive care structure (Colombia, 2007; 2008) from birth to six years of age, based on the recognition of their characteristics and the particularities of the contexts in which they live, favoring interactions generated in enriched environments through pedagogical experiences and care practices (Colombia, 2011).

Document produced in Brazil on day-to-day practices of day care centers and preschools (Barbosa, 2009) emphasizes that the objective of early childhood education, from the point of view of knowledge and learning, is to favor experiences that allow children to appropriate and immersion in their society through the social practices of their culture, the languages that this culture produces, and has produced, in order to build, express and communicate meanings and senses. It is evident that it is essential to offer,

from children's birth, practical situations and experiences that can be processed and systematized by a body that feels and thinks. For this reason, it is necessary to choose other ways to prioritize, select, classify and organize knowledge, closer to the children's dynamic experiences and not to the fragmented view of disciplinary specialization, problematized by science itself (Barbosa, 2009).

Research on babies in a day care center setting has also shown the babies' ability to create their own activities and games, exploring space, their bodies, and materials available. In this respect, the works of Barbosa (2009), Coutinho (2009, 2010, 2014, 2017), Schmitt (2008), Fochi (2015), and Tebet, Costa and Barros (2019) show the babies' creativity and even a certain independence from the proposals made by the teacher or the institutional routine. They reveal games created and developed by babies in a relatively autonomous manner in moments reserved for diaper changing, feeding or sleeping; and yet, they show how babies create in their own ways of exploring space and materials, often running away from what was initially proposed.

All of these authors point out that the relationship between babies and space is an important element, and this relationship is what is intended to be explored in this article from a perspective of the knowledge development in the field of mathematics. In particular, this article aims to present results from the topological notions of enclosure, proximity, division, and the projections in the baby's space.

But space has different connotations from different perspectives, it is not a unique concept brought from mathematical education. There is, for example, the notion of space proposed by Geography. According to Massey (2008), it is possible to (re) think the house, the day care center, the park or any other space not simply as surfaces, but as the result of encounters. Doreen Massey is responsible for a relational (and political) perspective of space, in which it acquires a social dimension and is considered a product of interrelations. It is a conjunctural encounter of trajectories in process (of human and non-human elements), "a simultaneity of stories-so-far" (Massey, 2008, p. 33). In this context, space is not given a priori, and trajectories play an important role. For the author, "place change [sic] us, not through

visceral belonging, […] but through the *practising* of place, the negotiation of intersecting trajectories" (Massey, 2008, p. 220). This approach allows us to (re) think the space records since in this relational perspective of space, "everything moves" (Massey, 2008, p. 199).

In this approach, space is not understood as a context where the baby moves and encounters different elements, but as a result of babies' encounter with such elements.

In this article, however, one will try to dialogue with mathematics concepts that can be very important to understand the babies' daily experiences and thus contribute to professionals' training for early childhood education, without disregarding other important perspectives for the study of space, as for example, the proposal by Massey (2008) or the debate about the directionality of the baby movement proposed by Impedovo and Tebet (2019).

Topological elements

Topology, one of the newest sub-areas of mathematics, represents a generalization of the properties of the open interval in the real line, the independent properties of others present in R, such as sum, order, and distance (Macho-Stadler, 2002). In other words, topology refers to studies of the properties of space that are not affected (or deformed) by continuous deformation, that is, it can stretch, bend, enlarge borders, but never break. To better understand the relevance of studying topology in babies, it is necessary to understand that topological space refers to the global properties of objects regardless of their shape and size (Lima, 2017). When we take these concepts brought from mathematics to the contexts of the space of the house, the day care center or the park, they can be understood from the conceptions of inside-outside, edge and border, near and far, among others. It should be noted that for topology, magnitudes, angles, or aligning points in space are not important, which relate more to the conception of a Euclidean space. Some important notions in the topological space correspond to the border, interior and exterior. These notions are visible from an early age but are limited to children's motor skills. Thus, the topological notions formed in babies are constituted from the "perceptual space" which evolves over time

(months in the case of babies) to the "concrete of space" (Ríos-Gallego, 2017, p.66) Therefore, simple notions such as up, down, about, under, above, below, before and behind, are developed from everyday experiences and contribute a great deal to achieving spatial notions. However, approaches to these notions occur since the first months. According to the proposal by Abarza et al. (2010), the evolutionary development of spatial notions starts from six months, when trying to approach the notions inside-outside, about, under, above, below, before and behind, which are consolidated until three years of age.

The authors also state that, at 18 months, they already understand the context; at two years old, until almost three years old, they develop symbolic thinking, recognize adults including adults behind barriers, and even find a toy; from the age of three, children can recognize and memorize short routes; at the age of four they can safely rotate sheets, and, it is considered that at six years of age children no longer has motor limitations to continue to develop their skills, and they are already able to differentiate far-near, inside-outside, together-separate, and the development of laterality begins. However, it is possible to affirm that even before six months, some topological notions are understood by babies, from experiences such as being placed (or not) on the lap, in the stroller, in the crib, on the carpet or in the rocking chair, and the possible movements to be made in each of these spaces.

According to Dienes and Holding (1972), four topological concepts are highlighted: enclosure, continuity, proximity, and division, which are important in the development of spatial notions in the baby until early childhood. *Enclosure* is directly linked to the notion of border, in which the baby can identify what is inside and what is outside, always stressing that being and belonging correspond to a reference, in this case, the border. The *border* is a concept that marks the development of spatial notions in the baby and that reverberates until adulthood. Regarding the border, it can be a real or imaginary line that separates one space from the other, distinguishing two non-material things (Quinn, 1994). Furthermore, a border divides space into regions. For example, the fence of a house delimits the space that the house has and what is outside it. Another example may be a small border, such as a box, and having toys inside or outside the box, but it also can be inside or outside the kitchen (median border), or even having a larger

border like the house or the park. Belonging relationships are important for the development of concepts such as: decimal numbering system (Vilotta et al., 2019), patterns (Alsina & Giralt, 2017), sequences (Rodríguez et al., 2019) and even for understanding intervals in functions at higher levels (Santos & Almouloud, 2014). *Continuity* refers to any transformation that does not eliminate the adjacency of the different parts of the figure. For example, there are continuity relationships in the body, because the parts are consecutive, but they also exist in a path without obstacles.

On the other hand, *proximity* is identified in the relationships between objects and corresponds to the management of notions as a neighborhood between elements. It can be understood in the baby as being close to a person or an object in space. The understanding of proximity in time is generated later. But some of these notions help with projective notions because when the baby knows whether a person or a toy is close to him/her, the distinction of far-near that is constituted at four years old begins. In this case, talking about the neighbor makes sense, because the neighbor (object or person) is the one on the side, but also in front of the baby. **Figure 1** bellow presents a scheme with the constitution of the baby's perceptual space, under the four topological, projective and geometric notions.

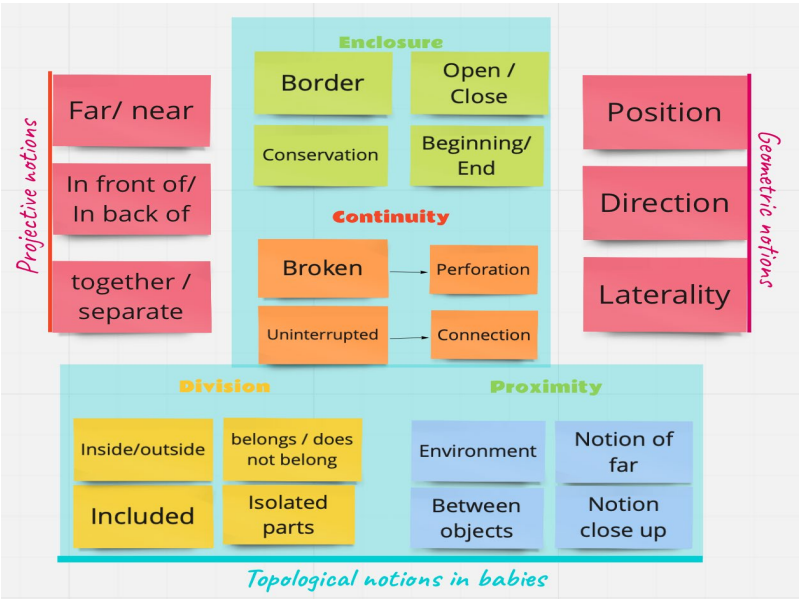


Figure 1. Constitution of perceptual space in the baby

Division corresponds to the relationship that allows the isolated parts of the objects to be recognized and identified. For example, is the toy inside or outside the crib? It is common to see very young babies exploring this notion, for example, when throwing out of their cribs a set of toys that were previously inside them. Is the mother in front of or behind the baby? Is she still there, behind the scarf when playing hide and seek with the baby? In particular, to conceive inside and outside, front and back, is necessary to recognize a point of reference (in itself, in general). But it also brings the reference of belonging, being inside or outside a border, delimited by a box, a vase, or a larger space that indicates something around the child. It is also possible to think of how the notions of belonging and border contribute to the constitution of the notion of group, linked to a limited space, such as being part of a specific family that lives in a house or apartment with its well-defined borders, or being part of a specific group that has a specific space (room) in the day care center.

In addition to the notions identified in babies, research such as that of Fennema and Sherman (1977), Lean and Clements (1981), and Castro (2004) also highlight the knowledge of projective and geometric notions, which are developed in later stages of the first approaches to the perception of space, formed up to approximately four years of age. However, they are formed from the first months when the baby faces different situations that propose different perspectives for the observer baby. For example, it is not the same to look at the mother from the floor when crawling as it is from the feeding chairs or from another adult's lap. This fact has implications for the constitution of the baby's perceptual space.

Methodology

This is a qualitative, exploratory, descriptive study (Hernández-Sampieri, 2014), which presents data that reveals that the information obtained from data records, collected through participant observation, is the report associated with cartography, an innovative methodology in the field of baby studies, inspired by Deligny's work The research adopts an interdisciplinary stance, which is based on the dialogue between contemporary authors who simultaneously mobilize important ideas from different disciplines

(sociology, geography, anthropology, philosophy, pedagogy and mathematics). The research protocol for the development of this study was approved by the Ethics Committee in Brazil and Colombia.

In addition, this research describes the topological notions of space that are developed by babies, which is within the focus of the research called "Spatiality," as shown in **figure 2**

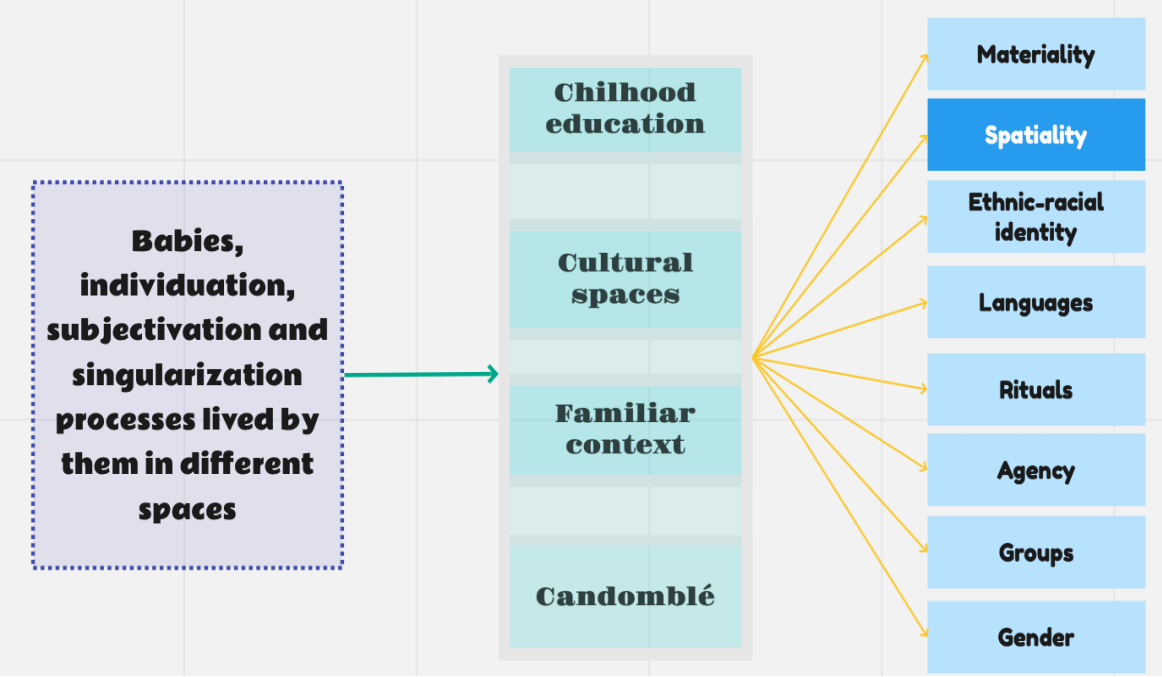


Figure 2. Project organization in the different research focuses[3]

Since the beginning of the project, the research involved 15 babies from Brazilian families and 10 babies from Colombian families, with observations and follow-up developed in different contexts, such as public and private environments in cities in different regions of Brazil (Southeast, Northeast and Center-West Regions) and Colombia (Bucaramanga, Barranquilla and Puerto Colombia). In addition, participant observations, photos, videos, field diaries and cartography made both in Brazil (since the beginning of the project in 2018) and in Colombia (since 2019) were used. In particular for this article, analyzes of the

[3] FAPESP Grant 15/10731-8. More information in gabrielatebet.com.br/projetobebes.

results of scenes collected in homes of families and day care centers, in São Carlos, Campinas, Hortolândia and Jundiaí (Brazil), and in Bucaramanga and Barranquilla (Colombia) are presented.

The records of the scenes observed during the research favor cartography inspired by the work developed by Deligny (2008), which allows showing the actions, relationships and interactions that the baby performs with the space and its elements or actors (that is, adult, baby, toy, food, etc.). In addition, the use of cartography allows relating spaces and movement (Tebet, 2015 a; 2015 b; Tebet, Costa and Barros, 2019). The relationships experienced by babies that produce them as subjects and individuals come together in cartography, which also produces a space that (re) signifies trajectories of belonging and interactions.

What the data show?

Below there are scenes collected in different contexts, but with similar babies' behaviors when interacting with the other (baby or adult), which relate to the categories: enclosure, proximity, division, and projective space. The following are some of the scenes chosen to present each of the categories proposed in the research.

Topological notion: enclosure

The borders demarcate two spaces and depend on the observer's point of view or point of reference point; the element or person is always "inside or outside." When observing the baby from the scene recorded in **figure 3**, the *notion of a border* projected by the edge of the basket is evidenced. There are two regions demarcated in this scene, the first corresponds to the basket, and the second, delimited by the colored carpet, corresponds to the space where the baby is trying to recognize what is on the other side of that border drawn by the basket. In this scene, the baby is outside the basket, given its position on the floor.

Figure 3. Baby realizing what is in the basket.

The barrier between the baby and the toys will remain until the adult helps, placing the infant in another position (on the lap or standing), in order to open the basket and take what was inside or taking an object out of the basket and offering to the baby. However, beyond this action, the baby can recognize that even being outside the basket and still close to the objects, it is clear for them what is in the basket. In this scene, therefore, the border between the baby and the objects inside the basket is real.

Figure 4 is an important record in our analysis, as there is a border proposed by the baby when placing the towel over his head. This gesture of hiding behind the towel places a barrier between the child and the outside world, which leads to identifying the *notion of border* in the baby.

Figure 4. Baby puts towel over his head.

The border is open, because it does not cover the whole baby, but only her head. However, this is not an imaginary border. Although it is small, it exists, and allows the baby to think that she can hide behind the object. The demarcation from inside the towel border allows verifying that even though the towel is small, it divides two regions that are between the baby and the outside world. Thus, from a game developed spontaneously by the baby, she autonomously explores important topological experiences, even without the intervention of any adult in the scene in question. Not all borders are visible to the eyes (of adults). Some of them are disregarded in their role of separating regions in the strict sense. But, when looking at Figure 4 carefully, only the baby can establish these bonds and give meaning to the game of hiding she proposed. In the case of the sequence of figures below (Figure 5), the border is not seen clearly as something real that divides the space, but rather a barrier that made it difficult to cross the path.

In an attempt to explore space, in the sequence of records of **figure 5**, it is possible to recognize the *notion of borders* between the cradle and the table chair, but here borders are invisible. There is a barrier, even if it is not clearly visible to the eye, and it does not allow the baby to walk the path to reach the wall. In this record, the baby, in addition to identifying the border, tries to cross it with movements and body accommodation.

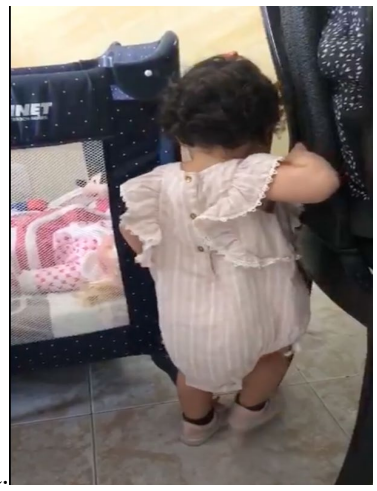
a.  b.  c. 

Figure 5. Border between the spaces marked by the cradle and the chair.

Similar scenes were found for other babies observed, in which they try to explore spaces and sometimes break the borders imposed by the adult world. For example, a fence, the place to get out of the house, or even the position of adults in front of them when protecting them from external situations. Those barriers perceived by them lead to the desire to explore the space and recognize what is behind them. In this way, these identified borders allow constituting the notion of enclosure formed from the babies' first months.

Topological notion: proximity

In everyday games, although they are simple, babies identify other relationships between people and objects, which also lead to expanding their knowledge of the world's size. As babies gain other movements, such as crawling or walking, their bodies will accommodate their perspective of distance through proximity. The movements allow babies to look at what is nearby and what is not, or also at what is between two points of reference or between two objects. Thus, the displacement of their bodies through space, with simple movements, such as rolling on the ground, creeping or crawling, allows them to bodily explore the notions of distance. It is with these simple movements that babies can code the location of objects (Newcombe et al. 1999); however, at one year of age they already perceive hidden objects and try to find them using the notions of distance and direction (Bushnell et al., 1995).

In the sequence of images of **figure 6**, the baby plays with the father, a game created by them, in which the father throws the blue cup in any direction, away from the baby, and he takes the cup and returns it to the father. When moving away from the father to reach the object, because the game involves taking the object, the baby shows the topological notion of proximity; finally, he recognizes that the object is far from him, and he needs to approach it to bring it back.

a.  b.  c. 

Figure 6. Baby playing with his father to catch the thrown object.

The concepts of near and far depend on the observer's disposition, as can be seen in Figures 6 and 7. For example, in Figure 6a, the cup is further away from the baby in pink than the baby in blue. Then, when the cup is thrown, it will be closer to the girl than to the boy. The same occurs with the babies in Figure 7, when the observer is the baby in the playpen; she is away from the teacher, and the other children are close to her. From the baby's perspective, shortening the distance to reach the teacher will depend directly on her movement.

Figure 7. Play Moment in the day care center; teacher and babies are on the scene.

The baby has to crawl to the teacher, following her signal to orient herself to the point of arrival. The notions of distance also allow arriving at babies' notion of comparing. For example, the more the baby

crawl to the point where the teacher is, the closer she is to her, therefore, the shortest the way to go. In addition, the comparison takes place between longer distances, and they are refined as babies reach the age of two (Huttenlocher Duffy & Levine, 2002). Over time, comparisons are made with smaller objects. For example, small and large dolls, which lead to the notions of comparing sizes. Thus, other discoveries begin to happen through the years. The assessment of the distance to be traveled is often used by babies to decide whether a given route will be followed by walking, crawling, or even if it is a very long distance that does not deserve the effort. In many situations it is possible to see babies who were learning to walk and took a few steps, and then they sat down and decided to end the route by crawling.

Topological notion: division

Perceptual space is also composed of *division*, which leads to the recognition of relationships between objects as parts of a set of elements. That is, these notions can be evidenced by identifying whether or not an object is part of the collections of objects. For this, the sense of belonging is central. For example, if there is a doll in the game box, the perception of it not being in the right place depends on the understanding of the notion of set and belonging, much more than the notion of border. It implies understanding that the outside, in this case, is related to the non-fulfillment of a certain characteristic.

This scene takes place on the terrace of the baby grandmother's house while she tries to catch the clothespin at the bottom of the bucket. The baby goes up every day to the terrace to walk and run, as it is a large space with objects that are attractive to her, such as clothespins, buckets, a basket of clothes, and even a dog that is sometimes afraid to approach her because the baby wants to "intensely" play with it, just like she does with the clothespins or the bucket.

An adult always supervises her movements, because there are stairs nearby. In this scene, there are two adults, one closer to her than the other. The mother is the baby's point of reference, and while she moves, she receives instructions from the mother, such as "be careful," "don't bother the dog," "can do it" or "cannot do it," "you are so smart." The baby takes some clothespins that are lying on the floor and takes

them to the bucket with clothes; when she takes them, she sees that there are two more clothespins and wants to pick them up. When making the first attempt, she takes one of them, her body gets stronger and then she gets into the bucket, as she insists on taking the two clothespins. However, she is not scare because the mother is close to her, reminding her how strong she is!

After picking them up, she smiles with her mother, forgets the clothespin set and goes towards the dog, with two clothespins, one in each hand. The mother's warnings - "put this in the bucket" or "don't take it for the dog," - make her go back to the bucket and try to play with the dog, and her uncle says in a strong voice "the dog is not for playing." She gives up on the dog and goes to the bucket, with which she finally plays in the center of the terrace.

The scene in **figure 8** describes the baby's notion of division related to inside and outside the basket of clothes. This particular scene refers to the discovery of other materialities in its context. In this scene, the baby approaches the basket, discovers that the clothespin is inside the basket, and it will stay out, if she persists in picking it up. Inside and outside the basket correspond to topological notions of the child's early years. This also refers to the perception of a point of reference that limits the space of the clothespin related to her point of view.

The clothespins that have to be removed in order for the basket to be empty cause the baby to change the projection of the visual line, and with this her position with respect to the floor; then, the first attempt was just to try to bend and reach the object, and since she was not successful, decided to take a little more risk with her body and stretch until she falls, goes out and sits down. Four distinct positions and one goal accomplished.

The border line, as a point of reference, causes the baby to fall into the same place where the two clothespins were. However, this was not a problem, and she did not cry because her mother was constantly supporting her through words. In this case, the observer is external, is outside the basket of

clothes, and analyzes turn around the elements that the observed tries to remove from the basket. While they are inside it, the border is defined by the edge.

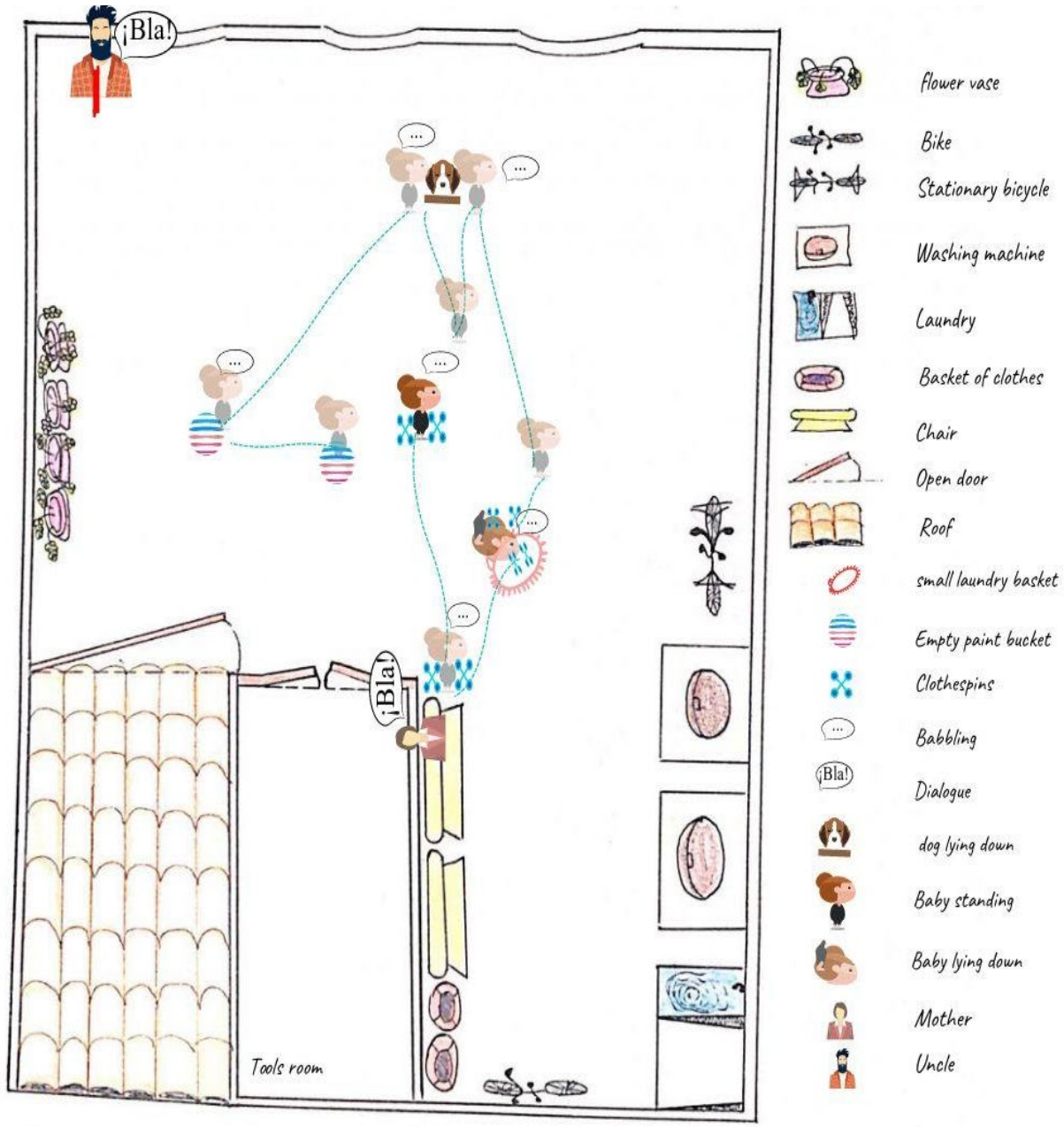


Figure 8. Cartography of the dialogue on the baby's family residence terrace[4].

Source: Elaborated by the authors

[4] https://youtu.be/5mGUzeNypRM

When experiencing the spatial language during these situations, such as that experienced in this scene, babies are able to relate the action to the words that express it (Gentner, 2003). For example, when someone says *put it in the bucket*, or *take it out of the bucket*, or also *look up*, *put it on*, or *put it under*, and makes the movement, babies can find points of reference in relation to their point of view.

On the other hand, **figure 9** presents the baby's routes when trying to fulfill the simple objective of "depositing the coins in the safe box." This time, it occurred on the first floor of the house, with a safe box she won as a birthday gift in March. This toy is pink and speaks up a phrase when large coins are deposited in its back side. At first, she did not understand the safe mechanism very well; afterwards she goes to where the mother is, babbling.

The scene begins with the baby sitting exploring the safe box and the coins, closing and opening the safe box to take out the coins. Each time the baby hears the phrases: "closed", "good!" "Inside, outside… opposites!", "Open and closed… opposites", her face expresses admiration, and she tries to relate the movements. It mainly occurs when she hears: "closed." But, when the safe box is closed and she cannot deposit coins in the cash outlet, she gets up, picks up the safe box and coins and throws the box at the floor in order for it to open due to the impact, and then she sits down again. However, this time the box safe falls on its side, then she tries to pick it up and go next to her mother, who helps to stop the safe box with her foot and the baby insists on depositing the coins in the cash outlet, not in the back side. She moves away from her mother, picks up the coins, and falls to the ground when picking more coins, taking the coins to her mother. She no longer insists on dealing with the safe box but keeps the coins and goes out to the hall to go to the kitchen.

Although the goal was not met, the baby looks at the coins and recognizes that the coins cannot be left out of the safe box. It is the notion of division that allows her to identify what is inside or outside the safe box, leading her to think about the coins belonging to the safe box. The baby's hurry to insert the coins,

even though it is through the wrong opening, demonstrates that she understands that the coins have to be inside the safe box.

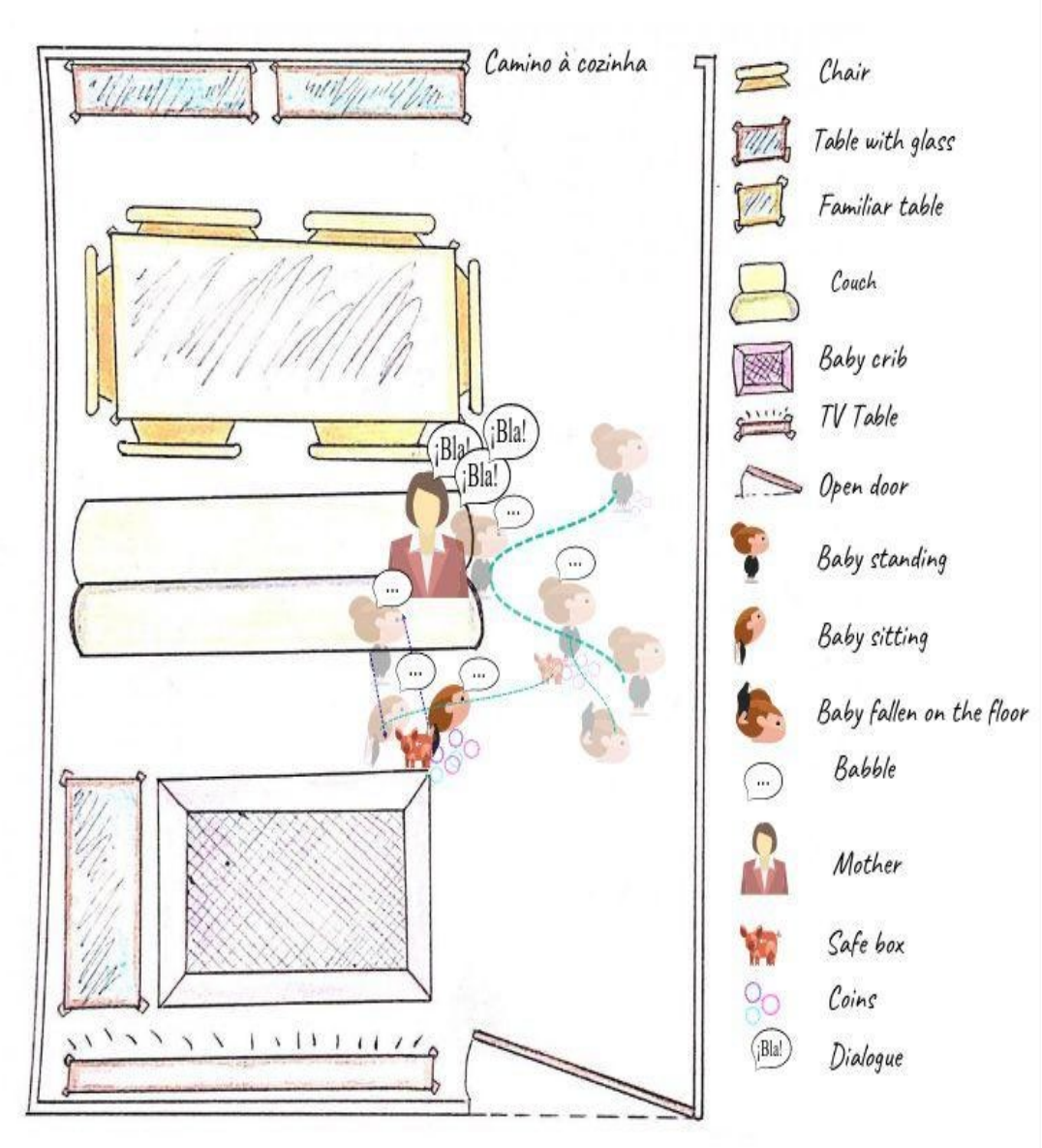


Figure 9. Cartography made in family residence' first floor room.

Source: Elaborated by the authors

The words spoken up by the toy do not reach her because they do not relate an action to a word, but all words arrive together. Although the objective of the toy is to "teach" the opposites with the song during

the action of depositing, the objective for her age (one year old) is fulfilled by identifying the collection of objects and the sense of their belonging to the interior of the border demarcated by the safe box.

Projections in space: approaches

As babies develop other possibilities for movement with their bodies, their perception of the world changes. For example, initially, the baby lies for the first few months looking at the ceiling, or at the mother, or even at his/her own body. It occurs always with a short perspective of the world. When trying to move from lying down to rolling on the bed or on the rug, the baby looks at something more than the ceiling; then, each new observation, in the same space but in another position, expands his/her perception of the world. And for each new object or person he/she sees, it is a new experience with which he/she becomes familiar and learns, based on the interactions that take place.

The change in the baby's different positions occurs according to the evolutionary stages, but on average, crawling usually occurs between eight and 12 months of age, although it may occur sooner or later, depending on each baby. One can consider the baby's advances from rolling to crawling, for example, as a learning process that involves movement coordination and neurological level development. These movements, linked to the recognition of space, have future implications for the child's development to coordinate movements, walk, and even read and write.

According to Cando (2011), crawling allows babies to develop their cognitive and social development when growing up. Thus, when the baby incorporates new movements and hence new points of reference, he/she realizes what he/she can and cannot see, depending on the position. In the scenes described below, babies appear in different places in the two countries, but they show the same regularities in their behavior in view of the perspectives they have developed.

In **figure 10** there is a group of babies inside a day care center in different positions, from which they have different perspectives. For example, the teacher is at the height of the baby on her lap, and the baby

looks at her shirt and what she sees at that moment is the teacher in front of her. And also, one baby who crawls over approaches the teacher trying to climb from her legs to her arms. These two babies have only the teacher in their nearest perspective, but also recognize the other children who are close to them. There are also two babies sitting, one behind the teacher trying to take her hand, which is within the reach of her outstretched arms, and the other focused on the baby bottle in her mouth, but who perceives the projection of what is on the floor.

Figure 10. Multiple perspectives (floor, standing and stroller).

She is next to the baby lying on the floor. The latter has, in his projective vision, a blue toy that he tries to reach, but needs to move to get to it. Finally, this scene has another baby in the stroller who looks down on the teacher. This means that this observation is made from another projection in space. Although they are all in the same space, not everyone has the same projection in space.

In **figure 11**, the projections in space correspond to the baby's horizontal view from the stroller, while waiting for his meal. This perspective is mainly turned to the table and the armchairs, which are far from the place he was sitting at the time of the record in question. An adult is closer to him, with whom he establishes contact. Even though there are two possible perspectives (of the adult and of the highchairs), there is another view if they are observed from the floor. This other experience then allows the baby to expand his notion of space through the projection of his perception.

Figure 11. Perspective from the feeding chair

Three-month-old babies are sensitive to up and down perspectives (Quinn, 1994) and also to left and right perception (for example, Quinn, 2004). The records in Figures 12 and 13 show perspectives from which babies have new experiences.

In **figure 12**, the baby climbs the chair to try to see from top to bottom an object that fell on the furniture in the room. If she were standing in front of the furniture, she would observe the same objects that she is observing from top to bottom. However, she does not observe the same phases of the same object.

Figure 12. Top-down perspective on high chair

Figure 13. Perspective opposite to the space of the feeding chairs

The baby in **figure 13** picks up a ball that is on the floor, within the space on which she crawls, but the baby decides to look up, it can be at an adult or at the ceiling from this point where she is, but this experience helps her form a space in which she can change position and observe different objects or people.

In short, babies learn about perspective at the same time that they experience different spatial positions: on the floor, in the stroller, in the armchair, on the lap, in the arms, crawling, standing, or trying to walk. Initially, the perception of space develops in the horizontal front perspective of one or more objects (or people). Posteriorly, they try to enlarge their visual field, when experiencing new displacements, and other experiences in general. The notions of laterality in the baby involve other relationships of the baby with two reference objects, and laterality is external to the recognition of the body itself. As it is more complex, it develops after the notion of up and down, although they are sensitive since three months old.

The spatial development of babies in the first months reverberates in the understanding of spatial transformations such as rotation, translation, amplifying and folding, and also to move easily somewhere and go back when in another place. According to the National Research Council of the National Academies (2014), this is achieved when the child is faced with different experiences and symbolic records. In addition, as described by the national curriculum guidelines (MEC, 2013), the school curriculum has to "ensure the understanding of the curriculum as school experiences that unfold around knowledge, permeated by social relationships, articulating students' experiences and knowledge with the knowledge historically accumulated" (MEC, 2013, p. 66). When referring to ways of organizing the school curriculum, MEC properly identifies the incorporation of experiences into school experiences. That is, what is experienced by the student before entering school, and in parallel to the time at school, contributes to the development of knowledge, understood as the "set of cultural experiences, common sense, behaviors, values, attitudes, in other words, all the knowledge acquired by the student in their relations with the family and with the society in movement" (MEC, 2013. p. 25).

Conclusions

The mapping of babies' actions and relationships, as well as experimentation with space lived in different contexts, is an important result in this research. The set of representations perceived by the baby constitute the images of the memories that remain in the mind and that are evoked later. Whenever the baby is faced with a new experience, he/she (re) organizes his/her perception of the world in relation to others (adults or children) or with objects with different characteristics from those he/she already knows in his/her daily life (Acevedo-Rincón, 2020). Therefore, the perceptual space becomes broader as experiences are offered in other perspectives that can be explored by the baby.

Learning of geometry (and mathematics) of primary and middle school are linked to the quality of development that is experienced since the first months according to the results of the research. However, authors have stressed the important contributions of approaches in the concepts of geometry through childhood experiences, as highlighted in research produced in Brazil (Nacarato & Passos, 2003; Biani, 2013; Leme da Silva & Valente, 2013; Rabaiolli, 2014; Acevedo -Rincón, 2020). Thus, as the baby incorporates a new movement, it will be a new learning that will help his/her neurodevelopment, and which reverberates in his/her childhood. This learning cannot be underestimated due to age, but it needs to be stimulated to motivate their creativity. This exploration of the world has to continue in early childhood education, and in the early years of education, whenever contact with other experiences allows them to learn and project their learning in an interdisciplinary manner. As suggested by Valente (2013), geometry goes through different historical stages and the current geometry is the product of systematic changes that intend to interweave non-school knowledge with that produced at school.

During early childhood education, babies experience the world, and produce knowledge that is the product of the spatial language acquired since the first months. This knowledge does not compose a formal curriculum modeled from learning objectives or specific content defined for each age group; they develop from the organization of an educational context that allows babies multiple explorations and free

playful experiences.

As evidenced by research by Barbosa (2009), Coutinho (2009, 2010, 2014, 2017); Schmitt (2008), and Fochi (2015), babies produce knowledge, even in the most unusual moments, and independent of a directed pedagogical action. They learn through interactions, games and everyday experiences, and the space, in this sense, assumes the role of a third educator capable of helping project learning beyond geometry, and of mathematics in general.

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