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Cyclic Cosmology from the Little Rip

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Abstract

We revisit a cyclic cosmology scenario proposed in 2007 to examine whether its hypotheses can be sustained if the underlying big rip evolution, which was assumed there, is replaced by the recently proposed little rip. We show that the separation into causal patches at turnaround is generally valid for a little rip, and therefore conclude that the little rip is equally as suitable a basis for cyclicity as is the big rip.

A few years ago [1], it was suggested, based on a suitably modified big-rip type of future evolution [2], that a separation into causal patches at turnaround and the subsequent contraction of an empty (except for dark energy) such patch with zero entropy can lead to a cyclic cosmology. In particular, the reduction to zero of the patch entropy could avoid conflict with the second law of thermodynamics. At the same time, and equally important, it could underly the low or vanishing entropy at the beginning of inflation in the next expansion era.

One of the oldest questions in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann [3] and especially by Tolman [4, 5] one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This led to the abandonment of the oscillatory universe by the majority of workers.

In the present article, we examine whether this type of cyclicity can be obtained in the little-rip scenario of future cosmic evolution [6] (see also [7, 8]), focusing at the beginning on the first model in [6], although our conclusion will apply to all little rip models.

An infinitely oscillatory universe is a very attractive alternative to the big bang. One new ingredient in the cosmic make-up is the dark energy discovered only in 1998, and so it is natural to ask whether this can avoid the difficulties with entropy.

Some work has been done on the exploitation of the dark energy in allowing cyclicity possibly without the need for inflation in [9–13]. Another approach is the use of branes and a fourth spatial dimension as in [14–17], which examined consequences for cosmology. The big rip and replacement of dark energy by modified gravity were explored in [18, 19].

If the dark energy has a constant super-negative equation of state, $\omega_{DE} = p_{DE}/\rho_{DE} < -1$, it leads to a big rip [2] at a finite future time where there exist extraordinary conditions with regard to density and causality as one approaches the rip. In [1], it was shown that these exceptional conditions can assist in providing an infinitely cyclic model. We consider here a different model where $\omega_{DE}(a)$ evolves with scale factor and where $\omega_{DE} \to -1$ asymptotically as $t \to \infty$, leading to a a little rip. As we approach the little rip, expansion stops due to a brane contribution and there is a turnaround at time $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently to separate universes. Turnaround takes place at a time when the universe is fractionated into many independent causal patches [19].

Contraction occurs with a very much smaller universe than in expansion and with vanishing entropy because the universe is assumed empty of dust, matter and black holes.

A bounce takes place a short time before a would-be big bang. After the bounce, entropy is injected by inflation [20], where it is assumed that an inflaton field is excited. Inflation is thus a part of the present model which is one distinction from the work of [10–13]. For cyclicity of the entropy, $S(t) = S(t + \tau)$, consistency with thermodynamics requires that the deflationary decrease by f^3 compensate the entropy increase acquired during expansion, including the increase during inflation.

Let us review the Friedmann equation for the expansion phase. Let the period of the universe be designated by τ, and let the bounce take place at $t = 0$ and turnaround at $t = t_T$. Thus the expansion phase is for times $0 < t < t_T$ and the contraction phase corresponds to times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\rho_{DE}(t) + \frac{\rho_{m_0}}{a(t)^3} + \frac{\rho_{r_0}}{a(t)^4}\right) - \frac{\rho_{total}(t)^2}{\rho_{crit}},\right] \tag{1}$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \simeq 14$ Gyr. ρ_{i_0} denotes the value of the density component ρ_i at time $t = t_0$. The first two terms are the contributions from dark energy and total matter (dark plus luminous), and $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the contribution from radiation. The final term $\sim \rho_{total}(t)^2$ is derivable from a brane setup [14,15,17]; we use a negative sign arising from negative brane tension (a negative sign can arise also from a second timelike dimension, but that gives difficulties with closed timelike paths). The constant critical density $\rho_{crit} = \rho_{total}(t_T) = \rho_{total}(\tau)$ is set appropriately so that the bounce occurs at the grand-unified-theory scale of our Universe, and $\rho_{total} = \Sigma_{i=DE,m,r}\ \rho_i$. As the turnaround is approached, the only significant terms in Eq. (1) are the first (where $\omega_{DE} < -1$) and the last. As the bounce is approached, the only important terms in Eq. (1) are the third and the last. (In [1], we argue that the second term, for matter, is absent during contraction.) In particular, the final term of Eq. (1), $\sim \rho_{total}(t)^2$, arising from the brane setup is insignificant for almost the entire cycle but becomes dominant as one approaches $t \to t_T$ for the turnaround and again for $t \to \tau$ approaching the bounce.

At the turnaround, to sustain the scenario of [1], we must check whether the causality structure is sufficiently close to that for the big rip. We consider Model 1 of [6] and investigate this. The dark energy density we use for $a \geq 1$ is [6]

$$\rho_{DE}(a) = \left(\frac{3A\ln a}{2} + \rho_{DE_0}^{1/2}\right)^2, \tag{2}$$

where $A = 3.46 \times 10^{-3}\ \mathrm{M}_\odot^{1/2}/\mathrm{pc}^{3/2}$. For $a \leq 1$, $\rho_{DE}(a) = \rho_{DE_0}$ (the usual concordance model), which matches up with Eq. (2) at $a = 1$.

Quarks in a proton will begin to dissociate when $\rho_{DE} = \rho_{proton} = 7.8 \times 10^{69}\ \mathrm{M}_\odot/\mathrm{pc}^3$, or when $a = e^{2(\rho_{proton}^{1/2} - \rho_{DE_0}^{1/2})/3A}$, assuming one fluid, the dominant dark energy. We assume that the physical distance between two quarks in a proton before dissociation is the diameter of a proton, which is 1.8 fm. This evolution of distance between two such quarks is governed by the timelike geodesic equation with a source equal to the acceleration due to the strong force trying to hold together the two quarks:

$$\begin{aligned}&\frac{1}{a}\frac{d\rho}{dt}\frac{d}{d\rho}\left(\frac{d\rho}{dt}\frac{dR}{d\rho}\right)\\&+\frac{R}{a}\frac{4\pi G}{3}\left[\rho-\frac{4\rho^2}{\rho_{crit}}-2p\left(\frac{2\rho}{\rho_{crit}}-1\right)\right]\\&=\frac{2\hbar c\alpha_s}{M_q}\left(\frac{1}{R^2}+\frac{1}{(6.48\times10^{-33}\text{ pc})^2}\right).\end{aligned}\tag{3}$$

The chosen critical density is $\rho_{crit} = 6.84 \times 10^{108}$ $\text{M}_\odot/\text{pc}^3$, $R = a(t)r$ is the physical distance, the righthand side of Eq. (3) is the experimentally determined expression for the proton's binding force [21] and M_q is the mass of a down quark.

Integrating the null geodesic equation, we see that light travels a comoving distance of

$$\begin{aligned}&\frac{ce^{2\rho_{DE_0}^{1/2}/3A}}{2A\sqrt{6\pi G}}\int_{\tilde{\rho}}^{\rho}\frac{e^{-2\rho'^{1/2}/3A}}{\rho'^{1/2}(\rho'-\frac{\rho'^2}{\rho_{crit}})^{1/2}}d\rho'\\&=3.22\times10^{8}\text{pc }[\text{Ei}(-192.68\text{ pc}^{3/2}/\text{M}_\odot^{1/2}\ \rho^{1/2})-\text{Ei}(-192.68\text{ pc}^{3/2}/\text{M}_\odot^{1/2}\ \tilde{\rho}^{1/2})],\end{aligned}\tag{4}$$

where $\text{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$ is the exponential integral function.

By using Eqs. (3) and (4), one sees that at some time before the turnaround, the quarks from all protons are dissociated to the extent that they are causally disconnected. Structures that are less strongly bound will dissociate earlier since the inertial force due to dark energy increases monotonically, so these structures will also be causally disconnected by the turnaround. Therefore, all bound structures will dissociate by the turnaround, and they will reside isolated in causally disconnected patches of spacetime. Other models in [6] all grow more quickly and dissociate bound structures sooner than Model 1 of [6] discussed here, so these models will also create causally disconnected patches of spacetime.

From this result, it follows that a cyclic cosmology along the lines of [1] can be sustained by a little rip equally as well as with a big rip. In other words, the little rip can be used as the basis for the turnaround between expansion and contraction eras because the causal patch structure is sufficiently similar to that for the big-rip case.

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