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Simplified physically-based modelling of overtopping induced levee breaching with TELEMAC-2D

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Abstract— **Flood events related to dike breach and failure are a major concern throughout the world due to the severe human and economic damages they cause. An important research effort is made to improve flood risk assessment through the detailed prediction of breach temporal and spatial evolutions, which remains essential to accurate prediction of the breach outflow discharge. In this study, a simplified modelling approach is demonstrated to reproduce a field scale experiment of fluvial levee breach induced by overtopping. The hydrodynamics module TELEMAC-2D is used with several empirical laws, now implemented within its breach module, to prescribe the gradual expansion (widening and deepening) of the breach. The numerical results are compared to measurements in terms of breach discharge and breach opening evolution and assess the performances of the implemented parametric models.**

I. INTRODUCTION

Levee breach and failure can lead to destructive floods with severe economic, social and environmental damages. Levees are commonly used as flood-defence structures but were mostly built with erodible material and designed for specific ranges of river discharges and water levels. Their breaching can occur due to several mechanisms, such as internal erosion with seepage flows and external erosion due to overflow. The latter being the most common failure mechanism [1, 2].

Populations living in flood-prone areas are continuously increasing, which makes the risk of important fatalities and property destruction in case of flooding even greater. Therefore, an important research effort is made in terms of physical and numerical modelling of levee breaching to improve available flood resilience and risk assessment techniques [3]. In the framework of numerical modelling of levee breach due to overflows, three different approaches for levee failure simulation emerge [1, 4]: (i) parametric models, which consist of simple regression equations for breach peak discharge and breach width and duration, resulting from the statistical analysis of reported historical failure events and mostly related to dams, (ii) simplified physically-based models, where flow variables are computed using one or two dimensional hydrodynamic models and breach expansion defined with parametric equations, and (iii) detailed physically-based models, which simulate both hydrodynamic and sediment transport processes.

The aim of this study is to simulate a field scale experiment of overtopping induced levee failure with a simplified physically based approach, which includes newly implemented empirical equations for gradual breach expansion in TELEMAC-2D. The capabilities and limitations of this model are discussed and the performance of each empirical equation to provide accurate estimation of breach discharge and expansion are evaluated.

II. FIELD SCALE EXPERIMENT MODELLING

A. Description of the field scale experiment

Kakinuma and Shimizu [5] presented full-scale experiments of side-overflow levee breaching performed in Chiyoda test channel, the largest river experimental facility in Japan built on Tokachi River, Hokkaido. Levee failure tests triggered by overtopping were conducted for various channel inflow discharges, dike soil composition and geometry as well as location. Further information on these experiments can be found in Shimada et al. [6].

In this paper, "Case 4" is considered (Fig. 1). This test was carried out in an 8 m wide and 176 m long main channel with a longitudinal bed slope of 1/500. The levee was built along the right side of the main channel toward an 80 m wide floodplain. The erodible part of the levee was 100 m long, 3 m high and its crown width was equal to 6 m with 1:2 (V:H) side slopes. Its soil composition consisted of non-cohesive sand (median diameter d_{50} = 0.74 mm) and 19% of silty soil and clay. In order to trigger overtopping an initial trapezoidal-shaped notch was carved 20 m far from the upstream end of the dike. The notch was 0.5 m deep, 3 m wide at the crest and 1 m wide at the bottom. The main channel inflow discharge was gradually increased to approximately 80 m^3/s to reach the required water level for overtopping at the notch location. Measurement data included breach outflow hydrographs, water levels, levee-breaching process estimated from acceleration sensors observations.

B. Numerical model

The two-dimensional shallow water equations code TELEMAC-2D is used to compute flow characteristics and combined with its BREACH module (Fig. 2). We implemented selected empirical laws within this module to simulate the gradual expansion of the breach in time (widening and deepening). Since sediment transport is not simulated, the use of such simplified models requires some user-defined parameters. First, the breach location is specified with a polygon created from a polyline along the dike crest and a bandwidth corresponding to dike bottom width. For the breaching A initiation, a criterion has to be selected among three different options implemented in the breach module : the user can directly specify (i) a start time, (ii) a threshold value for the average overflow water level above the entire breach location previously defined, or (iii) a threshold water level at a specific node. The user can then select a breach development mode among two options: (i) breach expansion is performed by lowering the breach bottom level for the complete breach zone (i.e. breach final width is reached instantaneously), or (ii) both breach widening and deepening are performed gradually. For this latter option, the user can choose a parametric model among newly implemented empirical laws in TELEMAC-2D (see Section C). Depending on that, information needed *a priori* can also include final breach dimensions, erosion rate, erosion duration and eventual empirical parameters values.

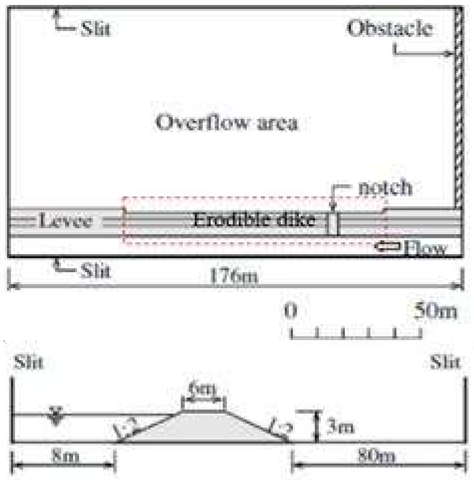

Figure 1. "Case 4" experiment - Plan view and cross-sectional profile (adapted from Kakinuma and Shimizu [5]).

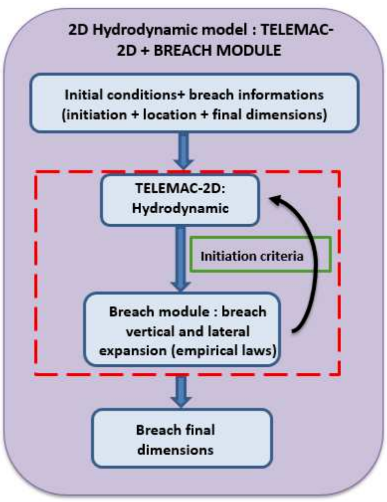

Figure 2. Simplified physically based approach for levee breach modelling in TELEMAC-2D.

C. *Computation of breach widening and deepening*

The breach lateral expansion is performed in a symmetrical way upstream and downstream of the initial notch location. The longitudinal breach cross-sectional profile is assumed rectangular, except for the Froehlich model [7] for which the profile is nearly trapezoidal (see Eqs.10-13).

Breach lateral expansion

The simplest description of breach widening is a time dependent linear equation: the breach width grows at a user-defined uniform rate. To mimic the real breach widening, another option is to split the process into two main phases (Eqs. 1 and 2), where the breach grows quickly in the first phase, and then slows down toward the end of the development time:

$$B(t) = E_{w1}t + B_0 \qquad \text{for } t \leq T_1 \qquad (1)$$

$$B(t) = E_{w1}T_1 + E_{w2}(t - T_1) + B_0 \quad \text{for } T_1 \leq t \leq T_f \quad (2)$$

where t is time in hours (after the breach initiation), B the breach width in meters, B_0 the initial breach width in meters, T_1 is the duration of phase 1 in hours, T_f total duration of the breach expansion (phase 1 and phase 2) in hours, E_{w1} and E_{w2} are breach growth rates (m/hr) for phase 1 and 2, respectively.

Growth rates could be obtained from literature or physically-based models ([2, 8, 9]). Using available datasets, Resio et al. [10] reported that the rate of breach widening is ranging between 9 m/hr for erosion-resistant soils (cohesive dikes) and 60 m/hr for erodible alluvial material (sand and gravel soils). The widening rate can rarely reach 300 m/hr for very erodible dikes.

USBR [11] recommended a single breach widening rate of 91 m/hr for embankment dams:

$$B(t) = 91t + B_0 \qquad \text{for } t \leq T_f \qquad (3)$$

Von Thun and Gillette [12] developed two equations for breach widening in dikes of low and high erodibility. For erodible dikes (*i.e.* non-cohesive dikes), the law reads as:

$$B(t) = (4h_w + 61)t + B_0 \qquad \text{for } t \leq T_f \qquad (4)$$

with h_w the water depth above the breach invert in meters at failure time and notch location. For resistant dikes (*i.e.* cohesive dikes), the law is:

$$B(t) = 4h_w t + B_0 \qquad \text{for } t \leq T_f \qquad (5)$$

Verheij [13] provided a simple relationship between the breach width B and time for sand and clay levees, based on field and laboratory data sets. For sand levees (*i.e.* non-cohesive dikes), the equation is:

$$B(t) = 37.2t^{0.51} + B_0 \qquad \text{for } t \leq T_f \qquad (6)$$

For clay levees (*i.e.* cohesive dikes), the law reads as:

$$B(t) = 13.4\sqrt{t} + B_0 \qquad \text{for } t \leq T_f \qquad (7)$$

Verheij and Van der Knaap [14] improved the previous formulations by including the effect of the difference in water levels at both sides of the dike at the breach location, and the critical flow velocity for the initiation erosion of the dike material. The empirical equation reads as:

$$B(t) = f_1 \frac{g^{0.5}\Delta H^{1.5}}{u_c} log\left(1 + f_2 \frac{g}{u_c} t\right) + B_0 \text{ for } t \leq T_f \quad (8)$$

with u_c the critical flow velocity for the initiation of erosion of dike material (m/s), f_1 and f_2 are empirical factors for breach width, g is the gravitational acceleration (m/s^2), and ΔH (m) denotes the difference in water level between the upstream and downstream sides of the breach.

In the implemented version within TELEMAC-2D, we consider the difference of water head instead of water level, *i.e.* ΔH (m) = $h_{up} - h_{down}$ with h_{up} the hydraulic head upstream of the breach (channel side) and h_{down} the hydraulic head downstream of the breach (floodplain side); this term allows a natural balance, meaning that breach width stabilises when the hydraulic head's difference is close to zero. Therefore, the user is not expected to give final breach width to run the model. Default values and ranges have been proposed for f_1 and f_2 (Table 1) [14]. Table 2 shows characteristic values of the critical velocity u_c for the surface erosion according to the dike material.

TABLE 1. DEFAULT AND RANGE OF VALUES FOR COEFFICIENTS F_1 AND F_2.

Type of Soil	Default	Range
f_1	1.3	0.5 – 5
f_2	0.04	0.01 – 1

TABLE 2. STRENGTH CHARACTERISTICS OF VARIOUS SOIL TYPES [13].

Type of Soil	u_c (m/s)	τ_c (Pa)	c_E (m^{-2}/s^2)
Grass, good	7	185	0.01×10^{-4}
Grass, moderate	5	92.5	0.02×10^{-4}
Grass, bad	4	62	0.03×10^{-4}
Clay, good (compact ; $\tau_{undrained}$ = 80-100 kPa)	1.0	4	0.50×10^{-4}
Clay with 60% sand (firm ; $\tau_{undrained}$ = 40-80 kPa)	0.80	2.5	0.60×10^{-4}
Good clay with less structure	0.70	2	0.75×10^{-4}
Good clay, heavily structured	0.60	1.5	1.5×10^{-4}
Bad clay (loose ; $\tau_{undrained}$ = 20-40 kPa)	0.40	0.65	3.5×10^{-4}
Sand with 17% silt	0.23	0.20	10×10^{-4}
Sand with 10% silt	0.20	0.15	12.5×10^{-4}
Sand with 0% silt	0.16	0.10	15×10^{-4}

τ_c: critical shear stress; c_E: strength coefficient. These characteristics are given as indicative values, as they are not used in Verheij and Van der Knaap's formula [14]).

Breach deepening

For the empirical models described above the time-evolution of the breach invert elevation is simulated according to the following linear-time progression law:

$$Z_B(\mathrm{t}) = Z_{B0} - \frac{Z_{B0} - Z_{Bmin}}{T_d} t \qquad \text{for } t \leq T_d \quad (9)$$

with Z_B the elevation of breach invert, Z_{B0} = initial elevation of breach invert, T_d is the required duration to reach Z_{Bmin} in hours. The breach minimum bottom level Z_{Bmin} (elevation of the dike foundation, main channel bottom or of a rigid layer) is reached in a shorter period than lateral expansion till ultimate breach width. By default, the duration T_d is taken 10 times smaller than the total duration of breach lateral expansion T_f.

Froehlich model (2008) (adapted)

Froehlich [7] proposed an empirical model, composed of three breach evolution variants [15] to approximate breach expansion (widening and deepening). Each of the three models assumes that a breach begins to form at the top and grows with time into a trapezoidal shape. Froehlich [7] used the concept of Brunner [16] who proposed a sine-curve time breach progression (instead of the common linear time evolution), reflecting slower growth at the start; then acceleration followed by another slow phase close to the end of breach development. The longitudinal cross-sectional profile of the breach is trapezoidal.

In TELEMAC-2D, an adapted version is implemented for two-dimensional simulations. The instantaneous top width of the breach is computed as:

$$B(t) = \beta(t)(B_f - B_0) + B_0 \quad \text{for } t \leq T_f \quad (10)$$

$$\text{with } \beta(t) = \frac{1}{2}\left\{1 + \sin\left[\pi\left(\frac{t}{T_f} - \frac{1}{2}\right)\right]\right\} \quad (11)$$

and B_f as the final top width of the breach in meters. The breach bottom elevation evolves as:

$$Z_B(t) = Z_{B0} - \beta_1(t)(Z_{B0} - Z_{Bmin}) \text{ for } t \leq T_d \quad (12)$$

$$\text{with } \beta_1(t) = \frac{1}{2}\left\{1 + \sin\left[\pi\left(\frac{t}{T_d} - \frac{1}{2}\right)\right]\right\} \quad (13)$$

D. Computational domain and parameters

The 2D computational domain was discretized into structured triangular elements with an edge of 0.5 m. Boundary conditions (Fig. 3) consisted in imposing the measured inflow discharge at the main channel inlet (Fig. 4) and a rating curve at the downstream end to achieve the required water level in the main channel and trigger levee overtopping at the notch location. A supercritical outflow with free water depth and velocity was set in the floodplain and a solid boundary was imposed elsewhere. The Strickler coefficient was set to 43 m$^{1/3}$ s^{-1}, and a constant eddy viscosity of 10^{-3} m$^{1/3}$ s^{-1} was applied for turbulence closure. The time step was set to 0.1 s. The solver was the conjugate gradient with an accuracy of 10^{6}, while the mass-conservative PSI scheme and NERD scheme were used for water depth and velocity, respectively.

The breach initiation time was 105 min and breach final width was predefined at 74.8 m as a criterion to stop breach widening (except for Verheij and Van der Knaap [14] formulation); These values were extracted from experimental data.

The simulation using the time dependent linear breach width growth law was performed with a lateral erosion rate E_w equal to 65 m/hr. The same value was adopted to describe breach evolution following two phases: during the first phase (E_{w1} = 65 m/hr) of the dual breach dynamics model during a period T_1 = 45 min, while the second phase was characterised by a slower growth rate E_{w2} = 30 m/hr.

For the Verheij and Van der Knaap [14] formulation, the critical erosion velocity u_c was taken equal to 0.23 m/s, as proposed by authors in Table 2 for sandy dikes with silt soil fractions. Default values were taken for f_1 (=1.3) and f_2 (= 0.06). Because the breach deepened faster than it widened, and because this model doesn't require user input breach final width or widening period, the duration of the breach deepening T_d was limited to 5 min, following the experimental observations. Finally, the total breach widening duration T_f was set to 50 min for Frohlich [7] model to achieve the best estimation of breach discharge.

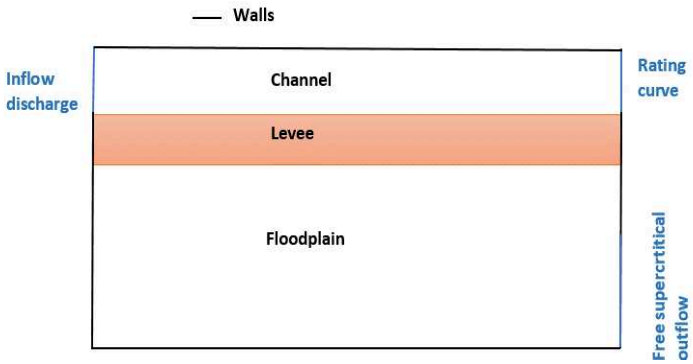

Figure 3. Boundary conditions imposed in TELEMAC-*2D.*

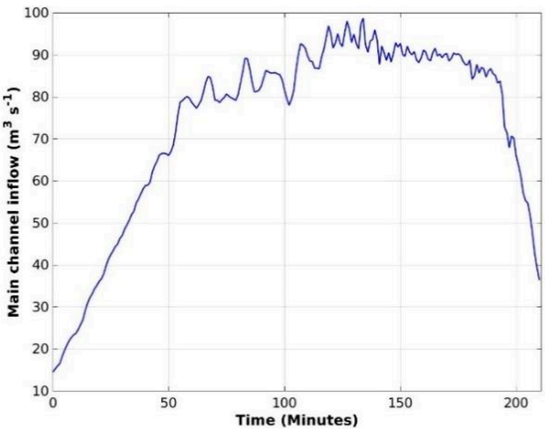

Figure 4. Measured main channel inflow discharge.

III. Results and discussion

In this section, simulated breach discharge and width time series are evaluated against experimental measurements and the performance of each parametric model is discussed. On Figures 5, one can see that the trend of fast increase in breach discharge followed by a quasi-stabilization step around a maximum value close to 70 m^3/s before dropping at test end (due to the limitation of channel inflow discharge at test end as shown in Figure 4) is well reproduced by the simplified modelling approaches. The results display a higher simulated amplitude of breach discharge with USBR formula as it estimates a higher breach width (Fig. 6), which induces a greater breach section to convey the flow.

Except for Froehlich's model, the breach discharge was slightly overestimated during the early stage of breach opening as the models predicted a faster breach lateral expansion. During quasi-stabilisation stage, computed breach dynamic with Verheij's (2002) model was too slow and the maximum breach discharge was missed, while USBR (1988) model overestimated breach widening rate and resulted in a higher breach outflow. The other laws successfully captured the breach discharge, although the breach width was underestimated by some laws such as Verheij and Van der Knaap's model (Run 5 in Table 4). This could be related to the preferential orientation of the flow through the breach section leaning toward the downstream end of the dike due to the lateral incident flow in the main channel, a dead water area usually forms near the upstream levee end [4, 5].

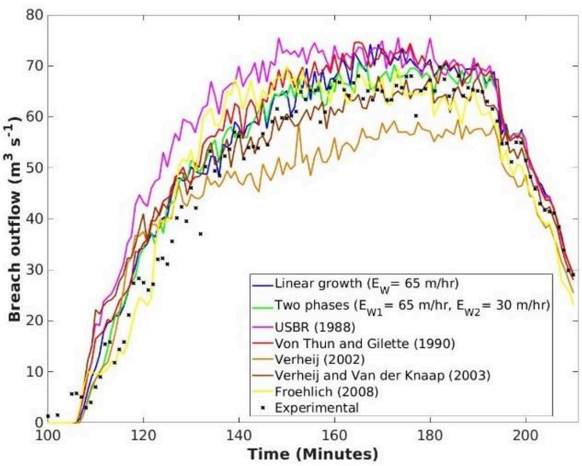

Figure 5. Computed and measured breach discharges.

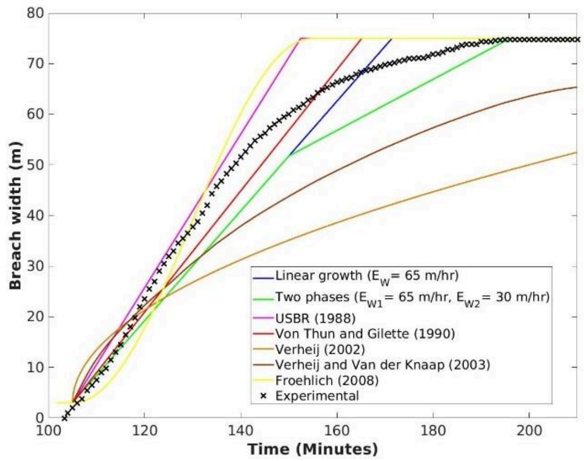

Figure 6. Computed and measured time-evolution of breach width.

For the user defined linear expansion models (both simple and two stages), a short sensitivity analysis was performed to assess the effect of the breach growth rate. Resulting breach outflow and widening are presented in Figures 7 and 8 and Normalized Root-Mean-Square Error (NRMSE) values are compared in Table 3. For the simple linear model three different widening rates were tested. In terms of NRMSE values test with E_w = 55 m/hr achieved the best agreement, but test with E_w = 65 m/hr displayed a more conservative estimate of breach outflow and was therefore retained. For the two phases linear representation of breach growth, two tests were performed and a good compromise between breach lateral growth and discharge prediction was obtained for E_{w1} =65 m/hr and E_{w2} =30 m/hr, although breach width is best estimated for E_{w1} =76 m/hr and E_{w2} =19 m/hr.

TABLE 3. LINEAR GROWTH MODEL INPUTS AND NRMSE.

Model	E_{w1} =76 m/hr	E_{w1} =65 m/hr	E_{w1} =55 m/hr	E_{w1} =76 m/hr, E_{w2} =19 m/hr	E_{w1} =65 m/hr, E_{w2} =30 m/hr
NRMSE	11.5%	7.5%	5%	10.2%	6.8%

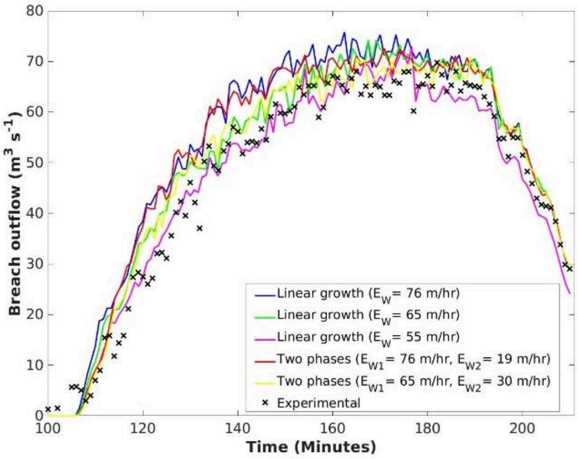

Figure 7. Computed breach discharges with the simple linear and two stages model compared to measurements.

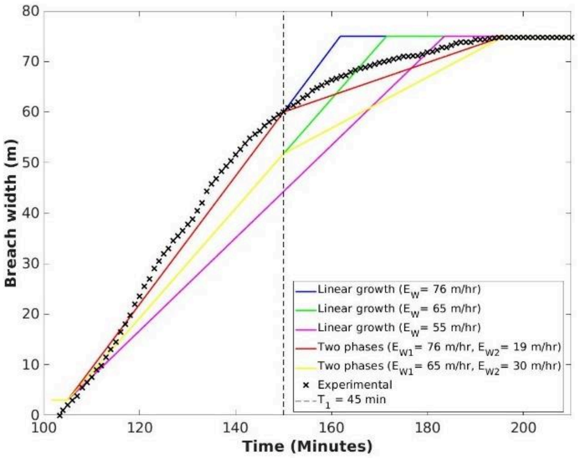

Figure 8. Computed breach widths with the simple linear and two stages model compared to measurements.

For the Froehlich model, a first run was performed with a lateral expansion period equal to that observed experimentally, as the ultimate breach width was reached 90 min after the initiation time, but this value did not allow a satisfactory concordance with measurements. As shown in Figures 9 and 10, the breach discharge fast increase was not captured using a duration of 90 min. The duration T_f was then adjusted and set to 50 min, which led to a better agreement with reported measurements. Figure 11 shows the computed longitudinal breach profile at the crest level (y = 89 m) for T_f = 50 min, evolving in a trapezoidal-like shape.

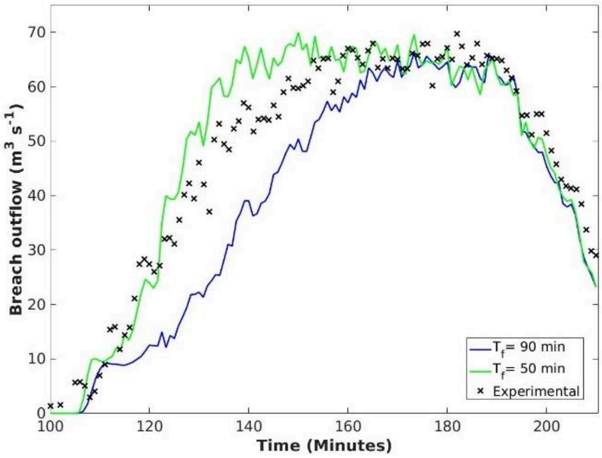

Figure 9. Computed breach discharge with Froehlich's model compared to measurements.

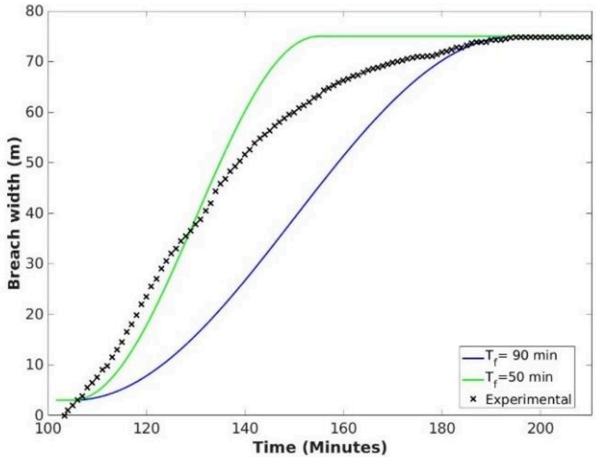

Figure 10. Computed breach width with Froehlich's model compared to measurements.

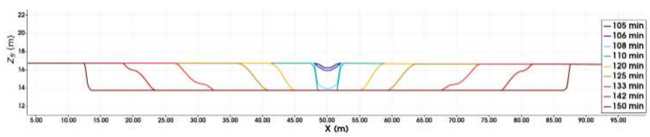

Figure 11. Longitudinal breach profile at the crest level (y=89 m) computed with Froehlich's model for T_f = 50 min.

For the Verheij and Van der Knaap formulation [14], a short analysis was carried out. The model was first tested in its original formulation considering water level difference upstream (channel side) and downstream (floodplain side) the dike. A comparison is made in Run 1 and 2 (Table 4) between two different methods to compute this difference. First, by taking the average value of the computed water level differences along the dike defined location from the channel and floodplain side (Run 1). In the second run, the maximum value is used (Run 2). The two additional runs (Run 3 and 4) used the hydraulic head instead of water level to account for the effect of flow velocity. In the same way as Runs 1 and 2, the average and maximum hydraulic head difference values were used in Run 3 and Run 4, respectively. The additional Run 5 is similar to Run 4, but the coefficient f_2 was set to 0.06 instead of 0.04 (Run 4). Figures 12 and 13 highlight an influence of both considered hydraulic variable (water level and hydraulic head) and computation method of the difference term (maximum or average value). One can see improved results in Run 2 and Run 4 when compared to Run 1 and Run 3, respectively. In the same way, breach dynamics and discharge are better captured in Run 4 and Run 3 than Run 2 and Run 1, respectively.

The formulation with maximum value of hydraulic head difference was the version conserved in the Breach module of TELEMAC-2D, as it is demonstrated here to achieve the best performance in terms of breach evolution and discharge prediction. Run 5 highlights the effect of breach width empirical parameters, which default values, can be adapted and bring further improvements into Verheij and Van der Knaap formula capabilities.

TABLE 4. PERFORMED RUNS WITH VERHEIJ AND VAN DER KNAAP [14] FORMULA.

Run 1	Average $(h_{up} - h_{down})$ h denotes water level	
Run 2	Max $(h_{up} - h_{down})$	$f_1 = 1.3, f_2 = 0.04,$ $u_c = 0.23$ m/s
Run 3	Average $(H_{up} - H_{down})$ H denotes head	
Run 4	Max $(H_{up} - H_{down})$	
Run 5	Max $(H_{up} - H_{down})$	$f_1 = 1.3, f_2 = 0.06,$ $u_c = 0.23$ m/s

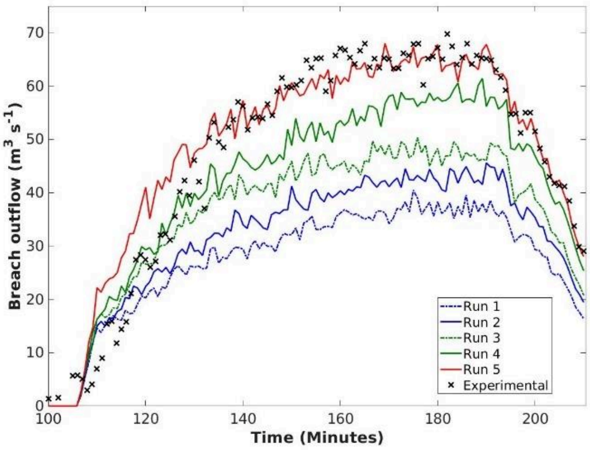

Figure 12. Verheij and Van der Knaap formulation - Computed and measured breach discharges.

Table 5 summarises the different parametric models required for each law as well as values of NRMSE calculated for the breach discharge. When the performances of these models are compared with each other, the best result is obtained from the two stages linear breach development model as it gives the lowest NRMSE values, while the highest values were calculated for the USBR (1988) and Verheij (2002) laws. When selecting a breach model, one should not only consider model performances on a particular case but also include the reliability and availability of data and information about the case of interest and their compatibility with available models. The Verheij and Van der Knaap (2003) formula led to satisfactory results and seems to be a good choice when poor information is available about breach dimensions and lateral expansion duration process as there is no need to pre-define the latter parameters.

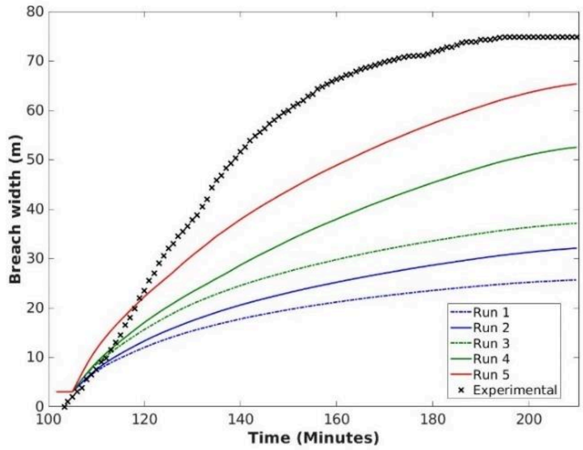

Figure 13. Verheij and Van der Knaap formulation - Computed and measured breach widths.

TABLE 5. EMPIRICAL LAW INPUTS AND BREACH DISCHARGE NRMSE

Empirical model	B_f or T_f	T_d	Other parameters	NRMSE Breach discharge
Linear		✖		7.5%
Two stages	✖	✖	T_1, E_{w1}, E_{w2}	6.7%
USBR [11]	✖	✖		15.6%
Von Thun and Gillette [12]	✖	✖		8.2%
Verheij [13]	✖	✖		12.4%
Verheij and Van der Knaap [14]		✖	f_1, f_2, u_c	8.3%
Froehlich [7]	✖	✖		8.6%

IV. CONCLUSION

In the present paper a set of parametric laws for breach expansion were presented and implemented in the breach module of TELEMAC-2D. Their performances were investigated through a field case experiment of a non-cohesive dike breaching highlighting the ability of the 2D hydrodynamic model and parametric breach models to predict breach discharge, which is a critical parameter for flood risk assessment and management. Although the simplified laws assume symmetrical breach widening and idealized breach longitudinal sectional profile (rectangular or trapezoidal shape), they have been shown to provide accurate results provided they are calibrated.

The use of simple geometric breach simulation methods is time efficient. On the other hand, they require user input choices and information about breach initiation, shape, and dimensions to be defined *a priori*. It is thus important to choose a model according to data availability and reliability about input parameters (reported erosion rates for similar cases, soil composition, breach duration and durations). It was also demonstrated that the modified Verheij and Von der Knaap formula, in which we used the head differences instead of the water level difference, performed better than the original formulation on the simulated case and represents an interesting possibility for levee breach modelling, as it only requires a few input parameters without the need to specify final breach width or widening duration. Further work will integrate additional field scale experiments and historical levee failure cases modelling to investigate the validity of the present conclusions.

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