

# Spacetime Emergence and General Covariance Transmutation

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## Abstract

Spacetime emergence refers to the notion that classical spacetime “emerges” as an approximate macroscopic entity from a non-spatio-temporal structure present in a more complete theory of interacting fundamental constituents. In this article, we propose a novel mechanism involving the “soldering” of internal and external spaces for the emergence of spacetime and the twin transmutation of general covariance. In the context of string theory, this mechanism points to a critical four dimensional spacetime background.

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## I. EMERGING THEORIES ON EMERGENCE

Is spacetime a necessary feature of a fundamental description of nature? This problem has long been debated among physicists and philosophers; and in recent years, it has been exacerbated by the unsolved nature of quantum gravity. For one thing, a straightforward quantization of general relativity leads to uncontrollable divergences. The non-renormalizability of gravity has naturally led to the suggestion that general relativity is perhaps only a low energy effective theory, and accordingly the metric and connection forms are nothing more than the collective or hydrodynamic fields of some fundamental degrees of freedom. Then, it is a logical possibility that the physical spacetime described by Einstein's theory of gravity is actually emergent — a feature akin to hydrodynamics emerging from molecular dynamics or nuclear physics from quantum chromodynamics.

The idea that spacetime is not a fundamental physical concept and that it emerges from some other more fundamental notions is not new. For example, Sakharov argued more than forty years ago that gravity is an induced force, and the dynamical spacetime is emergent from some more fundamental “atomic” structure [1, 2]. Adler et al. [3] explored an alternative approach to a microscopic theory of gravitation, in which the gravitational fields are identified with photon pairing amplitudes of a superconducting type.

In the 1970s, the remarkable discoveries regarding black hole thermodynamics by Bekenstein, Hawking and others [4, 5] have pointed in a more precise fashion towards an emergent nature of gravity and thus the emergent nature of spacetime [6–8]. If indeed gravitation can be understood as a thermodynamic effect, the statistical origin of such a course grained notion is immediately called into question. More recently, these intuitions from semiclassical quantum gravity have been reexamined by using the notion of the holographic principle [9, 10] in the context of the celebrated AdS/CFT correspondence in string theory [11]. More generally, various string dualities suggest that spacetime is indeed only a derived concept [12]. It has also been argued [13] that string gas cosmology favors four emergent macroscopic spacetime dimensions.

Emergent spacetime is also suggested in some other theory candidates of quantum gravity. The equations of loop quantum gravity [14, 15] do not presuppose spacetime; instead they are expected to give rise to spacetime at distances large compared to the Planck length. In the causal dynamical triangulation program [16], a non-perturbative sum over geometries has

yielded a fully dynamical emergence of a classical background (and solution to the Einstein equations) with the correct dimensionality of four at large scales (and effectively two at the Planck scale).

Last but not least, the concept of emergence, which is so natural in the context of many-body physics, has been argued to apply more generally in the context of emergence of all fundamental forces in physics [17–19]. Obviously, the idea of emergent spacetime has solid roots from many points of view.

In this article, we point out a new and explicit mechanism for the emergence of spacetime and general covariance. Then, we argue that, in the context of string theory, such a mechanism points to a critical four dimensional spacetime background.

## II. EMERGENT COSMOLOGICAL CONSTANT

Consider a  $D$ -dimensional spacetime with diagonal metric  $G_{\mu\nu}$ , where  $\mu, \nu = 0, 1, \dots, D-1$ . On this spacetime, suppose there exists a  $D$ -component non-linear sigma field  $\phi^a$ . The diagonal internal-space metric, which determines the dynamics of the non-linear sigma fields, is given by the metric  $h_{ab}$ , where  $a, b = 0, 1, \dots, D-1$ . Notice that we have chosen the number of non-linear sigma fields to be exactly the same as the dimension of the spacetime. Thus, throughout the entire article, both of the spacetime and internal-space indices run from 0 to  $D-1$ .

Let us consider a theory with the  $D$ -dimensional Einstein gravity coupled to the  $D$ -dimensional non-linear sigma model, with action given by

$$S = \int d^D x \sqrt{G} \left\{ -\frac{1}{2} M^{D-2} R + \frac{1}{\lambda^2} G^{\mu\nu} h_{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b \right\}, \quad (2.1)$$

where  $R$  is the Ricci scalar of the  $D$ -dimensional spacetime,  $1/\lambda^2$  is a coupling constant, and  $M$  is the  $D$ -dimensional Planck mass.

Now, suppose we impose the following ansatz:

$$\phi^0 = i \alpha t + C, \quad (2.2)$$

$$\phi^a = \alpha x^a + C, \quad \text{for } a \neq 0, \quad (2.3)$$

where  $\alpha$  and  $C$  are constants carrying mass dimensions of  $\frac{D-2}{2} + 1$  and  $\frac{D-2}{2}$  respectively. Notice that the extra factor of  $i$  in the identification (2.2) is required to properly match

the Lorentzian and Euclidean signatures of the D-dimensional spacetime and internal-space respectively. In fact, this is the same ansatz that was imposed in [20–23] in an attempt to induce dynamical compactification and hence inflation, although exponential inflation failed to emerge. A successful model using this ansatz to generate exponential inflation has recently been provided in [24]. Interestingly, 't Hooft made a similar ansatz in [25] in his consideration of Higgs mechanism for gravity. Physically, this ansatz is equivalent to identifying the components of the non-linear sigma fields with the spacetime coordinates, which is intuitively reasonable because the non-linear sigma fields themselves are “coordinates” of the internal-space manifold. Mathematically, the ansatz is also justified as the non-linear sigma fields are functions of the spacetime coordinates, and it simply specifies an explicit dependence of the non-linear sigma fields on the spacetime coordinates.

One may wonder if there is any dynamical basis that leads to the ansatz (2.2) and (2.3). We will not address this issue here; it is certainly beyond the scope of the present paper. In fact, even the papers by Gell-Mann and Zwiebach [21, 22] and 't Hooft [25] have not provided any dynamical basis for the ansatz. In what follows, we will just adopt this ansatz to see what it implies. But, as we will show in Section III, it is precisely the form of the ansatz shown in (2.2) and (2.3) that is required for general covariance. Hence the results will provide an *a posteriori* justification of the ansatz choice.

With this ansatz, the action  $S$  now becomes

$$S = \int d^D x \sqrt{G} \left\{ -\frac{1}{2} M^{D-2} R + \frac{1}{\lambda^2} \alpha^2 (-G^{00} h_{00} + G^{ij} h_{ij}) \right\}. \quad (2.4)$$

Actually, the identifications (2.2) and (2.3) should not affect the tensor character of  $h_{ab}$ . So we expect the components of  $h_{\mu\nu}$  to be proportional to the components of  $G_{\mu\nu}$ . For a given D-dimensional spacetime, if we assume that the internal-space metric is such that

$$h_{00} = \beta G_{00} \quad \text{and} \quad h_{ij} = -\beta G_{ij}, \quad (2.5)$$

with  $\beta > 0$  being a constant, then the action can be written as

$$S = \int d^D x \sqrt{G} \left\{ -\frac{1}{2} M^{D-2} R - \frac{1}{\lambda^2} \alpha^2 \beta G^{\mu\nu} G_{\mu\nu} \right\}. \quad (2.6)$$

Since  $\frac{1}{D} G^{\mu\nu} G_{\mu\nu} = 1$ , it follows that

$$S = -\frac{1}{2} M^{D-2} \int d^D x \sqrt{G} \left( R + 2 \frac{1}{\lambda^2} \frac{D \alpha^2 \beta}{M^{D-2}} \right). \quad (2.7)$$

Thus, a cosmological constant of magnitude  $\frac{1}{\lambda^2} \frac{D\alpha^2\beta}{M^{D-2}}$  emerges. Reversing the process, we can absorb the cosmological constant in a non-linear sigma field and gain conceptual advantage as we will discuss below.

As an illustrative and realistic example of the above idea, we consider a 4-dimensional spacetime with the Friedmann-Robertson-Walker (FRW) metric in Appendix A.

### III. GENERAL COVARIANCE TRANSMUTATION

Since both sets of indices  $\{\mu, \nu\}$  and  $\{a, b\}$  run from 0 to  $D-1$ , we can form the following interactions terms and add them to the action in (2.1):

$$(I) \quad f G_{\mu\nu} h^{ab} \partial_a \phi^\mu \partial_b \phi^\nu \quad (3.1)$$

$$(II) \quad g G^{\mu\nu} h_{\mu\nu} (\partial_a \phi^a)^2 \quad (3.2)$$

Obviously, both (I) and (II) break general covariance, because spacetime and internal-space indices have been contracted in a mixed way. However, general covariance can *re-emerge* as follows. If we impose the identifications (2.2) and (2.3), the non-linear sigma fields essentially become the spacetime coordinates. There is no longer a difference between internal-space indices and spacetime indices. Instead, the spacetime indices coincide exactly with the internal-space indices. Originally,  $h_{ab}(\phi)$  is a function of the non-linear sigma fields. But after making the identifications (2.2) and (2.3),  $h_{ab}$  becomes an explicit function of spacetime coordinates. This means that when we write  $h_{ab}$ , the indices  $a$  and  $b$  become spacetime indices. In this sense, all of the spacetime or internal-space indices in  $f G_{\mu\nu} h^{ab} \partial_a \phi^\mu \partial_b \phi^\nu$  and  $g G^{\mu\nu} h_{\mu\nu} (\partial_a \phi^a)^2$  have been contracted.

In light of the above identifications (2.2) and (2.3), the action becomes

$$S' = \int d^D x \sqrt{G} \left\{ -\frac{1}{2} M^{D-2} R + \left( \frac{1}{\lambda^2} + f \right) \alpha^2 \left( -G^{00} h_{00} + G^{ij} h_{ij} \right) \right\}, \quad (3.3)$$

where we are forced to set  $g = 0$  because the term  $g G^{\mu\nu} h_{\mu\nu} (\partial_a \phi^a)^2$  leads to a factor of  $(D-1+i)^2 \alpha^2$ , rendering the action non-Hermitian and hence non-unitary. One could argue that this term may still be unitary if it is PT-symmetric [26], although we will not consider this possibility here as it is not crucial to our discussions.

Since there is now no more uncontracted spacetime indices in  $S'$ , we conclude that general covariance has emerged upon the identifications (2.2) and (2.3). As a result, the action  $S'$  is generally covariant.

We can understand the origin of the emergent general covariance as follows. Originally, we have two different sets of diffeomorphisms. One is in the internal space and the other is the non-compact spacetime. Let's call their diffeomorphism algebras  $D_1$  and  $D_2$  respectively. Before the two sets of diffeomorphisms are coupled we have  $D_1 \otimes D_2$  as the symmetry algebra of the theory. Once they are coupled, only the diagonal subalgebra  $D \subset (D_1 \otimes D_2)$  is preserved. The purpose of adopting the ansatz (2.2) and (2.3) is precisely to extract the diagonal subalgebra, which leads to the general covariance in (3.3). Moreover, we note that the idea of ‘‘soldering’’ internal and external indices has been used by Polyakov in the context of non-critical string theory [27], where it is argued that general covariance emerges from the underlying (gauge) current algebra structure.

Similar to the case in the previous section, if we assume (2.5), then the action  $S'$  can be written as

$$S' = -\frac{1}{2} M^{D-2} \int d^D x \sqrt{G} \left\{ R + 2 \left( \frac{1}{\lambda^2} + f \right) \frac{D \alpha^2 \beta}{M^{D-2}} \right\}. \quad (3.4)$$

As a result, a shifted (with respect to eq.(2.7)) cosmological constant of magnitude  $\left( \frac{1}{\lambda^2} + f \right) \frac{D \alpha^2 \beta}{M^{D-2}}$  emerges.

Therefore, we have the following conclusions. First of all, even if  $f = 0$ , a cosmological constant can already emerge upon the identifications (2.2) and (2.3). When  $f \neq 0$ , the same identifications (2.2) and (2.3) lead to another contribution to the cosmological constant. In addition, the introduction of the term  $f G_{\mu\nu} h^{ab} \partial_a \phi^\mu \partial_b \phi^\nu$  breaks general covariance. However, with the identifications (2.2) and (2.3), the broken general covariance (which exists in the first place) can be restored. This ‘‘emergence’’ of general covariance can be called ‘‘general covariance transmutation’’. General covariance is a gauge symmetry and is the starting point of general relativity. With the transmutation of general covariance at hand, the stage is set for the emergence of spacetime itself. But for the latter, we need another theoretical input as we will see in Section IV.

Finally, we would like to provide an analysis of the equation of motion for the non-linear sigma fields in order to make sure that everything is consistent with the ansatz invoked in (2.2) and (2.3). Since the term  $g G^{\mu\nu} h_{\mu\nu} (\partial_a \phi^a)^2$  leads to a non-Hermitian factor of  $(D-1+i)^2 \alpha^2$  when the ansatz is applied, we will only consider the term  $f G_{\mu\nu} h^{ab} \partial_a \phi^\mu \partial_b \phi^\nu$  in addition to the action in (2.1). The equation of motion for a fixed component field  $\phi^\beta$  is

then given by

$$2 \partial_\mu (G^{\mu\nu} h_{\beta a} \partial_\nu \phi^a) + 2 f \lambda^2 \partial_a (G_{\beta\nu} h^{ab} \partial_b \phi^\nu) - \frac{\partial}{\partial \phi^\beta} (G^{\mu\nu} h_{ab} \partial_\mu \phi^a \partial_\nu \phi^b) - f \lambda^2 \frac{\partial}{\partial \phi^\beta} (G_{\mu\nu} h^{ab} \partial_a \phi^\mu \partial_b \phi^\nu) = 0. \quad (3.5)$$

One can easily verify that the equation of motion is satisfied by the ansatz in (2.2) and (2.3) only if  $h_{00} \propto G_{00}$  and  $h_{ij} \propto G_{ij}$  which are precisely what we imposed in (2.5).

#### IV. EMERGENT SPACE(-TIME)

Let us now apply the mechanism of “soldering” internal and external spaces to string theory. Imagine that we start with 2-dimensional (2d) gravity with a cosmological constant. This can be viewed as the sigma model action for 2d string theory (in what will turn out to be a 2d spacetime background, so  $c, d = 0, 1$ )

$$S_2 = \frac{T}{2} \int d^2\sigma \sqrt{g} [g^{\rho\tau} \xi_{cd}(X) \partial_\rho X^c \partial_\tau X^d + \dots], \quad (4.1)$$

where  $T$  is the string tension,  $g_{\rho\tau}$  is the worldsheet metric,  $\xi_{cd}(X)$  is the metric of the target manifold and “...” represents the remaining terms in the full 2d string action [28]. This starting 2d sigma model is crucial on three scores. Firstly it is renormalizable, secondly the 2d gravity is merely topological, and thirdly the  $\beta$ -function of the background metric viewed as a coupling gives, to leading order in the inverse of the string tension, the Einstein equations of motion (reflecting the famous fact of perturbative string theory [29, 30])

$$L \frac{d \xi_{cd}(X)}{dL} = R_{cd}, \quad (4.2)$$

where  $L$  is the renormalization group scale of the sigma model and  $R_{cd}$  is the Ricci tensor. The 2d string can also be given a non-perturbative formulation in terms of the  $c = 1$  matrix model [31], which represents a regularization of the Polyakov path integral.

Now, we can envision increasing the string coupling until the worldsheet description is replaced by something more fundamental. One indication, from M-theory, is that the string “thickens” into a membrane [32], described by the corresponding 3d sigma (membrane) model (where now it will turn out that  $c, d = 0, 1, 2$ )

$$S_3 = \frac{T}{2} \int d^3\sigma \sqrt{g} [g^{\rho\tau} \xi_{cd}(X) \partial_\rho X^c \partial_\tau X^d + \dots], \quad (4.3)$$

where  $g_{\rho\tau}$  becomes the metric of the worldvolume and “ $\dots$ ” represents the remaining terms in the full 3d membrane action [32]. Note that this 3d sigma model is non-renormalizable and it cannot be taken as the short-distance definition of the theory. A suitable definition is supplied by a regularization of the 3d sigma model action known as Matrix theory [33, 34].

As we saw in Section III, the mechanism of “soldering” internal and external indices could lead to the twin transmutation of general covariance. By this mechanism, the worldvolume general covariance could induce the target-space general covariance and thus the ambient target spacetime. Finally, by invoking the holographic principle, one can propose that the 3d membrane theory (coupled to some other degrees of freedom) is a holographic boundary dual of some emergent bulk 4d spacetime description. The radial direction in the 4d spacetime emerges without being a space dimension in the 3d field theory. It can be interpreted as the renormalizable group scale, or the energy scale used to probe the 3d theory.

In short, the emergence of spacetime comes about when we go from weak to strong coupling (strings to membranes) and when we reinterpret the renormalization group flow of this membrane theory coupled to some other matter holographically as a 4d theory that involves 4d gravity. All these insights are supported by what we know from string theory [29, 30]. It is pleasing that the mechanism of “soldering” internal and external indices discussed in this article not only points to emergent spacetime but, in the context of string theory, also to the critical 4d spacetime background.

We have just witnessed space emergence twice (with spatial dimensions grown by two). It is now natural to ask whether time can also emerge. Note that the current knowledge of string theory relies on the definition of the underlying sigma model in terms of a *Euclidean* quantum (conformal) field theory. The Lorentzian background is obtained after the usual Wick rotation, which in the case of a general time-dependent background (and dynamical causal structure), is problematic. One might say that holography [11] resolves the issues associated with dynamical causal structure. But holography relies on the existence of a large spacetime and the associated asymptotic regions which simply do not exist in general. Given what we know about string theory both in the perturbative and holographic contexts, our proposal should be understood in the same vein.

Of course, the unity of space and time demanded by relativity would seem to imply that time also emerges, given that space emerges. On the other hand, given the manifold issues associated with time-dependence in string theory, it has been repeatedly argued that the



emergence of time may violate locality and conceivably also causality [12]. This would imply that time is fundamental even if space is emergent [35–37]. Furthermore, it seems unlikely that any physical system can evolve without an underlying time. But this is not an air-tight argument against emergent time — even the dearth of examples of which may merely be due to our lack of imagination. In any case, the problem of time emergence has profound implications for the physics of black hole and cosmological singularities, and may well shed light on the structure of the Hilbert space of the Universe and the origin of the arrow of time [12, 35–38]. However, it is quite possible that time emergence is even harder to fathom than space emergence, and that the true physics of emergent time would require even more radical ideas [35–37, 39–42] than the ones presented in this paper.

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### Appendix A: An Example Involving the Friedmann-Robertson-Walker Metric

Consider a 4-dimensional spacetime with the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - a^2(t) g_{ij} dx^i dx^j, \quad (\text{A1})$$

where  $i, j = 1, 2, 3$  and  $a(t)$  is the scale factor. The various components of the Ricci tensor  $R_{\mu\nu}$  resulting from the above metric (A1) are

$$R_{00} = -3 \frac{\ddot{a}(t)}{a(t)} G_{00}, \quad (\text{A2})$$

$$R_{ij} = - \left( \dot{H}(t) + 3 H^2(t) \right) G_{ij}, \quad (\text{A3})$$

where  $G_{00} = 1$ ,  $G_{ij} = -a^2(t) g_{ij}$  and  $H(t) = \frac{\dot{a}(t)}{a(t)}$ .

On the other hand, the equations of motion following from the variation of the action  $S$  in (2.4) with respect to the metric  $G^{\mu\nu}$  are given by

$$R_{00} = - \frac{4 \alpha^2}{M^2} \frac{1}{\lambda^2} h_{00}, \quad (\text{A4})$$

$$R_{ij} = \frac{4 \alpha^2}{M^2} \frac{1}{\lambda^2} h_{ij}. \quad (\text{A5})$$

By Einstein's field equation, the components (A4) and (A5) represent the energy source that leads to the geometry realized by the metric (A1). Thus, we proceed to solve the equations of motion (A4) and (A5) by using the geometry of the metric (A1). This requires matching (A4) with (A2) and (A5) with (A3). Hence, the metric of the internal-space manifold is required to satisfy:

$$h_{00} = 3 \frac{\ddot{a}(t)}{a(t)} \frac{M^2}{4 \left(\frac{1}{\lambda^2}\right) \alpha^2} G_{00}, \quad (\text{A6})$$

$$h_{ij} = - \left( \dot{H}(t) + 3 H^2(t) \right) \frac{M^2}{4 \left(\frac{1}{\lambda^2}\right) \alpha^2} G_{ij}. \quad (\text{A7})$$

In particular, if  $3 \frac{\ddot{a}(t)}{a(t)} = \dot{H}(t) + 3 H^2(t) = \text{constant}$ , the internal-space metric is of the form  $h_{00} = \beta G_{00}$  and  $h_{ij} = -\beta G_{ij}$ . In this case, the only solution allowed is  $a(t) \propto e^{ht}$  with  $h$  being a constant. As shown above, the emergent cosmological constant is given by  $\Lambda = \frac{4}{\lambda^2} \frac{\alpha^2 \beta}{M^2}$ , for  $D = 4$ . This gives the conventional result, namely  $h = \sqrt{\frac{\Lambda}{3}}$ , when the universe is dominated by the cosmological constant.

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- [1] A. Sakharov, Doklady Acad. Nauk. USSR, **177** (1967) 70.
  - [2] M. Visser, gr-qc/0204061.
  - [3] S. L. Adler, J. Lieberman, Y. J. Ng and H.S. Tsao, Phys. Rev. D **17** (1976) 359.
  - [4] J. D. Bekenstein, Phys. Rev. D **7** (1973) 2333.
  - [5] S. W. Hawking, Comm. Math. Phys. **43** (1975) 199.
  - [6] T. Jacobson, Phys. Rev. Lett. **75** (1995) 1260.
  - [7] T. Padmanabhan, arXiv:0912.3165[gr-qc].
  - [8] E. P. Verlinde, JHEP **1104** (2011) 029.
  - [9] G. 't Hooft, gr-qc/9310026.
  - [10] L. Susskind, J. Math. Phys. **36** (1995) 6377.
  - [11] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rept. **323** (2000) 183, and references therein.
  - [12] N. Seiberg, hep-th/0601234.
  - [13] R. Brandenberger and C. Vafa, Nucl. Phys. B **316** (1989) 391.
  - [14] L. Smolin, Sci. Am. **290** (1) (2004) 66.
  - [15] C. Rovelli, arXiv:1102.3660 [gr-qc] and references therein.

- [16] J. Ambjorn, J. Jurkiewicz, and R. Loll, Phys. Rev. Lett. **93** (2004) 131301.
- [17] X.-G. Wen, *Quantum Field Theory and Many-Body Systems* (Oxford University Press, 2004).
- [18] G. E. Volovik, arXiv:0709.1258 [gr-qc].
- [19] B. L. Hu, arXiv:0903.0878 [gr-qc].
- [20] C. Omero and R. Percacci, Nucl. Phys. B **165** (1980) 351.
- [21] M. Gell-Mann and B. Zwiebach, Phys. Lett. B **141** (1984) 333.
- [22] M. Gell-Mann and B. Zwiebach, Nucl. Phys. B **260** (1985) 569.
- [23] B. Chakrabarty and R. Parthasarathy, Phys. Lett. A **142** (1989) 75.
- [24] C. M. Ho and T. W. Kephart, Int. J. Mod. Phys. A **27**, 1250151 (2012) [arXiv:1002.4044 [hep-ph]].
- [25] G. 't Hooft, arXiv:0708.3184 [hep-th].
- [26] C. M. Bender and S. Boettcher, Phys. Rev. Lett. **80**, 5243 (1998).
- [27] A. M. Polyakov, Int. J. Mod. Phys. A **5** (1990) 833.
- [28] J. Polchinski, hep-th/9411028.
- [29] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, 1987).
- [30] J. Polchinski, *String Theory* (Cambridge University Press, 1998).
- [31] P. Ginsparg and G. Moore, hep-th/930401, published in Proceedings of TASI 1992; J. Harvey and J. Polchinski (eds.), World Scientific, 1993.
- [32] M. J. Duff, P. S. Howe, T. Inami and K. S. Stelle, Phys. Lett. B **191** (1987) 70.
- [33] B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B **305** (1988) 545.
- [34] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D **55** (1997) 5112.
- [35] V. Jejjala, M. Kavic, and D. Minic, Int. J. Mod. Phys. A **22** (2007) 3317.
- [36] V. Jejjala, M. Kavic, D. Minic, and C. H. Tze, Int. J. Mod. Phys. A **25** (2010) 2515.
- [37] V. Jejjala, M. Kavic, D. Minic, C. H. Tze, Entropy **14** (2012) 614.
- [38] H. Dieter Zeh, *The Physical Basis of the Direction of Time* (Springer, 2007).
- [39] D. Minic and H. C. Tze, Phys. Rev. D **68**, 061501 (2003).
- [40] D. Minic and H. C. Tze, Phys. Lett. B **581**, 111 (2004).
- [41] V. Balasubramanian, N. Jokela, E. Keski-Vakkuri and J. Majumder, Phys. Rev. D **75**, 063515 (2007).
- [42] J. A. Hutasoit and N. Jokela, Phys. Rev. D **77**, 023521 (2008).