

Scattering of weakly interacting massive particles from ^{73}Ge

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Abstract

We use a “hybrid” method, mixing variationally-determined triaxial nuclear Slater determinants, to calculate the response of ^{73}Ge to hypothetical dark-matter particles such as neutralinos. The method is a hybrid in that rotational invariance is approximately restored prior to variation and then fully restored before the mixing of intrinsic states. We discuss such features of ^{73}Ge as shape coexistence and triaxiality, and their effects on spin-dependent neutralino cross sections. Our calculations yield a satisfactory quadrupole moment and an accurate magnetic moment in this very complicated nucleus, suggesting that the spin structure and thus the axial–vector response to dark matter particles is modeled well.

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The identity of the invisible stuff believed to constitute most of the material universe is still unknown [1,2]. For several years now, heavy weakly-interacting particles (WIMPs) have been an attractive candidate [2]. Although some recent evidence indicates the possible presence of macroscopic objects in the galactic halo [3], statistics are poor and conclusions uncertain, and the WIMP hypothesis is still quite plausible. A variety of experiments to detect WIMPs are in fact either already operating or in the planning/prototype stage. Among the most promising is a germanium-based detector that incorporates the odd-mass isotope ^{73}Ge [4], which carries spin. While scalar (spin-independent) cross sections for “neutralinos”, perhaps the most plausibly motivated WIMP, now appear usually to be larger than the axial-vector (spin-dependent) cross sections in this nucleus [5], there are still regions of parameter space for which this is not so. Furthermore, WIMPs with no scalar interactions, such as heavy Majorana neutrinos (perhaps with reduced coupling to the Z), have not yet been completely ruled out. A careful investigation of the spin-dependent response of ^{73}Ge is therefore desirable. Spin-independent scattering can be easily calculated following, e.g., the work of Ref. [6]

Several papers have addressed aspects of spin-dependent scattering from ^{73}Ge . Engel and Vogel [7] used data from magnetic moments to estimate the quenching of the neutron spin in several heavy nuclei, including germanium. Iachello, Krauss and Maino [8] employed the Interacting Boson Fermion Model, and Nikolaev and Klapdor-Kleingrothaus [9] used finite-fermi-systems theory to calculate the same quantities (Ref. [8] also included a calculation of the small proton spin). The most comprehensive study of ^{73}Ge to date appears in Ref. [10] where a large-basis shell-model calculation was performed and the full spin-dependent neutralino response, including the finite- q form factors [6], was calculated. Here we present an alternative, equally comprehensive calculation that, we argue, is better than any of the above and which leads to significantly different results from those obtained so far.

The principal difficulty in describing the spin of ^{73}Ge is its complicated collective structure. A few years ago, in a series of papers [11], the EXCITED VAMPIR method [12] was used to explore the even-isotopes of germanium. ^{72}Ge turned out to be particularly interesting and complicated. The authors concluded that both oblate and prolate shapes were represented in the ground state wave function, a scenario that most likely persists in ^{73}Ge .

As difficult as it is to describe collective shape coexistence in even-even nuclei, it is even more so in odd-mass systems. Shell-model methods have a hard time incorporating enough mixing of spherical configurations to properly describe such dynamics, particularly in odd-mass or odd-odd systems. While the IBFM can incorporate the dominant collective effects, it has trouble including the spin polarization that plays a crucial role in axial-vector scattering. In addition, it cannot be readily applied at non-zero momentum transfer. The VAMPIR method, though in principle well suited for such problems, cannot at present be used in odd-mass systems.

The method we use here is described in detail in Ref. [13]. It shares with VAMPIR the idea of mixing variationally determined Slater determinants, in which symmetries are broken but restored either before or after variation. In our calculation the symmetries broken in the intrinsic states are those associated with rotational invariance, parity, and axial shape. Ideally we would like to restore all these symmetries before variation; unfortunately that is too expensive computationally at present. Our “hybrid” procedure is to restore axial symmetry, parity invariance, and approximate rotational invariance (using a method similar

to that proposed by Kamlah [14]) prior to the variation of each intrinsic state, and then subsequently to fully restore rotational invariance before mixing the intrinsic states.

The procedure differs from VAMPIR, in that it allows fully triaxial Slater determinants at the expense of particle–number breaking [15]. Our recent work [13], along with that of other groups using completely different methods [16], indicates that the trading of number nonconservation for triaxiality is a good idea, despite the apparent loss of pairing correlations traditionally associated with the former. Pairing forces evidently induce effective triaxiality. Though the precise relationship of triaxiality to pairing needs to be further clarified, numerical results [13] in, e.g., the $0s,1d$ shell show that our approach is as accurate and efficient as VAMPIR for describing even–even systems while also providing a reliable reproduction of the collective dynamics of odd–mass systems, something VAMPIR cannot yet do precisely because of its BCS–like treatment of pairing.

In the calculations for ^{73}Ge that we report below, we assume a single–particle space for both protons and neutrons consisting of the full $0f, 1p$ shell plus the $0g_{9/2}$ and $0g_{7/2}$ orbitals. Our goal is to include all of the single–particle orbits that are important for low–energy properties of the nucleus ^{73}Ge plus all of their spin–orbit partners. Despite its size — no space this large can be fully treated in the shell model — our space imposes only a modest burden on the computer programs that implement the hybrid method.

The size of our space does, however, lead to one feature that is not present in the $0s,1d$ –shell tests reported in Ref. [13]. There all single–particle levels have the same parity. In our current work, the single–particle basis includes levels of both positive and negative parities, allowing parity invariance to be broken. At first glance, this may seem like an unfortunate complication; in fact it is a benefit. Consider a system of four identical particles interacting via a pairing force acting in two closely related model spaces. The first contains two degenerate single–particle levels, the $f_{7/2}$ and the $f_{5/2}$, with the same parities; the second contains the levels $g_{7/2}$ and $f_{5/2}$, with the same angular momenta as in the first but now with opposite parities. The exact ground–state energies in these two models are identical. A mixing of triaxial mean–field states can always describe the ground state of the two systems, the only issue being how many intrinsic states are required. We have carried out calculations for both models with the clear conclusion that convergence is more rapid in the second. The reason is also clear: the freedom to break (and subsequently restore) another symmetry permits us to build more correlations into the intrinsic states.

One caveat accompanies parity mixing: ideally it should be done democratically. Put another way, all of the dominant single–particle orbits should be allowed to benefit from parity mixing. At the practical level, this means that two complete oscillator shells should be included in a parity–mixed calculation. Since we are not currently able to include so large a single–particle space for germanium, we will somehow have to simulate the missing effects in our analysis. We will discuss how we do this shortly.

Returning to germanium, we note that a crucial ingredient in any realistic nuclear–structure calculation is an appropriate nuclear hamiltonian. The one– and two–body parts must be compatible with one another and also with the model space. This is difficult to achieve; microscopic two–body interactions, derived for example from a G–matrix, include monopole pieces that are unable to describe the movement of spherical single–particle levels as one passes from the beginning to the end of a shell [17,18]. In several papers, including a very recent one on neutrino scattering from iodine [18], we proposed a procedure for avoiding

this problem. It consists basically of removing from the two-body interaction *all* monopole components and shifting their effects to the single-particle energies. We follow the same procedure here; our two-body force, for example, is a fit to a Paris-potential G-matrix [19], modified as just described.

To determine spherical single-particle energies, we first carry out BCS fits with the above force to the spherical quasi-particle energies in the mass-71 isotopes ^{71}Ga and ^{71}Zn . These are the odd-mass nuclei closest to ^{73}Ge that are nearly spherical. For levels very far from the Fermi surface, where quasiparticle energies are ambiguous, we estimate the single-particle energies from general systematics. Since our model space does not include two full oscillator shells, however, we have found it necessary to adjust slightly some of the BCS-produced single-particle energies. The $0f_{7/2}$ and $0f_{5/2}$ orbits strongly mix with the $0g_{9/2}$ and $0g_{7/2}$ orbits, respectively, which have opposite parity, but our space does not include positive-parity orbits to mix with the $1p_{3/2}$ and $1p_{1/2}$. We therefore adjust the single-particle energies of the $1p$ levels to simulate the missing levels; our criterion is a reasonable reproduction of the occupation numbers obtained in the VAMPIR calculations of ^{72}Ge . The end result is the set of single-particle energies shown in Table I. We should note here that these energies are different from and more reasonable than those used in Ref. [10]. We believe the difference can be attributed to insufficient correlations in the shell model calculation [10], and to our removal of monopole forces.

The results of our calculations involve only the $J^\pi = 9/2^+$ ground state; for technical reasons [13], higher-lying states are not as well modeled. We have included in our analysis four intrinsic states, whose properties are summarized in Table II. The absolute energies are meaningless but the energy differences between configurations are important. The variables β and γ are the usual radial and angular measures of deformation in the collective model [20] (given here for both protons and neutrons). They imply that the lowest state is predominately oblate and slightly triaxial, and the second predominantly prolate and also slightly triaxial. These two intrinsic states mix very little with one another, however. This is a bit surprising since there seems to be significant oblate-prolate mixing in the VAMPIR results for ^{72}Ge . The other two intrinsic states, both predominantly oblate, do mix somewhat more strongly into the ground state, lowering its energy by 0.95 MeV. A summary of important quantities in the final mixed ground state is given in Table III.

The most important result, the ground-state magnetic dipole moment, is in excellent agreement with the experimental value. Such good agreement is difficult to achieve. The Schmidt magnetic moment (arising from a pure $0g_{9/2}$ neutron configuration) is $-1.91 n.m.$, very far from the experimental value of $-0.88 n.m.$. Ressel and collaborators [10], in their “large-space” shell-model calculation, were able to quench the magnetic moment significantly to $-1.24 n.m.$, but could not account for the remaining difference. Our calculation, despite the small number of intrinsic states, contains the full quenching required by experiment, a point on which we elaborate shortly. (It is interesting to note that we already achieve a magnetic moment of $\mu = -0.909 n.m.$ with just the lowest intrinsic state.)

Our calculated quadrupole moment is not quite as good (the experimental value is $-21 e - fm^2$), though still reasonable. The discrepancy is probably an indication that there should be somewhat more mixing of the prolate solution in our ground state, which would take us the right direction. Of course it is also possible that the inclusion of additional intrinsic states could modify the quadrupole moment.

To clarify the differences between our magnetic moment and that of Ref. [10], we compare in Table IV the results of both approaches for the various spin and orbital angular momenta that contribute. The most important difference occurs in the neutron spin, for which our result is significantly smaller. The large and negative neutron spin g -factor ($g_n^s = -3.826$) makes this the chief source of our improved result. The differences in the spins, unlike those in the orbital angular momenta, carry over into WIMP scattering cross sections.

Magnetic moments can be modified in two fundamentally different ways — through nuclear structure correlations not included in the model calculation, and through meson exchange corrections and/or nucleonic resonances. The non-nuclear contributions, though certainly important in a detailed theory of magnetic moments, rarely affect the final results by more than about 10%. Since our goal is a reliable description of the nuclear spin structure, we aim at (and achieve) agreement with experiment moment to within 10%. In the work of Ref. [10], as noted above, the magnetic moment is too large by roughly 30%. It appears to us that most of this discrepancy is due to the omission of important nuclear structure correlations. To compensate, Ref. [10] advocates quenching the isovector spin only (note: this is not done correctly everywhere in the paper; the two nominally equivalent prescriptions for quenching neutralino cross sections are in fact different), using the observation that the isovector spin is more strongly quenched from *its Schmidt value* than the isoscalar spin. It is not obvious, however, that the same statement should be true when the spins have already been partly quenched by nuclear correlations. Our results indicate that quenching both the isoscalar and isovector “large-space” spins would have been the best procedure in Ref. [10]. Of course, such a prescription should not be blindly extended to non-zero momentum transfer [21]. Our calculations, since they apparently correctly represent the spin structure, require no quenching at $q = 0$ and no arbitrary assumptions about how the form factor should change at $q \neq 0$. For these reasons we believe our neutralino cross sections, to which we now turn, to be the most reliable yet obtained.

In Fig. 1, we present the three functions that determine the spin-dependent cross sections for any neutralino at all momentum transfers, written in terms of $y = (bq/2)^2$, where $b = A^{1/6}$ fm = 2.04 fm is the oscillator parameter (the precise definitions of the three functions are given in refs. [6,21]). Comparing the results for $S_{00}(y)$ (the pure isoscalar form factor) and $S_{11}(y)$ (the isovector form factor) with the corresponding large-space results of Ref. [10] (see Fig. 4 of that reference), we conclude that *both* are reduced relative to theirs. Quenching the isovector cross sections alone does not seem appropriate at any momentum transfer.

To allow use of these functions (or form factors) we have made polynomial fits to them. They are very well represented over the full range of relevant momenta by the following sixth-order polynomials:

$$\begin{aligned}
S_{00}(y) &= 0.1606 - 1.1052 y + 3.2320 y^2 - 4.9245 y^3 \\
&\quad + 4.1229 y^4 - 1.8016 y^5 + 0.3211 y^6 \\
S_{11}(y) &= 0.1164 - 0.9228 y + 2.9753 y^2 - 4.8709 y^3 \\
&\quad + 4.3099 y^4 - 1.9661 y^5 + 0.3624 y^6 \\
S_{01}(y) &= - 0.2736 + 2.0374 y - 6.2803 y^2 + 9.9426 y^3 \\
&\quad - 8.5710 y^4 + 3.8310 y^5 - 0.6948 y^6
\end{aligned}$$

In Fig. 2, we show the full axial form factor for “B-ino” scattering from ^{73}Ge , the same choice considered in Ref. [10]. We assume that the spin structure of the nucleon is as given by the original EMC experiment, which, we should note, has recently been called into question [22]. For comparison, we also show the Independent Single Particle Shell Model (ISPSM) result, derived under the assumption that all the spin is carried by one $0g_{9/2}$ neutron. Importantly, nuclear correlations significantly quench the ISPSM form factor at *all* momentum transfers. The quenching at $q = 0$ by a factor of almost three persists out to very large momentum transfers and in fact even grows slightly stronger. This most likely reflects the prominent role played by the $0g$ orbits in building the correlations that lead to spin quenching.

Several improvements to our analysis should be considered in the future. The most important is an explicit incorporation of the remaining orbits from the $2s,1d,0g$ shell, whose effects were treated very roughly here. This should enable us to remove some of the arbitrariness in the single-particle energies that resulted from our incomplete treatment of parity-mixing effects. It may also lead to a slight lowering of the prolate intrinsic state, thereby permitting more mixing with the dominant oblate state. Shell-model calculations have more or less reached their limit for the time being; the improvements just outlined above, by contrast, can be incorporated with reasonable amounts of analysis, coding, and computer time. Should the spin-response of ^{73}Ge be needed more accurately, we will implement the improvements.

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FIGURES

FIG. 1. The calculated functions S_{00} , S_{01} and S_{11} for ^{73}Ge , as a function of $y = (bq/2)^2$ (b is the harmonic-oscillator length parameter). The solid line is S_{00} , the dashed line is S_{11} and the dotted line is S_{01} . These functions are defined in Refs. [3,21].

FIG. 2. The calculated spin structure function $S(y)$ for pure \tilde{B} scattering from ^{73}Ge , assuming EMC couplings. The solid curve gives the results of the calculation described in the text. The dashed curve gives the ISPSM results.

TABLES

TABLE I. Single-particle energies (in MeV) used in the calculation of the structure of ^{73}Ge described in the text.

orbit	protons	neutrons
$0f_{7/2}$	0	0
$1p_{3/2}$	1.5	4.2
$0f_{5/2}$	4.0	4.0
$1p_{1/2}$	3.3	5.3
$0g_{9/2}$	5.1	6.4
$0g_{7/2}$	13.0	13.5

TABLE II. Properties of the four intrinsic states for ^{73}Ge that are used in the calculations described in the text. The shape parameters are obtained in the intrinsic frame; the energy and magnetic moment in the lab frame.

Intrinsic State	Energy (MeV)	μ	β_p	γ_p (deg.)	β_n	γ_n (deg.)
1	48.27	-0.909	.14	50.3	.08	55.3
2	49.38	-0.279	.10	14.2	.05	1.7
3	49.71	-0.816	.12	53.2	.07	50.3
4	49.88	-0.857	.12	54.5	.08	55.3

TABLE III. Nuclear structure properties of the ground state of ^{73}Ge .

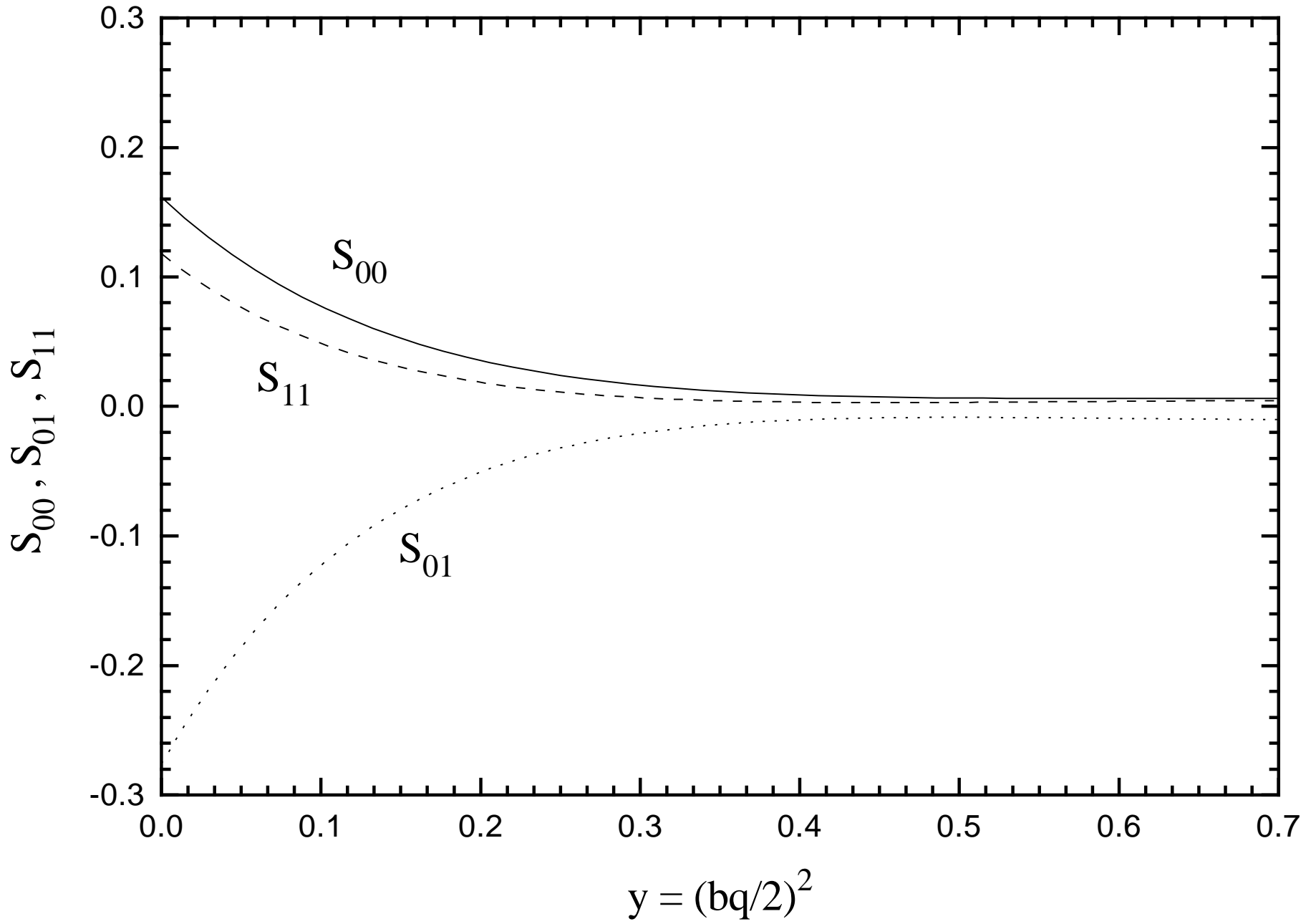
Energy (MeV)	47.32
μ (<i>n.m.</i>)	-0.920
Q ($e - fm^2$)	-40.98

TABLE IV. Comparison of spin and angular momenta of ^{73}Ge that derive from our “hybrid” variational approach and the large-basis shell-model approach of Ref. [10].

	$\langle S_p \rangle$	$\langle S_n \rangle$	$\langle L_p \rangle$	$\langle L_n \rangle$
Present Work	.030	.378	.361	3.732
Ref. [10]	.011	.491	.468	3.529

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