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Essays in Automation and Globalization

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Abstract

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This dissertation studies a range of topics in automation and globalization. Chapter 1 studies the role of industrial robots in the US in expanding the occupational wage polarization. Chapter 2 explores the effect of industrial robots in Japan on employment and wages in each industry and region. Chapter 3 investigates the impact of recent growth in multinational enterprises on headquarter country's labor demand and labor share.

Chapter 1 studies the distributional and aggregate effects of the rising use of industrial robots across occupations. I construct a novel dataset that tracks the cost of robots from Japan by occupations. The dataset reveals a relative one-standard deviation drop of Japan's robot cost induces a 0.2-0.3% drop in the US occupational wages. I develop a general equilibrium model where robots are internationally traded durable goods that may substitute for labor differently across occupations. The elasticities of substitution between robots and labor within an occupation drive the occupation-specific real-wage effects of robotization. I estimate the model using the robot cost shock from my dataset and the optimal instrumental variable implied by the model. I find that the elasticities of substitution between robots and labor are heterogeneous across occupations, and higher than those between general capital goods and labor in production occupations such as welding. The estimated model implies that the industrial robots explain a 0.9 percentage point increase in the 90-50th percentile ratio of US occupational wages, and a 0.2 percentage point increase of the US real income from 1990 to 2007.

Chapter 2 explores the impacts of industrial robots on employment in Japan, the country with the longest tradition of robot adoption. We employ a novel data set of robot shipments by destination industry and robot application (specified task) in quantity and unit values. These features allow us to use an identification strategy leveraging the heterogeneous application of robots across industries and heterogeneous price changes across applications. For example, the price drop of welding robots relative to assembling robots induced faster adoption of robots in the automobile industry, which intensively uses welding processes, than in the electric machine industry, which intensively uses assembling process. Our industrial-level

and commuting zone-level analyses both indicate that the decline of robot prices increased the number of robots as well as employment, suggesting that robots and labor are grossly complementary in the production process. We compare our estimates with the ones reported by existing studies and propose a mechanism that explains apparent differences between the results.

Chapter 3 investigates the impact of multinational enterprises (MNEs) on the source-country labor share. Our model shows that source-country factor demand elasticities with respect to foreign factor prices affect aggregate labor share. To identify these elasticities, we develop an estimator that leverages a foreign factor-productivity shock. We apply this estimator to a unique natural experiment: the 2011 Thailand Floods, which negatively impacted the foreign operation of Japanese MNEs. We employ a uniquely combined Japanese firm- and plant-level microdata and find that the Floods decreased fixed assets in Japan more than employment, suggesting that foreign factor productivity growth reduces Japan's labor share.

Essays in Automation and Globalization

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To my family Mai, Joe, Leopold, and Louis
For their unconditional support and everlasting love

Chapter 1

Robots and Wage Polarization: The Effects of Robot Capital by Occupations

1.1 Introduction

In the last three decades, the global market size of industrial robots has grown by 12% annually.[1] International trade of robots is also sizable, with 41% of all robots imported. Workers in different occupations are differentially susceptible to robots, raising concerns about the distributional effects of such trends. Motivated by this concern, policymakers have proposed various restrictions on automation, such as a robot tax.[2] An emerging literature has estimated the relative effects of robot penetration on employment and the potential impact of such taxes (e.g., Acemoglu and Restrepo, 2020, Humlum, 2019). However, due to the limited data measuring the cost of robots across occupations and the lack of a model capturing the trade of robots and their dynamic accumulation, our understanding of the distributional and aggregate impacts of industrial robots is still limited.

[1]Throughout the paper, industrial robots (or robots) are defined as multiple-axes manipulators and are measured by the number of such manipulators, or robot arms, a standard in the literature. A more formal definition given by ISO and example images of robots in such a definition are provided in Section A.1.2 of the Appendix. Such a definition implies that any automation equipment that does not have multiple axes is out of the scope of the paper, even though some of them are often called "robots" (e.g., Roomba, an autonomous home vacuum cleaner made by iRobot Corporation).

[2]The European Parliament proposed a robot tax on robot owners in 2015, although it eventually rejected the proposal [Delvaux et al., 2016]. South Korea revised the corporate tax laws that downsize the "Tax Credit for Investment in Facilities for Productivity Enhancement" for enterprises investing in automation equipment [MOEF, 2018].

In this paper, I study how industrial robots affect wage inequality between occupations and aggregate income. First, I assemble a new dataset of the cost of robots by 4-digit occupations that allows me to find stylized facts about the robot cost reduction and its impact on the US occupational labor market. Second, to better interpret these empirical facts, I develop a model where robots are internationally traded durable goods, are endogenously accumulated, and substitutes for labor within occupations. Third, using these data and model, I construct a model-implied optimal instrumental variable and provide the first estimate of the elasticity of substitution (EoS) between robots and workers heterogeneous across occupation groups as well as other model parameters. Finally, counterfactual exercises based on this estimated model reveal the distributional and aggregate implication of robotization in the US since 1990.

My dataset is unique in two ways. First, it tracks the robots' monetary value as well as their number. Second, the observation is disaggregated by the adopting country and the 4-digit occupation in which robots replace labor. To obtain such a dataset, I first use the information from the Japan Robot Association (JARA) about the shipment of Japanese robots to each country and by task, which comprises one-third of the world robot supply. I then combine the JARA data with the O*NET Code Connector's match score and the US Census/ACS data. Finally, I derive the robot cost shock by occupations from the average price variable after controlling for destination-country fixed effects. As a result, I obtain the dataset that links the US occupational labor market outcomes to the cost shock of robots imported from Japan (Japan robot shock).

The dataset reveals two stylized facts. First, over 1990-2007, the Japan robot shock exhibits that the average cost of a Japanese robot reduced and that the cost reduction is heterogeneous by occupations. Second, the Japan robot shock drives the drop in wages and employment by occupations in the US. A relative decrease in one standard deviation of Japanese robots' cost drives an annualized 0.2-0.3 percent decrease in occupational wages. This finding is robust to the control of non-robot occupational demand shocks, like the China trade shock, and thus suggests high responsiveness of relative robot demand to the cost reduction due to the strong substitutability of robots for labor. However, the Japan robot shock measure is subject to a concern that it may reflect robot quality upgrading during the sample period. Furthermore, the reduced-form empirical finding does not fully reveal the distributional and aggregate effects of the Japan robot shock. To overcome these issues and derive more conclusive statement, I employ a dynamic open-economy equilibrium model of automation.

I develop a general equilibrium (GE) model with robotics automation and the following three key features. First, I incorporate the Armington-style trade of robots, which fits well with the sizable robot trade in my dataset about Japanese robot export. Theoretically, trade of robots in a large-open economy implies that a robot tax affects the price of robots traded in the global market. Hence, a country may gain from the aggregate perspective if it can reduce the cost of adopting robots by imposing the robot tax. Second, the model describes the endogenous investment in robots with a convex adjustment cost, which implies sluggish accumulation of robot capital. Therefore, the aggregate income implication of the robot tax is nuanced and different over the time horizon. Finally, the model has a production function with occupation-specific EoS between robots and labor, which varies across occupations. This production function yields rich predictions regarding the real-wage effect of robot capital for the following two reasons. Firstly, the accumulated stock of robots is different across occupations. Secondly, a unit of robots can substitute for workers differentially in each occupation.

To better understand the role of the occupation-specific EoS between robots and workers, I consider an automation shock à la Acemoglu and Restrepo [2020] in which robots can exogenously perform a larger share of tasks compared to labor. I analytically show that, in the equilibrium, the automation shock's effect on occupational real wage is negatively related to the robot-labor EoS. Namely, the higher the EoS, the larger drop of labor demand given the automation shock because of the stronger substitution of labor with robots. The model also features the EoS between occupations, which affects the across-occupation effects of the automation shocks.

To identify the robot-labor EoS, I confront a challenge that the Japan robot shock is correlated with the automation shock, affecting the labor market outcomes simultaneously. To overcome this challenge, I use the GE structure and obtain the structural residual of labor market outcomes, which controls the effect of the automation shock. I then construct a moment condition in which this structural residual is orthogonal to the Japan robot shock. Using this moment condition, I generate an optimal instrumental variable implied by the model, which increases the estimation precision.

I apply this estimation method to the data on occupational labor market outcomes and robot adoption and find that the EoS between robots and workers is heterogeneous across occupation groups. For routine occupations that perform production and material moving, the estimates are as high as around 4. These estimates are significantly higher than the values of the EoS between labor and general capital like structure and equipment estimated in the literature, highlighting one of the main differences between robots and general capital goods. In contrast, the EoS in other occupations is close to 1, or robots and labor are neither substitutes nor complements in these other occupations.

The estimated model and shocks backed out from the model predict occupational US wage changes from 1990-2007. The high EoS between robots and workers in production and material moving occupations implies that the robotization in this period significantly decreased relative wage in these occupations. Since these occupations tend to be in the middle of the occupational wage distribution in 1990, this finding indicates that the automation shock compressed the wage growth of occupations in the middle deciles. Quantitatively, it explains 0.9 percentage point, or 11.7%, of the wage polarization measured by the change in the 90th-50th percentile wage ratio, a measure of wage inequality popularized by Goos and Manning [2007] and Autor et al. [2008]. The robotization also explains a 0.2 percentage point increase of the US real income, mostly accounted for by the rise in the firm profit due to the accumulation of robots.

Finally, I examine the counterfactual effect of introducing a tax on robot purchases. Such a robot tax could potentially increase the aggregate income of a country. Due to the trade of robots, a government can exert monopsony power in the global robot market by taxing robot purchases, leading to a decrease in the before-tax price of imported robots in each period. In contrast, the robot tax also disincentivizes the accumulation of robots in the long run, potentially reducing aggregate income. Quantitatively, the latter effect dominates the former in the long-run, and the robot tax decreases aggregate real income.

This paper contributes to the literature that studies the economic impacts of industrial robots by finding a sizable impact of robots on US wage inequality and a short-run positive aggregate effect of a robot tax. The closest papers are Acemoglu and Restrepo [2020] and Humlum [2019]. Acemoglu and Restrepo [2020] establishes that the US commuting zones experiencing penetration of robots over 1992-2007 also saw decreased wages and total employment.[3] Humlum [2019] uses firm-level data on robot adoption and firm-worker-

[3]Dauth et al. [2017] and Graetz and Michaels [2018] also use the industry-level aggregate data of robot adoption and its impact on labor markets.

level panel data and estimates a model that incorporates a small-open economy of robot importers, a binary decision of robot adoption, and an EoS between occupations. Using these data and model, he studies the distributional effect of robots and a counterfactual robot tax.[4]

In contrast to these papers, my study features the following three elements. First, I use the data about the Japan robot shock by occupation, which empirically reveals impacts on US occupations. Second, I consider the trade of robots in a large-open economy setting, which implies that the US real income effect of robots is positive in the short-run in my counterfactual exercise. Finally, these data and model allow estimating occupation-specific EoS between robots and labor. The estimated model implies that the wage-polarizing effect of the increase in robot use is larger than the prediction of the model with a conventional assumption on the robot-labor EoS, such as Leontief.

Occupations are receiving attention in the literature of automation as they matter when considering the distributional effects. While Jäger et al. [2016] finds no association between industrial robot adoptions and total employment at the firm level, Dinlersoz et al. [2018] report the cost share of workers in the production occupation dropped after the adoption of robots within a firm. Cheng [2018] studies the heterogeneous capital price decrease and its implication on job polarization. Jaimovich et al. [2020] construct a model to study the effect of automation on the labor market of routine and non-routine workers in the steady state. I contribute to this literature by estimating the within-occupation EoS between robots and labor with the occupation-level data of robot costs and labor market outcomes, as well as incorporating the endogenous trade of robots and characterizing the transition dynamics of the effect of robot tax.

Following the seminal work by Autor et al. [2003], there is a growing literature that attempts to detect the task contents of recent technological development. Webb [2019] provides a natural-language-processing method to match technological advances (e.g., robots, software, and artificial intelligence) embodied in the patent title and abstract to occupations. Montobbio et al. [2020] extends this approach to analyzing full patent texts by applying the topic modeling method of machine learning. My matching method between robot application and occupation complements these studies: On one hand, my methodology gives a

[4] There is also a growing body of studies that use the firm- and plant-level microdata to study the impact on workers in Canada [Dixon et al., 2019], France [Acemoglu et al., 2020, Bonfiglioli et al., 2020], Netherlands [Bessen et al., 2019], Spain [Koch et al., 2019], and the US [Dinlersoz et al., 2018].

list of matching scores. Combined with the robot data by application, my dataset yields the number and sales of robots for all 4-digit occupations. On the other hand, I do not provide such detailed textual analysis as the previous literature since I can only observe the title of robot applications.

Since robots are one type of capital goods, my paper is also related to the vast literature of estimating the EoS between capital and labor (to name a few, Arrow et al., 1961, Chirinko, 2008, Oberfield and Raval, 2014). Although the literature yields a set of estimates with a wide range, the upper limit appears around 1.5 [Hubmer, 2018, Karabarbounis and Neiman, 2014]. Therefore, the estimates as high as 4 in production and material-moving occupations are significantly higher than this upper limit. In this sense, my estimates highlight one of the main differences between robots and other capital goods: these workers' vulnerability to robots.

The rest of the paper is organized as follows. Section 1.2 describes my dataset of robots by occupations. I set out the general equilibrium model in 1.3, and estimate it using the model-implied instrumental variable in model 1.4. Using the estimated model, I study the effect of robotization and counterfactual robot taxes in Section 1.5. Section 1.6 concludes.

1.2 Data and Stylized Facts

This section begins with setting out two central data sources in Section 1.2.1: the Japan Robot Association survey and O*NET for matching the robot application code to the labor occupation code at the 4-digit level. Note that Japan has been a major robot innovator, producer, and exporter. For example, the US imports 5 billion-dollar worth of Japanese robots as of 2017, which comprises roughly one-third of robots in the US.[5] Therefore, Japanese robot cost reduction significantly affects robot adoption in the US and the world.

Using these data, I describe how to measure the robot cost, provide the matching method to obtain robot measures at the occupation level, and derive the Japan robot shock formally in Section 1.2.2. Section 1.2.3 provides stylized facts that suggest substitutability between robots and labor and motivate the model and estimation in later sections.

[5]Appendix A.1.3 shows the international robot flows, including Japan, the US, and the rest of the world.

1.2.1 Data Sources

The main part of my dataset is provided by Japan Robot Association (JARA), a general incorporated association composed of Japanese robot producing companies. The number of member companies is 381 as of August 2020. JARA annually surveys all these member and several non-member companies about the units and monetary values of robots sold for each destination country and robot application, or specified tasks of robots, which is discussed in detail in Section A.1.2 of the Appendix. JARA publishes summary cross-tables of the survey, which I digitize and use as one of the main data sources.

I also use Occupational Information Network OnLine (O*NET) Code Connector. O*NET is an online database of occupational definitions sponsored by the US Department of Labor, Employment, and Training Administration. O*NET Code Connector provides an occupational search service that helps workforce professionals determine relevant 4-digit level O*NET-SOC Occupation Codes for job orders. Along with the O*NET-SOC codes, the search algorithm provides (i) the textual description of each code and (ii) a match score that shows the relevance of the search target with the search query term. To match robot applications and labor occupations, I use these textual descriptions and match scores, which are further described in detail in Section A.1 of the Appendix.

1.2.2 Constructing the Dataset

Using these data, I construct a dataset that matches the cost of Japanese robots to the US labor market outcomes at the occupation level. After clarifying robot cost measurement, I describe the matching process between robot applications and labor occupations.

Measuring the Cost of Robots

To understand the measurement of the robot cost, I clarify how robots work. A modern industrial robot is typically not stand-alone hardware (e.g., robot joints and arms) but an ecosystem that includes the hardware and control units operated by software (e.g., computers and robot-programming language). Due to its complexity, installing robots in the production environment often requires hiring costly system integrators that offer specific engineering knowledge. A relevant cost of robots for adopters, therefore, includes hardware, software, and integration costs.[6]

In this paper, I measure the price of robots by average price, or the total sales divided by the quantity of hardware. In this sense, readers should interpret that my measure of robot price reflects a portion of overall robot costs. Since the literature has not established a method to deal with this issue, I will address this point in the model section by separately defining the observable hardware cost and unobserved components of the cost, and placing assumptions on the latter.

Another issue of this approach is that this price measure includes robot quality upgrading. Namely, innovation in robotics technology could entail both quality upgrading that makes robots perform more tasks at a greater efficiency and cost saving of producing robots that perform the same task as before. Inseparability of these two components poses an identification threat as I describe later in Section 1.4.2, which none of the previous studies could resolve. To work around this issue, I will use the general equilibrium model to predict the labor market effects of quality upgrading in Section 1.3.[7]

Matching Robot Applications and Labor Occupations

My dataset provides the employment of labor and robots at the occupation level, complementing data in the previous literature at the sector level or, more recently, firm level. This is made possible by having robot application-level data, and converting robot applications to labor occupations. I propose a method to match the JARA data and the Census 4-digit occupation level labor market outcomes.

There has not been formal concordance between application and occupation codes, although robot applications and labor occupations are close concepts. On the one hand, robot application is a task where the robot is applied. On the other hand, labor occupation

[6] As Leigh and Kraft [2018] pointed out, the current industry and occupation classifications do not allow separating system integrators, making it hard to estimate the cost from these classifications. Plus, there still remains apparently relevant costs of robot use, like maintenance fee, about which we also lack quantitative evidence. Although understanding these components of the costs is of first-order importance, this paper follows the literature convention and measure robots from market transaction of hardware.

[7] Note that this problem occurs because I consider the price of robots, as the past literature mainly focuses only on the quantity of robots [Acemoglu and Restrepo, 2020] or even firm's binary decision of robot adoption [Humlum, 2019]. One of the more data-driven approaches to this issue is to control the quality change by the hedonic approach as in Timmer et al. [2007], and in the application to digital capital in Tambe et al. [2019]. However, this strategy requires detailed information about the spec of each robot. Pursuing this direction is the next step of my research agenda, as I collaborate with JARA for retrieving catalog information of robots produced by major producers.

describes multiple types of tasks the person does on the job. Each task has different requirements for robotics automation. Therefore, a heterogeneous mix of tasks in each occupation generates a difference in the ease of automation across occupations and, thus, heterogeneous penetration of robots [Manyika et al., 2017]. I show examples of pairs of robot applications and labor occupations in Section A.1.2 in the Appendix.

More specifically, let a denote robot application and o labor occupation. JARA data measure robot sales quantity and total monetary transaction values for each application a. I write these as robot measures X_a^R, a generic notation that means both quantity and monetary values. The goal is to convert an application-level robot measure X_a^R to an O*NET-SOC occupation-level one X_o^R. First, I search occupations in O*NET Code Connector by the title of robot application a. Second, I web-scrape the match score m_{oa} between a and o.[8] Finally, I allocate X_a^R to each occupation o according to m_{oa}-weight by

$$X_o^R = \sum_a \omega_{oa} X_a^R \text{ where } \omega_{oa} \equiv \frac{m_{oa}}{\sum_{o'} m_{o'a}}.$$

As a result, X_o^R measures the occupation-level robot measures such as quantity and monetary values. Note $\sum_o \omega_{oa} X_a^R = X_a^R$ since $\sum_o \omega_{oa} = 1$. In other words, occupation-level robot measures sum back to the application level when summed across occupations, as a desired property of the allocation.

I then convert the O*NET-SOC-level occupation codes to OCC2010 occupation codes to match the labor market measures from the US Census, American Community Survey (ACS), retrieved from the Integrated Public Use Microdata Series (IPUMS) USA [Ruggles et al., 2018], described in detail in Appendix A.1.1.

[8] I focus on consistent occupations between the 1970 Census and the 2007 ACS that cover the sample period and pre-trend analysis period to obtain consistent data across periods. Therefore, this paper focuses on the intensive-margin substitution in occupations as opposed to the extensive-margin effect of automation that creates new labor-intensive tasks and occupations [Acemoglu and Restrepo, 2018c]. My dataset shows that 88.7 percent of workers in 2007 worked in the occupations that existed in 1990. It is an open question how to attribute the creation of new occupations to different types of automation goods like occupational robots in my case, although Autor and Salomons [2019] explore how to measure the task contents of new occupations.

Japan Robot Shock

To obtain the robot cost variation by occupation, write $p^R_{i,o,t}$ the average price of robots in occupation o in destination country i in year t. I fit the fixed-effect regression

$$\ln\left(p^R_{i,o,t}\right) - \ln\left(p^R_{i,o,t_0}\right) = \psi^D_{i,t} + \psi^J_{o,t} + \epsilon_{i,o,t},\ i \neq USA \tag{1.1}$$

where t_0 is the initial year, $\psi^D_{i,t}$ is destination-year fixed effect, $\psi^J_{o,t}$ is occupation-year fixed effect, and $\epsilon_{i,o,t}$ is the residual. To obtain the cost changes that are not affected by the US demand, I exclude the US prices from the sample. This regression controls any country-year specific effect $\psi^D_{i,t}$, which includes country i's demand shock or trade shock between Japan and i that are independent of occupations. I use the remaining variation across occupations $\psi^J_{o,t}$ as a cost shock of robots by occupations and term it as a "Japan robot shock."

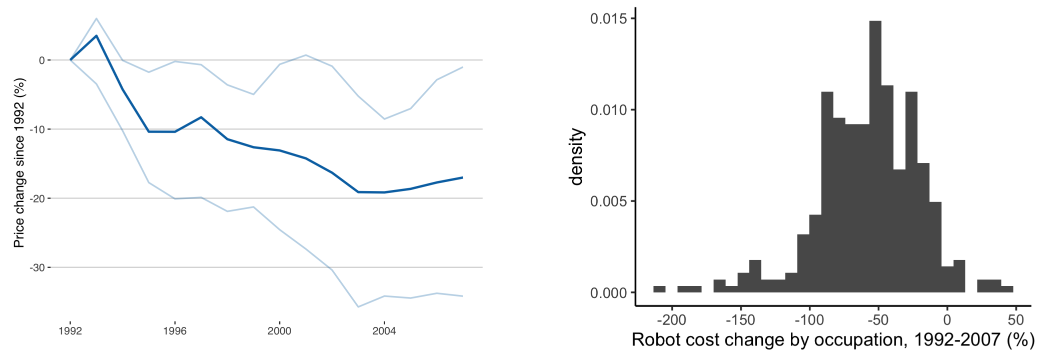
1.2.3 Stylized Facts

The resulting dataset permits me to study the robot cost variation by occupation, or Japan robot shock, and the corresponding occupation's labor market outcomes with demographic controls, which I explore in the next subsection. Throughout the paper, I define the initial year $t_0 = 1992$ (or for Census data $t_0 = 1990$), in which the JARA data starts tracking the destination-country level variable, and 1992-2007 as the sample period, with notation $t_1 = 2007$.

Fact 1: Trends of the Japan Robot Shock I show the patterns of average prices of robots across occupations that are not intensively studied in the literature. Figure 1.1a plots the distribution (10th, 50th, and 90th percentile) of the growth rates of the price of robots in the US relative to the initial year. The figure shows two patterns: (i) the robot prices show an overall decreasing trend, with the median growth rate of -17% from 1992 to 2007, or -1.1% annually, and (ii) a significant heterogeneity in the rate of price falls across occupations, with the 10th percentile occupation experienced -34% growth (-2.8% per annum), while the 90th percentile occupation almost did not change the price in the sample period. The price drop is consistent with the decreasing trend of prices of general investment goods since 1980, as Karabarbounis and Neiman [2014] report a 10% decrease per decade from their data sources. The large variation of the changes in prices by occupations persists even after

Figure 1.1: Distribution of the Cost of Robots

(a) US Robot Price by Occupations (b) Histogram of the Japan Robot Shock



Note: The author's calculation based on JARA and O*NET. The left panel shows the trend of prices of robots in the US by occupations, $p^R_{USA,o,t}$. The thick and dark line shows the median price in each year, and two thin and light lines are the 10th and 90th percentile. Three-year moving averages are taken to smooth out yearly noises. The right panel shows the histogram of long-run (1992-2007) cost shock of robots measured by the fixed effect ψ^C_{o,t_1} in equation (1.1).

controlling for the destination-year fixed effect $\psi^D_{i,t}$, as Figure 1.1b shows the distribution of the Japan robot shock in the long-run (1992-2007), or ψ^J_{i,t_1} in equation (1.1).

There are several interpretations of the price trend, including the reduction in the cost to produce robots and quality changes. First, if the cost of producing robots decreases, the measured prices naturally drop. In the model, I will capture this pattern by positive Hicks-neutral productivity shock to robot producers. Second, if the quality of the robots increased over the period, the quality-adjusted prices may experience a larger decrease than what is observed in the average price measure. They are hard to separate in my data and thus interpreted through the lens of the general equilibrium model in Section 1.3 by incorporating the quality change and examining its effects on robot prices and quantities. As a result, the differences in the robot cost shock and the quality change may affect the robot adoption and the labor market impacts by occupations.

Fact 2: Effects of the Japan robot shock on US occupations Using the variation of Japan robot shock, I study the effect on US labor market outcomes. Since the labor demand may be affected by the concurrent trade liberalization, notably the China shock, I control for the occupational China shock by the method developed by Autor et al. [2013], namely,

$$IPW_{o,t} \equiv \sum_s l_{s,o,t_0} \Delta m^C_{s,t}, \tag{1.2}$$

where l_{s,o,t_0} is sector-s share of employment for occupation o and $\Delta m^C_{s,t}$ is the per-worker Chinese export growth to non-US developed countries.[9] An occupation receives a high trade shock if sectors that experienced increased import competition from China intensively employ the occupation. With this measure of the trade shock, I run the following regression

$$\Delta \ln (Y_o) = \alpha_0 + \alpha_1 \times \psi^J_{o,t_1} + \alpha_2 \times IPW_{o,t_1} + \boldsymbol{X}_o \cdot \boldsymbol{\alpha} + \varepsilon_o, \tag{1.3}$$

where Y_o is a labor market outcome by occupations such as hourly wage and employment, $\boldsymbol{X}_o$ is the vector of baseline demographic control variables are the female share, the college-graduate share, the age distribution, and the foreign-born share, and Δ is the long-run time difference between 1990 and 2007.

Table 1.1 shows the result of regression (1.3). Columns 1-3 take hourly wages as the outcome, while columns 4-6 do employment. In columns 3 and 6, the main specifications that includes both the Japan robot shock and the China shock, I find that the negative Japan robot shock (reduction in the cost of Japanese robots) drives the drop of the labor market outcomes by occupation. Quantitatively, one standard-deviation decrease of the robot cost (annually, 2.8%) implies the fall of occupational wage by 0.2-0.3% in 95% confidence interval. This finding suggests substitutability between robots and workers because when the cost of robots falls in an occupation, the relative demand for robots (resp. labor) increases (resp. decreases) in the same occupation.

In Section A.1.6 of the Appendix, I compliment the findings in Table 1.1 by confirming the consistency with the result of Acemoglu and Restrepo [2020]. The section also shows that these analyses are robust to a number of sensitivity checks such as measuring robot stocks by quantity, quality adjustment following Khandelwal et al. [2013], and unweighted regression. Although these regressions are informative about the drivers of robot adoption, they do not give an answer to the distributional and aggregate effect of the Japan robot cost shock. To derive such more conclusive statements, I develop and estimate a general equilibrium model.

[9] Specifically, following Autor et al. [2013], I take eight countries: Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. Appendix A.1.1 shows the distribution of occupational employment l_{s,o,t_0} for each sector.

Table 1.1: Effects of the Japan robot shock on US occupations

VARIABLES	(1) $\Delta\ln(w)$	(2) $\Delta\ln(w)$	(3) $\Delta\ln(w)$	(4) $\Delta\ln(L)$	(5) $\Delta\ln(L)$	(6) $\Delta\ln(L)$
Japan Robot Cost (ψ^J)	0.0970***		0.1021***	0.0459***		0.0472***
	(0.0263)		(0.0266)	(0.0151)		(0.0142)
IPW		-0.0697**	-0.0748**		-0.0639***	-0.0663***
		(0.0348)	(0.0307)		(0.0143)	(0.0138)
Demographic controls	✓	✓	✓	✓	✓	✓
Observations	324	324	324	324	324	324
R-squared	0.379	0.320	0.394	0.103	0.073	0.178

Note: The table shows the coefficients in regression (1.3), based on the dataset constructed from JARA, O*NET, and the US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. ψ^C stands for the Japan robot shock from equation (1.1) and IPW stands for the occupation-level import penetration measure (in thousand USD) in equation (1.2). Demographic control variables are the female share, the college-graduate share, the age distribution (shares of age 16-34, 35-49, and 50-64 among workers aged 16-64), and the foreign-born share as of 1990. All time differences, Δ, are taken with a long difference between 1990 and 2007. All regressions are weighted by the employment in the initial year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). Robust standard errors are reported in the parentheses. *** p<0.01, ** p<0.05, * p<0.1.

1.3 Model

The open-economy dynamic general equilibrium model has three features: (i) occupation-specific substitution of robots for workers, (ii) robot trade in a large-open economy, and (iii) endogenous investment in robots with an adjustment cost. Section 1.3.1 states the assumptions, agents' optimization problems, and the equilibrium definition. After showing the solution method in Section 1.3.2, I discuss a key analytical result that shows the occupational wage implication of automation, which underscores the relevance of occupation-specific substitution in Section 1.3.3.

1.3.1 Setup

I formalize the model settings, assumptions, and key characterizations. I relegate discussions and comparisons to the literature in Section A.2.1 of the Appendix. Other standard characterizations of equilibrium conditions are given in Section A.2.4 of the Appendix.

Environment Time is discrete and has infinite horizon $t = 0, 1, \ldots$. There are N countries and O occupations. To clarify subscripts for countries, I use i, j, and l, where l is a robot-exporting country, i means a robot-importing and non-robot good-exporting country, and j indicates a non-robot good-importing country. There are two types of goods g, a non-robot good $g = G$ and robot $g = R$. Both goods are tradable. The non-robot good G is differentiated by origin countries and can be consumed by households, used as an intermediate good, invested to produce robots, and used as an input for integration, which I will discuss in detail. Robot R is differentiated by origin countries and occupations. There are bilateral and good-specific iceberg trade costs $\tau^g_{ij,t}$ for each $g = G, R$. I use notation Y for the total production, Q for the quantity arrived at the destination. For instance, non-robot good G shipped from i to j in period t satisfies $Y^G_{ij,t} = Q^G_{ij,t}\tau^G_{ij,t}$. There is no intra-country trade cost, thus $\tau^g_{ii,t} = 1$ for all i, g and t.

There are three factors for production of good G: labor by occupation L_o, robot capital by occupation K^R_o, and non-robot capital K. The stock of non-robot capital is exogenously given at any period for each country. There is no international movement of factors. Note that non-robot capital is not occupational. While producers rent non-robot capital from the rental market, they accumulate and own robot capital. All good and factor markets are perfectly competitive.

The government in each country exogenously sets the robot tax. Buyers of robot $Q^R_{li,o,t}$ have to pay ad-valorem robot tax $u_{li,t}$ on top of producer price $p^R_{li,o,t}$ to buy from l. The tax revenue is uniformly rebated to destination country i's workers.

Workers Workers solve a dynamic discrete choice problem to select an occupation [Humlum, 2019, Traiberman, 2019]. I follow the discrete sector choice problems in Dix-Carneiro [2014] and Caliendo et al. [2019] in that workers choose the occupations that maximize the lifetime utility based on switching costs and the draw of idiosyncratic shocks. The problem has a closed form solution when the idiosyncratic shocks follow a suitable extreme value distribution [McFadden, 1973].[10] In Section A.2.2 of the Appendix, I formally define the problem and show that the worker's problem can be characterized by, for each country i and period t, the transition probability $\mu_{i,oo',t}$ from occupation o in period t to occupation o' in

[10]One of the differences from these past studies is that I characterize the switching cost by an ad-valorem term, which makes the log-linearization simpler when solving the model.

period $t+1$, and the exponential expected value $V_{i,o,t}$ for occupation o that satisfy

$$\mu_{i,oo',t} = \frac{\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right)\left(V_{i,o'',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}, \tag{1.4}$$

$$V_{i,o,t} = \widetilde{\Gamma} C_{i,o,t}\left[\sum_{o'}\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}\right]^{\frac{1}{\phi}}, \tag{1.5}$$

respectively, where $C_{i,o,t+1}$ is the real consumption, $\chi_{i,oo',t}$ is an ad-valorem switching cost from occupation o to o', ϕ is the occupation-switch elasticity, $\widetilde{\Gamma} \equiv \Gamma\left(1-1/\phi\right)$ is a constant that depends on the Gamma function. For each i and t, employment level satisfies the law of motion

$$L_{i,o,t+1} = \sum_{o'} \mu_{i,o'o,t} L_{i,o',t}, \tag{1.6}$$

with the total employment satisfying an adding-up constraint

$$\sum_{o} L_{i,o,t} = \overline{L}_{i,t}. \tag{1.7}$$

Production Function I describe a production function in country i in period t. For each good g, there is a given mass of producers. Non-robot good-G producers produce by aggregating the tasks performed by either labor or robots within a given occupation $T^{O}_{i,o,t}$, intermediate goods $M_{i,t}$, and non-robot capital $K_{i,t}$ by

$$Y^{G}_{i,t} = A^{G}_{i,t}\left[\sum_{o}\left(b_{i,o,t}\right)^{\frac{1}{\beta}}\left(T^{O}_{i,o,t}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}\alpha_L}\left(M_{i,t}\right)^{\alpha_M}\left(K_{i,t}\right)^{1-\alpha_L-\alpha_M}, \tag{1.8}$$

where $Y^{G}_{i,t}$ is the production quantity, $A^{G}_{i,t}$ is a Hicks-neutral total factor productivity (TFP) shock, $b_{i,o,t}$ is the cost share parameter of occupation o, β is the elasticity of substitution between occupations from the production side, and $\alpha_{i,L}$, $\alpha_{i,M}$, and $1-\alpha_{i,L}-\alpha_{i,M}$ are Cobb-Douglas weights on occupations, intermediate goods, and non-robot capital, respectively. Parameters satisfy $b_{i,o,t} > 0$ for all i, o, and t, $\sum_o b_{i,o,t} = 1$, $\beta > 0$, and $\alpha_{i,L}, \alpha_{i,M}, 1-\alpha_{i,L}-\alpha_{i,M} > 0$. For simplification, I assume that robots R for occupation o are produced by investing non-robot goods $I^R_{i,o,t}$ with productivity $A^R_{i,o,t}$:[11]

$$Y^R_{i,o,t} = A^R_{i,o,t} I^R_{i,o,t}. \tag{1.9}$$

Note that the increase in the TFP term $A^R_{i,o,t}$ drives a reduction in the robot prices. To perform each occupation o, producers hire labor $L_{i,o,t}$ and robot capital $K^R_{i,o,t}$

$$T^O_{i,o,t} = \left[\left(1 - a_{o,t}\right)^{\frac{1}{\theta_O}} \left(L_{i,o,t}\right)^{\frac{\theta_O - 1}{\theta_O}} + \left(a_{o,t}\right)^{\frac{1}{\theta_O}} \left(K^R_{i,o,t}\right)^{\frac{\theta_O - 1}{\theta_O}}\right]^{\frac{\theta_O}{\theta_O - 1}}, \tag{1.10}$$

where $\theta_o > 0$ is the elasticity of substitution between robots and labor within occupation o, and $a_{o,t}$ is the cost share of robot capital in tasks performed by occupation o. In the following sections, I use the shift of $a_{o,t}$ as a source of automation. I will discuss real-world examples and the relationship to the models in the literature in Section A.2.1 The intermediate goods are aggregated by

$$M_{i,t} = \left[\sum_l \left(M_{li,t}\right)^{\frac{\varepsilon - 1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{1.11}$$

where ε the elasticity of substitution. Since intermediate goods are traded across countries and aggregated by equation (1.11), elasticity parameter ε serves as the trade elasticity. Given the iceberg trade cost $\tau^G_{ij,t}$, the bilateral price of good G that country j pays to i is $p^G_{ij,t} = p^G_{i,t}\tau^G_{ij,t}$.

Discussion–Production Function and Automation It is worth mentioning the relationship between production functions (1.8) and (1.10) and the way automation is treated in the literature. A common approach to modeling robots in the literature, called the task-based approach, constructs the production function (task-based production function) based on the producers' allocation problem of production factors (e.g., robot capital, labor) to a set of tasks (e.g., spot welding). A large body of literature develops the task-based approach to model industrial robots (e.g., Acemoglu and Restrepo, 2018c) and more general automation (e.g., Acemoglu and Autor, 2011, Autor et al., 2003). I show this task-based approach im-

[11]The assumption simplifies the solution of the model because occupations, intermediate goods, and non-robot capital are only used to produce non-robot goods. Furthermore, I can simply use the estimates measured at the unit of output dollar values when taking the budget constraint of the model to the data in log-linearized solution. To conduct the estimation and counterfactual exercises without this simplification, one would need to observe the cost shares of intermediate goods and non-robot capital for robot producers.

plies occupation production function (1.10) with a suitable distributional assumption of the efficiency of task performance for each production factor. Intuitively, one can regard tasks in the occupation o as simply the aggregate of inputs, robot capital and labor, abstracting away from allocating robots and workers to each exact task. More precisely, in Lemma 2 in Section C.2, I show that the solution to the factor allocation problem implies production functions (1.8) and (1.10).

The cost-share parameter $a_{o,t}$ of equation (1.10) has several interpretations. First, since the task-based approach consists of the allocation of factors to tasks, the cost-share parameter $a_{o,t}$ is the share of the space of tasks performed by robot capital as opposed to labor. Since automation improvements consist of expansion in the task space, I will log-linearize the equilibrium respect to $a_{o,t}$ and call the change as the *automation shock*. Second, following Khandelwal [2010], quality of goods can be regarded as a non-pecuniary "attribute whose valuation is agreed upon by all consumers." Therefore, the increase in the cost-share parameter $a_{o,t}$ can also be interpreted as quality upgrading of robots, when combined with a suitable adjustment in the TFP term I discuss in Section 1.3.3. In particular, equation (1.10) implies that in the long-run (hence dropping the time subscript) the demand for robot capital is

$$K_{i,o}^{R} = a_o \left(\frac{c_{i,o}^{R}}{P_{i,o}^{O}} \right)^{-\theta_o} T_{i,o}^{O},$$

where $c_{i,o}^{R}$ is the long-run marginal cost of robot capital formally defined in Section A.2.4 of the Appendix, and $P_{i,o}^{O}$ is the unit cost of performing occupation o. In this equation, a_o is the quality term defined above. For this reason, I use terms (positive) automation shocks and robot quality upgrading interchangeably to describe an exogenous increase in a_o.

The robot-labor substitution parameter θ_o is the key elasticity that affects the changes in real wages given the automation shocks. In Section 1.3.3, I show that θ_o is negatively related to the real wage changes conditional on the initial cost shares. Hence it is critical to know the value of the parameter to answer the welfare and policy questions. To the best of my knowledge, equation (1.10) is the most flexible formulation of substitution between robots and labor in the literature. For instance, I show that the unit cost function of Acemoglu and Restrepo [2020] can be obtained by $\theta_o \to 0$ for any o under specific assumptions about other parameters in Lemma 2 in Appendix A.2.3. I also show that my model can imply the production structure of Humlum [2019] in Lemma 3.

Producers' Problem The producers' problem comprises two tiers–static optimization about employment for each occupation and dynamic optimization about robot investment. The static optimization is to choose the employment and capital rental conditional on market prices and current stock of robot capital. Namely, for each i and t, conditional on the o-vector of stock of robot capital $\left\{K^R_{i,o,t}\right\}_o$,

$$\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) \equiv \max_{\left\{L_{i,o,t}\right\}_o, \left\{M_{li,t}\right\}_l, K_{i,t}} p^G_{i,t}Y^G_{i,t} - \sum_o w_{i,o,t}L_{i,o,t} - \sum_l p^G_{li,t}M_{li,t} - r_{i,t}K_{i,t}, \quad (1.12)$$

where $Y^G_{i,t}$ is given by production function (1.8).

The dynamic optimization is to choose the quantity of new robots to purchase, or robot investment, given the current stock of robot capital. It requires the following three assumptions. First, for each i, o, and t, robot capital $K^R_{i,o,t}$ accumulates according to

$$K^R_{i,o,t+1} = (1-\delta)\, K^R_{i,o,t} + Q^R_{i,o,t}, \quad (1.13)$$

where $Q^R_{i,o,t}$ is the amount of new robot investment and δ is the depreciation rate of robots. Second, I assume that the new investment is given by CES aggregation of robot arms from country l, $Q^R_{li,o,t}$, and the non-robot good input of integration $I^{int}_{i,o,t}$ that I discussed in Section 1.2,

$$Q^R_{i,o,t} = \left[\sum_l \left(Q^R_{li,o,t}\right)^{\frac{\varepsilon^R-1}{\varepsilon^R}}\right]^{\frac{\varepsilon^R}{\varepsilon^R-1}\alpha^R} \left(I^{int}_{i,o,t}\right)^{1-\alpha^R} \quad (1.14)$$

where l denotes the origin of the newly purchased robots, and α^R is the expenditure share of robot arms in the cost of investment. Note that equation (1.14) implies that the robots are traded because they are differentiated by origin country l. This follows the formulation of capital good trade in Anderson et al. [2019]. Furthermore, combined with equation (1.13), equation (1.14) implies that the origin-differentiated investment good is aggregated at first, and then added to the stock of capital. This specification helps reduce the number of capital stock variables and is also used in Engel and Wang [2011]. Given the iceberg trade cost $\tau^R_{ij,t}$, the bilateral price of robot R is $p^R_{ij,o,t} = p^R_{i,o,t}\tau^R_{ij,t}$. Write the unit investment price of robots as $P^R_{i,o,t}$. Third, installing robots is costly and requires a per-unit convex adjustment cost $\gamma Q^R_{i,o,t}/K^R_{i,o,t}$ measured in units of robots, where γ governs the size of adjustment cost (e.g., Cooper and Haltiwanger, 2006, Holt, 1960). This reflects the technological difficulty and sluggishness of robot adoption, as reviewed in Autor et al. [2020] and discussed in detail in Section A.2.1.

Given these settings, a producer of non-robot good G in country i solves the dynamic optimization problem

$$\begin{aligned} \max_{\left\{\left\{Q^R_{li,o,t}\right\}_l, I^{int}_{i,o,t}\right\}_o} & \sum_{t=0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \left[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right)\right. \\ & \left. - \sum_{l,o}\left(p^R_{li,o,t}\left(1+u_{li,t}\right)Q^R_{li,o,t} + P^G_{i,t} I^{int}_{i,o,t} + \gamma P^R_{i,o,t} Q^R_{i,o,t} \frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\right], \end{aligned} \tag{1.15}$$

subject to accumulation equation (1.13) and (1.14), and given $\left\{K^R_{i,o,0}\right\}_o$. Because producers are owned by households, the producer uses the household discount rate ι. Since this is a standard dynamic optimization problem, the standard method of Lagrangian multiplier yields the standard investment Euler equations, which I derive in Appendix A.2.4.

Equilibrium To close the model, the employment level must satisfy an adding-up constraint (1.7), and robot and non-robot good markets clear as described in Section A.2.4 of the Appendix. I first define a temporary equilibrium in each period and then a sequential equilibrium, which implies the steady-state definition. Some of the exact expressions are derived in Appendix A.2.4 to save a space.

Define the bold symbols as vectors of robot capital $\boldsymbol{K}^R_t \equiv \left\{K^R_{i,o,t}\right\}_{i,o}$, marginal values of robot capital $\boldsymbol{\lambda}^R_t \equiv \left\{\lambda^R_{i,o,t}\right\}_{i,o}$, employment $\boldsymbol{L}_t \equiv \left\{L_{i,o,t}\right\}_{i,o}$, workers' value functions $\boldsymbol{V}_t \equiv \left\{V_{i,o,t}\right\}_{i,o}$, non-robot good prices $\boldsymbol{p}^G_t \equiv \left\{p^G_{i,t}\right\}_i$ robot prices $\boldsymbol{p}^R_t \equiv \left\{p^R_{i,o,t}\right\}_{i,o}$, wages, $\boldsymbol{w}_t \equiv \left\{w_{i,o,t}\right\}_{i,o}$, bilateral non-robot good trade levels $\boldsymbol{Q}^G_t \equiv \left\{Q^G_{ij,t}\right\}_{i,j}$, bilateral non-robot good trade levels $\boldsymbol{Q}^R_t \equiv \left\{Q^R_{ij,o,t}\right\}_{i,j,o}$, and occupation transition shares $\boldsymbol{\mu}_t \equiv \left\{\mu_{i,oo',t}\right\}_{i,oo'}$. I write $\boldsymbol{S}_t \equiv \left\{\boldsymbol{K}^R_t, \boldsymbol{\lambda}^R_t, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$ as state variables.

Definition 1. In each period t, given state variables $\boldsymbol{S}_t$, a *temporary equilibrium* (TE) $\boldsymbol{x}_t$ is the set of prices $\boldsymbol{p}_t \equiv \left\{\boldsymbol{p}^G_t, \boldsymbol{p}^R_t, \boldsymbol{w}_t\right\}$ and flow quantities $\boldsymbol{Q}_t \equiv \left\{\boldsymbol{Q}^G_t, \boldsymbol{Q}^R_t, \boldsymbol{\mu}_t\right\}$ that satisfy: (i) given $\boldsymbol{p}_t$, workers choose occupation optimally by equation (1.4), (ii) given $\boldsymbol{p}_t$, producers maximize flow profit by equation (1.12) and demand robots by equation (A.15), and (iii) markets clear: Labor adds up as in equation (1.7), and goods market clear with trade balances as in equations (A.23) and (A.25).

The temporary equilibrium inputs all state variables and outputs other endogenous variables that are determined contemporaneously. The following sequential equilibrium determines all state variables given initial conditions.

Definition 2. Given initial robot capital stocks and employment $\left\{\boldsymbol{K}_0^R, \boldsymbol{L}_0\right\}$, a *sequential equilibrium* (SE) is a sequence of vectors $\boldsymbol{y}_t \equiv \{\boldsymbol{x}_t, \boldsymbol{S}_t\}_t$ that satisfies the TE conditions and employment law of motion (1.6), value function condition (1.5), capital accumulation equation (1.13), producer's dynamic optimization (A.19) and (A.18).

Finally, I define the steady state as a SE $\boldsymbol{y}$ that does not change over time.

1.3.2 Solution

I log-linearize around the initial equilibrium in order to solve the model. In particular, I study the effect of shocks on the sequential equilibrium $\boldsymbol{y}_t$. The log-linearization gives a sequence of matrices $\left\{\overline{\boldsymbol{F}_t}\right\}_t$ and a matrix $\overline{\boldsymbol{E}}$ that summarize the first-order effect on sequential equilibrium in transition dynamics and steady state, respectively. The steady state matrix $\overline{\boldsymbol{E}}$ is a key object in estimating the model in Section 1.4. Section A.4 of the Appendix gives the details of the derivation of these matrices.

In the economy described in Section 1.3.1, the shocks comprise changes in the economic environment and changes in policy. For instance, consider the increase of the robot task space $a_{o,t}$ in baseline period t_0 by Δ_o percent, or

$$a_{o,t} = \begin{cases} a_{o,t_0} & \text{if } t < t_0 \\ a_{o,t_0} \times (1 + \Delta_o) & \text{if } t \geq t_0 \end{cases}.$$

In this formulation, Δ_o is interpreted as the size of the expansion of the robot task space. I combine all these changes into a column vector $\boldsymbol{\Delta}$. I take the following three steps to solve the model. Write state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}_t^R, \boldsymbol{\lambda}_t^R, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, and use "hat" notation to denote changes from t_0: for any variable z_t, $\widehat{z_t} \equiv \ln(z_t) - \ln\left(z_{t_0}\right)$.

Step 1. For a given period t, I combine the vector of shocks $\boldsymbol{\Delta}$ and (given) changes in state variables $\widehat{\boldsymbol{S}_t}$ into a (column) vector $\widehat{\boldsymbol{A}_t} = \left\{\boldsymbol{\Delta}, \widehat{\boldsymbol{S}_t}\right\}$. Log-linearizing the TE conditions, I solve for matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ such that the log-difference of the TE $\widehat{\boldsymbol{x}_t}$ satisfies

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}_t} = \overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}_t}. \tag{1.16}$$

In this equation, $\overline{\boldsymbol{D}^x}$ is a substitution matrix and $\overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}_t}$ is a vector of partial equilibrium shifts in period t. Since the temporary equilibrium vector $\widehat{\boldsymbol{x}_t}$ includes wages $\widehat{\boldsymbol{w}_t}$, equation (1.16) generalizes the general equilibrium comparative statics formulation in Adao et al. [2019a]. Note that there exists a block separation of matrix $\overline{\boldsymbol{D}^A} = \left[\overline{\boldsymbol{D}^{A,\Delta}}|\overline{\boldsymbol{D}^{A,S}}\right]$ such that equation (1.16) can be written as

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}_t} - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}_t} = \overline{\boldsymbol{D}^{A,\Delta}}\boldsymbol{\Delta}. \tag{1.17}$$

Step 2. Log linearizing laws of motion and Euler equations around the old steady state, I solve for matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ such that $\overline{\boldsymbol{D}^{y,SS}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{D}^{\Delta,SS}}\boldsymbol{\Delta}$, where superscript SS denotes steady state. Combined with steady state version of equation (1.17), I have

$$\overline{\boldsymbol{E}^y}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^\Delta}\boldsymbol{\Delta}, \tag{1.18}$$

where

$$\overline{\boldsymbol{E}^y} \equiv \left[\begin{array}{cc} \overline{\boldsymbol{D}^x} & -\overline{\boldsymbol{D}^{A,T}} \\ \hline \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{array}\right], \text{ and } \overline{\boldsymbol{E}^\Delta} \equiv \left[\begin{array}{c} \overline{\boldsymbol{D}^{A,\Delta}} \\ \hline \overline{\boldsymbol{D}^{\Delta,SS}} \end{array}\right],$$

which implies the first-order steady state matrix $\overline{\boldsymbol{E}}$ that satisfies $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$.

Step 3. Log linearizing laws of motion and Euler equations around the new steady state, I solve for matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_t^{y,TD}}$ such that $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}_t^{y,TD}}\check{\boldsymbol{y}}_t$, where the superscript TD stands for transition dynamics. Log-linearized sequential equilibrium satisfies the following first-order difference equation

$$\overline{\boldsymbol{F}_{t+1}^y}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}_t^y}\widehat{\boldsymbol{y}_t} + \overline{\boldsymbol{F}_{t+1}^\Delta}\boldsymbol{\Delta}. \tag{1.19}$$

Using conditions in Blanchard and Kahn [1980], there is a converging matrix representing the first-order transitional dynamics $\overline{\boldsymbol{F}_t}$ such that

$$\widehat{\boldsymbol{y}_t} = \overline{\boldsymbol{F}_t}\boldsymbol{\Delta} \text{ and } \overline{\boldsymbol{F}_t} \to \overline{\boldsymbol{E}}. \tag{1.20}$$

1.3.3 Real-wage Effect of Automation

What does the occupation production function (1.10) imply about the effect of automation? This question is directly related to the distributional and aggregate effects of industrial robots. In this section, I show that the effect of automation on occupational real wages depends negatively on substitution elasticity parameters θ_o and β conditional on the changes in input and trade shares. The key insight is that the real wages are relative prices of labor to the bundle of factors, and the relative price changes are related to changes in the input shares and trade trade shares via the demand elasticities. These elasticities are among the target parameters of the estimation in Section 1.4.

I modify notations in equation (1.10) to express the result in a concise way. Define

$$A^K_{i,o,t} \equiv \left(A^G_{i,t}\right)^{\frac{\theta-1}{\alpha_{i,L}}} a_{o,t}, \; A^L_{i,o,t} \equiv \left(A^G_{i,t}\right)^{\frac{\theta-1}{\alpha_{i,L}}} \left(1 - a_{o,t}\right). \tag{1.21}$$

Substituting these into production functions (1.8) and (1.10), I have

$$Y^G_{i,t} = \left[\sum_o \left(b_{i,o,t}\right)^{\frac{1}{\beta}} \left(\widetilde{T}^O_{i,o,t}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}\alpha_{i,L}} \left(M_{i,t}\right)^{\alpha_{i,M}} \left(K_{i,t}\right)^{1-\alpha_{i,L}-\alpha_{i,M}},$$

where

$$\widetilde{T}^O_{i,o,t} = \left[\left(A^L_{i,o,t}\right)^{\frac{1}{\theta_O}} \left(L_{i,o,t}\right)^{\frac{\theta_O-1}{\theta_O}} + \left(A^K_{i,o,t}\right)^{\frac{1}{\theta_O}} \left(K^R_{i,o,t}\right)^{\frac{\theta_O-1}{\theta_O}}\right]^{\frac{\theta_O}{\theta_O-1}}.$$

Therefore, one can interpret the newly defined terms $A^K_{i,o,t}$ and $A^L_{i,o,t}$ as the productivity shock on robots and labor, respectively. The following proposition claims that the long-run real-wage implication of the robot productivity change $\widehat{A^K_{i,o}}$ can be expressed by changes in input and trade shares and elasticities of substitutions.[12]

Define the good G-producers' labor share within occupation $\widetilde{x}^L_{i,o,t}$, occupation cost share

[12] By equation (1.21), robot productivity change $\widehat{A^K_{i,o,t}}$ and automation shock $\widehat{a_{o,t}}$ satisfy that $\widehat{A^K_{i,o,t}} = \frac{\theta-1}{\alpha_{i,L}}\widehat{A^G_{i,t}} + \widehat{a_{o,t}}$. Namely, robot productivity change is the sum of total factor productivity change caused by robotics and the automation shock. I choose to use the automation shock in my main specification in equations (1.8) and (1.10) since it has a tight connection to the task-based approach, a common approach in the automation literature (e.g., Acemoglu and Restrepo, 2020), as I discussed in Section 1.3.1.

$\widetilde{x}^O_{i,o,t}$, and trade shares $\widetilde{x}^G_{ij,t}$ as

$$\widetilde{x}^L_{i,o,t} \equiv \frac{w_{i,o,t}L_{i,o,t}}{P^O_{i,o,t}T^O_{i,o,t}},\ \widetilde{x}^O_{i,o,t} \equiv \frac{P^O_{i,o,t}T^O_{i,o,t}}{P^O_{i,t}T^O_{i,t}},\ \widetilde{x}^G_{ij,t} \equiv \frac{p^G_{i,t}Q^G_{ij,t}}{P^G_{i,t}Q^G_{i,t}}, \tag{1.22}$$

where $P^O_{i,o,t}$, $P^O_{i,t}$, and $P^G_{i,t}$ are the price indices of occupation o, aggregated task $T^O_{i,t} \equiv \left[\sum_o \left(b_{i,o,t}\right)^{\frac{1}{\beta}} \left(T^O_{i,o,t}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}$, and non-robot goods consumed in country i, respectively. In Appendix A.2.4, I discuss how one can compute steady-state labor share $\widetilde{x}^L_{i,o}$. Given these, the following proposition characterizes the real-wage changes in the steady state.

Proposition 1. *Suppose robot productivity grows $\widehat{A^K_{i,o}} > 0$. For each country i and occupation o,*

$$\widehat{\left(\frac{w_{i,o}}{P^G_i}\right)} = \frac{1}{1-\alpha_{i,M}}\left(\frac{\widehat{\widetilde{x}^L_{i,o}}}{1-\theta_o} + \frac{\widehat{\widetilde{x}^O_{i,o}}}{1-\beta} + \frac{\widehat{\widetilde{x}^G_{ii}}}{1-\varepsilon}\right). \tag{1.23}$$

Proof. See Appendix A.2.5. □

Proposition 1 clarifies how the elasticity parameters and change of shares of input and trade affect real wages at the occupation level. Among the elasticity parameters, one can observe that if $\theta_o > 1$, then (i) the larger the fall of the labor share within occupation $\widehat{\widetilde{x}^L_{i,o}}$, the larger the real wage gains, and (ii) pattern (i) is stronger if θ_o is small and close to 1. Therefore, conditional on other terms, the steady state changes of occupational real wages depend on the elasticity of substitution between robots and labor θ_o.

The intuition of Proposition 1 comes from the series of revealed cost reductions, $\widehat{\widetilde{x}^L_{i,o}}$, $\widehat{\widetilde{x}^O_{i,o}}$, and $\widehat{\widetilde{x}^G_{ii}}$. The first term reveals the robot cost reduction relative to labor cost. If $\theta_o > 1$, then the reduction in the price index or cost savings dominates the drop in nominal wage, increasing the real wage. Similar intuition holds for the second and third terms. The second term reveals the relative occupation cost reduction, whereas the last term reveals the relative sectoral cost reduction.

Proposition 1 also extends the welfare sufficient statistic in the trade literature. In particular, Arkolakis et al. (2012, ACR) showed that under a large class of trade models, the welfare effect of the reduction in trade costs can be summarized into the well-known ACR formula, or log-difference of the trade shares times the negative of trade elasticity. In fact, by dropping the robots and non-robot capital and aggregating occupations into one, the model reduces to:

$$\widehat{\left(\frac{w_i}{P_i^G}\right)} = \frac{1}{1-\alpha_{i,M}}\frac{1}{1-\varepsilon}\widehat{\tilde{x}_{ii}^G},$$

which is a modified ACR formula with intermediate goods as in Caliendo and Parro [2015] and Ossa [2015].

Although Proposition 1 concisely represents the effect of the automation shock on real wages, it is not straightforward to take this equation directly to the data. The reason is that the observed data contain not only the automation shock but also other shocks such as trade shocks that significantly affect $\widehat{\tilde{x}_{ii}^G}$. To study robotization's role in the occupational wage effect, I estimate the model and back out the automation shock in the following.

1.4 Estimation

Using the occupation-level Japan robot shock described in Section 1.2 and the solution to the general equilibrium model in Section 1.3, I develop an estimation method based on the generalized method of moments (GMM), in particular, the model-implied optimal instrumental variable (MOIV, Adao et al., 2019a). To do so, Section 1.4.1 sets the stage for the structural estimation by giving the implementation detail. I formalize the MOIV estimator in Section 1.4.2, which gives the structural estimates in Section 1.4.3.

1.4.1 Bringing Model to Data

To simplify the notation and tailor to my empirical application, I stick to country labels $i = 1$ as the US (USA), $i = 2$ as Japan (JPN), $i = 3$ as the Rest of the World (ROW). Following my data, I interpret country $i = 1$ as the country of interest in terms of labor market outcome variables, country $i = 2$ as the source country of automation shocks by robots, and country $i = 3$ as the (set of) countries in which the measurement of robots proxies the technological changes in country 2.

In the estimation, I allow heterogeneity across occupations of the within-occupation EoS between robots and labor. To do so, I define the occupation groups as follows. I first separate occupations into three broad occupation groups, Abstract, Service, Routine following Acemoglu and Autor [2011]. Routines occupations include production, transportation and material moving, sales, clerical, and administrative support. Abstract occupations are professional, managerial and technical occupations; service occupations are protective service, food preparation, cleaning, personal care and personal services. Given the trend that production and transportation/material moving occupations introduced robots over the sample period, I further divide routine occupations into three sub-categories, Production (e.g., welders), Transportation (indicating transportation and material-moving, e.g., hand laborer), and Others (e.g., repairer), where Others include sales, clerical, and administrative support. As a result, I obtain five occupation groups, for each of which I assume a constant EoS between robots and labor.[13] With each occupation group (or mapping from 4-digit occupation o to the group) represented by g, notation θ_g denotes the robot-labor EoS for occupation group. In Section A.3.1 of the Appendix, I examine a different choice of occupation grouping.

The vector of structural parameters are denoted as $\mathbf{\Theta}$ and its dimension is $d \equiv \dim(\mathbf{\Theta})$. To formally define $\mathbf{\Theta}$, I fix a subset of parameters of the model at conventional values. In particular, I assume that the annual discount rate is $\iota = 0.05$ and the robot depreciation rate is 10 percent following Graetz and Michaels [2018].[14] I take trade elasticity of $\varepsilon = 4$ from the large literature of trade elasticity estimation (e.g., Simonovska and Waugh, 2014), and $\varepsilon^R = 1.2$ derived from applying the estimation method developed by Caliendo and Parro [2015] to the robot trade data, discussed in detail in Appendix A.3.2. Following Leigh and Kraft [2018], I assume $\alpha^R = 2/3$. With this parametrization, structural parameters to be estimated are $\mathbf{\Theta} \equiv \left\{\theta_g, \beta, \gamma, \phi\right\}$.

1.4.2 Estimation Method

I observe changes in endogenous variables, US occupational wages $\widehat{\boldsymbol{w}_1}$, US employment $\widehat{\boldsymbol{L}_1}$, robot shipment from Japan to the US $\widehat{\boldsymbol{Q}_{21}^R}$, and the corresponding unit values $\widehat{\boldsymbol{p}_{21}^R}$ between 1992 and 2007, as well as the initial equilibrium $\boldsymbol{y}_{t_0}$. I approximate the 15-year changes as the steady-state changes. To simplify, I focus on the expansion of robot task space $\widehat{a_o}$ and the efficiency gain to produce robots in Japan $\widehat{A_{2,o}^R}$ as the source of the occupational shocks

[13]In terms of OCC2010 codes in the US Census, Routine production occupations are ones in [7700, 8965], Routine transportation are in [9000, 9750], Routine others are in [4700, 6130], Service are in [3700, 4650], and Abstract are in [10, 3540].

[14]For example, see King and Rebelo [1999] for the source of the conventional value of ι who matches the discount rate to the average real return on capital. For ε, see Simonovska and Waugh [2014] or Caliendo and Parro [2015].

in this section. Note that the robot production function (1.9) implies that $\widehat{A^R_{2,o}}$ is negative of the cost shock to produce robots in Japan, I measure the robot efficiency gain by

$$\widehat{A^R_{2,o}} = -\psi^J_{o,t_1}, \tag{1.24}$$

where, again, ψ^J_{o,t_1} is the Japan robot shock defined in equation (1.1) and measured in my dataset.

To discuss the identification challenge and the countermeasure, I decompose the automation shock $\widehat{a_o}$ into observed component $\widehat{a_o^{\mathrm{obs}}}$ and unobserved error component $\widehat{a_o^{\mathrm{err}}}$ such that $\widehat{a_o} = \widehat{a_o^{\mathrm{obs}}} + \widehat{a_o^{\mathrm{err}}}$ for all o. The component $\widehat{a_o^{\mathrm{obs}}}$ is observed conditional on parameter θ_o–namely, it satisfies the steady-state change of relative demand of robots and labor implied by the Euler equation

$$\left(\widehat{\frac{p^R_{i,o}K^R_{i,o}}{w_{i,o}L_{i,o}}}\right) = (1-\theta_o)\left(\widehat{\frac{p^R_{i,o}}{w_{i,o}}}\right) + \frac{\widehat{a_o^{\mathrm{obs}}}}{1-a_{o,t_0}}. \tag{1.25}$$

Equation (1.25) highlights the issues in identifying θ. First, the observed relative price change $\left(\widehat{p^R_{i,o}/w_{i,o}}\right)$ does not identify θ_g because $\left(\widehat{p^R_{i,o}/w_{i,o}}\right)$ is endogenous and is correlated with the residual term $\widehat{a_o^{\mathrm{obs}}}/(1-a_{o,t_0})$ that represents the task-space expansion of robots [Hubmer, 2018, Karabarbounis and Neiman, 2014]. Second, the Japan robot shock ψ^J_{o,t_1} also does not also work as an instrumental variable (IV) in the linear regression model of (1.25) because of a potential correlation between ψ^J_{o,t_1} and observed task-space expansion shock $\widehat{a_o^{\mathrm{obs}}}$.

To overcome these identification issues, I employ a method based on the full GE model below. Conditional on $\widehat{a_o^{\mathrm{obs}}}$, the error component $\widehat{a_o^{\mathrm{err}}}$ can be inferred from each observed endogenous variable. Take the changes in occupational wages $\widehat{\boldsymbol{w}_1}$ for example. The steady-state solution matrix $\overline{\boldsymbol{E}}$ implies that there is a $O \times O$ sub-matrices $\overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{a}}$ and $\overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{A}^R_2}$ such that[15]

$$\widehat{\boldsymbol{w}} = \overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{a}}\widehat{\boldsymbol{a}} + \overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{A}^R_2}\widehat{\boldsymbol{A}^R_2}. \tag{1.26}$$

[15]I use the steady-state matrix $\overline{\boldsymbol{E}}$ instead of the transitional dynamics matrix $\overline{\boldsymbol{F}_t}$ for a computational reason, which is described in Appendix A.3.1 in detail.

Since $\widehat{\boldsymbol{a}} = \widehat{\boldsymbol{a}^{\text{obs}}} + \widehat{\boldsymbol{a}^{\text{err}}}$, I have

$$\boldsymbol{\nu_w} = \widehat{\boldsymbol{w}} - \overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{a}} \widehat{\boldsymbol{a}^{\text{obs}}} - \overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{A}_2^R} \widehat{\boldsymbol{A}_2^R},$$

where $\boldsymbol{\nu_w} \equiv \overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{a}} \widehat{\boldsymbol{a}^{\text{err}}}$ is the O-vector structural residual generated from the linear combination of the unobserved component of the automation shocks. Note that the structural residual depends on the structural parameters $\boldsymbol{\Theta}$. To clarify this, I occasionally write the structural residual as $\boldsymbol{\nu_w} = \boldsymbol{\nu_w}(\boldsymbol{\Theta})$. For other variables $\left(\widehat{\boldsymbol{L}_1}, \widehat{\boldsymbol{p}_{21}^R}, \widehat{\boldsymbol{Q}_{21}^R}\right)$, I perform the same process and obtain corresponding structural errors $\left(\boldsymbol{\nu_L}, \boldsymbol{\nu_{p^R}}, \boldsymbol{\nu_{Q^R}}\right)$. Then I stack these vectors into an $O \times 4$ matrix $\boldsymbol{\nu} \equiv \left[\boldsymbol{\nu_w}, \boldsymbol{\nu_L}, \boldsymbol{\nu_{p^R}}, \boldsymbol{\nu_{Q^R}}\right]$, and from its o-th row define 4×1 vector as $\nu_o = \left[\nu_{\boldsymbol{w},o}, \nu_{\boldsymbol{L},o}, \nu_{\boldsymbol{p}^R,o}, \nu_{\boldsymbol{Q}^R,o}\right]^\top$. Given these structural residuals and the Japan robot shock $\boldsymbol{\psi}_{t_1}^J \equiv \left\{\psi_{o,t_1}^J\right\}_o$, I assume the following moment condition.

Assumption 1. *(Moment Condition)*

$$\mathbb{E}\left[\nu_o | \boldsymbol{\psi}_{t_1}^J\right] = 0. \tag{1.27}$$

Assumption 1 puts restriction on structural residual $\boldsymbol{\nu}$ in that it should not be predicted by the Japan robot shock. Note that it allows that the automation shock $\widehat{a_o}$ may correlate with the robot efficiency change $\widehat{\boldsymbol{A}_2^R}$ which is likely as I discuss in Appendix A.1.2 in detail. Instead, the structural residual ν_o purges out all the predictions of the impacts of shocks $\widehat{\boldsymbol{a}}$ and $\widehat{\boldsymbol{A}_2^R}$ on endogenous variables, and I place the assumption that the remaining variation should not be predicted by the Japan robot shock from the data.

Under what circumstances does Assumption 1 break? Note that the answer to this question is not the correlation of the structural residuals with other shocks such as trade shocks because I have confirmed controlling for the trade shock does not qualitatively alter the reduced-form findings in Section 1.2.3. Instead, a candidate answer is a directed technological change, in which the occupational labor demand drives the changes in the cost of robots. Specifically, suppose a positive labor demand shock in an occupation o induces the research and development of robots in occupation o and drives cost down in the long run. This mechanism is not incorporated in my model where robots are produced with production function (1.9) with exogenous technological change. Therefore, the structural residual ν_o cannot remove this effect and is negatively correlated with ψ_{o,t_1}^J. In this sense, the positive impact of Japan robot costs found in Section 1.2.3 still prevails qualitatively even under the directed technological change.[16]

Using Assumption 1, I develop the two-step estimator that is consistent and asymptotically efficient, following Adao et al. (2019), who define the model-implied optimal instrumental variable (MOIV) and extend the estimator of Newey and McFadden (1994) to the general equilibrium environment. Namely, the optimal GMM estimator is based on the instrumental variable that depends on unknown structural parameters. The two-step estimator solves this unknown-dependent problem and achieves desirable properties of consistency and asymptotic efficiency. See Propositions 2 and 3 in Appendix A.2.6 for the detail.

1.4.3 Estimation Result

To apply the two-step estimator, I need to measure the initial equilibrium $\boldsymbol{y}_{t_0}$, which is an input to the solution matrix $\overline{\boldsymbol{E}}$ in equation (1.18). I take these data from JARA, IFR, IPUMS USA and CPS, BACI, and World Input-Output Data (WIOD). The measurement of labor market outcomes is standard and relegated to Section A.1.8 of the Appendix. I set the initial period robot tax to be zero in all countries.

Table 1.2a gives the estimates of the structural parameters. Panel 1.2a shows the estimation result when I restrict the EoS between robots and labor to be constant across occupation groups. The estimate of the within-occupation EoS between robots and labor, θ_g, implies that robots and labor are substitutes within an occupation, and rejects the Cobb-Douglas case $\theta_g = 1$ at a conventional significance level. The point estimate of the EoS between occupations, β, is 0.71, or occupation groups are complementary. The one-standard error bracket covers Humlum's (2019) central estimate of 0.49. The adjustment cost parameter γ is close to the estimate of Cooper and Haltiwanger [2006] when they restrict the model with only quadratic adjustment costs, like in my model. The one-standard error range of occupational dynamic labor supply elasticity ϕ is estimated to be $[0.55, 1.07]$, which contains an estimate of 0.6 in the dynamic occupation choice model in Traiberman [2019] in the case without the specific human capital accumulation.

Panel 1.2b shows the estimation result when I allow the heterogeneity across occupation

[16]With increasing returns for robot producers, I could model that the robot demand increase drives cost drop. Estimating such a model requires detailed data on robot producers and is left for future research.

Table 1.2: Parameter Estimates

(a) All Parameters

Parameter	$\theta_g = \theta$	Free θ_g
θ	2.96 (0.17)	[Table 1.2b]
β	0.71 (0.23)	0.73 (0.31)
γ	0.30 (0.11)	0.30 (0.14)
ϕ	0.81 (0.26)	0.81 (0.30)

(b) Heterogeneous EoS θ_g

Routine	Production	4.04 (0.24)
	Transportation	4.29 (0.28)
	Others	1.27 (0.53)
Service		1.35 (0.48)
Abstract		0.80 (0.60)

Note: The estimates of the structural parameters based on the estimator in Proposition 3. Standard errors are in parentheses. In the left panel, parameter θ is the within-occupation elasticity of substitution between robots and labor. Parameter β is the elasticity of substitution between occupations. Parameter γ is the capital adjustment cost. Parameter ϕ is the occupation switch elasticity. The column "$\theta_g = \theta$" shows the result with the restriction that θ_o is constant across occupation groups. The column "Free θ_g" shows the result with θ_g allowed to be heterogeneous across five occupation groups. In the right panel, estimates for parameters θ_g with heterogeneity are shown. Transportation indicates "Transportation and Material Moving" occupations in the Census 4-digit occupation codes (OCC2010 from 9000 to 9750). See the main text for other details.

groups. The other structural estimates, (β, γ, ϕ), do not change qualitatively. Table 1.2b shows the estimates of the within-occupation EoS between robots and labor, θ_g. I find that the EoS for routine production occupations and routine transportation occupations is around 4, while those for other occupation groups (other occupations in routine group, service, and abstract occupations) are not significantly different from 1, the case of Cobb-Douglas. The estimates for routine production and transportation indicate the susceptibility of workers in these occupations to accumulated robot capital.

What is the source of identification of these large and heterogeneous EoS between robots and labor identified? As in the literature of estimating the capital-labor substitution elasticity, the positive correlation between the robot price and the wage (labor market outcome) suggests robots and labor are substitutes, or large θ_g. Intuitively, if θ_g is large, then given a percentage decrease in the cost of robots, the steady-state relative robot (resp. labor) demand responds strongly in the positive (resp. negative) direction. Reducing the occupation wage through the labor demand equation, the large robot-labor EoS yields a positive correlation between the robot price trend and the wage trend, as found in Figure A.8. Appendix A.1.7 further discusses this source of identification of the EoS, the correlation between the Japan robot shock and the US wage change within each occupation group.

1.4.4 Measuring Shocks and Model Fit

To examine the plausibility of these parameter estimates, I simulate the model and check the model's fit. The simulation process comprises two steps. First, I back out the observed shocks from the estimated model for each year between 1992 and 2007. Namely, with the point estimates in Table 1.2b, I obtain the efficiency increase of Japanese robots $\widehat{A^R_{2,o,t}}$ using (1.24), equation the observed automation shock $\widehat{a^{\text{obs}}_{o,t}}$ using (1.25), and the US occupation demand shock $\widehat{b_{1,o,t}}$. To back out the efficiency shock of robots in the other countries, I assume that $\widehat{A^R_{i,o,t}} = \widehat{A^R_{i,t}}$ for $i = 1, 3$. Then by the robot trade prices $p^R_{ij,t}$ from BACI, I fit fixed effect regression $\Delta \ln \left(p^R_{ij,t}\right) = \widetilde{\psi}^D_{j,t} + \widetilde{\psi}^C_{i,t} + \widetilde{e}_{ij,t}$, and use $\widehat{A^R_{i,t}} = -\widetilde{\psi}^C_{i,t_1}$. The idea to back out the negative efficiency shock $\widetilde{\psi}^C_{i,t_1}$ is similar to the fixed-effect regression in Section 1.2, but without the occupational variation. Second, applying the backed-out shocks $\widehat{A^R_{i,o,t}}$, $\widehat{a^{\text{obs}}_{o,t}}$, and $\widehat{b_{1,o,t}}$ to the first-order solution of the GE in equation (1.20), I obtain the prediction of changes of endogenous variables to these shocks to the first-order. Finally, applying the predicted changes to the initial data in $t_0 = 1992$, I obtain the predicted level of endogenous variables.

I run the linear regression model (1.3) to examine the fit of the model and the role of the automation shock in estimating the robot-labor EoS.[17] First, I hit all the shocks generated in the above paragraph. In this case, the prediction is consistent with the moment condition (1.27) and thus I predict that the linear regression coefficient α_1 of equation (1.3) is close to the one in Table 1.1. I term the predicted wages in this way as the "targeted wage." Second, I hit all the shocks but the automation shock. In this case, the same moment condition is violated since the structural residual fails to incorporate the automation shock. Therefore, this exercise reveals how important taking into account the observed automation shock is in estimation. Namely, the larger the discrepancy of the regression coefficient of equation (1.3) between the data and this second simulation, the more severe the bias caused by the automation shock. I call the predicted wages in this way as the "untargeted wage."

Table 1.3 shows the result of these exercises. By comparing the first column that repeats column (3) of Table 1.1 and the second column based on the targeted wage, I confirm that the targeted moments match well as expected. The third column is the result based on the

[17] As another model validation exercise, I predict the stock of robots by occupation and find that the model predict the actual robot accumulation dynamics well, described in detail in Appendix A.3.3. Appendix A.3.4 gives further discussion about the Japan robot shock and the backed-out observed automation shocks.

Table 1.3: Model Fit: Linear Regression with Observed and Simulated Data

	(1)	(2)	(3)
VARIABLES	$\Delta \ln(w)$	Targeted $\widehat{\boldsymbol{w}}$	Untargeted $\widehat{\boldsymbol{w}}$
ψ^J	0.0984***	0.0980***	0.126***
	(0.0266)	(0.0077)	(0.0009)
Observations	324	324	324
R-squared	0.394	0.532	0.794

Note: The author's calculation based on the dataset generated by JARA, O*NET, and the US Census. Column (1) is the coefficient of the Japan robot shock ψ^J in the reduced-form regression with IPW. Column (2) takes the US wage change predicted by GE with ψ^J as well as other shocks such as the observed automation shock $\widehat{\boldsymbol{a}^{obs}}$. Column (3) takes the US wage change predicted by GE with shocks including the Japan robot shock, but excluding the observed automation shock. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

untargeted wage and shows stronger positive correlation between the simulated wage and the Japan robot shock. This is due to negative correlation between the Japan robot shock ψ_o^J and the observed automation shock $\widehat{a_o^{\text{obs}}}$, which is consistent with that robotic innovations that save cost (thus decreases ψ_o^J) and that upgrade quality (thus increases $\widehat{a_o^{\text{obs}}}$) are likely to happen at the same time, as exemplified in Appendix A.1.2.

More specifically, the real and simulated data with the targeted wage contain the negative bias due to this negative correlation. Since the untargeted wage is free from this bias, the linear regression coefficient α_1 of equation (1.3) is higher than the one obtained from the real data. In other words, if I have wrongfully assumed that the economy did not experience the automation shock and believed the regression coefficient in Table 1.1 is bias-free, I would have estimated higher EoS by ignoring the actual negative correlation between ψ_o^J and $\widehat{a_o^{\text{obs}}}$. This thought experiment reveals the importance of taking into account the automation shock in estimating the EoS between robots and labor using the robot cost shock.

1.5 Counterfactual Exercises

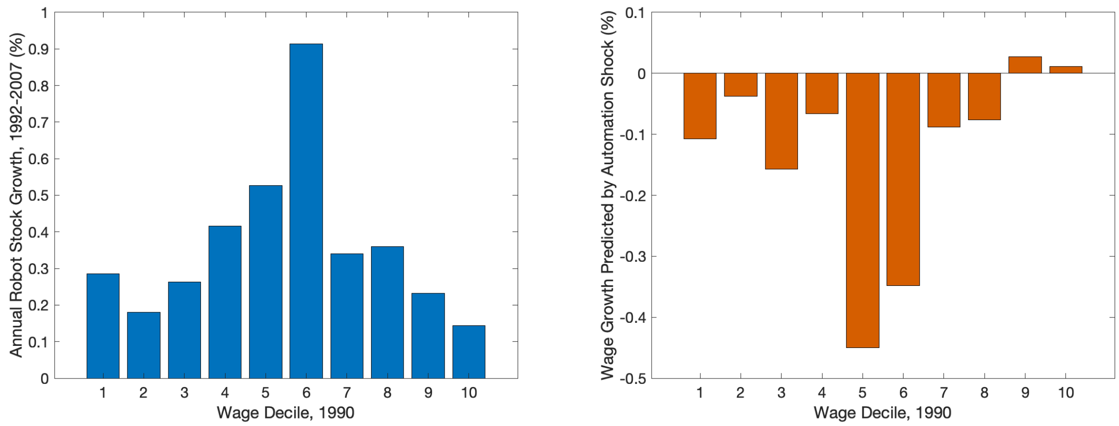
Using the estimated model and backed-out shocks in the previous section, I answer the following questions. The first question is the distributional effects of robots. Autor et al. [2008] argue that the wage inequality measured by the ratio of the wages between the 90th percentile and the 50th percentile (90-50 ratio) steadily increased since 1980.[18] Although

[18] As Heathcote et al. [2010] argue, a sizable part of the US economic inequality roots in the wage inequality. Furthermore, the polarization is not a unique phenomenon in the US, but found in the other context such as the UK [Goos and Manning, 2007].

Figure 1.2: Robots, Wage Inequality, and Polarization

(a) Robot growths by Wage Decile

(b) Robots' Effect on Occupational Wages



Notes: The left panel shows the average annual growth rates of the observed robot stock between 1992 and 2007 for every ten deciles of the occupational wage distribution in 1990. The right panel shows the annualized wage growth rates predicted by the backed-out shocks and the estimated model's first-order steady-state solution given in equation (1.18).

my data and model predicts the changes from 1990, can the increased use of industrial robots explain the 90-50 ratio? If so, how much? The second question concerns the policy implication of robot regulation. Due to the fear of automation, policymakers have proposed regulating industrial robots using robot taxes. What would be the effect of taxing on robot purchases?

1.5.1 The Distributional Effects of Robot Adoption

To study the contribution of robots to wage polarization, I begin by showing the pattern of robot accumulations over the occupational wage distribution. Figure 1.2a shows the average annual growth rates of observed robot stock between 1992 and 2007 for every ten deciles of the occupational wage distribution in 1990. The figure clarifies that the occupations in the middle deciles of the distribution received relatively many robots. Conditional on robot prices, this pattern implies there are relatively large automation shocks on these occupations.

The right panel shows the steady-state annualized predicted wage growths due to the shocks backed out in Section 1.4.4 and the estimated model with the first-order solution given in equation (1.20). Consistent with the high growth rate of robot stocks in the middle of the wage distribution and the estimation results that indicate the strong substitutability between robots and labor, I find that the wage effect in the middle deciles of the initial wage distribution is strongly negative. Quantitatively, the 90-50 ratio observed in 1990 and 2007 is, respectively, 1.588 and 1.668. On the other hand, the 90-50 ratio predicted by the initial 1990 data and the first-order solution (1.20) is 1.597. These findings indicate that 11.7 percent of the observed change in the 90-50 ratio between 1990 and 2007.

These results emerge from aggregating the effects on 4-digit occupational wages and my estimates of the robot-labor EoS. Specifically, relative occupational wages drastically drops in production and transportation (material-moving) occupations. This is a natural consequence of (i) the large quantities robots adopted in these occupations (Figure A.15) and (ii) the high estimates of EoS between robots and labor for these occupations (Table 1.2b). To confirm these observations, Appendix A.3.5 describes the wage changes for each of 5 occupation groups and Appendix A.3.6 performs the robotization exercise in case of low EoS as specified in the literature.

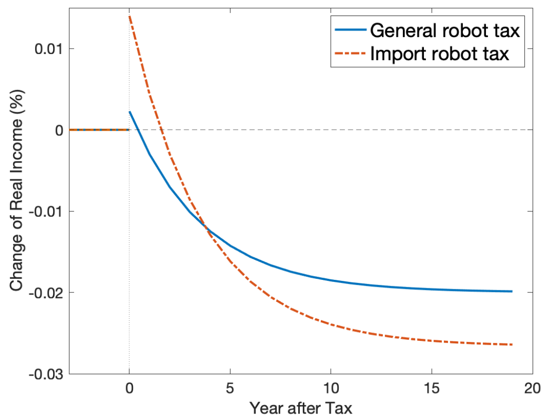
1.5.2 Robot Tax and Aggregate Income

Next, I consider a counterfactual rise of robot tax as well as the automation shock. In the baseline economy, all countries levied zero robot tax. On the one hand, consider an unexpected, unilateral, and permanent increase in the robot tax by 6% in the US, or the general tax scenario. I also consider the tax on only imported robots by 33.6%, or the import tax scenario, which implies the same amount of tax revenue as in the general tax scenario. Note that the 6% rate of the general tax is more modest than 30% considered in Humlum [2019] for the Danish case, and that the 33.6% import tax would make the tax revenue the same as the 6% general robot tax case, which makes the comparison straightforward between the scenarios. How does these robot tax schemes affect the US real income?[19] In Figure 1.3a, the solid line tracks the real-income effect of the general robot tax over a 20-year time horizon after the imposition. First, the magnitude of the effect is small because the cost of buying robots and the contribution of robot capital in the aggregate production are small. Second, in the short-run, there is a positive effect while the effect turns negative quickly and continues so in the long-run.

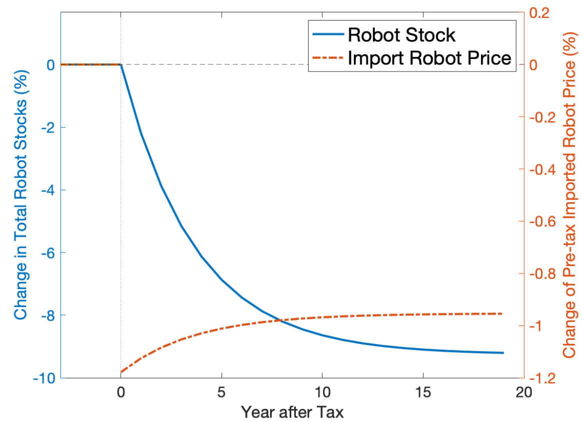
[19]Appendix A.3.7 provides analysis for occupational workers' welfare consequence and concludes that there is no general tax rate that makes all workers better off. There, I numerically show that the 6% general tax would roughly compensate the loss from robotization for production workers.

Figure 1.3: Effects of the Robot Tax

(a) US Real Income



(b) US Robot Stock and Import Robot Price



Notes: The left panel shows the counterfactual effect on the US real income of the two robot tax scenarios described in the main text over a 20-year time horizon. The right panel shows that of the import robot tax on the US total robot stocks (solid line) and the pre-tax robot price from Japan (dash-dot line) over the same time horizon.

Why is there a short-run positive effect on real income? A country's total income comprises the sum of workers' wages, the non-robot good producer's profit, and tax revenue. Since robots are traded, and the US is a large economy that can affect the robot price produced in other countries, there is a terms-of-trade effect of robot tax in the US. Namely, the robot tax reduces the demand for robots produced in the other country, let the equilibrium robot price go down along the supply curve. This reduction in the robot price contributes to the increase in the firm's profit, raising the real income in the short-run. The short-run positive effect is stronger in the import robot tax scenario because the higher tax rate induces a more substantial drop in the import robot price.

The terms-of-trade manipulation is well-studied in the trade policy literature. This paper offers the upward sloping export supply curve from the general equilibrium, as opposed to the supply curve that is assumed upward sloping (e.g., Broda et al., 2008). Namely, when the demand for robots in a robot exporter country decreases, the resource to produce robots in the exporter country is freed and reallocated to produce the non-robot goods. In my case, the resource is simply the non-robot goods that are input to robot production in equation (1.9). This increases the supply of non-robot goods in the robot-exporting country, depressing the price of non-robot goods. Again due to robot production function (1.9), this decrease in the non-robot goods price means the decrease of the cost of producing robots, which in turn reduces the price of robots produced in the exporter country.

Why do I have the different effect on real income in the long-run? The solid line in Figure 1.3b shows the dynamic impact of the import robot tax on robot stock accumulation. The tax significantly slows the accumulation of robot stocks, and decreases the steady-state stock of robots by 9.7 percent compared to the no-tax case. The smaller quantity of robot stocks reduces the firm profit, which contributes to smaller real income.[20] These results highlight the role of costly robot capital (de-)accumulation in the effect of the robot tax on aggregate income.

In Figure 1.3b, The dash-dot line shows a distinct dynamic effect: the effect of the robot tax on the price of robots imported from Japan in the US. In the short-run, the price decreases due to the decreased demand from the US. As the sequential equilibrium reaches the new steady state where the US stock of robots is decreased, the marginal value of the robots is higher. This increased marginal value partially offsets the reduced price of robots in the short-run, pushing back the cost of robots imported from Japan. This figure shows the effect of the international trade of robots in a large country as well as the accumulation of robots. As an extreme case, I also consider an alternative model with no trade of robots due to prohibitively high robot cost and give the robot tax counterfactual exercise in Section A.3.8 of the Appendix.

1.5.3 Other Exercises

What does the same robot tax do to each occupation? The robot tax rolls back the long-run real wage effect of automation. Workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) who would have been substituted by accumulated robots benefit from the tax, while the others lose. In Appendix A.3.9, I discuss the effect of the robot tax on occupational wages in detail. My model also allows the counterfactual exercise regarding robots trade liberalization. As mentioned in Appendix A.3.10, I find that trade liberalization would benefit all countries in the long-run. The benefit in the US appears immediately after the reduction in the trade cost.

[20]For each occupation, the counterfactual evolution of robot stocks is similar to each other in percentage and, thus, similar to the aggregate trend in percentage. This is not surprising since the robot tax is ad-valorem and uniform across occupations.

1.6 Conclusion

In this paper, I study the distributional and aggregate effects of industrial robots, emphasizing that robots perform specified tasks and are internationally traded. I make three contributions. First, I construct a first dataset that tracks the number of robot arms and unit values disaggregated by occupations that robots replace. Second, I develop a general equilibrium model that features the trade of robots in a large-open economy and endogenous robot accumulation with an adjustment cost. When estimating the model, to identify the occupation-specific EoS between robots and labor, I construct a model-implied optimal instrumental variable from the average price of robots in my dataset.

The estimates of within-occupation EoS between robots and labor is heterogeneous and as high as 4 in production and material moving occupations. These estimates are significantly larger than estimates of the EoS of capital goods and workers, with a maximum of about 1.5, revealing the susceptibility of workers in the occupations to robot adaptation. These estimates imply that robots contributed to the wage polarization across occupations in the US from 1990-2007. A commonly advertised robot tax could increase the US real income in the short-run but leads to a decline in the income in the long run due to the small steady-state robot stock. These findings indicate that the accumulated robots may have more massive distributional impacts than is considered in the previous literature, and regulating robots could have a positive effect from the aggregate perspective due to the trade of robots.

Chapter 2

Robots and Employment: Evidence from Japan, 1978-2017

2.1 Introduction

How does automation affect employment? Academic researchers, policymakers, and journalists alike all discuss this topic (Brynjolfsson and McAfee, 2014, Ford, 2015, Frey and Osborne, 2017, OECD, 2019). An example of automation is the use of industrial robots (robots hereafter).[1] This paper analyzes the effect of robot penetration on employment in Japan, relying on a novel identification strategy enabled by a newly digitized data set of robot shipments by destination industry and robot application (task, defined further below) in quantity and sales values.

To estimate the impact of robot penetration on employment, we exploit the variation in robot adoption due to heterogeneous changes in effective robot prices that differ by the characteristics of robot adopters. As our methodological benchmark, Acemoglu and Restrepo [2020] and Dauth et al. [2018] proxied the supply shock of robot to a domestic industry by the robot penetration of the same industry in other developed economies. The presumption is that the industry-level penetration of robots is similar within these economies, because both economies face the same industrial shock to the robot supply, such

[1] Other examples include the use of information and communication technologies (ICT, Autor et al., 2003, among others) and artificial intelligence (e.g., Agrawal et al., 2018). Research on automation in non-industrial contexts is also emerging. Png [2019], for example, finds positive welfare effects on routine cashier workers at supermarkets.

as innovation in a new robot model that is demanded by both countries. We propose a new identification strategy based on robot applications.

Robot applications are the tasks in the production process that the introduced robots are supposed to do, such as welding and parts assembling. This study substantiates the source of variation of robot adoption across industries by directly appealing to the differences in robot applications in the production process across industries. Exploiting this inter-industry technological variation, we construct a robot price index that each industry faces in each year by averaging the robot price by application, weighted by industry-level application shares of robots. We use this industry-year-level robot price index to estimate the impacts of robot price on robot adoption and employment. Subsequently, we estimate the impact of robot adoption on employment, using the price index as the instrumental variable (IV) for robot adoption. The strengths of our methods in comparison to the literature are as follows. First, our method does not require countries of comparison. Second, the regression coefficient reveals gross factor (robot and labor) demand elasticities with respect to robot prices, which are directly interpretable. Furthermore, our method has an additional benefit: The construction of different robot price indices across industries enables us to examine the experience of Japan, which has no comparable foreign counterparts, because robots were adopted in the production process as early as the late 1970s, about 20 years earlier than other developed countries.

We implement our identification strategy employing a new data set featuring a long panel (1978-2017), quantity and sales values, and disaggregation by destination industry and robot applications. Robot applications are the tasks in the production process that the introduced robots are supposed to perform, such as welding and parts assembling. We first demonstrate that various industries use robots in different production processes by showing that the composition of robot applications is substantially different across destination industries. For instance, welding robots and assembling robots consist of 46.3 percent and 7.1 percent of total shipments to the automobile industry, respectively, whereas the values for the electronics industry are 3.0 percent and 76.2 percent, respectively, in the years 1978-82. We next show that the unit price of welding robots fell consistently and substantially between 1978 and 2017, whereas the price drop of other robots was not as striking. Exploiting the heterogeneity of robot application intensity across industries and the different price trends by robot application, we construct the aggregate robot price index by industry that changed differentially across industries. We take this price variation across industries as the source of identification.

We find that robots are complementary to employment at both at the industry and region levels. At the industry level, we show that a 1 percent decrease in robot price increased robot adoption by 1.54 percent. Perhaps more surprisingly, we also find that a 1 percent decrease in robot price increased employment by 0.44 percent. This finding implies that robots and labor are gross complements. Therefore, taken together, our two-stage least squares estimates suggest that a 1 percent robot increase caused by price reduction increased employment by 0.28 percent.

To show the credibility of our identification assumption, we conduct robustness checks by dropping large robot-adopting industries and using different price measures. Robot prices may be directly affected by large adopters. For instance, large adopters, such as the automobile industry, may exert scale effects and reduce the prices of robots they purchase. In this case, prices are endogenous for these adopters. To deal with the concern, we drop these large industries and show that the results are qualitatively the same. To deal with further concerns about using prices as a direct shifter of our IV, we also report the results with alternative IVs based on measures of leave-one-out prices and prices demanded by foreign purchasers. The results are also robust to these considerations.

We move to the commuting-zone (CZ) level analysis to better compare our results with existing estimates in the literature. By constructing shift-share measures of robot exposure, we conduct a two-stage least square estimation resembling the one by Acemoglu and Restrepo [2020] but with our cost-based instrumental variable. Our result indicates that one robot unit per 1,000 workers increased employment by 2.2 percent, corroborating the finding that the robots and labor are gross complements. This contrasts with the finding by Acemoglu and Restrepo [2020], whose corresponding estimate was -1.6 percent. The difference of the results is not surprising, given the difference of the country and the time period covered. Particularly, considering the export-oriented nature of the automobile and electric machine sectors of Japan, robot adoption and its cost-reducing effect have contributed to the expansion of exports and increased the labor demand. This scale effect may well have dominated the substitution effect of robots for labor.

The CZ-level analysis enables us to conduct further analyses regarding spillovers from the manufacturing sector to the non-manufacturing sector, as advocated by Moretti [2010].

First, we find that the employment of non-manufacturing sectors neither increased nor decreased upon robot adoption. This fact shows that within-region-across-industry reallocation from service to manufacturing did not happen, suggesting that an across-region reallocation of workers (migration or lack thereof) happened. In other words, in the context of the non-increasing population in Japan, robots might work like a magnet that keeps workers from leaving for other regions. Second, we find that although the total employment increased when robots were adopted, the hours worked per worker decreased. This finding suggests that robots may have worked as work-sharing and time-saving technological changes. In turn, this implies that the hourly-wage effect might be even more positive, which we confirmed in our data.

Relative to the previous literature, for the first time, we propose and implement a robot-cost based identification strategy to estimate the causal effect of robot penetration on labor demand, drawing on newly digitized data covering the long period of 1978-2017 in Japan and the heterogeneous technology of robot use across industries. There is a growing empirical literature studying the impacts of robots on employment [Acemoglu and Restrepo, 2020, Artuc et al., 2020, Bessen et al., 2019, Dauth et al., 2018, Graetz and Michaels, 2018, Humlum, 2019, Koch et al., 2019]. The literature is finding mixed evidence from different contexts. Among them, Graetz and Michaels [2018] reported that robot penetration increased labor productivity as well as wages based on an Organisation for Economic Co-operation and Development (OECD) country-industry level analysis. Acemoglu and Restrepo [2020] analyzed US regional labor markets and concluded that robot penetration into a local labor market reduces the employment-to-population ratio and the earnings of all workers regardless of skill levels, implying that all workers lose from robot penetration. In contrast, Dauth et al. [2018] analyzed German regional labor markets to find that the penetration of robots decreased employment in the manufacturing sector but increased employment in the service sector, suggesting the coexistence of losers and winners in local economies. These conflicting empirical results from two major economies, in addition to the theoretical discussion by Berg et al. [2018] and Caselli and Manning [2019] that technological progress should benefit some workers under fairly standard assumptions, warrant further empirical study on other major economies.

The existing literature on the impact of robots on employment does not use robot cost as a robot demand shifter, because robot cost measures are not generally available. For example, the widely used data set compiled by the International Federation of Robotics does not record the sales values at a fine disaggregated level. Thus the unit value per robot is unknown. Graetz and Michaels [2018] is an exception, in that it reports the robot price taken from a survey of a subset of robot producers and shows that the cost reduction was a critical factor behind robot adoption. They did not, however, use the price information in their formal regression analysis.

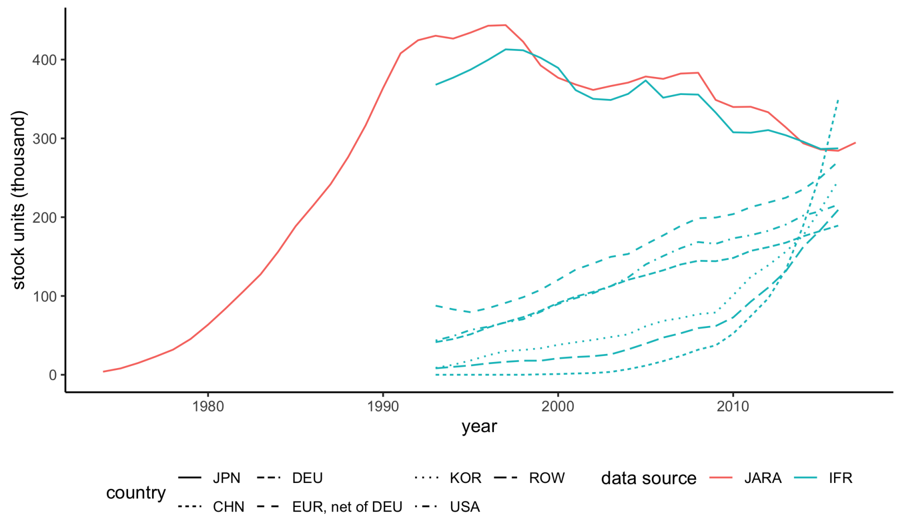
There is also an emerging literature that uses automation data with firm-level observations to study the margins of adjustment (Bessen et al., 2019, Koch et al., 2019). These papers typically are not aimed at studying a particular machine, but instead, an abstract "automation cost or technology." In contrast, we study the role of a particular well-defined machine, the robot, which makes a quantitative interpretation possible as opposed to a qualitative one.

The use of equipment prices to estimate the elasticity of substitution between production factors is not new.[2] Our paper may be thought of as a contribution to this literature in that it examines a particular type of capital, robots, that is expected to automate the production process and substitute for labor. Recent examples from this literature include Karabarbounis and Neiman [2014], who estimated capital-labor elasticity of substitution using cross-country and time-series variations to study drivers behind recent declines in labor shares in national income. Drawing on their arguments, our finding that robots and labor may be gross complements implies that caution should be taken in interpreting that the recent development in robot technology contributes to a decrease in labor shares.

This paper proceeds as follows. Section **??** reviews the necessary background for this paper. This includes the standard shift-share IV-based identification method, cross-country robot adoption trends, and robotics application classifications. Section 3.2.1 describes our datasets, with particular emphasis on the critical raw trends of our newly digitized dataset on robot adoption. Section 2.3 and 2.4 are the primary analysis sections of this paper, showing two different and complementary empirical strategies of industry-level and commuting-zone-level analyses, and their results. Section 2.5 discusses the efficiency adjustment over the long-run robot adoption and ensures the robustness of our results when those adjustments are accounted for. Section 2.6 concludes.

[2]See Antras [2004] for a list of papers.

Figure 2.1: Trends of Robot Stock Units, by Country



Note: Authors' calculation based on JARA and IFR data. The JARA data show the application-aggregated robot shipment units from Japan to Japan between 1974 and 2017. We supplement the shipment during the 1974-77 period by type-aggregated units. We assume zero units before 1973. To calculate the stock units, we assume eight (8) year immediate withdrawal method to match the stock unit trend of Japan observed in the IFR. The IFR data show stock unit trends for selected countries and aggregated rest of the world (ROW) between 1993 and 2016.

2.2 Data

2.2.1 Trends of Robot Stock Units

To grasp the cross-country trends of robot stock units, we leverage two data sources: the International Federation of Robotics and the Japan Robot Association (JARA). The IFR data are from a country-industry-level panel of operational stock of robots for years 1993 to 2016. This panel has been used intensively in the literature (Graetz and Michaels, 2018, Acemoglu and Restrepo, 2020, among others). The JARA data are based on an establishment-level survey completed by Japanese robot producers, which we will detail in Section 2.2.3. The JARA provides the statistics to IFR, and thus the Japanese series of the IFR is based on JARA statistics. Remarkably, the dataset is available since 1978. Based on these datasets, we construct the trends of robot stock units for each country in Figure 2.1.

In the figure, two colors are used to separate the data sources: one for JARA and the other for IFR. Different line types indicate different countries. Note that the solid line represents the trend of Japan, and the trends for Japan overlap well between the IFR and JARA datasets. This is not surprising, because the IFR creates its Japanese series based on JARA data. The two series do not overlap perfectly due to minor adjustments. Several findings stand out from Figure 2.1. First, Japan experienced a very different trajectory than other countries. There is a rapid increase in the 1980s, and the trend has been stable or even decreased from the 1990s and onward. Second, other large robot purchasers are Germany and other European countries, South Korea, the US, and recently, China. All of these countries increased the stocks rapidly in 1990s and 2000s. Although there are no data available for these countries before 1993, the relative novelty of the robotics technology suggest that the stocks before 1993 would have not been more than those of 1993. Therefore, Japan had a quite unique trend in introduction of robots.

Given that the standard regional shift-share IV (2.9) leverages the trends of similar comparison countries, how to apply it in our context is not straightforward. To solve the identification problem, we scrutinize robotics technology. In particular, we study the robot applications in the next subsection. The application-level variation turns out to be an additional source of variation that helps the identification in our context and with our dataset.

2.2.2 Robots and Robot Applications

Given the difficulty of finding countries of comparison, we employ a new identification strategy that is based on industry-level variation in the intensities of robot applications. To understand what this entails, we begin with the definition of an industrial robot (ISO 8373): "an automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications." Although well-defined, the definition is fairly broad and thus has different functionalities depending on the particular applications.

In fact, different robot applications are used in different industries with varying intensities. To understand this point better, consider an example task of spot welding (SW) in a certain manufacturing production process. SW combines two or more metal sheets together by applying pressure and heat to the weld area (spot). The task may be conducted in several ways, including using human labor or robots. Furthermore, different industries input the SW task differently. For instance, automobile industry intensively conducts SW, because it has thousands of metal parts that are combined together to make a car body. In contrast, in textile industries such tasks are seldom needed. Therefore, if the automation proceeds in SW task-intensive industries, it is likely that SW-application robots are adopted in such industries.

Another example of a robot application in our paper is surface mounting (SM). SM places surface-mount devices (SMDs) onto a printed circuit board (PCB). Since PCBs are primary inputs to a majority of electric machines, the electric machine industry uses SM robots intensively. Note that robots deployed for SM are different in the type or structure from those used for spot welding (SW). SM requires quick and accurate movement along a horizontal dimension for mounting SMDs. For this purpose, a typical structure of SM robots is called the Selective Compliance Assembly Robot Arm (SCARA), which is well suited to horizontal movements. In contrast, SW requires intensive movements in any directions within three-dimensional spaces. This feature helps welding the body sheets to assemble a complex automobile body shapes without losing an efficiency of production. Therefore, SW robots are typically structured as articulated robots, which are equipped with multiple joints (typically six) that enable smooth movements along any direction.

The difference of mechanical structures by robot applications creates price variation by robot applications. For example, the technological progress of the articulated robot reduced the price of welding robots (including SW robots) relatively faster than that of Assembly robots (including SM robots) in recent decades, as we will see in the data section.

Finally, we discuss the reasons behind robot adoption for such applications. Adoption of robots eventually depends on the productivity of robots relative to human labor, and the relative prices of robots and wages. For example, one of the first robot brands in Japan, the *Kawasaki-Unimate 2000*, was reported to be quite efficient in SW tasks: "The unmanned production line capable of spot welding 320 points per minute took over the work of 10 experienced welders. Including day and night shifts, it saved the labor of 20 people and as a result, the use of such highly versatile robots freed workers from welding, one of Japan's so-called '3K' (kitsui, or 'hard'; kitanai, or 'dirty'; and kiken, or 'dangerous') jobs." [KHI, 2018] The time-saving nature of robots indicates that the hours worked for each worker may decrease more than the headcount of workers when producers adopt robots. We consider this point thoroughly in Section 2.5. This enhanced productivity of robots relative to human labor and the decrease of the relative price of robots to labor explains robot adaptation.

We combine several data sources; two of the most important ones are the robot variables from the Japan Robot Association (JARA) and employment variables from the Employment Status Survey (ESS) from the Japan Statistic Bureau. We also complement these sources with data by Comtrade from the United Nations (UN), the Census of Manufacture, the Survey of Overseas Business Activities, and the Commodity Distribution Survey from Japan's government surveys. We will detail these below.

2.2.3 JARA Robot Data

Data Source

Our primary data source for robot purchases is the Appendix tables to *Survey Report on Company Conditions of Manipulators and Robots* (survey tables, henceforth), obtained from *the Japan Robot Association* (JARA, henceforth). The JARA is a non-profit organization of robot-producing member companies. As of October 2019, the number of full member companies is 53, and the associate member companies count 194.[3]

JARA sends an annual questionnaire to member companies and publicizes the survey tables to their member companies.[4] The survey is available starting from 1974, and we focus on the years after 1978 in our main analysis, because the shipment disaggregation by application became available from that year. The long period of coverage enables us to analyze the long-run and potentially time-varying effects of robots on labor markets. Furthermore, the sample period contains the year in which Japan introduced the industrial robots most intensively, as we will see.

The JARA's questionnaire asks for the unit and value of industrial robots that the firm ship by each application, type, structure, and destination industry. We focus on the disaggregation by applications in our main analysis, as we discussed in detail in the previous section. Robot types refer to a robot categorization based on the mechanical way in which robots work. Robot structures refer to a categorization based on the more particular dimensions and directions along which each joint of robots moves. Given the sophistication of robotic technologies, in 2004, the IFR and robot-producing companies

[3]See https://www.jara.jp/about/index.html. (In Japanese. Accessed on October 22, 2019)

[4]In 1996, the questionnaire was sent to 587 establishments and firms. Therefore, one establishment corresponds to one firm. Among them, 445 answered (for a response rate of 76 percent). Among the answering establishments, 231 actually produced robots in the year of the survey. Further details regarding the coverage of data are discussed in Section B.3.1.

Table 2.1: Classifications of Robots

	Classification by Application	Classification by Types
Classification	Handling operations/Machine tending (HO/MT)	Manual manipulator
	Welding and soldering (Welding)	Fixed sequence robot
	Dispensing	Variable sequence robot
	Processing	Playback robot
	Assembling and disassembling (Assembling)	Numerical control robot
	Others	SAL control robot
Available years	1978-2017	1974-2000

Note: Authors' aggregations based on consistently available classifications in JARA data for different years. "SAL control robot" stands for Sensory-, Adaptive-, or Learning-control robot.

agreed that the structure categorization should take over the type categorization. The full lists of the application and type categories are given in Table 2.1.[5]

We work with 13 industry classifications (12 manufacturing and one "others" aggregates) that are consistent with the other datasets we describe below. The list of industries are: Steel; Non-ferrous metal; Metal products; General machine (includes robot producers); Electric machine; Precision machine; Transport machine; Food, beverages, tobacco, feed; Pulp, paper, printing, publishing; Chemical, pharmaceuticals, cosmetics, etc.; Ceramics and earthwork products; Other manufacturing; and Non-manufacturing.

To do the long-run analysis, we construct a time-consistent aggregation of applications, types, and structures. Interestingly, there is no clear pattern of robot sophistication between the different applications while the robot types evolved as the technological frontier of robot-producing industry expanded, as we will see when we discuss the descriptive statistics. Therefore, the application-based classification is relatively stable for each destination industry. This feature provides a justification for our focus on applications. In particular, since we take a weighted average of each category's cost to construct industry-level robot cost measures, it is desirable that the weight is invariant and different across industries. Furthermore, the application-based classification is available for a more extended period, 1978-2017. For these reasons, our primary analysis hinges on the classification by applications.

The dimensions of the summary tables differ by years. The application-destination industry cross tables are available between 1978 and 2017. The details of the availability

[5]Each application is further discussed in Section B.1. The structure categorization is discussed in Section B.3.2.

are discussed in Section B.3.2.

Raw Trends

We overview some raw aggregate trends of the JARA data to highlight three novelties of our data: the length of the sample period, the availability of the unit value measures, and the availability of data by applications (and types up to 2000). To smooth the year-level volatility, we average observations with five-year bins by taking five-year observations prior to each year (e.g., the observation in 1982 is the simple average of those from 1978 to 1982). Further discussions and descriptive statistics are in Section B.3.3.

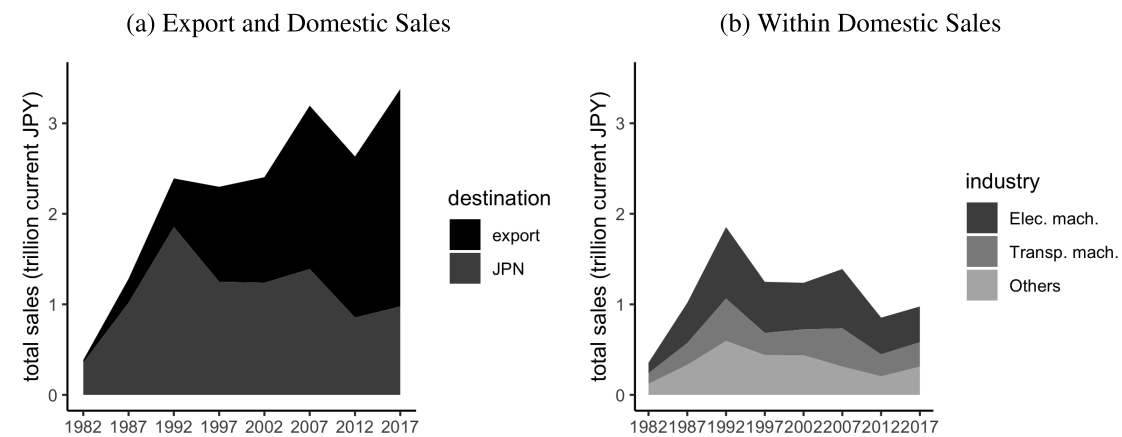
We first describe who buys robots from Japan. Figure 2.2 shows the industry decomposition of the aggregate sales of robots. The left panel 2.2a shows the decomposition into export and domestic sales, while the right panel 2.2b shows the disaggregation of domestic sales into destination industries. From Figure 2.2a, one can see that the growth of robot adoption within Japan expanded during 1980s, while the trend is stagnant afterward. Then starting around 1990s, the export trend expanded rapidly. This finding corroborates the trend from IFR data shown in Figure 2.1. Although the structural break before and after 1990 is an interesting phenomenon per se, our focus is on the domestic adoption trend and its industrial variation.

Figure 2.2b tells us that, within such domestic sales, electric machine and transportation machine (including automobiles) industries are significant purchasers of robots. These two industries represent 68.2 percent of the domestic absorption in 2017. This feature of the data leads our empirical strategy and indicate cautions in the identifications at the same time. Namely, our idea of the empirical strategy is driven by the salient differences in applications between the big buyers. At the same time, since these buyers are large, they might have market powers that affect observed unit values. We argue the first point in the next paragraph and address the concern in detail when discussing the empirical results.

At the same time, we also note that robot adoption was widespread in all manufacturing industries during the 1980s and was not concentrated in particular industries like electric and transportation machines. We confirm this point in a different slice of raw trends in Section B.3.3.

Next, Figure 2.3a shows the application-expenditure shares for the large robot purchasers, electric machines, and transportation machinery (including automobiles). We also show

Figure 2.2: Industry Decomposition



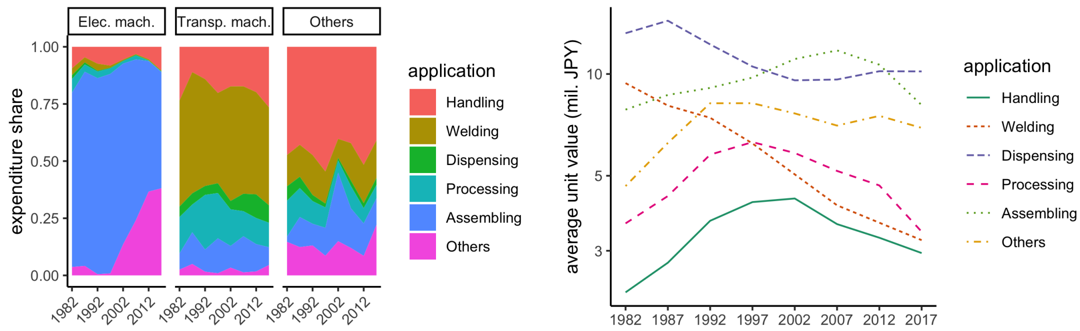
Note: Authors' calculation based on JARA data. The left panel compares the sales of exports (black) and aggregated domestic industries (dark gray). The right panel shows the decomposition of aggregated domestic industries into three categories: electric machine (Elec. mach.), transportation machine (Transp. mach.), and aggregated other domestic industries (Others).

the share trend for the other aggregated domestic industries. We highlight two empirical regularities. First, there is significant across-industry variation. The electric machine industry, for example, intensively purchased robots for assembling and disassembling, while the transportation machinery industry bought a significant amount of robots for welding and soldering. In particular, welding robots consisted of 46.3 percent of the total shipments to the transportation machinery industry in years 1978-82, whereas assembling robots consisted of 7.1 percent. In contrast, welding robots consisted of 3.0 percent of the total shipments to the electric machine industry, whereas assembling robots consisted of 76.2 percent during the same period. Second, within-industry patterns of expenditure shares are fairly stable.[6] These points suggest that industries engage in different tasks for which they demand robots. Therefore, the initial composition of applications and the different unit-value evolution afterward had differential effects on robot purchases across industries. We formalize this idea in the following sections.

[6]The only exception is the decrease in assembling robots and the increase of other robots shipped for the electric machinery industry. The reason is due to the classification. In the electric machine industry, the assembling process was integrated with the measuring and testing process and thus these robots became classified as other robots.

Figure 2.3: Application Trends

(a) Expenditure Shares by Industries (b) Unit-Value Trends by Applications



Note: Authors' calculation based on JARA data. The left panel shows the application-expenditure shares for the three industry aggregates: electric machine (Elec. mach.), transportation machine (Transp. mach.), and aggregated other domestic industries (Others). The right panel shows the aggregated unit-value trends for each application. The y-axis is scaled by the natural logarithm. The application list is discussed in the main text and shown in Table 2.1.

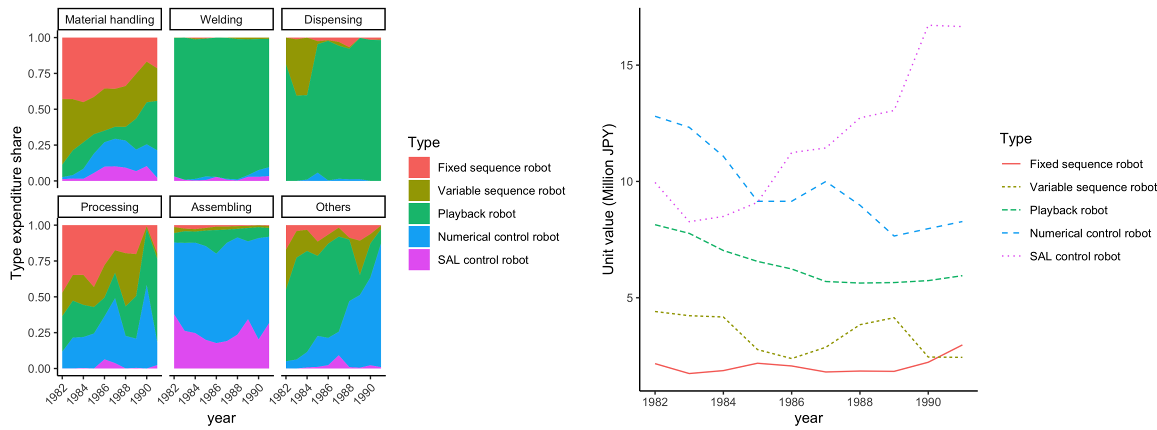
To see how the unit values evolved, Figure 2.3b shows the trends of the unit values for each application, aggregated across industries. The trends vary across applications. For example, robots for welding and soldering show the stable decline in the unit value for a broad set of industries. This suggests that the production technology for welding-soldering robots grew consistently over the sample period. More importantly, combined with the fact found in Panel (a), the decrease of the unit value of welding-soldering robots was *differentially* enjoyed by the transportation machinery industry, since the industry uses the welding-soldering robot intensively. These observations suggest that the heterogeneous reduction of robot-application price generates differential impacts across industries because of the heterogeneous composition of applications across industries.

The rapid drop of the unit price of welding robot is the key for identification and it is worth while to document the potential reasons why the welding robots became cheaper. The majority of welding robots is classified as the playback robot, a robot that repeats the same sequence of motions in all its operations, as a robot type. Figure 2.4a tabulates the relationships between the robot type and the robot application between 1982-1991 and the table shows that almost all of the welding robot consists of the playback robot.[7] Figure

[7]The type-application cross-tabulation tables are only available between 1982 and 1991. Further details of the JARA dataset is discussed in Appendix B.3.

Figure 2.4: Robot Type Trends, 1982-1991

(a) Expenditure Shares by Robot Type (b) Unit-Value Trends by Robot Type



Note: Authors' calculation based on JARA data. The left panel shows the type-expenditure shares for the five robot type aggregates: fixed sequence robot, variable sequence robot, playback robot, numerical control robot, intelligent robot. The right panel shows the aggregated unit-value trends for each robot type.

2.4b shows evolution of the unit price by robot types and we see that the unit price of the playback robot monotonically decreased. This monotonic price decline of the playback robot makes sharp contrast to the price trends of other robot types and reduced the price of welding robots. As a caveat we note that the price trends presented here do not adjust for the quality of robot and the price increase of intelligence robot may well reflect the quality improvement. We will discuss the issues related to quality adjustment in Section 2.5.

There are technological reasons behind the robot price decline concentrated in the welding robot or the playback robot. We reviewed the articles of Nekkei Telecom data base, a portal website collecting articles whose sources include domestic and international newspapers and magazines, between 1975 and 1985 and found two important technological developments during the period: the adoption of numerical control technology and the substitution of hydraulic actuator by electronic motor actuator [Nikkei, 1982, 1984]. These two developments presumably made the welding robot (or the playback robot) cheaper and gave Japanese manufacturers cost advantage.

In Section B.3.3, we examine the unit-value trends for each industry to find a relatively small variation across destination industries within an application. This suggests that the robot prices are not solely driven by demand shocks of particular industries, but by the technology for producing robots. Section B.3.3 further shows detailed descriptive statistics of JARA data and discuss the pre-trend before 1978, the distribution by robot types instead of applications, and some raw correlation between prices and quantities for each application.

In this section, we discuss the efficiency adjustment. This is motivated to address another interpretation of our primary findings in Sections 2.3 and 2.4. Note that our sample covers a very long period, 1978-2017. Therefore, it is likely that the unit efficiency of robots grew over that period, as we discussed in Section **??**. This may cause time series-variation based bias, as follows. Suppose the unit efficiency in later robots is higher than in earlier ones. This may have further set of tasks displaced by robots from labor due to relative cost efficiency [Acemoglu and Restrepo, 2018c]. In the later years, observed robot adoption units are smaller (even when efficiency-unit robot units grew), while at the same time, labor is displaced and employed less. This generates a *spurious* positive correlation between robots and employment that is not caused by gross complements, but by (gross) substitution and unit-efficiency growth.

To alleviate this concern, we estimate robot unit efficiency. Following and extending Khandelwal et al. [2013], we consider the following CES aggregation function for robots:

$$R_{it} = \left(\sum_{a} \widetilde{\iota_{ai}} \left(\lambda_{ait} R_{ait} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where R_{ait} is the robot stock of application a in industry i in year t, $\widetilde{\iota_{ai}}$ is the expenditure share parameter reflecting the applicability of application a in each industry i, λ_{ait} is unobserved quality, and σ>0 is the elasticity of demand substitution. The cost-minimizing demand for robot application a is then solved as

$$R_{ait} = \left(\widetilde{\iota_{ai}}\right)^{\sigma} \lambda_{ait}^{\sigma-1} r_{at}^{-\sigma} r_{it}^{\sigma-1} R_{it},$$

where r_{at} is the robot price of application a in year t, and $r_{it} = \prod_{a} \left(r_{at}\right)^{\widetilde{\iota_{ai}}}$. Taking logs, we obtain

$$\ln R_{ait} = -\sigma \ln r_{at} + \alpha_{ai} + \alpha_{it} + \eta_{ait},$$

where $\alpha_{ai} = \sigma \ln\left(\widetilde{\iota_{ai}}\right)$, $\alpha_{it} = \ln\left(C_{it}^{\sigma-1} R_{it}\right)$ and $\eta_{ait} = (\sigma - 1) \ln\left(\lambda_{ait}\right)$. Regressing $\ln R_{ait}$ on $\ln r_{at}$ and fixed effects α_{ai} and α_{it} gives $\widehat{\sigma}$ and the residual $\widehat{\eta}_{ait}$. Thus, we obtain the efficiency estimate $\widehat{\lambda_{ait}} = \exp\left(\widehat{\eta}_{ait}/(\widehat{\sigma} - 1)\right)$.

Several discussions follow in terms of our model choice. First, we model quality as the application-augmenting shocks. We view this is as a natural interpretation in the case of robotics, because robot quality may be conceptualized by the speed of task performance relative to that of old types of robots or human hands. Second, our extended specification (2.2.3) nests the Cobb-Douglas case. Namely, with $\sigma \to 1$ and $\lambda_{ait} = 1$, we revert to our initial specification $R_{it} = \prod_a r_{ait}^{\widetilde{\iota_{ai}}}$, with $\widetilde{\iota_{ai}} = \iota_{ai}$. Third, relative to Khandelwal et al. [2013], we have an additional expenditure share term ι_{ai}, because we have a clear pattern of applicability for each industry. Finally, compared to the IFR's quality adjustment, our treatment of the efficiency estimation is more systematic and based on a standard demand theory. Recall that Graetz and Michaels [2018] also reports quality-adjusted prices. In their data source, quality adjustment is not backed up based on a demand function. Instead, the method is called a "production-cost mark-up" method, which is "subjective but with a certain amount of knowledge through experience" [IFR, 2006].

With the unit efficiency estimate, we augment the application-level robot quantity measurement from R_{ait} to $\widehat{\lambda_{at}} R_{ait}$ and construct the industrial robot stocks. Using such a measure of efficiency-augmented robot stocks, we rerun the industry-level regression (2.2). Table 2.2 reports the 2SLS result. The column structures remain the same as in Table 2.5. The first-stage IV F-statistic is somewhat weaker, especially in the specification without control variables.[8] The estimated coefficients are similar with and without quality adjustments of robots after conditioning on the control variables. In our preferred specification of column 4, we still find a significant positive effect of robot adoption on employment. Therefore, we conclude that our results are robust to the efficiency adjustment.

2.2.4 Other Data

For the employment-side variables, we take the Employment Status Survey (ESS) administered by the Ministry of Internal Affairs and Communications (MIC). The ESS has

[8]The first-stage regression results are reported in Section B.8.2. The reason for the weaker first stage in column 1 of Table 2.2 than column 1 of Table 2.5 is mechanical. The quality adjustment factor λ_{ait} is estimated by the residual of the regression of lnR_{ait} on $\ln r_{at}$ and fixed effects, α_{ai} and α_{it}. By this construction, the adjustment factor $\widehat{\ln \lambda_{ait}}$ is orthogonal to the application price $\ln r_{at}$. The price aggregate, which is the weighted average of $\ln r_{at}$, is thus orthogonal to the quality adjustment factor. Therefore, using the log of the quality-adjusted robot as the dependent variable in the first stage is adding a factor orthogonal to the instrument variable to the dependent variable and reduces the first-stage F-Statistics. Conditioning on the control variables mitigates this mechanical relationship.

Table 2.2: Industry-level, 2SLS, Efficiency-adjusted Quantity

	Dependent variable:			
	$\ln(L_{it})$			
	(1)	(2)	(3)	(4)
$\ln(\widetilde{R}_{it})$	2.587	0.297^{***}	0.219^{*}	0.378^{**}
	(2.739)	(0.099)	(0.121)	(0.183)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	0.855	14.131	8.195	5.860
Observations	104	104	104	104
R^2	−0.440	0.981	0.986	0.980

*p<0.1; **p<0.05; ***p<0.01

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log efficiency-adjusted robot stock measure and log employment across industries and years, with the instrument of log efficiency-adjusted robot cost measure. The employment measure includes the employment of robot-producing plants. Efficiency adjustment is performed by the method described in the main text. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

conducted the survey as of October 1979 and in every five years that end with digit 2 or 7 since 1982. It samples roughly one million persons who live in Japan and are age 15 or above, which is roughly one percent of this population in Japan.

We obtain the regional and industrial employment and population variables for each demographic group. In particular, the ESS asks for basic demographic information, such as physical address, age, gender, education attainment, and employment status. For workers, we also obtain the industry classification of the workplace, hours worked, and income. For details of these variables, see Section B.6.

Our main datasets are made from the JARA and the ESS. We relate robot adoption taken from the JARA to employment and other labor-market variables from the ESS. In addition to these, we supplement the main dataset with the Census of Manufacture (CoM), the Basic Survey on Overseas Business Activities (BSOBA), and the Japan Industrial Database (JIP). These are used mainly to construct control variables. The details are provided in Section 2.3.3.

The CoM, which is conducted by Japan's Ministry of Economy, Trade and Industry (METI), is an annual survey of manufacturing establishments in Japan. Depending on the year, it takes the universe of establishments. The strength of the dataset is the fine industry coding employed. The ESS surveys the 4-digit Japan Standard Industry Classification (JSIC) for each establishment. Furthermore, the survey asks each respondent for their major products and product codes. Thus, for each establishment, we can observe the intensity of production for each product. This allows us to obtain a fairly accurate measure of employment in robot production. We report some of the results with the employment variable net of such robot-producing workers, as it helps clarify the mechanism. We will come back to this issue when we discuss the regression results. See Section B.7.1 for details.

The BSOBA, which is also administered by the METI, is an annual survey of the universe of Japanese multinational enterprises (MNEs). For all MNEs, all children and grandchildren subsidiaries must be reported. For each of these headquarters and subsidiaries, information on basic variables, including financial variables from balance sheets, are recorded. This dataset enables us to measure the offshore sales variables for each industry and headquarters location. Since offshoring was a concurrent phenomenon that changed the labor demand in Japan (see, for example, Hummels et al., 2014), we control it with variables constructed from the BSOBA. Section B.7.2 provides the details.

The JIP, which is administered by Japan's Research Institute of Economy, Trade and Industry (RIETI), releases long-run industrial data for the Japanese economy starting from 1970, assembled from several sources of administrative data [Fukao et al., 2008]. The industrial data contain KLEMS variables and variables regarding trade. We use the gross export and import variables to further control the labor-market impacts of offshoring, trade, and technological changes. Detailed discussions about constructing the variables based on the JIP are relegated to Section B.7.3.

2.3 Industry-level Analysis

We first analyze the robot impacts at the industry level. By so doing, we discuss the effect with minimal data transformation and explore a proper empirical specification. We then move on to a region-level analysis in the next section, which aims to identify regional implications and provide better comparison to the literature. We find qualitatively similar results between these two approaches.

The similar findings are appealing, given the recent development of econometric theories. For instance, Borusyak et al. [2018] discuss the identification assumptions behind the shift-share instrumental variables (SSIVs) based on exogeneity of the "shift" component and show that the identification variation is at industry level (and time level). Adao et al. [2019b] shows that in the presence of cross-regional correlations, the standard SSIV estimators over-reject the null of no effects. Our industrial results indicate that our findings are robust to these concerns, because our findings are robust to the use of industrial variations.

2.3.1 Robot Aggregation

In this section, we describe the method for constructing the cost indicators for each industry. We will employ the cost indicator as an exogenous shifter of the robot introduction to each industry.

We begin by constructing stock measures of robots. Recall that the JARA data have only the flow shipment variable, whereas we are concerned about the stock of robots and the impact on employment. To construct the robot stock, we follow the immediate withdrawal method, which is also applied by the IFR [IFR, 2018]. In particular, we aggregate the robot of a particular application for 12 years to obtain the robotś capital stock of the robot. The assumption is that the robot begins to perform the capital service immediately upon purchase and depreciates completely in 12 years. The choice of 12 years is within the range of conventions. We discuss the robustness to other choices of stock measure construction in the robustness section below.

Suppose each industry i produces with labor and robots as primary factors. We assume that the robots are aggregated across applications a and perform service for production in each industry. We then consider the substitutability between labor and robots. Motivated by the observation that the expenditure shares are roughly constant across applications for major industries, such as the electric machine and transportation machine industries (Figure B.3), the robot aggregation is assumed to be Cobb-Douglas:

$$R_{it} = \prod_a (R_{ait})^{\iota_{ai}},$$

where R denotes the quantity of the robots, i indicates industry, t indicates the year, and a indicates the robot application. The aggregation weight ι_{ai} is assumed to be constant over time. Furthermore, given that the variation of trends of unit values are similar across industries within applications (Figure B.4), we assume that the robot market for each application is competitive and sets the same price r_{at} for all buyer industries i. Section B.8.1 discusses the case without this assumption. Then the price indicator for robots in industry i is given by

$$r_{it} = \prod_a (r_{at})^{\iota_{ai}}.$$

To measure r_{at} in the data, we simply sum the values and quantities across all industries to calculate the average unit value for each application and year. Formally,

$$r_{at} = \frac{\sum_i v^A_{ait}}{\sum_i R_{ait}}, \tag{2.1}$$

where v^A_{ait} is the sales value of application a to industry i in year t. Furthermore, given the idea that the initial expenditure share is the exogenous source of variation of price changes, we obtain ι_{ai} by the expenditure share on application a for each industry i in year 1982, our earliest year of analysis. Formally, $\iota_{ai} = v^A_{ai,1982}/\sum_a v^A_{ai,1982}$.

2.3.2 Effect of Robot Adoption on Employment

We consider the following regression specification:

$$\ln(L_{it}) = \alpha_i + \alpha_t + \beta \ln(R_{it}) + X'_{it}\gamma + \varepsilon_{it}, \tag{2.2}$$

where L_{it} is employment of industry i in year t, R_{it} is the stock of robots in industry i in year t and X_{it} is the vector of control variables explained in Subsection 2.3.3. Our coefficient of interest is β, the gross elasticity of robot adoption on the employment at industry level. To capture the robot production shock that is not correlated to error term ε_{it}, we instrument

$\ln(R_{it})$ with $\ln(r_{it})$, a measure of exogenous change in robot price for each industry in each year. The coefficient of interest does not have a direct structural interpretation, but it makes possible a high-level interpretation of the employment impacts of robots.

The first-stage and reduced-form equations of the two-stage least square (2SLS) estimation of the model (2.2) are:

$$\ln(R_{it}) = \alpha_i^{FS} + \alpha_t^{FS} + \beta^{FS} \ln(r_{it}) + X_{it}\gamma^{FS} + \varepsilon_{it}^{FS}, \tag{2.3}$$

$$\ln(L_{it}) = \alpha_i^{RF} + \alpha_t^{RF} + \beta^{RF} \ln(r_{it}) + X_{it}\gamma^{RF} + \varepsilon_{it}^{RF}, \tag{2.4}$$

respectively, where X_{it}'s are control variables described in Subsection 2.3.3. If the robot producers' technological innovation drove the cost reduction, then our cost measures and robot purchase in quantity would be correlated negatively; thus β^{FS} is expected to be negative in (2.3). The reduced form (2.4), in contrast, expresses the relationship between the robot price and employment, conditional on fixed effects and control variables.

We assume that the variation in r_{it} is exogenous to demand industries. Note that the i-level variation comes from the initial share $\iota_{ai} = s_{ait_0}$ across industries. Our identification assumption is thus that the initial applications shares are uncorrelated with unobserved labor-market factors after conditioning on fixed effects and control variables. In other words, the endogeneity of r_{at} does not give industry i-level variation [Goldsmith-Pinkham et al., 2018], which alleviates the concern about using application prices as shifters of the instrument. We also conduct a robustness check using several price measures that are less endogenous to robot adopters, such as leave-one-out robot prices and export robot prices in the following section.

Interpreting r_{it} as an exogenous price indicator of robots, the first-stage and reduced-form regressions also have an interpretation relevant to the conditional gross own-price elasticity and the conditional gross cross-price elasticity. In this sense, one may view the first-stage (FS) and reduced-form (RF) regressions as giving "low-level" interpretations. Note that standard factor demand theory does not make a particular restriction on these elasticities, because they are a mix of the substitution effect and scale effect [Cahuc et al., 2014].

Note that all the elasticity estimates in equations (2.2), (2.3) and (2.4) are gross concepts. Namely, they contain both the displacement effect, keeping the scale of output constant, and the scale effect, due to the productivity gain made possible by robot adoption. As we show in Section B.2, the displacement effect is mechanically negative, while the productivity effect is positive if robot adoption improves the overall productivity and the output demand is not perfectly inelastic. Although it would be informative if one could separate the gross estimates into each mechanism, there are few methods for such a purpose to date.[9] For this purpose, we would need to have an exogenous determinant of production scale apart from the exogenous source of robot price variation. For instance, a standard SSIV method by Acemoglu and Restrepo [2020] also estimates the gross effect of robots. The gross effect, however, is still informative for policy makers that are concerned about the total labor market effects of robot adoption. Therefore, we stick to our estimation method and leave addressing this problem as a future work.

2.3.3 Control Variables

We discuss potential confounders and how we deal with them. First and most importantly, recall that we control for industry-specific time-fixed characteristics and aggregate shocks common across industries. As further possible industry- and time-varying explanations for industrial employment changes, we consider demographics, globalization, and technology, as follows.

First, demographic dynamics change the labor-supply conditions that may correlate with robot adoption incentives for producers and employment at the same time. In particular, in Japan, labor shortages due to aging and low population growth rates have been an acute problem that partly pushes firms to adopt robots [Acemoglu and Restrepo, 2018b]. This may bias the 2SLS estimates downward. To alleviate this concern, we control for detailed demographic variables, including education (high-school/4-year university graduate shares), sex ratio (female share), and the age distribution (age under 35/over 50 shares). All of these variables are taken from the ESS.

Second, concurrent globalization alters both the labor demand and robot adoptions. Given the complexity of modern manufacturing production, it is likely that easier access to foreign markets of both outputs and inputs may alter the incentive to adopt new technology and workers (see, for instance, Fort et al., 2018). Depending on the substitutability (e.g.,

[9] Dekle [2020] sets up a general equilibrium model and suggests the decomposition of the total effect into the displacement effect, the scale effect, and the general equilibrium income effect.

foreign cheap workers replaces the necessity of adopting robots and labor domestically) or complementarity (e.g., high availability of foreign cheap energies enhance the efficiency of robots), the bias may go upward or downward. To alleviate this concern, we control for offshoring, import competitions, and outsourcing. In particular, we take the total import values of each sector from the JIP database. This variable controls for the role of import competition (e.g., Autor et al., 2013) and outsourcing (e.g., Hummels et al., 2014). From the BSOBA data, we take the total gross sales value for each industry. This variable controls the changes in labor demand due to global sourcing (e.g., Antras et al., 2017) or export platforms (e.g., Arkolakis et al., 2018).

Third and finally, technological changes other than robots, such as increases in ICT adoptions [Autor et al., 2003], may also alter the labor demand and robot adoptions simultaneously. In fact, robots need to be programmed rather than human-operated; as such, robots and ICT adoptions are complementary. Since our interest is the direct impact of robot-based automation on employment, we control for other technological progress. We do so by using intangible capital stock values from the JIP database. In fact, all explanatory variables (i.e., robot, globalization, and technology) are positively correlated. These variables are explained in detail in Section B.7.3.

2.3.4 Main Results

With the industry-level aggregate robot service measure and cost indicators, we proceed to the analysis of the effect of robots on employment at the industry level. Several past studies focus on region-level analysis. We conduct a industry-level analysis for two reasons. First, our result is not driven solely by the increased observations by constructing the local-level outcomes by allocating the industry-level variables with fewer observations [Adao et al., 2019b, Borusyak et al., 2018]. Second, we devote the section to discuss the preferred specifications in several dimensions. We then stick to the preferred specification in the CZ-level analysis. Other specification results can be found in the Appendix.

Table 2.3 shows the first-stage result of regression (2.3). Four columns show results with alternative sets of control variables. Column 1 controls only industry and year fixed effects, column 2 adds demographic controls, column 3 globalization, and column 4 technology. The sample size is 104, interacting 13 industries and eight 5-year periods (quinquennially from 1982 to 2017). We report the first-stage IV F-statistics, indicating that those in the

Table 2.3: Industry-level, First Stage

	Dependent variable:			
	$\ln(R_{it})$			
	(1)	(2)	(3)	(4)
$\ln(r^Z_{it})$	-1.413^{***}	-1.852^{***}	-1.322^{***}	-1.542^{***}
	(0.469)	(0.557)	(0.400)	(0.377)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	9.1	11.047	10.912	16.703
Observations	104	104	104	104
R^2	0.971	0.979	0.984	0.986

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot cost measure and log robot stock measure across industries and years. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

specifications from columns 2 to 4 exceed 10, a conventional value for checking weak instruments. In all specifications, we find consistently negative and significant estimates. This result is not surprising, given that the price reductions made by different application compositions drive increases in demand. Our preferred specification is column 4, because it includes all potential confounders discussed in Section 2.3.3 and has the highest first stage F-statistics among the four specifications. According to column 4, controlling for fixed effects and other factors, a one-percent decline in robot prices drive a 1.54 percent increase in robot adoption.

Table 2.4 shows the reduced-form result of regression (2.4). Each of the four columns controls fora different set of control variables, as in Table 2.3. Again, in all specifications,

Table 2.4: Industry-level, Reduced Form

	Dependent variable:			
	$\ln(L_{it})$			
	(1)	(2)	(3)	(4)
$\ln(r_{it}^{Z})$	-0.853^{***}	-0.465^{***}	-0.272^{*}	-0.437^{**}
	(0.130)	(0.144)	(0.151)	(0.171)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	9.1	11.047	10.912	16.703
Observations	104	104	104	104
R^2	0.975	0.984	0.985	0.987

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot cost measure and log employment measure across industries and years. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

we find consistently negative and significant estimates, though specification in column 3 shows statistical significance at only the 10 percent level. To clarify, the negative coefficient implies that the fall of the effective robot price induced employment growth, implying that robots and employment are *gross complements*. Our preferred estimate from column 4 indicates that a one percent decrease in robot price *increases* employment by 0.44 percent.

Table 2.5 shows the 2SLS result of regression (2.2). Four columns control different sets of control variables. Given the results in Tables 2.3 and 2.4, it is not surprising that we find *positive* and significant point estimates in the four specifications. This confirms that robots and employment are gross complementary. Again, our preferred specification (column 4) indicates that a one-percent increase in robot adoption, caused by a drop in the robot prices,

Table 2.5: Industry-level, 2SLS

	Dependent variable:			
	$\ln(L_{it})$			
	(1)	(2)	(3)	(4)
$\ln(R_{it})$	0.716***	0.251***	0.206*	0.283**
	(0.154)	(0.071)	(0.105)	(0.108)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	9.1	11.047	10.912	16.703
Observations	104	104	104	104
R^2	0.941	0.986	0.988	0.988

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

increases employment by 0.28 percent.

Validating Identification Since the novelty of this paper is the new identification method based on robot prices, we argue that our choice of robot price IV is appropriate. In this section, we discuss two robustness exercises: other price measures and dropping big robot buyers. The first robustness check partly addresses the concern about the endogenous initial industry share of robot applications, as we will discuss.

First, we consider other price measures that do not depend on the own industries' robot prices. Although in the previous section we explained that the source of identification comes from initial application share variations for each industry, our IV does depend on the actual aggregate trends of prices of robots. If the initial application share is endogenous, however, the endogeneity of actual price trends may add bias to our estimates [Goldsmith-Pinkham et al., 2018]. To alleviate this concern, we consider the following two alternative price measures. The first measure leaves out the own industry from the calculation of robot prices. Namely, we construct the following robot price:

$$r_{ait}^{LOO} = \frac{\sum_{i' \neq i} v_{ai't}^{A}}{\sum_{i' \neq i} R_{ai't}}. \tag{2.5}$$

Then we construct the industrial price index based on equation (2.3.1). This measure is supposed to take the variation that is external to each industry.

Since we have 12 manufacturing industries, however, the leave-one-out price measure may contain feedback bias (dropping one industry in turn affects the price of own industry). To address this further concern, we consider the second measure using export price as follows:

$$r_{ait}^{EXP} = \frac{v_{a,exp,t}^{A}}{R_{a,exp,t}}. \tag{2.6}$$

Then we aggregate to the industrial price index by equation (2.3.1). This measure is supposed to take the variation that is further external to each domestic industry, while the concern is that the relevance of IV may be attenuated, because it takes different prices than the actual price each domestic industry faces. We report the results based on these prices in Table 2.6. Column 1 shows the baseline estimates (thus the same as Table 2.5, column 4). Columns 2 and 3 are based on leave-one-out prices (2.5) and export prices (2.6), respectively. We find that the estimates are robust to these price measures. Therefore, the endogeneity of the actual price measure poses minimal threat to our identification strategy. Furthermore, the robust findings also support our identification idea, the exogenous initial application shares. This is because of the discussion in Goldsmith-Pinkham et al. [2018]–if initial application shares are exogenous, the choice of price measures merely changes the weighting of the Generalized Method of Moments (GMM), affecting the precision of the estimates but not the consistency. This prediction is consistent with our robust findings in Table 2.6.

Second, we drop large robot buyer industries. The endogeneity of robot prices may be most severe to big robot buyers, such as the transportation industry and electrics, because

Table 2.6: Industry-level, 2SLS, Different Price Measurement

	Dependent variable:		
	$\ln(L_{it})$		
	(1)	(2)	(3)
$\ln(R_{it})$	0.283^{**}	0.303^{***}	0.318^{**}
	(0.108)	(0.101)	(0.139)
Industry FE	✓	✓	✓
Year FE	✓	✓	✓
Demographic Controls	✓	✓	✓
Globalization Controls	✓	✓	✓
Technology Controls	✓	✓	✓
Price Measurement	Industry Aggregate	Leave-one-out	Export
Observations	104	104	104
R^2	0.988	0.987	0.987

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects, demography controls, the logarithm import values from JIP database and logarithm offshoring value added from SOBA, logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Column 1 shows the result with the instrumenal variable of industry-aggregate robot prices (benchmark). Column 2 shows the result with the instrumenal variable of leave-one-out robot prices. Column 3 shows the result with both the instrumenal variable of export robot prices. *p<0.1; **p<0.05; ***p<0.01.

they may have market power and affect robot prices (recall Figure B.3). If their output demand surges, it may enhance both robot and labor demands. The increase in robot demand decreases prices due to market power. In this case, there is a spurious negative correlation between robot price and labor. While this concern is already addressed by using an arguably exogenous price series in the above analysis, to address this concern in a more direct way, we drop the two large industries, transportation and electrics. The resulting sample is made from smaller industries, whose robot prices are thus less endogenous to robot demand-side shocks. Table 2.7 shows the 2SLS estimates of these regressions. In

Table 2.7: Industry-level, 2SLS, Dropping Major Industries

	Dependent variable:			
	$\ln(L_{it})$			
	(1)	(2)	(3)	(4)
$\ln(R_{it})$	0.283^{**}	0.283^{**}	0.579^{**}	0.351^{**}
	(0.108)	(0.118)	(0.270)	(0.146)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓
Industries	All	Net Elec.	Net Transp.	Net E&T
Observations	104	96	96	88
R^2	0.988	0.989	0.979	0.988

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects, demography controls, the logarithm import values from JIP database and logarithm offshoring value added from SOBA, logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Column 1 shows the result with the full sample (benchmark). Column 2 and 3 show the results with the electronic machine industry and transportation machine industry dropped, respectively. Column 4 shows the result with both the electronic and transporation machine inustry dropped together. *p<0.1; **p<0.05; ***p<0.01.

column 1, we show the baseline estimate. Columns 2-4 show the dropping of each of the large buyers: Column 2 drops electrics, column 3 drops transportation, and column 4 drops both. We find that the positive results are consistent across columns. The finding suggests that the concern about endogeneity due to market powers of demand industries is minimal.

2.3.5 Heterogeneity

How does our aggregate finding mask heterogeneous impacts to different groups of workers? To see this, we consider our main empirical specification (2.2) with several subgroups of workers as outcome variables. Table 2.8 shows the results. In column 1, we show our baseline estimate (the same as Table 2.5, column 4). We then consider the impacts of robots on the employment of high-school graduates (column 2), four-year university graduates (column 3), female workers (column 4), young workers (less than or equal to 35 years old, column 5), and elder workers (more than or equal to 50 years old, column 6). Perhaps surprisingly, our finding of the positive labor-market implications of robots is robust to all of these subgroups. Moreover, the sizes of the estimated coefficients do not differ substantially across demographic groups.

Table 2.8: Industry-level, 2SLS, Effects on Subgroups

	Dependent variable:					
	$\ln(L_{it})$	$\ln(L_{it}^{HS})$	$\ln(L_{it}^{CG})$	$\ln(L_{it}^{Fem})$	$\ln(L_{it}^{U35})$	$\ln(L_{it}^{O50})$
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(R_{it})$	0.283**	0.279**	0.303**	0.278**	0.328***	0.398***
	(0.108)	(0.107)	(0.117)	(0.115)	(0.114)	(0.125)
Industry FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓	✓	✓
Group of Worker	All	High School	4-year Univ.	Female	Age $\leq$ 35	Age $\geq$ 50
Observations	104	104	104	104	104	104
R^2	0.988	0.987	0.989	0.994	0.988	0.986

Notes: Authors' calculation based on JARA, ESS, CoM, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects, demography controls, the logarithm import values from JIP database and logarithm offshoring value added from SOBA, logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Column 1 shows the result with the outcome variable of all workers (benchmark). Column 2 and 3 show the results with the outcome variables of high-school graduates and 4-year university graduates (and more), respectively. Column 4 shows the result with the outcome variable of female workers. Columns 5 and 6 show the results with the outcome variable of workers with age equal or lower than 35, and with age higher than 50, respectively. *p<0.1; **p<0.05; ***p<0.01.

2.3.6 Further Robustness Checks

We briefly discuss additional robustness checks. In particular, we consider (i) netting out of robot-producing workers, (ii) industry-specific robot prices, and (iii) alternative stock measures. All of these results suggest the validity of our main definitions above.

First, we net out robot-producing workers from employment. Although our primary interest is the robots' gross impact on employment, it may mask a heterogeneity of impacts. As Acemoglu and Restrepo [2018c] clarifies, robot innovation is likely to increase the set of tasks that are input in production. One such example is clearly the robot-producing workers themselves. By measuring the number of such workers, we may gain more insights from our positive estimates found in Section 2.3.4. Section B.7.1 discusses in detail how to generate robot-producing worker employment and regression results when robot producers are netted out of the outcome variable.

Second, we consider industry-specific robot prices r_{ait}; in the main specification (2.1), the robot prices are aggregated and not industry-specific. We also consider industry-specific r_{ait}, calculated by v^A_{ait}/R_{ait}. The use of this variable can be viewed as a robustness check that complements the main specification; the industry-specific prices are likely to reflect the actual prices to each industry. We do not, however, consider it as a main specification. Recall the above discussion regarding the endogeneity concern. The industry-specific-price specification is more vulnerable to this concern, because it varies across industries by definition. In fact, as shown in Section B.8.1, it is less robust to the inclusion of control variables, suggesting a negative bias due to negative correlations between industry-specific prices and the unobserved component of employment.

Third and finally, we check several measures of robot stocks. Recall that our data come with purchase of robots, which is a flow quantity and monetary values. Robots are durable investment goods, however, and thus the flow values may understate the actual robot stock active in production. To construct the stock variable, our main specifications follow the literature and IFR suggestion and aggregate the 12-year flows of robots, which we call the immediate withdrawal method (IWM). As several authors argue, it is also natural to follow the standard perpetual inventory method (PIM) that is also used in capital formation in National Accounts [Artuc et al., 2020, Graetz and Michaels, 2018]. We thus create the robot stock variables based on different-year IWM and PIM with some depreciation rates. When we use alternative measures, we find that the variation in the definition of stock variables has minimal impacts on the regression results. We discuss further details and empirical results in Section B.5.

Note that the invariance of results with respect to alternative stock measures provides further support for our empirical strategy. The invariance suggests that industry-level robot adoption variation is more significant than time-level variations. Recall that, in our empirical specification, the main identifying variation does not come from the one in application-year prices r_{at}, but instead from that of the initial application shares ι_{ait_0}. Since we take a stand that the latter variable is more exogenous than the former, the invariance to stock measures indicates our method has a minimal concern regarding endogeneity. Section B.5 discusses the detailed construction of variables and regression results.

2.4 Region-level Analysis

Our industry-level analysis reveals that at the national level, robots and employment are gross complements in our context. In this section, we consider converting the data and regression analysis to the region level. We do so for three purposes: (i) to discuss robots' impacts on subnational and local economies, (ii) to make better comparisons to the literature, and (iii) to delve into spillover effects.

In particular, as the definition of local labor markets, we employ Commuting Zones (CZs) developed by Adachi et al. [2020], who defined CZs following the method employed in, among others, Tolbert and Sizer [1996].[10] CZs made via this method have several strengths. First, they represent the local labor-market delineation better and contain more observations than administrative divisions, such as prefectures. Second and more importantly, they partition the overall national area in a mutually exclusive and exhaustive manner. This point is in contrast to city-level delineations, such as MSAs.[11] This point is especially relevant in our study, because the impacts of robots may not be constrained in urban areas, but also penetrate into rural ones. Rural areas may by overlooked by non-exhaustive regional delineations, such as MSAs.

Based on the industrial robot adoption and price measures and region definitions, we

[10]In particular, the hierarchical agglomerative clustering method with average linkages and dissimilarity measures based on bilateral flows of commuters and populations.

[11]A famous regional classification based on city-level density inhabited districts (DIDs) in Japan is proposed and popularized by Kanemoto and Tokuoka [2002].

employ the shift-share method to construct a CZ-level robot exposure measure [Acemoglu and Restrepo, 2020]:

$$\Delta R_{ct} = \sum_i l_{cit} \frac{\Delta R_{it}}{L_{it}},$$

where c is commuting zones. Following Acemoglu and Restrepo [2020], we call this variable robot exposures. To construct CZ-level IVs, we use $t_0 \equiv 1979$ as the base year and similarly generate the shift-share measures but based on price changes:

$$\Delta \ln (r_{ct}) = \sum_i l_{cit_0} \Delta \ln (r_{it}) .$$

With these region-level variables, we then study the following specification:

$$\Delta Y_{ct} = \alpha_c^{CZ} + \alpha_t^{CZ} + \beta^{CZ} \Delta R_{ct} + X_{ct} \gamma^{CZ} + \varepsilon_{ct}^{CZ}, \tag{2.7}$$

where ΔR_{ct} is instrumented by $\Delta \ln (r_{ct})$. Note that in the region-level specification, we measure changes in robot adoption and prices. We do so for the following two purposes. First, the fixed effect in specification (2.7) controls the differential growths trends in each location. This is more flexible than just controlling for level differences, and it is preferable when studying regional differences, given that the various regions experience different trends in labor-market characteristics (e.g., Diamond, 2016). Second, it makes the specification more comparable to the ones in the literature, such as Acemoglu and Restrepo [2020]. Note that our specification also allows differential trends across CZs, because we access many (in particular, seven time-first differences) time periods.

Table 2.9 shows the first-stage results. Each column shows the results with a different set of control variables: Column 1 controls only fixed effects, column 2 adds demographic controls, column 3 adds globalization, and column 4 adds technology. The results show a statistically significant and negative correlation between robot price changes and robot exposures, both with and without the covariates. The F test indicates that the use of our price measure passes the test of weak instruments. From now on, we focus on our preferred specification with full controls, the one in column 4.

Table 2.10 shows the 2SLS results of specification (2.7).[12] All columns report the preferred specification with the full set of our control variables. The standard deviation

[12]Since the reduced-form results are redundant, they are not reported but are available upon request.

Table 2.9: CZ-level, First Stage

	Dependent variable:			
	ΔR_{ct}			
	(1)	(2)	(3)	(4)
Δr^{Z}_{ct}	-30.074^{***}	-30.928^{***}	-27.330^{***}	-21.179^{***}
	(1.864)	(1.846)	(1.763)	(1.424)
CZ FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-statistics	260.305	280.586	240.268	221.249
Observations	1,466	1,466	1,466	1,466
R^2	0.938	0.941	0.949	0.971

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share logarithm robot prices and shift-share measures of changes in robot stock per thousand workers. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. Column 1 controls the industry and year fixed effects. Column 2 controls the demographic variables: share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 controls the logarithm import values from JIP database and logarithm offshoring value added from SOBA as well as control variables in Column 2. Column 4 (baseline specification) controls logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database in addition to the controls in column 3. *p<0.1; **p<0.05; ***p<0.01.

of robot exposure measures is 5.25 at the raw level and 2.17 when residualized. For columns 1-6, the outcome variables are log manufacturing employment (column 1), log total employment (column 2, baseline estimate), log total population (column 3), (non-log) employment-to-population ratio (column 4, the main outcome variable of Acemoglu and Restrepo, 2020), log non-manufacturing employment (column 5), and log non-working population (column 6), respectively. First, as our primary interest, column 2 tells us that overall employment increased in CZs that were exposed to robots, confirming the findings in Section 2.3, even in the first difference-based specification (2.7) with two-way fixed effects. Perhaps surprisingly, column 1 shows that the impacts to manufacturing employment are even stronger and positive. This may suggest that the scale effects of robot adoption rather than the substitution effects that drive the labor-market impacts.

Table 2.10: CZ-level, 2SLS

	Dependent variable:					
	$\Delta \ln(L_{ct}^{MAN.})$	$\Delta \ln(L_{ct})$	$\Delta \ln(P_{ct})$	$\Delta \frac{L_{ct}}{P_{ct}}$	$\Delta \ln(L_{ct}^{SER.})$	$\Delta \ln(P_{ct}^{DEP})$
	(1)	(2)	(3)	(4)	(5)	(6)
ΔR_{ct}	4.658**	2.203**	2.003**	0.133	1.101	1.617
	(2.062)	(1.017)	(0.929)	(0.230)	(1.036)	(1.103)
CZ FE	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓	✓	✓
Observations	1,413	1,466	1,466	1,466	1,466	1,466
R^2	0.619	0.817	0.811	0.732	0.831	0.768

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are manufacturing employment, total employment (baseline), total population, (non-log) employment-to-population ratio, non-manufacturing employment (L_{ct}^{SER}), and non-working population (L_{ct}^{DEP}) in columns 1-6, respectively. *p<0.1; **p<0.05; ***p<0.01.

Turning to columns 3 and 4, we discuss the relationship with the estimates obtained in the literature and the mechanism. In column 4, we do not find a statistically significant effect on the employment-population ratio. This can be understood from column 3 (and column 2): The population positively responded to robot exposure. Thus, both employment (column 2) and population (column3) receive positive effects, cancelling the impacts to the ratio by increasing both the numerator and denominator. This finding, however, is in contrast to part of the literature. In fact, using their similar-country shift-share IV strategy, Acemoglu and Restrepo [2020] found -0.388 with the standard error of 0.091 for column (4).[13]

Note that our findings in columns 2-4 may be regarded in consistent with the findings in "local multiplier" literature [Moretti, 2010]: An increase in demand for a group of people positively spillovers to other group of people in a locality. Indeed, Moretti [2010] discusses the positive spillover of local manufacturing labor demand on service labor demand, such as local restaurants, and bars worker demands for manufacturing workers there. In contrast, our results indicate that the increase in labor demand in general even has an implication for increasing the local population in general, such as housekeepers for general employees.

To further compare our results with the literature and highlight the relevance of our price-based instruments, we also conduct an analysis based on a similar country. We choose Germany as the country of comparison and take the robot adoption trends from the IFR. As Table B.9 shows, the regression results are imprecise. This is not surprising, given the discussion in Section **??**: Even compared with a close innovator of robots, Germany, the trend of robot adoptions in Japan has been very unique. Therefore, the conventional identification method in the literature is not applicable in our context. In contrast, our cost-based identification method is robust to contexts.

Finally, columns 5 and 6 show suggestive evidence on spillover effects. Column 5 takes non-manufacturing employment (labeled as "L^{SER}" as abbreviation for service), defined as total employment (cf. column 2) net of manufacturing employment (cf. column 1) as the outcome variable. Column 6 takes the non-working population (labeled as "P^{DEP}" as abbreviation for dependent), defined as the total population (cf. column 4) net of employed population (cf. column 2), as the outcome variable. These results show insignificant effects on non-manufacturing employment or non-employed population and suggest the

[13] See their Table 7, panel A, column 3. They also report the effect on changes in log employment in the Appendix. The point coefficient is -1.656 with a standard error of 0.411.

following. First, we find little evidence of spillover effects to demand for these groups of people. Second, given that there are negative effects to demand for these groups, there is no evidence about reallocations within CZ across sectors. If this was the case, the reallocation happened across CZs.[14]

So far, our main outcome variables have been regarding headcounts of people, such as employment and population. Given that robot technology is characterized by the time-saving nature of routine tasks (e.g., spot welding), it is possible that the impacts on hours worked may be different from the headcount impacts. Furthermore, such hour effects may have implications for hourly wages, which reflects the hourly productivity of workers. We explore these dimensions in Table 2.11. In column 1, we show the baseline result (cf. Table 2.10, column 2), which takes log employee headcounts as the outcome variable. For columns 2 and 3, we take per-capita hours worked and hourly wages as the outcome variables, respectively. Column 2 reveals that the average hours worked decreased dramatically due to the adoption of robots. In contrast, and partly as a result of hour-reducing effects, column 3 shows that hourly wages increased due to robot adoption. These findings suggest that robots enable work-sharing or time-saving technological changes, which enhance the hour-unit productivity of employed workers.

Finally, we conduct the CZ-level analysis by subgroup of workers. We overview the takeaways, as follows. Robot-exposed regions increased the employment of both educated (more than four-year university) and non-educated (high-school graduates) workers, increased female workers more than male workers, and increased middle-aged workers (workers aged between 35 and 49) more than the other age groups of workers. Regarding wages and hours, robot-exposed regions increased wages and decreased hours worked of workers with all educational backgrounds. The impacts of robot adoption were homogeneous as found by Acemoglu and Restrepo [2020] but in the opposite direction; in Japan in 1978-2017, robot adoption improved working conditions of workers across the board while it was opposite in the US in 1992-2007. Detailed regression results are reported in Section B.9.

[14] Since the first and second implications contrast their predicted sign of the effect (positive spillover effects and negative reallocation effects), we acknowledge the possibility that both effects worked but canceled each other out.

Table 2.11: CZ-level, 2SLS

	Dependent variable:		
	$\Delta\ln(L_{ct})$	$\Delta\ln(h_{ct})$	$\Delta\ln(w_{ct})$
	(1)	(2)	(3)
ΔR_{ct}	2.213^{**}	-1.963^{***}	4.117^{***}
	(1.000)	(0.585)	(0.913)
CZ FE	✓	✓	✓
Year FE	✓	✓	✓
Demographic Controls	✓	✓	✓
Globalization Controls	✓	✓	✓
Technology Controls	✓	✓	✓
Variable	# Workers	Average Hours	Average Hourly Wage
Observations	1,453	1,453	1,453
R^2	0.818	0.827	0.948

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are total employment (baseline), average weekly hours, and average hourly wages in columns 1-3, respectively. *p<0.1; **p<0.05; ***p<0.01.

2.5 Discussion

As background for our research, we overview a standard identification approach using shift-share instruments, raw trends of robot stock units by country, and the description of robots and robot applications.

By showing cross-country raw trends, we not only convey the significance of our context as the users of robots, but also show why a new identification strategy is needed. Namely, since Japan has a very distinct trajectory in introducing robots for industrial production, it is hard to find similar countries of comparison that would be necessary to construct the shift-share-type instrumental variable prevailing in the literature.

We then move on to describe robots and robot applications. There are two reasons for providing this description. First, it gives readers an understanding of the technological context of our research. Second, by so doing, we introduce a core idea behind our novel identification strategy–an industrial variation in the intensity of different tasks, or applications of robots. Throughout the paper, we use the terms tasks and applications interchangeably.

2.5.1 Standard Identification Method–Shift-share Instrument

Although we do not employ the standard shift-share identification method in this paper, it is worth mentioning it to highlight the features of our method. When estimating the effect of robots on employment, one confronts a classical identification problem: Observed robot absorption is endogenous to employment. Consider a standard model that attempts to identify the impact of robot adoption in commuting zone (CZ) c in year t on the employment of the industry in the year:

$$\ln(L_{ct}) = \alpha_c + \alpha_t + \beta \ln(R_{ct}) + X'_{ct}\gamma + \varepsilon_{ct}, \tag{2.8}$$

where L_{ct} is the employment in CZ c in year t, R_{ct} is the stock of robots in CZ c in year t, and X_{ct} is the set of time-varying control variables in CZ c in year t.

The challenge of estimating this equation is the endogeneity of R_{ct}, as the unobserved product demand shock to the CZ included in ε_{ct} is correlated with R_{ct}. For example, the depreciation of JPY disproportionately increases the product demand in Toyota city, where factories of Toyota Motor Co. are located, which increases both the employment and robot stock of the area. A typical workaround to this problem is to construct a region-level shift-share instrumental variable (IV) Z_{ct}. Formally, the IV is defined as follows:

$$Z_{ct} \equiv \sum_i l_{cit_0} \frac{\Delta R_{it}^{\text{comparison countries}}}{L_{it}}, \tag{2.9}$$

where c is a region (e.g., commuting zone), t is year, i is industry, l_{cit_0} is base-year (t_0) industrial employment share in (c,t), $R_{it}^{\text{comparison countries}}$ is the operational robot stock in (i,t) of *comparison countries*, and L_{it} is total employment in (i,t). A researcher should use the stock value in comparison countries rather than the country of interest, because the observed robot stocks in the country of interest are subject to endogeneity problem; for example, the increase in the robot stock may be due to a demand boom in (i,t), which raises the industrial labor demand at the same time. In this case, the unobserved error term is positively correlated with the robot stock variable. By using values from comparison countries and assuming that the industrial demands in these countries are uncorrelated, the researcher can take an exogenous source of variation, such as robotics technological growth.

It is not obvious which countries of comparison should be selected. An intuitive method is to take countries that are similar on some measure. Acemoglu and Restrepo [2020], for example, takes Denmark, Finland, France, Italy, and Sweden as comparison countries for the US, based on the similarity of robot absorption trends, while Dauth et al. [2018] takes Spain, France, Italy, the United Kingdom, Finland, Norway, and Sweden for Germany. To see potential comparison countries, the next subsection reviews the trends of robot stock units in several countries. This will demonstrate the difficulty in finding natural comparison countries in our research context.

2.6 Conclusion

How does automation affect labor demand? We study this question by focusing on a context with a long tradition in robot adoption, Japan 1978-2017. We develop a cost-based identification method for studying the impact of robots on employment, given the difficulty of applying a conventional identification method, and a newly digitized data of robot adoption. The data are characterized by three novelties: (i) a long panel covering 1978-2017 that suits the study of our context, (ii) adoption units and total values that allow systematic calculation of robot unit costs, and (iii) disaggregation of these by robot applications. Armed with these unique features, our identification hinges on industry-level application share variations during the initial period of robot adoption. We conduct a industry-level analysis and a region-level analysis. The industry-level analysis suggests that robots and labor are gross complements. Our preferred estimates mean that a one-percent decrease in robot price increased labor demand by 0.44 percent, which implies that robots and labor are gross complements. This thus implies that a one percent increase in robot adoption made by the robot price reduction increased employment by 0.28 percent. We show that these results are robust to other definitions of price measures and dropping major robot purchasers, indicating the validity of our cost-based IV. The CZ-level shift-share regressions confirm this, which shows a clear distinction from the findings in the literature and suggest across-region reallocations and time-saving technical changes.

We suggest a potential reason why we find a different result than the one in the literature: the differences in scale effects. To see this, note that our context might have been unique in strong robot demands. This may be seen, for example, in a biography of a robot innovator, Kawasaki Heavy Industry, as the incentive for producing robots: "Japan's rapid economic growth increased demands for automobiles and factories faced serious labor shortages" [KHI, 2018]. If the demand for the robot-intensive industry is strong due to either domestic or international demands, then robot adoption may invoke the scale effect to the industry, mitigating the displacement effect found in, among others, Acemoglu and Restrepo [2020]. We leave testing this hypothesis or other potential explanations as a future work.

Chapter 3

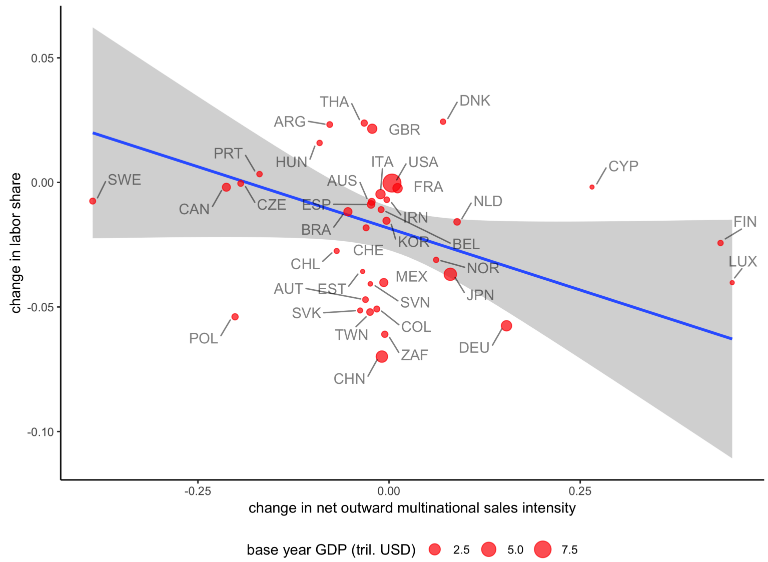
Multinational Production and Labor Share

3.1 Introduction

A growing body of evidence suggests that in recent decades, the labor share of national income has decreased in several developed countries [Karabarbounis and Neiman, 2013], raising concerns both for policymakers and economists. For policymakers, the decrease might be interpreted as an increase in income inequality between capital holders and laborers. For economists, it challenges one of the stylized facts of growth models [Kaldor, 1961].

The broad question of the current paper is what drove the decrease in the labor share. Several potential explanations have been proposed in the literature, including the role of bias in technological change [Oberfield and Raval, 2014], for if changes in productivity augment capital more than labor, then the total payment to labor will decrease relative to capital. Although this is a theoretically coherent and straightforward explanation, there are several potential mechanisms behind such factor-specific augmentation. For example, the production processes are fragmenting across the world [Johnson and Noguera, 2012, 2017]. If international direct investment and employment of foreign labor by multinational enterprises (MNEs) complements capital in the source country, there can be a relative increase in the demand for capital relative to labor. In this paper, we formalize this idea and ask if and to what extent it may explain the labor share trend in the source country of

Figure 3.1: Net Outward Multinational Sales and Labor Share



Note: The authors' calculation using data from Karabarbounis and Neiman [2013] and UNCTAD. Details of the construction of the data are discussed in Section C.1.5. The horizontal axis is the change in the average sums of bilateral net outward multinational sales between 1991-1995 and 1996-2000. The vertical axis is the change in labor share from 1991 to 2000. Singapore was dropped because it has an outlier value for outward multinational sales.

foreign direct investment (FDI).

This surge in MNE international factor employment has been brought about by a wide range of changes in the economic environment, including technological change, policy and institutional reform, and growth of developing economies. For example, better global communication technologies, removal of political barriers regarding international direct investment and employment of foreign labor, and increasing demand by external economies all may have increased the availability and productivity of foreign factors. We take these events as exogenous augmentations of foreign factors for MNEs and study the effects on the labor share of the source country.

As first-pass evidence, Figure 3.1 shows a significant negative ralationship between the change in net MNE sales and labor share among countries. Although we do not suggest that the correlation is causal, this negative relationship is consistent with the idea that the outward activities of MNEs complement capital demand more than labor in the source country.

To formally answer the question if and how much foreign factor augmentation decreased the labor share in the source country, we proceed in three steps. First, we develop a general equilibrium model that features heterogeneous and non-parametric production functions that include the employment and exogenous augmentations of the foreign factors. In the model, the factor market-clearing wages are the key endogenous variables determining labor share, and we show that to the first order, the elasticity structure of factor demand is critical to the the direction and size of the change in labor share. This first order approximation is robust across several existing models of multinational production and many factors, including models of offshoring (e.g., Feenstra and Hanson, 1997) and multinational production with export platforms (e.g., Arkolakis et al., 2017). We then show how the key elasticities may be identified given the foreign augmentation shocks, describing the moment conditions and explaining how to back out the structural errors that depend on the elasticity matrix which contains our parameters of interest.

As our baseline model spans broad production technologies, we discuss the example of homogeneous nested CES production in order to provide the intuition behind the theoretical results and some guidance for the empirical application. Out of the specified model, our key theoretical results are twofold. First, the difference in the elasticities of substitution between nests is key to the impact of foreign factor augmentation on home labor share. Namely, if home labor is more substitutable with the foreign factor than home capital, then foreign factor augmentation implies a reduction in the home labor share. Intuitively, if foreign factor augmentation occurs, both home labor and capital are substituted away for a given level of output. But if the substitution is larger for labor, so is the *relative* decline in home labor demand, causing the labor share to decrease. One of the challenges is to identify the novel parameter of our model, the foreign factor-home factor elasticity of substitution. To proceed, we assume the existence of an exogenous change in foreign factor productivity. Our second key theoretical result is that the shock-induced derivative of home-foreign employment with respect to the shock is informative of the key parameter.

Armed with these theoretical results, we estimate the model empirically by applying it to the *2011 Thailand Floods*. These floods, which extended throughout the latter half of 2011, causing 65 of 76 provinces to be declared disaster zones, affecting more than 13 million people, and creating an estimated 1,425 trillion baht (USD 46.5 billion) in economic losses (World Bank), were a severe negative foreign factor productivity shock to Japanese MNEs operating in the flooded regions.

To study this unique event, we employ and combine firm- and establishment-level microdata sourced from the *Basic Survey on Japanese Business Structure and Activities* (BSJBSA)*, the Basic Survey on Overseas Business Activities* (BSOBA)*,* and the *Orbis* database from Bureau van Dijk (Orbis BvD). First, we calibrate the capital-labor substitutability by the first order condition and a shift-share-type instrumental variable [Raval, 2019]. Consistent with the previous literature, we find that capital-labor are gross complements. Second, by regressing the log-home employment on the log-foreign employment (with the instrumental variable relating the intensity of the flood damage) and a firm-level fixed effect, we obtain a two-stage least squares estimate which indicates that home labor and foreign labor are gross substitutes, in keeping with our theory. This result is based on our homogeneous nested CES specification.

Given the home labor-foreign labor substitutability implied by our estimate, we find that foreign factor augmentation (from the perspective of Japan) did contribute to a decrease in labor share in Japan. We thus have shown that foreign factor augmentation decreases home labor share. To quantify this, we exploit the nested CES specification to back out the aggregate evolution of foreign factor-augmenting productivities. Applying our implied elasticities of substitution, we find that foreign factor augmentation alone explains 59 percent of the decrease in Japan's labor share between 1995 and 2007.

We also derive an elasticity estimate from a more general production function based on the method of moments. As in the nested CES case, the result shows that foreign factor augmentation increases the *relative* demand for capital, which implies that the relative labor wage decreases, as does the labor share.

The paper is structured as follows. After discussing the related literature below, Section 3.2 provides some of the motivating facts about labor share and MNEs. Section 3.3 then provides the conceptual framework modeling labor share and foreign factor augmentation, presenting both a general foreign factor demand model and a nested CES specification. Section 3.4 discusses the empirical setting, data, specification and reduced-form estimates, while Section 3.5 shows the derived structural parameters and estimation under both the general and nested CES models. Section 3.6 concludes the paper.

3.1.1 Related Literature

This paper relates to two strands of literature: research concerning the recent decreasing labor share (or increasing capital share) of production, and studies on MNEs and their impact on the source country's labor market.

Changing Factor Shares

Seminal research on changing factor shares that empirically finds a decreasing labor share includes Elsby et al. [2013] in the U.S and Karabarbounis and Neiman [2013], whose comparable cross-country data show a declining labor share in many developed countries in particular, which Piketty and Zucman [2014] have suggested leads to expanding within-country income inequalities. However, there are several conceptual qualifications and labor share measurement issues that raise concerns about these empirical findings. For instance, Bridgman [2018], Rognlie [2016, 2018] stress that the treatment of capital depreciation and the self-employee income allotment to each factor entails strong assumptions. Making a different argument, Koh et al. [2018] claim that the decline in the labor share in the U.S is attributable mainly to the capitalization of intellectual property, while Cette et al. [2019] discuss a similar effect regarding the accounting of real estate income. In our view, even after taking these qualifications into account, Japan's labor share between 1995 and 2007 still decreased, as discussed further in Section C.1.2.

There are several possibilities suggested in the literature as to why the labor share is decreasing, including bias in technological change. For instance, Oberfield and Raval [2014] emphasize the role of "technology, broadly defined, including automation and offshoring, rather than mechanisms that work solely through factor prices", and Elsby et al. [2013] conclude that the offshoring of labor-intensive activities within the supply chain is "the leading potential explanation of the decline in the U.S. labor share over the past 25 years." On the technology side, automation technology may have contributed to the labor share decrease, as reviewed in Acemoglu and Restrepo [2019] for the example of industrial robots. To these, we add globalization as a partial explanation, particularly in our context of Japan from 1995-2007. We offer evidence based on a particular mechanism that features the role of MNEs, providing a novel identification strategy and empirical evidence from the 2011 Thailand Floods.[1]

[1] Other potential explanations for declining labor share include lower capital and increasing risk premiums,

An increase in market power (and thus profit) by dominant corporations has also been proposed as a reason for decreasing labor share, as market power can emerge both in the goods market [Autor et al., 2017a,b, Barkai, 2017, De Loecker and Eeckhout, 2017] and in the labor market [Berger et al., 2019, Gouin-Bonenfant et al., 2018].[2] Although market power is not our main focus, we explore its potential role to help validate our mechanism. Applying the method of De Loecker and Eeckhout [2017] to our Japanese data, we find that the markup did not increase as much in Japan as in the U.S.

Our contribution to this literature is adding multinational enterprises (MNEs) to the discussion of potential reasons for the observed decrease in labor share. Among this growing literature, we regard our work to be closest to Oberfield and Raval [2014] who, as mentioned above, emphasize the role of biased technological change in the production function as opposed to the factor-price story of Karabarbounis and Neiman [2013]. For this purpose, they employ plant-level microdata in the U.S. and estimate the capital-labor substitution elasticity by estimating the first order condition for factor demand with a Bartik-type instrument. They find that since the 1970s, the aggregate capital-labor elasticity has been constant at around 0.7, which indicates that capital and labor are indeed gross complements, while a capital cost decline like that emphasized by Karabarbounis and Neiman [2013] would imply an increasing labor share. Our analysis draws upon and extends Oberfield and Raval [2014], first applying their method of estimating capital-labor elasticity to firm- and plant-level data in Japan to confirm that the elasticity is below one. We depart from their analysis to the extent that we explicitly incorporate the foreign factor employment of home firms in our model in order to consider the effect of foreign factor augmentation on the home labor share. Section 3.3.1 explains how our production function nests that of Oberfield and Raval [2014].

Labor Market Impact of MNEs

This paper also relates to a second line of research that examines the impact of foreign production on the source country's labor market [Boehm et al., 2020, Desai et al., 2009, Ebenstein et al., 2014, Harrison and McMillan, 2011, Kovak et al., 2017, Muendler and

as discussed in detail in Section C.1.4.

[2] Berger et al. [2019], however, argue that labor market concentration contributed in the opposite direction, if at all, as the U.S. local labor market concentration has been decreasing since the 1970s.

Becker, 2010].[3] Notably, as does the current paper, this literature explicitly uses exogenous variation that changes the profitability from engaging in foreign production. For example, Desai et al. [2009] take the source country's economic indicators and construct a shift-share-style instrumental variable, whereas Kovak et al. [2017] use a change in bilateral tax treaties between the U.S. and other countries to construct the IVs. Bernard et al. [2018] study the effect on firms' occupational organization using a shift-share-type instrument from a detailed set of firm-specific variables regarding changes in import share, while Setzler and Tintelnot [2019] use the geographic clustering of MNEs as the source of exogenous variation and find that when foreign MNEs enter the U.S., there are two effects on local wages: a direct foreign MNE premium on worker wages and an indirect wage-bidding-up effect on incumbent domestic firms. Our contribution to this literature is both empirical and theoretical. Empirically, our approach adds another piece of evidence based on a negative productivity shock resulting from an unexpected natural disaster that affects firms' foreign production and domestic employment decisions.[4]

More importantly, unlike these studies, we are able to provide a clear structural interpretation of the shock and its impact on employment by specifying the foreign production model in Section 3.3. As will be clear, this is indeed the key to identifying the model parameters.[5] Specifically, both Desai et al. [2009] and Kovak et al. [2017] find positive causal effects of foreign employment on home employment, which is consistent with our finding that the decrease in employment in the foreign country due to flooding is accompanied by a *decrease* in home employment. Our model further implies that this positive association caused by an exogenous shock to foreign productivity can be interpreted as home labor and foreign labor being gross substitutes.

Although the literature on the relationship between MNEs and factor intensities is

[3]Earlier observational studies include Brainard and Riker [1997], Head and Ries [2002], Slaughter [2000].

[4]Kato and Okubo [2017] study the same event to learn about the loss of vertical linkages in the destination country, while Boehm et al. [2018] study another dimension of the impact of a shock on multinational firms. While they consider the effect of the 2011 Tohoku Earthquake in Japan on the Japanese HQ and its subsequent impact on foreign (U.S.) affiliates, this paper is interested instead in the impact of a foreign shock on foreign affiliates in Thailand and its impact on domestic headquarters in Japan.

[5]For example, while Desai et al. [2009] use a Bartik shock with GDP growth rates as the 'shift' component, if we do not know whether the GDP growth is due to increased consumer demand or firm productivity in the short-run, then it is not clear how to interpret their coefficients. On the other hand, Kovak et al. [2017] employ the enforcement of a bilateral tax treaty, which is arguably a supply-side shock. However, their approach does not permit backing out substitution parameters, as they use a different sourcing model and therefore do not estimate the 2SLS specification but only an event-study regression.

not extensive, different factor intensities across firms may have quantitative implications for labor shares through the reallocation of factors across firms. Like Sun [2020], who studies the differential capital intensities between MNEs and other firms, we also study heterogeneous factor intensities by firm-level MNE data and provide some evidence as to the role of the heterogeneity. While Sun [2020] focuses on host countries, our focus is on the labor share of the source country of FDI. Furthermore, Sun [2020] does not focus on estimating the elasticity between home and foreign factors, but calibrates his model of export platforms to global affiliate data using the cross-section variation. In contrast, we use a natural experiment to formally identify the elasticity between factors in the MNE-source and destination countries.

Lastly, our paper is related to the literature which argues that automation may displace labor demand and thus reduce labor share. Our goal is to establish that instead of technological changes in automation, MNE activities play a substantial role in decreasing labor share. However, it is difficult to separate the effect of globalization from that of technological change, both conceptually and empirically [Fort et al., 2018]. To partially address the concern, in Section C.1.4, we provide evidence that the growth in automation technology in Japan during the 1990s and 2000s was not rapid relative to other countries.

3.2 Motivating Facts

In this section, we provide further evidence that suggests that a surge in MNE growth was behind the decrease in labor share in recent decades. For this purpose, we focus on our context of Japan from 1995-2007, as reviewed in Section 3.1. In Section 3.2.1 we describe the major data source for our analysis and then, using this data, we show relevant facts in Section 3.2.2.

3.2.1 Data Sources

This study utilizes four main datasets, the first being the *Basic Survey on Japanese Business Structure and Activities* (BSJBSA), an annual survey of large firms in Japan administered by the Ministry of Economy, Trade, and Industry (METI).[6] BSJBSA has a detailed set of

[6]On March 31st each year, all firms with more than 50 employees and assets of JPY 30 million (USD 0.3 million) are asked to complete the questionnaire.

variables providing Japanese firm-level information such as the firm's address, division-level employment distribution, holding relationships, balance sheet components, itemized sales by goods, costs by type, export and import by region, outsourcing, research and development, technology and patents, among others. The data spans the years from 1995-2017.

In order to match foreign production information to the BSJBSA, we employ a second dataset, the *Basic Survey of Overseas Business Activities* (BSOBA), another annual survey administered since 1995 by METI in which *all* (i.e. both private and public) Japanese MNEs are asked about their domestic and foreign business information as of March each year.[7] The BSOBA is comprised of a Headquarter File and a Subsidiary File, and this study employs mostly the Subsidiary File which asks for information about all child and grandchild foreign subsidiaries of each MNE.[8] The questions consist of the destination country, local employment and sales, which are allocated to the following destination categories: Japan (home country), Asia, Europe, and America. To check the quality of its employment and labor compensation data, we compare the variables to those obtained from PWT, as discussed in Section C.1.7.

Although the BSOBA contains the country in which each subsidiary is located, it does not provide a detailed address. As we saw in Section 3.4.1, while the Thai floods caused extensive damage, it was mainly concentrated in Ayutthaya and Pathum Thani provinces, which we define as the flood-affected regions, following JETRO (2012). The exact location in Thailand of the Japanese MNE subsidiaries is thus necessary to properly assign treatment status. To do this, we use the address variable from the Bureau van Dijk *Orbis* database.

Finally, as the above datasets do not share firm IDs, we matched the firm names, locations, and phone numbers using a firm-level dataset collected by private credit agency *Tokyo Shoko Research* (TSR). The match rate from BSOBA is 93.0%. Because the scope of BSOBA is exclusively Japanese MNEs, we interpret each firm in TSR as an MNE if and only if the firm also appears in BSOBA for that year. The details of the matching

[7]BSOBA defines MNEs and foreign activities as the following: A firm is defined as an MNE if it has a foreign subsidiary, which can be either a "child subsidiary" or "grandchild subsidiary". A child subsidiary firm is a foreign corporation whose Japanese ownership ratio is 10% or more, while a grandchild subsidiary is a foreign corporation whose ownership ratio is 50% or more by the foreign subsidiaries whose Japanese ownership ratio is 50% or more. Therefore, the definition of foreign production is not limited to greenfield investment but also includes purchases of foreign companies such as M&A.

[8]We drop subsidiaries located in tax-haven countries, following the Gravelle [2015] definition. We thank Cheng Chen for kindly sharing the code for the sample selection.

process can be found in Section C.1.7. Since TSR access is limited only back to 2007, the BSJBSA-BSOBA match can occur only for years 2007-2016.

3.2.2 Time-series of Labor Share and Multinational Activities

Armed with these data, we provide the negative time-series correlation between labor share and multinational activities in Japan from 1995-2007, which will guide us in developing the model of labor share and multinational activities in the following sections.

Figure 3.2 shows trends in Japan's labor share and aggregate payment to foreign employees. The red line shows the overall decreasing labor share in Japan since 1995.[9] As outlined above, the current paper aims to explain such a decrease by foreign factor augmentation from the perspective of Japan, which potentially increases both factor prices and employment, as detailed in the section describing our model. Therefore, one of the augmentation measures is the aggregated payment from Japanese multinational enterprises (MNEs) to foreign workers. The blue line shows this trend since 1995, which is the first year of our dataset. As can be seen, the trend increased rapidly, at least partly indicating the augmentation of the foreign factor. Given this finding, the current paper asks if, how, and under which conditions is there a logical link between foreign factor augmentation and labor share, and quantitatively what portion of the observed labor share decline can be attributed to this foreign factor augmentation.

Due to the space constraint, we relegate robustness checks to the Appendix. Section C.1.2 shows that the decrease in labor share is robust to other measures discussed in the literature. To provide another perspective on the role of MNEs, we also compare trends in labor share among MNEs and non-MNEs, and the composition of MNEs in Section C.1.3. To provide partial evidence about other mechanisms discussed above, we show in Section C.1.4 that (i) the increase in the stock of industrial robot is likely to have happened *before* the period of our analysis, and (ii) market power in Japan was low and relatively constant during 1995-2007.

[9]A figure since 1980 is shown in Section C.1.1.

Figure 3.2: Labor Share and Payment to Foreign Employment, Japan

Note: The authors' calculation based on Japan Industrial Productivity (JIP) Database 2015 from Research Institute of Economy, Trade and Industry (RIETI) and the Basic Survey on Overseas Business Activities (BSOBA) 1996-2008. The labor share is calculated by the share of the nominal labor cost in the value added of JIP market economies in nominal terms. The compensation for foreign labor is the sum of compensation to workers in foreign subsidiaries of all Japanese multinational corporations in BSOBA.

3.3 Conceptual Framework

In this section, we set up our conceptual framework for the estimation and quantification that follows. First, in Section 3.3.1, we develop a general framework that features foreign production which has the same first order implications for labor share as several offshoring [Feenstra and Hanson, 1997] and MNE models (among others, Arkolakis et al. 2017, Ramondo and Rodríguez-Clare 2013). This framework contains foreign factor augmentation representing a reduction in the cost of multinational production, and factor market clearing conditions to discuss labor share. A detailed discussion of the relationship between these models is in Section C.2 of the Appendix. Section 3.3.2 discusses the implications of foreign factor augmentation on labor share to the first order. Out of this section, readers will know which statistics from the model are necessary to qualitatively and quantitatively understand the impact on labor share. In Section 3.3.3, we construct a methodology for identifying these statistics given a foreign factor augmentation shock, and then in the estimation section, we apply this methodology to the 2011 Thailand Floods. In Section 3.3.4, we discuss a specific version of our general model; namely, a homogeneous nested CES production function.

This setup allows us to discuss the solution to the labor share in a closed form, the necessary statistic in the form of constant parameters, and the identification through a simple linear regression. By doing so, we aim to offer a simpler intuition of the mechanisms of our model and guide our estimation and quantitative exercise, as we discuss later.

3.3.1 Setup

The environment is static, with two countries H and F. There are no trade costs but factors cannot move between countries. Firms originate from both countries and produce firm-specific output $i \in I$. Each good i is contestable and so firms are perfectly competitive in the output market. Country H is small-open in the sense that the set I_H of firms from country H constitutes zero measure among all firms I and the total demand is determined by F demand. Next, we describe the consumption setup, production setup, and equilibrium in turn. The representative consumer has a CES preference across all goods $i \in I$. Hence the demand function is

$$q_i = \left(\frac{p_i}{P}\right)^{-\varepsilon} Q, \tag{3.1}$$

where $P = \int_{i \in I} (p_i)^{1-\varepsilon} di$ is the ideal price index. Since H is small-open, P and Q are determined exogenously to country H. Firm i in H may hire intermediate inputs m_i. Note that foreign inputs are so general that they may entail foreign-produced intermediate inputs (e.g., Feenstra and Hanson, 1997) or foreign factors of production (e.g., Arkolakis et al., 2017, Helpman et al., 2004, Ramondo and Rodríguez-Clare, 2013). A firm i from H produces a firm-specific output with domestic factors k_i and l_i, and foreign inputs m_i according to the constant returns to scale production function:

$$F\left(\widetilde{k_i}, \widetilde{l_i}, \widetilde{m_i}\right),$$

where $\widetilde{k_i} \equiv a_i^K k_i$, $\widetilde{l_i} \equiv a_i^L l_i$, and $\widetilde{m_i} \equiv a_i^M m_i$ are the augmented factors. We assume F is increasing, strictly concave, and twice continuously differentiable (so that Young's theorem applies in Section C.2.7). Note that there are factor augmentations $\left(a_i^K, a_i^L, a_i^M\right)$. Below, our theoretical interest is the effect of changes in foreign factor augmentation a_i^M. In particular, in our comparative statics, we are concerned about positive log-augmentation $d \ln a_i^M > 0$, whereas in our identification argument and empirical application, we consider a negative log-augmentation. One may interpret the foreign factor augmentation as policy or institutional changes that reduce the cost of firms' multinational activities, or technological or economic growth in country F that increases the productivity of the factors.[10]

Factors from country H are hired competitively in each factor market. As we will see, capital and labor markets in country H clear at prices r and w respectively, but F's factor prices are given to small-open country H. Firms solve the augmented factor demands by the standard cost-minimizing problem given the quantity q_i in terms of augmentation-controlled prices $\widetilde{r_i} \equiv r/a_i^K$, $\widetilde{w_i} \equiv w/a_i^L$, and $\widetilde{p_i^M} \equiv p_i^M/a_i^M$, with $\widetilde{p_i^f} \equiv \left(\widetilde{r_i}, \widetilde{w_i}, \widetilde{p_i^M}\right)'$ the vector of the augmentation-controlled factor prices. Given CES world demand (3.1), we have the quantity q_i that depends on firm i's price p_i. Given perfect competition, p_i further depends on augmentation-controlled factor prices $\widetilde{p_i^f}$. Substituting this relationship between q_i and $\widetilde{p_i^f}$ in the cost-minimizing factor demand, we have the *reduced* factor demand functions that only depend on augmentation-controlled factor prices:

$$\widetilde{k_i} = \widetilde{k_i}\left(\widetilde{p_i^f}\right),\ \widetilde{l_i} = \widetilde{l_i}\left(\widetilde{p_i^f}\right), \widetilde{m_i} = \ \widetilde{m_i}\left(\widetilde{p_i^f}\right). \tag{3.2}$$

The factor prices are determined by market clearing. The small-open H assumption implies that the hiring of foreign input m_i by $i \in I_H$ does not affect the prices p_i^M, so that we write $p_i^M = \overline{p_i^M}$ and $\widetilde{\overline{p_i^M}} = \overline{p_i^M}/a_i^M$. In H, capital and labor are supplied inelastically at level K and L. The H factor markets are cleared at the factor market conditions

$$K = \int_{i\in I_H} \frac{\widetilde{k_i}\left(\widetilde{p_i^f}\right)}{a_i^K} di,\ L = \int_{i\in I_H} \frac{\widetilde{l_i}\left(\widetilde{p_i^f}\right)}{a_i^L} di. \tag{3.3}$$

Hence, the small-open equilibrium is $\left(\{k_i, l_i, m_i\}_{i\in I_H}, r, w\right)$ that satisfies (i) the factor demands given by (3.2) and (ii) the factor prices solving (3.3).

Under the environment with continuous demand and supply functions like ours, it is routine to show the existence of an equilibrium. Furthermore, as consumers are homogeneous (since H is small-open), the uniqueness of the equilibrium can be proven (see Section C.2.1 of the Appendix). Given this unique equilibrium, the following subsections discuss the first-order properties of labor share and identification, and then provide an example in

[10] Sun [2020] conducts a counterfactual analysis of bilateral multinational production cost, but notes that the calibration strategy does not identify bilateral productivity. Thus, this counterfactual analysis corresponds to our study in that either a decline in multinational production cost or productivity growth in the foreign country may induce a change in equilibrium, including the labor share.

the form of a nested CES specification.

3.3.2 Labor Shares

With a unique equilibrium, we can analyze the implication of a foreign productivity shock to labor share as follows. Labor share is defined as

$$LS \equiv \frac{wL}{wL + rK}. \tag{3.4}$$

Note that the endogenous variables in expression (3.4) are w and r, so the solution to the model should transform the endogenous expression into an exogenous one. Taking the log-first order approximation of labor share definition (3.4) gives us

$$dLS = LS_0 \left(1 - LS_0\right) \left(d \ln w - d \ln r\right). \tag{3.5}$$

Thus, the labor share increases if and only if the Home wage grew more than the Home rental rate. To study this, we revisit the factor market clearing condition (3.3) and take the log-first order approximation to obtain:

$$0 = \int_{i \in I_H} s_i^K \left[\sigma_{\widetilde{kr},i} d \ln r + \sigma_{\widetilde{kw},i} d \ln w - \sigma_{\widetilde{kp^M},i} d \ln a_i^M\right] di,$$

$$0 = \int_{i \in I_H} s_i^L \left[\sigma_{\widetilde{lr},i} d \ln r + \sigma_{\widetilde{lw},i} d \ln w - \sigma_{\widetilde{lp^M},i} d \ln a_i^M\right] di,$$

or

$$\overline{\Sigma_H} \times \begin{pmatrix} d \ln r \\ d \ln w \end{pmatrix} = \begin{pmatrix} \int_{i \in I_H} s_i^K \sigma_{\widetilde{kp^M},i} d \ln a_i^M di \\ \int_{i \in I_H} s_i^L \sigma_{\widetilde{lp^M},i} d \ln a_i^M di \end{pmatrix}, \tag{3.6}$$

where $s_i^K \equiv rk_i/rK$ (resp. $s_i^L \equiv wl_i/wL$) is the capital (resp. labor) employment share of firm i among all H-origin firms I_H.

$$\overline{\Sigma_H} \equiv \begin{pmatrix} \int_{i \in I_H} s_i^K \sigma_{\widetilde{kr},i} di & \int_{i \in I_H} s_i^K \sigma_{\widetilde{kw},i} di \\ \int_{i \in I_H} s_i^L \sigma_{\widetilde{lr},i} di & \int_{i \in I_H} s_i^L \sigma_{\widetilde{lw},i} di \end{pmatrix} \tag{3.7}$$

is the weighted Home factor elasticity matrix. If $\overline{\Sigma_H}$ is negative definite, then

$$d\ln w - d\ln r = \begin{pmatrix} -1 & 1 \end{pmatrix} \left(\overline{\Sigma_H}\right)^{-1} \begin{pmatrix} \int_{i\in I_H} s_i^K \sigma_{\widetilde{k}\widetilde{p^M},i} d\ln a_i^M di \\ \int_{i\in I_H} s_i^L \sigma_{\widetilde{l}\widetilde{p^M},i} d\ln a_i^M di \end{pmatrix}. \tag{3.8}$$

It is worth noting the relationship between our general equilibrium setup and the offshoring and multinational production models in the literature. Specifically, our general equilibrium model nests modified versions of models of offshoring [Feenstra and Hanson, 1997] and multinational production (e.g., Arkolakis et al., 2017) in terms of the first order approximation of factor prices (3.6). This equivalence is formally shown in Section C.2.2 of the Appendix. Consequently, for our purposes, we may turn away from these detailed models of international trade and multinational production to focus exclusively on the sufficient statistics of the elasticity matrix given by equation (3.7). We discuss how to identify these elasticities in the following section.

3.3.3 Identification

In this section, we discuss our identification strategy by formally introducing a foreign factor-augmenting negative productivity shock to a measure-zero subset of firms.[11] In Section 3.4.1, we apply the model to empirically estimate the effect on labor share of the devastating 2011 Thailand Floods. To begin, first assume that there exists an instrumental variable Z_i that correlates with the Foreign productivity shock $d\ln a_i^M$ but not with Home productivity shocks $d\ln a_i^K$ and $d\ln a_i^L$. Then we may construct the moment conditions:

$$E\left[Z_i \begin{pmatrix} d\ln a_i^K \\ d\ln a_i^L \end{pmatrix}\right] = 0. \tag{3.9}$$

[11] This simplifying assumption is helpful because otherwise the equilibrium effect on factor prices (r, w^H, w^L) would emerge and complicate the analysis. This assumption is realistic because the set of Japanese firms hit by the flooding is small relative to the population. Having said that, mulitnational firms are larger and comprise a significant portion of factor employment both theoretically (Arkolakis et al., 2017, Helpman et al., 2004 among others) and empirically [Ramondo et al., 2015], so a quantitatively relevant extension is to allow the shock to extend to a positive-measured set of firms for identification.

To obtain the structural productivity shocks $d\ln a_i^K$ and $d\ln a_i^L$, consider the following model inversion. By factor demand functions (3.2), we have

$$\begin{aligned}
d\ln(rk_i) = d\ln\left(\widetilde{r_i}\widetilde{k_i}\right) &= \left(1+\sigma_{\widetilde{k}\widetilde{r},i}\right)d\ln\widetilde{r_i} + \sigma_{\widetilde{k}\widetilde{w},i}d\ln\widetilde{w_i} + \sigma_{\widetilde{k}\widetilde{p^M},i}d\ln\widetilde{p_i^M},\\
d\ln(wl_i) = d\ln\left(\widetilde{w_i}\widetilde{l_i}\right) &= \sigma_{\widetilde{l}\widetilde{r},i}d\ln\widetilde{r_i} + \left(1+\sigma_{\widetilde{l}\widetilde{w},i}\right)d\ln\widetilde{w_i} + \sigma_{\widetilde{l}\widetilde{p^M},i}d\ln\widetilde{p_i^M},\\
d\ln\left(p^M m_i\right) = d\ln\left(\widetilde{p_i^M}\widetilde{m_i}\right) &= \sigma_{\widetilde{m}\widetilde{r},i}d\ln\widetilde{r_i} + \sigma_{\widetilde{m}\widetilde{w},i}d\ln\widetilde{w_i} + \left(1+\sigma_{\widetilde{m}\widetilde{p^M},i}\right)d\ln\widetilde{p_i^M},
\end{aligned}$$

or, in matrix form,

$$\begin{pmatrix} d\ln(rk_i)\\ d\ln(wl_i)\\ d\ln\left(p^M m_i\right)\end{pmatrix} = (I+\Sigma_i)\begin{pmatrix} d\ln(\widetilde{r_i})\\ d\ln(\widetilde{w_i})\\ d\ln\left(\widetilde{p_i^M}\right)\end{pmatrix} = (I+\Sigma_i)\begin{pmatrix} d\ln r - d\ln\left(a_i^K\right)\\ d\ln w - d\ln\left(a_i^L\right)\\ d\ln p^M - d\ln\left(a_i^M\right)\end{pmatrix},$$

where

$$\Sigma_i \equiv \begin{pmatrix} \sigma_{\widetilde{k}\widetilde{r},i} & \sigma_{\widetilde{k}\widetilde{w},i} & \sigma_{\widetilde{k}\widetilde{p^M},i}\\ \sigma_{\widetilde{l}\widetilde{r},i} & \sigma_{\widetilde{l}\widetilde{w},i} & \sigma_{\widetilde{l}\widetilde{p^M},i}\\ \sigma_{\widetilde{m}\widetilde{r},i} & \sigma_{\widetilde{m}\widetilde{w},i} & \sigma_{\widetilde{m}\widetilde{p^M},i}\end{pmatrix}. \tag{3.10}$$

Thus we have

$$\begin{pmatrix} d\ln\left(a_i^K\right)\\ d\ln\left(a_i^L\right)\\ d\ln\left(a_i^M\right)\end{pmatrix} = \begin{pmatrix} d\ln r\\ d\ln w\\ d\ln p^M\end{pmatrix} - (I+\Sigma_i)^{-1}\begin{pmatrix} d\ln(rk_i)\\ d\ln(wl_i)\\ d\ln\left(p^M m_i\right)\end{pmatrix}. \tag{3.11}$$

Therefore, conditional on parameter restrictions, we may identify two elasticity parameters from elasticity matrix (3.10) given moment condition (3.9). The empirical details are discussed in Section 3.4.

At this point, we discuss the nature of our identification strategy. A typical method for identifying labor demand-side elasticity such as our Σ_i is to use labor-supply side elasticity such as a short-run surge in migration. In fact, the idea that a labor supply shock can identify the supply elasticity is through the change in wages exogenous to producers [Ottaviano and Peri, 2012]. The benefit of our approach is that we do not need to have such exogenous changes observed, since our approach does not rely on factor price changes but instead on the change in *effective* factor prices, which are specific to firms. This implies that our identification method is free of any assumptions about labor market delineation or competition structure within the labor market.

3.3.4 Example: Nested CES

In this section, we discuss a special case of our non-parametric and heterogeneous framework presented in Section 3.3.1. This clarifies the intuition of our general setup and simplifies the identification strategy. We rely on this example to calibrate some aspects of the general model in later sections.

Setup In this example, we maintain the same setup for countries and consumer preferences but assume that there is a homogeneous set of firms. We continue denoting I as the set of firms in any country and I_H as that from country H. To achieve a different elasticity of substitution between the foreign factor and home factors, we assume a parsimonious nested CES production function. Namely, each firm produces output q with nested CES production function:

$$F(k,l,m) = \left(\left(a^K k\right)^{\frac{\sigma-1}{\sigma}} + x^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 0$ is an upper substitution of elasticity and

$$x = \left(\left(a^L l\right)^{\frac{\lambda-1}{\lambda}} + \left(a^M m\right)^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}}, \tag{3.12}$$

where $\lambda > 0$ is a lower elasticity substitution. Note that the special case of $\lambda = \sigma$ implies the single-nest CES production function. Finally, suppose factor supplies (K, L, M) are fixed. Factor market clearing $k = K, l = L$, and $m = M$ gives the factor prices w^c and r, and the labor share is given by equation (3.4).

Several discussions follow. First, to relate our production function choice with the one in Oberfield and Raval [2014], note that firms need not hire foreign factor m in reality. If this is the case, we can define the production function with $m = 0$, which would yield CES $q = \left(\left(a^K k\right)^{\frac{\sigma-1}{\sigma}} + \left(a^L l\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. On the other hand, Oberfield and Raval [2014] consider the CES production function with heterogeneous augmentation a^K and a^L at the firm level. Note also that firms need not hire the foreign factor, which guides us to the restriction that labor and the foreign factor are gross substitutes, or $\lambda > 1$.

As for further potential parameter restrictions, to the extent that typical firms hold some form of capital to produce output, we have an educated guess that capital and aggregate labor are gross complements, or $\sigma < 1$. Because the parameter restrictions so far turn out to be useful for some of the theoretical arguments, we formalize these as the following assumption:

Assumption 2. $\lambda > 1 > \sigma > 0$.

In what follows, we show that under Assumption 2, foreign labor (log-)augmentation $d \ln a_i^M > 0$ implies that the reduction in labor share $dLS < 0$. As explained in Section 3.1.1, Oberfield and Raval [2014] indeed estimate that σ is well-below unity using U.S. plant-level microdata. In our empirical and quantitative exercise, we apply this method, modified to our nested CES assumption and with Japanese firm- and plant-level data, and confirm that $\sigma < 1$ is also the case in Japan. Moreover, our identification method applied to the natural experiment- based IV estimate reveals that, in fact, $\lambda > 1$. Therefore, we find that Assumption 2 holds empirically. Notwithstanding this, a number of results in the following section do not depend on a particular parameter restriction 2.

Labor Share Our proof in Section C.2.3 of the Appendix shows that under our setting, the labor share expression may be solved analytically as

$$LS = \frac{\left(a^L L\right)^{1-\lambda^{-1}} X^{\lambda^{-1}-\sigma^{-1}}}{\left(a^L L\right)^{1-\lambda^{-1}} X^{\lambda^{-1}-\sigma^{-1}} + \left(a^K K\right)^{1-\sigma^{-1}}}, \tag{3.13}$$

where

$$X \equiv \left(\left(a^L L\right)^{\frac{\lambda-1}{\lambda}} + \left(a^M M\right)^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}}$$

is the aggregate value of the factor supplies to country H measured by function (3.12). Note that X is increasing in a^M. Hence, if $\lambda^{-1} - \sigma^{-1} < 0$ or $\lambda > \sigma$, LS is decreasing in a^M. Intuitively, if within the lower nest the factors are more substitutable, then the increase in the productivity of the offshore worker relatively strongly substitutes away domestic labor. More specifically, note that cost-minimizing factor demand (C.50) implies $d \ln L = WS_0^M d \ln A^M$ to the first order, where $WS^M \equiv p^M m / \left(wl + p^M m\right)$ is the aggregate share of payments to foreign labor within the total payments to home and foreign labor. Throughout the paper, subscript 0 denotes the variable at the base year. Hence WS_0^M is the base-year value of the aggregate share of payments to foreign labor.

It is worthwhile to highlight the role of the simplifying assumptions, homogeneous nested CES and fixed foreign factor supplies, in deriving the closed form equation (3.13). Note that wage rate w and capital rental rate r are endogenous objects that may be solved according to factor market clearing conditions. With homogeneous and nested CES structures, we may solve these relative factor prices analytically. Furthermore, the assumption that the foreign factor market clears with the foreign factor supply fixed at M plays a crucial role in deriving the closed form in exogenous terms. Namely, without it, the lower aggregate term L contains the foreign factor supply M, which is endogenous and needs to be solved as other exogenous elements in the model.

With these assumptions, other exogenous variables being fixed, the first order approximation with respect to $d \ln a^M$ implies

$$\begin{aligned} dLS &= LS_0 \left(1 - LS_0\right) \left(\lambda^{-1} - \sigma^{-1}\right) d \ln L \\ &= LS_0 \left(1 - LS_0\right) \left(\lambda^{-1} - \sigma^{-1}\right) WS_0^M d \ln a^M. \end{aligned} \tag{3.14}$$

Therefore, it is again crucial to know the relative values of λ and σ to learn the sign of the effects of foreign labor augmentation on labor share. Thus, our informed guess in Section 3.3.1 establishes the following formal result.

Lemma 1. *Under assumption 2, foreign factor augmentation $d \ln a^M$ implies $dLS < 0$.*

One implication is that, in a special case, the single-nest CES production function $\lambda = \sigma$ would not permit a discussion of the effect of foreign factor augmentation on home labor share. In other words, this is one of the manifestations of the restrictive implication of *independence from irrelevant alternatives* (IIA) that the CES function features. Namely, since the foreign factor is an irrelevant alternative to home capital and labor, the augmentation does not affect the *relative* factor demands of home capital and labor. Thus, we need the nested structure in the production function in our simplest setting.

Quantitatively, identifying the value of $\lambda^{-1} - \sigma^{-1}$ is critical to understanding the labor share implication. In what follows, we obtain an even stronger result for identification – we identify the absolute value of λ and σ – by employing a shift-share instrument to identify σ [Oberfield and Raval, 2014, Raval, 2019]. In contrast, the foreign negative productivity shock is used for identification of λ as described below.

Since the homogeneous nested CES case is a special case of the general setup in Section 3.3.1, we may also relate the theoretical implications in terms of the general substitution elasticity matrix (3.10). This is discussed in Section C.2.5 of the Appendix.

Identification We then show the identification result given the foreign factor augmentation shocks as in Section 3.3.3. In Section C.2.4, we prove the following equations:

$$d \ln l = \left[(\lambda - \sigma) WS_0^M + (\sigma - \varepsilon) CS_0^M\right] d \ln a^M, \tag{3.15}$$

$$d \ln m = \left[-\lambda + (\lambda - \sigma) WS_0^M + (\sigma - \varepsilon) CS_0^M\right] d \ln a^M. \tag{3.16}$$

These equations mean that the elasticities of the foreign factor-augmenting productivity shock are summarized by three parameters $\lambda, \sigma, \varepsilon$ and constants WS_0^M and CS_0^M. The intuition is as follows. A negative foreign factor-augmenting productivity shock has both *direct* and *indirect* effects on factor employment. The direct effect speaks to the *biasedness* of the factor-augmenting shock, such as when the shock is biased to foreign factor demand, or $\lambda > 1$, whereby the lower nest features gross substitutes, as formalized in Assumption 2.[12] It then has the force to decrease foreign factor demand given the negative shock by the elasticity of $\lambda - 1$. On the other hand, the indirect effect is as follows. To the first order, a one percent decrease in the foreign factor-augmenting productivity shock increases the lower nest aggregate cost by WS_0^M percent and the marginal cost by CS_0^M percent. These increases have the effect on the total demand through the elasticities governed by the nested CES structure, $\lambda - \sigma$ and $\sigma - \varepsilon$, respectively. Due to the CES production function, this effect applies to labor demand l and foreign factor demand m alike. Therefore, the direct effect matters for foreign labor employment, whereas the indirect one matters for the demand for all factors.

However, it is not trivial to observe the size of the foreign factor-augmenting productivity shock $d \ln a^M$ empirically. Therefore, we consider the elasticity of foreign labor with respect to domestic labor given the arbitrary size of the shock $d \ln a^M$.[13] Namely, if elasticity σ_{lm,a^M}

[12] A detailed discussion of factor augmentation and bias is given in Acemoglu [1998].

[13] Although here we consider the foreign factor-augmenting productivity shock that does not change any domestic productivities, in estimation, we conjecture it is possible to relax this assumption and consider more formal moment conditions. In this line of argument, Adao et al. [2018] offer a rigorous derivation.

is

$$\sigma_{lm,a^M} \equiv \frac{d \ln l / d \ln a^M}{d \ln m / d \ln a^M}, \tag{3.17}$$

then by equations (3.15) and (3.16), we have

$$\sigma_{lm,a^M} = \frac{(\lambda - \sigma) \, WS_0^M + (\sigma - \varepsilon) \, CS_0^M}{-\lambda + (\lambda - \sigma) \, WS_0^M + (\sigma - \varepsilon) \, CS_0^M}. \tag{3.18}$$

In equation (3.18), readers might wonder why an increase in λ would result in a decrease in shock-induced elasticity σ_{lm,a^M}. To clarify, we discuss an extreme case when $\lambda < 1$, which means that the foreign factor augmentation is biased to *country-H* labor. In such a case, the negative foreign factor-augmenting productivity shock would *increase* hiring of the foreign factor. This is because the home and foreign factors would be gross complements, which in turn means that the foreign factor compression would result in the need to replenish the physical foreign factor rather than substituting for it. Therefore, relative to the case $\lambda > 1$, the denominator of equation (3.18) would be small, which would result in a *large* value of σ_{lm,a^M}, so long as $-\lambda + (\lambda - \sigma) \, WS_0^M + (\sigma - \varepsilon) \, CS_0^M > 0$.

To summarize the discussion, by equation (3.18), we can identify λ given the knowledge of σ_{lm,a^M}, constants $\left(WS_0^M, CS_0^M\right)$, and other parameters (σ, ε). We discuss how to obtain these constants and parameters in detail in the following sections as well as how to identify and estimate σ_{lm,a^M}. Finally, in the following sections, we use the identification arguments based on both equations (3.9) and (3.18).

3.4 Empirical Application–the *2011 Thailand Flood*

Given our theoretical result of identification in Section 3.3, we next discuss how we may obtain Σ_i in equation (3.10). Finding a plausibly exogenous shock to the multinational activities of MNEs is not trivial. For example, there are challenges that emerge from the small number of MNEs relative to the total number of firms. In particular, Boehm et al. [2020] mentions "the notorious difficulty to construct convincing instruments with sufficient power at the firm level."[14] In this paper, our approach is to focus on a unique natural experiment, the *2011 Thailand Floods*. We first describe the event and the interpretation for

[14] Such difficulty is reaffirmed in Section C.1.11 by analyzing substitution parameter λ by means of shift-share instruments in the manner of Desai et al. [2009] and Hummels et al. [2014].

our purpose in Section 3.4.1, and then discuss how the firm- and plant-level data described in Section 3.2.1 capture the event.

For our main empirical results, we rely on the homogeneous nested CES specification and identify parameters of interest based on the moment condition (3.18). Section 3.4.2 details the process. Section 3.5.3 discusses the alternative general approach based on the moment condition (3.9).

3.4.1 Background

Between July 2011 and January 2012, massive flooding occurred along the Mekong and Chao Phraya river basins in Thailand, which caused numerous factories in the area to halt operation. The magnitude of this shock to the production economy was extraordinary, causing an estimated $46.5 billion in economic damage, which was then the fourth costliest disaster in history (World Bank, 2011).[15] To the extent that the firms could not anticipate the flooding beforehand, we take this event as an exogenous shock. Section C.1.7 describes the results of a balancing test to confirm that there are not large systematic differences between the Japanese MNEs that had subsidiaries located in the flooded regions and those that did not.

Next, we argue that the flood can be interpreted as a *negative foreign productivity shock* for Japanese MNEs. First, as to whether or not it can be seen as a productivity shock, it is worthwhile to confirm that the shock was local. Thailand is subdivided into several provinces, and among them, Ayutthaya and Pathum Thani provinces along the flood-prone Chao Phraya river suffered severely from the flood. In these areas, the flood inundation reached its peak in October 2011. Adachi et al. [2016] find in their survey of local firms that in Ayutthaya and Pathum Thani provinces, the maximum days of inundation were 84 and 77, respectively, with maximal depth of flooding of 6 and 4 meters. In contrast, no firms from regions outside of Ayutthaya or Pathum Thani provinces claimed any days or height of inundation due to the 2011 floods.

In the severely damaged localities of Ayutthaya province and Pathum Thani province,

[15]To the extent that the economic damage includes the values of property damaged, natural disasters in developed countries are likely to be costlier. In fact, the three disasters whose economic damage surpassed the 2011 Thailand Floods at that time were the *2011 Tohoku Earthquake and tsunami* (Japan), the *1995 Great Hanshin Earthquake* (Japan), and the *2005 Hurricane Katrina* (U.S.). Given the extent of the economic damage, the physical shock to a developing country like Thailand should be regarded as even larger.

there were seven industrial clusters where roughly 800 factories were located [Tamada et al., 2013].[16] A large proportion of firms in these industry clusters are engaged in the automobile and electronics industries [Haraguchi and Lall, 2015]. Therefore, the flood shook local regions within Thailand that are intensively involved in industrial production, particularly automobile and electronics.

To further provide suggestive evidence that the flood was shock on the production side of the economy, we observe that, after the Floods, Thailand experienced a decrease in exports but not in imports. The observed pattern can be seen as evidence that the production side was hit by a shock rather than the demand side, as Benguria and Taylor [2019] argue in their interpretation of the shock origin of the global financial crisis of 2008. Section C.1.6 of the Appendix discusses this in detail.

What makes this event unique for our study is that although it hit localized regions of Thailand, it can be considered to be a *sizable foreign productivity shock* from the perspective of Japanese MNEs. To see this, we describe the close relationship between the two countries as investment destination and source country. Among the roughly 800 factories in the heavily flood-inundated industrial clusters, roughly 450 are Japanese subsidiaries.[17] Section C.1.7 describes the industrial patterns of factories that are subsidiaries of Japanese MNEs from our dataset, and the flood indeed created a large negative shock for Japanese producers. In Section C.1.7, we also show from our dataset that Thailand is a major destination country of Japanese MNEs.

Recall that our dataset in Section 3.2.1 covers the information well needed to study the impact of the flooding shock on factor employment. BSJBSA contains comprehensive data on firms, with domestic factor employment including employment, labor compensation, fixed assets and net income. BSOBA includes data on the universe of factories worldwide of Japanese MNEs. The plant-level variables contain the plant name, the parent firm name, employment, labor compensation, and net income. Orbis provides the exact addresses of these overseas plants, and TSR data facilitates the matching of these datasets since it contains the universe of Japanese firms. Together, these datasets allow us to analyze the

[16]Reinsurance broker Aon Benfield reported on the flood area and the locations of the inundated industrial clusters. Exhibit 16 of the report (available at http://thoughtleadership.aonbenfield.com/Documents/20120314_impact_forecasting_thailand_flood_event_recap.pdf) shows the relevant map.

[17]Exhibit 15 of the Aon Benfield report mentioned above shows a photo of the inundated Honda Ayutthaya Plant, which is located in Rojana Industrial Park, one of the seven severely flooded industrial clusters.

flood shock that hit a subset of firms in our analysis sample by micro-econometric methods.

The Thai Floods and Aggregate Trends

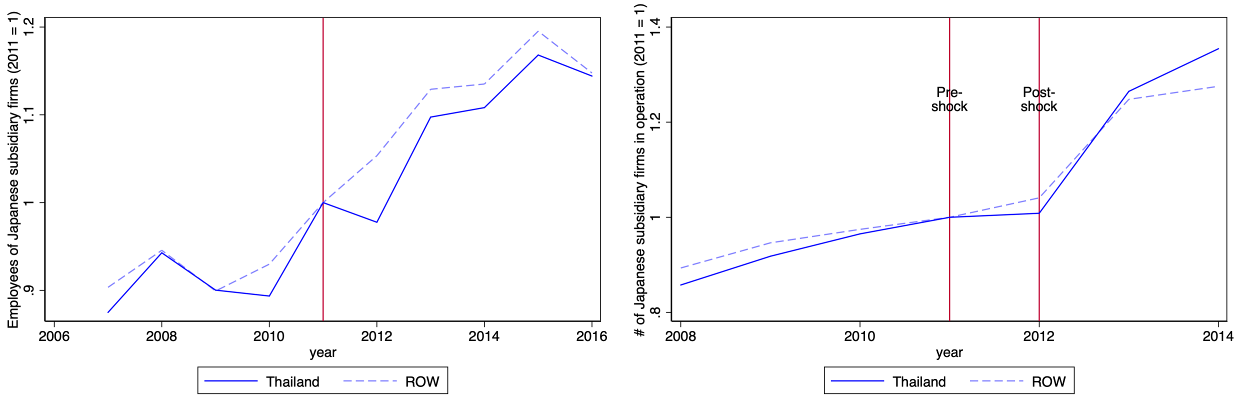
In this section, we provide the first-pass evidence of the effect of the floods on Japanese MNEs from our combined dataset described in Section 3.2.1. Figure 3.3 shows the relative trends for total employment (Panel 3.3a) and number of subsidiaries (Panel 3.3b) in the flooded regions versus the rest of the world, excluding Japan. In both panels, the solid line shows the trend for the flooded region and the dashed line shows that for the other regions excluding Japan (labeled ROW). Both trends are normalized at one in 2011. One can see that, for both statistics, the ROW trend is increasing over the sample period. This reflects the fact that more firms are entering the pool of MNEs, investing in Thailand, opening subsidiaries and hiring local workers, while MNEs already with Thai subsidiaries are expanding and hiring more local workers. When we turn to the flooded regions, however, the pattern is noticeably different. While the trend until 2011 is similarly rapid or even slightly faster than the ROW, indicating that those flooded regions had taken measures to attract foreign capital before the floods, the 2011 floods abruptly stopped this trend, creating a peak that year or with a year lag, after which both variables declined in magnitude.

What is further noteworthy is the *persistence* of the decrease. Even though the flood itself had subsided by early 2012, the decline in both total employment and number of subsidiaries continued at least up to 2016. A potential explanation can be found in news articles and academic discussions. Because the one-time event was large enough for firms to update their risk perception regarding future flooding, companies "move[d] to avoid potential supply chain disruptions" (Nikkei Asian Review, 2014). Similar arguments can be found in academic discussions of the negative effects of policy uncertainty on international trade and investment [Handley and Limão, 2017, Pierce and Schott, 2016, Steinberg et al., 2017]. See Section C.1.7 for other trends in investment and intermediate purchases beyond those discussed here.

Given the findings in Figure 3.3, we regard the elasticity findings below as the medium- to long-run elasticities as opposed to short-run. We return to this issue when we set out our empirical specification.

Figure 3.3: Relative Trends of Aggregate Variables in Flooded Regions

(a) Total employment (b) Count of subsidiaries



Note: Authors' calculation from BSOBA 2007-2016. "Flooded" shows the evolution of total employment in factories located in the flooded area of *Ayutthaya* and *Pathum Thani* provinces. "ROW" shows that from outside the flooded area. Both trends are normalized to 1 in 2011.

3.4.2 Estimation

For our main empirical results, we apply the identification strategy based on a linear regression (3.18). Specifically, we first calibrate the standard parameter values σ, ε and constants CS_0^F and WS_0^F as discussed in Section 3.4.2. Given these values and the reduced-form parameter of σ_{lm,a^M}, we may back out λ from equation (3.18). Section 3.4.2 discusses how we identify and estimate σ_{lm,a^M}.

Step 1: Calibration under Homogeneous Nested CES

We first discuss how we back out σ and ε from the data and then how to obtain λ from equation (3.18). As for the capital-aggregate labor elasticity σ, we employ the method developed in recent studies [Oberfield and Raval, 2014, Raval, 2019]. Specifically, the cost-minimizing factor demands (C.47) and (C.48) imply $\ln\left(rk/p^X x\right) = (\sigma - 1)\ln\left(p^X/r\right) +$ const. Furthermore, recall that for non MNEs, $p^X = w$ and $p^X x = wl$. Therefore, for non MNEs,

$$\ln \frac{rk}{p^X x} = (\sigma - 1)\ln \frac{p^X}{r} + \text{const.} \tag{3.19}$$

Based on this estimation equation, if we can obtain the coefficient on the log-relative factor price term, we can back out the substitution parameter σ. To operationalize the first-order condition relationship (3.19) to the data, we use the location m-level variation of each plant i, or $m(i)$. Because local regions constitute labor markets due to commuting immobility, wages vary across m's. Moreover, these variations are empirically persistent, so the coefficient obtained by this variation reveals the medium- to long-run elasticity of substitution. Note that the location-level variation in wage can be sourced from many shocks. Oberfield and Raval [2014], Raval [2019] let the location-level wage vary by a shift-share instrument. Therefore, the regression specification is

$$\ln\left(\frac{rk}{wl}\right)_i = b_0 + b_1 \ln\left(w_{m(i)}\right) + X_i b_2 + e_i, \tag{3.20}$$

with a shift-share instrument $z_m = \sum_{j\in\mathscr{J}^{NM}} \omega_{mj,-10} g_j$, where X_i is a plant-level control variable, j is an industry, $\mathscr{J}^{NM}$ is the set of non-manufacturing industries, $\omega_{mj,-10}$ is the employment share of industry j in location m in the ten-year prior to the analysis period, and g_j is the leave-m-out growth rate of national employment in industry j over the ten years that preceded the analysis year.

We apply this method to Japan's Census of Manufacture plant-level data, selecting only firms that do not have international factories, as the estimation equation (3.19) applies to non-MNEs. We define the unit of location m as the municipality.[18] There are several sources of municipality-level wage data, including the Japan Cabinet Office (CO) which provides municipality-level average wages. In addition, the *Basic Survey on Wage Structures* (BSWS) administered by Japan's Ministry of Health, Labour and Welfare offers national survey-based estimates of average municipal wages for each industry. Therefore, we have three alternatives for the w_m variable, and details of the estimation procedure are discussed in Section C.1.8.

The estimation results for our main specification are shown in Table 3.1. Depending on the choice of regressors, our estimates imply a *lower* substitution parameter $\sigma \leq 0.2$ than Oberfield and Raval [2014] (see Section C.1.8 of the Appendix for detailed comparison of this estimate to the values in the literature). The low substitution parameter would imply a larger effect of $d\ln A^F$ on dLS according to equation (3.14). We take the conservative

[18]The total number of municipalities was roughly 1700 as of 2005. This is a fairly small definition of the local labor market, resembling counties in the U.S., of which there are roughly 3000. Another potential choice for local labor markets in Japan are commuting zones recently used by Adachi et al. [2019], following the seminal method introduced and popularized by Tolbert and Sizer [1996]. In 2005, there were 331 commuting zones in Japan.

Table 3.1: Estimates of $\sigma - 1$

	IV, CO	IV, BSWS, all	IV, BSWS, manuf.
$\log(w_{m(i)})$	-1.15	-1.24	-0.88
	(0.18)	(0.18)	(0.13)
Num. obs.	51477	51477	51477

Notes: CO indicates that the wage data is from the Cabinet Office."BSWS, all" indicates that the wage variable is taken from all industries from the Basic Survey on Wage Structures (BSWS), while "BSWS, manuf." indicates that the wage variable is taken from only manufacturing industries from the BSWS. All regressions include industry FE and multiunit status indicators. Standard errors are clustered at the municipality level.

result by choosing the upper bound $\sigma = 0.2$ in the following quantitative exercise.

As for the demand elasticity ε, we again employ Japan's 2011 Census of Manufacture. Following Oberfield and Raval [2014], we back out ε by $\varepsilon = m/(m-1)$, where $m \equiv sales/cost$ is the measured markup.[19] The measured markup distribution is shown in Figure C.17. The implied average markup implies $\varepsilon \in [3.98, 4.88]$, depending on the treatment of extreme values. This lies well within the range of the demand elasticity estimate using firm-level markups. For a conservative impact on the labor share, we choose the low-end $\varepsilon = 4$, which is within the range of demand elasticities of different industries in the U.S. reported in Oberfield and Raval [2014].

To obtain the value for the foreign labor cost share of total cost $CS^M \equiv p^M m/\left(rk + wl + p^M m\right)$, we use the 2011 BSOBA survey data. Since our purpose is to back out λ from our estimate of ε_{lm,a^M}, we focus on firms located in the flooded region and then calculate, for each headquarter firm i,

$$CS^M_{i,2011} = \frac{\sum_{l \in flooded} \text{total payroll}^l_{f,2011}}{\sum_{l \in world} \text{total cost}^l_{f,2011}},$$

where $flooded$ is the set of locations that were hit by the Floods. The definitions of total payroll and total cost are provided in Section C.3.1. We then obtain the 2011 firm-level average value of $CS^M = 2.4\%$ (see Figure C.18 for the complete CS^M distribution). Accordingly, we obtain the 2011 average value of $WS^M = 4.0\%$ by replacing the denominator of CS^M_i with the sum of payrolls at all locations l in the world. Section C.1.8 provides estimation results for ε and CS^M.

[19] Note that we use the 2011 survey because equations (3.15) and (3.16) should be evaluated at the time of the shock, which is 2011 in our case.

Step 2: Estimating λ by Natural Experiment

For obtaining λ by Equation (3.18), our goal is to estimate the left-hand side parameter σ_{lm,a^M}. In our empirical application, we specify $H = JPN$ and $F = ROW$. We measure foreign factor employment m_{it} by total foreign labor employment since in our data, the quantity of factor employment is only available for labor. We thus specify the factor substitution between country H and F in the model as the substitution of labor across countries. As discussed further below, our result is robust to other choices for measuring m_{it}. Hence, we use the notation l^{JPN} as employment in Japan and l^{ROW} as employment in the rest of the world, measuring factor employment in the rest of the world.

We run the following regression

$$\ln\left(l_{it}^{JPN}\right) = a_i + a_t + b\ln\left(l_{it}^{ROW}\right) + e_{it}, \tag{3.21}$$

where $\ln\left(l_{it}^{JPN}\right)$ is log of firm i in year t, $\ln\left(l_{it}^{ROW}\right)$ is the log factor employment in the rest of the world, a_i and a_t are firm- and year-fixed effects, and e_{it} is the error term.

It is critical to control these rich fixed effects. In fact, controlling firm fixed effects restricts ourselves to leverage within-firm variations, since high-productivity firms are likely to hire workers in the ROW (or conduct FDI and become an MNE, as in Helpman et al., 2004). On the other hand, controlling for year fixed effects enhances the validity of our analysis given the economic environment in which an increasing number of firms become MNEs and hire more local workers in foreign countries, as we saw in Figure 3.3.

In the data, the variation in the explanatory variable $\ln\left(l_{it}^{ROW}\right)$ can emerge from many sources, one of which is the firm-specific exchange rate that occurs through demand shocks or total factor productivity. Since we look for a foreign factor-augmenting productivity shock in equation (3.17), we construct an instrumental variable (IV) based on the 2011 Thailand Floods. For this purpose, note that the flooding was local, it occurred during one limited period of time relative to the coverage of our dataset and, most importantly, it was unexpected. We construct an IV that interacts in the Thailand location before the flood and the year after the flood. Because the shock was unexpected, the IV is exogenous to firms' foreign production decisions, after controlling for the firm and year fixed effects. To leverage the variation in $\ln\left(l_{it}^{ROW}\right)$ caused by this foreign productivity shock, an IV of a shock intensity measure is used:

$$Z_{it} \equiv \frac{l_{i,2011}^{flooded}}{l_{i,2011}^{JPN} + l_{i,2011}^{ROW}} \times \mathbf{1}\{t \geq 2012\}, \tag{3.22}$$

where $l_{i,2011}^{flooded}$ is firm-i's total employment in the flooded regions in year 2011, right before the flooding. This measure captures how much each MNE i relies on employment in the flooded region. Namely, if a firm hires a relatively large number of workers in the flooded region immediately before the flood, the firm is likely to be hit by the flood severely, thus receiving a large negative (firm-i) foreign factor-augmenting productivity shock.

Given this instrument, the two-stage least square (2SLS) estimator is based on the following equations:

$$\ln\left(l_{it}^{ROW}\right) = \widetilde{a_i} + \widetilde{a_t} + \widetilde{b}Z_{it} + \widetilde{e_{it}}, and \tag{3.23}$$

$$\ln\left(l_{it}^{JPN}\right) = a_i + a_t + bZ_{it} + e_{it}. \tag{3.24}$$

Therefore, we expect the first stage regression will yield a negative correlation between $\ln\left(l_{it}^{ROW}\right)$ and Z_{it} conditional on the fixed effects. Given the validity of the first stage, we interpret

$$\widehat{b_{IV}} = \widehat{\sigma_{lm,a^M}}.$$

As shown in Section 3.4.1, the floods had medium- to long-run effects rather than short-run effects on employment, so coefficient $\widehat{b}_{IV}$ or, in turn, λ as meduim- to long-run elasticity rather than short-run. Specifically, we relate the decline in employment found in aggregate in Panel 3.3a as an exogenous-sourced decline in ROW log-employment $\ln\left(l_{it}^{ROW}\right)$, and relate that to the change in log-employment in Home $\ln\left(l_{it}^{JPN}\right)$. This point is crucial when we move on to the quantitative exercise, because our concern is a relatively long-run change in labor share. In the discussion below, we conduct a robustness check of the long-difference specification and extension to the event-study regressions.

Table 3.2 shows the estimation result from the 2SLS specification (3.23) and (3.24), with the robust standard errors reported in parentheses. All columns show that the coefficients are statistically significant at the two-sided one-percent level.

Column 1 shows the result of the OLS specification without any fixed effects; that is,

Table 3.2: Estimating σ_{lm,a^M}

VARIABLES	(1) $\ln l_{it}^{JPN}$	(2) $\ln l_{it}^{JPN}$	(3) $\ln l_{it}^{JPN}$	(4) $\ln l_{it}^{ROW}$	(5) $\ln l_{it}^{JPN}$
$\ln l_{it}^{ROW}$	0.446	0.0604	0.192		
	(0.00686)	(0.0106)	(0.0502)		
Z_{it}				-0.728	-0.140
				(0.108)	(0.0367)
Observations	5,563	5,563	5,563	5,563	5,563
Model	OLS	FE	2SLS	2SLS-1st	2SLS-reduced
Firm FE	-	YES	YES	YES	YES
Year FE	-	YES	YES	YES	YES

Robust standard errors in parentheses.

under the restriction $a_i = a_t = 0$ for all i and t, column 2 the fixed effect specification, and column 3 the instrumental variable specification in equation (3.22). Comparing Columns 1 and 2, we see that while both coefficients are positive, the coefficient magnitude becomes small after controlling for fixed effects as to heterogeneity in the productivity of firms.

Column 3 shows that a one-percent decrease in employment in the rest of the world due to the 2011 Thailand Floods caused Japanse MNEs to *decrease* home employment by 0.192 percent. Although this coefficient is a composite of model parameters as shown in equation (3.18) without any meaningful interpretation by itself, it is produced by the 2SLS first stage and reduced-form regressions shown in columns 4 and 5. Column 4 shows that a firm that did not rely on employment in the flooded region in 2011 would have reduced its employment in the rest of the world by 72.8 percent had it relied completely on the employment there.[20] Given that a few firms resumed operations relatively quickly after the flooding subsided, a reduction less than 100 percent is reasonable. As mentioned in Section 3.3.3, this number is proportional to the composite direct and indirect effect of the flood shock, related to equation (3.16). More interestingly, column 5 reveals that the same hypothetical increase in the reliance in Thailand employment would cause the firm to *reduce* employment in Japan by 14.0 percent. Again, this cross effect of a foreign factor-augmenting productivity shock on home employment is indicative of the indirect effect

[20]The standard deviation of our IV is 0.157, so a one standard deviation increase in our IV translates to an 11.4% decrease in employment in foreign countries.

described in equation (3.15). Thus, the fraction of these two coefficients, 0.192 in column 3, indicates the direct effect of the flood, or the implied elasticity of substitution between home and foreign labor inputs. Since the sign of the regression coefficient in column 5 is the key to the sign of the 2SLS estimate and thus the value of λ, we conduct a robustness check in Section C.1.10.

Before turning to the backing out of our parameter of interest λ, it is worthwhile to note that the column 3 estimates of foreign employment are higher than in column 2. This is due to the difference in the source of identifying variation and is indicative of the benefit of our natural experiment approach. Namely, in the 2SLS specification, the source of variation is the offshore productivity shock, so the coefficient reflects only the structural interpretation (i.e. the function of structural parameters). However, the fixed effect specification does not identify any structural parameters. For example, if the underlying variation is an increase in wages in the offshore country relative to Japan caused by, for example, TFP growth in the offshore country, then the substitution from offshore labor to Japanese labor (so-called inshoring) would have a negative correlation between offshore labor and Japanese labor. This potentially explains why the coefficient for the fixed effect specification would be smaller in magnitude than that for 2SLS.

Based on the calibrations of σ and ε from Section 3.4.2, equation (3.17) implies that $\lambda = 1.4$. Since ε_{lm,a^M} has a valid standard error from the 2SLS estimation, we can obtain the standard error of λ as 0.13 by the Delta method, indicating that we reject the gross-complementary home and foreign labor $\lambda \leq 1$ at the 0.1 percent significance level, as discussed in detail in Section C.1.9. Therefore, our calibrations $\sigma = 0.2$ and $\lambda = 1.4$ imply that Assumption 2 is satisfied empirically. Applying Lemma 1, we conclude that a positive foreign factor augmentation causes a qualitative decrease in labor share. To proceed to the quantitative implication, in what follows we back out the foreign factor augmentation from the aggregate data.

In Section C.1.10 of the Appendix, we discuss a number of robustness checks, including use of other measures of MNEs like foreign operation at extensive margins and subsidiary sales, choice of control groups, and the role of *Tohoku Earthquake and tsunami*. In Section C.1.11, we also show some extension exercises, such as the use of widely-used shift-share instruments, event study-style regression analysis, substitution between Thailand and other third countries, and industry-specific effects.

3.5 Discussions

3.5.1 Quantitative Implication of Foreign Factor Augmentations

In Section 3.4, we discussed how we backed out the *elasticity* of labor share with respect to the foreign factor augmentation. To derive the implication quantitatively, we need to know how much foreign factor-augmenting productivity grew over the period of interest. For this purpose, we invert the factor demand functions (C.49) and (C.50) in the aggregate. We proxy the quantity of employment of foreign factors M by the foreign employment L^{ROW} and accordingly the foreign factor prices by labor wage w^{ROW}. We apply $H = JPN$ and $F = ROW$ to obtain[21]

$$\frac{a^M}{a^L} = \left(\frac{L^{ROW}}{L^{JPN}}\right)^{\frac{1}{\lambda-1}} \left(\frac{w^{ROW}}{w^{JPN}}\right)^{\frac{\lambda}{\lambda-1}}. \tag{3.25}$$

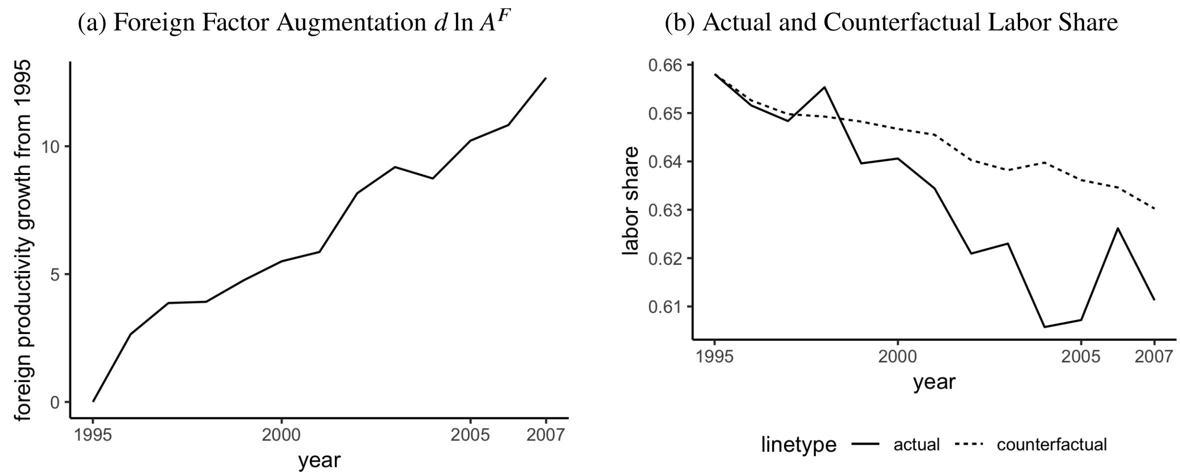
Therefore, given the measures of aggregate employment and average wage in the home and foreign countries, and the backed-out elasticity between home and foreign labor, we can obtain the implied relative productivity in the rest of the world. The intuition behind the relationship is as follows. Suppose, as we calibrated, $\lambda > 1$, or factor augmentation is biased to that factor. Then given the wage structure in the rest of the world and Japan, relatively large employment in the rest of the world implies relatively augmentingly productive labor in the rest of the world. On the other hand, given the employment structure, a relatively high wage in the rest of the world also reflects relatively augmentingly productive labor in the rest of the world.[22] We measure $\left(L^{ROW}, w^{ROW}\right)$ by BSOBA. In particular, we calculate L^{ROW} by aggregating all foreign employment in all countries except for Japan and w^{ROW} by dividing the aggregate total labor compensation by L^{ROW}. For $\left(L^{JPN}, w^{JPN}\right)$, we apply the JIP database, a Japanese project for assembling KLEMS database.

To obtain the absolute productivity a^M as opposed to the relative productivity a^M/a^L, we calibrate Japan's labor-augmenting productivity growth by JIP database's Quality of Labor measure. The JIP measure reflects changes in the composition of workers–gender, age, education, employment status– which is standard representation of factor augmentation.

[21]It is an important extension to consider a multi-country version of our model and its empirical implementations. Section C.1.7 reviews the country-level aggregate trends.

[22]Note our qualification $\lambda > 1$. In fact, if $\lambda < 1$, then the discussion in the text reverses, which means that an observed increase in foreign employment and wages would imply *decreasing* relative foreign factor augmentation. To the extent that such a decrease is implausible, this observation is more supporting evidence for $\lambda > 1$. Section C.3.1 shows the implication to A^{ROW} when it is nonetheless assumed that $\lambda < 1$.

Figure 3.4: Quantitative Implications



We argue that this affects the efficiency units of labor, thereby Japan's labor-augmenting productivity. Given this trend, we can separate the trends of factor augmentation in Japan and the rest of the world. As a result, in Figure 3.4a, the left panel shows the trend of the evolution of $d \ln a_t^M \approx \ln a_t^M - \ln a_{1995}^M$, with our base year 1995, while Figure C.22 shows the growth in Japan's labor augmentation.

We emphasize the relative importance of $d \ln a_t^M$ and $d \ln a_t^L$. By comparing the y-axes, we confirm that the growth $d \ln a^L$ is relatively minor in the relative factor augmentation evolution $d \ln \left(a^M / a^L\right) = d \ln a^M - d \ln a^L$ in equation (3.25). This could be interpreted as due to at least two factors. First, the relative augmentation was fast in the rest of the world relative to in Japan. This is plausible given that our BSOBA-based measure of L^{ROW} and w^{ROW}, the ingredients in equation (3.25), comes from employment data in many rapidly expanding economies including Thailand. Second, globalization may contribute to the relative importance of $d \ln a^M$, comprising declining transportation and communication costs as well as the removal of political barriers to investment. These imply higher employment in the rest of the world by Japanese MNEs given the factor costs, according to our estimated elasticity of substitution $\lambda = 1.4$.

With these calibrations, we may learn the evolution of the effect of foreign factor augmentation on labor share. We calculate WS_0^M as the fraction of total payment to foreign labor taken from BSOBA over the sum of that and the total payment to workers in Japan in the base year, which gives $WS_0^M = 0.8\%$. In Figure 3.4b, we show the counterfactual labor share had there been only the foreign augmentation from equation (3.14). Again, we set the base year to 1995, the first year of the BSOBA data. The solid line shows the actual evolution, whereas the dashed line shows the counterfactual one, and we see that foreign factor augmentation played a substantial role in explaining the observed labor share decline from 1995-2007. Quantitatively, it explains 59 percent of the decline during the period.

Discussion One might naturally ask whether this large effect is an artifact of our choice of sample period. We chose a baseline final year of 2007 for our analysis both because the Great Recession would complicate the interpretation and also because the international System of National Accounts (SNA) changed drastically after 2008. However, Section C.3.2 shows the corresponding results for a more recent trend in labor share up to 2015,[23] and we find that 58 percent of the decrease in labor share from 1995 to 2015 may be attributed to our mechanism of increased foreign productivity. Our mechanism is also successful in explaining the countercyclical component of labor share trend in the data.

We also conduct the same quantitative analysis from peak to peak to further control for potential effects of business cycle. We take peak years, 1997, 2000, and 2008, from the Cabinet Office of Japan. When we conduct the quantitative analysis from 1997 to 2008 and 2000 to 2008, our mechanism could explain the decrease in the labor share by 77 percent and 112 percent. The reason we obtain large values for the recent waves of the business cycle is that the actual decrease in the labor share was small, especially between 2000 and 2008, so the fraction that our mechanism explains becomes large.

Finally, we investigate how our quantitative results may differ given the error in the estimate of our key parameter, λ. To do this, we use the standard error estimate $\widehat{se(\lambda)} = 0.13$ that we derive in Section C.1.9 of the Appendix. In particular, we conduct the same quantitative analysis with values of λ one standard error smaller and larger. The results indicate that the quantitative magnitude relative to the observed decrease in the labor share varies between 48 percent to 83 percent depending on the value of λ. Although there is an uncertainty in the exact size of the impact, we conclude that there was clearly a significantly negative effect on the labor share in Japan due to foreign factor augmentation.

[23] As a reservation, the quantitative result is sensitive to our parameter values, in particular the estimate of σ. Section C.3.1 shows different results under several parameter values of λ and σ.

3.5.2 Role of Firm Heterogeneity

Our estimation and quantitative implications have relied on the homogeneous production function derived in Section 3.3.4. An implication of this restriction is the coincidence of micro- and macro-elasticities [Oberfield and Raval, 2014]. However, since we observe rich heterogeneity at the firm level, foreign factor augmentation could potentially *reallocate* factor resources from one firm to another. Since factor prices clear the factor markets, such heterogeneity and reallocation may matter for the relative wage, which through equation (3.5) affects the labor share. To examine this effect clearly, we consider a modified version of the model in Section 3.3.4; a general equilibrium with the same nested CES production function but with firm-level *heterogeneous* augmentations. To facilitate the comparison, we also consider a case with the same foreign factor augmentation across firms, $d \ln a_i^M = d \ln a^M$ for any i. This is consistent with the interpretation that the foreign factor augments because policy and institutional changes or technological progress in country F affects all firms in country H alike.

In Section C.2.6, we prove that the change in the relative wage in this case may be solved as

$$d \ln w - d \ln r \propto \left[-(\lambda - \sigma) WS_l^M + (\sigma - \varepsilon) \left(CS_k^M - CS_l^M \right) \right] d \ln a^M, \tag{3.26}$$

where

$$WS_l^M \equiv \int \frac{wl_i}{wL} \frac{p^M m_i}{wl_i} di, CS_k^M \equiv \int \frac{rk_i}{rK} \frac{p^M m_i}{p_i q_i} di, CS_l^M \equiv \int \frac{wl_i}{wL} \frac{p^M m_i}{p_i q_i} di.$$

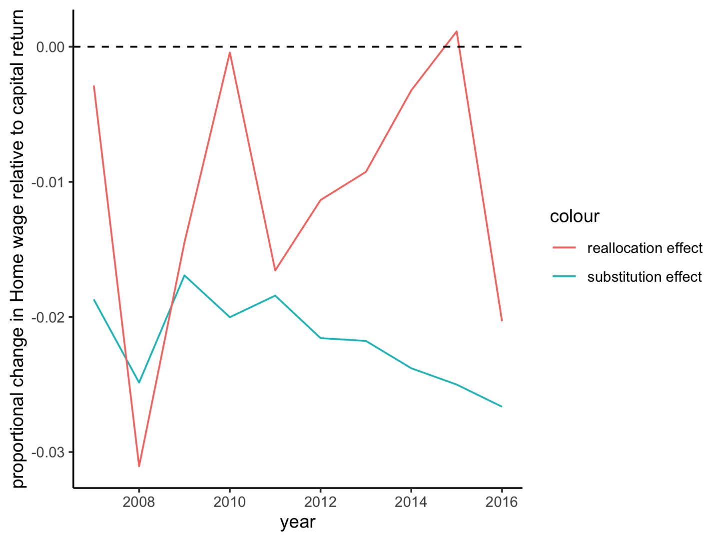
To interpret the terms in equation (3.26), note that the first term reflects the differences in the elasticities in the upper σ and lower nest λ. If $\lambda > \sigma$, then the increase in foreign productivity substitutes labor in country H relatively more than capital in country H, which results in a downward pressure to the wage relative to capital return and, in turn, the labor share. This effect depends on the average wage payment share to the foreign factor WS_l^M. These arguments do not involve the firm heterogeneity and appear in the homogeneous model as well. We call this term the *substitution effect.*

On the other hand, the second term involves the weighted averages of the cost share to the foreign factor CS_k^M and CS_l^M. More specifically, the difference between the differentially weighted averages $CS_k^M - CS_l^M$ matters. To understand this term, suppose that (i) demand elasticity ε is so elastic that $\varepsilon > \sigma$, and (ii) the foreign factor share distribution is *more skewed* to capital intensive firms than it is to Home labor intensive firms, or $CS_k^M - CS_l^M > 0$. Note that assumption (ii) is consistent with the finding by Sun [2020] that "multinational firms are on average larger firms and larger firms on average use more capital-intensive technologies." Given these two assumptions, the increase in the foreign factor productivity results in a reduction in the marginal cost of production, which causes high demand if ε is large. Across firms, this is moreso for firms that use relatively more foreign factors. If such firms are more capital intensive than Home labor intensive, the aggregate demand for capital increases more than that for Home labor. This causes the capital return to increase more than the Home wage, which results in a reduction in the labor share. We call this effect the *reallocation effect.* Note that the argument above crucially depends on the heterogeneity in firms' factor intensities. Indeed, if firms are homogeneous, the weight of the weighted average of cost shares does not matter, so that $CS_k^M - CS_l^M = CS^M - CS^M = 0$. Therefore, when there is no heterogeneity in firm factor intensity, there is no reallocation effect and the effect on the Home relative wage only emerges as the substitution effect.

To measure the size of the substitution and reallocation effects, we employ our matched dataset. BSJBSA data contains the variable of cost of labor compensation in the domestic firms and net operating surplus. We take these measures as wl_i and rk_i, respectively. From BSOBA, we can obtain the total compensation of MNEs in each foreign plant, which we aggregate to form a measure of $p^M m_i$. We match these datasets and calculate CS_k^M, CS_l^M, and WS_l^M that are relevant in equation (3.26). The matched data is available for 2007-2016. With values $\lambda = 1.4, \sigma = 0.2, \varepsilon = 4$, we plot the size of the substitution and reallocation effects according to data from each year (Figure 3.5).

Figure 3.5 provides two takeaways. First, both the substitution and reallocation effects are negative for most years. Indeed, except for the reallocation effect in 2015, all effects head to the negative side. This implies that when foreign factor productivity increases, the relative Home wage decreases, which pushes down the Home labor share. Moreover, introducing heterogeneity into the model *strengthens* the negative effect of a given increase in the foreign productivity due to the reallocation effect. Note that the substitution effect is negative because $\lambda > \sigma$. The reallocation effect is mostly negative both because (i) $\varepsilon > \sigma$ and (ii) in most of the years $CS_k^M > CS_l^M$ in the data. Second, in many years,

Figure 3.5: Relative Size of the Effects



Note: The 2007-2016 trend of the relative size of the reallocation effect and substitution effects among the drop in the labor share. The reallocation effect is the component due to firms' heterogeneous factor inputs, and the substitution effect shows the capital-labor substitution given the relative factor price changes within a firm. See the main text for the detail.

the reallocation effect is smaller than the substitution effect in absolute value. Therefore, although the reallocation amplifies the negative substitution effect, it does not contribute to the overall effect in a dominant way.[24] Combining these two observations, we conclude that firm heterogeneity and the reallocation effect does not alter our conclusion that foreign factor augmentation worked as a force that decreased the labor share in Japan from 1995-2007.

3.5.3 Method of Moments Estimation

In this section, we apply the estimation method based on the general moment conditions (3.9). For this purpose, we take a two-step approach. First, we separate the set of parameters estimated by equation (3.9) and other nuisance parameters.[25] We then fix the nuisance parameters by the method based on the nested CES specification discussed so far. Second, given these parameters, we identify and estimate the parameters of our interest.

In particular, given the elasticity matrix discussed in Section C.2.5, for all i, we set $\sigma_{\widetilde{k}\widetilde{r},i} = -\sigma + (\sigma - \varepsilon)\, CS_i^K$, $\sigma_{\widetilde{k}\widetilde{w},i} = (\sigma - \varepsilon)\, CS_i^L$, $\sigma_{\widetilde{l}\widetilde{r},i} = (\sigma - \varepsilon)\, CS_i^K$, $\sigma_{\widetilde{l}\widetilde{w},i} = -\lambda +$

[24]One may see relatively high volatility of the reallocation effect. This is due in part to the fact that we measure the capital payment by accounting net operating surplus which is volatile.

[25]Section C.3.3 discusses the estimation of standard errors of the estimator defined by equation (3.9).

$(\lambda - \sigma) WS_i^L + (\sigma - \varepsilon) CS_i^L$, $\sigma_{\widetilde{m}\widetilde{p^M},i} = -\lambda + (\lambda - \sigma) WS_i^M + (\sigma - \varepsilon) CS_i^M$, where σ and λ are constants reflecting the substitution parameters under nested CES, CS_i^f are firm i's factor-f cost share for $f = K, L, M$, and WS_i^f are firm i's factor-f payment share between L and M for $f = L, M$. We take CS_i^f and WS_i^f from our firm-level Japanese MNE data, and calibrate $\lambda = 1.4$, $\sigma = 0.2$, and $\varepsilon = 4$.

We then estimate the remaining parameters, elasticities of country-H capital and labor demand with respect to foreign factor prices. We employ the method of moments (3.9) and the flood-based instrumental variable. In particular, first, to have restrictions on $\sigma_{\widetilde{m}\widetilde{r},i}$ and $\sigma_{\widetilde{m}\widetilde{w},i}$, note that by the symmetry of the Hessian matrix of the cost function, demand structure (3.1), and Shephard's lemma, we have

$$CS_i^M \sigma_{\widetilde{m}\widetilde{r},i} = CS_i^K \sigma_{\widetilde{k}\widetilde{p^M},i}, \tag{3.27}$$

$$CS_i^M \sigma_{\widetilde{m}\widetilde{l},i} = CS_i^L \sigma_{\widetilde{l}\widetilde{p^M},i}. \tag{3.28}$$

Proof of these equations is given in Section C.2.7. Finally, to implement the estimation, we assume the following constant parameter assumption.

Assumption 3. *$\sigma_{\widetilde{k}\widetilde{p^M},i} = \sigma_{\widetilde{k}\widetilde{p^M}}$ and $\sigma_{\widetilde{l}\widetilde{p^M},i} = \sigma_{\widetilde{l}\widetilde{p^M}}$ for all $i \in I_H$.*

Given this setup, we implement the method of moments estimation based on equation (3.9) as follows.

1. Set $n = 0$. Guess $\left(\sigma_{\widetilde{k}\widetilde{p^M}}, \sigma_{\widetilde{l}\widetilde{p^M}}\right) = \left(\sigma^{(n)}_{\widetilde{k}\widetilde{p^M}}, \sigma^{(n)}_{\widetilde{l}\widetilde{p^M}}\right)$ and generate implied firm-level elasticity matrix $\Sigma^{(n)}$ based on equation (3.10) and our set of partial identification assumptions and the factor augmentation

$$\begin{pmatrix} d\ln\left(a_{it}^{K,(n)}\right) \\ d\ln\left(a_{it}^{L,(n)}\right) \\ d\ln\left(a_{it}^{M,(n)}\right) \end{pmatrix}$$

based on equation (3.11).

2. Remove the year fixed effects from the factor augmentation and instrumental variables Z_{it}.

3. Evaluate the sample-analog of the moment condition (3.9).

4. If a closed-form solution is not obtained, update n, go back to process 1 and iterate until convergence.

Out of the above algorithm, we obtain

$$\widehat{\sigma_{\widetilde{kp^M}}} = -0.20 \text{ (std. err. 0.13)}, \tag{3.29}$$

$$\widehat{\sigma_{\widetilde{lp^M}}} = -0.09 \text{ (std. err. 0.04)}. \tag{3.30}$$

Recall that our finding that the capital demand is more elastic, $\widehat{\sigma_{\widetilde{kp^M}}} < \widehat{\sigma_{\widetilde{lp^M}}}$, suggests that a negative cost shock (or positive factor augmentation) increases the demand for Home capital *relatively more than* Home labor. Given that capital demand is more negatively elastic to the capital rental rate and labor demand is to wage, the capital rental rate must increase while the wage for labor must decrease to restore the factor market clearing conditions (3.3).

Note that the last part of this logic is reminiscent of the celebrated Stolper-Samuelson theorem: Consider a two-factor economy. If the factor demand increases *differentially* for one factor, then the relative wage of that factor increases (more than the increase in the factor demands) while the wage of the other decreases. In the Stolper-Samuelson setup, the factor demand increase is caused by an exogenous change in the terms of trade. On the other hand, in our setup, what drives the changes in factor demands in H is the combination of foreign factor augmentation and (total) elasticity with respect to it of factors in H. If (same-sized) foreign factor augmentation elastically increases the relative demand for one factor (in our empirical case, capital relative to labor), that means that the demand for the factor increases differentially, or the factor is complementary to the foreign factor that was augmented.

3.6 Conclusion

What impact does increased employment of foreign factors of production by an MNE have on labor share in the home country? To address this question, we proceeded in three steps. First, we developed an equilibrium model of production featuring augmented employment of foreign factors with varying elasticities of substitution. From this theory, we then pinpointed the key elasticities relevant to labor share: the relative size of labor, and capital substitution with foreign factors, and showed how a foreign factor-augmenting productivity shock to a small set of firms may help to identify these elasticities. Armed with these theoretical results, we then applied the theory to the 2011 Thailand Floods, which was experienced as a large negative foreign factor productivity shock for a subset of Japanese MNEs. Using firm- and plant-level data from Japan, we estimated the reduced-form parameter with the instrumental variable related to the flood shock, finding that home and foreign labor are gross substitutes. This estimate, combined with the estimates of capital-labor elasticity based on our Japanese plant-level data, indicates that foreign factor augmentation contributed to the decline in the labor share in Japan from 1995-2007. Our further quantitative counterfactual analyses show that 59 percent of the observed labor share decline in the period can be attributed to foreign factor augmentation.

There are at least three different directions for extending the model. First, we may incorporate rich heterogeneity at the firm level as discussed in Section C.2.6 and at the destination country level as we empirically suggest in Section C.1.7. Second, our production model could potentially generalize an influential model of factor offshoring such as Feenstra and Hanson [1997], which would tighten the connection with the literature. For this purpose, we suggest a generalized production function in Section C.2.2. Finally, while our model extends to a more complete general equilibrium with factor supplies and a large open economy, all of the results of this paper in terms of identification, empirical application, and quantification depend on the specific model choice, so this is another theoretical development left for future study.

Appendix A

Appendix for Chapter 1

A.1 Data Appendix

A.1.1 Data Sources in Detail

I complement data from JARA data and O*NET data by the ones from IFR, BACI, IPUMS USA and CPS. IFR is a standard data source of industrial robot adoption in several countries (e.g., Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020, AR thereafter), to which JARA provides the robot data of Japan. I use IFR data to show the total robot adoption in each destination country as opposed to the import from Japan. I use Federal Reserve Economic Data (FRED) to convert JARA variables denominated in JPY to USD. BACI provides disaggregated data on trade flows for more than 5000 products and 200 countries and is a standard data source of international trade [Gaulier and Zignago, 2010]. I use BACI data to obtain the measure of international trade of industrial robots and baseline trade shares. IPUMS USA collects and harmonizes US census microdata [Ruggles et al., 2018]. I use Population Censuses (1970, 1980, 1990, and 2000) and American Community Surveys (ACS, 2006-2008 3-year sample and 2012-2016 5-year sample). I obtain occupational wages, employment, and labor cost shares from these data sources. To obtain the intermediate inputs shares, I take data from the World Input-Output Data (WIOD) in the closest year to the initial year, 1992.

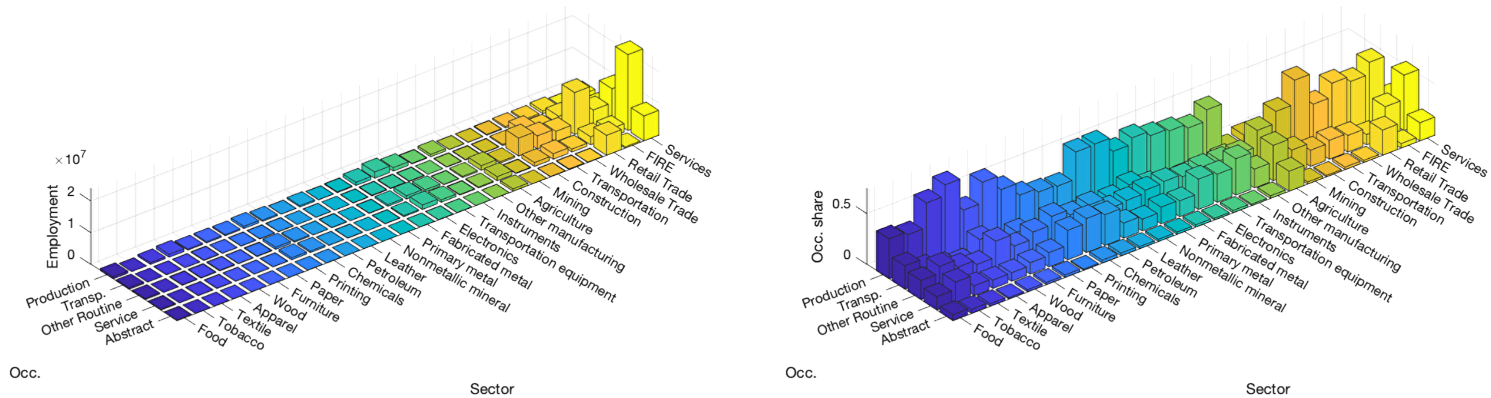
I use the match score from the O*NET Code Connector that contains detailed textual descriptions of 4-digit occupations. The match score is an output of the *weighted search algorithm* used by the O*NET Code Connector, which is the internal search algorithm developed and employed by O*NET and since September 2005. Since then, the O*NET has continually updated the algorithm and improved the quality of the search results. Morris [2019] reports that the updated weighted search algorithm scored 95.9% based on the position and score of a target best 4-digit occupation for a given query.

I followed Autor et al. [2013] for Census/ACS data cleaning procedure. Namely, I extract the 1970, 1980, 1990, 2000 Censuses, the 2006-2008 3-year file of American Community Survey (ACS), and the 2012-2016 5-year file of ACS from Integrated Public Use Micro Samples. For each file, I select all workers with the OCC2010 occupation code whose age is between 16 and 64 and who is not institutionalized. I compute education share in each occupation by the share of workers with more than "any year in college," and foreign-born share by the share of workers with BPL (birthplace) variable greater than 150, or those whose birthplace is neither in the US nor in US outlying areas/territories. I compute hours worked by multiplying usual weeks worked and hours worked per week. For 1970, I use the median values in each bin of the usual weeks worked variable and assume all workers worked for 40 hours a week since the hour variable does not exist. To compute hourly wage, I first impute each state-year's top-coded values by multiplying 1.5 and divide by the hours worked. To remove outliers, I take wages below first percentile of the distribution in each year, and set the maximum wage as the top-coded earning divided by 1,500. I compute the real wage in 2000 dollars by multiplying CPI99 variable prepared by IPUMS. I use the person weight variable for aggregating all of these variables to the occupation level. Figure A.1 shows the occupational employment distribution for each sector, a variable used for creating the occupational China shock in equation (1.2).

To estimate the model with workers' dynamic discrete choice of occupation, I further use the bilateral occupation flow data following the idea of Caliendo et al. [2019]. Specifically, I obtain the Annual Social and Economic Supplement (ASEC) of the CPS since 1976. For each year, I select all workers with the 2010 occupation code for the current year (OCC2010) and the last year (OCC10LY) whose age is between 16 and 64 and who is not institutionalized, and treated top-coded wage income, converted nominal wage income to real one, and computed labor hours worked, education, foreign born flag variable with the same method as the one used for Census/ACS above. When computing the occupation switch probability, note that the 4-digit occupations are too disaggregated to precisely estimate with the small sample size of CPS-ASEC, as pointed out by Artuç et al. [2010].

Figure A.1: Occupational Employment Distribution

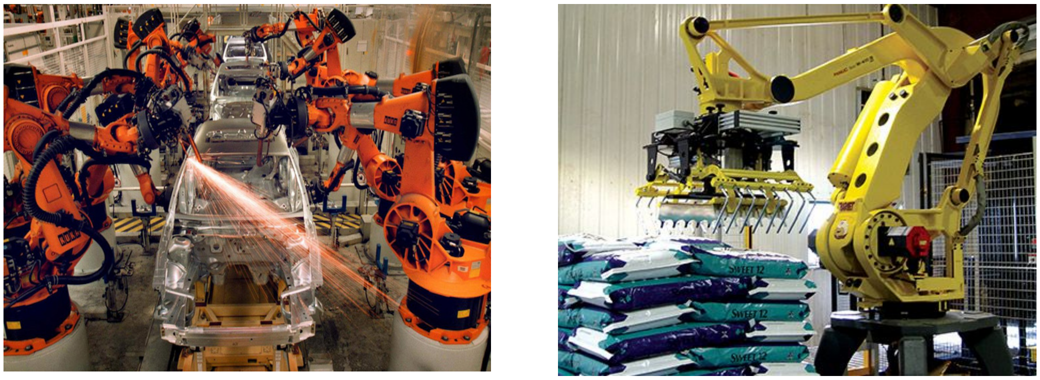
(a) Employment size L_{s,o,t_0} (b) Employment share l_{s,o,t_0}



Note: The author's calculation from the 1990 US Census. The axis on the left indicates the 5 occupation groups defined in Section 1.4.1, and the one on the right shows sectors (roughly 4-digit for manufacturing sectors and 2-digit for the non-manufacturing). The left panel shows the size of employment, and the right one indicates the occupation share for each given sector.

Figure A.2: Examples of Industrial Robots

(a) Spot Welding (b) Material Handling



Sources: Autobot Systems and Automation (https://www.autobotsystems.com) and PaR Systems (https://www.par.com)

Therefore, I assume that the occupations do not flow between 4-digit occupations within the 5 groups defined in Section 1.4.1, but do between the 5 groups. I assume that workers draw a destination 4-digit occupation occupation from the initial-year occupational employment distribution within the destination group when switching occupations. With these data and assumptions, I compute the occupation switching probability by year.

A.1.2 Details in Industrial Robots

Definition and Examples

As defined in Footnote 1, industrial robots are defined as multiple-axes manipulators. More formally, following International Organization for Standardization (ISO), I define robots as "automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications" (ISO 8373:2012). This section gives a detailed discussion about such industrial robots. Figure A.2 shows the pictures of examples of industrial robots that are intensively used in the production process and considered in this paper. The left panel shows spot-welding robots, while the right panel shows the material-handling robots. The spot welding robots are an example of robots in routine-production occupations, while the material-handling robots are that in routine-transportation (material-moving) occupations.

It is also worthwhile to give an example of technologies that are *not* robots according to the definition in this paper. An example of a growth in technology in the material-handling area is autonomous driving. Mehta and Levy [2020] predicts that such automation will grow strong and result in the reduction of total number of jobs in this area in eight to ten years since 2020. However, since autonomous vehicles do not operate multiple-axes, they are not treated in this paper at all. A similar observation applies for computers or artificial intelligence more generally.

JARA Robot Applications

In addition to applications in Section A.1.2, the full list of robot applications avaliable in JARA data is Die casting; Forging; Resin molding; Pressing; Arc welding; Spot welding; Laser welding; Painting; Load/unload; Mechanical cutting; Polishing and deburring; Gas cutting; Laser cutting; Water jet cutting; General assembly; Inserting; Mounting; Bonding; Soldering; Sealing and gluing; Screw tightening; Picking alignment and packaging; Palletizing; Measurement/inspection/test; and Material handling.

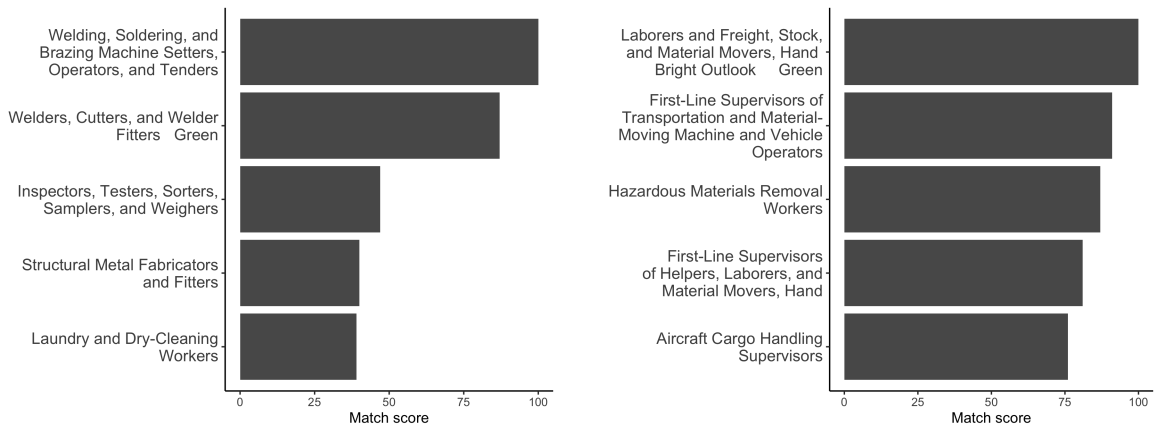
Can robots be classified as one of these applications? If one is familiar with the history of industrial robots, (s)he might wonder that robots are characterized by versatility as opposed to older specified industrial machinery [KHI, 2018]. Although it is true that a robot may be reprogrammed to perform more than one task, I claim that robots are well-classified to one of the applications listed above since the layer of dexterity is different. Robots might be able to adjust a model change of the products, but are not supposed to perform different tasks across the 4-digit occupation level. To support this point, recall that "SMEs are mostly high-mix/low-volume producers. Robots are still too inflexible to be switched at a reasonable cost from one task to another" [Autor et al., 2020]. These technological bottlenecks still make it hard for producers to have such a versatile robot that can replace a wide range of workers at the 4-digit occupation level even today, all the more for the sample period of my study.

Details in Application-Occupation Matching

Concrete examples of the pairs of an application and an occupation that are close are spot welding and material handling. On the one hand, spot welding is a task of welding two or more metal sheets into one by applying heat and pressure to a small area called spot. In contrast, O*NET-SOC Code 51-4121.06 has the title "Welders, Cutters, and Welder Fitters" ("Welders" below). Therefore, both spot welding robots and welders perform the welding task. On the other hand, Material handling is a short-distance movement of heavy materials. It is another major application of robots. In comparison, ONET-SOC Code 53-

Figure A.3: Examples of Match Scores

(a) Spot Welding (b) Material Handling



Note: The author's calculation from the search result of O*NET Code Connector. The bars indicate match scores for the search query term "Spot Welding" in panel (a) and "Material Handling" in panel (b). Occupations codes are 2010 O*NET SOC codes. In each panel, occupations are sorted descendingly with the relative relevance scores, and the top 5 occupations are shown. See the main text for the detail of the score.

7062.00 has the title "Laborers and Freight, Stock, and Material Movers, Hand" ("Material Handler" below). Therefore, both material handling robots and material handlers perform the material handling task.

Figure A.3 shows examples of the O*NET match scores for spot welding and material handling. On the left panel, welding occupations are listed as relevant occupations for spot welding. In contrast, on the right panel, a material-moving laborer is a top occupation in terms of the relevance to the material-handling task, as I described above.

Examples of Robotics Innovation

In the model, I call a change in the robot task space $a_{o,t}$ as the automation shock, and that in robot producer's TFP $A^R_{l,o,t}$ as the cost shock to produce robots. In this section, I show some examples of changes of robot technology and new patents to facilitate understandings of these interpretation. An example of task space expansion is adopting *Programmed Article Transfer* (PAT, Devol, 1961). PAT was machine that moves objects by a method called "teaching and playback". Teaching and playback method needs one-time teaching of how to move, after which the machine playbacks the movement repeatedly and automatically. This feature frees workers of performing repetitive tasks. PAT was intensively introduced in spot welding tasks. KHI [2018] reports that among 4,000 spot welding points, 30% were done be human previously, which PAT took over. Therefore, I interpret the adoption of PAT as the example of the expansion of the robot task space, or increase in $a_{o,t}$. Note that AR also analyze this type of technological change.

An example of cost reduction is adopting *Programmable Universal Manipulator for Assembly (PUMA)*. PUMA was designed to quickly and accurately transport, handle and assemble automobile accessories. A new computer language, *Variable Assembly Language (VAL)*, made it possible because it made the teaching process less work and more sophisticated. In other words, PUMA made tasks previously done by other robots but at cheaper unit cost per unit of task.

It is also worth mentioning that introduction of a new robot brand typically contains both components of innovation (task space expansion and cost reduction). For example, PUMA also expanded task space of robots. Since VAL allowed the use of sensors and "expanded the range of applications to include assembly, inspection, palletizing, resin casting, arc welding, sealing and research" [KHI, 2018].

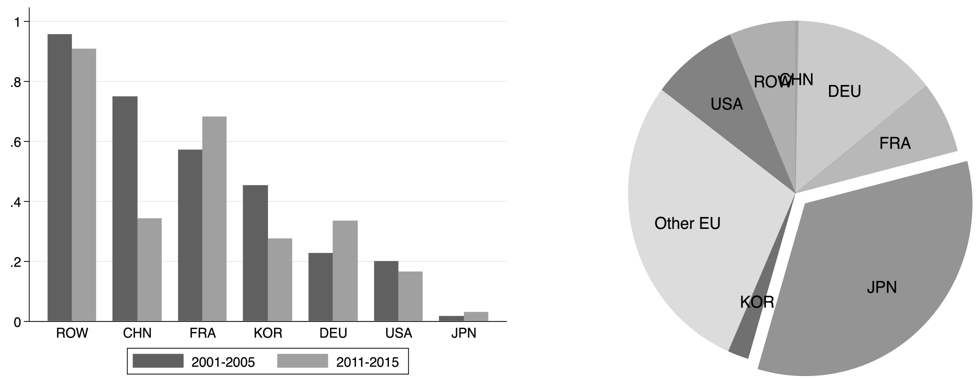
A.1.3 Trade of Industrial Robots

To compute the trade shares of industrial robots, I combine BACI and IFR data. In particular, I use the HS code 847950 ("Industrial Robots For Multiple Uses") to measure the robots, following Humlum [2019]. I approximate the initial year value by year of 1998, when the this HS code of robots is first available. To calculate the total absorption value of robots in each country, I use the IFR data's robot units (quantities), combined with the price indices of robots occasionally released by IFR's annual reports for selected countries. These price indices do not give disaggregation by robot tasks or occupations, highlighting the value added of the JARA data. Figure A.4 the pattern of international trade of international robots. In the left panel, I compute the import-absorption ratio. To remove the noise due to yearly observations and focus on a long-run trend, I aggregate by five-year bins 2001-2005 and 2011-2015. The result indicates that many countries import robots as opposed to produce in their countries. Japan's low import ratio is outstanding, revealing that its comparative advantage in this area. It is noteworthy that China largely domesticated the production of robots over the sample period. Another way to show grasp the comparative advantage of the robot industry is to examine the share of exports as in the right panel of Figure A.4. Roughly speaking, the half of the world robot market was dominated by EU and

Figure A.4: Trade of Industrial Robots

(a) Robot Import-Absorption Ratio

(b) World Robot Export Share, 2001-2005



Note: The author's calculation from the IFR, and BACI data. The left panel show the fraction of import in the total absorption value. The import value is computed by aggregating trade values across origin country in the BACI data (HS-1996 code 847950), and the absorption value is computed by the price index and the quantity variable available for selected countries in the IFR data. The data are five-year aggregated in 2001-2005 and 2011-2015, and countries are sorted according to the import shares in 2001-2005 descendingly. The right panel shows the export share for 2001-2005 aggregates obtained from the BACI data.

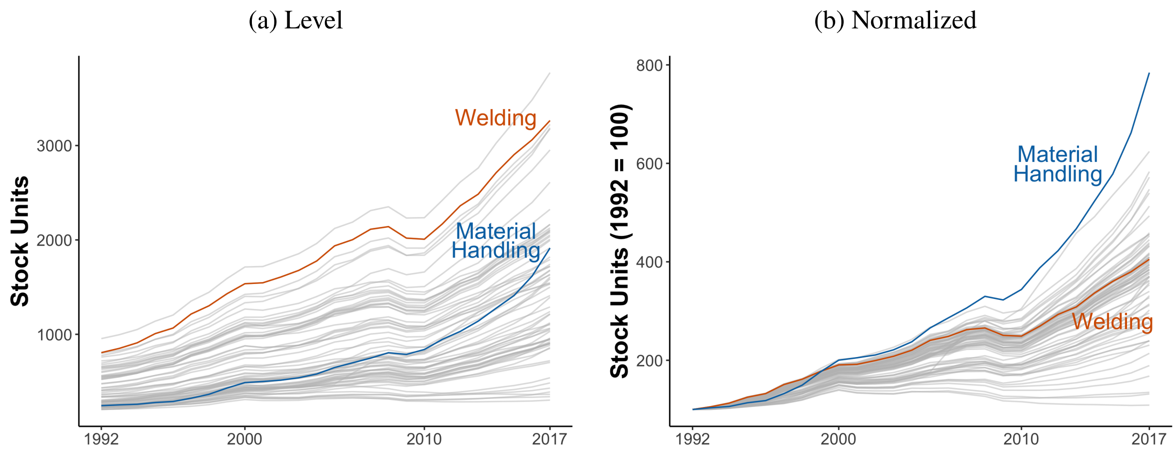
one-third by Japan in 2001-2005. The rest 20% is shared by the rest of the world, mostly by the US and South Korea.

A.1.4 Trends of Robot Stocks and Prices

I will show that different occupations experienced different trends in robot adoption. Figure A.5 shows the trend of US robot stocks at the occupation level. In the left panel, I show the trend of raw stock. First, the overall robot stocks increased rapidly in the period, as found in the previous literature. The panel also shows that the increase occurred in many occupations, but at differential rates. To highlight such a difference, in the right panel, I plot the normalized trend at 100 in the initial year. There is significant heterogeneity in the growth rates, ranging from a factor of one to eight.

For example, I color in the figure two occupations, robots that correspond to "Welding, Soldering, and Brazing Workers" (or "Welding") and "Laborers and Freight, Stock, and Material Movers, Hand" (or "Material handling"). On the one hand, welding is an occupation where the majority of robots were applied continuously throughout the period, as can be confirmed in the left panel. However, the growth rate of the stocks is not outstanding, but within the range of growth rates of other occupations. On the other hand, material handling was not a majority occupation as of the initial year, but it grew at the most rapid pace in the

Figure A.5: US Robot Stocks at the Occupation Level



Notes: The author's calculation based on JARA and O*NET. The figure shows the trend of stocks of robots in the US for each occupation. The left panel shows the level, whereas the right panel shows the normalized trend at 100 in 1992. In both panels, I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises.

period.

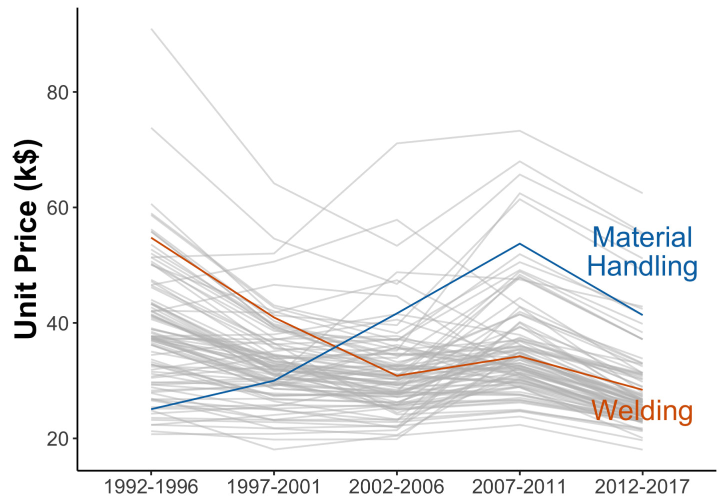
These findings indicate the difference between the automation shocks each occupation received. Some occupations were already somewhat automated by robotics as of the initial year, and the automation process continued afterward (e.g., welding). There are a few occupations where robotics automation was not achieved initially, but the innovation and adoption occurred rapidly in the period (e.g., material handling). I propose a model that incorporates this heterogeneity and discuss how to exploit it in estimation in the following sections.

Figure A.6 shows the trend of prices of robots in the US for each occupation. In addition to the overall decreasing trend, there is significant heterogeneity in the pattern of price falls across occupations. For instance, although the welding robots saw a large drop in the price during the 1990s, the material handling robots did not but increased the price over the sample period.

A.1.5 Robots from Japan in the US, Europe, and the Rest of the World

I review the international comparison of the pattern of robot adoption. I generate the growth rates of stock of robots between 1992 and 2017 at the occupation level for each

Figure A.6: Robot Prices at the Occupation Level



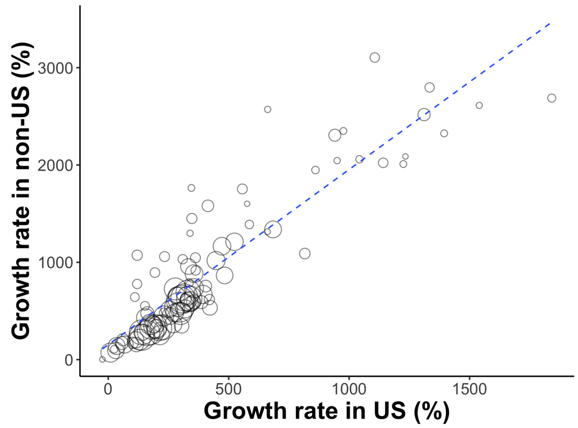
Notes: The author's calculation based on JARA and O*NET. The figure shows the trend of prices of robots in the US for each occupation. I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises. The dollars are converted to 2000 real US dollar using CPI.

group of destination countries. The groups are the US, the non-US countries, (namely, the world excluding the US and Japan), and five European countries (or "EU-5"), Denmark, Finland, France, Italy, and Sweden used in AR. To calculate the stock of robots, I employ the perpetual inventory method with depreciation rate of $\delta = 0.1$, following Graetz and Michaels [2018].

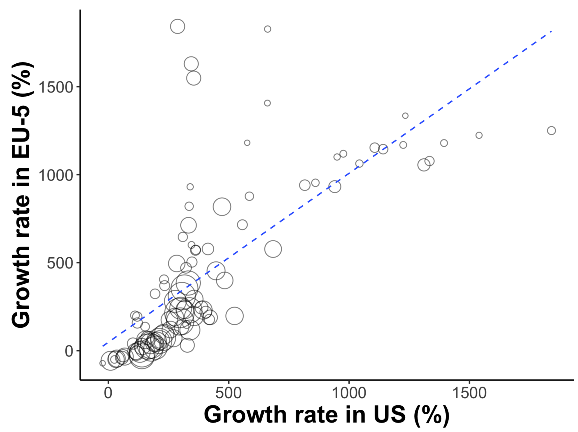
Figure A.7 shows scatterplots of the growth rates at the occupation level. The left panel shows the growth rates in the US on the horizontal axis and the ones in non-US countries on the vertical axis. The right panel shows the same measures on the horizontal axis, but the growth rates in the set of EU-5 countries on the vertical axis. These panels show that the stocks of robots at the occupation level grow (1992-2017) between the US and non-US proportionately relative to those between the US and EU-5. This finding is in contrast to AR, who find that the US aggregate robot stocks grew at a roughly similar rate as those did in EU-5. It also indicates that non-US growth patterns reflect growths of robotics technology at the occupation level available in the US. I will use these patterns as the proxy for robotics technology available in the US. In Section 1.3 and on, I take a further step and solve for the robot adoption quantity and values in non-US countries in general equilibrium including

Figure A.7: Growth Rates of Robots at the Occupation Level

(a) Comparison between the US and non-US



(b) Comparison between the US and EU-5



Notes: The author's calculation based on JARA, and O*NET. The left panel shows the correlation between occupation-level growth rates of robot stock quantities from Japan to the US and the growth rates of the quantities to the non-US countries. The right one shows the correlation between growth rates of the quantities to the US and EU-5 countries. Non-US are the aggregate of all countries excluding the US and Japan. EU-5 are the aggregate of Denmark, France, Finland, Italy, and Sweden used in Acemoglu and Restrepo [2020]. Each bubble shows an occupation. The bubble size reflects the stock of robot in the US in the baseline year, 1992. See the main text for the detail of the method to create the variables.

the US and non-US countries.

It is worth mentioning that a potential reason for the difference between my finding and AR's is the difference in data sources. In contrast to the JARA data I use, AR use IFR data that include all robot seller countries. Since EU-5 is closer to major robot producer countries other than Japan, including Germany, the robot adoption pattern across occupations may be influenced by their presence. If these close producers have a comparative advantage in producing robots for a specific occupation, then EU-5 may adopt the robots for such occupations intensively from close producers. In contrast, countries out of EU-5, including the US, may not benefit the closeness to these producers. Thus they are more likely to purchase robots from far producers from EU-5, such as Japan.

A.1.6 Further Analysis about Fact 2

Figure A.8 plots the correlation between the changes in robot measures and the changes in log labor market outcomes in the US at the occupation level, weighted by the size of occupation measured by initial the employment level. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in non-US countries as a robot measure on the left panel, and the right two take the log Japan robot shock ψ^{J}_{o,t_1}.

First, I offer a piece of evidence that robots have replaced workers at the occupation level. To control the demand factor in the US, Acemoglu and Restrepo [2020] used the robot stock changes in the other countries that show a similar trend of robot stocks as a proxy for the robot technological change and find the negative impact on the US regional labor market. Following this approach, using the changes in robot stocks in non-US countries (all countries except for the US and Japan), I find that the robot penetration measure negatively affects and labor market outcomes of wages and employment by occupation.[1] This result provides direct evidence of the substitution of robots for workers who perform the same task as robots, as well as corroborating the finding of Acemoglu and Restrepo [2020].

Furthermore, it also implies the reallocation of workers across occupations due to technological changes in robotics. Namely, suppose that an occupation experiences a rapid adoption of robots and workers in the occupation switch to other occupations that did not experience the adoption. This mechanism also works as a force that makes the negative correlation found in Figure A.8. In Section 1.3, I model these points by considering the dynamic occupation choice by workers. The characterizing parameter is the occupation switch elasticity ϕ, which is one of the target elasticity in estimation.

Figure A.9 shows the results of a set of robustness checks with an emphasis on the

[1] In Section A.1.5, I show that the robot stock growths are similar between the US and the non-US countries by occupations. In contrast, the occupation-level trend in the five countries Acemoglu and Restrepo [2020] used as comparison (Denmark, Finland, France, Italy, and Sweden) is less similar to the US trend than the non-US countries.

correlation between the wage changes and the changes in robot measures. Figures A.9a and A.9b show the wage correlation after residualizing the demographic control variables (initial-year female share, college graduates share, age 35-49 share, age 50-64 share, and foreign-born share in each occupation) for the robot stock and robot prices, respectively. Figures A.9c and A.9d show the correlation with the robot measures in the rest of the world (namely, world excluding the US and Japan) after residualizing the demographic control variables. The motivation of this exercise follows the intuition of AR–using the technological change proxied by the rest-of-the-world change in robot measures to move away from the occupational demand shocks since US occupational robot adoption may be affected by occupational demand shocks such as occupational productivity changes. Figure A.9e shows the result with the measurement of the robot stock by quantities of machines as opposed to monetary value, which follows the approach in the past literature such as AR. Figure A.9f shows the result of correlation using quality-adjusted robot prices, where the method of quality adjustment follows the spirit of Khandelwal et al. [2013]. Namely, I fit the following equation with the fixed-effect regression:

$$\ln\left(X^R_{JPN\to i,o,t}\right) = -\varsigma \ln\left(p^R_{JPN\to i,o,t}\right) + a^R_{o,t} + e^R_{i,o,t},$$

from which I obtain the fixed effect $a^R_{o,t}$, which absorbs the occupation-o specific log sales component that is not explained by the prices. I then proxy the quality change by the change in such fixed effects, $\Delta a^R_{o,t} \equiv a^R_{o,t} - a^R_{o,t_0}$. The (log) quality-adjusted price is then obtained by $\ln\left(p^R_{JPN\to i,o,t}\right) - \Delta a^R_{o,t}$. All the results are robust to these considerations–wage growths are negatively correlated with stock growths, and positively correlated with price growths, both across occupations.

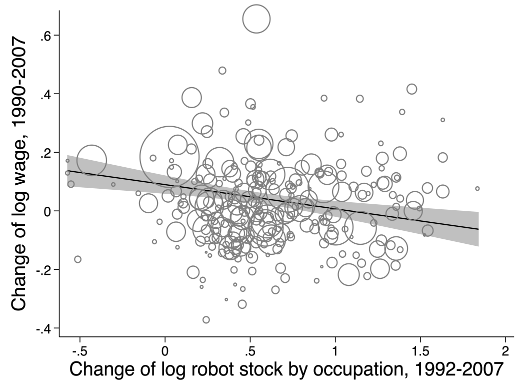
To examine if the positive correlation between the US wage would be unaffected absent of the Japan robot shock, I examine the pre-trend correlation between them. To do so, I take the 20-year difference of occupational wage since 1970 to 1990 as the outcome variable and run regression (1.3). The result is shown in Figure A.10. As expected, there is no significant relationship between (lagged) wage change and robot cost reduction.

To further check the correlation systematically, I run the following regressions and report the results in Table A.1:

$$\Delta \ln\left(y_o\right) = a_R \Delta \ln\left(R_o\right) + \left(X_o\right)^\top \boldsymbol{a} + e_o,$$

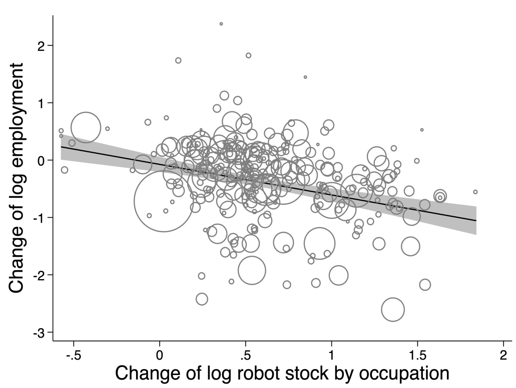
Figure A.8: Correlation between Wages and Robot Measures

(a) Wage vs Robot Stock

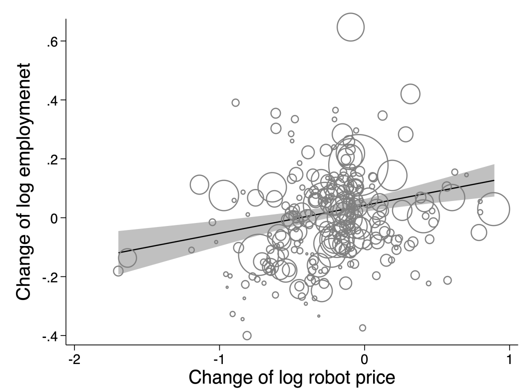


(b) Wage vs Robot Price

(c) Employment vs Robot Stock



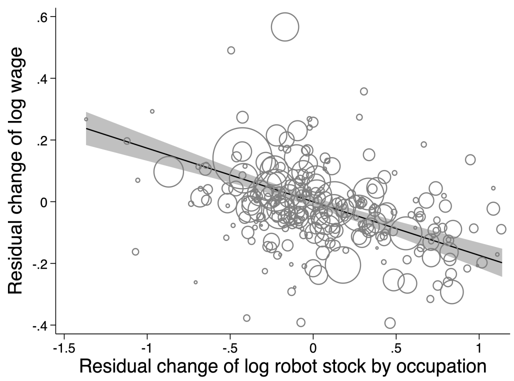
(d) Employment vs Robot Price



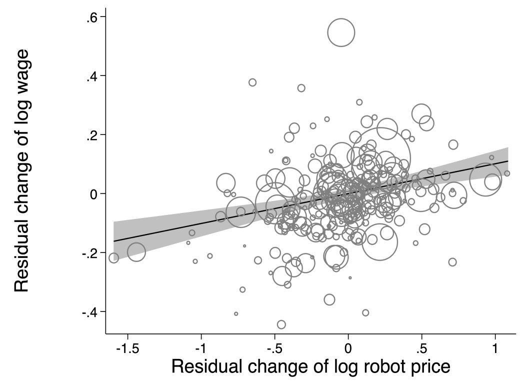
Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. On the y-axis, the top figures take occupational wages, while the bottom figures take occupational employment. The left panels take the change in log robot stocks (measured in monetary value) on the x-axis, while the right panels take the change in log robot average prices on the x-axis. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992).

Figure A.9: Correlation between Occupational Wage and Occupational Robot Measures

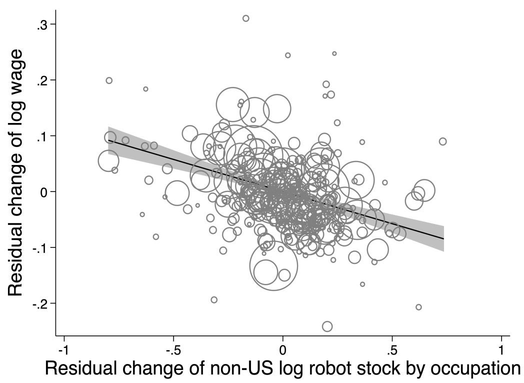
(a) Residualizing Control Variables, Stocks



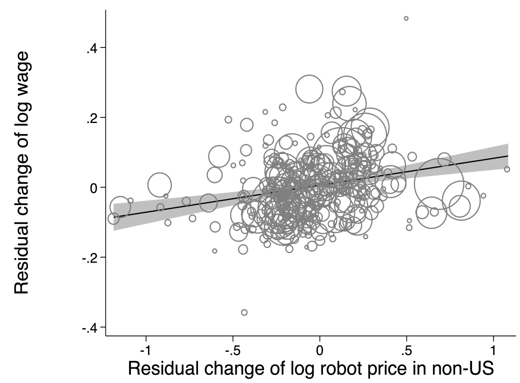
(b) Residualizing Control Variables, Prices



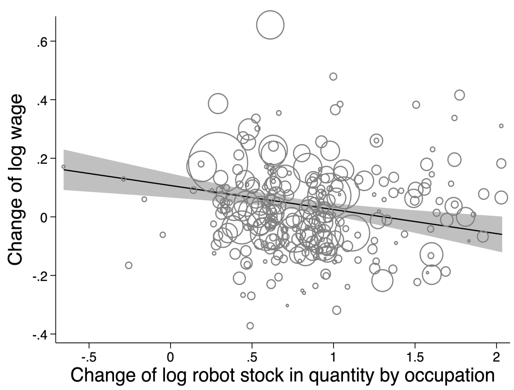
(c) ROW Stocks



(d) ROW, Prices



(e) Stock Quantity



(f) Quality-adjusted prices

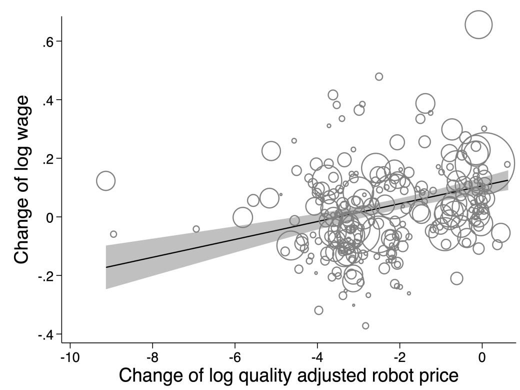


Figure A.10: Correlation between Occupational Wage and Occupational Robot Measures

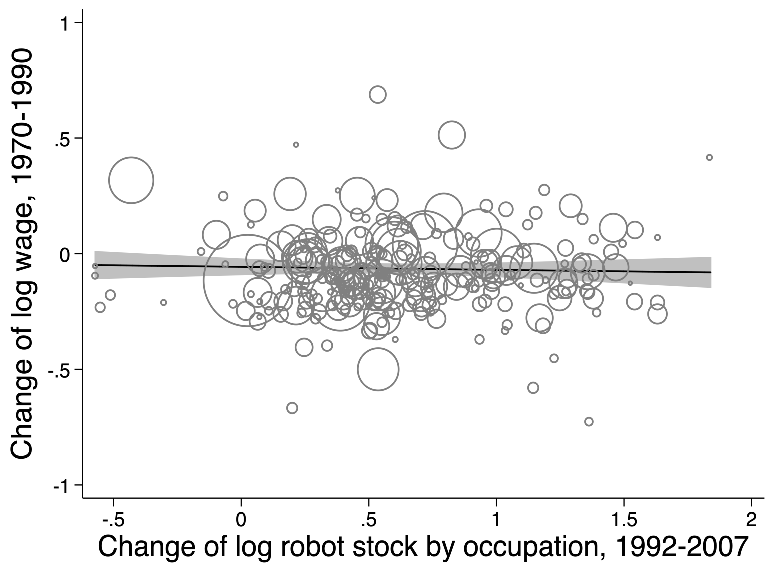


Table A.1: Regression Result of Labor Market Outcome on Robot Measures

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(w)$	(4) $\Delta \ln(w)$	(5) $\Delta \ln(L)$	(6) $\Delta \ln(L)$	(7) $\Delta \ln(L)$	(8) $\Delta \ln(L)$
$\Delta \ln(K^R_{USA})$	-0.174***				-0.532***			
	(0.0251)				(0.203)			
$\Delta \ln(p^R_{USA})$		0.0969***				0.507***		
		(0.0263)				(0.141)		
$\Delta \ln(K^R_{ROW})$			-0.116***				-0.575***	
			(0.0162)				(0.0953)	
$\Delta \ln(p^R_{ROW})$				0.0999***				0.458***
				(0.0257)				(0.148)
Female share	0.0366	0.0391	0.0383	0.0361	-0.0658	-0.0663	-0.0563	-0.0616
	(0.0320)	(0.0340)	(0.0328)	(0.0335)	(0.175)	(0.178)	(0.175)	(0.181)
Col. grad. share	0.399***	0.379***	0.401***	0.399***	0.114	0.113	0.119	0.107
	(0.0684)	(0.0673)	(0.0707)	(0.0691)	(0.284)	(0.285)	(0.281)	(0.287)
Age 35-49 share	-0.768*	-0.594	-0.697*	-0.672*	0.399	0.449	0.325	0.427
	(0.395)	(0.405)	(0.405)	(0.404)	(1.281)	(1.331)	(1.308)	(1.320)
Age 50-64 share	0.778**	0.797**	0.787**	0.765**	-1.636	-1.650	-1.541	-1.576
	(0.345)	(0.345)	(0.365)	(0.376)	(1.166)	(1.134)	(1.208)	(1.170)
Foreign-born share	-0.0905	-0.0250	-0.0241	-0.00227	-0.255	-0.221	-0.322	-0.197
	(0.225)	(0.213)	(0.230)	(0.221)	(1.142)	(1.073)	(1.074)	(1.044)
Observations	324	324	324	324	324	324	324	324
R-squared	0.467	0.344	0.398	0.367	0.138	0.104	0.199	0.106

Notes: The author's calculation based on JARA, O*NET, and US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. In each country $i \in \{USA, ROW\}$, K_i^R stands for the 2000-dollar value of the robot stock in the occupation and p_i^R stands for the average price of robot transacted in each year. All time differences (Δ) are taken with a long difference between 1990 and 2007. All demographic control variables are as of 1990. "Col. Grad. Share" stands for the college graduate share. Robust standard errors are reported in the parentheses. *** p<0.01, ** p<0.05, * p<0.1.

where y_o is labor-market outcome of occupation o (wage and employment), R_o is the measure of robots (stocks and prices), X_o are the demographic control variables, e_o is the regression residual, and Δ indicates the long-difference between 1990 (1992 for $\Delta \ln(R_o)$) and 2007. The coefficient of interest is a_R–I expect negative a_R if I take robot stocks as the explanatory variable, while I expect positive a_R when I take robot price as the right-hand side variable.

A.1.7 Robot Price Trends by Occupation Groups

In this section, I examine the facts discussed in Section 1.2.3 for each occupation group described in Section 1.4.1. First, Figure A.11 shows the plot of the trend of the robot price distribution since 1992 for each occupation group, a version of Figure 1.1a, disaggregated by occupation groups. The top three panels show the trends for routine occupations, namely, from the left, routine-production, routine-transportation, and routine-others. The bottom two panels show the trends for service occupations and abstract occupations, from the left. All these panels show the overall decreasing trend of robot prices, and the dispersion of prices within each occupation group. Having such a dispersion is important because in Section 1.4 when I estimate heterogeneous EoS between robots and labor, I use the price variation within each occupation group. Next, Figure A.12 shows the correspondent of Figure A.8 for each occupation group. The alignment of occupation groups is the same as Figure A.11. Interestingly, the positive correlation between occupational wage changes and occupational robot price changes, robustly found in Figure A.8 and Section A.1.6, is seen only in the group of production occupations and transportation occupations. Given that strong positive correlation yields a high elasticity of substitution, the finding in this figure is consistent with the heterogeneous elasticity of substitution between robots and labor found in Table 1.2b.

A.1.8 Initial Share Data

Since the log-linearized sequential equilibrium solution depends on several initial share data generated from the initial equilibrium, I discuss the data sources and methods for measuring these shares. I define $t_0 = 1992$ and the time frequency is annual. I consider the world that consists of three countries $\{USA, JPN, ROW\}$. Table A.2 summarizes overview of the

Table A.2: List of Data Sources

Variable	Description	Source
$\widetilde{y}^G_{ij,t_0}, \widetilde{x}^G_{ij,t_0}, \widetilde{y}^R_{ij,t_0}, \widetilde{x}^R_{ij,t_0}$	Trade shares of goods and robots	BACI, IFR
$\widetilde{x}^O_{i,o,t_0}$	Occupation cost shares	IPUMS
l_{i,o,t_0}	Labor shares within occupation	JARA, IFR, IPUMS
$s^G_{i,t_0}, s^V_{i,t_0}, s^R_{i,t_0}$	Robot expenditure shares	BACI, IFR, WIOT
$\alpha_{i,M}$	Intermediate input share	WIOT

Figure A.11: Robot Price Trends by Occupation Groups

Figure A.12: Correlation between Wage and Robot Prices by Occupation Groups

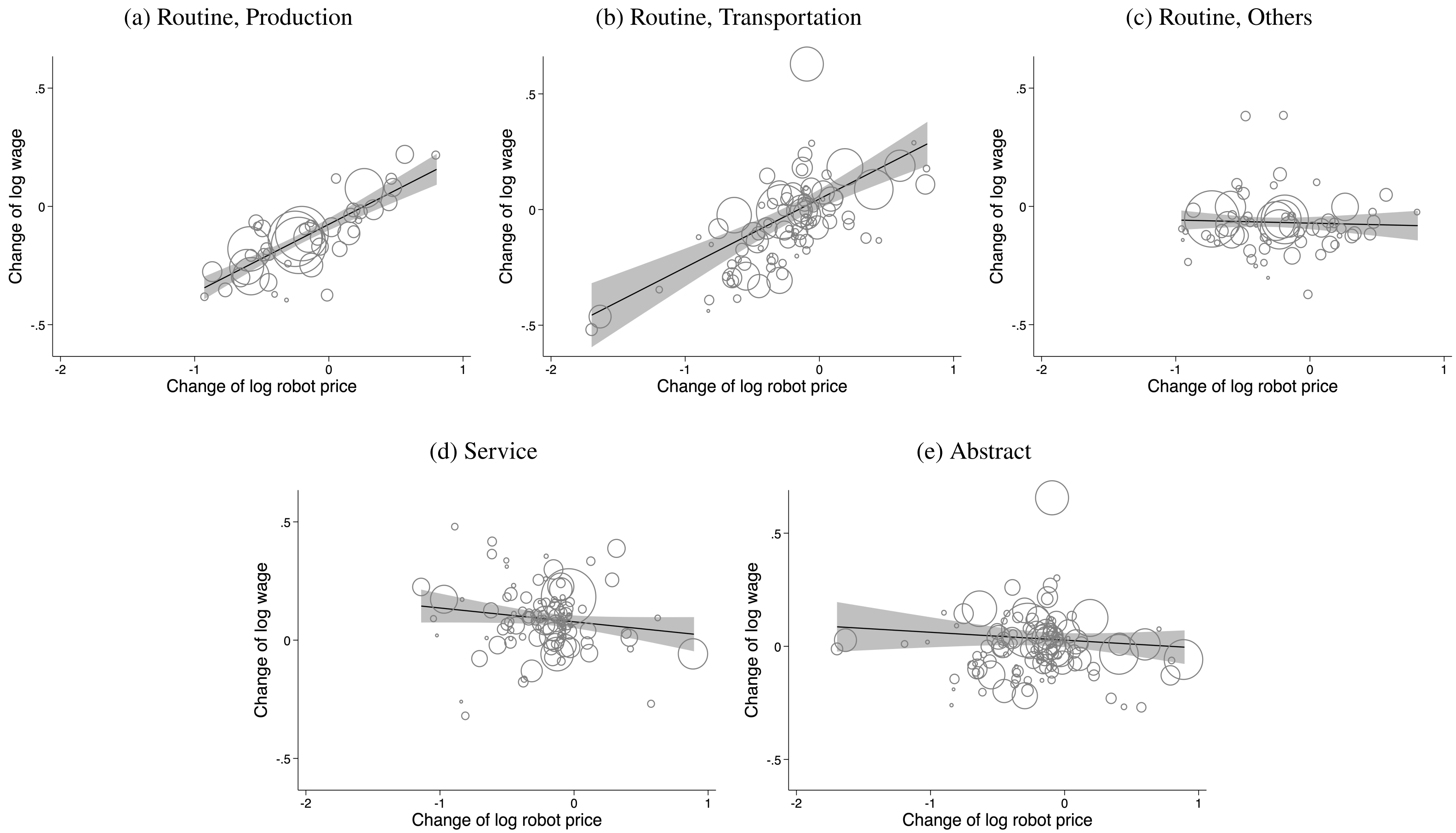
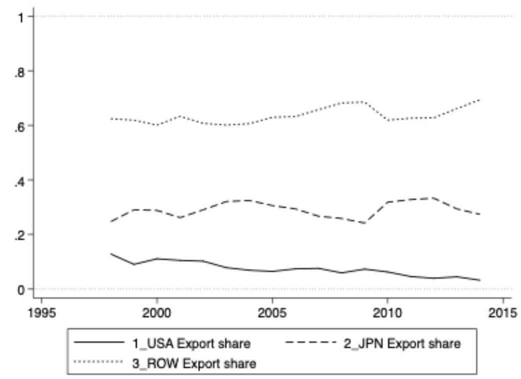
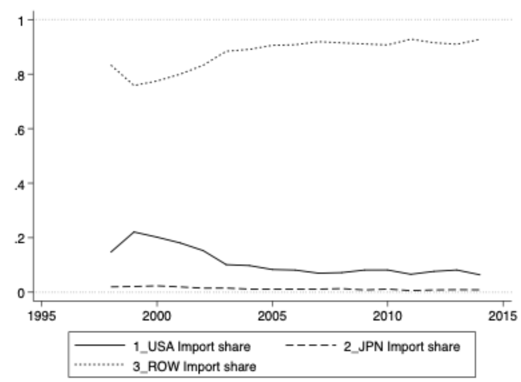


Figure A.13: Robot Trade Share Trends

(a) Exports



(b) Imports



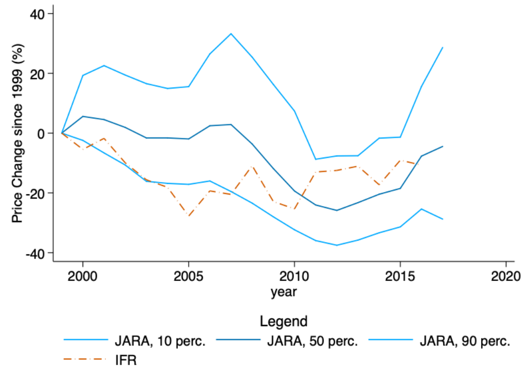
Note: The author's calculation of world trade shares based on the BACI data. Industrial robots are measured by HS code 847950 (Industrial robots for multiple uses).

variable notations, descriptions, and data sources.

I take matrices of trade of goods and robots by BACI data. As in Humlum [2019], I measure robots by HS code 847950 ("Industrial Robots For Multiple Uses") and approximate the initial year value by year of 1998, in which the robot HS code is first available. Figure A.13 shows the trend of export and import shares of robots among the world for the US, Japan, and the Rest Of the World. The trends are fairly stable for the three regions of the world, except that the import share of the US has declined relative to the ROW.

To obtain the domestic robot absorption data, I take from IFR data the flow quantity variable and the aggregate price variable for a selected set of countries. I then multiply these to obtain USA and JPN robot adoption value. For robot prices in ROW, I take the simple

Figure A.14: Comparison of US Price Indices between JARA and IFR



Note: The author's calculation of US robot price measures in JARA and IFR. The JARA measures are disaggregated by 4-digit occupations, and the figure shows the 10th, 50th (median), and 90th percentiles each year. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data.

average of the prices among the set of countries (France, Germany, Italy, South Korea, and the UK, as well as Japan and the US) for which the price is available in 1999, the earliest year in which the price data are available. Graetz and Michaels [2018] discuss prices of robots with the same data source. Figure A.14 shows the comparison of the US price index measure available between JARA and IFR. The JARA measures are disaggregated by 4-digit occupations. The figure shows the 10th, 50th (median), and 90th percentiles each year, as in Figure 1.1a. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data. Overall, the JARA price trend variation tracks the overall price evolution measured by IFR reasonably well: The long-run trends from 1999 to the late 2010s are similar between the JARA median price and the IFR price index. During the 2000s, the IFR price index drops faster than the median price in the JARA data. It compares with the JARA 10th percentile price, which could be due to robotic technological changes in other countries than Japan in the corresponding period.

I construct occupation cost shares $\widetilde{x}^O_{i,o,t_0}$ and labor shares within occupation l_{i,o,t_0} as follows. To measure $\widetilde{x}^O_{i,o,t_0}$, I aggregate the total wage income of workers that primarily works in each occupation o in year 1990, the Census year closest to t_0. I then take the share of this total compensation measure for each occupation. To measure l_{i,o,t_0}, I take the total compensation as the total labor cost and a measure of the user cost of robots for each occupation. The user cost of robots is calculated with the occupation-level robot price data available in IFR and the set of calibrated parameters in Section 1.4.1. Table A.3 summarizes these statistics for the aggregated 5 occupation groups in the US. One can

Table A.3: Baseline Shares by 5 Occupation Group

Occupation Group	$\widetilde{x}^{O}_{1,o,t_0}$	l^{O}_{1,o,t_0}	y^{R}_{2,o,t_0}	x^{R}_{1,o,t_0}	x^{R}_{2,o,t_0}	x^{R}_{3,o,t_0}
Routine, Production	17.58%	99.81%	64.59%	67.49%	62.45%	67.06%
Routine, Transportation	7.82%	99.93%	12.23%	11.17%	13.09%	11.04%
Routine, Others	28.78%	99.99%	10.88%	9.52%	11.68%	10.40%
Service	39.50%	99.99%	8.87%	8.58%	9.17%	8.32%
Abstract	6.32%	99.97%	3.43%	3.24%	3.60%	3.18%

Note: The author's calculation of initial-year share variables based on the US Census, IFR, and JARA. As in the main text, country 1 indicates the US, country 2 Japan, and country 3 the rest of the world. See the main text for the construction of each variable.

see that the cost for production occupations and transportation occupations comprise 18% and 8% of the US economy, respectively, totaling more than one-fourth. Furthermore, the share of robot cost in all occupations is still quite low with the highest share of 0.19% in production occupations, revealing still small-scale adoption of robots from the overall US economy perspective.

To calculate the effect on total income, I also need to compute the sales share of robots by occupations $y^R_{i,o,t_0} \equiv Y^R_{i,o,t_0}/\sum_o Y^R_{i,o,t_0}$ and the absorption share $x^R_{i,o,t_0} \equiv X^R_{i,o,t_0}/\sum_o X^R_{i,o,t_0}$. To obtain y^R_{i,o,t_0}, I compute the share of robots by occupations produced in Japan $y^R_{2,o,t_0} = Y^R_{2,o,t_0}/\sum_o Y^R_{2,o,t_0}$ and assume the same distribution for other countries due to the data limitation: $y^R_{i,o,t_0} = y^R_{2,o,t_0}$ for all i. To have x^R_{i,o,t_0}, I compute the occupational robot adoption in each country by $X^R_{i,o,t_0} = P^R_{i,t_0} Q^R_{i,o,t_0}$, where Q^R_{i,o,t_0} is the occupation-level robot quantity obtained by the O*NET concordance generated in Section 1.2.2 applied to the IFR application classification. As mentioned above, the robot price index P^R_{i,t_0} is available for a selected set of countries. To compute the rest-of-the-world price index P^R_{3,t_0}, I take the average of all available countries weighted by the occupational robot values each year. The summary table for these variables y^R_{i,o,t_0} and x^R_{i,o,t_0} at 5 occupation groups are shown in Table A.3. All values in Table A.3 are obtained by aggregating 4-digit-level occupations, and raw and disaggregated data are available upon request.

I take a more standard measure, the intermediate input share $\alpha_{i,M}$, from World Input-Output Tables (WIOT Timmer et al., 2015). Finally, I combine the trade matrix generated above and WIOT to construct the good and robot expenditure shares s^G_{i,t_0}, s^V_{i,t_0}, and s^R_{i,t_0}. In particular, with the robot trade matrix, I take the total sales value by summing across importers for each exporter, and total absorption value by summing across exporters for each importers. I also obtain the total good absorption by WIOT. From these total values,

Table A.4: 1990 Occupation Group Switching Probability

		Routine Production	Routine Transportation	Routine Others	Service	Abstract
Routine	Production	0.961	0.011	0.010	0.006	0.012
	Transportation	0.020	0.926	0.020	0.008	0.025
	Others	0.005	0.006	0.955	0.020	0.014
Service		0.003	0.002	0.020	0.967	0.007
Abstract		0.014	0.014	0.036	0.015	0.922

Note: The author's calculation from the CPS-ASEC 1990 data. The conditional switching probability to column occupation group conditional on being in each row occupation.

I compute expenditure shares. are obtained by aggregating 4-digit occupations, and the disaggregated data are available upon request.

As initial year occupation switching probabilities μ_{i,oo',t_0}, I take 1990 flow Markov transition matrix from the cleaned CPS-ASEC data created in Section A.1.1. Table A.4 shows this initial-year conditional switching probability. The matrix for the other years are available upon request. As for other countries than the US, although Freeman et al. [2020] has begun to develop occupational wage measures consistent across country, world-consistent occupation employment data are hard to obtain. Therefore, I assign the same flow probabilities for other countries in my estimation.

A.2 Theory Appendix

A.2.1 Further Discussion of Model Assumptions

Capital-Skill Complementarity Occupation production function (1.10) also nest the one in the literature of capital-skill complementarity (Krusell et al., 2000 among others). To simplify, I focus on individual producer's production function in the steady state. Thus I drop subscripts and superscripts of country i and time period t. Suppose the set of occupations is $O \equiv \{R, U\}$ and $a_U = 0$. R stands for the robotized occupation (e.g., spot welding) and U stands for "unrobotized" (e.g., programming). Note that since U is unrobotized $a_U = 0$. Then the unit cost of occupation aggregate (1.10), P^O, is

$$P^O = \left[(b_R)^{\frac{1}{\beta}} \left((1 - a_R) (w_R)^{1-\theta_R} + a_R (c_R)^{1-\theta_R} \right)^{\frac{1-\beta}{1-\theta_R}} + (b_U)^{\frac{1}{\beta}} (w_U)^{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

Thus different skills R and U are substituted by robots with different substitution parameters θ_R and β, respectively. Since the literature of capital-skill complementarity studies the rising skill premium, the current model also has an ability to discuss the occupation (or skill) premium given the different level of automation across occupations.

Adjustment Cost of Robot Capital To interpret another key feature of the model, the convex adjustment cost of robot adoption, consider the cost of adopting new technology and integration. With the convex adjustment cost, the model predicts the staggered adoption of robots over years that I observe in the data (see Figure 1.3b), and implies a rich prediction about the short- and long-run effects of robotization.

First, when adopting new technology including robots, it is necessary to re-optimize the overall production process since the production process is typically optimized to employ workers. More generally, the literature of organizational dynamics studies the difficulty, not to say the impossibility, of changing strategies of a company due to complementarities (see Brynjolfsson and Milgrom, 2013 for a review). Such a re-optimization incurs an additional cost of adoption in addition to the purchase of robot arms. Moreover, even within a production unit, there is a variation of this difficulty of adopting robots across production processes. In this case, the part where the adjustment is easy adopts the robots first, and vice versa. This allocation of robot adoptions over years may aggregate to make the robot stock increase slowly [Baldwin and Lin, 2002]. Waldman-Brown [2020] also finds that the incremental and sluggish automation is particularly well-observed in small and medium-sized firms, as they add "a machine here or there, rather than installing whole new systems that are more expensive to buy and integrate" [Autor et al., 2020].

The second component of the adjustment cost may come from the cost of integration as I discussed in Section 1.2.1. The marginal integration cost may increase as the massive upgrading of robotics system may require large-scale overhaul of production process, which increases the complexity and so is costly. The adjustment cost may capture the increasing marginal cost component of the integration cost. It explains an additional component of the integration cost implied by constant returns-to-scale robot aggregation in equation (1.14).

Another potential choice of modeling a staggered growth of robot stocks is to assume a fixed cost of robot adoption and lumpy investment. Humlum [2019] finds that many plants buy robots only once during the sample period. Since JARA data does not observe plant-level adoptions, I do not separately identify lumpy investment from the staggered growth of robot stocks in the data. To the extent that fixed cost of investment may make the policy intervention less effective (e.g., Koby and Wolf, 2019), the counterfactual analysis in this paper may overestimate the effect of robot taxes since it does not take into account the fixed cost and lumpiness of investment.

A.2.2 Derivation of Worker's Optimality Conditions

In this section, I formalize the assumptions behind the derivation and show equations (1.4) and (1.5). Fix country i and period t. There is a mass $\overline{L}_{i,t}$ of workers. In the beginning of each period, worker $\omega \in \left[0, \overline{L}_{i,t}\right]$ draws a multiplicative idiosyncratic preference shock $\left\{Z_{i,o,t}\left(\omega\right)\right\}_o$ that follows an independent Fréchet distribution with scale parameter $A^V_{i,o,t}$ and shape parameter $1/\phi$. Note that one can simply extend that the idiosyncratic preference follows a correlated Fréchet distribution to allow correlated preference across occupations, as in Lind and Ramondo [2018]. To keep the expression simple, I focus on the case of independent distribution. A worker ω then works in the current occupation, earns income, consumes and derives logarithmic utility, and then chooses the next period's occupation with discount rate ι. When choosing the next period occupation o', she pays an ad-valorem switching cost $\chi_{i,oo',t}$ in terms of consumption unit that depends on current occupation o. She consumes her income in each period. Thus, worker ω who currently works in occupation o_t maximizes the following objective function over the future stream of utilities by choosing occupations $\{o_s\}_{s=t+1}^{\infty}$:

$$E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1 - \chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]$$

where $C_{i,o,s}$ is a consumption bundle when working in occupation o in period $s \geq t$, and E_t is the expectation conditional on the value of $Z_{i,o_t,t}\left(\omega\right)$. Each worker owns occupation-specific labor endowment $l_{i,o,t}$. I assume that her income is comprised of labor income $w_{i,o,t}$ and occupation-specific ad-valorem government transfer with rate $T_{i,o,t}$. Given the consumption price $P^G_{i,t}$, the budget constraint is

$$P^G_{i,t} C_{i,o,t} = w_{i,o,t} l_{i,o,t} \left(1 + T_{i,o,t}\right)$$

for any worker, with $P^G_{i,t}$ being the price index of the non-robot good G.

By linearity of expectation,

$$
\begin{aligned}
& E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right] \\
& = \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right].
\end{aligned}
$$

By monotone transformation with exponential function,

$$
\begin{aligned}
& \exp\left\{\sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right].\right\} \\
& = \prod_{s=t}^{\infty} \exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]\right\}.
\end{aligned}
$$

Write the value function conditional on the realization of shocks at period t as follows:

$$
V_{i,o_t,t}(\omega) \equiv \max_{\{o_s\}_{s=t+1}^{\infty}} \prod_{s=t}^{\infty} \exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]\right\}.
$$

I apply Bellman's principle of optimality as follows:

$$
\begin{aligned}
& V_{i,o_t,t}(\omega) \\
& = \max_{\{o_s\}_{s=t+1}^{\infty}} \prod_{s=t}^{\infty} \exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]\right\} \\
& = \max_{o_{t+1}} \exp\left\{\ln\left(C_{i,o_t,t}\right) + \ln\left(1-\chi_{i,o_t o_{t+1},t}\right) + \ln\left(Z_{i,o_{t+1},t}(\omega)\right)\right\} \times \\
& \max_{\{o_s\}_{s=t+2}^{\infty}} \prod_{s=t+1}^{\infty} \exp\left\{\left(\frac{1}{1+\iota}\right)^{s-(t+1)} \left[E_{t+1} \ln\left(C_{i,o_s,s}\right) + E_{t+1} \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_{t+1} \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]\right\} \\
& = \max_{o_{t+1}} \exp\left\{\ln\left(Z_{i,o_t,t}(\omega)\right) + \ln\left(C_{i,o_t,t}\right) + \ln\left(1-\chi_{i,o_t o_{t+1},t}\right)\right\} V_{i,o_{t+1},t+1},
\end{aligned}
$$

where $V_{i,o_t,t}$ is the unconditional expected value function $V_{i,o_t,t} \equiv E_{t-1} V_{i,o_t,t}(\omega)$. Changing the notation from (o_t, o_{t+1}) into (o, o'), I have

$$
V_{i,o,t}(\omega) = \max_{o'} C_{i,o,t} \left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega) V_{i,o',t+1}.
$$

Solving the worker's maximization problem is equivalent to finding:

$$\begin{aligned}
\mu_{i,oo',t} &\equiv \Pr\left(\text{worker } \omega \text{ in } o \text{ chooses occupation } o'\right) \\
&= \Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o'',t}\left(\omega\right) V_{i,o'',t+1} \le C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1}\right).
\end{aligned}$$

By the independent Fréchet assumption, we have the maximum value distribution

$$\begin{aligned}
\Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1} \le v\right) &= \prod_{o'} \Pr\left(Z_{i,o',t}\left(\omega\right) \le \frac{v}{C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}}\right) \\
&= \prod_{o''} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \\
&= \exp\left(\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right).
\end{aligned}$$

Therefore, the conditional choice probability satisfies, again by the independent Fréchet assumption,

$$\begin{aligned}
&\mu_{i,oo',t} \\
=&\int_0^\infty \Pr\left(\max_{o''\neq o'} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o'',t+1} \le v\right) d\Pr\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1} \ge v\right) \\
=&\int_0^\infty \exp\left(\sum_{o''\neq o'} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right)\times \\
&\qquad \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right)\times\left(-\phi v^{-\phi-1}\right) dv \\
=&\frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}}\times \\
&\qquad \int_0^\infty \exp\left(\sum_{o'''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi} \times\left(-\phi v^{-\phi-1}\right) dv.
\end{aligned}$$

The last integral term is one by integration and the definition of distribution. Therefore, I arrive at

$$\mu_{i,oo',t} = \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} = \frac{\left(\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}},$$

$$V_{i,o,t+1} = \widetilde{\Gamma} C_{i,o,t} \left(\sum_{o'} \left(\left(1 - \chi_{i,oo',t+1} \right) V_{i,o',t+2} \right)^{\phi} \right)^{\frac{1}{\phi}}.$$

A.2.3 Relationship with Other Models of Automation

The model in Section 1.3 is general enough to nest models of automation in the previous literature. In particular, I show how the production functions (1.8) and (1.10) imply to specifications in AR and Humlum [2019]. Throughout Section A.2.3, I fix country i and focus on steady states and thus drop subscripts i and t since the discussion is about individual producer's production function.

Relationship with the model in Acemoglu and Restrepo (2020, AR)

Following AR that abstract from occupations, I drop occupations by setting $O = 1$ in this paragraph. Therefore, the EoS between occupations β plays no role, and $\theta_o = \theta$ is a unique value. AR show that the unit cost (hence the price given perfect competition) is written as

$$p^{AR} \equiv \frac{1}{\widetilde{A}} \left[(1 - \widetilde{a}) \frac{w}{A^L} + \widetilde{a} \frac{c^R}{A^R} \right]^{\alpha_L} r^{1-\alpha_L},$$

for each sector and location (See AR, Appendix A1, equation A5). In this equation, c^R is the steady state marginal cost of robot capital defined in equation (A.27) and A^L and A^R represent per-unit efficiency of labor and robots, respectively. In Lemma 2 below, I prove that my model implies a unit cost function that is strict generalization of p^{AR} with proper modification to the shock terms and parameter configuration. I begin with the modification that allows per-unit efficiency terms in my model.

Definition 3. For labor and robot per-unit efficiency terms $A^L > 0$ and $A^R > 0$ respectively, modified robot task space $\widetilde{a}$ and TFP term $\widetilde{A}$ are

$$\widetilde{a} \equiv \frac{a \left(A^L\right)^{\theta-1}}{a \left(A^L\right)^{\theta-1} + (1-a) \left(A^R\right)^{\theta-1}}, \tag{A.1}$$

$$\widetilde{A} \equiv \frac{A}{\left[(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1} \right]}. \tag{A.2}$$

Lemma 2. *Set the number of occupations $O = 1$. In the steady state,*

$$p^G = \frac{1}{\widetilde{A}} \left[(1 - \widetilde{a}) \left(\frac{w}{A^L} \right)^{1-\theta} + \widetilde{a} \left(\frac{c^R}{A^R} \right)^{1-\theta} \right]^{\frac{\alpha_L}{1-\theta}} \left(p^G \right)^{\alpha_M} r^{1-\alpha_M-\alpha_L}. \tag{A.3}$$

Proof. Note that modified robot task space (A.1) and modified TFP (A.2) can be inverted to have

$$a \equiv \frac{\widetilde{a} \left(A^R\right)^{\theta-1}}{(1 - \widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1}}, \tag{A.4}$$

$$A \equiv \left[(1 - \widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1} \right] \widetilde{A}. \tag{A.5}$$

Cost minimization problem with the production functions (1.8) and (1.10) and perfect competition imply

$$p^G = \frac{1}{A} \left(P^O\right)^{\alpha_L} p^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

and

$$P^O = \left[(1 - a) w^{1-\theta} + a^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

where P^O is the unit cost of tasks performed by labor and robots. Substituting equations (A.4) and (A.5) and rearranging, I have

$$p^G = \frac{1}{\widetilde{A}} \left(\widetilde{P^O}\right)^{\alpha_L} \left(p^G\right)^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

where $\widetilde{P^O}$ is the cost of the tasks performed by labor and robots:

$$\widetilde{P^O} = \left[(1 - \widetilde{a}) \left(\frac{w}{A^L} \right)^{1-\theta} + a \left(\frac{c^R}{A^R} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

□

Lemma 2 immediately implies the following corollary that shows that the steady state modified unit cost (A.3) strictly nests the unit cost formulation of AR as a special case of Leontief occupation aggregation.

Corollary 1. *Suppose $\alpha_M = 0$. Then as $\theta \to 0$, $p^G \to p^{AR}$.*

Relationship with the model in Humlum [2019]

I show that production functions (1.8) and (1.10) nest the production function used by Humlum [2019]. Since the setting of Humlum [2019] does not have non-robot capital, in this section, I simplify the notation for robot capital K^R by dropping the superscript and denote as K. For each firm in each period, Humlum [2019] specifies

$$Q^D = \exp\left[\varphi_H^D + \gamma_H^D K\right]\left[\sum_o \left(\exp\left[\varphi_o^D + \gamma_o^D K\right]\right)^{\frac{1}{\beta}} (L_o)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}, \tag{A.6}$$

where $K = \{0, 1\}$ is a binary choice, $\varphi_H^D, \gamma_H^D, \varphi_o^D$ and γ_o^D are parameters, and superscript D represents the discrete adoption problem of Humlum [2019]. As normalization, suppose that

$$\sum_o \exp\left(\varphi_o^D + \gamma_o^D K\right) = 1.$$

I will start from production function (1.8) and (1.10), place restrictions, and arrive at equation (A.6). As a key observation, relative to the discrete choice of robot adoption in Humlum [2019], the continuous choice of robot *quantity* in production function (1.10) allows significant flexibility. In this paragraph, I assume away with intermediate inputs. This is because Humlum [2019] assumes that intermediate inputs enter in an element of CES, while production function (1.8) implies that intermediate inputs enter as an element of the Cobb-Douglas function.

Now, given our production functions (1.8) and (1.10), suppose producers follow the binary decision rule defined below.

Definition 4. A binary decision rule of a producer is that producers can choose between two choices: adopting robots $K = 1$ or not $K = 0$. If they choose $K = 1$, they adopt robots at the same unit as labor $K_o = L_o \geq 0$ for all occupation o. If they choose $K = 0$, $K_o = 0$ for all o.

Note that the binary decision rule is nested in the original choice problem from $K_o^R \geq 0$ for each o. Set

$$A_o^D\left(K^R\right) \equiv \begin{cases} A_o\left((1-a_o)^{\frac{1}{\theta}} + (a_o)^{\frac{1}{\theta}}\right)^{\frac{\theta}{\theta-1}(\beta-1)} & \text{if } K^R = L_o \\ A_o\left(1-a_o\right)^{\frac{1}{\theta-1}(\beta-1)} & \text{if } K^R = 0 \end{cases}.$$

Then I have

$$Q=\left[\sum_{o}\left(A_{o}^{D}\left(K_{o}\right)\right)^{\frac{1}{\beta}}\left(L_{o}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}.$$

To normalize, define

$$\widetilde{A_{o}^{D}}\equiv\left(\sum_{o}A_{o}^{D}\left(K_{o}\right)\right)^{\frac{1}{\beta-1}}$$

and

$$a_{o}^{D}\left(K_{o}^{R}\right)\equiv\frac{A_{o}^{D}\left(K_{o}\right)}{\sum_{o'}A_{o'}^{D}\left(K_{o'}\right)}.$$

Then I have

$$Q=\widetilde{A_{o}^{D}}\left[\sum_{o}\left(a_{o}^{D}\left(K_{o}\right)\right)^{\frac{1}{\beta}}\left(L_{o}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}. \tag{A.7}$$

Finally, let

$$A_{o,0}\equiv\left[\exp\left(\varphi_{H}^{D}+\varphi_{o}^{D}\right)\right]^{\frac{\theta_{o}-1}{\beta-1}}$$

and

$$A_{o,1}\equiv\left[\left(\exp\left(\varphi_{H}^{D}+\varphi_{o}^{D}+\gamma_{H}^{D}+\gamma_{o}^{D}\right)\right)^{\frac{1}{\theta_{o}}\frac{\theta_{o}-1}{\beta-1}}-\left(\exp\left(\varphi_{H}^{D}+\varphi_{o}^{D}\right)\right)^{\frac{1}{\theta_{o}}\frac{\theta_{o}-1}{\beta-1}}\right]^{\theta_{o}}.$$

and also let A_o and a_o satisfy

$$A_{o}=\left(A_{o,0}+A_{o,1}\right)^{\frac{\beta-1}{\theta_{o}-1}} \tag{A.8}$$

and

$$a_{o}=\frac{A_{o,1}}{A_{o,0}+A_{o,1}}. \tag{A.9}$$

Then one can substitute equations (A.8) and (A.9) to equation (A.7) and confirm that $Q=Q^{D}$. Summarizing the discussion above, I have the result that my model can be restricted to produce the production side of the model of Humlum [2019] as follows.

Lemma 3. *Suppose that (i) producers follow the binary decision rule in Definition 4 and that (ii) occupation productivity A_o and robot task space a_o satisfy equations (A.8) and (A.9) for each o. Then $Q = Q^D$.*

A.2.4 Equilibrium Characterization

To characterize the producer problem, I show the static optimization conditions and then the dynamic ones. To solve for the static problem of labor, intermediate goods, and non-robot capital, consider the FOCs of equation (1.12)

$$p_{i,t}^{G}\alpha_{i,L}\frac{Y_{i,t}^{G}}{T_{i,t}^{O}}\left(b_{i,o,t}\frac{T_{i,t}^{O}}{T_{i,o,t}^{O}}\right)^{\frac{1}{\beta}}\left(\left(1-a_{o,t}\right)\frac{T_{i,o,t}^{O}}{L_{i,o,t}}\right)^{\frac{1}{\theta_{O}}}=w_{i,o,t}, \tag{A.10}$$

where $T_{i,t}^{O}$ is the aggregated occupations $T_{i,t}^{O}\equiv\left[\sum_{o}\left(T_{i,o,t}^{O}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$,

$$p_{i,t}^{G}\alpha_{i,M}\frac{Y_{i,t}^{G}}{M_{i,t}}\left(\frac{M_{i,t}}{M_{li,t}}\right)^{\frac{1}{\varepsilon}}=p_{li,t}^{G}, \tag{A.11}$$

and

$$p_{i,t}^{G}\alpha_{i,K}\frac{Y_{i,t}^{G}}{K_{i,t}}=r_{i,t}, \tag{A.12}$$

where $\alpha_{i,K}\equiv 1-\alpha_{i,L}-\alpha_{i,M}$. Note also that by the envelope theorem,

$$\frac{\partial\pi_{i,t}\left(\left\{K_{i,o,t}^{R}\right\}\right)}{\partial K_{i,o,t}^{R}}=p_{i,t}^{G}\frac{\partial Y_{i,t}}{\partial K_{i,o,t}^{R}}=p_{i,t}^{G}\left(\alpha_{L}\frac{Y_{i,t}^{G}}{T_{i,t}^{O}}\left(b_{i,o,t}\frac{T_{i,t}^{O}}{T_{i,o,t}^{O}}\right)^{\frac{1}{\beta}}\left(a_{o,t}\frac{T_{i,o,t}^{O}}{K_{i,o,t}^{R}}\right)^{\frac{1}{\theta}}\right). \tag{A.13}$$

Another static problem of producers is robot purchase. Define the "before-integration" robot aggregate $Q_{i,o,t}^{R,BI}\equiv\left[\sum_{l}\left(Q_{li,o,t}^{R}\right)^{\frac{\varepsilon^{R}-1}{\varepsilon^{R}}}\right]^{\frac{\varepsilon^{R}}{\varepsilon^{R}-1}}$ and the corresponding price index $P_{i,o,t}^{R,BI}$. By the first order condition with respect to $Q_{li,o,t}^{R}$ for equation (1.14), I have $p_{li,o,t}^{R}Q_{li,o,t}^{R}=\left(\frac{p_{li,o,t}^{R}}{P_{i,o,t}^{R,BI}}\right)^{1-\varepsilon^{R}}P_{i,o,t}^{R,BI}Q_{i,o,t}^{R,BI}$, and $P_{i,o,t}^{R,BI}Q_{i,o,t}^{R,BI}=\alpha P_{i,o,t}^{R}Q_{i,o,t}^{R}$. Thus $p_{li,o,t}^{R}Q_{li,o,t}^{R}=\alpha\left(\frac{p_{li,o,t}^{R}}{P_{i,o,t}^{R,BI}}\right)^{1-\varepsilon^{R}}P_{i,o,t}^{R}Q_{i,o,t}^{R}$. Hence

$$Q_{li,o,t}^{R}=\alpha\left(p_{li,o,t}^{R}\right)^{-\varepsilon^{R}}\left(P_{i,o,t}^{R,BI}\right)^{\varepsilon^{R}-1}P_{i,o,t}^{R}Q_{i,o,t}^{R}.$$

Writing $P^R_{i,o,t} = \left(P^{R,BI}_{i,o,t}\right)^{\alpha^R} \left(P_{i,t}\right)^{1-\alpha^R}$, I have

$$Q^R_{li,o,t} = \alpha \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{-\varepsilon^R} \left(\frac{P^{R,BI}_{i,o,t}}{P_{i,t}}\right)^{-\left(1-\alpha^R\right)} Q^R_{i,o,t}.$$

Alternatively, one can define the robot price index by $\widetilde{P}^R_{i,o,t} = \alpha^{\frac{1}{\varepsilon^R}} \left(P^{R,BI}_{i,o,t}\right)^{\frac{\varepsilon^R-\left(1-\alpha^R\right)}{\varepsilon^R}} P_{i,t}^{\frac{1-\alpha^R}{\varepsilon^R}}$ and show

$$Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{\widetilde{P}^R_{i,o,t}}\right)^{-\varepsilon^R} Q^R_{i,o,t}, \tag{A.14}$$

which is a standard gravity representation of robot trade.

To solve the dynamic problem, set up the (current-value) Lagrangian function for non-robot goods producers

$$\begin{aligned}\mathscr{L}_{i,t} = \sum_{t=0}^{\infty} \Bigg\{ \left(\frac{1}{1+\iota}\right)^t \Bigg[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) - \sum_{l,o}\left(p^R_{li,o,t}\left(1+u_{li,t}\right) Q^R_{li,o,t} + P^G_{i,t} I^{int}_{i,o,t} + \gamma P^R_{i,o,t} Q^R_{i,o,t} \frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\Bigg]\Bigg\}.\\ - \lambda^R_{i,o,t}\left\{K^R_{i,o,t+1} - (1-\delta) K^R_{i,o,t} - Q^R_{i,o,t}\right\}\end{aligned}$$

Taking the FOC with respect to the hardware from country l, $Q^R_{li,o,t}$, I have

$$p^R_{li,o,t}\left(1+u_{li,t}\right) + 2\gamma P^R_{i,o,t}\left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}} = \lambda^R_{i,o,t}\frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}}. \tag{A.15}$$

Taking the FOC with respect to the integration input $I^{int}_{i,o,t}$, I have

$$P^G_{i,t} + 2\gamma P^R_{i,o,t}\left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}} = \lambda^R_{i,o,t}\frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}}, \tag{A.16}$$

Taking the FOC with respect to $K^R_{i,o,t+1}$, I have

$$\left(\frac{1}{1+\iota}\right)^{t+1}\left[\frac{\partial\pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}_o\right)}{\partial K^R_{i,o,t+1}}+\gamma P^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2+(1-\delta)\,\lambda^R_{i,o,t+1}\right]-\left(\frac{1}{1+\iota}\right)^t\lambda^R_{i,o,t}=0, \tag{A.17}$$

and the transversality condition: for any j and o,

$$\lim_{t\to\infty} e^{-\iota t}\lambda^R_{j,o,t}K^R_{j,o,t+1}=0. \tag{A.18}$$

Rearranging equation (A.17), I obtain the following Euler equation.

$$\lambda^R_{i,o,t}=\frac{1}{1+\iota}\left[(1-\delta)\,\lambda^R_{i,o,t+1}+\frac{\partial}{\partial K^R_{i,o,t+1}}\pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}\right)+\gamma p^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2\right]. \tag{A.19}$$

Turning to the demand for non-robot good, I will characterize bilateral intermediate good trade demand and total expenditure. Write $X^G_{j,t}$ the total purchase quantity (but not value) of good G in country j in period t. By equation (1.11), the bilateral trade demand is given by

$$p^G_{ij,t}Q^G_{ij,t}=\left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon}P^G_{j,t}X^G_{j,t}, \tag{A.20}$$

for any i, j, and t. In this equation, $P^G_{j,t}X^G_{j,t}$ is the total expenditures on non-robot goods. The total expenditure is the sum of final consumption $I_{j,t}$, payment to intermediate goods $\alpha_M p^G_{j,t}Y^G_{j,t}$, input to robot productions $\sum_o P^G_{j,t}I^R_{j,o,t}=\sum_{o,k}p^R_{jk,o,t}Q^R_{jk,o,t}$, and payment to robot integration $\sum_o P^G_{j,t}I^{int}_{j,o,t}=\left(1-\alpha^R\right)\sum_o P^R_{j,o,t}Q^R_{j,o,t}$. Hence

$$P^G_{j,t}X^G_{j,t}=I_{j,t}+\alpha_M p^G_{j,t}Y^G_{j,t}+\sum_{o,k}p^R_{jk,o,t}Q^R_{jk,o,t}+\left(1-\alpha^R\right)\sum_o P^R_{j,o,t}Q^R_{j,o,t}.$$

For country j and period t, by substituting into income $I_{j,t}$ the period cash flow of non-robot good producer that satisfies

$$\Pi_{j,t}\equiv\pi_{j,t}\left(\left\{K^R_{j,o,t}\right\}_o\right)-\sum_{i,o}\left(p^R_{ij,o,t}\left(1+u_{ij,t}\right)Q^R_{ij,o,t}+\sum_o P^G_{j,t}I^{int}_{j,o,t}+\gamma P^R_{j,o,t}Q^R_{j,o,t}\left(\frac{Q^R_{j,o,t}}{K^R_{j,o,t}}\right)\right)$$

and robot tax revenue $T_{j,t} = \sum_{i,o} u_{ij,t} p^R_{ij,o,t} Q^R_{ij,o,t}$, I have

$$I_{j,t} = (1-\alpha_M) \sum_k p^G_{jk,t} Q^G_{jk,t} - \left(\sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} + \left(1-\alpha^R\right) \sum_o P^R_{j,o,t} Q^R_{j,o,t} \right), \quad \text{(A.21)}$$

or in terms of variables in the definition of equilibrium,

$$I_{j,t} = (1-\alpha_M) \sum_k p^G_{jk,t} Q^G_{jk,t} - \frac{1}{\alpha^R} \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t}.$$

Hence, the total expenditure measured in terms of the production side as opposed to income side is

$$P^G_{j,t} X^G_{j,t} = \sum_k p^G_{jk,t} Q^G_{jk,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} \left(1 + \gamma \frac{Q^R_{ij,o,t}}{K^R_{j,o,t}}\right). \quad \text{(A.22)}$$

Note that this equation embeds the balanced-trade condition. By substituting equation (A.22) into equation (A.20), I have

$$p^G_{ij,t} Q^G_{ij,t} = \left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon^G} \left(\sum_k p^G_{jk,t} Q^G_{jk,t} + \sum_{k,o} p^R_{jk,o,t} Q^R_{jk,o,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} \right). \quad \text{(A.23)}$$

The good and robot-o market-clearing conditions are given by,

$$Y^R_{i,t} = \sum_j Q^G_{ij,t} \tau^G_{ij,t}, \quad \text{(A.24)}$$

for all i and t, and

$$p^R_{i,o,t} = \frac{P^G_{i,t}}{A^R_{i,o,t}} \quad \text{(A.25)}$$

for all i, o, and t, respectively.

Conditional on state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}^R_t, \boldsymbol{\lambda}^R_t, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, equations (1.4), (A.10), (A.15), (A.23), (A.24), and (A.25) characterize the temporary equilibrium $\left\{\boldsymbol{p}^G_t, \boldsymbol{p}^R_t, \boldsymbol{w}_t, \boldsymbol{Q}^G_t, \boldsymbol{Q}^R_t, \boldsymbol{L}_t\right\}$. In addition, conditional on initial conditions $\left\{\boldsymbol{K}^R_0, \boldsymbol{L}_0\right\}$, equations (1.13), (A.19), and (A.18) characterize the sequential equilibrium.

Finally, the steady state conditions are given by imposing the time-invariance condition to equations (1.13) and (A.19):

$$Q_{i,o}^{R}=\delta K_{i,o}^{R}, \tag{A.26}$$

$$\frac{\partial}{\partial K_{i,o}^{R}}\pi_{i}\left(\left\{K_{i,o}^{R}\right\}\right)=(\iota+\delta)\,\lambda_{i,o}^{R}-\sum_{l}\gamma p_{li,o}^{R}\left(\frac{Q_{li,o}^{R}}{K_{i,o}^{R}}\right)^{2}\equiv c_{i,o}^{R}. \tag{A.27}$$

Note that equation (A.27) can be interpreted as the flow marginal profit of capital must be equalized to the marginal cost term. Thus I define the steady state marginal cost of robot capital $c_{i,o}^{R}$ from the right-hand side of equation (A.27). Note that if there is no adjustment cost $\gamma=0$, the steady state Euler equation (A.27) implies

$$\frac{\partial}{\partial K_{i,o}^{R}}\pi_{i}\left(\left\{K_{i,o}^{R}\right\}\right)=c_{i,o}^{R}=(\iota+\delta)\,\lambda_{i,o}^{R},$$

which states that the marginal profit of capital is the user cost of robots in the steady state [Hall and Jorgenson, 1967].

A.2.5 Proof of Proposition 1

The proof takes the following four conceptual steps. First, I will write the real wage change $\left(\widehat{w_{i,o}/P_{i}^{G}}\right)$ in terms of the weighted average of relative price changes, making use of the fact that the sum of shares equals one. Second, I rewrite relative price change into layers of relative price changes with the technique of addition and subtraction. Third, I show that each layer of relative price changes is a change of relevant input or trade shares controlled by elasticity substitution. In other words, an input or trade shares reveals a layer of relative price changes. Finally, I make use of the fact that the sum of shares do not change after the shock to arrive at equation (1.23).

Cost minimization given production functions (1.8), (1.10), and (1.11) imply

$$\widehat{\left(\frac{w_{i,o}}{P_{i}^{G}}\right)}=\frac{1}{1-\alpha_{i,M}}\sum_{l}\widetilde{x}_{li,t_0}^{G}\sum_{o'}\widetilde{x}_{l,o',t_0}^{O}\left[\widetilde{x}_{l,o',t_0}^{L}\left(\widehat{w_{i,o}}-\widehat{w_{l,o'}}\right)+\left(1-\widetilde{x}_{l,o',t_0}^{L}\right)\left(\widehat{w_{i,o}}-\left(\frac{\widehat{A_{l,o'}^{K}}}{1-\theta_{o}}+\widehat{c_{l,o'}^{R}}\right)\right)\right]. \tag{A.28}$$

Note that by additions and subtractions, I can rewrite

$$\widehat{w_{i,o}} - \widehat{w_{l,o'}} = \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\widehat{w_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_i}\right), \tag{A.29}$$

where $\widehat{P^O_{i,o}}$, $\widehat{P^O_i}$, and $\widehat{P^G_i}$ are the price (cost) index of occupation o, occupation aggregate $T^O_{i,t} \equiv \left[\sum_o \left(T^O_{i,o,t}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$, and consumption of non-rogot good G, and

$$\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}}\right) = \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_i}\right). \tag{A.30}$$

Note that the cost minimizing input and trade shares satisfy

$$\begin{cases} \widehat{\tilde{x}^L_{i,o}} = (1-\theta_o)\left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right), \ \widehat{1-\tilde{x}^L_{i,o}} = \widehat{A^K_{i,o}} + (1-\theta_o)\left(\widehat{c^R_{i,o}} - \widehat{P^O_{i,o}}\right) \\ \widehat{\tilde{x}^O_{i,o}} = (1-\beta)\left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right), \ \widehat{\tilde{x}^G_{li}} = (1-\varepsilon)\left(\widehat{p^G_l} - \widehat{P^G_i}\right) \end{cases} \tag{A.31}$$

Combined with the Cobb-Douglas assumption of production function (1.8), equations (A.29), (A.30), and (A.31) imply

$$\widehat{w_{i,o}} - \widehat{w_{l,o'}} = \frac{\widehat{\tilde{x}^L_{i,o}}}{1-\theta_o} - \frac{\widehat{\tilde{x}^L_{l,o'}}}{1-\theta_o} + \frac{\widehat{\tilde{x}^O_{i,o}}}{1-\beta} - \frac{\widehat{\tilde{x}^O_{l,o'}}}{1-\beta} + \frac{\widehat{\tilde{x}^G_{ii}}}{1-\varepsilon} - \frac{\widehat{\tilde{x}^G_{li}}}{1-\varepsilon}$$

$$\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta_o} + \widehat{c^R_{l,o'}}\right) = \frac{\widehat{\tilde{x}^L_{i,o}}}{1-\theta_o} - \frac{\widehat{\left(1-\tilde{x}^L_{l,o'}\right)}}{1-\theta_o} + \frac{\widehat{\tilde{x}^O_{i,o}}}{1-\beta} - \frac{\widehat{\tilde{x}^O_{l,o'}}}{1-\beta} + \frac{\widehat{\tilde{x}^G_{ii}}}{1-\varepsilon} - \frac{\widehat{\tilde{x}^G_{li}}}{1-\varepsilon}.$$

Substituting these in equation (A.28) and using the facts that $\tilde{x}^L_{i,o,t_0}\widehat{\tilde{x}^L_{i,o}} + \left(1-\tilde{x}^L_{i,o,t_0}\right)\widehat{\left(1-\tilde{x}^L_{i,o}\right)} = 0$ for all i and o, $\sum_o \tilde{x}^O_{i,o,t_0}\widehat{\tilde{x}^O_{i,o}} = 0$, and $\sum_l \tilde{x}^G_{li,t_0}\widehat{\tilde{x}^G_{li}} = 0$ for all i, I have equation (1.23).

A.2.6 Details of the Two-step Estimator

Assumption 1 implies that, for any d-dimensional functions $\boldsymbol{H} \equiv \{H_o\}_o$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\right] = 0$. The GMM estimator based on $\boldsymbol{H}$ is

$$\boldsymbol{\Theta}_{\boldsymbol{H}} \equiv \arg\min_{\boldsymbol{\Theta}} \sum_{o=1}^{O} \left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\boldsymbol{\Theta}\right)\right]^\top \left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\boldsymbol{\Theta}\right)\right], \tag{A.32}$$

which is consistent under the moment condition (1.27) if $\boldsymbol{H}$ satisfies the rank conditions in Newey and McFadden [1994]. The exact specification of $\boldsymbol{H}$ determines the optimality, or the minimal variance, of estimator (A.32). To specify $\boldsymbol{H}$, I apply the approach that achieves the asymptotic optimality developed in Chamberlain [1987]. Formally, define the instrumental variable Z_o as follows:

$$Z_o \equiv H_o^*\left(\boldsymbol{\psi}_{t_1}^J\right) \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}, \tag{A.33}$$

and assume the regularity conditions 4 in Section A.2.7 of the Appendix.

Proposition 2. *Under Assumptions 1 and 4, $\boldsymbol{\Theta}_{\boldsymbol{H}^*}$ is asymptotically normal with the minimum variance among the asymptotic variances of the class of estimators in equation* (A.32).

Proof. See Section A.2.7. □

To understand the optimality of the IV in equation (A.33), note that it has two components. The first term is the conditional expected gradient vector $\mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$, which takes the gradient with respect to the structural parameter vector. Thus, it assigns large weight to occupation that changes the the predicted outcome variable sensitively to the parameters. The second term is the conditional inverse expected variance matrix $\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, which put large weight to occupation that has small variance of the structural residuals.

Substituting equation (A.33) to the general GMM estimator (A.32), I have an estimator $\boldsymbol{\Theta}_{\boldsymbol{H}^*} = \arg\min_{\boldsymbol{\Theta}}\left[\sum_o Z_o\nu_o\left(\boldsymbol{\Theta}\right)\right]^\top\left[\sum_o Z_o\nu_o\left(\boldsymbol{\Theta}\right)\right]$. Since Z_o depends on unknown parameters $\boldsymbol{\Theta}$, I implement the estimation by the two-step feasible method, or the model-implied optimal IV [Adao et al., 2019a]. I first estimate the first-step estimate $\boldsymbol{\Theta}_1$ from arbitrary initial values $\boldsymbol{\Theta}_0$. Since the IV is a function of the Japan robot shock $\boldsymbol{\psi}_{t_1}^J$, $\boldsymbol{\Theta}_1$ is consistent by Assumption 1. However, it is not optimal. To achieve the optimality, in the second step, I obtain the optimal IV using the consistent estimator $\mathbf{\Theta}_1$. To summarize the discussion so far, define IVs $Z_{o,n}$ where $n = 0, 1$ as follows:

$$Z_{o,n} \equiv H_{o,n}\left(\boldsymbol{\psi}_{t_1}^J\right) = \mathbb{E}\left[\nabla_{\mathbf{\Theta}} \nu_o\left(\mathbf{\Theta}_n\right) | \boldsymbol{\psi}_{t_1}^J\right] \mathbb{E}\left[\nu_o\left(\mathbf{\Theta}_n\right)\left(\nu_o\left(\mathbf{\Theta}_n\right)\right)^\top | \boldsymbol{\psi}_{t_1}^J\right]^{-1}. \tag{A.34}$$

Then I have the following result.

Proposition 3. *Under Assumptions 1 and 4, the estimator $\mathbf{\Theta}_2$ obtained in the following procedure is consistent, asymptotically normal, and optimal:*

Step 1: With a guess $\mathbf{\Theta}_0$, estimate $\mathbf{\Theta}_1 = \mathbf{\Theta}_{\boldsymbol{H}_0}$ using $Z_{o,0}$ defined in equation (A.34).

Step 2: With $\mathbf{\Theta}_1$, estimate $\mathbf{\Theta}_2$ by $\mathbf{\Theta}_2 = \mathbf{\Theta}_{\boldsymbol{H}_1}$ using $Z_{o,1}$ defined in equation (A.34).

Proof. See Section A.2.8. □

A.2.7 Proof of Proposition 2

To prove Proposition I follow the arguments made in Sections 2 and 3 of Newey and McFadden [1994]. The proof consists of four sub results in the following Lemma. Proposition 2 can be obtained as a combination of the four results. The formal statement requires the following additional assumptions.

Assumption 4. *(i) A function of $\widetilde{\mathbf{\Theta}}$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\right] \neq 0$ for any $\widetilde{\mathbf{\Theta}} \neq \mathbf{\Theta}$. (ii) $\underline{\theta} \leq \theta_o \leq \overline{\theta}$ for any o, $\underline{\beta} \leq \beta \leq \overline{\beta}$, $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$, and $\underline{\phi} \leq \phi \leq \overline{\phi}$ for some positive values $\underline{\theta}, \underline{\beta}, \underline{\gamma}, \underline{\phi}, \overline{\theta}, \overline{\beta}, \overline{\gamma}, \overline{\phi}$. (iii) $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\|\right] < \infty$. (iv) $\mathbb{E}\left[\| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\|^2\right] < \infty$ (v) $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nabla_{\widetilde{\mathbf{\Theta}}}\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\|\right] < \infty$.*

Lemma 4. *Assume Assumptions 1 and 4(i)-(iii).*

(a) The estimator of the form (A.32) is consistent.

Additionally, assume Assumptions 4(iv)-(v).

(b) The estimator of the form (A.32) is asymptotically normal.

(c) $\sqrt{O}\left(\mathbf{\Theta}_{\boldsymbol{H}^*} - \mathbf{\Theta}\right) \to_d \mathcal{N}\left(0, \left(\boldsymbol{G}^\top \mathbf{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\right)$, and the asymptotic variance is the minimum of that of the estimator of the form (A.32) for any function $\boldsymbol{H}$.

Proof. (a) I follow Theorems 2.6 of Newey and McFadden [1994], which implies that it suffices to show conditions (i)-(iv) of this theorem are satisfied. Assumption 4(i) guarantees condition (i). Condition (ii) is implied by Assumption 4(ii). Condition (iii) follows because all supply and demand functions in the model is continuous. Condition (iv) is implied by Assumption 4(iii).

(b) I follow Theorem 3.4 of Newey and McFadden [1994], which implies that it suffices to show conditions (i)-(v) of this theorem are satisfied. Condition (i) is satisfied by Assumption 4(i). Condition (ii) follows because all supply and demand functions in the model is continuously differentiable. Condition (iii) is implied by Assumption 1 and Assumption 4(iv). Assumption 4(v) implies condition (iv). Finally, the gradient vectors of the structural residual is linear independent, guaranteeing the non-singularity of the variance matrix and condition (v).

(c) By Theorem 3.4 of Newey and McFadden [1994], for an arbitrary IV-generating function $\boldsymbol{H}$, the asymptotic variance of the GMM estimator $\boldsymbol{\Theta_H}$ is

$$\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1},$$

where $G_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$. Therefore, if $H_o\left(\boldsymbol{\psi}_{t_1}^J\right) = Z_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, then this expression is equal to $\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, where

$$\boldsymbol{G} \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)\right] \text{ and } \boldsymbol{\Omega} \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right].$$

To show that this variance is minimal, I will check that

$$\begin{aligned}\boldsymbol{\Delta} \equiv &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\\ &-\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\end{aligned}$$

is positive semi-definite. In fact, note that

$$\begin{aligned}\boldsymbol{\Delta} = &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\times\\ &\left\{\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right] - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right\}\times\\ &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}.\end{aligned}$$

Define

$$\widetilde{\nu}_o = H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left((G_o)^\top \Omega_o^{-1}\nu_o\right)^{-1}\right]\mathbb{E}\left((G_o)^\top \Omega_o^{-1}\nu_o\right)^{-1}(G_o)^\top \Omega_o^{-1}\nu_o,$$

where $\Omega_o \equiv \mathbb{E}\left[\nu_o(\boldsymbol{\Theta})(\nu_o(\boldsymbol{\Theta}))^\top |\boldsymbol{\psi}_{t_1}^J\right]$. Applying Theorem 5.3 of Newey and McFadden [1994], I have

$$\mathbb{E}\left[\widetilde{\nu}_o(\widetilde{\nu}_o)^\top\right] = \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right] - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right].$$

Since $\mathbb{E}\left[\widetilde{\nu}_o(\widetilde{\nu}_o)^\top\right]$ is positive semi-definite, so is $\boldsymbol{\Delta}$, which completes the proof. □

A.2.8 Proof of Proposition 3

I apply arguments in Section 6.1 of Newey and McFadden [1994]. Namely, I define the joint estimator of the first-step and second-step estimator in Proposition 3 that falls into the class of general GMM estimation, and discuss the asymptotic property using the general result about GMM estimation. In the proof, I modify the notation of the set of functions that yield optimal IV, $\boldsymbol{H}^*$, to clarify that it depends on parameters $\boldsymbol{\Theta}$ as follows:

$$H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right) = \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o(\boldsymbol{\Theta})|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o(\boldsymbol{\Theta})(\nu_o(\boldsymbol{\Theta}))^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}.$$

Define the joint estimator as follows:

$$\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix} \equiv \arg\min_{\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1}\left[\sum_o e_o(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1)\right]^\top\left[\sum_o e_o(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1)\right],$$

where

$$e_o(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1) \equiv \begin{pmatrix}H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o(\boldsymbol{\Theta}_2)\\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o(\boldsymbol{\Theta}_1)\end{pmatrix}.$$

Since for any $\boldsymbol{\Theta}$, IV-generating function $H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)$ gives the consistent estimator for $\boldsymbol{\Theta}$, I have $\boldsymbol{\Theta}_1 \to \boldsymbol{\Theta}$ and $\boldsymbol{\Theta}_2 \to \boldsymbol{\Theta}$. I also have the asymptotic variance

$$\mathrm{Var}\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix} = \left[\left(\widetilde{\boldsymbol{G}}\right)^\top\widetilde{\boldsymbol{\Omega}}\widetilde{\boldsymbol{G}}\right]^{-1},$$

where

$$\widetilde{\boldsymbol{G}} \equiv \mathbb{E}\left[\nabla_{(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1)^\top} e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right] = \mathbb{E}\begin{bmatrix} H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nabla\nu_o\left(\boldsymbol{\Theta}_2\right) & \nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right) \\ \boldsymbol{0} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nabla\nu_o\left(\boldsymbol{\Theta}_1\right) \end{bmatrix}$$

and

$$\widetilde{\boldsymbol{\Omega}} \equiv \mathbb{E}\left[e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\left[e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]^\top\right] = \mathbb{E}\begin{bmatrix} H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\right]^\top \\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\right]^\top \end{bmatrix}.$$

Note that Assumption 1 implies that any function of $\boldsymbol{\psi}_{t_1}^J$ is orthogonal to ν_o, implying $\mathbb{E}\left[\nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right] = 0$. Therefore, $\widetilde{\boldsymbol{G}}$ is a block-diagonal matrix and thus the marginal asymptotic distribution of $\boldsymbol{\Theta}_2$ is normal with variance $\mathrm{Var}\left(\boldsymbol{\Theta}_2\right) = \left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, noting that $\boldsymbol{G} = \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nabla\nu_o\left(\boldsymbol{\Theta}\right)\right]$ and $\boldsymbol{\Omega} \equiv \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nu_o\left(\boldsymbol{\Theta}\right)\left(H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right]$. By Proposition 2, this asymptotic variance is minimal among the GMM estimator (A.32).

A.3 Further Estimation and Simulation Results

A.3.1 Estimation at 2 Digit-level Occupation Groups

In this section, I study how the occupation grouping defined in Section 1.4.1 affects the estimation result. Specifically, I apply the estimation method in Proposition 3 for 2-digit occupation groups provided by the US Census "[f]or users who wish to further aggregate occupation to broader categories[.]"[2] Table A.5 shows the result. I find that the elasticity estimates for "Production" and "Transportation and Material Moving" occupations remain around 4 with small standard errors. However, the estimates for the other occupations become different from the case with the 5-occupation aggregates, and the standard errors are larger and volatile. This exercise reveals that the 5-occupation aggregation in Section 1.4.1

[2]Further details can be found at https://usa.ipums.org/usa-action/variables/OCC2010 (Accessed on December 6, 2020).

provides the conservative grouping and tightly estimated elasticities of substitution. The reason is that I use the 4-digit occupational variation for estimation, and 2-digit occupation grouping often yields only a small number of 4-digit occupations, reducing estimation power (see “# Occ.” column in Table A.5).

At this point, it is also worth noting the time-series variation. Since I have annual observation for occupational robot costs, it is potentially possible to leverage this rich variation for the structural estimation, which may permit me to estimate the EoS θ_o at a narrower occupation group level. However, the bottleneck of this approach is the computational burden to compute the dynamic solution matrix $\overline{\boldsymbol{F}_t}$. Specifically, dynamic substitution matrix $\overline{\boldsymbol{F}_{t+1}^y}$ in equation (1.19) is based on the conditions of Blanchard and Kahn [1980]. This requires computing the eigenspace, as described in detail in Section A.4. This is computationally hard since we cannot rely on the sparse structure of the matrix $\overline{\boldsymbol{F}_{t+1}^y}$. In contrast, the estimation method in Proposition 3 does not involve such computation, but only requires computing the steady-state solution matrix $\overline{\boldsymbol{E}}$. Then I only need to invert steady-state substitution matrix $\overline{\boldsymbol{E}^y}$, which is feasible given the sparse structure of $\overline{\boldsymbol{E}^y}$. Therefore, a future potential breakthrough on computation technology could make it possible to estimate the model based on the dynamic solution matrix $\overline{\boldsymbol{F}_t}$ and annual observation of my dataset.

A.3.2 Robot Trade Elasticity

To estimate robot trade elasticity ε^R, I apply and extend the trilateral method of Caliendo and Parro [2015]. Namely, decompose the robot trade cost $\tau_{li,t}^R$ into $\ln \tau_{li,t}^R = \ln \tau_{li,t}^{R,T} + \ln \tau_{li,t}^{R,D}$, where $\tau_{li,t}^{R,T}$ is tariff on robots taken from the UNCTAD-TRAINS database and $\tau_{li,t}^{R,D}$ is asymmetric non-tariff trade cost. The latter term is assumed to be $\ln \tau_{li,t}^{R,D} = \ln \tau_{li,t}^{R,D,S} + \ln \tau_{l,t}^{R,D,O} + \ln \tau_{i,t}^{R,D,D} + \ln \tau_{li,t}^{R,D,E}$, where $\tau_{li,t}^{R,D,S}$ captures symmetric bilateral trade costs such as distance, common border, language, and FTA belonging status and satisfies $\tau_{li,t}^{R,D,S} = \tau_{il,t}^{R,D,S}$, $\tau_{l,t}^{R,D,O}$ and $\tau_{i,t}^{R,D,D}$ are the origin and destination fixed effects such as non-tariff barriers respectively, and $\tau_{li,t}^{R,D,E}$ is the random error that is orthogonal to tariffs. From the robot gravity equation (A.14) that I derive in Section A.2.4, I have

$$\ln \left(\frac{X_{li,t}^R X_{ij,t}^R X_{jl,t}^R}{X_{lj,t}^R X_{ji,t}^R X_{il,t}^R} \right) = \left(1 - \varepsilon^R\right) \ln \left(\frac{\tau_{li,t}^{R,T} \tau_{ij,t}^{R,T} \tau_{jl,t}^{R,T}}{\tau_{lj,t}^{R,T} \tau_{ji,t}^{R,T} \tau_{il,t}^{R,T}} \right) + e_{lij,t}, \tag{A.35}$$

Table A.5: Estimates of Elasticities of Substitution between Robots and Workers, θ_o, at the 2-digit Occupation Level

OCC2010 2-dig. Label	OCC2010 Range	# Occ.	2-dig. EoS	2-dig. SE	Group	5-group EoS	5-group SE
Management, Business, Science, And Arts	[30, 430]	9	0.15	1.57	Abstract	0.80	0.60
Business Operations Specialists	[500, 730]	9	1.98	0.91			
Financial Specialists	[800, 1240]	8	0.04	1.29			
Architecture And Engineering	[1300, 1560]	15	-0.31	1.48			
Life, Physical, And Social Science	[1600, 1960]	14	-0.05	1.49			
Community And Social Services	[2000, 2140]	6	0.23	0.69			
Education, Training, And Library	[2200, 2540]	9	0.24	0.64			
Arts, Design, Entertainment, Sports, And Media	[2600, 2910]	13	0.98	0.77			
Healthcare Practitioners And Technical	[3010, 3650]	23	0.6	1.48			
Protective Service	[3720, 4130]	13	1.94	0.58	Service	1.35	0.48
Building And Grounds Cleaning And Maintenance	[4200, 4650]	16	0.5	0.9			
Sales And Related	[4700, 4965]	15	2.07	1.49	Routine, Others	1.27	0.53
Office And Administrative Support	[5000, 5940]	38	-0.24	1.06			
Farming, Fishing, And Forestry	[6005, 6130]	7	2.15	1.22			
Construction	[6200, 6765]	22	1.03	0.85			
Extraction	[6800, 6940]	5	1.44	1.22			
Installation, Maintenance, And Repair	[7000, 7610]	22	-0.7	1.36			
Production	[7700, 8965]	55	3.91	0.24	Routine, Production	4.04	0.24
Transportation And Material Moving	[9000, 9750]	25	4.41	0.36	Routine, Transportation	4.29	0.28

Note: The estimates of the structural parameters based on the estimator in Proposition 3. In header, "OCC2010 2-dig. Label" shows the label of 2-digit occupations groups in the OCC2010 coding scheme, "OCC2010 Range" shows the range of OCC2010 codes that fall into the 2-digit occupation group, "Num. Occ." shows the number of 4-digit level occupations in the 2-digit occupation group, "2-dig. EoS" shows the point estimate of the elasticity of substitution between robots and workers in the 2-digit occupation group, "2-dig. SE" shows the standard error estimate of the elasticity of substitution between robots and workers in the 2-digit occupation group, "Group" shows the 5-group I defined in Section 1.4.1, "5-group EoS" shows the point estimate of the elasticity of substitution in the "5-group", and "5-group SE" shows the the standard error estimate of the elasticity of substitution in the "5-group."

Table A.6: Coefficient of equation (A.35)

	(1) HS 847950	(2) HS 847950	(3) HS 8479	(4) HS 8479
Tariff	-0.272***	-0.236***	-0.146***	-0.157***
	(0.0718)	(0.0807)	(0.0127)	(0.0131)
FEs	h-i-j-t	ht-it-jt	h-i-j-t	ht-it-jt
N	4610	4521	88520	88441
r2	0.494	0.662	0.602	0.658

Note: The author's calculation based on BACI data from 1996 to 2018 and equation (A.35). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses), while the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, while the second and fourth the bilateral fixed effect. See the text for the detail.

where $X^R_{li,t}$ is the bilateral sales of robots from l to i in year t and $e_{lij,t} \equiv \ln \tau^{R,D,E}_{li,t} + \ln \tau^{R,D,E}_{ij,t} + \ln \tau^{R,D,E}_{jl,t} - \ln \tau^{R,D,E}_{lj,t} - \ln \tau^{R,D,E}_{ji,t} - \ln \tau^{R,D,E}_{il,t}$. The benefit of this approach is that it does not require symmetry for non-tariff trade cost $\tau^{R,D}_{li}$, but only requires the orthogonality for the asymmetric component of the trade cost. My method also extends Caliendo and Parro [2015] in using the time-series variation as well as trilateral country-level variation to complement the relatively small number of observations in robot trade data.

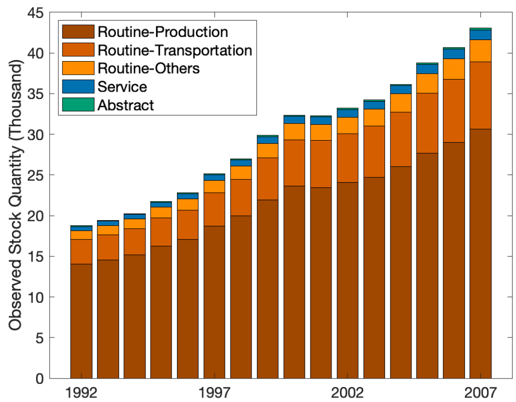
When implementing regression of equation (A.35), I further consider controlling for two separate sets of fixed effects. The first set is the unilateral fixed effect indicating if a country is included in the trilateral pair of countries, and the second set is the bilateral fixed effect for the twin of countries is included in the trilateral pair. These fixed effects are relevant in my setting as a few number of countries export robots, and controlling for these exporters' unobserved characteristics is critical.

Table A.6 shows the result of regression of equation (A.35). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses, the definition of robots used in Humlum, 2019), and the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, and the second and fourth the bilateral fixed effect. The implied trade elasticity of robots ε^R is fairly tightly estimated and ranges between 1.13-1.34. Given these estimation results, I use $\varepsilon^R = 1.2$ in the estimation and counterfactuals.

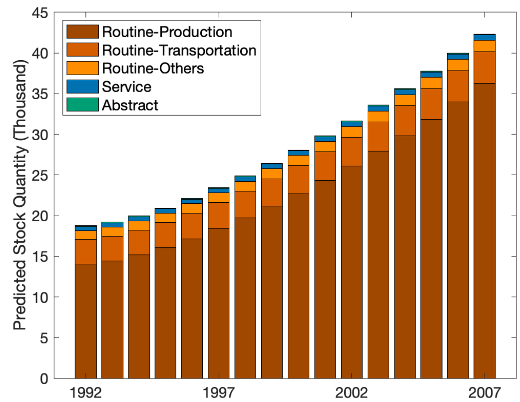
To assess the estimation result, note that Caliendo and Parro [2015] show in Table 1 that the regression coefficient of equation (A.35) is 1.52, with the standard error of 1.81,

Figure A.15: Trends of Robot Stocks

(a) Data



(b) Model



Notes: Figures show the trend of the observed (left) and predicted (right) stock of robots for each occupation group measured by quantities. The predicted robot stocks are computed by shocks backed out from the estimated model and applying the first-order solution to the general equilibrium described in equation (1.20).

for "Machinery n.e.c", which roughly corresponds to HS 84. Therefore, my estimate for industrial robots falls in the one-standard-deviation range of their estimate for a broader category of goods.

Note that the average trade elasticity across sectors is estimated significantly higher than these values, such as 4 in Simonovska and Waugh [2014]. The low trade elasticity for robots ε^R is intuitive given robots are highly heterogeneous and hardly substitutable. This low elasticity implies small gains from robot taxes, with the robot tax incidence almost on the US (robot buyer) side rather than the robot-selling country.

A.3.3 Actual and Predicted Robot Accumulation Dynamics

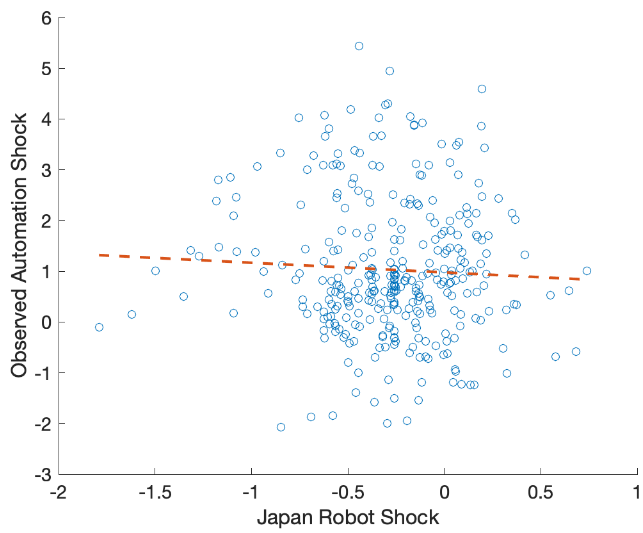
Figure A.15 shows the trends of robot stock in the US in the data and the model. Although I do not match the overall robot capital stocks, the estimated model tracks the observed pattern well between 1992 and the late 2010s, consistent with the fact that I target the changes between 1992 and 2007. There is a slight over-prediction of the growth of production robots and under-prediction of the growth of transportation (material moving) robots between occupation groups.

Table A.7: Shocks Aggregated at 5 groups

Group	ψ^J	$\widehat{a^{obs}}$
Routine, Production	-0.305	2.453
Routine, Transportation	-0.497	3.428
Routine, Others	-0.460	0.335
Service	-0.378	0.623
Abstract	-0.289	0.133

Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The Japan robot shock ψ^J is based on the regression of equation (1.1). The observed automation shock $\widehat{a_o^{obs}}$ is backed out from equation (1.25) with the estimated parameters in Table 1.2. Both measures are aggregated from the 4-digit level to 5 groups using the initial employment weight.

Figure A.16: Correlation between ψ_o^J and $\widehat{a_o^{obs}}$



Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The Japan robot shock is taken from the regression of equation (1.1). The observed automation shock is backed out from equation (1.25) with the estimated parameters in Table 1.2. Each circle is 4-digit occupation and dashed line is the fitted line.

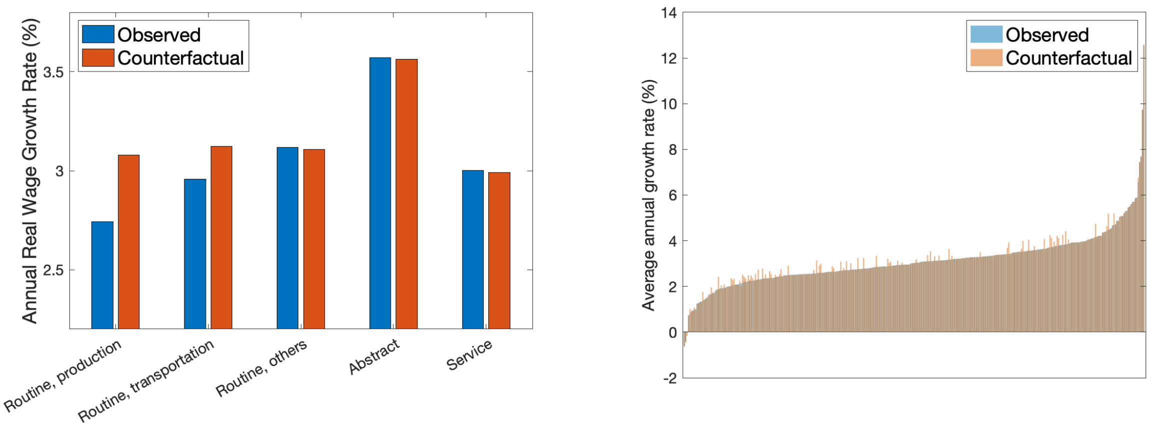
A.3.4 Japan Robot Shock and Observed Automation Shock

Table A.7 shows the Japan robot shock and observed automation shock backed out from the estimated model. On the one hand, one can see that the Japanese robots' cost declined similarly across occupation groups. On the other hand, the observed automation shock shows a significant variation, namely, larger in production and transportation occupations than other occupations. In turn, Figure A.16 shows a further detailed scatter plot between the two shocks, delivering a mild negative relationship. This negative correlation is consistent with the example of robotic innovations in Appendix A.1.2.

Figure A.17: The Steady-state Effect of Robots on Wages

(a) Occupation Groups

(b) All Occupations



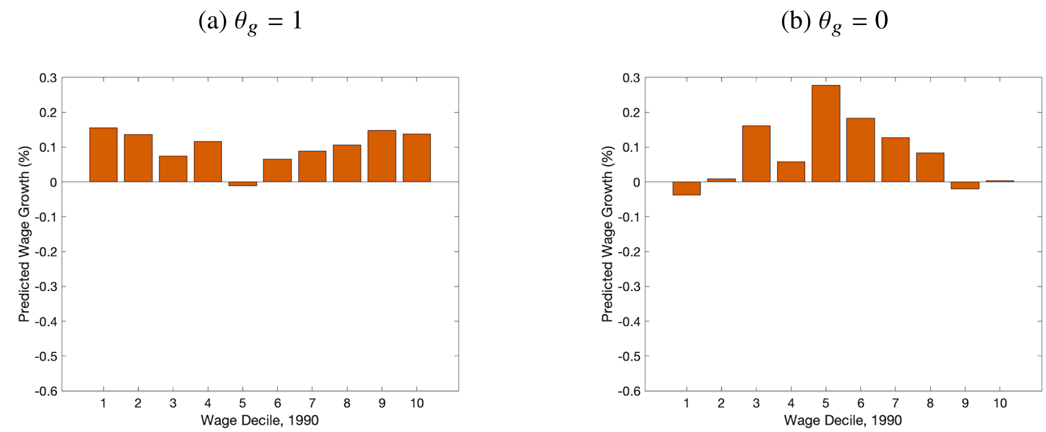
A.3.5 Automation and Wages at Occupations

Figure A.17 shows the observed and counterfactual growth rate of real wages for each occupation, where the counterfactual change means the simulated change absent the automation shock. Figure A.17a shows the results aggregated at the 5 occupations groups defined in Section 1.4.1. I compute the counterfactual growth rate from the observed rate of the wage change, subtracted by the change predicted by the first-order steady-state solution $\overline{\boldsymbol{E}}$ and the observed automation shock $\widehat{\boldsymbol{a}^{\text{obs}}}$. The result is based on the observed high growth rates of robots in routine production and transportation (material moving) occupations, and these occupations' high EoS estimates between robots and workers. In particular, at the 5-occupation aggregate level, most of the observed differences in the real wage growth rates in the three routine occupation groups are closed absent the automation shock. Applying the similar exercise for all occupations in my sample, Figure A.17b shows a more granular result, where occupations are sorted by the observed changes of wages from 1990-2007.

A.3.6 Wage Polarization Exercise under Different Robot-labor EoS

To learn the role of my robot-labor EoS estimates in deriving the wage-polarizing effect of robotization, I perform the same robotization exercise as in Section 1.5.1 under different values of the EoS θ_g and study the occupational wage consequence. Specifically, I consider the following two cases: $\theta_g = 0$ for any occupation group g as in Acemoglu and Restrepo

Figure A.18: Wage Polarization Exercise under Different Elasticity of Substitution, θ_g



Notes: The annualized wage growth rates predicted by the backed-out shocks and the estimated model's first-order steady-state solution given in equation (1.18) under specific value of the elasticity of substitution between robots and labor, θ. The left panel shows the case with $\theta = 1$ and the right $\theta = 0$. See Figure 1.2b for comparison to the case under my parameter estimates.

[2020] as described in Section A.2.3, and Cobb-Douglas case of $\theta = 1$ for any g. The results are shown in Figure A.18. Compared with the right panel of Figure 1.2b, we do not find that the observed robotization shock does not contribute to wage polarization when ϑ is as low as 0 or 1. This finding is because the increased robot use in the middle of the distribution does not reduce wage in such a case. In this sense, it is critical to have a cost shock measure to estimate the robot-labor EoS to derive the wage-polarizing effect of robotization.

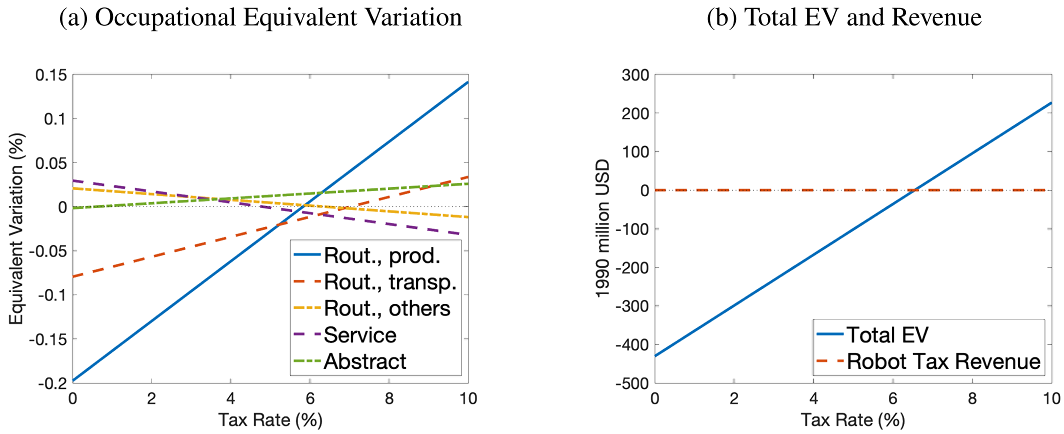
A.3.7 Robot Tax and Workers' Welfare

To examine how the robot tax affects workers in different occupations, I define the equivalent variation (EV) implicitly as follows:

$$\sum_{t=t_0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \ln\left(\left[C'_{i,o,t}\right]\right) = \sum_{t=t_0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \ln\left(C_{i,o,t}\left[1+EV_{i,o}\right]\right). \tag{A.36}$$

Namely, the EV is the fraction of the occupation-specific subsidy that would make the present discounted value (PDV) of the utility in the robotized and taxed equal to the PDV of the utility if the occupation-specific subsidy were exogenously given in the initial equilibrium. On the left-hand side, I hit the robotization shock backed out in Section 1.4.4. As in Section 1.5.2, I consider the US unilateral (not inducing a reaction in other countries), unexpected, and permanent tax on robot purchases. By this definition, the worker in occupation o prefers

Figure A.19: Robot Tax and Workers' Welfare



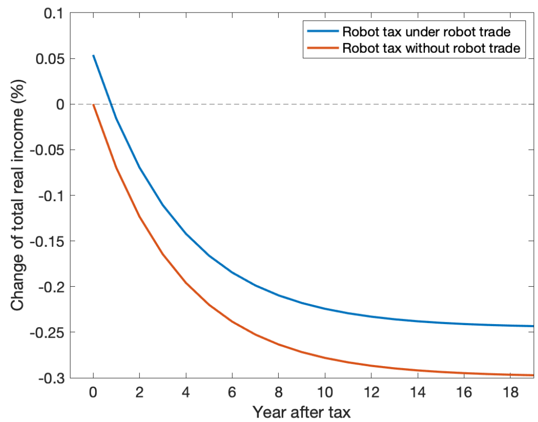
Note: The left panel shows the US workers' equivalent variation defined in equation (A.36) as a function of the US robot tax rate. Labels "Rout., prod.", "Rout., transp.", and "Rout., others" mean routine, production; routine, transportation; and routine, others occupations, respectively. The right panel shows monetary values of equivalent variations aggregated across workers and robot tax revenue as a function of the robot tax rate, measured in 1990 million USD.

the robotized and taxed world if and only if the EV is positive for o.

Figure A.19a shows this occupation-specific EV as a function of the tax rate. The far-left side of the figure is the case of zero robot tax, thus a case of only the robotization shock. Consistent with the occupational wage effects (cf. Figure A.17), workers in production and transportation occupations lose significantly due to robotization. In contrast other workers are roughly indifferent between the robotized world and the non-robotized initial equilibrium or slightly prefer the former world. Going right through the figure, the production and transportation workers' EV improves as the robot tax reduces competing robots. The EV of production workers turns positive when the tax rate is around 6%, and that of transportation workers is positive when the rate is about 7%. However, these tax rates are too high and would make EVs in other occupations negative. In fact, in production and transportation occupations, robots do not accumulate and adversely affect labor demand in the other occupations.

To study if the reallocation policy by robot tax may work, I also compute the equivalent variation in terms of monetary value aggregated by occupation groups (total EV) and compare it with the robot tax revenue, both as a function of robot tax. Figure A.19b shows the result. One can confirm that the marginal robot tax revenue is far from enough to compensate for workers' loss that concentrates on production and transportation workers, at the initial equilibrium with zero robot tax rate. The robot tax revenue is negligible at this

Figure A.20: Effects of the Robot Tax on the US Real Income



margin compared with the workers' loss due to robotization. It is true that as the robot tax rate increases, the total EV rises: When the rate is as large as 6-7%, the sum of the total EV and the robot tax revenue is positive. However, one should be cautious that my solution to the model is to the first order. Thus the approximation error may play an important role when the robot tax rate is significantly higher than the one in the initial equilibrium, zero. Extending my solution to the higher-order or even finding the exact solution is left for future research.

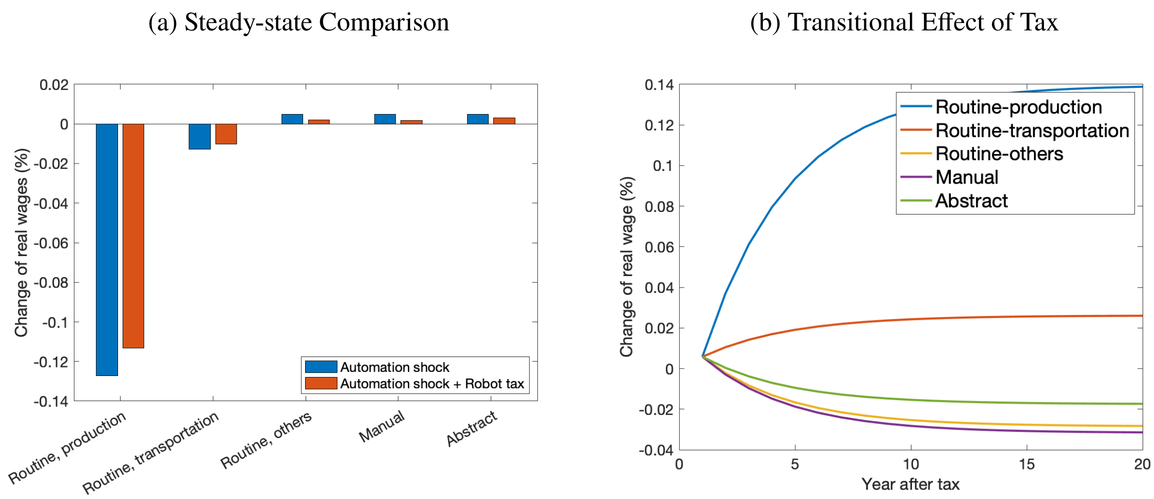
A.3.8 The Role of Trade that Plays in the Robot Tax Effect

Figure A.20 shows the dynamic effect of the robot tax on the US real income. If the robot trade is not allowed, the robot tax does not increases the real income in any period since the terms-of-trade effect does not show up, but only the long-run capital decumulation effect does. On the other hand, once I allow the robot trade as observed in the data, the robot tax may increase the real income because it decreases the price of imported robots. The effect is concentrated in the short-run before the capital decumulation process matures. In the long run, the negative decumulation effect dominates the positive terms-of-trade effect.

A.3.9 The Robot Tax Effect on Occupations

In Figure A.21a, I show two scenarios of the steady-state changes in occupational real wages. On the one hand, I shock the economy only with the automation shocks. On the other hand,

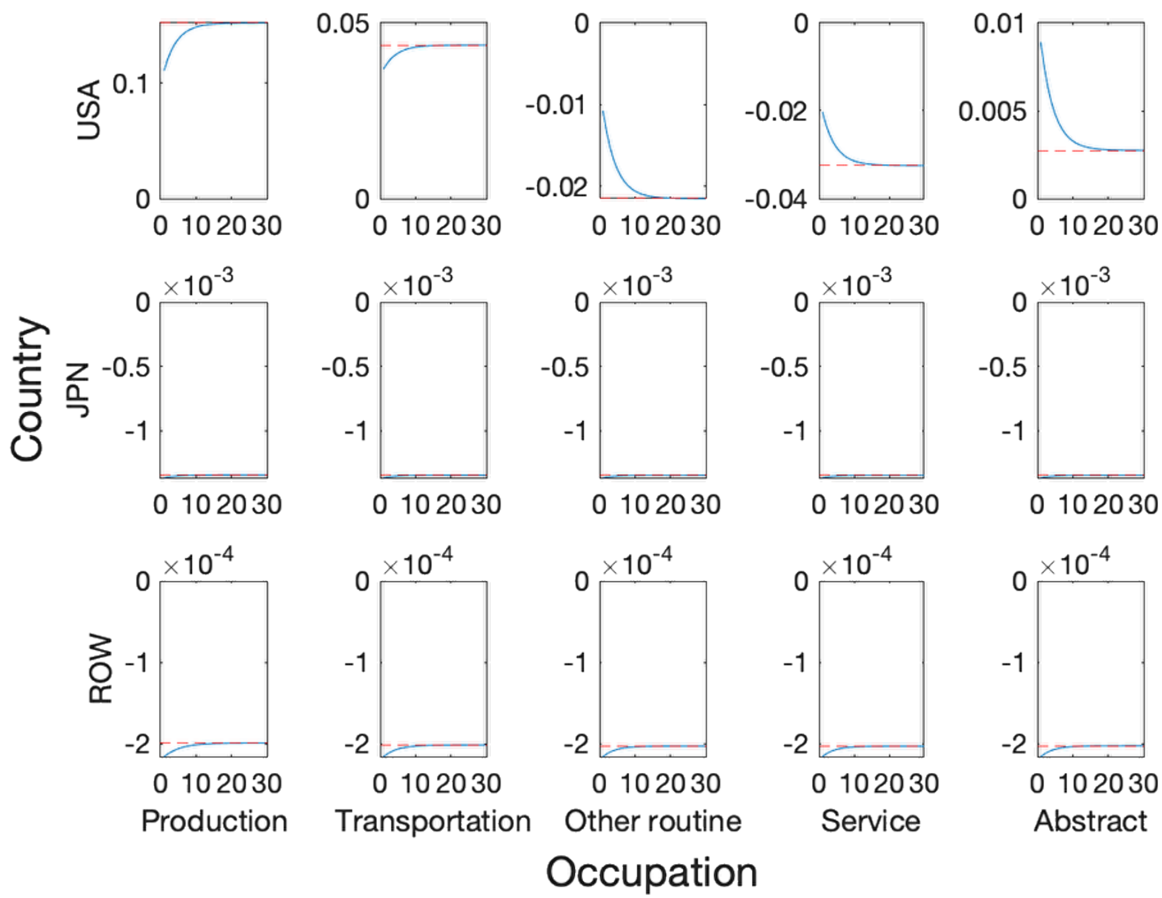
Figure A.21: Effects of the Robot Tax on Occupational Real Wages



I shock the economy with both the automation shocks and the robot tax. The result shows heterogenous effects on occupational real wages of the robot tax. The tax mitigates the negative effect of automation on routine production workers and routine transportation workers, while the tax marginally decreases the small gains that workers in the other occupations would have enjoyed. Overall, the robot tax mitigates the large heterogeneous effects of the automation shocks, that could go negative and positive directions depending on occupation groups, and compresses the effects towards zero. Figure A.21b shows the dynamics of the effects of robot tax, net of the effects of automation shocks. Although the steady-state effects of robot tax were heterogeneous as shown in Figure A.21a, the effect is not immediate but materializes after around 10 years, due to the sluggish adjustment in the accumulation of the robot capital stock. Overall, I find that since the robot tax slows down the adoption of robots, it rolls back the real wage effect of automation–workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) benefit from the tax, while the others lose.

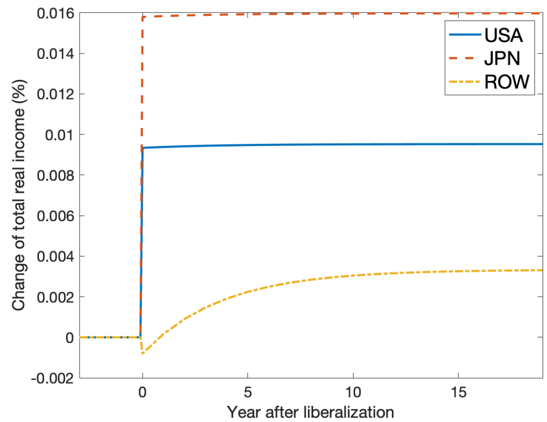
To study how the occupational effects unfold over time and if the US policy affects third countries, I study occupational value evolution given the US general robot tax. Figure A.22 shows the impact of the US's unilateral, unexpected, and permanent 6% general robot tax on the world's occupational values in the short run and the long run. In the first row, panels show the US occupational values and corroborate the finding in Figure A.21 that production and transportation workers gain from the tax but not other workers. As can be seen

Figure A.22: US General Robot Tax and Global Occupational Value Evolution



Note: Transition dynamics of workers' occupation-specific values given the US's unexpected, unilateral, and permanent 6% general robot tax at the initial steady-state (period 0) are shown. Blue solid lines are the transitional dynamics, and red dashed lines are the steady-state values.

Figure A.23: The Effect of Robot Trade Cost Reduction



from the figure, it takes roughly 10 years until the worker values reach steady states. In other countries than the US, the US robot tax effect is negative but quantitatively limited.

A.3.10 Trade Liberalization of Robots

Following Ravikumar et al. [2019], I consider unexpected and permanent 20% reduction in the bilateral trade costs to study the effect trade liberalization of capital good and dynamics gains from trade. Figure A.23 shows the result of such a simulation for a 20-years time horizon. All country groups in the model gain from the trade liberalization. The US gain materialize almost immediately after the trade cost change. A possible explanation is the combination of the following two observation. First, it takes time to accumulate robots after the trade liberalization, which makes the gains from trade liberalization sluggish. Second, by exporting robots to ROW, the US increases the revenue of robot sales immediately after the trade cost drop, improving the short-run real income gain. The real income gain is the largest for Japan, a large net robot exporter. It is noteworthy that ROW loses from the reduction in the robot trade cost, possibly due to the terms-of-trade deterioration in the short-run.

A.4 Detail of the GE Solution

I discuss the derivation log-linearization in equations (1.16), (1.18), and (1.20), so that I can bring the theory with computation. Throughout the section, relational operator $\circ$ is Hadamard product, $\oslash$ indicates Hadamard division, and $\otimes$ means Kronecker product.

It is useful to show that the CES production structure implies the share-weighted log-change expression for both prices and quantities. Namely, I have a formula for the change in destination price index $\widehat{P^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\widehat{p^G_{ij,t}}$ and one for the change in destination expenditure $\widehat{P^G_{j,t}} + \widehat{Q^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\left(\widehat{p^G_{ij,t}} + \widehat{Q^G_{ij,t}}\right)$. These imply that

$$\widehat{Q^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\widehat{Q^G_{ij,t}},$$

or the changes of quantity aggregate $\widehat{Q^G_{j,t}}$ are also share-weighted average of changes of origin quantity $\widehat{Q^G_{ij,t}}$.

By log-linearizing equation (A.24) for any i ,

$$\begin{aligned}
&-\alpha_M \widehat{p^G_{i,t}} + \alpha_M \sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{l,t}} + (1-\alpha_M)\sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij,t}} - \alpha_L \sum_o \widetilde{x}^O_{i,o,t_0} l^O_{i,o,t_0}\widehat{L_{i,o,t}} \\
&= \frac{\alpha_L}{\theta-1}\sum_o \frac{\widetilde{x}^O_{i,o,t_0}}{1-a_{o,t_0}}\left(-a_{o,t_0}l^O_{i,o,t_0} + \left(1-a_{o,t_0}\right)\left(1-l^O_{i,o,t_0}\right)\right)\widehat{a_{o,t}} + \alpha_L\sum_o \widetilde{x}^O_{i,o,t_0}\frac{1}{\beta-1}\widehat{b_{i,o,t}} \\
&\quad + \widehat{A^G_{i,t}} + (1-\alpha_L-\alpha_M)\widehat{K_{i,t}} - \alpha_M\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li,t}} - (1-\alpha_M)\sum_j\widetilde{y}^G_{ij,t_0}\widehat{\tau^G_{ij,t}} + \alpha_L\sum_o\widetilde{x}^O_{i,o,t_0}\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,}}
\end{aligned}$$

To write a matrix notation, write

$$\overline{\boldsymbol{M}^{yG}} \equiv \begin{bmatrix} \left[\widetilde{y}^G_{11,t_0},\ldots,\widetilde{y}^G_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^G_{N1,t_0},\ldots,\widetilde{y}^G_{NN,t_0}\right] \end{bmatrix}$$

a $N \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)^\top \end{bmatrix}$$

a $N \times NO$ matrix where

$$\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \equiv \left(\widetilde{x}^O_{1,o,t_0}\right)_o \text{ and } \widetilde{\boldsymbol{l}}_{1,\cdot,t_0} \equiv \left(l^O_{1,o,t_0}\right)_o \tag{A.37}$$

are $O \times 1$ vectors, $\overline{\boldsymbol{M}^{al}}$ as a matrix with its element

$$M^{al}_{i,o} = \frac{-a_{o,t_0} l^O_{i,o,t_0} + \left(1 - a_{o,t_0}\right)\left(1 - l^O_{i,o,t_0}\right)}{1 - a_{o,t_0}},$$

and a $N \times O$ matrix,

$$\overline{\boldsymbol{M}^{xO}} \equiv \begin{bmatrix} \left[\widetilde{x}^O_{1,1,t_0}, \ldots, \widetilde{x}^O_{1,O,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^O_{N,1,t_0}, \ldots, \widetilde{x}^O_{N,O,t_0}\right] \end{bmatrix},$$

a $N \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xG}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^G_{1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\widetilde{x}^G_{N\cdot,t_0}\right) \end{bmatrix},$$

a $N \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xOl,2}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)\right)^\top \end{bmatrix},$$

a $N \times NO$ matrix where $\widetilde{\boldsymbol{x}}_{1,\cdot,t_0}$ and $\widetilde{\boldsymbol{l}}_{1,\cdot,t_0}$ are defined in equation (A.37). Then I have

$$\begin{aligned}
&-\alpha_M \left(\overline{\boldsymbol{I}} - \left(\overline{\widetilde{\boldsymbol{x}}_{t_0}^G} \right)^\top \right) \widehat{\boldsymbol{p}_t^G} + (1-\alpha_M)\, \overline{\boldsymbol{M}^{yG}} \widehat{\boldsymbol{Q}_t^G} - \alpha_L \overline{\boldsymbol{M}^{xOl}} \widehat{\boldsymbol{L}_t} \\
&= \frac{\alpha_L}{\theta - 1} \left(\widetilde{\boldsymbol{x}}_{t_0}^O \circ \overline{\boldsymbol{M}^{al}} \right) \widehat{\boldsymbol{a}}_t + \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}} \widehat{\boldsymbol{b}}_t + \widehat{\boldsymbol{A}_t^G} + (1 - \alpha_L - \alpha_M)\, \widehat{\boldsymbol{K}_t} \\
&\quad - \left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1-\alpha_M)\, \overline{\boldsymbol{M}^{yG}} \right] \widehat{\boldsymbol{\tau}_t^G} + \alpha_L \overline{\boldsymbol{M}^{xOl,2}} \widehat{\boldsymbol{K}_t^R},
\end{aligned}$$

By log-linearizing equation (A.25) for any i and o,

$$\widehat{p_{i,o,t}^R} = \widehat{P_{i,t}^G} - \widehat{A_{i,o,t}^R}$$

$$-\sum_l \widetilde{x}_{li,t_0}^G \widehat{p_{l,t}^G} + \widehat{p_{i,o,t}^R} = -\widehat{A_{i,o,t}^R} + \sum_l \widetilde{x}_{li,t_0}^G \widehat{\tau_{li,t}^G}.$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xG,2}} \equiv \begin{bmatrix} \boldsymbol{1}_O \left[\widetilde{x}_{11,t_0}^G, \ldots, \widetilde{x}_{N1,t_0}^G \right] \\ \vdots \\ \boldsymbol{1}_O \left[\widetilde{x}_{1N,t_0}^G, \ldots, \widetilde{x}_{NN,t_0}^G \right] \end{bmatrix}$$

a $NO \times N$ matrix, and

$$\overline{\boldsymbol{M}^{xG,3}} \equiv \begin{bmatrix} \widetilde{x}_{11,t_0}^G & \ldots & \widetilde{x}_{N1,t_0}^G & & & \boldsymbol{0} \\ & & & \ddots & & \\ & \boldsymbol{0} & & \widetilde{x}_{1N,t_0}^G & \ldots & \widetilde{x}_{NN,t_0}^G \end{bmatrix} \otimes \boldsymbol{1}_O$$

a $NO \times N^2$ matrix. Then I have

$$-\overline{\boldsymbol{M}^{xG,2}} \widehat{\boldsymbol{p}_t^G} + \widehat{\boldsymbol{p}_t^R} = -\widehat{\boldsymbol{A}_t^R} + \overline{\boldsymbol{M}^{xG,3}} \widehat{\boldsymbol{\tau}_t^G}.$$

By log-linearizing equations (1.4), (1.5), and (1.6) for any i, o, and o', I have

$$\widehat{\mu_{i,oo',t}} = \phi \left(-d\chi_{i,oo',t} + \frac{1}{1+\iota} \widehat{V_{i,o',t+1}} \right) - \sum_{o''} \mu_{i,oo'',t_0} \left(-d\chi_{i,oo'',t} + \frac{1}{1+\iota} \widehat{V_{i,o'',t+1}} \right), \quad \text{(A.38)}$$

$$\widehat{V_{i,o,t+1}} = \widehat{w_{i,o,t+1}} + dT_{i,o,t+1} - \widehat{P_{i,t+1}} + \sum_{o'} \mu_{i,oo',t_0} \left(-d\chi_{i,oo',t+1} + \frac{1}{1+\iota} \widehat{V_{i,o',t+2}} \right), \quad \text{(A.39)}$$

and

$$\widehat{L_{i,o,t+1}} = \sum_{o'} \frac{L_{i,o',t_0}}{L_{i,o,t_0}} \mu_{i,o'o,t_0} \left(\widehat{\mu_{i,o'o,t}} + \widehat{L_{i,o',t}} \right). \quad \text{(A.40)}$$

In matrix notation, by equation (A.38),

$$\widehat{\boldsymbol{\mu}_t^{\text{vec}}} = -\phi \left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}} \right) d\boldsymbol{\chi}_t^{\text{vec}} + \frac{\phi}{1+\iota} \left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}} \right) \left(\overline{\boldsymbol{I}_{NO}} \otimes \mathbf{1}_O \right) \widehat{\boldsymbol{V}_{t+1}}.$$

where

$$\overline{\boldsymbol{M}^{\mu}} \equiv \overline{\boldsymbol{M}^{\mu,3}} \otimes \mathbf{1}_O,$$

$$\overline{\boldsymbol{M}^{\mu,3}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{i,1\cdot,t_0}\right)^\top & & & & & & \\ & \ddots & & & & \mathbf{0} & \\ & & \left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right)^\top & & & & \\ & & & \ddots & & & \\ & & & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top & & \\ & \mathbf{0} & & & & \ddots & \\ & & & & & & \left(\boldsymbol{\mu}_{i,O\cdot 1,t_0}\right)^\top \end{bmatrix},$$

$$d\boldsymbol{\chi}_t^{\text{vec}} \equiv \begin{bmatrix} d\boldsymbol{\chi}_{1,1\cdot,t} & \dots & d\boldsymbol{\chi}_{1,O\cdot,t} & \dots & d\boldsymbol{\chi}_{N,1\cdot,t} & \dots & d\boldsymbol{\chi}_{N,O\cdot,t} \end{bmatrix}^\top,$$

and

$$\boldsymbol{\mu}_{i,o\cdot,t_0} \equiv \left(\mu_{i,oo',t_0}\right)_{o'} \text{ and } d\boldsymbol{\chi}_{1,o\cdot,t} \equiv \left(d\chi_{1,oo',t}\right)_{o'} \quad \text{(A.41)}$$

are $O \times 1$ vectors. By equation (A.39),

$$\frac{1}{1+\iota} \overline{\boldsymbol{M}^{\mu,2}} \boldsymbol{V}_{t+2}^{\vee} = \overline{\boldsymbol{M}^{xG,2}} \boldsymbol{p}_{t+1}^{\check{G}} - \boldsymbol{w}_{t+1}^{\vee} + \boldsymbol{V}_{t+1}^{\vee}.$$

where

$$\overline{\boldsymbol{M}^{\mu,2}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right)^\top & & \\ \vdots & & \mathbf{0} \\ \left(\boldsymbol{\mu}_{1,O\cdot,t_0}\right)^\top & & \\ & \ddots & \\ & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top \\ \mathbf{0} & & \vdots \\ & & \left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right)^\top \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (A.41) for any i and o. By equation (A.39),

$$\check{\boldsymbol{L}_{t+1}} = \overline{\boldsymbol{M}^{\mu L,2}} \boldsymbol{\mu}_t^{\check{\text{vec}}} + \overline{\boldsymbol{M}^{\mu L}} \check{\boldsymbol{L}}_t$$

where $\overline{\boldsymbol{M}^{\mu L}}$ being the $NO \times NO$ matrix

$$\overline{\boldsymbol{M}^{\mu L}} = \overline{\boldsymbol{M}^{\mu,2}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \mathbf{1}_O \right) \oslash \left(\begin{bmatrix} \boldsymbol{L}_{1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \boldsymbol{L}_{N,\cdot,t_0} \end{bmatrix} \otimes \left(\mathbf{1}_O\right)^\top \right)$$

and $\overline{\boldsymbol{M}^{\mu L,2}}$ being the $NO \times NO^2$ matrix

$$\overline{\boldsymbol{M}^{\mu L,2}} = \overline{\boldsymbol{M}^{\mu,4}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \overline{\boldsymbol{I}_O} \right) \oslash$$
$$\begin{pmatrix} \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{1,o,t_0}\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{N,o,t_0}\right) \end{pmatrix},$$

where

$$\overline{\boldsymbol{M}^{\mu,4}} \equiv \begin{bmatrix} \operatorname{diag}\left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right) \ \dots \ \operatorname{diag}\left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \operatorname{diag}\left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right) \ \dots \ \operatorname{diag}\left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right) \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (A.41) for any i and o.

By log-linearizing equation (A.23) for each i and j,

$$\widehat{Q^G_{ij,t}} = -\varepsilon^G \widehat{p^G_{ij,t}} - \left(1-\varepsilon^G\right)\widehat{P^G_{j,t}} + \left[s^G_{j,t_0} \sum_k \widehat{p^G_{jk,t} Q^G_{jk,t}} + s^V_{j,t_0} \sum_{i,o} \widehat{p^R_{ij,o,t} Q^R_{ij,o,t}} + s^R_{j,t_0} \sum_{o,k} \widehat{p^R_{jk,o,t} Q^R_{jk,o,t}} \right]$$

where

$$s^G_{j,t_0} \equiv \frac{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}}$$

is the baseline share of non-robot good production in income,

$$s^R_{j,t_0} \equiv \frac{\sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}},$$

is the baseline share of robot production, and

$$s^V_{j,t_0} \equiv -\frac{\sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}},$$

is the (negative) baseline absorption share of robots. Thus

$$\begin{aligned}
&\left[\varepsilon^G \widehat{p^G_{i,t}} + \left(1-\varepsilon^G\right) \sum_l \widetilde{x}^G_{lj,t_0} \widehat{p^G_{l,t}} - s^G_{j,t_0} \widehat{p^G_{j,t}}\right] - \left[s^V_{j,t_0} \sum_{l,o} \widetilde{x}^R_{lj,o,t_0} \widetilde{x}^R_{j,o,t_0} \widehat{p^R_{l,o,t}} + s^R_{t_0} \sum_o \widetilde{y}^R_{j,o,t_0} \widehat{p^R_{j,o,t}} \right] \\
&+ \left(\widehat{Q^G_{ij,t}} - s^G_{j,t_0} \sum_k \widetilde{y}^G_{jk,t_0} \widehat{Q^G_{jk,t}} \right) - \left(s^V_{j,t_0} \sum_{l,o} \widetilde{x}^R_{lj,o,t_0} \widetilde{x}^R_{j,o,t_0} \widehat{Q^R_{lj,o,t}} + s^R_{j,t_0} \sum_{k,o} \widetilde{y}^R_{jk,o,t_0} \widetilde{y}^R_{j,o,t_0} \widehat{Q^R_{jk,o,t}} \right) \\
&= -\left[\varepsilon^G \widehat{\tau^G_{ij,t}} + \left(1-\varepsilon^G\right) \sum_l \widetilde{x}^G_{lj,t_0} \widehat{\tau^G_{lj,t}} - s^G_{j,t_0} \sum_k \widetilde{y}^G_{jk,t_0} \widehat{\tau^G_{jk,t}} \right] + \left[s^V_{j,t_0} \sum_{l,o} \widetilde{x}^R_{lj,t_0} \widehat{\tau^R_{lj,t}} + s^R_{j,t_0} \sum_{k,o} \widetilde{y}^R_{jk,t_0} \widehat{\tau^R_{jk,t}} \right]
\end{aligned}$$

where

$$\widetilde{x}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{P^R_{j,o,t_0} Q^R_{j,o,t_0}}, \widetilde{x}^R_{j,o,t_0} \equiv \frac{P^R_{j,o,t_0} Q^R_{j,o,t_0}}{P^R_{j,t_0} Q^R_{j,t_0}}, \widetilde{x}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{P^R_{j,t_0} Q^R_{j,t_0}},$$

$$\widetilde{y}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}, \widetilde{y}^R_{i,o,t_0} \equiv \frac{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}{\sum_{k,o'} p^R_{ik,o',t_0} Q^R_{ik,o',t_0}}, \widetilde{y}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_{k,o} p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}.$$

In matrix notation, define

$$\overline{\boldsymbol{M}^{xR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot 1,\cdot,t_0} & \dots & \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot N,\cdot,t_0} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{y}^R_{1,1} & \dots & \widetilde{y}^R_{1,O} & & & \mathbf{0} & \\ & & & \ddots & & & \\ & \mathbf{0} & & & \widetilde{y}^R_{N,1} & \dots & \widetilde{y}^R_{N,O} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yG,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{yG}}.$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{11,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{N1,o,t_0}\right]_o & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{1N,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2 O$ matrix ,

$$\overline{\boldsymbol{M}^{yR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{11,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{1N,o,t_0}\right]_o & & & \mathbf{0} & \\ & & & \ddots & & & \\ & \mathbf{0} & & & \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{N1,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2 O$ matrix,

$$\overline{\boldsymbol{M}^{xG,4}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xG}}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,t_0}\right) \end{bmatrix}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{yR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{11,t_0}, \ldots, \widetilde{y}^R_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^R_{N1,t_0}, \ldots, \widetilde{y}^R_{NN,t_0}\right] \end{bmatrix}$$

a $N^2 \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xO,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xO}},$$

a $N^2 \times NO$ matrix. Then I have

$$\begin{aligned}
&\left(\varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N\right] + \left(1 - \varepsilon^G\right) \left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top\right] - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right) \left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N}\right]\right) \widehat{\boldsymbol{p}^G_t} \\
&- \left(\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right) \overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right) \overline{\boldsymbol{M}^{yR}}\right) \widehat{\boldsymbol{p}^R_t} \\
&+ \left(\overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right) \overline{\boldsymbol{M}^{yG,2}}\right) \widehat{\boldsymbol{Q}^G_t} - \left[\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right) \overline{\boldsymbol{M}^{xR,2}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right) \overline{\boldsymbol{M}^{yR,2}}\right] \widehat{\boldsymbol{Q}^R_t} \\
&= -\left(\varepsilon^G + \left(1 - \varepsilon^G\right) \overline{\boldsymbol{M}^{xG,4}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right) \overline{\boldsymbol{M}^{yG,2}}\right) \widehat{\boldsymbol{\tau}^G_t} \\
&\qquad + \left(\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right) \overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right) \overline{\boldsymbol{M}^{yR,3}}\right) \widehat{\boldsymbol{\tau}^R_t}
\end{aligned}$$

By log-linearizing equation (A.15) for each i, j, and o,,

$$\begin{aligned}
&\left(1 - \alpha^R\right) \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} \sum_l \widetilde{x}^G_{lj,t_0} \widehat{p^G_{l,t}} + \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} \widehat{p^R_{i,o,t}} \\
&+ \left[\frac{2\gamma\delta}{1 + u_{ij,t_0} + 2\gamma\delta} - \left(1 - \alpha^R\right) \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta}\right] \sum_l \widetilde{x}^R_{lj,o,t_0} \widehat{p^R_{l,o,t}} \\
&+ \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} \frac{1}{\varepsilon^R} \widehat{Q^R_{ij,o,t}} + \left[-\frac{1}{\varepsilon^R} \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} + \frac{2\gamma\delta}{1 + u_{ij,t_0} + 2\gamma\delta}\right] \sum_l \widetilde{x}^R_{lj,o,t_0} \widehat{Q^R_{lj,o,t}} \\
&= -\frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} du_{ij,t} - \left(1 - \alpha^R\right) \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} \sum_l \widetilde{x}^G_{lj,t_0} \widehat{\tau^G_{lj,t}} - \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta} \widehat{\tau^R_{ij,t}} \\
&- \left[\frac{2\gamma\delta}{1 + u_{ij,t_0} + 2\gamma\delta} - \left(1 - \alpha^R\right) \frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta}\right] \sum_l \widetilde{x}^R_{lj,o,t_0} \widehat{\tau^R_{lj,t}} + \widehat{\lambda^R_{j,o,t}} + \frac{2\gamma\delta}{1 + u_{ij,t_0} + 2\gamma\delta} \widehat{K^R_{j,o,t}}.
\end{aligned}$$

In matrix notation, write a preliminary $N \times N$ matrix $\overline{\widetilde{\boldsymbol{u}_{t_0}}}$ as such that the (i, j)-element is

$$\frac{1 + u_{ij,t_0}}{1 + u_{ij,t_0} + 2\gamma\delta}.$$

Then $\mathbf{1}_N\,(\mathbf{1}_N)^\top - \overline{\widetilde{\boldsymbol{u}_{t_0}}}$ is a matrix that is filled with $2\gamma\delta/\left(1 + u_{ij,t_0} + 2\gamma\delta\right)$ for its (i, j) element and

$$\overline{\boldsymbol{M}^u} \equiv \operatorname{diag}\left(\left[\widetilde{u_{1\cdot,t_0}}, \ldots, \widetilde{u_{N\cdot,t_0}}\right]^\top\right).$$

Using these, write

$$\overline{\boldsymbol{M}^{xG,5}} \equiv \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O}\right)\left(\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top \otimes \mathbf{1}_O\right)$$

a $N^2O \times N$ matrix,

$$\overline{\boldsymbol{M}^{u,2}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{u}_{1\cdot,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{u}_{N\cdot,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O},$$

a $N^2O \times NO$ matrix where $\widetilde{\boldsymbol{u}_{i\cdot,t_0}} \equiv \left(\widetilde{u_{i\cdot,t_0}}\right)_j$ is a $N \times 1$ vector,

$$\begin{aligned}\overline{\boldsymbol{M}^{xR,4}} \equiv & \left\{\left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u}\right) - \left(1 - \alpha^R\right)\overline{\boldsymbol{M}^u}\right] \otimes \overline{\boldsymbol{I}_O}\right\} \times \\ & \left(\mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\left\{\widetilde{x}^R_{11,o,t_0}\right\}_o\right) & \ldots & \operatorname{diag}\left(\left\{\widetilde{x}^R_{N1,o,t_0}\right\}_o\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\left\{\widetilde{x}^R_{1N,o,t_0}\right\}_o\right) & \ldots & \operatorname{diag}\left(\left\{\widetilde{x}^R_{NN,o,t_0}\right\}_o\right) \end{bmatrix}\right)\end{aligned}$$

a $N^2O \times NO$ matrix,

$$\begin{aligned}\overline{\boldsymbol{M}^{xR,5}} \equiv & \left\{\left[-\frac{1}{\varepsilon^R}\overline{\boldsymbol{M}^u} + \left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u}\right)\right] \otimes \overline{\boldsymbol{I}_O}\right\} \times \\ & \left\{\mathbf{1}_N \otimes \left[\operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{11,1,t_0} \\ \vdots \\ \widetilde{x}^R_{11,O,t_0} \\ \vdots \\ \widetilde{x}^R_{1N,O,t_0} \end{bmatrix}\right) \ldots \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{N1,1,t_0} \\ \vdots \\ \widetilde{x}^R_{N1,O,t_0} \\ \vdots \\ \widetilde{x}^R_{NN,O,t_0} \end{bmatrix}\right)\right]\right\}\end{aligned}$$

a $N^2O \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,6}} \equiv \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O}\right) \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{11,t_0} \\ \vdots \\ \widetilde{x}^G_{1N,t_0} \end{bmatrix} \right) & \dots & \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{N1,t_0} \\ \vdots \\ \widetilde{x}^G_{NN,t_0} \end{bmatrix} \right) \end{bmatrix} \otimes \mathbf{1}_O \right\}$$

a $N^2O \times N^2$ matrix,

$$\begin{aligned} \overline{\boldsymbol{M}^{xR,6}} \equiv & \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^{u}} \right) - \left(1 - \alpha^R\right) \overline{\boldsymbol{M}^{u}} \right] \otimes \overline{\boldsymbol{I}_O} \right\} \\ & \times \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{11,o,t_0}\right]_o & \mathbf{0} & \mathbf{0} & \dots & \left[\widetilde{x}^R_{N1,o,t_0}\right]_o & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^R_{1N,o,t_0}\right]_o & & \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^R_{N3,o,t_0}\right]_o \end{bmatrix} \right\} \end{aligned}$$

a $N^2O \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{u,3}} \equiv \begin{bmatrix} 1 - \widetilde{u_{11,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1 - \widetilde{u_{1N,t_0}} \\ & \vdots & \\ 1 - \widetilde{u_{N1,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1 - \widetilde{u_{NN,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O}$$

a $N^2O \times NO$ matrix. Finally, I have

$$\begin{aligned} & \left(1 - \alpha^R\right) \overline{\boldsymbol{M}^{xG,5}} \widehat{\boldsymbol{p}^G_t} + \left[\overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}} \right] \widehat{\boldsymbol{p}^R_t} + \left\{ \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O} \right) + \overline{\boldsymbol{M}^{xR,5}} \right\} \widehat{\boldsymbol{Q}^R_t} \\ & = - \left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O \right) d\boldsymbol{u}_t - \left(1 - \alpha^R\right) \overline{\boldsymbol{M}^{xG,6}} \widehat{\boldsymbol{\tau}^G_t} - \left[\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O \right) + \overline{\boldsymbol{M}^{xR,6}} \right] \widehat{\boldsymbol{\tau}^R_t} + \left(\mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}} \right) \widehat{\boldsymbol{\lambda}^R_t} + \overline{\boldsymbol{M}^{u,3}} \widehat{\boldsymbol{K}^R_t}. \end{aligned}$$

By log-linearizing equation (A.10) for each i and o,

$$\widehat{p^G_{i,t}} + \sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij,t}} - \widehat{w_{i,o,t}} + \left[-\frac{1}{\theta} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) l^O_{i,o,t_0}\right]\widehat{L_{i,o,t}} + \left(-1 + \frac{1}{\beta}\right)\sum_{o'} \widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0}\widehat{L_{i,o',t}}$$

$$= -\frac{1}{\beta}\widehat{b_{i,o,t}} + \frac{1}{\theta}\frac{a_{o,t_0}}{1-a_{o,t_0}}\widehat{a_{o,t}} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right)\frac{1}{\theta-1}\left[-\left(1 - l^O_{i,o,t_0}\right) + l^O_{i,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right]\widehat{a_{o,t}}$$

$$+ \left(-1 + \frac{1}{\beta}\right)\frac{1}{\theta-1}\sum_{o'} \widetilde{x}^O_{i,o',t_0}\left[-\left(1 - l^O_{i,o',t_0}\right) + l^O_{i,o',t_0}\frac{a_{o',t_0}}{1-a_{o',t_0}}\right]\widehat{a_{o',t}}$$

$$- \sum_j y^G_{ij,t_0}\widehat{\tau^G_{ij,t}} - \left(-\frac{1}{\beta} + \frac{1}{\theta}\right)\left(1 - l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}} - \left(-1 + \frac{1}{\beta}\right)\sum_{o'} \widetilde{x}^O_{i,o',t_0}\left(1 - l^O_{i,o',t_0}\right)\widehat{K^R_{i,o',t}},$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{yG,3}} \equiv \overline{\boldsymbol{M}^{yG}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl,3}} \equiv \overline{\boldsymbol{M}^{xOl}} \otimes \mathbf{1}_O$$

a $NO \times NO$ matrix,

$$\overline{\boldsymbol{M}^{a}} \equiv \mathbf{1}_N \otimes \operatorname{diag}\left(\frac{a_{o,t_0}}{1-a_{o,t_0}}\right)$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,2}} \equiv \begin{bmatrix} \operatorname{diag}\left(-\left(1 - l^O_{1,o,t_0}\right) + l^O_{1,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \\ \vdots \\ \operatorname{diag}\left(-\left(1 - l^O_{N,o,t_0}\right) + l^O_{N,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \end{bmatrix}$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,3}} \equiv \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \otimes \mathbf{1}_O$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{xOl,4}} \equiv \overline{\boldsymbol{M}^{xOl,2}} \otimes \mathbf{1}_O,$$

a $NO \times NO$ matrix. I have

$$
\begin{aligned}
&\left(\boldsymbol{I}_N \otimes \mathbf{1}_O\right) \widehat{\boldsymbol{p}_t^G} - \widehat{\boldsymbol{w}_t} + \overline{\boldsymbol{M}^{yG,3}} \widehat{\boldsymbol{Q}_t^G} + \left(-\frac{1}{\theta} \overline{\boldsymbol{I}_{NO}} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \operatorname{diag}\left(\boldsymbol{l}_{t_0}^O\right) + \left(-1 + \frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,3}}\right) \widehat{\boldsymbol{L}_t} \\
&= -\frac{1}{\beta} \widehat{\boldsymbol{b}}_t + \left[\frac{1}{\theta} \overline{\boldsymbol{M}^a} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \frac{1}{\theta - 1} \overline{\boldsymbol{M}^{al,2}} + \left(-1 + \frac{1}{\beta}\right) \frac{1}{\theta - 1} \overline{\boldsymbol{M}^{al,3}}\right] \widehat{\boldsymbol{a}}_t - \overline{\boldsymbol{M}^{yG,3}} \widehat{\boldsymbol{\tau}_t^G} \\
&\quad + \left[-\left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \operatorname{diag}\left(1 - l_{i,o,t_0}^O\right) - \left(-1 + \frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,4}}\right] \widehat{\boldsymbol{K}_t^R}.
\end{aligned}
$$

Hence the log-linearized temporary equilibrium system is

$$\overline{\boldsymbol{D}^x} \widehat{\boldsymbol{x}} = \overline{\boldsymbol{D}^A} \widehat{\boldsymbol{A}}$$

where matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ are defined as

$$
\overline{\boldsymbol{D}^x} \equiv \begin{bmatrix}
\overline{\boldsymbol{D}_{11}^x} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{14}^x} & \mathbf{0} & \overline{\boldsymbol{D}_{16}^x} \\
-\overline{\boldsymbol{M}^{xG,2}} & \overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\phi \overline{\boldsymbol{M}^{xG,2}} & \mathbf{0} & -\phi \overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{M}^l} \\
\overline{\boldsymbol{D}_{41}^x} & \overline{\boldsymbol{D}_{42}^x} & \mathbf{0} & \overline{\boldsymbol{D}_{44}^x} & \overline{\boldsymbol{D}_{45}^x} & \mathbf{0} \\
\overline{\boldsymbol{D}_{51}^x} & \overline{\boldsymbol{D}_{52}^x} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{55}^x} & \mathbf{0} \\
\overline{\boldsymbol{D}_{61}^x} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}_{66}^x}
\end{bmatrix},
$$

where

$$\overline{\boldsymbol{D}_{11}^x} \equiv -\alpha_M \left(\overline{\boldsymbol{I}_N} - \left(\widetilde{\boldsymbol{x}}_{t_0}^G\right)^\top\right),\ \overline{\boldsymbol{D}_{14}^x} \equiv \left(1 - \alpha_M\right) \overline{\boldsymbol{M}^{yG}},\ \overline{\boldsymbol{D}_{16}^x} \equiv -\alpha_L \overline{\boldsymbol{M}^{xOl}},$$

$$\overline{\boldsymbol{D}_{41}^x} \equiv \varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N\right] + \left(1 - \varepsilon^G\right) \left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}_{t_0}^G\right)^\top\right] - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^G\right) \left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N}\right],$$

$$\overline{\boldsymbol{D}_{42}^x} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^V\right) \overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^R\right) \overline{\boldsymbol{M}^{yR}},$$

$$\overline{\boldsymbol{D}_{44}^x} \equiv \overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^G\right) \overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{45}^x} \equiv -\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^V\right) \overline{\boldsymbol{M}^{xR,2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^R\right) \overline{\boldsymbol{M}^{yR,2}},$$

$$\overline{\boldsymbol{D}_{51}^x} \equiv \left(1 - \alpha^R\right) \overline{\boldsymbol{M}^{xG,5}},\ \overline{\boldsymbol{D}_{52}^x} \equiv \overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}},\ \overline{\boldsymbol{D}_{55}^x} \equiv \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O}\right) + \overline{\boldsymbol{M}^{xR,5}},$$

$$\overline{\boldsymbol{D}_{61}^{x}} \equiv \boldsymbol{I}_N \otimes \mathbf{1}_N,\ \overline{\boldsymbol{D}_{66}^{x}} \equiv -\frac{1}{\theta} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \operatorname{diag}\left(\boldsymbol{l}_{t_0}^{O}\right) + \left(-1 + \frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,3}},$$

and

$$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{D}_{12}^{A}} & \overline{\boldsymbol{D}_{13}^{A}} & \overline{\boldsymbol{I}_N} & \mathbf{0} & \overline{\boldsymbol{D}_{16}^{A}} & \overline{\boldsymbol{D}_{17}^{A}} & \mathbf{0} & \alpha_L \overline{\boldsymbol{M}^{xOl,2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \overline{\boldsymbol{M}^{xG}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\phi \overline{\boldsymbol{M}^{xG,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{47}^{A}} & \overline{\boldsymbol{D}_{48}^{A}} & \mathbf{0} & \mathbf{0} \\ \overline{\boldsymbol{D}_{51}^{A}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{57}^{A}} & \overline{\boldsymbol{D}_{58}^{A}} & \overline{\boldsymbol{M}^{u,3}} & \overline{\boldsymbol{D}_{5,10}^{A}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{62}^{A}} & -\frac{1}{\beta}\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}_{69}^{A}} & \mathbf{0} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{12}^{A}} \equiv \frac{\alpha_L}{\theta - 1} \left(\widetilde{\boldsymbol{x}}_{t_0}^{O} \otimes \overline{\boldsymbol{M}^{al}}\right),\ \overline{\boldsymbol{D}_{13}^{A}} \equiv \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}},$$

$$\overline{\boldsymbol{D}_{16}^{A}} \equiv (1 - \alpha_L - \alpha_M) \overline{\boldsymbol{I}_N},\ \overline{\boldsymbol{D}_{17}^{A}} \equiv -\left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1 - \alpha_M) \overline{\boldsymbol{M}^{yG}}\right],$$

$$\overline{\boldsymbol{D}_{47}^{A}} \equiv -\boldsymbol{\varepsilon}^{G} + \left(1 - \boldsymbol{\varepsilon}^{G}\right) \overline{\boldsymbol{M}^{xG,4}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{G}\right) \overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{48}^{A}} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{V}\right) \overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{R}\right) \overline{\boldsymbol{M}^{yR,3}},$$

$$\overline{\boldsymbol{D}_{51}^{A}} \equiv -\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right),\ \overline{\boldsymbol{D}_{57}^{A}} \equiv -\left(1 - \alpha^{R}\right) \overline{\boldsymbol{M}^{xG,6}},$$

$$\overline{\boldsymbol{D}_{58}^{A}} \equiv -\left[\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right) + \overline{\boldsymbol{M}^{xR,6}}\right],\ \overline{\boldsymbol{D}_{5,10}^{A}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}},$$

$$\overline{\boldsymbol{D}_{62}^{A}} \equiv \frac{1}{\theta} \overline{\boldsymbol{M}^{a}} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \frac{1}{\theta - 1} \overline{\boldsymbol{M}^{al,2}} + \left(-1 + \frac{1}{\beta}\right) \frac{1}{\theta - 1} \overline{\boldsymbol{M}^{al,3}},$$

and

$$\overline{\boldsymbol{D}_{69}^{A}} \equiv -\left(-\frac{1}{\beta} + \frac{1}{\theta}\right) \operatorname{diag}\left(1 - l_{i,o,t_0}^{O}\right) - \left(-1 + \frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,4}}.$$

To normalize the price, one of the good-demand equation must be replaced with log-linearized numeraire condition $\widehat{P_{1,t}^{G}} = \sum_i x_{i1,t_0}^{G} \left(\widehat{p_{i,t}^{G}} + \widehat{\tau_{i1,t}^{G}}\right) = 0$, or

$$\overline{\boldsymbol{M}^{xG,num}} \widehat{\boldsymbol{p}_t^{G}} = -\overline{\boldsymbol{M}^{xG,num}} \widehat{\boldsymbol{\tau}_t^{G}},$$

where $\overline{\boldsymbol{M}^{xG,num}} \equiv \left[x_{11,t_0}^{G}, x_{21,t_0}^{G}, x_{31,t_0}^{G}\right]$.

To analyze the steady state conditions, first note that the steady state accumulation condition (A.26) implies $\widehat{Q^R_{i,o}} = \widehat{K^R_{i,o}}$. Using robot integration function, integration demand and unit cost formula, I have

$$\widehat{Q^R_{i,o}} = \sum_l x^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left(1-\alpha^R\right)\left(\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{p^R_{li,o}} - \sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{li,t}}\right) \tag{A.42}$$

Thus the condition is

$$\begin{aligned}
&\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left(1-\alpha^R\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{p^R_{l,o}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_l} - \widehat{K^R_{i,o}} \\
&= \left(1-\alpha^R\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{\tau^R_{li}}.
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{xR,7}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xR,8}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{11,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N1,\cdot,t_0}\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\widetilde{x}^R_{1N,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{NN,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times NO$ matrix, and

$$\overline{\boldsymbol{M}^{xG,7}} \equiv \begin{bmatrix} \widetilde{x}^G_{11,t_0} & & & \dots & \widetilde{x}^G_{N1,t_0} & & \mathbf{0} \\ & \ddots & & & & \ddots & \\ \mathbf{0} & & \widetilde{x}^G_{1N,t_0} & & & & \widetilde{x}^G_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix.

$$\overline{\boldsymbol{M}^{xR,9}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{11,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{N1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{1N,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{NN,\cdot,t_0} \end{bmatrix},$$

a $NO \times N^2$ matrix, where $\widetilde{\boldsymbol{x}}^R_{ij,\cdot,t_0} \equiv \left(\widetilde{x}^R_{ij,o,t_0}\right)_o$ is an $O \times 1$ vector for any i and j. Then I have

$$-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G}+\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\widehat{\boldsymbol{p}^R}+\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R}-\widehat{\boldsymbol{K}^R} = \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G}-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}$$

Next, to study the steady state Euler equation (A.27) , note that by equation (A.13),

$$\begin{aligned}\frac{\partial \pi_{i,t}\widehat{\left(\left\{K^R_{i,o,t}\right\}\right)}}{\partial K^R_{i,o,t}} &= \sum_j \widetilde{y}^G_{ij,t}\left(\widehat{p^G_{ij,t}}+\widehat{Q^G_{ij,t}}\right)+\left[-\frac{1}{\beta}\sum_{o'}x^O_{i,o',t_0}\widehat{b_{i,o',t}}+\frac{1}{\beta}\widehat{b_{i,o,t}}\right]\\
&+\left\{\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\sum_{o'}\frac{\widetilde{x}^O_{i,o',t_0}}{1-a_{o,t_0}}\left[-l^O_{i,o',t_0}a_{o,t_0}+\left(1-l^O_{i,o',t_0}\right)\left(1-a_{o,t_0}\right)\right]\widehat{a_{o',t}}\right.\\
&+\left.\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\frac{-l^O_{i,o,t_0}a_{o,t_0}+\left(1-l^O_{i,o,t_0}\right)\left(1-a_{o,t_0}\right)}{1-a_{o,t_0}}+\frac{1}{\theta}\right\}\widehat{a_{o,t}}\right\}\\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}x^O_{i,o',t_0}l^O_{i,o',t_0}\widehat{L_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)l^O_{i,o,t_0}\widehat{L_{i,o,t}}\right]\\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0}\left(1-l^O_{i,o',t_0}\right)\widehat{K^R_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}}+\left(-\frac{1}{\theta}\right)\widehat{K^R_{i,o,t}}\right].\end{aligned} \tag{A.43}$$

Note that by the steady state accumulation condition (A.26), $Q^R_{i,o,t_0}/K^R_{i,o,t_0} = \delta$. Note also that investment function implies that, in the steady state,

$$\frac{\lambda^R_{j,o}}{P^R_{j,o}} = \left(\sum_i \frac{x^R_{ij,o}}{\left(1+u_{ij}\right)^{1-\varepsilon^R}}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta. \tag{A.44}$$

To simplify the notation, set

$$\widetilde{u}^{SS}_{j,o,t_0} \equiv \frac{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]}{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]-\gamma\delta^2},$$

Then by log-linearizing equation (A.27) implies, after rearranging,

$$
\begin{aligned}
&\left[\widehat{p_i^G} + 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{l,t}}\right] - \left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\widehat{p^R_{i,o}} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{p^R_{l,o}} \\
&+ \sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij}} - 2\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0}\widehat{L_{i,o'}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right) l^O_{i,o,t_0}\widehat{L_{i,o}}\right] \\
&+ \left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0}\left(1-l^O_{i,o',t_0}\right)\widehat{K^R_{i,o'}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o}} + \left(-\frac{1}{\theta}\right)\widehat{K^R_{i,o}} + 2\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\widehat{K^R_{i,o}}\right] \\
&- \widetilde{u}^{SS}_{i,o,t_0}\widehat{\lambda^R_{i,o}} \\
&= -\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\sum_{o'}\frac{\widetilde{x}^O_{i,o',t_0}}{1-a_{o,t_0}}\left[\left(1-l^O_{i,o',t_0}\right)\left(1-a_{o',t_0}\right) - l^O_{i,o',t_0}a_{o',t_0}\right]\widehat{a_{o'}} \\
&- \left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\frac{1}{1-a_{o,t_0}}\left[\left(1-l^O_{i,o,t_0}\right)\left(1-a_{o,t_0}\right) - l^O_{i,o,t_0}a_{o,t_0}\right] + \frac{1}{\theta}\right\}\widehat{a_o} \\
&- \left[-\frac{1}{\beta}\sum_{o'}\widetilde{x}^O_{i,o',t_0}\widehat{b_{i,o'}} + \frac{1}{\beta}\widehat{b_{i,o}}\right] + \left[-\sum_j \widetilde{y}^G_{ij,t_0}\widehat{\tau^G_{ij}} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li,t}}\right] \\
&+ 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{\tau^R_{li}}
\end{aligned}
$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xO,3}} \equiv \overline{\boldsymbol{M}^{xO}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix. Then

$$
\begin{aligned}
&\left[\left(\overline{\boldsymbol{I}_N}\otimes\mathbf{1}_O\right) + 2\left(1-\alpha^R\right)\mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,2}}\right]\widehat{\boldsymbol{p}}^G - \mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\left(\overline{\boldsymbol{I}_{NO}} - 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\widehat{\boldsymbol{p}}^R \\
&+ \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G} - 2\mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R} + \left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\mathrm{diag}\left(l^O_{\cdot,\cdot,t_0}\right)\right]\widehat{\boldsymbol{L}} \\
&+ \left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\mathrm{diag}\left(1-l^O_{\cdot,\cdot,t_0}\right) - \frac{1}{\theta}\overline{\boldsymbol{I}_{NO}} + 2\mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\right]\widehat{\boldsymbol{K}^R} - \mathrm{diag}\left(\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\widehat{\boldsymbol{\lambda}^R} \\
&= -\left[\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}} - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}}\right]\widehat{\boldsymbol{a}} - \frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{xO,3}}\right)\widehat{\boldsymbol{b}} \\
&+ \left[-\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,7}}\right]\widehat{\boldsymbol{\tau}^G} + 2\left(1-\alpha^R\right)\mathrm{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}
\end{aligned}
$$

In the steady state, I write equations (A.39) and (A.40) as

$$\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^{G}} - \widehat{\boldsymbol{w}} + \left[\overline{\boldsymbol{I}_{NO}} - \frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\right]\widehat{\boldsymbol{V}} = -\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^{G}} + d\boldsymbol{T} - \overline{\boldsymbol{M}^{\mu,3}}d\boldsymbol{\chi}^{\text{vec}}$$

and

$$\left[\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{\mu L}}\right]\widehat{\boldsymbol{L}} - \overline{\boldsymbol{M}^{\mu L,2}}\widehat{\boldsymbol{\mu}^{\text{vec}}} = \boldsymbol{0}.$$

respectively.

Hence the log-linearized steady state system is

$$\overline{\boldsymbol{E}^{y}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^{\Delta}}\boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \hline \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \hline \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,T}} & \overline{\boldsymbol{D}^{A,\Delta}} \end{bmatrix}$, and matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ are defined as

$$\overline{\boldsymbol{D}^{y,SS}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{y,SS}_{11}} & \overline{\boldsymbol{D}^{y,SS}_{12}} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{M}^{xR,7}} & \boldsymbol{0} & -\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}^{y,SS}_{21}} & \overline{\boldsymbol{D}^{y,SS}_{22}} & \boldsymbol{0} & \overline{\boldsymbol{M}^{yG,3}} & \overline{\boldsymbol{D}^{y,SS}_{25}} & \overline{\boldsymbol{D}^{y,SS}_{26}} & \overline{\boldsymbol{D}^{y,SS}_{27}} & \overline{\boldsymbol{D}^{y,SS}_{28}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^{y,SS}_{11}} \equiv -\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{12}} \equiv \left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{21}} \equiv \left(\overline{\boldsymbol{I}_{N}} \otimes \boldsymbol{1}_{O}\right) + 2\left(1-\alpha^{R}\right)\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{22}} \equiv -\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{25}} \equiv -2\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{26}} \equiv \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(l^{O}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{27}} \equiv \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(1-l^{O}_{\cdot,\cdot,t_0}\right) - \frac{1}{\theta}\overline{\boldsymbol{I}_{NO}} + 2\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}_{28}^{y,SS}} \equiv -\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right),$$

and

$$\overline{\boldsymbol{D}^{\Delta,SS}} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{17}^{\Delta,SS}} & \overline{\boldsymbol{D}_{18}^{\Delta,SS}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{22}^{\Delta,SS}} & \overline{\boldsymbol{D}_{23}^{\Delta,SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{27}^{\Delta,SS}} & \overline{\boldsymbol{D}_{28}^{\Delta,SS}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{17}^{\Delta,SS}} \equiv \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}},$$

$$\overline{\boldsymbol{D}_{18}^{\Delta,SS}} \equiv -\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}},$$

$$\overline{\boldsymbol{D}_{22}^{\Delta,SS}} \equiv \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} - \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}},$$

$$\overline{\boldsymbol{D}_{23}^{\Delta,SS}} \equiv -\frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{xO,3}}\right),$$

$$\overline{\boldsymbol{D}_{27}^{\Delta,SS}} \equiv -\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\mathrm{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,7}},$$

and

$$\overline{\boldsymbol{D}_{28}^{\Delta,SS}} \equiv 2\left(1-\alpha^R\right)\mathrm{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,9}}.$$

If $\overline{\boldsymbol{E}^y}$ is invertible, I have $\overline{\boldsymbol{E}} \equiv \left(\overline{\boldsymbol{E}^y}\right)^{-1}\overline{\boldsymbol{E}^\Delta}$ such that $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$. Write dimensions of $\boldsymbol{y}$ and $\boldsymbol{\Delta}$ as $n_y \equiv N + 3NO + N^2 + N^2O$ and $n_\Delta \equiv 3N^2 + O + 2NO + 2N$, respectively.

Finally, to study the transitional dynamics, the capital accumulation dynamics (1.13) implies

$$\check{K_{i,o,t+1}^{R}} = -\delta\left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,t_0}^G \check{p}_{l,t}^G + \delta\left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,o}^R \check{p}_{l,o,t}^R + \delta\sum_l \widetilde{x}_{li,o}^R \check{Q}_{li,o,t}^R + (1-\delta)\check{K}_{i,o,t}^R.$$

In a matrix form, write

$$\check{\boldsymbol{K}}_{t+1}^R = -\delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\check{\boldsymbol{p}}_t^G + \delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\check{\boldsymbol{p}}_t^R + \delta\overline{\boldsymbol{M}^{xR,7}}\check{\boldsymbol{Q}}_t^R + (1-\delta)\overline{\boldsymbol{I}_{NO}}\check{\boldsymbol{K}}_t^R.$$

Next, to study the Euler equation, define

$$\widetilde{u}_{i,o}^{TD,1} \equiv \frac{-\left(\iota+\delta\right)\left[\left(\sum_{l} x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]+\gamma\delta^{2}}{\left(1-\delta\right)\left[\left(\sum_{l} x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]}$$

and

$$\widetilde{u}_{i,o}^{TD,2} \equiv \frac{-\gamma\delta^{2}}{\left(1-\delta\right)\left[\left(\sum_{l} x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]}.$$

Then I have

$$\begin{aligned}
&\left[-\widetilde{u}_{i,o}^{TD,1} p_{i,t+1}^{\check{G}}+2\left(1-\alpha^{R}\right)\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li}^{G} p_{l,t+1}^{\check{G}}\right]+\left[-\widetilde{u}_{i,o}^{TD,2} p_{i,o,t+1}^{\check{R}}-2\left(1-\alpha^{R}\right)\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li,o}^{R} p_{l,o,t+1}^{\check{R}}\right] \\
&-\widetilde{u}_{i,o}^{TD,1}\sum_{j}\widetilde{y}_{ij}^{G} Q_{ij,t+1}^{\check{G}}-2\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li,o}^{R} Q_{li,o,t+1}^{\check{R}}-\widetilde{u}_{i,o}^{TD,1}\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^{O}\left(1-l_{i,o'}^{O}\right) K_{i,o',t+1}^{\check{R}} \\
&-\widetilde{u}_{i,o}^{TD,1}\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^{O} l_{i,o'}^{O} \check{L}_{i,o',t+1}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right) l_{i,o}^{O} \check{L}_{i,o,t+1}\right] \\
&-\left[\widetilde{u}_{i,o}^{TD,1}\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l_{i,o}^{O}\right)+\left(-\frac{1}{\theta}\right)\right\}-2\widetilde{u}_{i,o}^{TD,2}\right] K_{i,o,t+1}^{\check{R}}+\lambda_{i,o,t+1}^{\check{R}}=\frac{1+\iota}{1-\delta}\lambda_{i,o,t}^{\check{R}}
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{u,4}}=\begin{bmatrix}\widetilde{\boldsymbol{u}}_{1,\cdot}^{TD,1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{u}}_{N,\cdot}^{TD,1}\end{bmatrix},$$

a $NO \times N$ matrix where $\widetilde{\boldsymbol{u}}_{i,\cdot}^{TD,1} \equiv \left(\widetilde{u}_{i,o}^{TD,1}\right)_o$ is an $O \times 1$ vector for any i. Then

$$\begin{aligned}
&\left(-\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^R\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}}\right)\boldsymbol{p}_{t+1}^{\check{G}} - \text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\boldsymbol{p}_{t+1}^{\check{R}} \\
&\quad - \left[\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_N)^\top\right) \circ \overline{\boldsymbol{M}^{yG,3}}\right]\boldsymbol{Q}_{t+1}^{\check{G}} - 2\left((\boldsymbol{1}_N)^\top \otimes \text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right) \circ \overline{\boldsymbol{M}^{xR,7}}\boldsymbol{Q}_{t+1}^{\check{R}} \\
&\quad + \left[-\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_O)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^{O}\right)\right]\boldsymbol{L}_{t+1}^{\check{}} \\
&\quad + \left\{\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_O)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,4}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^{O}\right)\right)\right. \\
&\quad \left. + \frac{1}{\theta}\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right\}\boldsymbol{K}_{t+1}^{\check{R}} + \overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_{t+1}^{\check{R}} = \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_t^{\check{R}}.
\end{aligned}$$

Hence the log-linearized transitional dynamic system is $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}_t^{y,TD}}\check{\boldsymbol{y}}_t$, where matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_t^{y,TD}}$ are defined as

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}} = \left[\begin{array}{cccccccc} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}_{21,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{22,t+1}^{y,TD}} & \boldsymbol{0} & \overline{\boldsymbol{D}_{24,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{25,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{26,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{27,t+1}^{y,TD}} & \overline{\boldsymbol{I}_{NO}} \end{array}\right],$$

where

$$\overline{\boldsymbol{D}_{21,t+1}^{y,TD}} \equiv -\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^R\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}_{22,t+1}^{y,TD}} \equiv -\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}_{24,t+1}^{y,TD}} \equiv -\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_N)^\top\right) \circ \overline{\boldsymbol{M}^{yG,3}},$$

$$\overline{\boldsymbol{D}_{25,t+1}^{y,TD}} \equiv -2\left((\boldsymbol{1}_N)^\top \otimes \text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right) \circ \overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}_{26,t+1}^{y,TD}} \equiv -\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_O)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^{O}\right),$$

$$\begin{aligned}
\overline{\boldsymbol{D}_{27,t+1}^{y,TD}} \equiv &\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\boldsymbol{1}_O)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,4}}\right) \\
&- \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^{O}\right)\right) + \frac{1}{\theta}\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\text{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right),
\end{aligned}$$

and

$$\overline{\boldsymbol{D}_t^{y,TD}} = \begin{bmatrix} -\delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}} & \delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}} & \boldsymbol{0} & \boldsymbol{0} & \delta\overline{\boldsymbol{M}^{xR,7}} & \boldsymbol{0} & (1-\delta)\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}} \end{bmatrix}. \tag{A.45}$$

Since $\check{\boldsymbol{y}}_t = \widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}$ for any $t \geq t_0$ and $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$, I have

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}}\left(\widehat{\boldsymbol{y}_{t+1}} - \widehat{\boldsymbol{y}}\right) = \overline{\boldsymbol{D}_t^{y,TD}}\left(\widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}\right)$$

$$\Longleftrightarrow \overline{\boldsymbol{D}_{t+1}^{y,TD}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{D}_t^{y,TD}}\widehat{\boldsymbol{y}}_t - \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_t^{y,TD}}\right)\overline{\boldsymbol{E}}\boldsymbol{\Delta}.$$

Recall the temporary equilibrium condition $\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}}_t - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}}_t = \overline{\boldsymbol{D}^{A,\Delta}}\widehat{\boldsymbol{\Delta}}$ for any t. Thus

$$\overline{\boldsymbol{F}_{t+1}^y}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}_t^y}\widehat{\boldsymbol{y}}_t + \overline{\boldsymbol{F}_{t+1}^{\Delta}}\boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{F}_{t+1}^y} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^x} \quad -\overline{\boldsymbol{D}^{A,T}} \\ \overline{\boldsymbol{D}_{t+1}^{y,TD}} \end{bmatrix}, \ \overline{\boldsymbol{F}_t^y} \equiv \begin{bmatrix} \boldsymbol{0} \\ \overline{\boldsymbol{D}_t^{y,TD}} \end{bmatrix}, \ \overline{\boldsymbol{F}_{t+1}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_t^{y,TD}}\right)\overline{\boldsymbol{E}} \end{bmatrix},$$

or with $\overline{\boldsymbol{F}^y} \equiv \left(\overline{\boldsymbol{F}_{t+1}^y}\right)^{-1}\overline{\boldsymbol{F}_t^y}$ and $\overline{\boldsymbol{F}^{\Delta}} \equiv \left(\overline{\boldsymbol{F}_{t+1}^y}\right)^{-1}\overline{\boldsymbol{F}_{t+1}^{\Delta}}$, one can write

$$\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{y}}_t + \overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}. \tag{A.46}$$

It remains to find the initial values of the system (A.46) that converges to the steady state. To this end, I apply a standard method in Stokey and Lucas [1989]. In particular, I first homogenize the system: Note that equation (A.46) can be rewritten as $\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{y}}_t + \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)\left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}$ and thus

$$\widehat{\boldsymbol{z}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{z}}_t \tag{A.47}$$

where

$$\widehat{\boldsymbol{z}}_t \equiv \widehat{\boldsymbol{y}}_t - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}. \tag{A.48}$$

The system (A.47) must not explode, or it must be that $\widehat{z_t} \to \mathbf{0} \iff \widehat{\boldsymbol{y}_t} \to \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1} \overline{\boldsymbol{F}^\Delta}\boldsymbol{\Delta}$. I follow Blanchard and Kahn [1980] to find such a condition. Write Jordan decomposition of $\overline{\boldsymbol{F}^y}$ as $\overline{\boldsymbol{F}^y} = \overline{\boldsymbol{B}}^{-1}\overline{\boldsymbol{\Lambda}\boldsymbol{B}}$. Then Theorem 6.4 of Stokey and Lucas [1989] implies that it must be that out of n_y vector of $\overline{\boldsymbol{B}}\widehat{z_{t_0}}$, n-th element must be zero if $|\lambda_n| > 1$. Since $\widehat{\boldsymbol{K}^R_{t_0}} = \mathbf{0}$, I can write

$$\widehat{z_{t_0}} = \overline{\boldsymbol{F}^\Delta_{t_0}}\boldsymbol{\Delta} + \overline{\boldsymbol{F}^\lambda_{t_0}}\widehat{\boldsymbol{\lambda}^R_{t_0}},$$

where

$$\overline{\boldsymbol{F}^\Delta_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^x}\right)^{-1} \overline{\boldsymbol{D}^{A,\Delta}} \\ \mathbf{0}_{2NO\times n_\Delta} \end{bmatrix} - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1} \overline{\boldsymbol{F}^\Delta} \text{ and } \overline{\boldsymbol{F}^\lambda_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^x}\right)^{-1} \overline{\boldsymbol{D}^{A,\lambda}} \\ \mathbf{0}_{NO\times NO} \\ \overline{\boldsymbol{I}_{NO}} \end{bmatrix}$$

and $\overline{\boldsymbol{D}^{A,\lambda}}$ is the right block matrix of $\overline{\boldsymbol{D}^A} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,K}} & \overline{\boldsymbol{D}^{A,\lambda}} \end{bmatrix}$ that corresponds to vector $\widehat{\boldsymbol{\lambda}^R}$. Extracting n-th row from $\overline{\boldsymbol{F}^\Delta_{t_0}}$ and $\overline{\boldsymbol{F}^\lambda_{t_0}}$ where $|\lambda_n| > 1$ and writing them as a $NO \times n_\Delta$ matrix $\overline{\boldsymbol{G}^\Delta_{t_0}}$ and $NO \times NO$ matrix $\overline{\boldsymbol{G}^\lambda_{t_0}}$, the condition of the Theorem is

$$\mathbf{0} = \overline{\boldsymbol{G}^\Delta_{t_0}}\boldsymbol{\Delta} + \overline{\boldsymbol{G}^\lambda_{t_0}}\widehat{\boldsymbol{\lambda}^R_{t_0}},$$

or $\widehat{\boldsymbol{\lambda}^R_{t_0}} = \overline{\boldsymbol{G}_{t_0}}\boldsymbol{\Delta}$ where $\overline{\boldsymbol{G}_{t_0}} \equiv -\left(\overline{\boldsymbol{G}^\lambda_{t_0}}\right)^{-1} \overline{\boldsymbol{G}^\Delta_{t_0}}$. Finally, tracing back to obtain the initial conditions for $\widehat{\boldsymbol{y}_t}$, it must be $\widehat{\boldsymbol{y}_{t_0}} = \overline{\boldsymbol{F}^y_{t_0}}\boldsymbol{\Delta}$,where

$$\overline{\boldsymbol{F}^y_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^x}\right)^{-1} \left(\overline{\boldsymbol{D}^{A,\Delta}} + \overline{\boldsymbol{D}^{A,\lambda}}\overline{\boldsymbol{G}_{t_0}}\right) \\ \mathbf{0}_{NO\times n_\Delta} \\ \overline{\boldsymbol{G}_{t_0}} \end{bmatrix}.$$

Appendix B

Appendix for Chapter 2

B.1 Other Application Examples

Out of the application list Table 2.1, we list some of example tasks for each category. Handling operations/Machine tending (or Handling) includes material handling, picking, and packaging. Recent developments enable food industries to package foods automatically by robots. Welding and soldering (Welding) includes several welding technologies, such as spot welding discussed in the main text, and arc welding. Dispensing includes painting and plating. Processing includes loading and unloading, polishing, and deburring. Assembling and disassembling (Assembling) includes surface mounting, as discussed in Section 2.2.2, and bonding. Finally, Others include robots used for education and research and to clean rooms.

B.2 Net Elasticity of Factor Prices

The purpose of this section is to show that the compensated (net) elasticity of factor price is always negative. This implies that the net elasticity concepts cannot explain the empirical findings of positive elasticities in Sections 2.3 and 2.4. The net elasticity with respect to robot prices must be negative out of the standard profit-maximizing factor demand. To simplify the notations, we drop all subscripts in this section. The Allen-Uzawa Elasticity of Substitution (AUES) is

$$\sigma^A_{CW} \equiv \frac{D_{CW}D}{D_C D_W}.$$

We estimated the cross-price elasticity

$$\frac{\partial \ln L}{\partial \ln C} = \frac{C}{L}\frac{\partial L}{\partial C} = \frac{C}{D_W} D_{CW},$$

where the last equality holds by Shephard's lemma. We have

$$C = \frac{CR}{R} = \frac{D}{R}\frac{CR}{D} = \frac{D}{R}\theta_R. \tag{B.1}$$

Thus we have, with Shephard's lemma $R = D_C$,

$$\frac{\partial \ln L}{\partial \ln C} = \frac{1}{D_W} D_{CW} \frac{D}{R}\theta_R = \sigma_{CW}\theta_R. \tag{B.2}$$

In contrast, we have the own-price elasticity

$$\frac{\partial \ln R}{\partial \ln C} = \frac{C}{D_C} D_{CC} = \sigma_{CC}\theta_R$$

where the last equation by (B.1). By Euler's formula we have

$$\sigma_{CC} = \frac{-\sigma_{CW}\theta_L}{\theta_R}.$$

Thus we have

$$\frac{\partial \ln R}{\partial \ln C} = -\sigma_{CW}\theta_L. \tag{B.3}$$

Equations (B.2) and (B.3) imply

$$\frac{\partial \ln L}{\partial \ln C} = -\frac{\theta_R}{\theta_L}\frac{\partial \ln R}{\partial \ln C}.$$

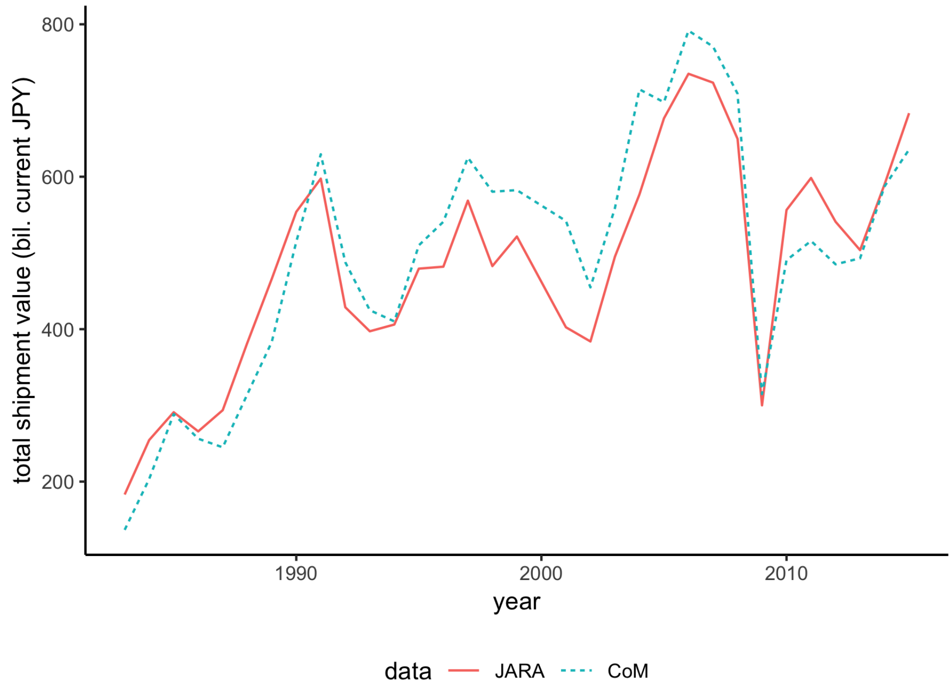
Thus, the sign must be flipped.

B.3 Detail in the JARA

B.3.1 Coverage of the JARA

JARA data cover most robot producers in Japan. In 1996, 587 establishments were asked to answer the survey, among which 445 answered, for a response rate of 76 percent. To

Figure B.1: Comparison of JARA and Census of Manufacture



Note: Authors's calculation based on JARA and CoM/ECBA data. The trend for the JARA is the sum of shipment value of any robot applications to any industries. The trend for the CoM is the sum of shipment value (net of VAT) of all products categorized as industrial robots (available from 1983 to 2016). The CoM was not conducted in 2011 and 2015. Instead, in these years, we employ data from the ECBA.

show the coverage trend of the JARA data in Japan's robot production, we compare the aggregate trend with government-based statistics. In particular, we employ Japan's Census of Manufacture (CoM) and Economic Census for Business Activity (ECBA), the latter of which was conducted jointly by METI and MIC. From these data sources, we take the aggregated total sales of industrial robots each year. The construction of aggregate statistics by CoM and ECBA is discussed in detail in Appendix B.7.1. Figure B.1 shows the comparison of total shipment values taken from the JARA and CoM/ECBA data. As one can see, overall, the two trends are parallel. In some years, the JARA data even surpass the total shipment values observed in the CoM/ECBA data. Therefore, the JARA covers most of the robot transactions measured in government statistics.

B.3.2 JARA Cross Tables

The cross tables we were able to access are as follows. The cross tables by application by buyer industry are available between 1978 and 2017 and are the data source for our primary analysis. The cross tables by types and buyer industry are also available, but only for the years between 1974 and 2000. From 2001 to 2017, cross tables of robot structures and buyer industries are available. This is consistent with the statement of the IFR that since 2004, the robot classifications should only be done in structures.[1] For 1982-1991, we can also access the cross tables by application and types.

In our study, we leverage the heterogeneity in these robot classifications. Robots may be categorized by several dimensions, such as applications, types, and structures. Applications are the classification of robots due to the tasks (applications) that robots are expected to perform by each user. Examples include Welding and soldering (WS) that are intensively needed tasks in the automobile industry, and Assembling and disassembling (AD) that are intensive in the electric machine industry. Types refer to the physical structure and features of robots. For instance, a playback robot is a type of robot that remembers pre-specified moves and plays them back over and over again. Numerical control robots receive the input by programs and move without memory based on the moves performed beforehand. Playback robots are relatively intensively used in automobile industry, while numerical control robots tend to be in electric machine industry.

Starting in 2004, the IFR and major robot producers agreed that robots should not be classified according to the above types but instead, by structures that represent the physical feature of robots. In the JARA data, the type classification discontinues in 2000 and the structure classification begins in the following year. The classifications are as follows: articulated robot, SCARA robot, polar coordinate robot, cylindrical robot, cartesian robot, and parallel link robot.

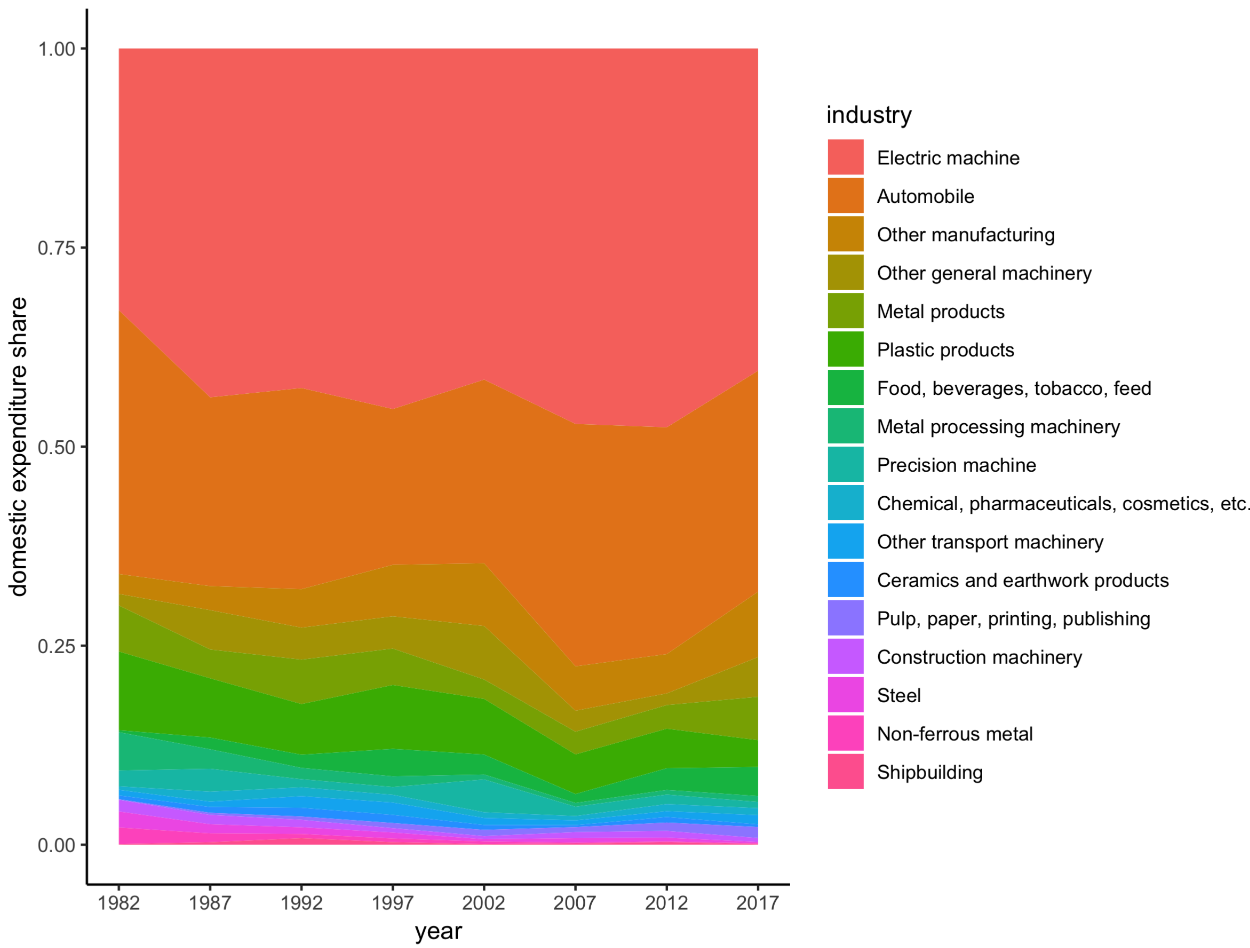
B.3.3 Further Raw Trends of the JARA

Figure B.2 shows the trend of expenditure shares across industries in Japan taken from the JARA data. The industries are sorted according to their value in 2017. As one can see, the distribution is historically highly skewed to particular industries within manufacturing.

[1] See https://www.ifr.org/downloads/press2018/SourcesandMethodsWRIndustrialRobots2018.pdf. (Accessed on October 23, 2019)

In particular, in 2017, the top 10 industries (industries above Chemical, pharmaceuticals, cosmetics, etc. in Figure B.2) constitute more than 97 percent of the domestic expenditure. Furthermore, the electric machine and automobile industries are noticeable significant purchasers. These two industries represent 68.2 percent of the absorption in Japan in 2017.

Figure B.2: Industry Shares



Note: Authors' calculation based on JARA data. Industries are sorted by the total quantity in 2017.

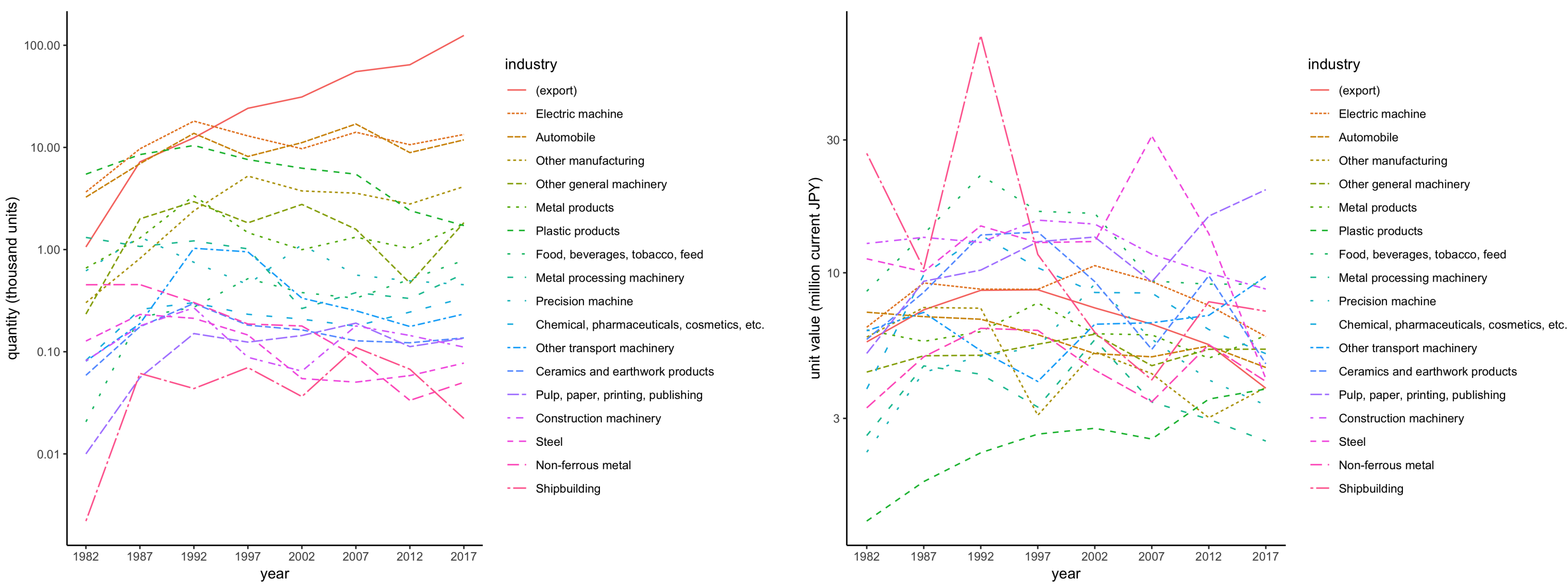
We then show the raw trends of shipment quantity and unit values for each industry, aggregated by all applications for the industry. To calculate unit values, we take the sales value and the corresponding quantity units and divide the former measure by the latter. In addition to the domestic adsorptions, we also show the trend of exports from now on for comparison. Figure B.3 shows the result. The left panel shows the quantity unit of robots, while the right shows the unit values. First, note that our data spans 1978-2017. The coverage of our data allows us to study the early period in which the quantity shipment grew rapidly in Japan. Our data show that the growth did not concentrate in a particular industry, but instead occurred across a large set of industries. As the other data source confirms, since 1990, shipments to domestic industries shrank in general, while the export trends continued to grow at a somewhat slower pace than before. These constant growth shares across domestic industries suggest that the robot penetration to domestic industries is caused not only by demand shocks to a particular set of buyer industries, but by the overall growth of the robot-producing industry.

Second, in the right panel, we see that the unit-value trends for each industry are relatively constant or slightly decreasing over time, with some exceptions. The constant or decreasing unit prices would suggest that the robot producers become efficient and offer lower prices as time goes on. Note that these patterns hold for the two large buyers of industrial robots, electric machines, and automobiles.

Figure B.3: Raw Trends by Industry

(a) Quantity Units

(b) Unit Values



Note: Authors' calculation based on JARA data. Industries are sorted by the total quantity in 2017. Exports are not classified by industries in the survey.

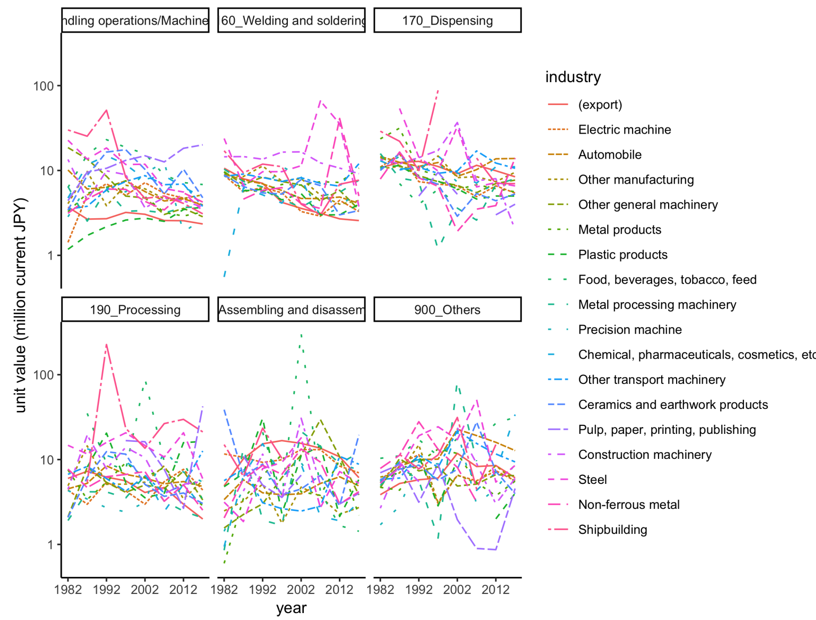
With the JARA data, we are able to show the trends by industry by applications according to Table 2.1 in Figure B.4. Panel (a) shows the quantity and Panel (b) shows the unit value. From Panel (a), one can see that the shipment quantity increases for broad applications and broad industry during 1980s, while the increasing trend stops after the 1990s, confirming the trend we found in Figure B.3. In contrast, there is significant within-application and across-industry variation. For example, the electric machine industry intensively purchased robots for Assembling and disassembling, while the Automobile industry bought a significant amount of robots for Welding and soldering. This suggests that industries have different tasks for which they demand robots.

Panel (b) of Figure B.4 reveals the following two features. First, it shows relatively small within-application and across-industry variations. This suggests that the robot prices are not driven solely by particular industries, but by the technology for producing robots. Second, there are significant variations in trends across different applications. For example, robots for Welding and soldering show a stable decline in the unit value for a broad set of industries. This suggests that the production technology for Welding-soldering robots grew consistently over the sample period.

Figure B.4: Raw Trends by Industry and Applications

(a) Quantity Units

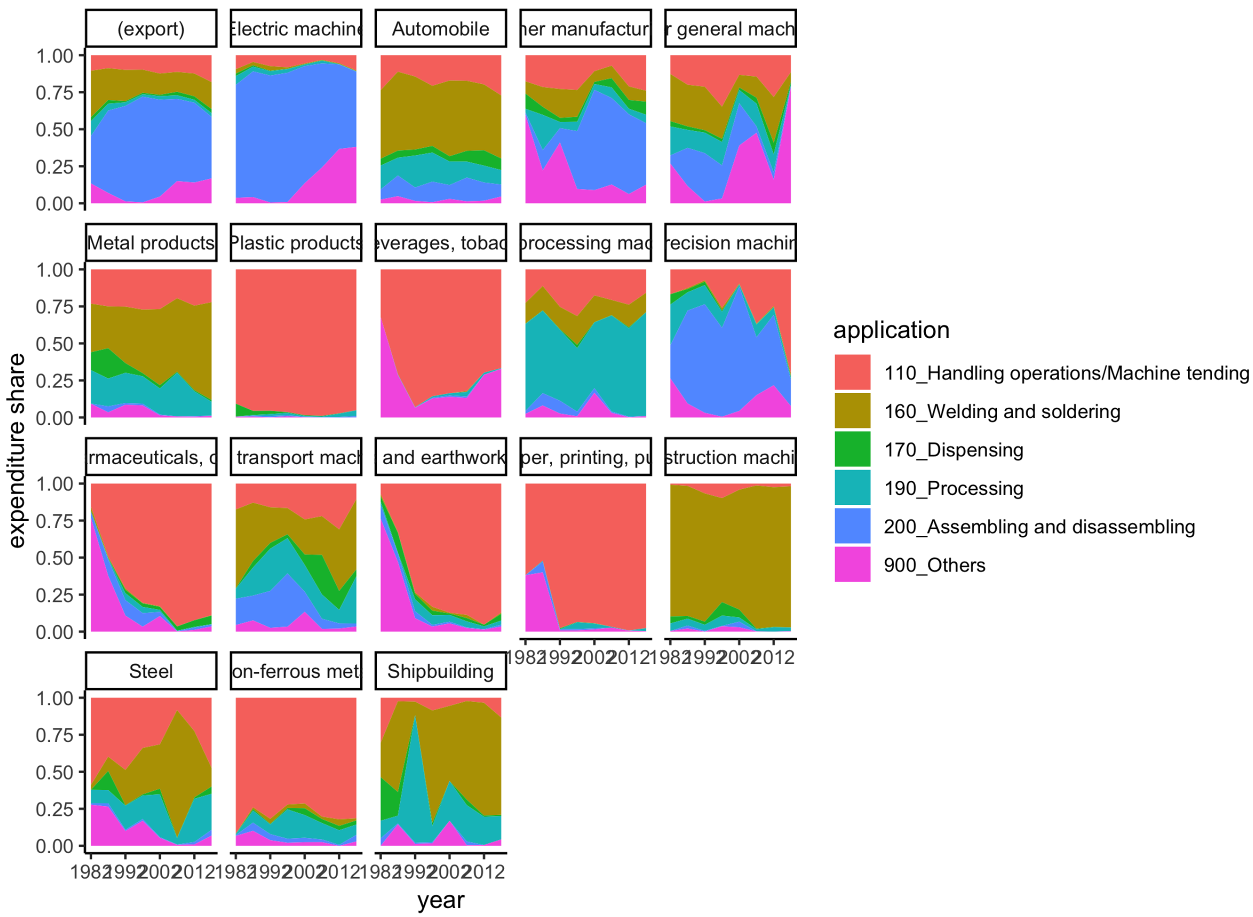
(b) Unit Values



Note: Authors' calculation based on JARA data. Industries are sorted by the total quantity in 2017. Exports are not classified by industries in the survey. The y-axis is log scale and lines reaching the x-axis imply zero quantity in level.

To further highlight the application compositions, we show the expenditure share of applications for each industry in Figure B.5. One may see significant variations across industries. As we have discussed, the Electric machine industry and the Automobile industry intensively buy robots for Assembling and disassembling, and Welding and soldering, respectively. Furthermore, Figure B.5 reveals that the within-industry patterns of expenditure shares are stable with some exceptions in minor industries. This confirms that the expenditure share is driven by the feature of industries.

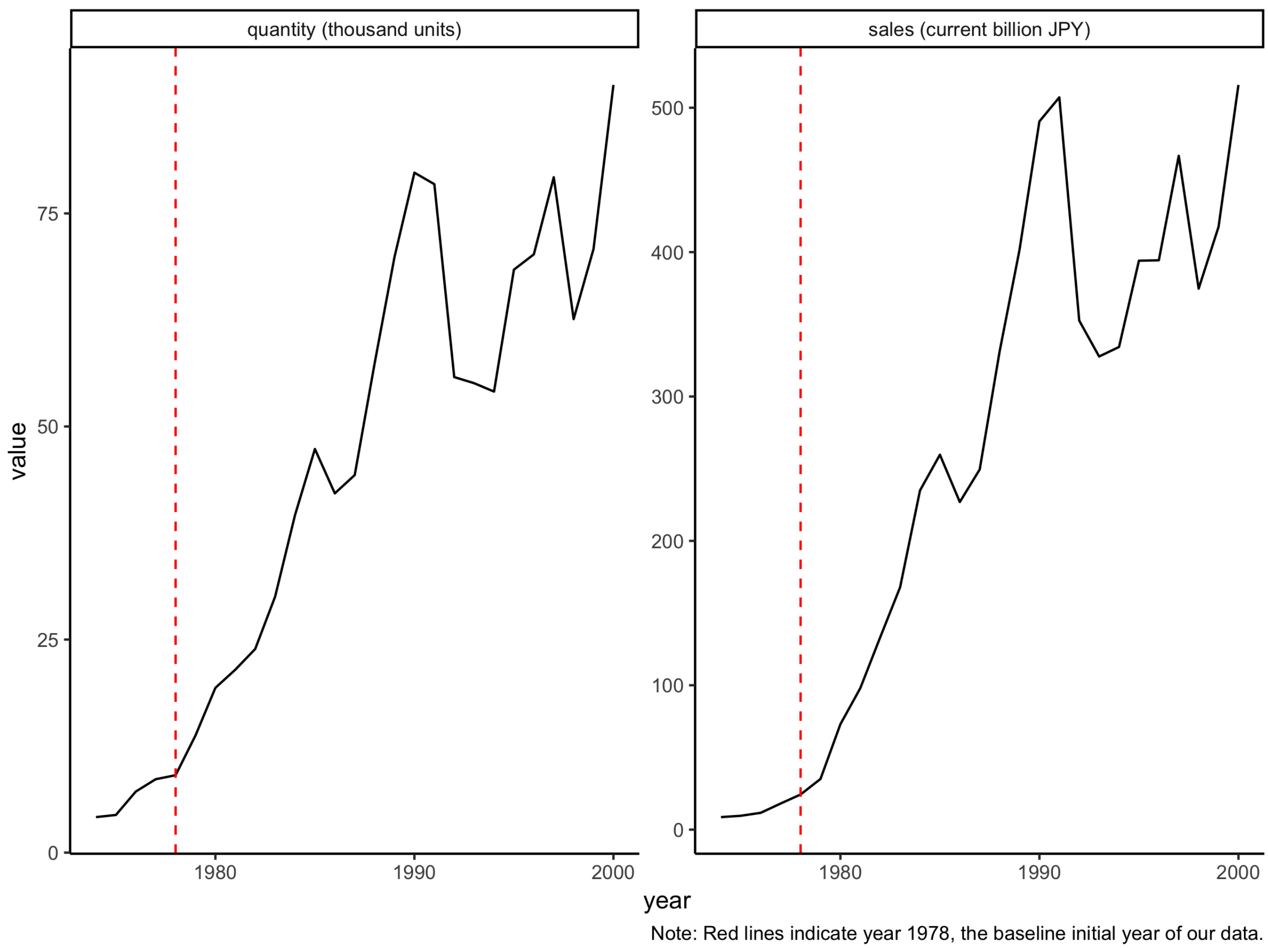
Figure B.5: Raw Trends by Industry and Applications



Note: Industries are ordered by the quantity of shipment as of 2017.

Note: Authors' calculation based on JARA data. Industries are sorted by the total quantity in 2017. Exports are not classified by industries in the survey.

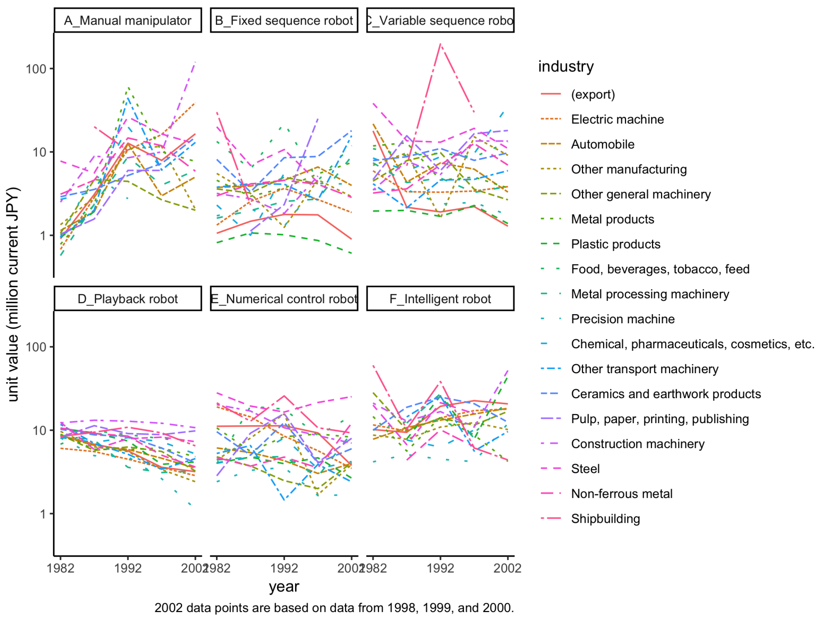
Figure B.6: Robot Trends Before and After 1978



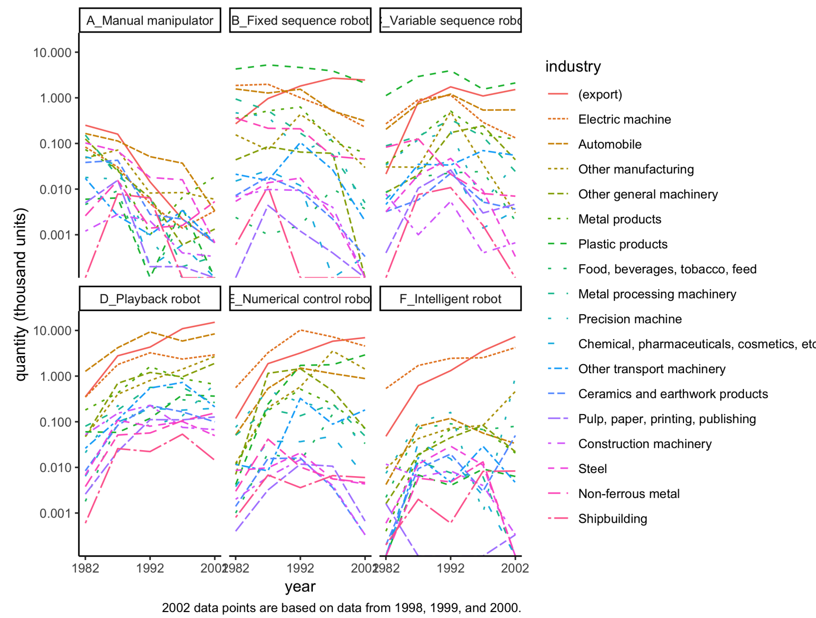
Note: Authors' calculation based on JARA data. The red dashed line indicates 1978, the initial year of our primary analysis. Trends before 1977 are taken by aggregating type-buyer industry across tables.

Figure B.7: Raw Trends by Industry and Types

(a) Quantity Units



(b) Unit Values



Note: Authors' calculation based on JARA data. Industries are sorted by the total quantity in 2017. Exports are not classified by industries in the survey. The y-axis is log scale and lines reaching the x-axis imply zero quantity in level.

Table B.1: Pooled Regression

	Dependent variable:						
	log stock units (thousand)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log unit value	−0.728***	−1.449***	−0.344	0.200	−1.250***	−0.081	−0.230
	(0.126)	(0.253)	(0.426)	(0.301)	(0.232)	(0.318)	(0.266)
Sample	All	110, Handling	160, Welding	170, Dispensing	180, Processing	200, Assembling	900, Others
Industry FE							
Year FE							
Application FE							
Observations	818	144	129	134	138	135	138
R^2	0.039	0.188	0.005	0.003	0.176	0.0005	0.005

Note: *p<0.1; **p<0.05; ***p<0.01
Log unit values are in the unit of current million JPY.

Table B.2: Industry-Fixed Effects

	Dependent variable:						
	log stock units (thousand)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log unit value	−0.370***	−0.210	−0.739***	−0.237	−0.135	−0.105	−0.101
	(0.098)	(0.188)	(0.188)	(0.191)	(0.153)	(0.162)	(0.176)
Sample	All	110, Handling	160, Welding	170, Dispensing	180, Processing	200, Assembling	900, Others
Industry FE	✓	✓	✓	✓	✓	✓	✓
Year FE							
Application FE							
Observations	818	144	129	134	138	135	138
R^2	0.499	0.785	0.893	0.753	0.823	0.813	0.720

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Log unit values are in the unit of current million JPY.

Table B.3: Industry- and Year- Fixed Effects

	Dependent variable:						
	log stock units (thousand)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log unit value	-0.504^{***}	-0.388^{***}	-0.572^{***}	0.010	-0.333^{***}	-0.512^{***}	-0.333^{*}
	(0.097)	(0.139)	(0.140)	(0.198)	(0.123)	(0.138)	(0.183)
Sample	All	110, Handling	160, Welding	170, Dispensing	180, Processing	200, Assembling	900, Others
Industry FE	✓	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓	✓
Application FE							
Observations	818	144	129	134	138	135	138
R^2	0.545	0.912	0.950	0.828	0.903	0.897	0.770

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Log unit values are in the unit of current million JPY.

Table B.4: Industry-Year-Application Fixed Effects

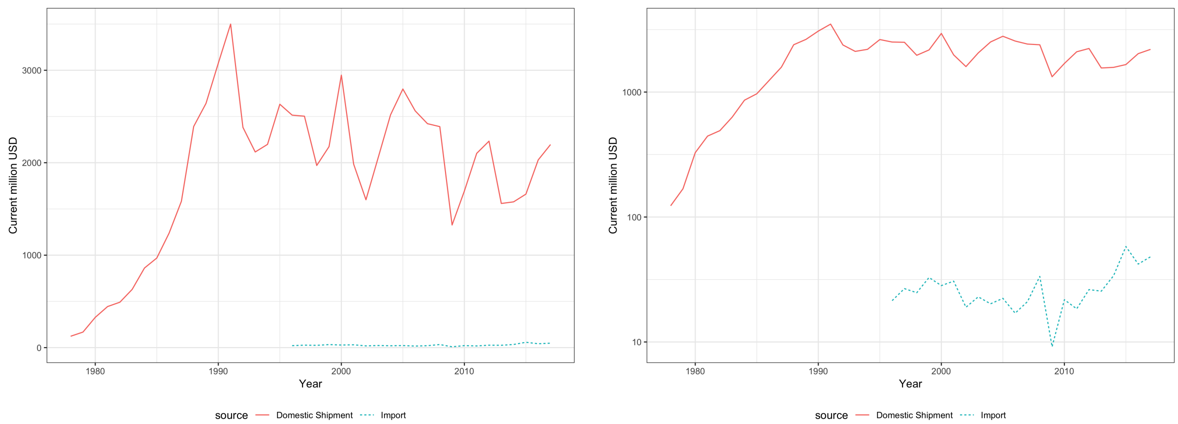
	Dependent variable:
	log stock units (thousand)
log unit value	-0.245^{***}
	(0.086)
Sample	All
Industry FE	✓
Year FE	✓
Application FE	✓
Observations	818
R^2	0.659
Note:	*p<0.1; **p<0.05; ***p<0.01

B.4 Robot Imports in Japan

Japan produces most of its robots domestically (Acemoglu and Restrepo, 2018b, among others). To confirm this, we compare the import (from the rest of the world to Japan) and the domestic sales measures of robots (from Japan to Japan). In particular, we visit Comtrade data, take HS Code 847950 (Industrial Robots for Multiple Uses), and compare the trend with domestic shipment trends from our main data source, the JARA, discussed in detail in the next section. Trade data for the HS code are available only since 1996, while JARA data exist from 1978. Table B.8 shows the result. We also calculate the shipment share by domestic producers. We obtained 97.9 percent to 99.3 percent shares, depending on the year, between 1996 and 2017. Therefore, we interpret that most of robot purchases in Japan have been domestic-sourced. In our paper, we focus only on JARA data, from which we may exploit a rich set of information that is crucial for our analysis, as we will discuss in detail.

Figure B.8: Domestic Shipments and Import of Robots in Japan

(a) Level Scale (b) Log-10 Scale



Note: Authors' calculation based on Comtrade and JARA data. The Comtrade trends show the total import value (reported by importer, Japan) of HS Code 847950 (Industrial Robots for Multiple Uses). The JARA trends show the total shipment from the domestic producers, aggregated by all applications and industries. The Comtrade data is denominated by current USD, while the JARA data are denominated by current JPY. To convert the monetary values, we use the FRED data, the annual current JPY-USD exchange rate.

B.5 Calculating Robot Stock

As we briefly discussed in Section 2.3.6, we assume alternative and flexible assumptions on the robot stock calculation. The first set of assumptions is based on the immediate withdrawal method (IWM). The IWM assumes the shipped robots are in use immediately after purchase and not in use in a specified length of years. IFR follows this method with the withdrawal period of 12 years. To better compare the results with the literature, our primary specification follows the stock definition based on IWM with 12 years. The 12-year assumption is debatable, however, as IFR admits: "This assumption was investigated in an UNECE/IFR pilot study, carried out in 2000 among some major robot companies ... This investigation suggested that an assumption of 12 years of average life span might be too conservative and that the average life/ service life was closer to 15 years." (IFR, 2018). German and US tax authorities, in contrast, suggest the standard depreciation schedules be shorter. Given these discussions, we consider three alternatives: 10, 12 (baseline), and 15 years of depreciations.

The second set of assumptions is based on the perpetual inventory method (PIM). The PIM is a standard method used when calculating capital stocks, adopted in National

Accounts [OECD, 2009]. A key parameter in the method is depreciation rates. There is no systematic empirical study on the value. As one measure, following Artuc et al. [2020], we use an annual 10 percent depreciation rate. As a more context-based estimate, we employ the result from Nomura and Momose [2008]. Based on disposal asset data in Japan (Survey on Capital Expenditures and Disposables), Nomura and Momose [2008] estimated the depreciation rate of machinery, with the category of machinery and equipment, as 18 percent. Admitting that machinery is a broader category than industrial robots, we employ 18 percent as a larger alternative than 10 percent.

Table B.5 shows the baseline regression result of specification (2.2) with these alternative stock measures. Column 2 shows the main regression result based on a 12-year IWM. Columns 1 and 3 show the different-year based IWM, 10 years and 15 years, respectively. Columns 4 and 5 show the results with a PIM with depreciation rates of 10 percent and 18 percent, respectively. The regression coefficients are robust to alternative choices of stock measurement of robots.

B.6 Detail in ESS

Employment status is based on the usual employment status (the "usual method"). Any answer of mostly worked, worked besides doing housework, worked besides attending school, and worked besides doing housework and attending school are recorded as employed. For the education attainment, we define four values that are consistent across surveys: less than high-school diploma (LHS), high-school diploma (HS), technical/vocational school diploma (TVS), four-year college diploma or more (FC). For age, we define the five-year bins from age 15 up to age 79 and aggregate the age groups over 80. The survey records the annual earnings, annual days worked, and weekly hours worked in categories. Industry and occupations are encoded according to the Japan Standard Industry Classifications (JSIC) and the Japan Standard Occupation Classifications (JSOC). We convert these categorical variables into continuous variables using the mid-point of the range. Taking the 2007 survey as an example, the mean annual earnings is 3,170,263 current JPY (27,330 current USD), and the mean hours worked is 1,516 hours.

Another set of representative labor statistics is from the Basic Survey of Wage Structure that is based on the random sampling of payroll records for June. For the purpose of

Table B.5: Industry-level, 2SLS, Different Stock Measurement

	Dependent variable:				
	$\ln(L_{it})$				
	(1)	(2)	(3)	(4)	(5)
$\ln(R_{it})$	0.255^{***}	0.283^{**}	0.322^{**}	0.349^{***}	0.284^{***}
	(0.096)	(0.108)	(0.123)	(0.114)	(0.093)
Industry FE	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓	✓
Stock Measurement	10 Years	12 Years	15 Years	$\delta = 0.1$	$\delta = 0.18$
Observations	104	104	104	104	104
R^2	0.987	0.988	0.987	0.988	0.988

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are total employment (baseline), under age-35 employment, age 35-50 employment, and over age-50 employment in columns 1-4, respectively.*p<0.1; **p<0.05; ***p<0.01.

comparison, we report the statistics of 2007. The mean of the estimated annual salary based on the June salary and bonus payment of the previous year is 3,858,233 yen, and the mean of the estimated annual hours worked is 1,869 hours.

B.7 Details in Other Data

B.7.1 Census of Manufacture

The Census of Manufacture (CoM) annually surveys manufacturing establishments in Japan.[2] The CoM asks each establishment for its product-level shipment values. The product code for industrial robots used in CoM exists since 1977. We take the CoM data since 1983 to 2016. The survey was not conducted in 2011 and 2015, because a substituting government survey, the Economic Census of Business Activity (ECBA), was conducted. We also take ECBA data to construct the complete observations of Japan's establishments that shipped robots in any years between 1983 and 2016.

The CoM and the ECBA treat the VAT in the following way. For CoM, before 2014, respondents were forced to report the shipment value gross of VAT. Since 2016, they have been allowed to choose to gross or net VAT. For the ECBA, both surveys allowed respondents to select. For consistency, we net out the VAT from all data by the legislative VAT rate from the total sales value. The VAT rate is 0 before 1988, 0.03 between 1989 and 1996, 0.05 between 1997 and 2013, and 0.08 since 2014 and onward.

The primary purpose of using the CoM and the ECBA is to take robot-producers' employment. For this purpose, we follow the following steps. First, we calculate each establishment's intensity of robot production by taking the share of robot sales among total sales. To take robot sales, we aggregate shipment values and processing fees of all products under the 1976 Japan Standard Industrial Code (JSIC) of 3498, 1984 and 1993 JSIC of 2998, 2002 JSIC of 2698 ("Industrial Robot Manufacturing," all above), and 2007 and 2013 JSIC of 2694 ("Robot Manufacturing"). Second, assuming a proportional allocation of workers for dollar sales, we multiply total workers by robot production intensity for each establishment. These steps generate robot-producing workers for each establishment, aggregating up to industry-level robot-producing workers.

Table B.6 shows the result of the specification (2.4) with employment, net of robot-producing workers as an outcome variable. The column structure is the same as that of our main result Table 2.4. One should note the very close estimates to the ones in Table 2.4. This solely comes from the fact that the number of robot-producing workers is very

[2]The survey was conducted for all establishments in years with last digit 0, 3, 5, or 8 until the Economic Census of Business Activity (ECBA) started in 2011. In other years and after 2011, all establishments with the number of employees more than three are surveyed.

Table B.6: Industry-level, Reduced Form, net of Robot Producing Workers

	Dependent variable:			
	$\ln(L_{it}^{NRP})$			
	(1)	(2)	(3)	(4)
$\ln(r_{it}^{Z})$	-0.853^{***}	-0.466^{***}	-0.274^{*}	-0.437^{**}
	(0.130)	(0.144)	(0.151)	(0.171)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	9.1	11.047	10.912	16.703
Observations	104	104	104	104
R^2	0.975	0.984	0.985	0.987

Notes: Authors' calculation based on JARA, ESS, CoM, SOBA and JIP data. The table presents estimates of the relationship between log robot cost measure and log robot stock measure across industries and years. The employment measure excludes the employment of robot-producing plants. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

small relative to the size of the Japanese manufacturing industry. In fact, the share of robot-producing workers in manufacturing is between 0.001 and 0.004 percent throughout the sample period. Therefore, as far as the directly robot-producing workers are concerned, the reinstatement effect of automation is quite small [Acemoglu and Restrepo, 2018a].

B.7.2 Basic Survey on Overseas Business Activities

Basic Survey on Overseas Business Activities (BSOBA) is a firm-level census of Japanese multinational enterprises (MNEs) and their foreign subsidiaries. We take offshoring intensity measures by aggregating operating revenues in foreign subsidiaries. We take subsidiaries' industry codes when allocating revenues to each industry.

B.7.3 Japan Industrial Productivity Database

Japan Industrial Productivity (JIP) database is a long-run industrial aggregate of several measures starting in 1970. Among them, we use import and intangible capital. The intangible capital measure is composed of following items: ICT assets (ordered and packaged software, own-developed software), innovation assets (R and D expenditures, mineral exploration, copyright and trademark right, other product/design/research development), and competition assets (brand capital, firm-specific human capital, expenditure for restructuring). Basic concepts of these variables follow National Accounts. Detailed discussion is provided in Fukao et al. [2008].

B.8 Further Industry-level Results

B.8.1 Industry-specific Prices

In this section, we assume that the prices at which each industry purchases robots may differ. In particular, we assume that the robot price of application a in industry i and year t is given by

$$r_{ait} = \frac{v^A_{ait}}{R_{ait}}, \tag{B.4}$$

where v^A_{ait} is purchase value of application a in industry i and year t. Armed with these variables, in Table B.7, we report the regression results of the specification (2.2) based on the price index (B.4). The results are not robust across control variables. In particular, in our preferred specification of column 4, we find a positive but insignificant coefficient estimate.

Table B.7: Industry-level, 2SLS, industry-specific prices

	Dependent variable:			
	$\ln(L_{it})$			
	(1)	(2)	(3)	(4)
$\ln(R_{it})$	0.429***	0.199***	0.103	0.139
	(0.079)	(0.061)	(0.101)	(0.101)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	27.262	17.462	12.754	15.404
Observations	104	104	104	104
R^2	0.968	0.987	0.987	0.988

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure, based on equation (B.4). The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

B.8.2 First Stage for Efficiency-adjusted Regressions

B.9 Further CZ-level Analysis

B.9.1 Similar Country SSIV Result

Table B.9 shows the results of regression (2.7) with the standard geographic SSIV. Among the three specifications (log changes in total employment in column 1, log changes in total population in column 2, and changes in employment-to-population ratio in column 3), the point estimates show signs consistent with those of our main results in Table 2.10. They

Table B.8: Industry-level, First Stage, Efficiency-adjusted Quantity

	Dependent variable:			
	$\ln(\widetilde{R}_{it})$			
	(1)	(2)	(3)	(4)
$\ln(r_{it}^{Z})$	-0.325	-1.564^{***}	-1.244^{***}	-1.156^{**}
	(0.352)	(0.416)	(0.435)	(0.477)
Industry FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls		✓	✓	✓
Globalization Controls			✓	✓
Technology Controls				✓
IV F-stasitic	0.855	14.131	8.195	5.860
Observations	104	104	104	104
R^2	0.949	0.969	0.973	0.977

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between log robot stock measure and log employment across industries and years, with the instrument of log robot cost measure. The employment measure includes the employment of robot-producing plants. All columns control the industry and year fixed effects. All regressions are weighted by purchase values of robots in each year. The standard errors are shown in the parenthesis. Columns 1 shows the result without other control variables. Column 2 includes the demography controls. Demography controls include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. Column 3 includes the logarithm import values from JIP database and logarithm offshoring value added from SOBA. Column 4 includes logarithm stock value measures for ICT capital, innovation capital, competition capital from the JIP database. *p<0.1; **p<0.05; ***p<0.01.

are not precise enough, however, to conclude the positive impacts of robots.

B.9.2 Heterogeneous Impacts

Table B.9: Regressions with Exposure to Robots IV

	Dependent variable:		
	$\Delta \ln(L)$	$\Delta \ln(P)$	$\Delta(L/P)$
	(1)	(2)	(3)
ΔR	1.661	0.835	0.472
	(1.479)	(1.287)	(0.386)
Controls	✓	✓	✓
CZ and year FEs	✓	✓	✓
Observations	906	906	906
R^2	0.423	0.358	0.587

Notes: Authors' calculation based on IFR, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. The dependent variable is instrumented by the shift-share measure whose shift is taken from German robot adoption trends and share is taken by the baseyear industrial employment share in each CZ. As outcome variables, column 1 takes log total employment, column 2 takes log total population, and column 3 takes employment-to-population ratio. All regressions are weighted by base-year populations in each CZ. Control variables include demographic, industry, trade and capital controls in the base year. Demographic variables consists of CZ's female share and elderly (age 65 and above) share. Industry variables are CZ's manufacturing and service employment shares. Trade variable includes the import exposure from China as in Autor, Dorn, Hanson (2013). Capital control is made from the information-technology capital in each industry from JIP database. *p<0.1; **p<0.05; ***p<0.01.

Table B.10: CZ-level, 2SLS. Results by Education Groups

	Dependent variable:		
	$\Delta \ln(L_{ct})$	$\Delta \ln(L_{ct}^{HS})$	$\Delta \ln(L_{ct}^{CG})$
	(1)	(2)	(3)
ΔR_{ct}	2.203**	2.426**	3.320**
	(1.017)	(1.200)	(1.616)
CZ FE	✓	✓	✓
Year FE	✓	✓	✓
Demographic Controls	✓	✓	✓
Globalization Controls	✓	✓	✓
Technology Controls	✓	✓	✓
Group	All	High School Grad.	4-year Univ. Grad.
Observations	1,466	1,466	1,439
R^2	0.817	0.841	0.759

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are total employment (baseline), high-school graduate employment, and 4-year university graduate-or-more employment in columns 1-3, respectively. *p<0.1; **p<0.05; ***p<0.01.

Table B.11: CZ-level, 2SLS. Results by Sex Groups

	Dependent variable:		
	$\Delta \ln(L_{ct})$	$\Delta \ln(L_{ct}^{Female})$	$\Delta \ln(L_{ct}^{Male})$
	(1)	(2)	(3)
ΔR_{ct}	2.203**	3.112***	1.816*
	(1.017)	(1.122)	(1.028)
CZ FE	✓	✓	✓
Year FE	✓	✓	✓
Demographic Controls	✓	✓	✓
Globalization Controls	✓	✓	✓
Technology Controls	✓	✓	✓
Group	All	Female	Male
Observations	1,466	1,466	1,466
R^2	0.817	0.802	0.802

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are total employment (baseline), female employment, and male employment in columns 1-3, respectively. *p<0.1; **p<0.05; ***p<0.01.

Table B.12: CZ-level, 2SLS. Results by Age Groups

	Dependent variable:			
	$\Delta\ln(L_{ct})$	$\Delta\ln(L_{ct}^{a\leq 34})$	$\Delta\ln(L_{ct}^{35\leq a\leq 49})$	$\Delta\ln(L_{ct}^{50\leq a})$
	(1)	(2)	(3)	(4)
ΔR_{ct}	2.203**	1.463	4.656***	0.499
	(1.017)	(1.428)	(1.467)	(1.209)
CZ FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓
Group	All	Age $\leq$ 34	35 $\leq$ Age $\leq$ 49	50 $\leq$ Age
Observations	1,466	1,458	1,465	1,466
R^2	0.817	0.818	0.804	0.881

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of outcome variables multiplied by 100. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The outcome variables are total employment (baseline), under age-35 employment, age 35-50 employment, and over age-50 employment in columns 1-4, respectively. *p<0.1; **p<0.05; ***p<0.01.

Table B.13: CZ-level, 2SLS, Wage Effects By Education Level

	Dependent variable:			
	$\Delta \ln(w_{ct}^{MS})$	$\Delta \ln(w_{ct}^{HS})$	$\Delta \ln(w_{ct}^{TC})$	$\Delta \ln(w_{ct}^{4U})$
	(1)	(2)	(3)	(4)
ΔR_{ct}	2.275^{*}	3.595^{***}	4.842^{***}	3.213^{**}
	(1.295)	(0.929)	(1.445)	(1.613)
CZ FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓
Education	Middle School	High School	Technical College	4-year Univ.
Observations	1,402	1,402	1,402	1,402
R^2	0.899	0.951	0.836	0.879

*p<0.1; **p<0.05; ***p<0.01

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of hourly wages multiplied by 100 for different education group of workers. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The first column shows the result for the sample of middle-school graduates, the second for high-school graduates, the third for technical and vocational college graduates, and the fourth for four-year university graduates or more education. *p<0.1; **p<0.05; ***p<0.01.

Table B.14: CZ-level, 2SLS, Hours Effects By Education Level

	Dependent variable:			
	$\Delta \ln(h_{ct}^{MS})$	$\Delta \ln(h_{ct}^{HS})$	$\Delta \ln(h_{ct}^{TC})$	$\Delta \ln(h_{ct}^{4U})$
	(1)	(2)	(3)	(4)
ΔR_{ct}	-2.160^{*}	-2.253^{***}	-3.336^{***}	-0.932
	(1.117)	(0.649)	(1.017)	(0.896)
CZ FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓
Demographic Controls	✓	✓	✓	✓
Globalization Controls	✓	✓	✓	✓
Technology Controls	✓	✓	✓	✓
Education	Middle School	High School	Technical College	4-year Univ.
Observations	1,402	1,402	1,402	1,402
R^2	0.446	0.780	0.734	0.748

*p<0.1; **p<0.05; ***p<0.01

Notes: Authors' calculation based on JARA, ESS, SOBA and JIP data. The table presents estimates of the relationship between shift-share measures of changes in robot stock per thousand workers and log difference of hours worked multiplied by 100 for different education group of workers. All regressions control demographic variables, globalization controls, and technology controls as well as the industry and year fixed effects. All regressions are weighted by initial-year population. The standard errors are shown in the parenthesis. The demographic variables include share of high school graduates, share of 4-year university graduates, share of female workers, share of workers under age of 35, and share of workers above age of 50 from ESS. The globalization controls contain the logarithm import values from JIP database and logarithm offshoring value added from SOBA. The technology controls include logarithm stock value measures for ICT capital, innovation capital, and competition capital from the JIP database. The first column shows the result for the sample of middle-school graduates, the second for high-school graduates, the third for technical and vocational college graduates, and the fourth for four-year university graduates or more education. *p<0.1; **p<0.05; ***p<0.01.

Appendix C

Appendix for Chapter 3

C.1 Data and Empirical Result Appendix

C.1.1 Further Data on Labor Share and Multinational Activities

Figure C.1 shows the longer-run trend of labor share in Japan since 1980 to complement Figure 3.2 in the main text. Although there is a countercyclical component and a sag around 1990, the overall long-run trend reveals the same message as Figure 3.2: the persistent decline in the labor share for the three decades.

While Figure C.1 is concerned about labor compensation, Figure C.2a shows the trends of both labor and capital earnings by Japanese MNEs from the BSOBA data. Note that capital income has been increasing since 1995, but the trend is more volatile than payment to labor. This increase can partly explain the more rapid increase in GNI than GDP in Japan because the income from the foreign capital is accounted for under the positive net primary income from abroad. It will be a crucial step to explicitly allow foreign capital in our model.

C.1.2 Robust Labor Share Decrease

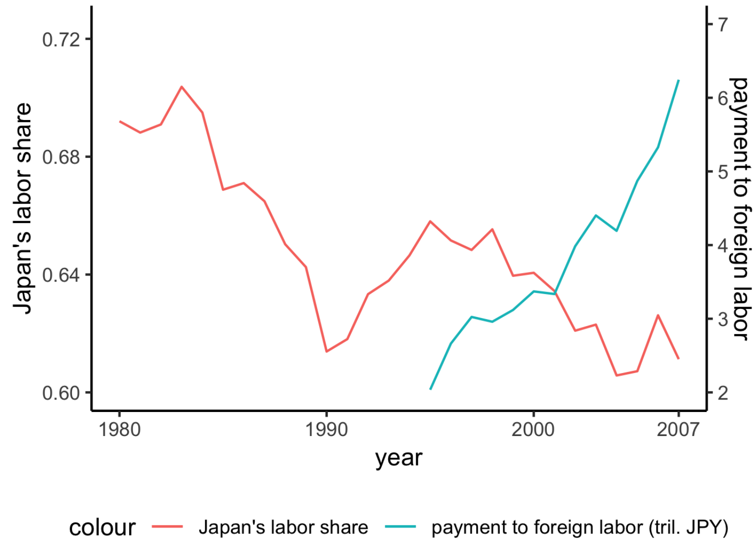
As explained in Section 3.1.1, neither the conceptual or operational measurement of labor share is trivial. In this section, we provide several measures of the labor share in Japan between 1995 and 2007 to see that irrespective of the measurement process, we have robust evidence that the labor share has been decreasing. First, the green line in Figure C.2b shows our preferred measure, the total labor cost divided by GDP. However, since GDP or value added includes capital depreciation, it overstates net capital income [Bridgman, 2018]. To overcome this, we take the SNA data from the Japan Cabinet Office Long-run Economic Statistics and calculate the trend in the share of nominal employee compensation over nominal national income, which excludes capital depreciation (as well as indirect tax, while including subsidies). This trend is shown by the blue line. Another issue is the treatment of the mixed income of self-employees. Since self-employees are typically owners of both the production capital and labor, the allocation of the generated income to labor and capital can be distorted (e.g., Rognlie, 2018). To remove any biases due to the misallocation of such mixed income, we take the trend of domestic corporate factor income and their compensation payment to the labor, which is shown by the red line. We can see in Figure C.2b that all of these trends are decreasing.

C.1.3 Comparison of MNEs and Non-MNEs

To further analyze the role of MNEs in the decrease in labor share, we conduct a simple decomposition analysis across MNEs and non-MNEs by aggregating total sales, labor compensation and net income separately for MNEs and non-MNEs. We then calculate the labor share for the two groups. Figure C.3 shows the trends for labor share and MNE composition. The blue line depicts the trends in the share of the sum of MNE HQ sales relative to the sum of sales of all firms. In 1995, the share was roughly 12 percent, which rose to close to 15 percent in 2007. Therefore, the composition of sales became skewed toward MNEs over the period. The red lines show the labor share trends among MNEs (solid line) and non-MNEs (dashed line), and we can see that the labor share of MNEs decreases more rapidly. Additionally, throughout the period, MNEs had a lower labor share than non-MNEs. Since the labor share composition by MNEs increased (as shown by the blue line), both of these facts have contributed to the decrease in aggregate labor share.

One of the interpretations of these facts is that within MNEs, the payment to labor relative to capital decreased over the period, *and* more firms become MNEs. In the model section, we develop a framework in which *both* of these may be explained by foreign factor augmentation.

Figure C.1: Labor Share and Payment to Foreign Employment, Japan



Note: The authors' calculation based on Japan Industrial Productivity (JIP) Database 2015 from Research Institute of Economy, Trade and Industry (RIETI) and the Basic Survey on Overseas Business Activities (BSOBA) 1996-2008. The labor share is calculated by the share of the nominal labor cost in the value added of JIP market economies in nominal terms. The payment to foreign labor is the sum of compensation to workers in foreign subsidiaries of all Japanese multinational corporations in BSOBA.

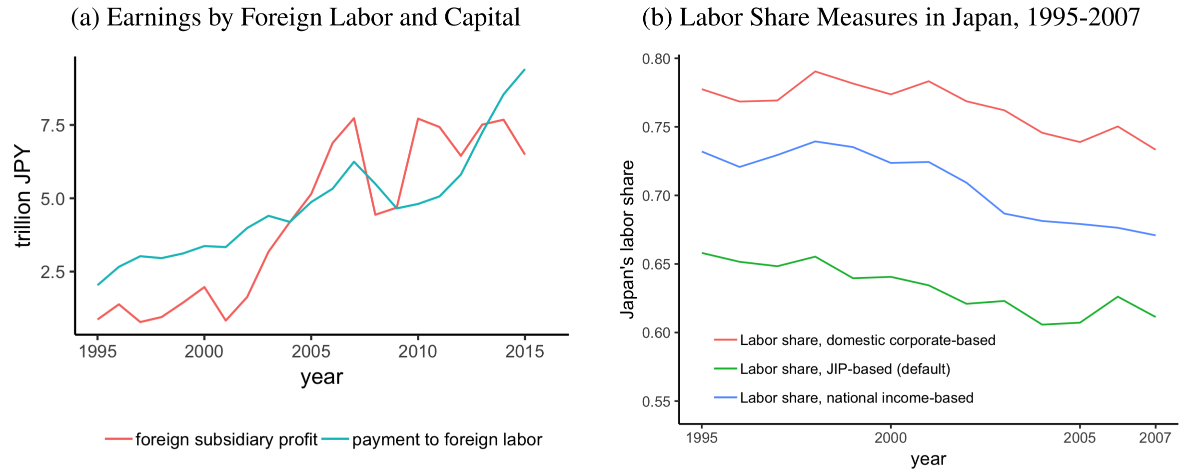
C.1.4 Other Potential Mechanisms

There are several mechanisms behind decreasing labor shares proposed in the literature. I review some first-pass evidence that neither of recent automation and surge in market powers is likely to play a main role in explaining shrinking labor share.

Automation in Japan As discussed in Section 3.1.1, bias in technological change can arise from at least two sources: automation and offshoring. Since the current paper focuses on offshoring; in particular, factor offshoring as opposed to goods offshoring [Hummels et al., 2014], the evolution of automation technology in Japan is not our main focus. However, by examining the aggregate data used in automation literature (e.g., Acemoglu and Restrepo, 2019), we see suggestive evidence that automation acceleration was not observed in Japan as rapidly as in other highly automating countries from the 1990s to 2010s.

To examine this, we employ data from the *International Federation of Robotics* (IFR) to obtain the operational stock of industrial robots in each country by year. Figure C.4a

Figure C.2: MNE Activities and Labor Shares, Detailed

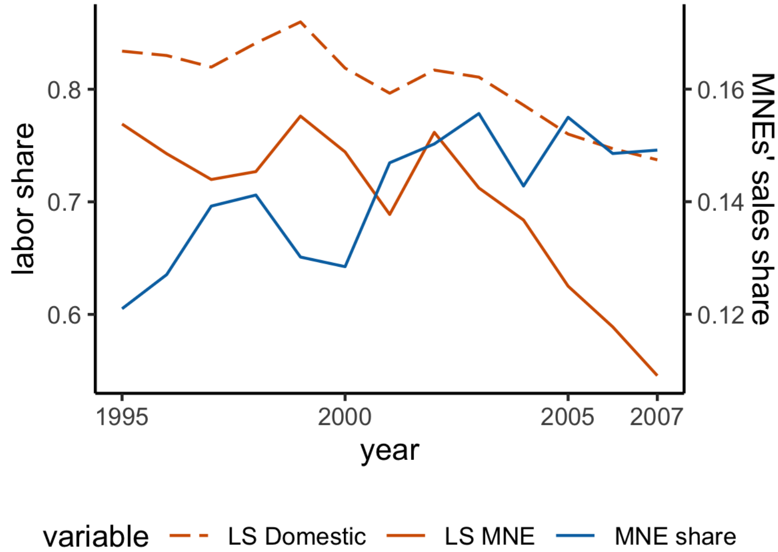


Note: The authors' calculation based on the Basic Survey on Overseas Business Activities (BSOBA) 1996-2017, the 2015 Japan Industrial Productivity (JIP) Database, Cabinet Office Long-Run Economic Statistics (COLES, https://www5.cao.go.jp/j-j/wp/wp-je12/h10_data01.html, accessed on May 13, 2019), and Japan SNA, Generation of Income Account, 2009. The left panel shows the trends of the total profit and labor compensation in the foreign subsidiaries of all Japanese MNEs. Foreign subsidiary profit is the sum of current net profit of all subsidiaries of Japanese multinational corporations in BSOBA, and payment to offshore labor is the sum of worker compensation to foreign subsidiaries of all Japanese multinational corporations in BSOBA. The right panel shows labor share trends in Japan from several sources. JIP-based labor share is calculated by the share of nominal labor cost in the nominal value added of JIP market economies. National income-based labor share is the fraction of nominal employee compensation over nominal national income from COLES. Domestic corporate-based labor share is the net labor share of domestic corporate factor income, calculated from the SNA by the fraction of the item "Wages and salaries" over the sum of "Wages and salaries" and "Operating surplus, net."

shows the trends for the five most robot-adopting countries (China, Japan, United States, South Korea, Germany). All countries other than Japan rapidly introduced industrial robots from 1993 to 2016. In particular, China's absorption has been extraordinarily fast since the mid 2000s, when it entered the international trade markets after being granted membership in the WTO following the removal of a number of political and institutional barriers. In contrast, Japan's automation capital stock was declining during this period.

In a related argument, Fort et al. [2018] emphasizes the impact of globalization on country-level labor markets, but qualifies this statement because "providing a definitive accounting of the amount of employment change attributable to either factor is extraordinarily difficult." Given this difficulty, focusing on Japan might offer insight because the analysis is not contaminated by concurrent extensive technological growth.

Figure C.3: Labor Shares of MNEs and Non-MNEs



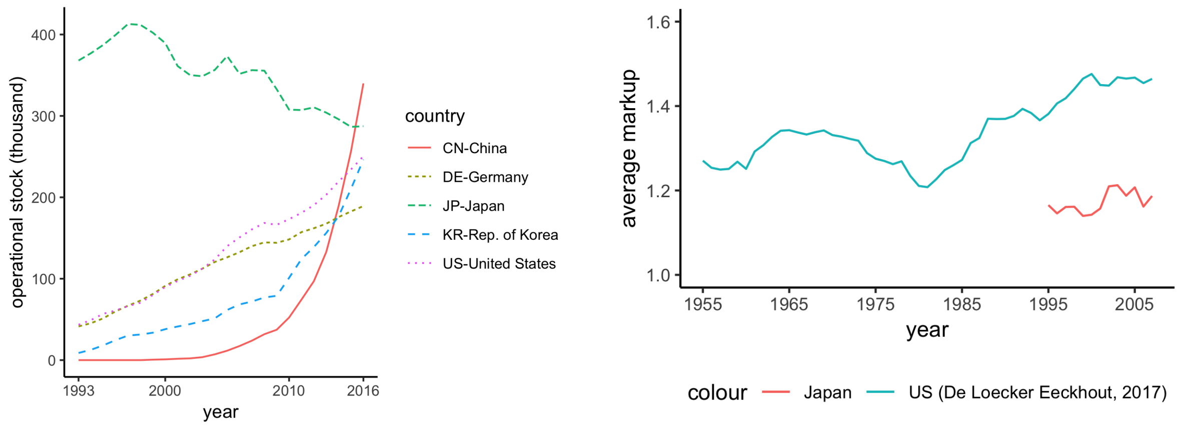
Note: The authors' calculation based on the Basic Survey on Overseas Business Activities (BSOBA) 2007-2016, the Basic Survey on Japanese Business Structure and Activities (BSJBSA) 1996-2017, and the JIP 2015. The red lines indicate labor shares of MNEs (solid line) and non-MNE (domestic, dashed line) firms in Japan. The blue line shows the sales share of MNEs in Japan.Multinational enterprise (MNE) is defined as firms that appeared in BSOBA at least once during 2007-2016. For each firm, labor share is calculated as the fraction of total payroll over the sum of total payroll and current profit from BSJBSA. Share of MNE size is calculated as the fraction of aggregated sales of all multinational firms over the total nominal output from JIP 2015.

A Surge in Market Powers Another potential explanation for the labor share decline is provided by De Loecker and Eeckhout [2017], who argue that it is explained by a surge in market power. They develop a parsimonious but versatile method to back out the markups from the firm- or plant-level data and conclude that the markup in the U.S. has been increasing remarkably since around 1980. Applying this method to our Japanese firm-level data (BSJBSA), we find a much smaller increase in markups relative to the U.S., a finding that is in line with De Loecker and Eeckhout [2018]. The result is shown in Figure C.4b.

Remaining Culprits Other potential explanations for declining labor share include lower capital prices. For example, Hubmer [2018], Karabarbounis and Neiman [2013] explore if and how much a decrease in capital goods prices causes demand for labor to be substituted by demand for capital. Also, since capital income is intrinsically related to the financial sector, some authors discuss the possibility that labor share may have fallen due to financial-

Figure C.4: Other Potential Mechanisms

(a) Industrial Robot Stock, Top 5 Countries in 2016 (b) Markup Estimates for Japan



Note: The left panel shows the trend of operational robot stocks in the largest 5 countries of robot adoption as of 2017, sourced from the International Federation of Robotics. The right panel shows the trend of markups computed by applying the method of De Loecker and Eeckhout [2017] using data from the Basic Survey on Japanese Business Structure and Activities (BSJBSA) 1995-2016. Variable input cost is the sum of labor compensation and intermediate purchases. Output elasticity is estimated by the method of Olley and Pakes [1996] for each JSIC 4-digit industry. The average is calculated using the weight of each firm's sales.

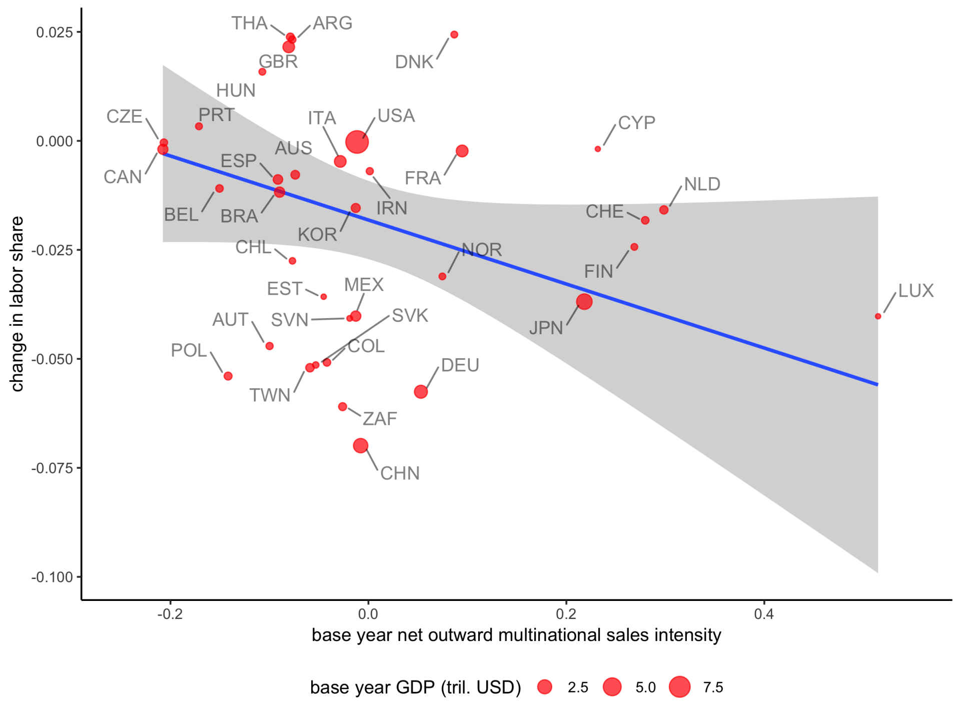
market features such as increasing risk premiums [Caballero et al., 2017] or equity values [Greenwald et al., 2019].

C.1.5 MNEs and Labor Share, Cross Country

This section describes the construction of Figure 3.1 and some further results from it. To see the first pass evidence that multinational activities have an impact on labor share, we take the cross country variation in the change of outward multinational activities and the change in labor share using labor share data from Karabarbounis and Neiman [2013] and assembling data on multinational activities from UNCTAD. We calculate the level and change in outward multinational activities as follows. For the level, we take a snapshot (1996-2000 average) of net outward multinational sales, taking a five-year average to control the noise in the raw data. For the change, we obtain the difference in net outward multinational sales between the 1991-1995 average and the 1996-2000 average.

Figure 3.1 shows the results in the form of a correlation plot. To allow for a lagged response in the labor share to multinational intensity changes, we take the change in the labor share between 1995 and 2007. The left panel shows the results for the level of

Figure C.5: Net Outward Multinational Sales and Labor Share



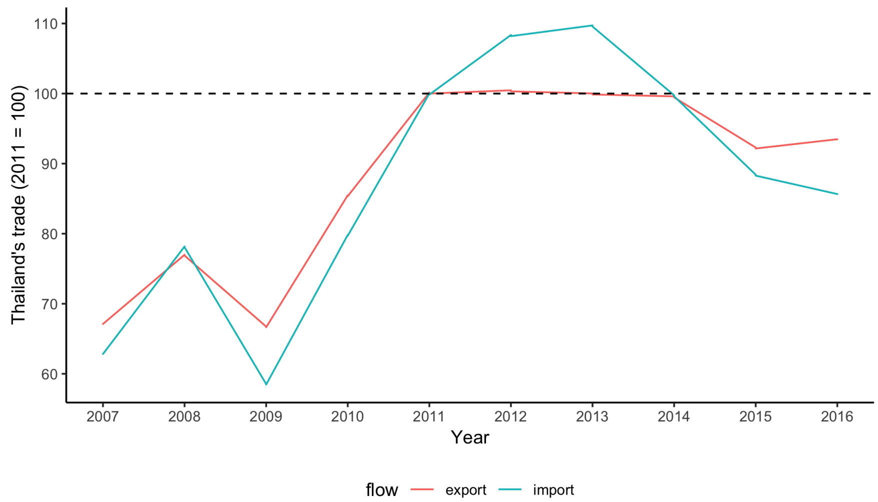
Note: The authors' computation from the data from Karabarbounis and Neiman [2013] and UNCTAD.
Note: The horizontal axis is the 1991-1995 average sum of bilateral net outward multinational sales. The vertical axis is the change in labor share from 1991 to 2000. Singapore was dropped because it had an outlier value for the outward multinational sales measure.

outward multinational sales and the right panel shows the change. We fit the correlation by the weighted regression by country size measured by the base-year GDP. The number of countries in both plots is 36, and even with such a small sample, we see a remarkably significant negative relationship both in the level of multinational intensity and the change. Both regression slope coefficients are significantly negative at the two-sided 95 percent level.

An interpretation of this negative relationship is that the outward multinational activities substitute labor in the home country more than capital. Hence, the demand for labor in the home country decreases more than proportionately to the decrease in the demand for capital. The theory is detailed in Section 3.3.

Figure C.5 shows a plot of the levels of net multinational sales (1991-1995) and the changes in labor share (1991-2000). Again, the countries that have higher multinational sales have relatively larger decreases in labor share in the next 10 years.

Figure C.6: Trend of Thailand's Trade



Note: The authors' computation from the Comtrade data.

C.1.6 Thailand's Gross Export and Import Trends

Figure C.6 shows the trend in Thailand's exports and imports using data from Comtrade. Recalling that the floods occurred in 2011, exports and imports show a roughly parallel trend before the floods, but after 2011, export growth falters and the trend breaks from that of imports. This is consistent with our interpretation that the flooding hit the supply-side of the economy heavily, given that several large-scale manufacturing industrial parks were inundated. Also, this is consistent with Benguria and Taylor [2019], who present a method for identifying demand and supply shocks from gross export and import data in the context of financial crises. They claim that "firm deleveraging shocks are mainly supply shocks and contract exports," while leaving imports largely unchanged.

Next, we provide an overview of Thailand's economic policies. Thailand began international liberalization ahead of other Southeast Asian countries, being one of the original member countries of the *Association of Southeast Asian Nations* (ASEAN) and entering GATT in 1982. In the early 2000's, it formed FTAs with several large economies (India in 2003, the U.S. in 2004, Australia and Japan in 2005). This is in addition to some major internal and external FTAs made by ASEAN. The internal FTA among member countries went into effect in 1993, and by 2003, internal tariffs had been driven down to below five percent. External FTAs with other large economies include one with China in 2003. Given this history, over the period of our analysis from 2007 to 2016, we do not see a large degree of institutional internationalization, as much of this had occurred beforehand.[1] The gross trade trends shown in Figure C.6 are consistent with this fact, as the drivers behind the changes in trade trends were external business cycles (e.g., the global great recession after 2008) or political upheaval (e.g., a coup d'etat in 2014) rather than large trade policy changes.

C.1.7 Discussion of Data Sources

Country-level Analysis from BSOBA

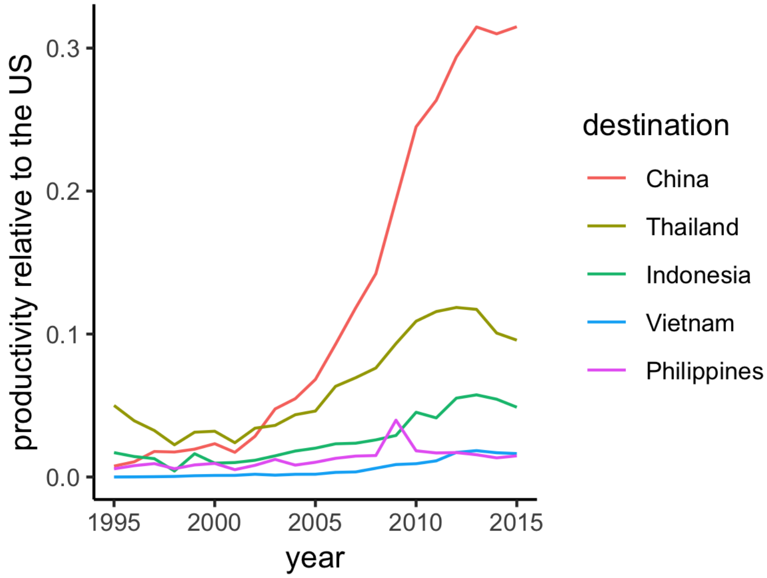
Throughout the main sections, we maintained the assumption of an aggregated rest of the world (ROW). Although this simplifies the analysis greatly, there are at least two reasons why we believe this may not be consistent with empirical findings. First, ample empirical evidence suggests that the motivation for MNE investment is different for high-income and low-income countries (see, e.g., Harrison and McMillan, 2011). Second, to the extent that our natural experiment involves a local natural disaster, a model in which the factors in the affected area can be separated from factors in other regions would be more realistic. To move in this direction, this section discusses several country-level data. By doing so, we provide the motivation for the future development of multi-country analyses.

Our major data source for MNEs is BSOBA which, as described in Section 3.2.1, contains the universe of offshore plants owned by headquarter firms located and registered in Japan, covering both private and public firms. As each plant-level observation has the country-level location, total employment and total labor compensation, we may aggregate the country-level average wage and labor productivity measure. These variables are analyzed in detail below.

Productivity Growth in Each Destination Country Applying the model inversion (3.25) for each destination country, Figure C.7 shows the results for Japan's five most intensive MNE destination countries measured by size of total employment, taking the U.S. as the base country in equation (3.25). Therefore, observed productivity growth relative to the base country shows the relative augmentation of factors in these developing economies.

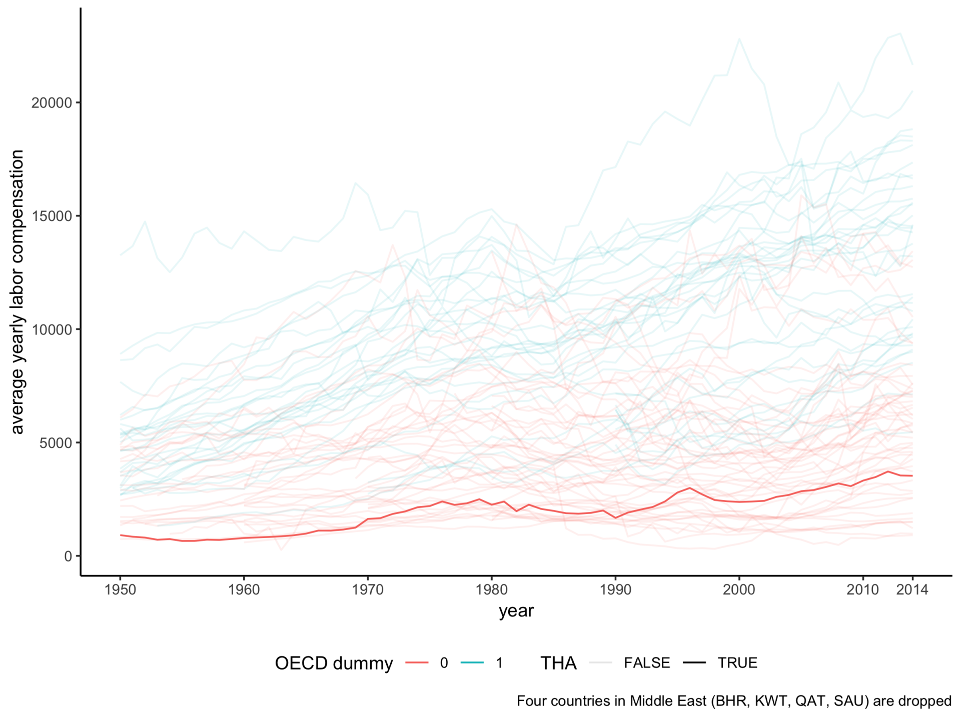
[1] Several exceptions include the ASEAN-South Korea FTA that reduced the tariff between South Korea and Thailand in 2010 and the Chile-Thailand FTA that went into effect in 2015.

Figure C.7: Country-level Measured Productivity



Broader Country-level Wage Trends–Evidence from PWT Figure C.8 shows country-level wages generated from *Penn World Table* (PWT) data. Each line indicates the country-level average wage, where wage is calculated as per-capita average labor compensation. To highlight the differences between developed and developing countries, blue lines indicate OECD countries and red lines non-OECD countries. Since our empirical application focuses on Thailand, it is highlighted in bold. Several points can be taken from the figure. First, there is significant wage variation across countries. In particular, OECD countries on average have historically paid higher wages than their non-OECD counterparts. This in itself might indicate that the reasons behind multinational activities differ, which would make a theory based on multiple countries more realistic. For example, Harrison and McMillan [2011] point out that cost changes in high-income and low-income countries result in different effects in home-country employment, so future research should take this into consideration. Second, average wages have grown in many countries, in both OECD and non-OECD countries alike. This may make it more difficult to derive conclusions about the desirability for multinational firms to hire foreign workers based on differences in labor costs. Our approach is therefore to invert the factor demand to obtain the implied factor-augmenting productivity shocks, as detailed in Section 3.5.1. Third, our natural experiment shock in Thailand in 2011 did not appear to change the average wage drastically.

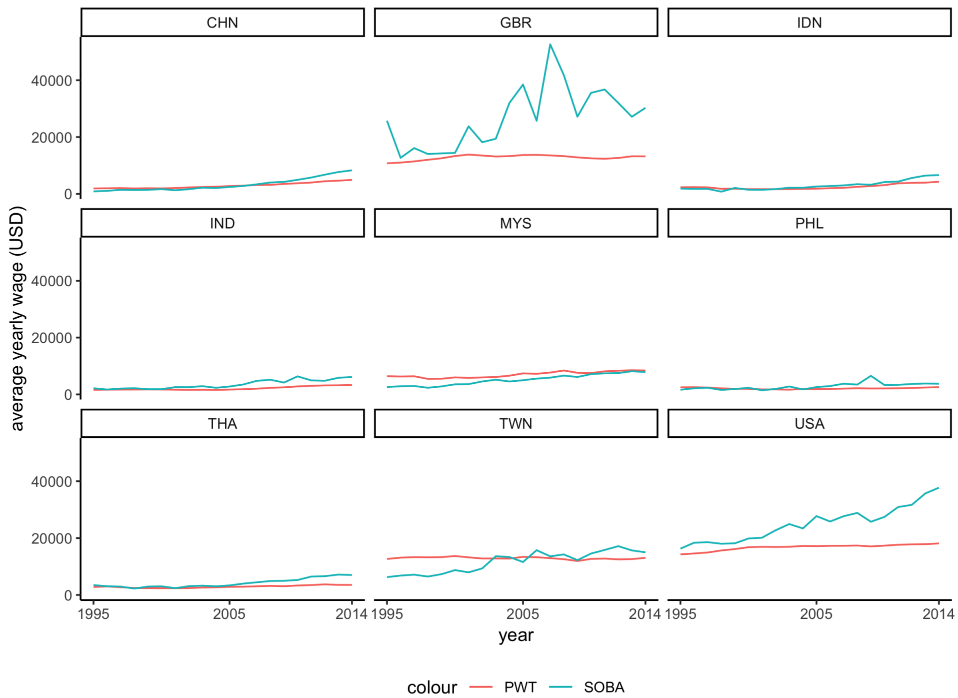
Figure C.8: Offshore Labor Cost in Thailand and Other Countries



This might be due to the fact that the flood shock was local and short-lived, which might not be well-captured in coarse country-year aggregate data. We thus employ microdata set out in Section 3.2.1 to study the exogenous negative foreign factor productivity shock.

Comparison of BSOBA and PWT In this section, we check the differences in the aggregate (average) wage measures from PWT and BSOBA, our primary source of data on multinational production. Note that PWT aggregate wage is calculated from the total labor cost and total employment in each country. Thus, a wage difference emerges if Japanese-parented subsidiaries hire a different type of worker than the typical firm in each country. Figure C.9 shows the comparison of BSOBA and PWT for a selected set of nine countries chosen by their ranking in terms of total employment by Japanese subsidiaries at the end of FY2015. From the Figure, one can see that BSOBA and PWT show a similar trend for each country overall. This suggests that Japanese subsidiary firms hire workers from a similar labor market as other firms in each country. However, there are several interesting deviations from this pattern, particularly in high-income countries such as the U.K. and the U.S. This might reflect the fact that the subsidiaries in these countries focus

Figure C.9: Comparison of BSOBA and PWT



on high-value added activities such as finance, which might cause the hiring structure of Japanese subsidiaries there to be different from that of other firms. We also show in Table C.1 the results of a regression of the PWT wage on the BSOBA wage with and without fixed effects to confirm that the goodness of fit is remarkably high for cross-section-cross-year data.

Table C.1: Discrepancies between SOBA and PWT

	All	All	Top 9	Top 9
	(1)	(2)	(3)	(4)
PR	0.332	0.038	0.772	0.409
	(0.014)	(0.007)	(0.028)	(0.045)
Country FE		YES		YES
Year FE		YES		YES
Observations	1,350	1,350	180	180
R^2	0.300	0.950	0.805	0.983

Data-linking strategy

We match the BSJBSA and BSOBA datasets in the following way. First, for each firm, we pick up from both BSOBA and TSR the firm name, the headquarter address and phone number for each year. To match the datasets using this information, we first need to deal with a problem of spelling variation. Due to language translation, there are several potential spelling variations that refer to the same company or address, so we prepare a spelling variation table and applied it to the data to obtain a variation-proof dataset.[2] In addition, to further improve the precision of the addresses, we geocode the written address to the layered system of all Japanese addresses using the CSV Geocoding Service offered by the University of Tokyo Center for Spatial Information Science.[3] The layers are defined as follows. 1 and 2: "todofuken" (prefecture), 3: city, 4: district in large cities, 5 and 6: coarse street address, and 7: fine street address.

Using the obtained data for each year, we conduct the following matching. To break the potential multiple matches within and across years, for each match in each year we assign a match score that measures the quality of the match. The match score is defined as follows: a firm name match receive a score of 1,000, a firm phone number match receive 100, a firm address match before geocoding receive 10, and a firm address match after geocoding at the layer of $l = 2, \ldots, 7$ received l.[4] We considered the match successful if the match score is strictly larger than 1,000. In other words, this means that we require the following two criteria. First, the firm names have to match between the two datasets up to any spelling variation. Second, either the address or the phone number have to match. If there are multiple successful matches, we pick the one with the highest match score. Then we compare the matching results across years and use the results with the highest score. This procedure results in a match rate of 93.0% from all firm IDs in BSOBA to firm IDs in TSR.

We match BSJBSA and TSR data by a similar method except that we have BSJBSA data from the year of 1995. The resulting match rate for all firm IDs in BSJBSA in year 2007-2016 is 88.9%. Figure C.10 provides a schematic diagram that summarizes the data-linking strategy set out above.

[2] The spelling variation table is available upon request.

[3] http://newspat.csis.u-tokyo.ac.jp/geocode/ (in Japanese, accessed on July 27, 2018)

[4] We did not assign any points for the match at the level of todofuken because it is too coarse.

Figure C.10: Schematic Data-linking Strategy

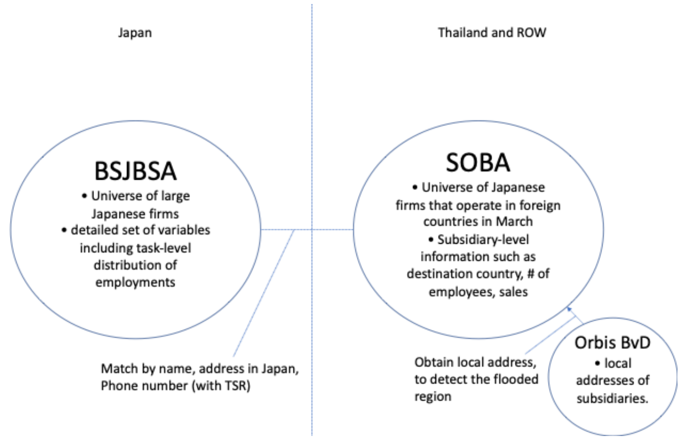
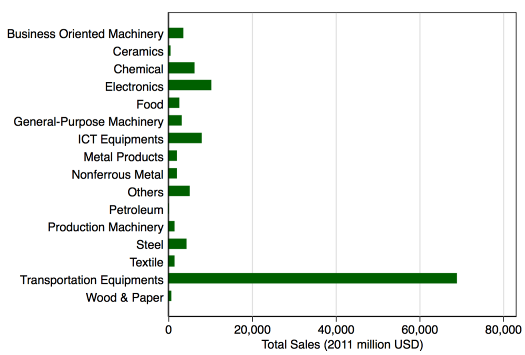
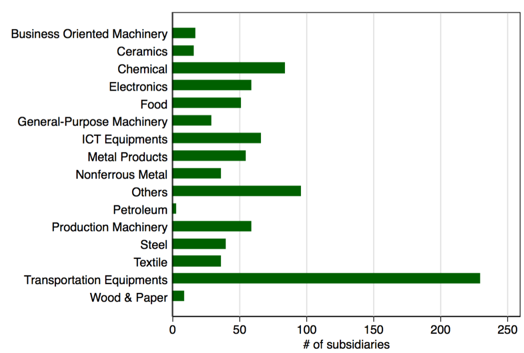


Figure C.11: Industry Distribution of the Treated Subsidiaries of Japanese firms, 2011

(a) Total Sales



(b) Number of Subsidiaries



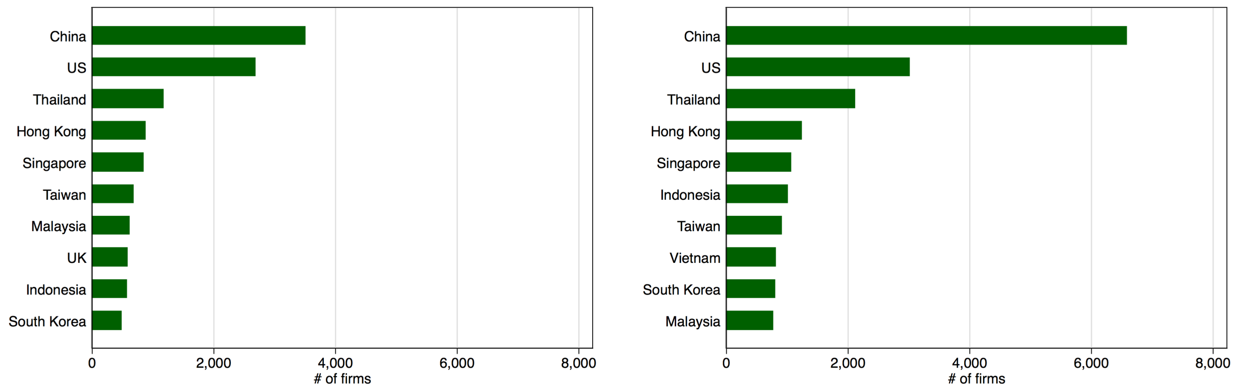
Overview of Japanese Subsidiaries in Thailand

Using the datasets described above, we here show statistics about production in the flooded region. First, to understand the industry clustering patterns in detail, Figure C.11 shows the industry distribution of Japanese subsidiaries in the flooded region in Thailand by sales in 2011. As mentioned earlier, most of the subsidiaries in the flooded region produced Transportation Equipment, including automobiles, whether measured by total sales or number of local subsidiaries. The second and third largest sectors were Electronics and ICT Equipment when measured by total sales. Transportation Equipment dominates other industries in terms of total sales because the unit value is high, but the difference is slightly less dramatic in terms of the number of subsidiaries.

Figure C.12: Top 10 Countries in which Japanese Firms Have Subsidiaries

(a) 2007

(b) 2016



Destination Countries of Japanese MNEs

To further support our empirical analysis, we discuss here the strong relationship with respect to foreign direct investment (FDI) from Japan to Thailand. In 2011, Japan was the largest country investing in Thailand, while Thailand was the third largest destination country for Japanese FDI. Therefore, the flooding in Thailand was not only a local shock but also had a non-negligible impact on the Japanese MNEs and their employment as well.

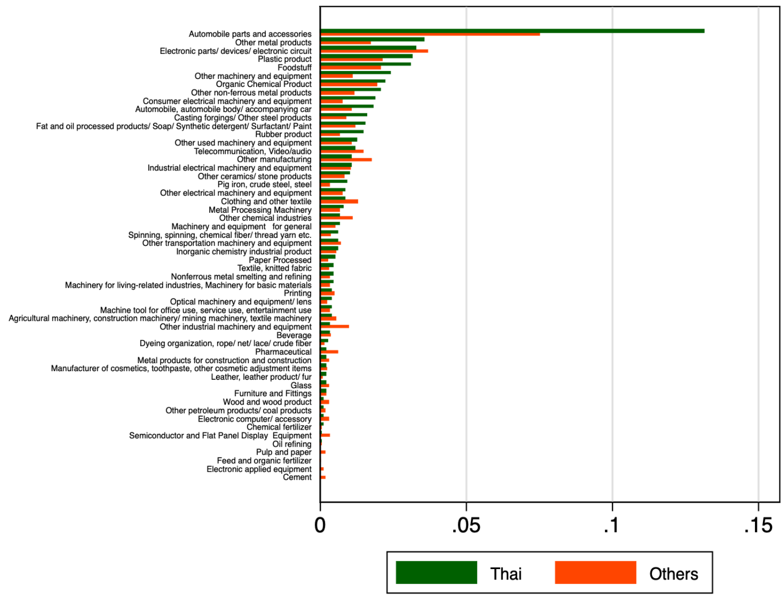
To confirm the importance of the Japan-Thailand relationship, Figure C.12 shows the top 10 countries in which Japanese firms had subsidiaries in 2007 and 2016. During these 10 years, the ranking of the top five countries did not change (China, U.S., Thailand, Hong Kong, and Singapore), which indicates the stable economic relationship between these countries and Japanese firms. The number of Japanese firms with links to Thailand actually grew during this period.

Balancing Analysis

To check the similarity between the firms that experienced and did not experience flooding, we conduct several balancing tests. Figure C.13 shows the industry distribution of subsidiary manufacturing firms for the group of parent firms that invested in 2011 in the flooded region in Thailand (labeled Thai) and other regions of Thailand and the ROW (labeled Others).

Qualitatively, industries that had many firms in the flooded region of Thailand are likely to also have had a higher number of firms in other countries, but the flooded region had

Figure C.13: Industry Distributions, Flooded Region of Thailand vs Other Regions



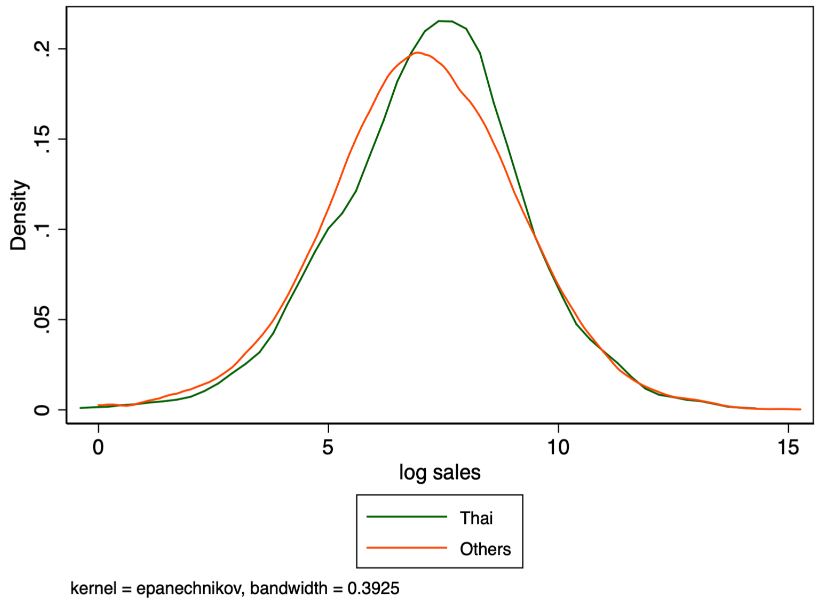
relatively more "Automobile parts and accessories" industry firms relative to other countries.

Figure C.14 shows the estimated distribution of the size of subsidiary firms of the parent firms that invested in the flooded region of Thailand versus other regions. We measure the size by log sales in 2011, and the estimation is a kernel density estimation with an Epanechinikov kernel. At the top of the distribution, the densities of the flooded group (labeled Thai) and the other group (labeled Others) are similar, while at the bottom of the distribution, the Thailand group dominates the Others group. The Kolmogorov-Smirnov test rejects the null hypothesis that the distribution of the two groups is the same.

Other Trends

Complementing the relative aggregate trend analysis in Section 3.4.1, the left and right panels of Figure C.15 show the trends in investment and sales. The red vertical line indicates 2011, the year of the floods. Interestingly, the investment trend in the flooded region and the rest of the world follows a parallel path before the flood, but the trend breaks sharply after the flood. Intuitively, this is reasonable because plants in the damaged area would have needed to substantially increase investment to repair flood damage. On the

Figure C.14: Sales Distributions, Flooded Region of Thailand vs Others



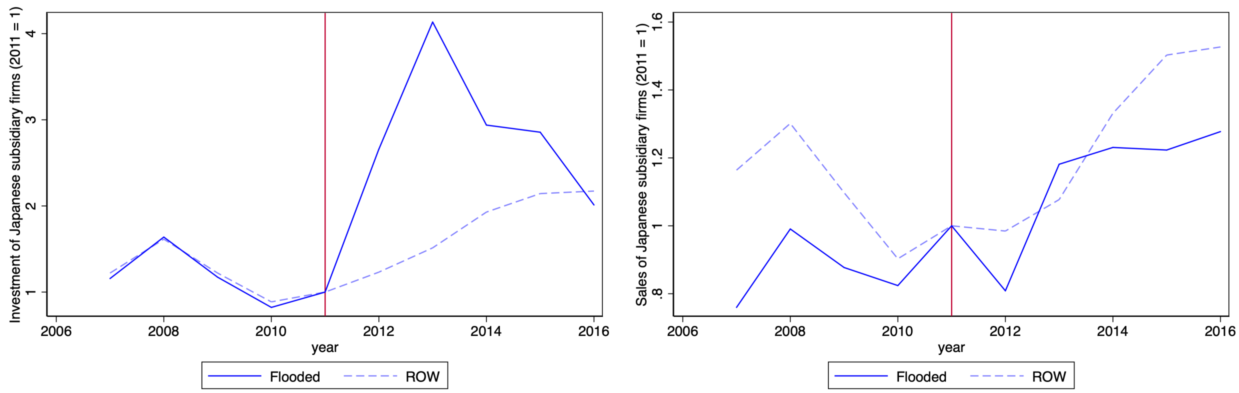
other hand, the sales trend in the right panel does not show a parallel pattern even before the flood.

Choice of Treatment Groups

As described in Section 3.4.1, the flood severely affected *Ayutthaya* and *Pathum Thani* provinces. However, it is important to acknowledge these particular provinces because Thailand overall was relatively unaffected in terms of employment or number of subsidiaries of Japanese MNEs after the flood, as seen in Figure C.16, which shows the trends in employment and number of Japanese subsidiaries inside and outside of Thailand instead of only in the flooded provinces. One can see that the impact on total Thai employment and number of subsidiaries is not as stark as when we compare the flooded provinces versus the rest of the world in Figure 3.3. Therefore, in our main analysis, we take these two heavily flooded provinces as the shocked regions and construct the IV based on that idea. Note again that, for this purpose, it is critical to link our BSOBA data, which only contains the country information of each Japanese MNE overseas factory, to Orbis BvD that contains specific factory addresses.

Figure C.15: Relative Trends in Investment and Sales of Japanese MNEs

(a) Investment (b) Sales



C.1.8 Calibration Details

Estimation of σ

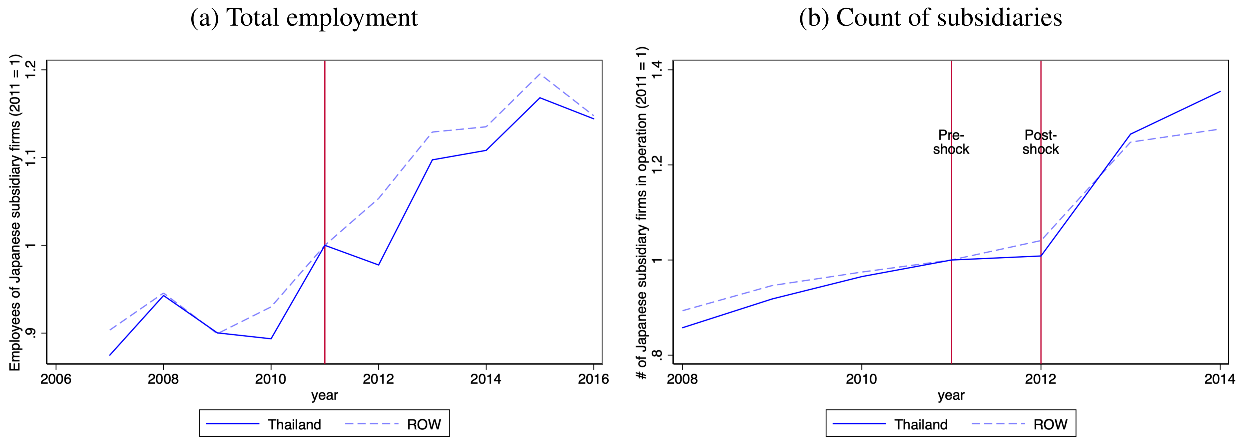
Following recent developments in estimating capital-labor elasticities [Oberfield and Raval, 2014, Raval, 2019], we estimate σ using the regression equation (3.20). Our coefficient of interest is b_1 since $\sigma = b_1 + 1$.

To obtain the factor payment ratio $(rk/wl)_i$, we use the annual *Census of Manufacture* (CoM) survey. For our purposes, we use the initial stock of tangible assets in the next year survey. To obtain the total payment to workers, we use the variable total payroll for all workers, which includes both full-time and part-time workers. Since we can obtain the rental rate of capital at the industry level, it drops with the industry-fixed effect in specification (3.20). Finally, CoM also offers municipality, 4-digit industry, and multiplant status variables. The multiplant status includes three values: no other plants or headquarter office; no other plant but a headquarter office; has other offices. We include the fixed effect for all of these values in specification (3.20).

For the local wage, we use the municipality-level wage taken from the long-run economic database of the Japan Cabinet Office. The long-run trend data offers the taxpayer-per-capita taxable income from 1975 to 2013.[5] The municipality unit is as of the last day of April 2014. We convert the municipality code in each analysis year to the one as of April 2014

[5]The primary data source of this dataset is the *Survey of Municipality Taxation* administered by Ministry of Internal Affairs and Communications, Japan.

Figure C.16: Trends in Employment and Japanese MNE Subsidiaries, Thailand versus ROW



using *Municipality Map Maker* [Kirimura et al., 2011].

To control the endogeneity in equation (3.20), we use a shift-share instrument [Bartik, 1991, Goldsmith-Pinkham et al., 2018]. Specifically, estimation equation (3.20) may be biased with the existence of labor-augmenting productivity shocks to locality $m(i)$. If $m(i)$ receives a positive shock, then $w_{m(i)}$ increases and $(rK/wL)_i$ decreases, so β would be negatively biased. To obtain the exogenous shifter that changes $w_{m(i)}$ but not $(rK/wL)_i$ without the influence through $w_{m(i)}$, we take the average of national growth in employment weighted by the base-year employment share of non-manufacturing industries. In particular, from the Employment Status Survey (ESS), we take the ten year growth in employment in industry n as $g_{n,t} = \ln\left(L_{n,t}/L_{n,t-10}\right)/10$. We then take the base-year industry-n share of employment $\omega_{m,n,t-10}$ in municipality m, and calculate our shift-share instrument by

$$z_{j,t} = \sum_n \omega_{m,n,t-10} g_{n,t}.$$

Table C.2 shows the results of regression (3.20) for 1997 since it is the nearest year to that of the ESS survey.

Table C.2: Estimates of $\sigma - 1$

	OLS, CO	OLS, BSWS, all	OLS, BSWS, manuf.	IV, CO	IV, BSWS, all	IV, BSWS, manuf.
$\log(w_{m(i)})$	-0.60	-0.20	-0.13	-1.15	-1.24	-0.88
	(0.05)	(0.04)	(0.03)	(0.18)	(0.18)	(0.13)
Num. obs.	51477	51477	51477	51477	51477	51477

CO indicates that the wage data is from the Cabinet Office. BSWS indicates that the wage data is from the Basic Survey of Wage Structures. "BSWS, all" indicates that the wage variable is taken from all industries, while "BSWS, manuf." indicates that the wage variable is taken from manufacturing industries. See the text for detail. All regressions include industry FE and multiunit status indicator. Standard errors are clustered at municipality level.

Comparison to the Literature The central estimate of Oberfield and Raval [2014] was around 0.7. However, this value includes industry-level heterogeneity in substitutability and the caused reallocation mechanism. Oberfield and Raval [2014] reports that plant-level elasticity estimates range from 0.4-0.7 depending on the industry, which is closer to our value. Incorporating heterogeneity into the model is our high-priority next research step.

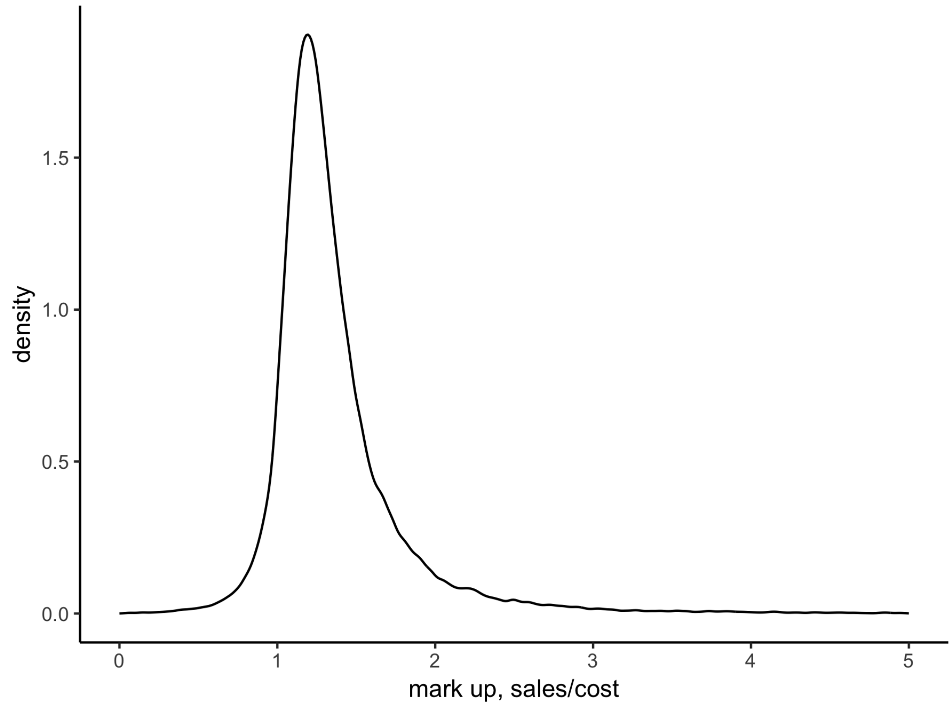
Moreover, the capital-labor substitution parameters at micro and macro-level estimates sometimes disagree. In fact, although microdata-based findings by ourselves and Oberfield and Raval [2014] both point to gross-complementary capital and labor, the macro estimates often indicate that they are gross substitutes [Hubmer, 2018, Karabarbounis and Neiman, 2013]. Note that these macro estimates typically rely on U.S. data. In Japan, even at the macro level, Hirakata and Koike [2018] show that the capital-labor elasticity is below one, which is qualitatively consistent with our findings.

Detailed Results regarding ε and CS^M

Figure C.17 shows the distribution of measured markups, which exhibits a spike at a value slightly larger than one and a longer tail on the right side than the left. Our estimate $\varepsilon = 4$ is based on the peak value of measured markup $m = 4/3$, which is standard in the literature [Oberfield and Raval, 2014].

To calculate CS^M at the firm level, global total cost is calculated as the sum of domestic cost and multinational cost. The domestic cost is the sum of the following items: advertising expense, information processing communication cost, mobile real estate rent, packing and transportation costs, total payroll, depreciation expense, welfare expense, taxes, interest expense, and lease payments. The international cost is the sum of each subsidiary's total costs, which is its total sales minus the total purchase of intermediate goods.

Figure C.17: Distribution of Measured Markup m



Note: The authors' computation from Census of Manufacture, 2011. Estimates are obtained by inverting the markup, following Oberfield and Raval (2014). The markup is defined as sales divided by the sum of costs from capital, labor, and materials.

C.1.9 Delta Method for $se\left(\widehat{\lambda}\right)$

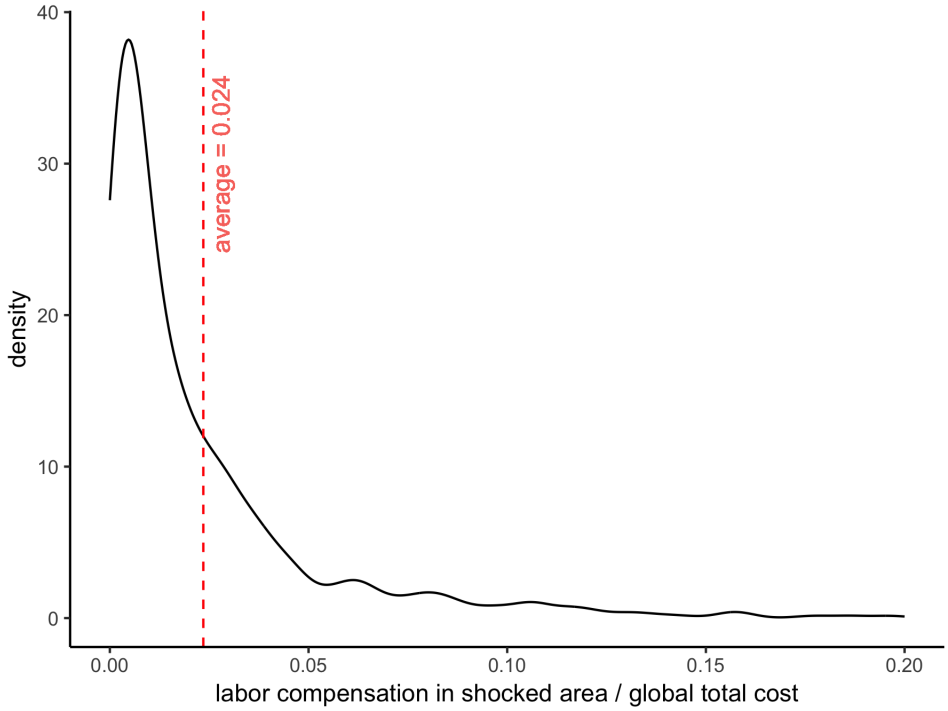
This section derives the standard error estimate of our estimator of λ obtained by

$$
\begin{aligned}
\widehat{b_{IV}} &= \frac{\left(\sigma - \widehat{\lambda}\right) WS^F + (\varepsilon - \sigma)\, CS^F}{\widehat{\lambda} - 1 + \left(\sigma - \widehat{\lambda}\right) WS^F + (\varepsilon - \sigma)\, CS^F} \\
\Leftrightarrow \widehat{\lambda} &= \frac{\widehat{b_{IV}}\left(1 - \sigma WS^F + (\sigma - \varepsilon)\, CS^F\right) + \sigma WS^F - (\sigma - \varepsilon)\, CS^F}{\widehat{b_{IV}}\left(1 - WS^F\right) + WS^F}.
\end{aligned}
$$

Recall that a standard argument holds for our standard two-stage least square estimate $\widehat{b}$. Hence, it satisfies $\sqrt{n}\left(\widehat{b} - b_0\right) \rightarrow_d N(0, \Sigma)$ so, by the delta method, we have

$$
\sqrt{n}\left(\widehat{\lambda} - \lambda_0\right) \rightarrow_d N\left(0, \left(\frac{\varepsilon s_0^F}{b_0^2}\right)^2 \Sigma\right).
$$

Figure C.18: Share of Foreign Labor Cost in Total Cost CS^M



Note: Global total cost is calculated by the sum of domestic cost and multinational cost, where domestic cost is the sum of the following items: advertising expense, information processing communication cost, mobile real estate rent, packing and transportation costs, total payroll, depreciation expense, welfare expense, taxes, interest expense, and lease payments. The international cost is the sum of each subsidiary's total cost, which is total sales minus total purchases of intermediate goods.

Thus, in our case, $se\left(\widehat{\lambda}\right) = \left(\varepsilon s_0^F/\widehat{b}^2\right)\sqrt{\widehat{\Sigma}} = \left(4 \times 0.024/(0.19)^2\right) \times 0.05 \approx 0.13$. Given our point estimate $\widehat{\lambda} = 1.4$, we can test if $H_0 : \lambda \leq 1$; namely, that home labor and foreign labor are gross complements. As the standard t-value is $t = 0.40/0.13 \approx 3.08.$, we reject H_0 at the 0.1 percent level of significance.

C.1.10 Robustness Checks

Alternative Extensive-Margin Instrument

We consider the following instrumental variable:

$$Z_{it}^{EXT} = \mathbf{1}\left\{L_{i,2011}^{treated} > 0 \cap t \geq 2012\right\}. \tag{C.1}$$

Table C.3: Extensive Margin Estimates

VARIABLES	(1) $\ln l_{it}^{JPN}$	(2) $\ln l_{it}^{JPN}$	(3) $\ln l_{it}^{JPN}$	(4) $\ln l_{it}^{ROW}$	(5) $\ln l_{it}^{JPN}$
$\ln l_{ft}^{ROW}$	0.284	0.0271	0.212		
	(0.00394)	(0.00435)	(0.0921)		
Z_{ft}				-0.321	-0.0681
				(0.0854)	(0.0291)
Observations	22,795	22,795	22,795	22,795	22,795
Model	OLS	FE	2SLS	2SLS-1st	2SLS-reduced
Firm FE	-	YES	YES	YES	YES
Year FE	-	YES	YES	YES	YES

Robust standard errors in parentheses.

The idea is to take the extensive margin of the shock, meaning that a firm located in the flooded region changes employment relative to those in other regions. Table C.3 shows the results and, notably, the estimate for $\widehat{b}_{IV}$ in column 3 here (0.212) is remarkably similar to the 0.192 in column 3 of Table 3.2. As there is no statistical difference between these two estimates, our result is robust to the choice of IV.

It is also worth noting the reasons underlying the similar estimates. In columns 4 and 5 of Table C.3, we find significantly smaller first stage and reduced-form estimates than the corresponding values in Table 3.2. This is because we define the shock Z_{it}^{EXT} at the extensive margin taking the binary value of zero or one, which has a larger variance than $Z_{it} \in [0, 1]$ in equation (3.22), and this larger variance in the regressor results in smaller estimates in columns 4 and 5. This signifies our choice of target reduced-form parameter (3.18); since we do not know the exact size of the flood shock, we do not have a precise measure of its size. Therefore, arbitrary definitions of the shocks such as (3.22) or (C.1) would result in quantitatively different estimates of equations (3.15) and (3.16), recalling that they are *proportionally* corresponding to columns 4 and 5 up to the choice of the shock measure or IV. Our choice of the target parameter (3.18), however, does not depend on such a choice. Specifically, as can be seen in the 2SLS formula, the variance of the instrument does not affect the estimator, but the *relative* covariance of the regressand and regressor does. Therefore, our 2SLS produces stable and robust estimates.

Different Control Groups

In our main specification, we consider only MNEs in our sample, a choice that is justified because MNE and non-MNE firms differ substantially, and also because the inclusion of fixed effects in regression equations (3.23) and (3.24) makes the variation in the IV for firms that are never MNEs irrelevant since then $Z_{it} = 0$ for any t by definition (3.22), which is absorbed by the fixed effect. However, to ensure that the home employment trend did not differ significantly between MNEs and non-MNEs, Table C.4 shows the reduced form of 2SLS (3.23) and (3.24) with different control groups. Formally, the specification is

$$\ln\left(l_{it}^{JPN}\right) = a_i^{robustness} + a_t^{robustness} + b^{robustness} Z_{it} + e_{it}^{robustness},$$

with different samples and different IV definitions.

Columns 1-4 show the results including all firms in Japan. Since our data is an unbalanced panel, we select firms that are observed throughout the period 2007-2016 to construct a balanced panel, and columns 5-8 show these results. Columns 1 and 2 show the results based on the IV Z_{it}^{EXT} defined in equation (C.1) (labeled "Extensive"), and columns 3 and 4 show Z_{it} in equation (3.22) (labeled "Intensive"). Note that both of these IVs leverage the shock induced by the 2011 Thailand Floods, though the precise definition differs. Column 1 defines the flooded region as Ayutthaya and Pathum Thani provinces, which is our preferred definition (labeled "Flooded"), while column 2 defines the flooded area as all of Thailand (labeled "Thailand"). The other columns show the results based on the specifications following this basic structure. Overall, we find a robust result irrespective of the choice of sample and IV definition; that is, the flood-affected firms *reduced* employment in Japan.

Table C.4: Different Reduced Form Specifications

VARIABLES	(1) Extensive Flooded	(2) Extensive Thailand	(3) Intensive Flooded	(4) Intensive Thailand	(5) Extensive Flooded	(6) Extensive Thailand	(7) Intensive Flooded	(8) Intensive Thailand
shock	-0.0497	-0.0159	-0.172	-0.127	-0.0490	-0.00747	-0.249	-0.101
	(0.0126)	(0.00585)	(0.0667)	(0.0242)	(0.0139)	(0.00612)	(0.0774)	(0.0248)
Observations	185,703	185,703	185,703	185,703	91,690	91,690	91,690	91,690
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Balanced panel?					YES	YES	YES	YES

Robust standard errors in parentheses.

Indeed, it is crucial to have the IV defined according to the flood-induced variation, despite minor differences in definitions between equations (3.22) and (C.1). In Section C.1.11, we see that another widely-used IV definition, the Bartik instrument, does not work in our context, which highlights our choice based on the natural experiment and its power of identification.

The 2011 Tohoku Earthquake

As another check of the robustness of our results, we now consider the potential impact on our results of the 2011 Tohoku Earthquake, another severe natural disaster that affected Japanese industry that year. It's impact on the country was so profound that it remains in the minds of Japanese today almost a decade later. Carvalho et al. [2016] summarizes the event as follows: "On March 11, 2011, a magnitude 9.0 earthquake occurred off the northeast coast of Japan. This was the largest earthquake in the history of Japan and the fifth largest in the world since 1900. The earthquake brought a three-fold impact on the residents of northeast Japan: (i) the main earthquake and its aftershocks, directly responsible for much of the material damage that ensued; (ii) the resulting tsunami, which flooded 561 square kilometers of the northeast coastline; and (iii) the failure of the Fukushima Dai-ichi Nuclear Power Plant that led to the evacuation of 99,000 residents of the Fukushima prefecture."

Since our data is annual and the flooding in Thailand began about four months after the earthquake in Japan, we want to ensure that our results using the Thai Floods as a natural experiment are not qualitatively affected by the co-ocurrence of the Tohoku earthquake that same year. There are two ways to address this concern. First, our specifications (3.23) and (3.24) include the fixed effects of firms, which means that we are leveraging the variation within a firm across years, not the differences between firms that may or may not have experienced the earthquake. Second, in order to further mitigate any concern that the existence of flooded firms in the main sample still biases the fixed effect estimator, we conduct the following robustness check exercise. We drop firms located in the four most severely hit prefectures in Japan–Aomori, Iwate, Miyagi, and Fukushima (called "damaged prefectures" below). These damaged prefectures include 36 municipalities that were designated as disaster areas by the Ministry of Land, Infrastructure, Transport and Tourism (MLIT) of Japan after the earthquake, and this is also used by Carvalho et al.

Table C.5: Specification Without Earthquake-hit Firms

VARIABLES	(1) $\ln l_{it}^{JPN}$	(2) $\ln l_{it}^{JPN}$	(3) $\ln l_{it}^{JPN}$	(4) $\ln l_{it}^{ROW}$	(5) $\ln l_{it}^{JPN}$
$\ln l_{it}^{ROW}$	0.448	0.0601	0.192		
	(0.00684)	(0.0107)	(0.0502)		
Z_{it}				-0.730	-0.140
				(0.108)	(0.0368)
Observations	5,551	5,551	5,551	5,551	5,551
Model	OLS	FE	2SLS	2SLS-1st	2SLS-reduced
Firm FE	-	YES	YES	YES	YES
Year FE	-	YES	YES	YES	YES

Robust standard errors in parentheses.

[2016], which an interested reader may refer to for the rationale.[6]

Table C.5 shows the results of the 2SLS regression based on equations (3.23) and (3.24) for the sample that omits firms in the damaged prefectures. We do not find any statistically significant difference in the estimates between the two samples, which is expected given the similar samples considered. Further, because the 2011 Tohoku Earthquake hit the more rural northeast regions of Japan most severely while firms that intensively engage in FDI and multinational activities are skewed to large cities such as the Tokyo and Osaka metropolitan areas, dropping the firms that suffered from the earthquake did not substantially alter our original estimation sample of multinational firms. Consequently, we find similar estimates in Tables 3.2 and C.5, which indicates that the effect of the 2011 Tohoku Earthquake on our estimate is limited at most.

Other Measures of Foreign Factors

In our main empirical specification, we use foreign labor employment as the measure of the foreign factor due to the data limitation that other factor employment quantities are difficult

[6]Although the propagation of the shock due to the input-output linkages makes it not trivial to measure the exact impact of the earthquake on each firm, we view our choice as a conservative test for the existence of an earthquake effect on our estimator. Namely, since the firms located in the defined four prefectures suffered most from the earthquake, if there are any confounding effects of the earthquake on our estimator, dropping such firms should substantially alter the estimate. Thus, if we find no difference between our full sample and the sample omitting these four prefectures, this indicates that any potential earthquake effects are not significant.

to measure and not readily available. However, our model shows that the foreign factor is more general than just labor employment. For example, the foreign factor may contain foreign capital and land that produce additional value added to the output of the MNE from country H. To capture this feature of the model, we now consider the value added measure by subtracting the value of all intermediate good purchases from the total sales of each subsidiary. We then aggregate these subsidiary-level sales for each MNE to construct the MNE-level foreign value added measure VA_{it}^{ROW}. Then we conduct the regression with specification

$$\ln\left(l_{it}^{JPN}\right) = a_i^{VA} + a_t^{VA} + b^{VA}\ln\left(VA_{it}^{ROW}\right) + e_{it}^{VA}. \tag{C.2}$$

As an additional check, we also substitute a raw sales measure for the value added measure. Here, we construct the flood shock-based IV as in the main text, but with our value added measure instead:

$$Z_{it}^{VA} = \frac{VA_{i,2011}^{flooded}}{VA_{i,2011}^{JPN} + VA_{i,2011}^{ROW}} \times \mathbf{1}\{t \geq 2012\}. \tag{C.3}$$

Since both IVs (3.22) and (C.3) satisfy the standard requirements for IVs under our maintained assumption that the flood was an unexpected augmentation shock to the MNEs located in the damaged region, we conduct the robustness exercise based on both definitions.

Tables C.6 and C.7 show the results from our VA-based regressor specification (C.2) using our preferred IV (3.22) and the IV defined by equation (C.3), respectively. Both tables share the same structure; Columns 1-3 show the results using VA_{it}^{ROW} as the regressor, while columns 4 and 5 use the raw sales measure. Columns 1 and 4 show the first stage results to check the relevance of the IV, and column 2 shows the reduced form relating the IV to the outcome variable. Note that the value added and crude sales regressors share the same reduced form. Finally, columns 3 and 5 show the 2SLS results, and particularly worth mentioning is that they are all qualitatively similar *no matter what regressors and IVs were used*. Therefore, we view our preferred result reported in Table 3.2 as robust to the choice both of variables for foreign factor employment and IVs.

Long-difference Specification

Our main empirical specification (3.23) and (3.24) is meant to identify the medium- to long-run elasticity for five years after the flood. Another way to approach this is to conduct a long-difference specification by taking the difference between variables after the flood and

Table C.6: VA-based Regressor with IV (3.22)

VARIABLES	(1) $\ln VA_{it}^{ROW}$	(2) $\ln l_{it}^{JPN}$	(3) $\ln l_{it}^{JPN}$	(4) $\ln sales_{it}^{ROW}$	(5) $\ln l_{it}^{JPN}$
Z_{it}	-0.762	-0.132		-0.549	
	(0.105)	(0.0374)		(0.0849)	
$\ln(VA_{it}^{ROW})$			0.173		
			(0.0494)		
$\ln(sales_{it}^{ROW})$					0.240
					(0.0685)
Observations	5,460	5,460	5,460	5,460	5,460
Model	2SLS-1st	2SLS-reduced	2SLS	2SLS-1st	2SLS
Firm FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Robust standard errors in parentheses.

before:

$$\Delta \ln \left(l_i^{JPN} \right) = a^{LD} + b^{LD} \Delta \ln \left(l_i^{ROW} \right) + \Delta e_i^{LD},$$

where the time difference Δ takes the difference between the 2012-2016 average after the flood and the 2007-2011 average before the flood. We instrument the regression with the long-difference IV

$$\Delta Z_i = \frac{l_{i,2011}^{flooded}}{l_{i,2011}^{JPN} + l_{i,2011}^{ROW}}.$$

The results are shown in Table C.8, with column 1 the 2SLS first stage, column 2 the 2SLS reduced form, and column 3 the 2SLS result. Although the sample size reduced significantly due to time-averaging, the qualitative result of the regressions remain the same as in Table 3.2: strong first-stage correlation (column 1), weaker but significant negative correlation in the reduced form (column 2), and the positively significant 2SLS estimate (column 3) which is not significantly different from the preferred 2SLS estimate (column 3 of Table 3.2).

Table C.7: VA-based Regressor with IV (C.3)

VARIABLES	(1) $\ln VA_{it}^{ROW}$	(2) $\ln l_{it}^{JPN}$	(3) $\ln l_{it}^{JPN}$	(4) $\ln sales_{it}^{ROW}$	(5) $\ln l_{it}^{JPN}$
Z_{it}^{VA}	-0.957	-0.144		-0.791	
	(0.210)	(0.0749)		(0.173)	
$\ln(VA_{it}^{ROW})$			0.173		
			(0.0494)		
$\ln(sales_{it}^{ROW})$					0.183
					(0.0846)
Observations	5,460	5,460	5,460	5,460	5,460
Model	2SLS-1st	2SLS-reduced	2SLS	2SLS-1st	2SLS
Firm FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Robust standard errors in parentheses.

C.1.11 Further Empirical Results

Next, we conduct several extension exercises of the linear regression results discussed in Section 3.4.2.

Shift-share-type Instrument and Identifying the Substitution Elasticity

Hummels et al. [2014] considers a type of offshoring whereby a firm does not hire foreign factors directly but imports the intermediate good instead of producing it in the home country, which we call *output offshoring*. Our concern, however, is multinational firms' foreign employment, or *factor offshoring*. The firm-level specification of Hummels et al. [2014] is

$$\ln l_{it} = a_i^{HJMX} + a_t^{HJMX} + b^{HJMX} \ln m_{it} + e_{it}^{HJMX},$$

where l_{it} is firm f's employment in year t and m_{it} is the value of offshoring, measured by the value of imports.[7] They address the endogenous offshoring value by adopting the

[7] More specifically, this corresponds to the row named "log employment," with point coefficient 0.044 in Table 3, Column 2 ofHummels et al. [2014].

Table C.8: Long-difference Specification

VARIABLES	(1) $\Delta \ln l_i^{ROW}$	(2) $\ln \Delta l_i^{JPN}$	(3) $\ln \Delta l_i^{JPN}$
ΔZ_i	-0.731	-0.121	
	(0.159)	(0.0570)	
$\Delta \ln l_i^{ROW}$			0.165
			(0.0787)
Observations	674	674	674
Model	2SLS-1st	2SLS-reduced	2SLS

Robust standard errors in parentheses.

shift-share instrument at the country and product level,

$$I_{it} = \sum_{c,k} s_{ick} I_{ckt},$$

where s_{ick} is the pre-sample year (1994) share of the country c-product k pair in total material imports of firm f, and I_{ckt} is a world export shifter such as world export supply or transport costs of country c, product k, in year t.

Analogously, in our *factor offshoring* framework, the relevant regression is

$$\ln l_{it}^{JPN} = a_i^{FO} + a_t^{FO} + b^{FO} \ln l_{it}^{ROW} + e_{ft}^{FO}, \tag{C.4}$$

where l_{it}^{ROW} is employment of offshore workers and the instrument is

$$I_{it}^{ROW} = \sum_{c} s_{ic}^{O} L_{ct}^{ROW},$$

where s_c^O is the pre-sample year (in our case, 2007) share of offshore employment in country c of firm i, and $emp_{ct}^{O,-JPN}$ is the stock of workers at subsidiaries of multinational firms excluding those in Japan. However, as we do not find a relevant measure for L_{ct}^{ROW}, we need a proxy for that.

Note that Desai et al. [2009] takes the GDP growth rate in each country as the instrumental variable, based on the idea that "national economic growth is associated with productivity gains that correspond to declining real input costs" (p. 186). A problem that one can imagine with this proxy is that the GDP growth rate does not necessarily reflect the offshorability or availability of offshore factors in the destination country. For example, the GDP growth rate might not reflect political instability [Pierce and Schott, 2016]. If a country has high growth but high political instability that makes employment difficult for multinational firms, then GDP growth might overstate the actual ability to offshore in the country. Moreover, the Desai et al. [2009] measure is the GDP growth rate rather than the level of GDP, whereas Hummels et al. [2014] take world trade value levels to construct the instrumental variable. With these cautions in mind, we calculate the instrumental variable as follows:

$$\widetilde{I}_{it}^{ROW} = \sum_{c} s_{ic}^{O} g_{ct},$$

where g_{ct} is the GDP growth rate of country c from year $t-1$ to year t.

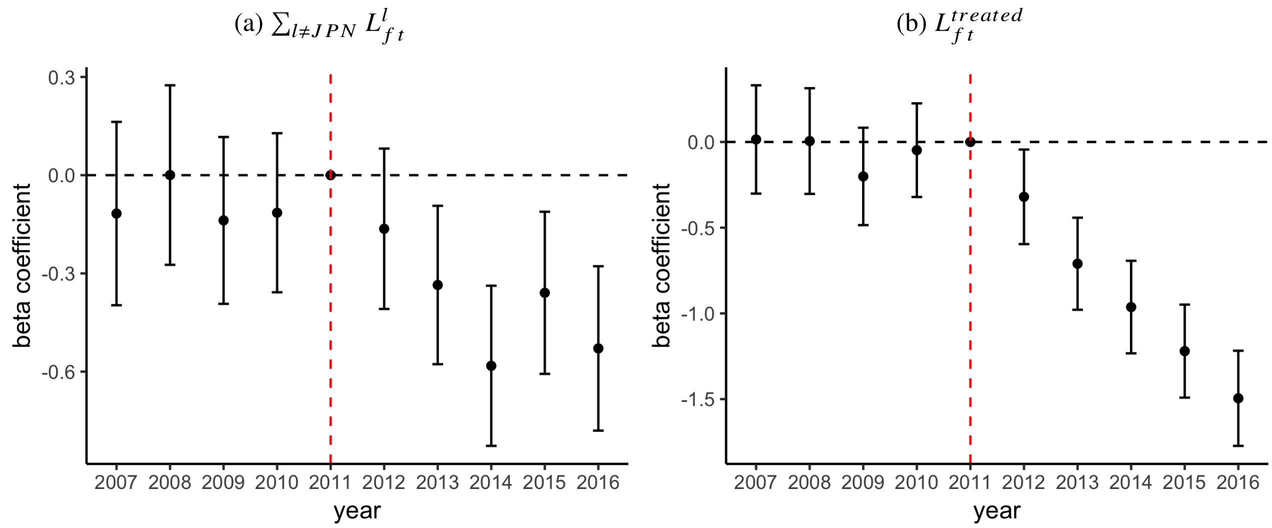
While Hummels et al. [2014] find a statistically significant substitution of home labor with the foreign imported intermediate input, in contrast, we do not find a similar substitution result (Table C.9). This highlights the difficulty in using a Bartik-type instrument to identify the effect of factor-usage offshorability on home employment, as we emphasized in the beginning of Section 3.4.

Table C.9: Results with GDP Growth-based Bartik Instruments

VARIABLES	(1) OLS	(2) FE	(3) 2SLS	(4) 2SLS first stage	(5) 2SLS reduced form
Log Subsidiary Employment	0.304	0.0314	0.0770		
	(0.00364)	(0.00245)	(0.0896)		
Shift-share Shock				0.435	0.0335
				(0.123)	(0.0388)
Observations	20,317	20,317	20,317	20,317	20,317
Firm FE		YES	YES	YES	YES
Year FE		YES	YES	YES	YES

Standard errors in parentheses.

Figure C.19: Results of Event Study Regression on Foreign Employment



Long-run Impact on Foreign Employment

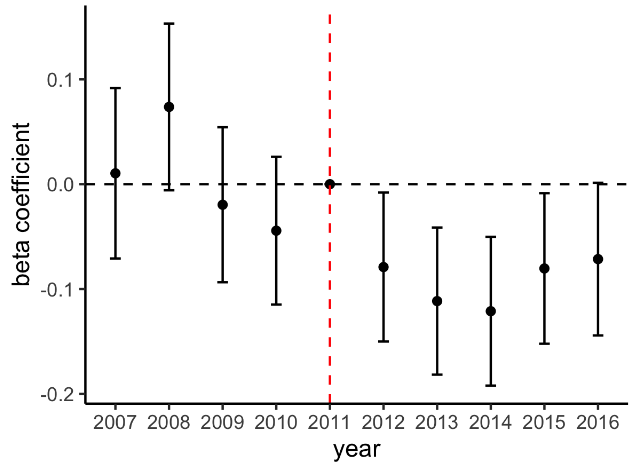
In our main specification (3.23) and (3.24), we consider the average long-run, but not time-varying, impacts. As we saw in Figure 3.3, the flood had long-lasting effects on employment in Thailand, so to see if this is true at the firm-level of those affected by the flood, we consider the following event-study regression:

$$y_{it} = a_i^{ES} + a_t^{ES} + \sum_{\tau \neq 2011} b_\tau^{ES} \frac{l^{flooded}_{i,2011}}{l^{JPN}_{i,2011} + l^{ROW}_{i,2011}} \times \mathbf{1}\{t = \tau\} + e_{it}^{ES}. \tag{C.5}$$

Figure C.19 shows the event-study plots of b_t^{ES},which confirm that the effect of the flood persisted for at least five years in terms of foreign employment.

We next turn to the long-run effect on Home employment. Namely, we take log employment in Japan as y_{it} and run regression (C.5), with the results shown in Figure C.20. Consistent with the reduced form results of our main specification in Table 3.2, we find significantly negative effects four years after the flood at the 5 percent significance level. We can also confirm that the pre-trends are balanced since no coefficient before the year of the flood is statistically significantly different from zero. In 2016, the coefficient is negative but only marginally different from zero (significant only at the 10 percent level). This reversion to zero may imply recovery from the flood for the firms that were severely hit. However,

Figure C.20: Results of Event Study Regression on Home Employment



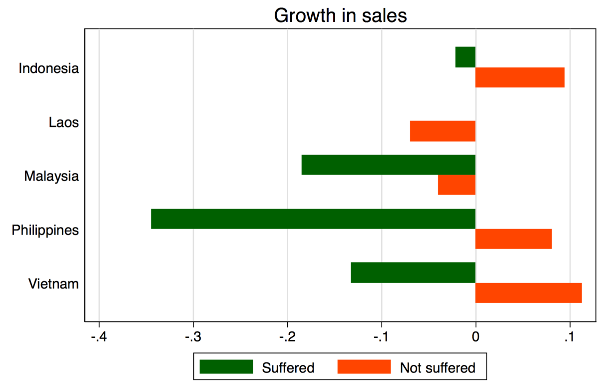
even in the Home country (Japan), the recovery in terms of employment took at least five years after the flood. In a nutshell, the event-study analysis supports our interpretation of the shock as having a medium-run impact on worldwide factor employment by affected firms.

Third Country Substitution

Where do subsidiaries substitute production after the flood? To answer this in a brief manner, we can observe the sales growth of subsidiaries in nearby countries (Indonesia, Laos, Malaysia, Philippines, and Vietnam) between 2011 and 2012 for those firms affected and unaffected by the flooding. If production substitution occurred in nearby countries, then the MNEs affected by the floods would increase sales in nearby countries relative to unaffected MNEs. Figure C.21 shows the sales growth rates of foreign subsidiaries in each country near Thailand for those MNEs with subsidiaries in Thailand (labeled as "suffered") and those without (labeled as "not suffered"). As one can see, for all countries except Laos, there is no relative increase in sales for firms hit by the flooding. Therefore, on average, production substitution to nearby countries did not occur very strongly, which supports the validity of our main analysis of production substitution between Japan and Thailand.

To more formally and systematically study the substitution after the flood, we conduct the same regression specification as in the main text with a modified coefficient notation to

Figure C.21: Growth in Sales Across Firms Located in Thailand in 2011 or Not



indicate Third Country Substitution (TCS):

$$y_{it} = a_i^{TCS} + a_t^{TCS} + b^{TCS} Z_{it} + e_{it}^{TCS}, \tag{C.6}$$

and also with the outcome variable about operations in third countries. Specifically, we take as an outcome variable log employment in Southeast Asian countries other than Thailand (Myanmar, Malaysia, Singapore, Indonesia, Phillipines, Cambodia, Laos, with notation emp_{it}^{SEA}), log employment in the world other than in Thailand and Japan (emp_{it}^{World}), log sales in Southeast Asian countries other than Thailand ($sales_{it}^{SEA}$), and log sales in the world other than in Thailand and Japan ($sales_{it}^{World}$). To best control any unobserved subsidiary heterogeneity, we restrict the sample to subsidiaries located in Thailand, and compare the headquarters that have subsidiaries in the flooded regions versus those that do not, using our IV Z_{it}. If third country substitution were significant, we would observe the positive coefficient $b^{TCS} > 0$.

Table C.10 shows the results of regression (C.6), and throughout columns 1-4, we do not find positive substitution from the flooded regions to non-flooded third countries. Perhaps surprisingly, we do find some strong *negative* effects on third country employment and sales. In fact, this is consistent with our interpretation that the flood decreased overall productivity of the affected MNEs, so that they decreased factor employment and sales *everywhere* in the world, including third countries and Japan. Recall that this productivity effect could be seen in our main regression result 3.2.

Table C.10: Third Country Substitution

VARIABLES	(1) $\ln emp_{it}^{SEA}$	(2) $\ln emp_{it}^{World}$	(3) $\ln sales_{it}^{SEA}$	(4) $\ln sales_{it}^{World}$
Z_{it}	0.443	-1.039	-0.600	-0.951
	(0.354)	(0.270)	(0.253)	(0.184)
Observations	2,434	5,549	2,364	5,489
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES

Robust standard errors in parentheses.

Regression by Industries

Table C.11 shows the results of the headquarter industry-regression for specification (3.22) and (C.1), with panel C.11a showing the 2SLS results and C.11b and C.11c showing the corresponding first stage and reduced-form regressions. In each panel, column 1 refers to the aggregated manufacturing sector and columns 2-6 the Chemical, Metal, General Machinery, Electronic Machinery, and Automobile industries, respectively.

C.2 Theory Appendix

This appendix provides proofs and extensions of the model described in Section 3.3.

C.2.1 Uniqueness of the Equilibrium

A desirable property of a general equilibrium is its uniqueness, as uniqueness guarantees that the equilibrium is robust across shocks and parameter values. Although it is well-known that the uniqueness result is difficult to obtain [Debreu, 1974, Mantel, 1974, Sonnenschein, 1972], our constant returns to scale model allows us to adopt the *generic* uniqueness approach described in Chapter 17 of Mas-Colell et al. [1995]. In the explanation below, proposition numbers that begin with "17" are all taken from that chapter.

We begin with the following facts: (i) Under a regularity condition, any equilibrium factor price vector (r, w) is locally isolated (Proposition 17.D.1) and, furthermore, the regularity condition holds generically (Proposition 17.D.5); (ii) If the weak axiom of revealed preference (WARP) is satisfied, then under constant returns to scale technology, the set of equilibrium price vectors (r, w) is convex (Proposition 17.F.2). Note that as our factor demand functions are obtained by solving a cost-minimization problem, so they satisfy WARP. Thus, the set of equilibrium factor price vectors is both locally isolated convex and a singleton. Thus, we may conclude the following:

Proposition 4. *(Generic Uniqueness) The general equilibrium defined in Section 3.3.1 is generically unique.*

C.2.2 Equivalence Results

We consider a special case of our model within offshoring and multinational models, beginning the equilibrium analysis from modified versions of Feenstra and Hanson [1997] and Arkolakis et al. [2017] to arrive at equation (3.6).

Offshoring

First, we propose a modified version of Feenstra and Hanson [1997] in which we do not distinguish between high-skill and low-skill workers but we do distinguish between factor augmenting productivity shocks in the Home country H and the Foreign country F. Each country $c = H, F$ is endowed with (L^c, K^c), and the factor prices are (w^c, r^c). Competitive producers in H and F produce a single numeraire final good q by combining continuum intermediate inputs $z \in [0, 1]$. To produce the intermediate good, producers may use the factors in either H or F. In production place c, the intermediate good z is produced by CES technology[8]

$$x^c(z) = \left(\left(\frac{A^{c,L} l^c(z)}{a(z)} \right)^{\frac{\sigma-1}{\sigma}} + \left(A^{c,K} k^c(z) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{C.7}$$

[8]Note that this formulation is more general than the nesting of Cobb-Douglas and Leontief production as in Feenstra and Hanson [1997].

where $a(z)$ is increasing and $\sigma < 1$. That is, labor and capital are gross complements. The final good q is then costlessly assembled according to the Cobb-Douglas function[9]

$$\ln q = \int_0^1 \alpha(z) \ln x^c(z)\, dz. \tag{C.8}$$

There is no cost to trade the output good. Consider for now that world income $E = \overline{E}$ is fixed and spent on the output so that

$$q = \overline{E}. \tag{C.9}$$

The equilibrium is characterized by factor prices $\left(w^H, r^H, w^F, r^F\right)$ that solve the market clearing condition. To formally derive such conditions, suppose that foreign labor L^F is abundant enough so that

$$\frac{w^H}{r^H} > \frac{w^F}{r^F},$$

or H has a comparative advantage in producing the capital-intensive intermediate good. Further assume L^F is large so that $\widetilde{w^H} > \widetilde{w^F}$ where $\widetilde{w^c} \equiv w^c / A^{c,L}$ and $\widetilde{r^c} \equiv r^c / A^{c,K}$ are augmented factor prices.[10] We will consider our case of CES output demand and income effects after solving the current simple version.

To solve the model, consider the cost-minimizing factor demand given factor prices $\left(w^H, r^H, w^F, r^F\right)$. First, conditional on country $c = H, F$, the unit cost function is

$$c^c(z) = \left(\left(a(z)\,\widetilde{w^c}\right)^{1-\sigma} + \left(\widetilde{r^c}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{C.10}$$

Given the comparative advantage assumption, in equilibrium, there is $z^* \in [0,1]$ that satisfies z being produced in the Home country if and only if $z \le z^*$. z^* satisfies $c^H(z^*) = c^F(z^*)$, or

$$\left(a(z^*)\,\widetilde{w^H}\right)^{1-\sigma} + \left(\widetilde{r^H}\right)^{1-\sigma} = \left(a(z^*)\,\widetilde{w^F}\right)^{1-\sigma} + \left(\widetilde{r^F}\right)^{1-\sigma}. \tag{C.11}$$

The marginal cost is thus $c(z) \equiv \min_c \{c^c(z)\}$. Given such z^*, by Shepherd's lemma, the

[9]Note that the same production function with costless intermediate good trade makes the trade in the final good irrelevant. Specifically, countries do not have an incentive to specialize (or not specialize) in the production of the final good. In this sense, the model is all about intermediate good trade. As we see in equation (C.11), H exports $z < z^*$ and imports $z > z^*$.

[10]This assumption is not essential, but allows us to proceed without unnecessary complications.

factor demands are characterized by

$$\widetilde{l^c}(z) = \left(\frac{a(z)\,\widetilde{w^c}}{c^c(z)}\right)^{-\sigma} a(z)\,x^c(z)\,, \tag{C.12}$$

$$\widetilde{k^c}(z) = \left(\frac{\widetilde{r^c}}{c^c(z)}\right)^{-\sigma} x^c(z) \tag{C.13}$$

for $c = H, F$, where $\widetilde{l^c}(z) \equiv A^{c,L} l^c(z)$ and $\widetilde{k^c}(z) \equiv A^{c,K} k^c(z)$ are the augmented factor demands for variety z in country c. Hence, the market clearing conditions are

$$\widetilde{L^H} = \int_0^{z^*} \widetilde{l^H}(z)\,dz, \tag{C.14}$$

$$\widetilde{K^H} = \int_0^{z^*} \widetilde{k^H}(z)\,dz, \tag{C.15}$$

$$\widetilde{L^F} = \int_{z^*}^{1} \widetilde{l^F}(z)\,dz, \tag{C.16}$$

$$\widetilde{K^F} = \int_{z^*}^{1} \widetilde{k^F}(z)\,dz, \tag{C.17}$$

where $\widetilde{L^c} \equiv A^{c,L} L^c$ and $\widetilde{K^c} \equiv A^{c,K} K^c$ are the augmented endowments for $c = H, F$. To solve $x^c(z)$, by Cobb-Douglas assumption (C.8), we have $p(z)\,x^c(z) = \alpha(z)\,q$, where $p(z)$ is the price of the intermediate good z. Moreover, the perfect competition assumption implies that $p(z)$ is given by the (minimum) marginal cost $c(z)$. Thus, by good market clearing condition (C.9), we have

$$x^c(z) = \frac{\alpha(z)}{c(z)}\overline{E}. \tag{C.18}$$

Thus, the equilibrium is $\left(z^*, \left(\widetilde{w^c}, \widetilde{r^c}\right)_{c\in(H,F)}\right)$ that solves equations (C.11), (C.14), (C.15), (C.16), and (C.17). To study the Home labor share, we still have $LS \equiv w^H L^H / \left(w^H L^H + r^H K^H\right)$ that is to the first order

$$dLS = LS_0\,(1 - LS_0)\left(d\ln w^H - d\ln r^H\right).$$

Hence, it remains to study $d\ln w^H$ and $d\ln r^H$. For this purpose, we have log first-order approximations with respect to $d\ln A^{F,L}$ and $d\ln A^{F,K}$ to equations (C.11), (C.14), (C.15), (C.16), and (C.17) as follows:

$$\begin{aligned} &CS^{H,L}(z^*)\left(\sigma_{az} d\ln z^* + d\ln \widetilde{w^H}\right) + CS^{H,K}(z^*)\, d\ln \widetilde{r^H} \\ &= CS^{F,L}(z^*)\left(\sigma_{az} d\ln z^* + d\ln \widetilde{w^F}\right) + CS^{F,K}(z^*)\, d\ln \widetilde{r^F} \end{aligned}, \tag{C.19}$$

$$\begin{aligned} 0 &= \frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}} d\ln z^* - \sigma d\ln \widetilde{w^H} + \int_0^{z^*} \frac{\widetilde{l^H}(z)}{\widetilde{L^H}} \left(\sigma d\ln c^H(z) + d\ln x^H(z)\right) dz, \\ 0 &= \frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}} d\ln z^* - \sigma d\ln \widetilde{r^H} + \int_0^{z^*} \frac{\widetilde{k^H}(z)}{\widetilde{K^H}} \left(\sigma d\ln c^H(z) + d\ln x^H(z)\right) dz, \\ d\ln A^{F,L} &= -\frac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^F}} d\ln z^* - \sigma d\ln \widetilde{w^F} + \int_{z^*}^{1} \frac{\widetilde{l^F}(z)}{\widetilde{L^F}} \left(\sigma d\ln c^F(z) + d\ln x^F(z)\right) dz, \\ d\ln A^{F,K} &= -\frac{z^*\widetilde{k^F}(z^*)}{\widetilde{K^F}} d\ln z^* - \sigma d\ln \widetilde{r^F} + \int_{z^*}^{1} \frac{\widetilde{k^F}(z)}{\widetilde{K^F}} \left(\sigma d\ln c^F(z) + d\ln x^F(z)\right) dz. \end{aligned}$$

Note that by equation (C.10), $d\ln c^c(z) = CS^{c,L}(z)\, d\ln \widetilde{w^c} + CS^{c,K}(z)\, d\ln \widetilde{r^c}$ and by equation (C.18), we have

$$d\ln x^c(z) = -d\ln c(z). \tag{C.20}$$

Note also that z is produced in H if and only if $z < z^*$. Hence, the conditions are further reduced to

$$0 = \frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}} d\ln z^* + \left(-\sigma + (\sigma-1)\, ACS^{H,L}(z^*)\right) d\ln \widetilde{w^H} + (\sigma-1)\, ACS^{H,K}(z^*)\, d\ln \widetilde{r^H}, \tag{C.21}$$

$$0 = \frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}} d\ln z^* + (\sigma-1)\, ACS^{H,L}(z^*)\, d\ln \widetilde{w^H} + \left(-\sigma + (\sigma-1)\, ACS^{H,K}(z^*)\right) d\ln \widetilde{r^H}, \tag{C.22}$$

$$\begin{aligned} &d\ln A^{F,L} = \\ &-\tfrac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^H}} d\ln z^* + \left(-\sigma + (\sigma-1)\, ACS^{F,L}(z^*)\right) d\ln \widetilde{w^F} + (\sigma-1)\, ACS^{F,K}(z^*)\, d\ln \widetilde{r^F}, \end{aligned} \tag{C.23}$$

$$\begin{aligned} &d\ln A^{F,K} = \\ &-\tfrac{z^*\widetilde{k^F}(z^*)}{\widetilde{K^F}} d\ln z^* + (\sigma-1)\, ACS^{F,L}(z^*)\, d\ln \widetilde{w^F} + \left(-\sigma + (\sigma-1)\, ACS^{F,K}(z^*)\right) d\ln \widetilde{r^F}, \end{aligned} \tag{C.24}$$

where

$$ACS^{H,L}(z^*) \equiv \int_0^{z^*} \frac{\widetilde{l^H}(z)}{\widetilde{L^H}} CS^{H,L}(z)\,dz,$$

$$ACS^{H,K}(z^*) \equiv \int_0^{z^*} \frac{\widetilde{k^H}(z)}{\widetilde{K^H}} CS^{H,K}(z)\,dz,$$

$$ACS^{F,L}(z^*) \equiv \int_{z^*}^{1} \frac{\widetilde{l^F}(z)}{\widetilde{L^F}} CS^{F,L}(z)\,dz,$$

$$ACS^{F,K}(z^*) \equiv \int_{z^*}^{1} \frac{\widetilde{k^F}(z)}{\widetilde{K^F}} CS^{F,K}(z)\,dz$$

are the average cost shares of each factor in each country. Assume for now that $\widetilde{w^H} > \widetilde{w^F}$. We can then show that $CS^{H,L}(z^*) > CS^{F,L}(z^*)$,[11] as we prove below. Also, we will come back to the condition that guarantees $\widetilde{w^H} > \widetilde{w^F}$. By (C.19), we have

$$d\ln z^* = \frac{\left(CS^{F,L}(z^*)\,d\ln\widetilde{w^F} + CS^{F,K}(z^*)\,d\ln\widetilde{r^F}\right) - \left(CS^{H,L}(z^*)\,d\ln\widetilde{w^H} + CS^{H,K}(z^*)\,d\ln\widetilde{r^H}\right)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}}. \tag{C.25}$$

By substituting equation (C.25) into equations (C.21), (C.22), (C.23), and (C.24), we have

$$\begin{aligned} 0 = {} & \underbrace{\left[-\frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}}\frac{CS^{H,L}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}} + \left(-\sigma + (\sigma-1)\,ACS^{H,L}(z^*)\right)\right]}_{\equiv\sigma_{\widetilde{l^H}\widetilde{w^H}}} d\ln\widetilde{w^H} \\ & + \underbrace{\left[-\frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}}\frac{z^*\widetilde{l^H}(z^*)\,CS^{H,K}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}} + (\sigma-1)\,ACS^{H,K}(z^*)\right]}_{\equiv\sigma_{\widetilde{l^H}\widetilde{r^H}}} d\ln\widetilde{r^H} \\ & + \underbrace{\frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}}\frac{CS^{F,L}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{l^H}\widetilde{w^F}}} d\ln\widetilde{w^F} + \underbrace{\frac{z^*\widetilde{l^H}(z^*)}{\widetilde{L^H}}\frac{CS^{F,K}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{l^H}\widetilde{r^F}}} d\ln\widetilde{r^F}, \end{aligned}$$

[11] To see this, note that at $z = z^*$ we have $c^H(z^*) = c^F(z^*)$. Hence, it remains to show $\widetilde{w^H}\widetilde{l^H}(z^*) > \widetilde{w^F}\widetilde{l^F}(z^*)$. By substituting the factor demand functions and noting that $c^H(z^*) = c^F(z^*)$ and thus $x^H(z^*) = x^F(z^*)$, the inequality is equivalent with $\left(\widetilde{w^H}\right)^{1-\sigma} > \left(\widetilde{w^F}\right)^{1-\sigma}$. Our assumption $\sigma < 1$ means that this is equivalent to $\widetilde{w^H} > \widetilde{w^F}$.

$$
\begin{aligned}
0 = & \underbrace{\left[-\frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}}\frac{CS^{H,L}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}+(\sigma-1)\,ACS^{H,L}(z^*)\right]}_{\equiv\sigma_{\widetilde{k^H}\widetilde{w^H}}} d\ln\widetilde{w^H} \\
& + \underbrace{\left[-\frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}}\frac{CS^{H,K}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}+\left(-\sigma+(\sigma-1)\,ACS^{H,K}(z^*)\right)\right]}_{\equiv\sigma_{\widetilde{k^H}\widetilde{r^H}}} d\ln\widetilde{r^H} \\
& + \underbrace{\frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}}\frac{CS^{F,L}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{k^H}\widetilde{w^F}}} d\ln\widetilde{w^F} + \underbrace{\frac{z^*\widetilde{k^H}(z^*)}{\widetilde{K^H}}\frac{CS^{F,K}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{k^H}\widetilde{r^F}}} d\ln\widetilde{r^F},
\end{aligned}
$$

$$
\begin{aligned}
& d\ln A^{F,L} = \\
& \underbrace{\frac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^H}}\frac{CS^{H,L}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{l^F}\widetilde{w^H}}} d\ln\widetilde{w^H} + \underbrace{\frac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^H}}\frac{CS^{H,K}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv\sigma_{\widetilde{l^F}\widetilde{r^H}}} d\ln\widetilde{r^H} \\
& + \underbrace{\left[-\frac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^H}}\frac{CS^{F,L}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}+\left(-\sigma+(\sigma-1)\,ACS^{F,L}(z^*)\right)\right]}_{\equiv\sigma_{\widetilde{l^F}\widetilde{w^F}}} d\ln\widetilde{w^F} \\
& + \underbrace{\left[-\frac{z^*\widetilde{l^F}(z^*)}{\widetilde{L^H}}\frac{CS^{F,K}(z^*)}{\left(CS^{H,L}(z^*)-CS^{F,L}(z^*)\right)\sigma_{az}}+(\sigma-1)\,ACS^{F,K}(z^*)\right]}_{\equiv\sigma_{\widetilde{l^F}\widetilde{r^F}}} d\ln\widetilde{r^F},
\end{aligned}
$$

$$
\begin{aligned}
&d \ln A^{F,K} = \\
&\underbrace{\frac{z^* \widetilde{k^F}(z^*)}{\widetilde{K^F}} \frac{CS^{H,L}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv \sigma_{\widetilde{k^F}\widetilde{w^H}}} d \ln \widetilde{w^H} + \underbrace{\frac{z^* \widetilde{k^F}(z^*)}{\widetilde{K^F}} \frac{CS^{H,K}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}}}_{\equiv \sigma_{\widetilde{k^F}\widetilde{r^H}}} d \ln \widetilde{r^H}, \\
&\underbrace{\left[-\frac{z^* \widetilde{k^F}(z^*)}{\widetilde{K^F}} \frac{CS^{F,L}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}} + (\sigma - 1) ACS^{F,L}(z^*)\right]}_{\equiv \sigma_{\widetilde{k^F}\widetilde{w^F}}} d \ln \widetilde{w^F} \\
&\underbrace{\left[-\frac{z^* \widetilde{k^F}(z^*)}{\widetilde{K^F}} \frac{CS^{F,K}(z^*)}{\left(CS^{H,L}(z^*) - CS^{F,L}(z^*)\right)\sigma_{az}} + \left(-\sigma + (\sigma - 1) ACS^{F,K}(z^*)\right)\right]}_{\equiv \sigma_{\widetilde{k^F}\widetilde{r^F}}} d \ln \widetilde{r^F}.
\end{aligned}
$$

Therefore, the elasticity matrix Σ is given as

$$
\Sigma \equiv \begin{pmatrix}
\sigma_{\widetilde{l^H}\widetilde{w^H}} & \sigma_{\widetilde{l^H}\widetilde{r^H}} & \sigma_{\widetilde{l^H}\widetilde{w^F}} & \sigma_{\widetilde{l^H}\widetilde{r^F}} \\
\sigma_{\widetilde{k^H}\widetilde{w^H}} & \sigma_{\widetilde{k^H}\widetilde{r^H}} & \sigma_{\widetilde{k^H}\widetilde{w^F}} & \sigma_{\widetilde{k^H}\widetilde{r^F}} \\
\sigma_{\widetilde{l^F}\widetilde{w^H}} & \sigma_{\widetilde{l^F}\widetilde{r^H}} & \sigma_{\widetilde{l^F}\widetilde{w^F}} & \sigma_{\widetilde{l^F}\widetilde{r^F}} \\
\sigma_{\widetilde{k^F}\widetilde{w^H}} & \sigma_{\widetilde{k^F}\widetilde{r^H}} & \sigma_{\widetilde{k^F}\widetilde{w^F}} & \sigma_{\widetilde{k^F}\widetilde{r^F}}
\end{pmatrix}. \tag{C.26}
$$

Now we consider extension cases of non-homogeneous output and income effects. In particular, consideration of non-homogeneous output makes the model more isomorphic to ours. Again, this helps us to think of the heterogeneous firm case and identification given the productivity shocks. Suppose that output is demanded by the following CES demand

$$
q = \left(\frac{p}{P}\right)^{-\varepsilon} Q,
$$

where P and Q are given exogenously (as H is a small open economy). Then the change in cost structure $d \ln p$ further has a feedback loop through the reduction in output demanded $d \ln q$ with elasticity ε. With this effect taken into consideration in the demand for the intermediate good, equation (C.18) becomes

$$
d \ln x^c(z) = -d \ln c(z) - \varepsilon d \ln p.
$$

Also, by the perfect competition assumption, $d \ln p$ =

$\sum_c \left(ACS^{c,L}(z^*)\, d\ln\widetilde{w^c} + ACS^{c,K}(z^*)\, d\ln\widetilde{r^c}\right)$. With these considerations, we have our expression (3.6) with Σ defined in equation (C.26) and

$$\boldsymbol{\sigma}_q = \begin{pmatrix} \varepsilon ACS^{H,L}(z^*) \\ \varepsilon ACS^{H,K}(z^*) \\ \varepsilon ACS^{F,L}(z^*) \\ \varepsilon ACS^{F,K}(z^*) \end{pmatrix}.$$

Next, we consider income effects by relaxing the small open economy assumption. This is desirable even when we are willing to assume that Japan is small-open because since the reduction in the cost of multinational production and offshoring are global phenomena and not limited to a particular country such as Japan, it affects world income even if Japan is small-open. To put it differently, only in the case of Japan being small-open *and* an isolated country that experienced a reduction in multinational production cost does the no-income-effect assumption hold. To put it simply, we come back to the homogeneous outcome case and consider the changes in income due to productivity growth. Particularly, suppose instead of equation (C.9) that final demand is given by

$$q = \sum_c \left(w^c L^c + r^c K^c\right).$$

This implies that $d\ln q = \sum_c IS^c \left(LS^c d\ln w^c + (1 - LS^c)\, d\ln r^c\right)$, where $IS^c \equiv w^c L^c + r^c K^c / \sum_{c'} \left(w^{c'} L^{c'} + r^{c'} K^{c'}\right)$ is the income share of country c. This again modifies equation (C.20) to

$$\begin{aligned} d\ln x^c(z) &= -d\ln c(z) + d\ln q \\ &= -d\ln c(z) + \sum_c IS^c \left(LS^c d\ln w^c + (1 - LS^c)\, d\ln r^c\right). \end{aligned}$$

Hence, the relevant modification to the elasticity matrix (C.26) would accommodate this.

Multinational Production and Export Platforms

Following Arkolakis et al. [2017], we consider a different type of offshoring whereby firms from different source countries may produce a good in a different production country and sell to different destination countries. In particular, we take a special case of Arkolakis et al.

[2017] in which is there are two distinct factors, which we interpret as labor and capital. To simplify matters, consider the two country case with H and F. There are no trade costs, whereas there could be an iceberg-type cost for producing in different countries $\gamma^{il} > 1$ for $i \neq l$. In each country, there are two types of factors, labor and capital, with endowment (L^c, K^c), where $c = H, F$.[12] Labor is used for production and capital for innovation. A potential firm in country i can enter the market and create a new variety by paying fixed entry cost $f^{e,i}$ units of capital. When a firm enters, it draws a productivity vector $\boldsymbol{z} \equiv \left(z^H, z^F\right)$ from the multivariate Pareto distribution

$$\Pr\left(Z^H \leq z^H, Z^F \leq z^F\right) = G^i\left(z^H, z^F\right) \equiv 1 - \left(\sum_{l \in \{H,F\}} \left[A^{il}\left(z^l\right)^{-\theta}\right]^{\frac{1}{1-\rho}}\right)^{1-\rho},$$

with support $z^l \geq \left(\widetilde{A}^i\right)^{\theta^{-1}}$ for all l, where $\widetilde{A}^i \equiv \left[\sum_l \left(A^{il}\right)^{(1-\rho)^{-1}}\right]^{1-\rho}$. $\rho \in [0, 1)$ is a parameter that governs the correlation between countries, and $\rho = 0$ is the case of uncorrelated Z^H and Z^F while $\rho \to 1$ is the perfect collinear extreme. We assume $A^{il} = A^{e,i}A^{p,l}$ so that $\widetilde{A}^i = \left[\sum_l \left(A^{p,l}\right)^{(1-\rho)^{-1}}\right]^{1-\rho} A^{e,i}$.[13] We call $A^{e,i}$ the quality of innovation in country i and $A^{p,l}$ the productivity in production of country l.

$$\xi^{iln} \equiv \gamma^{il} w^l \tau^{ln}, \tag{C.27}$$

$$\Psi^{in} \equiv \left[\sum_{l'} \left(A^{il'}\left(\xi^{il'n}\right)^{-\theta}\right)^{(1-\rho)^{-1}}\right]^{1-\rho} \tag{C.28}$$

$$\psi^{iln} \equiv \left(\frac{T^{il}\left(\xi^{iln}\right)^{-\theta}}{\Psi^{in}}\right)^{(1-\rho)^{-1}} \tag{C.29}$$

$$X^{iln} = \psi^{iln}\lambda^{E,in}X^n \tag{C.30}$$

$$M^{iln} = \frac{\theta - \sigma + 1}{\sigma\theta}\frac{X^{iln}}{w^n F^n}, \tag{C.31}$$

$$\lambda^{E,in} \equiv \frac{\sum_l X^{iln}}{X^n}. \tag{C.32}$$

[12]This is a special case of the treatment in Arkolakis et al. [2017]; namely, inelastic supply of production and innovation workers, by letting $\kappa \to 1$.

[13]This is without loss of generality, as γ^{il} may have the necessary variation as is clear.

$$\lambda^{T,ln} \equiv \sum_i \frac{X^{iln}}{X^n} = \sum_i \psi^{iln} \lambda^{E,in}. \tag{C.33}$$

We now characterize the equilibrium conditions that pin down $\left(w^H, r^H, w^F, r^F\right)$ as follows. The labor market equilibrium is given by the payment to workers equalling the sum of the value of labor demand for production and marketing, as

$$\frac{1}{\tilde{\tilde{\sigma}}} \sum_n \lambda^{T,ln} X^n + \left(1 - \eta - \frac{1}{\tilde{\tilde{\sigma}}}\right) X^l = w^l L^l \tag{C.34}$$

for each $l = H, F$. The capital market equilibrium condition is such that the payment to capital equals the value of capital demand for innovation, as

$$\eta \sum_n \lambda^{E,in} X^n = r^i K^i \tag{C.35}$$

for each $i = H, F$. Finally, the budget balance condition is

$$w^i L^i + r^i K^i + \Delta^i = X^i \tag{C.36}$$

for each $i = H, F$, where Δ^i is the transfer to country i from the ROW.

Consider a multinational production shock to the economy $d \ln \gamma^{HF} < 0$. The log first-order approximations to the system (C.34), (C.35), and (C.36) give

$$LDS^l \sum_n PS^{ln} \left(d \ln \lambda^{T,ln} + d \ln X^n\right) + \left(1 - LDS^l\right) d \ln X^l = d \ln w^l, \tag{C.37}$$

$$\sum_n IS^{in} \left(d \ln \lambda^{E,in} + d \ln X^n\right) = d \ln r^i, \tag{C.38}$$

$$LS^i d \ln w^i + \left(1 - LS^i\right) d \ln r^i = d \ln X^i, \tag{C.39}$$

where

$$LDS^l \equiv \frac{\frac{\sigma-1}{\sigma} \sum_n \lambda^{T,ln} X^n}{\frac{\sigma-1}{\sigma} \sum_n \lambda^{T,ln} X^n + \left(1 - \eta - \frac{\sigma-1}{\sigma}\right) X^l}, \; PS^{ln} \equiv \frac{\lambda^{T,ln} X^n}{\sum_{n'} \lambda^{T,ln'} X^{n'}}, \; IS^{in} \equiv \frac{\lambda^{E,in} X^n}{\sum_{n'} \lambda^{E,in'} X^{n'}}$$

are the (value) labor demand share in production of each country, the production sales share to country n, and innovation sales share to country n. Substituting equation (C.39) into equations (C.37) and (C.38), we have

$$\sum_n PS^{ln}\left(d\ln\lambda^{T,ln} + LS^n d\ln w^n + (1-LS^n)\,d\ln r^n\right) + LS^l d\ln w^l + \left(1-LS^l\right) d\ln r^l = d\ln w^l, \tag{C.40}$$

$$\sum_n IS^{in}\left(d\ln\lambda^{E,in} + LS^n d\ln w^n + (1-LS^n)\,d\ln r^n\right) = d\ln r^i. \tag{C.41}$$

To study $d\ln\lambda^{T,ln}$ and $d\ln\lambda^{E,in}$, first, by equation (C.33),

$$d\ln\lambda^{T,ln} = \sum_i \frac{\psi^{iln}\lambda^{E,in}}{\sum_{i'}\psi^{i'ln}\lambda^{E,i'n}}\left(d\ln\psi^{iln} + d\ln\lambda^{E,in}\right),$$

which, by equation (C.30), further simplifies to

$$d\ln\lambda^{T,ln} = \sum_i \frac{X^{iln}}{X^n}\left(d\ln\psi^{iln} + d\ln\lambda^{E,in}\right). \tag{C.42}$$

On the other hand, by equation (C.32), we have

$$d\ln\lambda^{E,in} = \left(1-\lambda^{E,in}\right)\left(d\ln M^i + d\ln\Psi^{in}\right) - \lambda^{E,\bar{i}n}\left(d\ln M^{\bar{i}} + d\ln\Psi^{\bar{i}n}\right). \tag{C.43}$$

To further deduce, it remains to know $d\ln\psi^{iln}$, $d\ln M^i$, and $d\ln\Psi^{in}$. By equations (C.27), (C.28), (C.29), and (C.31), they are

$$\begin{aligned} d\ln\psi^{iln} &= \frac{1}{1-\rho}\left(-\theta d\ln\xi^{iln} - d\ln\Psi^{in}\right), \\ &= \frac{1}{1-\rho}\left(-\theta d\ln\left(d\ln w^l + d\ln\gamma^{il}\right) - d\ln\Psi^{in}\right), \end{aligned} \tag{C.44}$$

$$\begin{aligned} d\ln\Psi^{in} &= \sum_{l'}\psi^{il'n}\left(-\theta d\ln\xi^{il'n}\right), \\ &= \sum_{l'}\psi^{il'n}\left(-\theta\left(d\ln w^{l'} + d\ln\gamma^{il'}\right)\right), \end{aligned} \tag{C.45}$$

$$d \ln M^i = \sum_{l',n'} \frac{M^{il'n'}}{M^i} \left(d \ln X^{il'n'} - d \ln w^{n'} \right) \tag{C.46}$$
$$= \sum_{l',n'} \frac{M^{il'n'}}{M^i} \left(d \ln \psi^{il'n'} + d \ln \lambda^{E,in'} + d \ln X^{n'} - d \ln w^{n'} \right).$$

By substituting equations (C.42), (C.43), (C.44), (C.45), and (C.46), into equations (C.40) and (C.41), we obtain an expression for

$$\Sigma \begin{pmatrix} d \ln w^H \\ d \ln r^H \\ d \ln w^F \\ d \ln r^F \end{pmatrix} = b,$$

for a reduced demand elasticity matrix Σ and shock vector b.

C.2.3 Derivation of Equation (3.13)

The cost minimization problem under the upper nest implies

$$k = \left(a^K\right)^{\sigma-1} \left(\frac{r}{p}\right)^{-\sigma} q, and \tag{C.47}$$
$$l = \left(\frac{w}{p}\right)^{-\sigma} q, \tag{C.48}$$

where $p^X \equiv \left(\left(w/a^L\right)^{1-\lambda} + \left(p^M/a^M\right)^{1-\lambda}\right)^{1/(1-\lambda)}$ is the aggregate wage index and $p \equiv \left(\left(r/a^K\right)^{1-\sigma} + \left(p^X\right)^{1-\sigma}\right)^{1/(1-\sigma)}$ is the marginal cost index. Similarly,

$$l = a^L \left(\frac{w}{p^X}\right)^{-\lambda} l, and \tag{C.49}$$
$$m = a^M \left(\frac{p^M}{p^X}\right)^{-\lambda} l. \tag{C.50}$$

By equating total demand to total supply, we have

$$
\begin{aligned}
K &= \left(a^K\right)^{\sigma-1}\left(\frac{r}{p}\right)^{-\sigma} q \Leftrightarrow r = p\left(a^K\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{q}{K}\right)^{\frac{1}{\sigma}}, \\
X &= \left(\frac{p^X}{p}\right)^{-\sigma} q \Leftrightarrow p^X = p\left(\frac{q}{X}\right)^{\frac{1}{\sigma}}, \\
L &= \left(a^H\right)^{\lambda-1}\left(\frac{w}{p^X}\right)^{-\lambda} X \Leftrightarrow w = p^X\left(a^H\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{L^H}\right)^{\frac{1}{\lambda}}, \\
M &= \left(a^M\right)^{\lambda-1}\left(\frac{p^M}{p^X}\right)^{-\lambda} X \Leftrightarrow p^M = p^X\left(a^M\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{M}\right)^{\frac{1}{\lambda}}.
\end{aligned}
$$

By substituting these expressions,

$$
\begin{aligned}
LS &\equiv \frac{wL}{wL+rK} \\
&= \frac{w\left(a^L\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{L}\right)^{\frac{1}{\lambda}} L}{w\left(a^L\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{L}\right)^{\frac{1}{\lambda}} L + c\left(a^K\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{q}{K}\right)^{\frac{1}{\sigma}} K} \\
&= \frac{c\left(\frac{q}{L}\right)^{\frac{1}{\sigma}}\left(a^L\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{L}\right)^{\frac{1}{\lambda}} L}{c\left(\frac{q}{X}\right)^{\frac{1}{\sigma}}\left(a^L\right)^{\frac{\lambda-1}{\lambda}}\left(\frac{X}{L}\right)^{\frac{1}{\lambda}} L + c\left(a^K\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{q}{K}\right)^{\frac{1}{\sigma}} K} \\
&= \frac{\left(a^L L\right)^{1-\lambda^{-1}} X^{\lambda^{-1}-\sigma^{-1}}}{\left(a^L L\right)^{1-\lambda^{-1}} X^{\lambda^{-1}-\sigma^{-1}} + \left(a^K K\right)^{1-\sigma^{-1}}}.
\end{aligned}
$$

C.2.4 Derivation of Equations (3.15) and (3.16)

We prove the effect of the shock with the following property: The shock decreases only the foreign augmentation $d\ln a^M < 0$ but not capital and labor $d\ln a^K = d\ln a^L = 0$, and hits only an infinitesimal set of firms so that factor prices are not affected. Note that perfect competition implies the price is the marginal cost. Consider the following overall marginal cost function

$$
\begin{aligned}
p &= \left(\left(\frac{r}{a^K}\right)^{1-\sigma} + \left(p^X\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&= \left(\left(\frac{r}{a^K}\right)^{1-\sigma} + \left(\left(\frac{w}{a^L}\right)^{1-\lambda} + \left(\frac{p^M}{a^M}\right)^{1-\lambda}\right)^{\frac{1-\sigma}{1-\lambda}}\right)^{\frac{1}{1-\sigma}}.
\end{aligned}
$$

Then the (first-order) elasticity of the cost with respect to log foreign factor-augmenting productivity $d\ln A^F$ is

$$d\ln p = -CS_0^M d\ln a^M,$$

where $CS^M \equiv p^M m/\left(rk + wl + p^M m\right)$ is the share of foreign labor cost in the overall cost. In addition, by equation (3.1) we have $d\ln q = -\varepsilon d\ln p$. This in turn reduces the factor demand $d\ln x = d\ln q$ according to aggregate labor demand (C.48). Lower-nest labor demands (C.49) and (C.50) imply that $d\ln l = d\ln x$ and $d\ln m = (\lambda - 1)\,d\ln a^M + d\ln x$. In a nutshell, we arrive at equations (3.15) and (3.16).

C.2.5 Elasticity Matrix under Nested CES

Under the homogeneous nested CES, by factor demand functions (C.47), (C.48), (C.49), and (C.50), we may obtain that the reduced elasticity matrix

$$\Sigma \equiv \begin{pmatrix} -\sigma + (\sigma-\varepsilon)\,CS^K & (\sigma-\varepsilon)\,CS^L & (\sigma-\varepsilon)\,CS^M \\ (\sigma-\varepsilon)\,CS^K & -\lambda + (\lambda-\sigma)\,WS^L + (\sigma-\varepsilon)\,CS^L & (\lambda-\sigma)\,WS^M + (\sigma-\varepsilon)\,CS^M \\ (\sigma-\varepsilon)\,CS^K & (\lambda-\sigma)\,WS^L + (\sigma-\varepsilon)\,CS^L & -\lambda + (\lambda-\sigma)\,WS^M + (\sigma-\varepsilon)\,CS^M \end{pmatrix}.$$

C.2.6 Derivation of Equation (3.26)

Next, we consider rich heterogeneity in factor augmenting productivities. The production function is

$$q_i = \left(\left(a_i^K k_i\right)^{1-\sigma^{-1}} + x_i^{1-\sigma^{-1}}\right)^{\left(1-\sigma^{-1}\right)^{-1}}, \textit{and}$$

$$x_i = \left(\left(a_i^L l_i\right)^{1-\lambda^{-1}} + \left(a_i^M m_i\right)^{1-\lambda^{-1}}\right)^{\left(1-\lambda^{-1}\right)^{-1}}.$$

We assume that Assumption 2 is satisfied for the purpose of the matrix inversion exercise below. For simplicity, assume additionally that the competitive environment features constant markups such as in perfect competition or monopolistic competition. The factor supply (K, L, M) and total demand Q are fixed.[14] Thus, the following factor market clearing

[14]Note that the fixed total demand Q means that Q is independent of the income of factors. This further implies that the treatment of the economic profits of firms is immaterial because so long as their shares enter as incomes. An assumption that justifies this would be that the Home country is a small-open *exporter* of the

conditions determine the market wages $\left(r, w, p^M\right)$.

$$K = \int_0^1 k_i di, L = \int_0^1 l_i di, M = \int_0^1 m_i di. \tag{C.51}$$

Unless otherwise noted, the integrals below are with respect to each firm and so from 0 to 1, which are omitted to keep the notation concise.

The cost-minimizing factor demands of firm i are

$$k_i = \left(a_i^K\right)^{\sigma-1}\left(\frac{r}{p_i}\right)^{-\sigma} q_i, x_i = \left(\frac{p_i^X}{p_i}\right)^{-\sigma} q_i, l_i = \left(a_i^L\right)^{\lambda-1}\left(\frac{w_i}{p_i^X}\right)^{-\lambda} x_i, m_i = \left(a_i^F\right)^{\lambda-1}\left(\frac{p_i^M}{p_i^X}\right)^{-\lambda} x_i,$$

where the marginal cost and aggregate labor-foreign input cost index are

$$p_i = \left(\left(\frac{r}{a_i^K}\right)^{1-\sigma} + \left(p_i^X\right)^{1-\sigma}\right)^{(1-\sigma)^{-1}}, p_i^X = \left(\left(\frac{w}{a_i^L}\right)^{1-\lambda} + \left(\frac{p^M}{a_i^M}\right)^{1-\lambda}\right)^{(1-\lambda)^{-1}}. \tag{C.52}$$

Substituting these into conditions (C.51), we have

$$K = \int \left(a_i^K\right)^{\sigma-1}\left(\frac{r}{p_i}\right)^{-\sigma}\left(\frac{p_i}{P}\right)^{-\varepsilon} Q di, \tag{C.53}$$

$$L = \int \left(a_i^L\right)^{\lambda-1}\left(\frac{w}{p_i^X}\right)^{-\lambda}\left(\frac{p_i^X}{p_i}\right)^{\sigma}\left(\frac{p_i}{P}\right)^{-\varepsilon} Q di, \tag{C.54}$$

$$M = \int \left(a_i^M\right)^{\lambda-1}\left(\frac{p^M}{p_i^X}\right)^{-\lambda}\left(\frac{p_i^X}{p_i}\right)^{\sigma}\left(\frac{p_i}{P}\right)^{-\varepsilon} Q di. \tag{C.55}$$

Since the system involves integrals of non-linear equations of unknowns $\left(r, w, p^M\right)$, the closed-form solution is hardly tractable. We thus rely on our first order approximation obtained in (3.6). Therefore, it suffices to know the log first-order approximation on w and r. The log first-order approximations with respect to a^M to the system (C.53), (C.54), and

goods each Home firm produces. In this case, the factor income of the Home country is negligible relative to the World income, which makes Q virtually fixed. We will come back to this issue when we analyze the general equilibrium.

(C.55) are

$$0 = \int \frac{rk_i}{rK} \left(-\sigma d \ln r + (\sigma - \varepsilon) d \ln p_i\right) di,$$
$$0 = \int \frac{wl_i}{wL} \left(-\lambda d \ln w + (\lambda - \sigma) d \ln p_i^X + (\sigma - \varepsilon) d \ln p_i\right) di,$$
$$0 = \int \frac{p^M m_i}{p^M M} \left((\lambda - 1) d \ln a^M - \lambda d \ln p^M + (\lambda - \sigma) d \ln p_i^X + (\sigma - \varepsilon) d \ln p_i\right) di,$$

where, by equations (C.52) and cost-minimizing demand functions,

$$d \ln p_i = \frac{rk_i}{p_i q_i} d \ln r + \frac{p_i^X x_i}{p_i q_i} d \ln w_i, and$$
$$d \ln w_i = \frac{wl_i}{p_i^X x_i} d \ln w^H + \frac{p^M m_i}{p_i^X x_i} d \ln w^F.$$

With slight abuse of notation, for $Z = K, L, M$ and $y = k, l, m$, write CS_y^Z as the employment of factor y-weighted average of payment share to factor Z. For instance, with the home labor (l) employment as the weight, the weighted average of the capital (K) payment share is $CS_l^K \equiv \int \frac{wl_i}{wL} \frac{rk_i}{p_i q_i} di$. Accordingly, for $Z = L, M$ and $y = l, m$, write WS_y^Z as the employment of factor y-weighted average of wage payment share to factor Z.[15] With these notations, we can summarize the first-order approximation as the linear algebraic problem $Az = b$, where

$$A \equiv \begin{pmatrix} (\sigma - \varepsilon) CS_k^K - \sigma & (\sigma - \varepsilon) CS_k^L & (\sigma - \varepsilon) CS_k^M \\ (\sigma - \varepsilon) CS_l^K & (\lambda - \sigma) WS_l^L + (\sigma - \varepsilon) CS_l^L - \lambda & (\lambda - \sigma) WS_l^M + (\sigma - \varepsilon) CS_l^M \\ (\sigma - \varepsilon) CS_m^K & (\lambda - \sigma) WS_m^L + (\sigma - \varepsilon) CS_m^L & (\lambda - \sigma) WS_m^M + (\sigma - \varepsilon) CS_m^M - \lambda \end{pmatrix}, \tag{C.56}$$

$$z \equiv \begin{pmatrix} d \ln r \\ d \ln w \\ d \ln p^M \end{pmatrix},$$

[15]Note the similarity of the definition to the one used in the homogeneous case in equation (3.14). WS_y^X is the version that accommodates heterogeneity and contains the homogeneous case as a special case–with homogeneous firms, $WS_y^X = WS^X$ for any y, and WS^F coincides with the one in equation (3.14).

$$b \equiv \begin{pmatrix} (\sigma - \varepsilon)\, CS_k^M \\ (\lambda - \sigma)\, WS_l^M + (\sigma - \varepsilon)\, CS_l^M \\ (\lambda - \sigma)\, WS_l^M + (\sigma - \varepsilon)\, CS_l^M - \lambda - (\lambda - 1) \end{pmatrix} d \ln a^M. \tag{C.57}$$

Hence, if A is non-singular, z can be solved as $z = A^{-1} b$ and the log first-order approximation of labor share (3.6) can be solved accordingly. Indeed, equation (3.6) shows that only $x_2 - x_1$ is our variable of interest.

To proceed, note that

$$z_2 - z_1 = \sum_{j=1}^{3} \left(a_{2j}^{-1} - a_{1j}^{-1} \right) b_j,$$

where a_{ij}^{-1} is (i, j)-element of inverse matrix A^{-1}. According to the formula of inverting 3×3 matrices, we have

$$\begin{aligned} z_2 - z_1 \propto\ & \{(a_{31} + a_{32})\, a_{23} - (a_{21} + a_{22})\, a_{33}\}\, b_1 \\ & + \{(a_{11} + a_{12})\, a_{33} - (a_{31} + a_{32})\, a_{13}\}\, b_2 \\ & + \{(a_{21} + a_{22})\, a_{13} - (a_{11} + a_{12})\, a_{23}\}\, b_3 \\ = &\ (a_{11} + a_{12})\,(a_{33} b_2 - a_{23} b_3) + (a_{21} + a_{22})\,(a_{13} b_3 - a_{33} b_1) \\ & (a_{31} + a_{32})\,(a_{23} b_1 - a_{13} b_2)\,. \end{aligned}$$

Note that from equations (C.56) and (C.57), we have $a_{13} = b_1, a_{23} = b_2$, and $a_{33} = b_3 + (\lambda - 1)$. Therefore, $a_{33} b_2 - a_{23} b_3 = [b_3 + (\lambda - 1)]\, b_2 - b_2 b_3 = (\lambda - 1)\, b_2$, $a_{13} b_3 - a_{33} b_1 = b_1 b_3 - [b_3 + (\lambda - 1)]\, b_1 = -(\lambda - 1)\, b_1$, and $a_{23} b_1 - a_{13} b_2 = b_2 b_1 - b_1 b_2 = 0$. Using these facts, we further deduce that

$$z_2 - z_1 \propto (a_{11} + a_{12})\, b_2 + (a_{21} + a_{22})\, b_1.$$

Substituting the elements in expressions (C.56) and (C.57), we finally have expression (3.26).

C.2.7 Derivation of Equations (3.27) and (3.28)

Here, we prove only equation (3.27), as (3.28) follows the same logic. First, recall that the quantity-conditional factor demand cross derivatives $\partial \widetilde{k}_i / \partial \widetilde{p_i^M}|_{q_i}$ and $\partial \widetilde{m}_i / \partial \widetilde{r_i}|_{q_i}$ are equal because (i) due to Shephard's lemma, factor demands are the partial derivative of the (quantity-conditional) cost function and (ii) the Hessian matrix is symmetric due to Young's theorem. Thus we have

$$\sigma_{\widetilde{m}\widetilde{r},i}|_{q_i} \equiv \frac{\widetilde{r_i}}{\widetilde{m_i}}\frac{\partial \widetilde{m}_i}{\partial \widetilde{r_i}}|_{q_i} = \frac{\widetilde{r_i}}{\widetilde{m_i}}\frac{\partial \widetilde{k}_i}{\partial \widetilde{p_i^M}}|_{q_i} = \frac{rk_i}{p^M m_i}\sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i},$$

where the last equality follows by definition $\sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i} \equiv \left(\widetilde{p_i^M}/\widetilde{k}_i\right)\left(\partial\widetilde{k}_i/\partial\widetilde{p_i^M}|_{q_i}\right)$ and the definitions of factor augmentation notations (tilde). By rearranging, we have

$$CS_i^M \sigma_{\widetilde{m}\widetilde{r},i}|_{q_i} = CS_i^K \sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i}. \tag{C.58}$$

Finally, we have

$$\begin{aligned}\sigma_{\widetilde{m}\widetilde{r},i} &\equiv \sigma_{\widetilde{m}\widetilde{r},i}|_{q_i} + \sigma_{qp}\frac{\partial \ln \widetilde{p}_i}{\partial \ln \widetilde{r_i}} \\ &= \sigma_{\widetilde{m}\widetilde{r},i}|_{q_i} + \sigma_{qp} CS_i^K,\end{aligned}$$

where the last equality again follows Shepard's lemma. Similarly, we have $\sigma_{\widetilde{k}\widetilde{p^M},i} = \sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i} + \sigma_{qp}CS_i^M$. Hence, we have

$$CS_i^M\sigma_{\widetilde{m}\widetilde{r},i} = CS_i^M\sigma_{\widetilde{m}\widetilde{r},i}|_{q_i} + CS_i^M\sigma_{qp}CS_i^K = CS_i^K\sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i} + CS_i^K\sigma_{qp}CS_i^M = CS_i^K\sigma_{\widetilde{k}\widetilde{p^M},i},$$

where the second equality holds by equation (C.58).

C.2.8 Properties of the Factor Demand Elasticity Matrix

Generalizing the results in Section C.2.7, we may establish the following result regarding off-diagonal elements of the (quantity-controlled and -uncontrolled) elasticity matrix. We further establish additional results below.

Proposition 5. *(i) (Off-diagonal elements) For any $f, g \in \{k, l, m\}$ with $f \neq g$,*

$$CS_i^f\sigma_{\widetilde{f}\widetilde{g},i}|_{q_i} = CS_i^g\sigma_{\widetilde{g}\widetilde{f},i}|_{q_i},$$

$$CS_i^f\sigma_{\widetilde{f}\widetilde{g},i} = CS_i^g\sigma_{\widetilde{g}\widetilde{f},i}.$$

(ii) (Singularity) $\Sigma_i|_{q_i}$ *is singular.*

Proof. (i) is a generalization of other factor-factor price pairs of the arguments in Section C.2.7. To show (ii), recall that by zero-th order homogeneity of the cost-minimizing factor demand functions, Euler's theorem implies that

$$\frac{\partial \widetilde{k_i}}{\partial \widetilde{r_i}}|_{q_i}\widetilde{r_i} + \frac{\partial \widetilde{k_i}}{\partial \widetilde{w_i}}|_{q_i}\widetilde{w_i} + \frac{\partial \widetilde{k_i}}{\partial \widetilde{p_i^M}}|_{q_i}\widetilde{p_i^M} = 0.$$

Hence, we have

$$\sigma_{\widetilde{k}\widetilde{r},i}|_{q_i} + \sigma_{\widetilde{k}\widetilde{w},i}|_{q_i} + \sigma_{\widetilde{k}\widetilde{p^M},i}|_{q_i} = \frac{1}{\widetilde{k_i}}\left(\frac{\partial \widetilde{k_i}}{\partial \widetilde{r_i}}|_{q_i}\widetilde{r_i} + \frac{\partial \widetilde{k_i}}{\partial \widetilde{w_i}}|_{q_i}\widetilde{w_i} + \frac{\partial \widetilde{k_i}}{\partial \widetilde{p_i^M}}|_{q_i}\widetilde{p_i^M}\right) = 0.$$

Similar arguments apply for augmented-factor demand functions $\widetilde{l_i}$ and $\widetilde{m_i}$, so we have

$$\left(\Sigma_i|_{q_i}\right)\mathbf{1} = 0.$$

Hence, $\Sigma_i|_{q_i}$ is singular. □

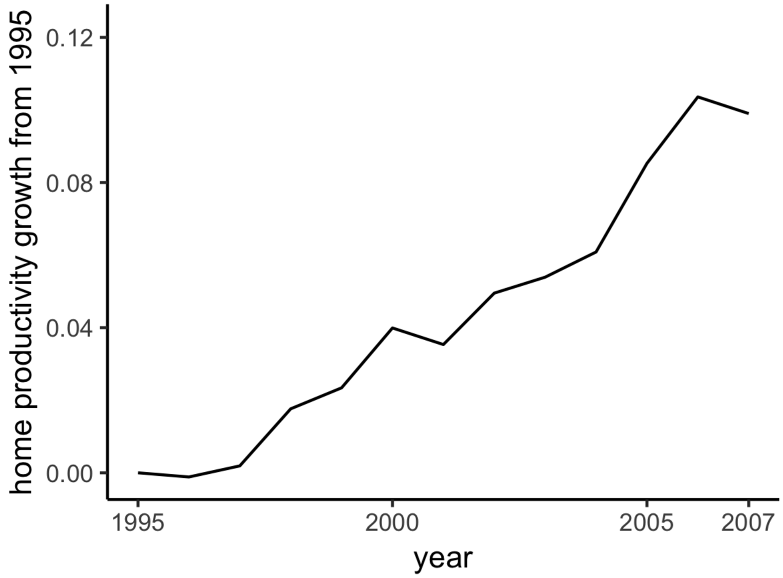
C.3 Further Simulation Results

C.3.1 Calibration Details

Home Labor Productivity Growth since 1995

Figure C.22 shows the evolution of $d\ln a_t^L \approx \ln a_t^L - \ln a_{1995}^L$, with the base year 1995. Although labor productivity in Japan has grown somewhat since 1995, it is less than the foreign productivity growth shown in the left panel of Figure 3.4, with foreign factor productivity growing by more than three log points from 1995 to 2007 while labor productivity in Japan grew only by 0.1 log point. This is consistent with our interpretation of foreign factor augmentation; that is, that over the period, other countries grew relatively more quickly than Japan due to relatively faster technological growth and several international trade liberalization events such as China's entry into the WTO.

Figure C.22: Evolution of $d\ln a^L$



Factor-bias and Implied Factor Augmentation

Here we provide additional suggestive evidence that λ is likely to be greater than one, which indicates that foreign factor augmentation is foreign factor-*biased* technological change. Note that relative aggregate employment L^F/L^H and wages w^F/w^H increased over the period of interest. Then $\lambda < 1$ would imply that our model inversion

$$\frac{a_t^M}{a_t^L} = \left(\frac{M_t}{L_t}\right)^{\frac{1}{\lambda-1}} \left(\frac{p_t^M}{w_t}\right)^{\frac{\lambda}{\lambda-1}}$$

means *decreasing* relative foreign productivity. As a numerical example, Figure C.23 shows $d\ln\left(a_t^M/a_t^L\right) = d\ln a_t^M - d\ln a_t^L$ when $\lambda = 0.2\,(=\sigma)$. Therefore, $\lambda < 1$ would imply observed relative foreign factor *compression* over the period 1995-2007.

Sensitivity to Parameters

Our numerical results in Section 3.5.1 are sensitive to parameter values λ and σ, as we show some implied counterfactual results for different values in Figure C.24.

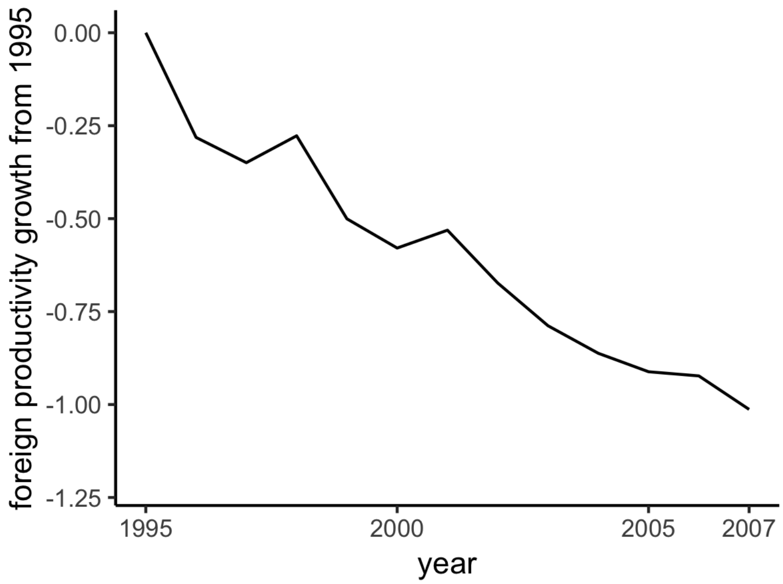


Figure C.23: Implied a^M Trend When $\lambda = \sigma = 0.2$

C.3.2 Implications of a More Recent Trend, 1995-2015

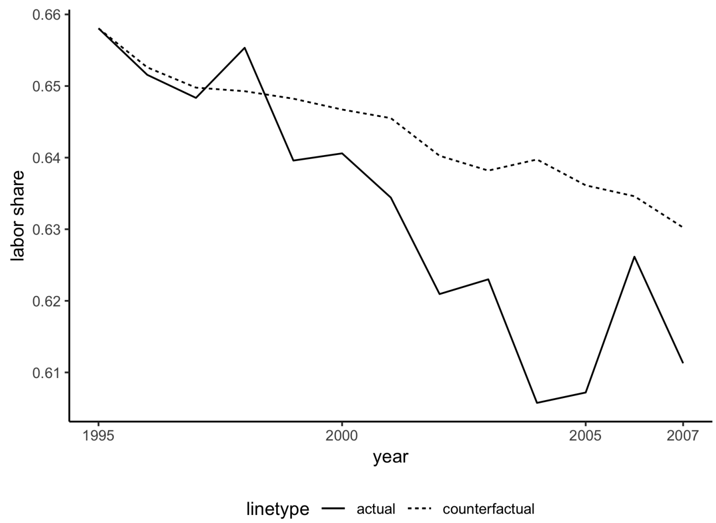
The 2008SNA from the Japan Cabinet Office provides System of National Accounts (SNA) data for 1994-2015. The 2008SNA introduced many modifications to the previous SNAs, among them the capitalization of Research and Development (R&D) expenditures which, for our purposes, bumps up value added and drives down labor share.

Another qualification regarding the use of the more recent trend is the Great Recession that began in 2008 which empirically shows a halt in the reduction in labor share in the mid 2000s that is consistent with a widely found fact that labor share is countercyclical [Schneider, 2011]. Since our focus is the structural change in foreign factor augmentation, we do not emphasize this period, but as described below, our mechanism of *relative* factor substitution of labor might help to explain this countercyclicality.

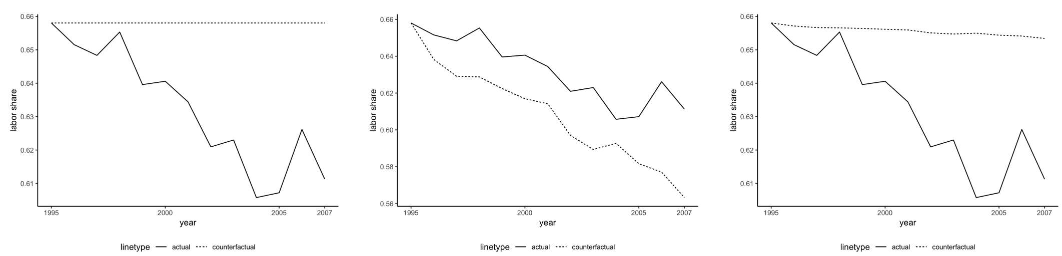
Figure C.25 shows the actual and counterfactual labor share trends derived by the same exercise as in Figure 3.4b. As mentioned above, while labor share has generally been decreasing, there was a halt in the decline in the mid 2000s to early 2010s. Remarkably, the counterfactual trend shows a similar pattern of an overall decline that paused during the Great Recession. To understand this, recall that an important factor driving the changes in the counterfactual trend is the observed foreign factor augmentation (e.g., equation (3.25)),

Figure C.24: Sensitivity Analysis to Parameter Values λ and σ

(a) (Preferred Calibration) $\lambda = 1.4$, $\sigma = 0.2$

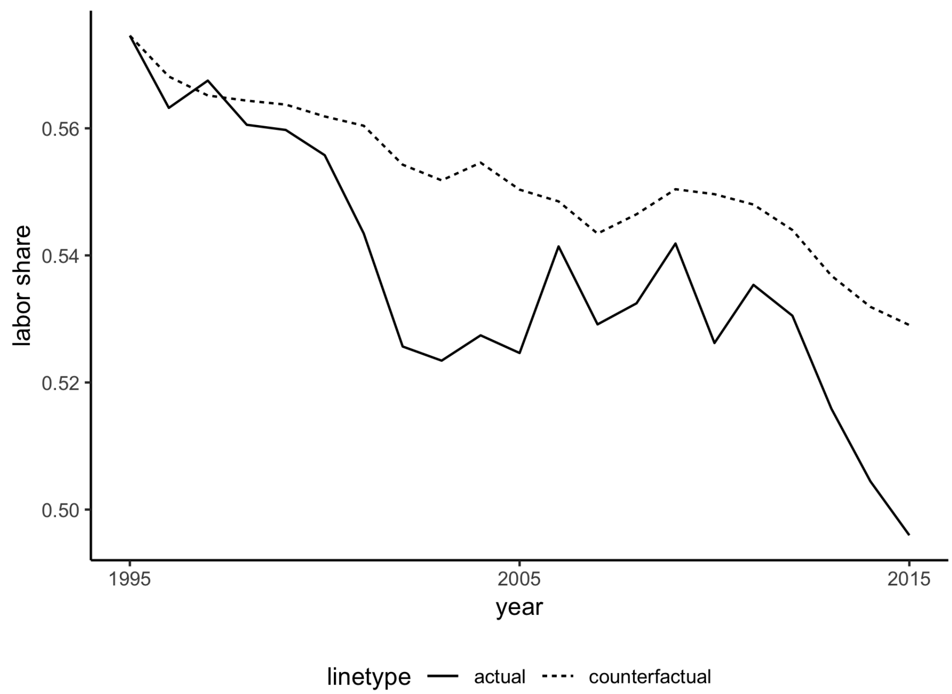


(b) $\lambda = \sigma = 0.2$ (c) $\lambda \to 1$, $\sigma = 0.2$ (d) $\lambda = 1.4$, $\sigma = 0.7$



which is backed out by relative foreign employment and wages. To the extent that during the Great Recession globalization halted and MNE multinational activities stalled, the measured foreign factor augmentation process slowed down and even reversed, which is consistent with the slow-down and increase in counterfactual labor share seen in Figure C.25. Although we do not centralize this hypothesis in the current paper, a more thorough examination of the validity of attributing the countercyclicality in labor share to the countercyclicality of globalization would be a promising future research project. Finally, from the quantitative analysis, we find that during period 1995-2015, 57.9 percent of the decrease in the labor share may be explained by increased productivity of foreign factors.

Figure C.25: Actual and Counterfactual Labor Shares, 1995-2015



C.3.3 Standard Errors of the Method of Moments Estimator

To find the standard errors of the method of moments estimator given by (3.9), we refer to Greene [2003], Section 13.2.2, and conclude that

$$\sqrt{n}\,(\widehat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}_0) \rightarrow_d N\left(0, [\Gamma(\boldsymbol{\sigma}_0)]^{-1}\,\Phi\,[\Gamma(\boldsymbol{\sigma}_0)]^{-1}\right) \tag{C.59}$$

under a set of regularity conditions, where n is the sample size, $\boldsymbol{\sigma} \equiv \left(\sigma_{\widetilde{k}\widetilde{p^M}}, \sigma_{\widetilde{l}\widetilde{p^M}}\right)'$ is the vector of target parameters, $\boldsymbol{\sigma}_0$ is the true value and $\widehat{\boldsymbol{\sigma}}$ is the estimator implied by the sample analog of equation (3.9). Furthermore, the 2×2 matrices $\Gamma(\boldsymbol{\sigma})$ and Φ's are such that

$$\sqrt{n}\left(\frac{1}{n}\sum_i Z_i \boldsymbol{a}_i(\boldsymbol{\sigma}_0)\right) \rightarrow_d N(0, \Phi),$$

$$\frac{1}{n}\sum_i Z_i \nabla_{\boldsymbol{\sigma}} \boldsymbol{a}_i(\boldsymbol{\sigma}) \rightarrow_p \Gamma(\boldsymbol{\sigma}),$$

for any $\boldsymbol{\sigma}$, where n is the effective sample size after removal of fixed effects, $\boldsymbol{a}_i(\boldsymbol{\sigma}) = \left(d\ln a_i^K(\boldsymbol{\sigma}), d\ln a_i^L(\boldsymbol{\sigma})\right)'$ and the dependence on $\boldsymbol{\sigma}$ is given by equations (3.10), (3.11), and Assumption 3. To learn more about $\nabla_{\boldsymbol{\sigma}}\boldsymbol{a}_i(\boldsymbol{\sigma})$, recall that the derivative of the inverse matrix is given by

$$\frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1} = -(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\mathbf{0}_{(1,3)}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}$$

where $\mathbf{0}_{(i,j)}$ is the 3×3 matrix filled with one in its (i,j) element and zeros elsewhere. Note also that

$$\begin{aligned}0 &= \frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}I = \frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}\left[(I+\Sigma_i(\boldsymbol{\sigma}))(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\right]\\ &= \left[\frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}(I+\Sigma_i(\boldsymbol{\sigma}))\right](I+\Sigma_i(\boldsymbol{\sigma}))^{-1} + (I+\Sigma_i(\boldsymbol{\sigma}))\left[\frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\right].\end{aligned}$$

Using these, we may solve

$$\nabla_{\boldsymbol{\sigma}}\boldsymbol{a}_i(\boldsymbol{\sigma}) = \left(\frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}\boldsymbol{a}_i(\boldsymbol{\sigma}), \frac{\partial}{\partial\sigma_{\widetilde{lp^M}}}\boldsymbol{a}_i(\boldsymbol{\sigma})\right)$$

and

$$\begin{aligned}\frac{\partial}{\partial\sigma_{\widetilde{kp^M}}}\boldsymbol{a}_i(\boldsymbol{\sigma}) &= I_{-(3,\cdot)}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\mathbf{0}_{(1,3)}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\boldsymbol{p}_i,\\ \frac{\partial}{\partial\sigma_{\widetilde{lp^M}}}\boldsymbol{a}_i(\boldsymbol{\sigma}) &= I_{-(3,\cdot)}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\mathbf{0}_{(2,3)}(I+\Sigma_i(\boldsymbol{\sigma}))^{-1}\boldsymbol{p}_i,\end{aligned}$$

where

$$I_{-(3,\cdot)} \equiv \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix},\ \boldsymbol{p}_i = \begin{pmatrix}d\ln(rk_i)\\ d\ln(wl_i)\\ d\ln\left(p^M m_i\right)\end{pmatrix}.$$

To estimate Φ and $\Gamma(\boldsymbol{\sigma})$, we use their sample analogs $\widehat{\Phi}$ and $\widehat{\Gamma}(\boldsymbol{\sigma})$

$$\widehat{\Phi}_{j_1,j_2} = \frac{1}{n-1}\sum_i Z_i^2 d\ln a_i^{j_1}(\widehat{\boldsymbol{\sigma}})\, d\ln a_i^{j_2}(\widehat{\boldsymbol{\sigma}}),$$

$$\widehat{\Gamma}(\boldsymbol{\sigma}) = \frac{1}{n}\sum_{i} Z_i \nabla_{\boldsymbol{\sigma}} \boldsymbol{a}_i(\boldsymbol{\sigma}),$$

where $j_1, j_2 \in \{K, L\}$. We evaluate $\widehat{\Gamma}$ at the estimated parameter value $\boldsymbol{\sigma} = \widehat{\boldsymbol{\sigma}}$, and then estimate the finite approximation of the asymmetric distribution of the estimator (C.59) by

$$\frac{1}{n}\left[\widehat{\Gamma}(\widehat{\boldsymbol{\sigma}})\right]^{-1}\widehat{\Phi}\left[\widehat{\Gamma}(\widehat{\boldsymbol{\sigma}})\right]^{-1}.$$

Table C.11: Industry-level Regression

(a) 2SLS

VARIABLES	(1) 2SLS	(2) 2SLS	(3) 2SLS	(4) 2SLS	(5) 2SLS	(6) 2SLS
Log Subsidiary Employment	0.120	-0.00447	0.168	0.0774	-0.184	0.507
	(0.0501)	(0.610)	(0.0486)	(0.0694)	(0.162)	(0.292)
Observations	3,704	773	540	563	521	915
Firm FE	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES
Industry	manuf	chem	metal	machine	elec	auto

Robust standard errors in parentheses.

(b) First Stage

VARIABLES	(1) 1st	(2) 1st	(3) 1st	(4) 1st	(5) 1st	(6) 1st
Thai Flood Shock	-0.730	-0.152	-1.655	-2.223	-0.655	-0.303
	(0.169)	(0.173)	(0.358)	(1.101)	(0.161)	(0.132)
Observations	3,704	773	540	563	521	915
Firm FE	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES
Industry	manuf	chem	metal	machine	elec	auto

Robust standard errors in parentheses.

(c) Reduced Form

VARIABLES	(1) reduced	(2) reduced	(3) reduced	(4) reduced	(5) reduced	(6) reduced
Thai Flood Shock	-0.0874	0.000677	-0.277	-0.172	0.120	-0.154
	(0.0428)	(0.0923)	(0.0594)	(0.225)	(0.105)	(0.0700)
Observations	3,704	773	540	563	521	915
Firm FE	YES	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES	YES
Industry	manuf	chem	metal	machine	elec	auto

Robust standard errors in parentheses.

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