Strong-Electroweak Unification at About 4 TeV

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I show how an $SU(N)^M$ quiver gauge theory can accommodate the standard model with three chiral families and unify all of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ couplings with high accuracy at one unique scale estimated as $M \simeq 4$ TeV.

Conformal invariance in two dimensions has had great success in comparison to several condensed matter systems. It is an interesting question whether conformal symmetry can have comparable success in a four-dimensional description of high-energy physics.

Even before the standard model (SM) $SU(2) \times U(1)$ electroweak theory was firmly established by experimental data, proposals were made [1,2] of models which would subsume it into a grand unified theory (GUT) including also the dynamics [3] of QCD. Although the prediction of SU(5) in its minimal form for the proton lifetime has long ago been excluded, *ad hoc* variants thereof [4] remain viable. Low-energy supersymmetry improves the accuracy of unification of the three 321 couplings [5,6] and such theories encompass a "desert" between the weak scale ~ 250 GeV and the much-higher GUT scale $\sim 2 \times 10^{16}$ GeV, although minimal supersymmetric $SU(5)$ is by now ruled out [7].

Recent developments in string theory are suggestive of a different strategy for unification of electroweak theory with QCD. Both the desert and low-energy supersymmetry are abandoned. Instead, the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is embedded in a semi-simple gauge group such as $SU(3)^N$ as suggested by gauge theories arising from compactification of the IIB superstring on an orbifold $AdS_5 \times S^5/\Gamma$ where Γ is the abelian finite group Z_N [8]. In such nonsupersymmetric quiver gauge theories the unification of couplings happens not by logarithmic evolution [3] over an enormous desert covering, say, a dozen orders of magnitude in energy scale. Instead the unification occurs abruptly at $\mu = M$ through the diagonal embeddings of 321 in $SU(3)^N$ [9]. The key prediction of such unification shifts from proton decay to additional particle content, in the present model at $\simeq 4$ TeV.

Let me consider first the electroweak group which

in the standard model is still un-unified as $SU(2) \times U(1)$. In the 331-model [10,11] where this is extended to $SU(3) \times U(1)$ there appears a Landau pole at $M \simeq 4$ TeV because that is the scale at which $\sin^2\theta(\mu)$ slides to the value $\sin^2(M) = 1/4$. It is also the scale at which the custodial gauged $SU(3)$ is broken in the framework of [12].

Such theories involve only electroweak unification so to include QCD I examine the running of all three of the SM couplings with μ as explicated in *e.g.* [6]. Taking the values at the Z-pole $\alpha_Y(M_Z) = 0.0101, \alpha_2(M_Z) = 0.0338, \alpha_3(M_Z) = 0.118 \pm 0.003$ (the errors in $\alpha_Y(M_Z)$ and $\alpha_2(M_Z)$ are less than 1%) they are taken to run between M_Z and M according to the SM equations

$$\alpha_Y^{-1}(M) = (0.01014)^{-1} - (41/12\pi)\ln(M/M_Z)$$
$$= 98.619 - 1.0876y \qquad (1)$$

$$\alpha_2^{-1}(M) = (0.0338)^{-1} + (19/12\pi)\ln(M/M_Z)$$
$$= 29.586 + 0.504y \qquad (2)$$

$$\alpha_3^{-1}(M) = (0.118)^{-1} + (7/2\pi)\ln(M/M_Z)$$
$$= 8.474 + 1.114y \qquad (3)$$

where $y = \log(M/M_Z)$.

The scale at which $\sin^2\theta(M) = \alpha_Y(M)/(\alpha_2(M)+\alpha_Y(M))$ satisfies $\sin^2\theta(M) = 1/4$ is found from Eqs.(1,2) to be $M \simeq 4$ TeV as stated in the introduction above.

I now focus on the ratio $R(M) \equiv \alpha_3(M)/\alpha_2(M)$ using Eqs.(2,3). I find that $R(M_Z) \simeq 3.5$ while $R(M_3) = 3$, $R(M_{5/2}) = 5/2$ and $R(M_2) = 2$ correspond to $M_3, M_{5/2}, M_2 \simeq 400\text{GeV}, 4\text{TeV}, \text{and} 140\text{TeV}$ respectively. The proximity of $M_{5/2}$ and M, accurate to a few percent, suggests strong-electroweak unification at $\simeq 4$ TeV.

There remains the question of embedding such unification in an $SU(3)^N$ of the type described in [8,9]. Since the required embedding of $SU(2)_L \times U(1)_Y$ into an $SU(3)$ necessitates $3\alpha_Y = \alpha_H$ the ratios of couplings at $\simeq 4$ TeV is: $\alpha_{3C} : \alpha_{3W} : \alpha_{3H} :: 5 : 2 : 2$ and it is natural to examine $N = 12$

with diagonal embeddings of Color (C), Weak (W) and Hypercharge (H) in $SU(3)^2, SU(3)^5, SU(3)^5$ respectively.

To accomplish this I specify the embedding of $\Gamma = Z_{12}$ in the global $SU(4)$ R-parity of the $\mathcal{N} = 4$ supersymmetry of the underlying theory. Defining $\alpha = \exp(2\pi i/12)$ this specification can be made by $\mathbf{4} \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ with $\Sigma A_\mu = 0 (\text{mod}12)$ and all $A_\mu \neq 0$ so that all four supersymmetries are broken from $\mathcal{N} = 4$ to $\mathcal{N} = 0$.

Having specified A_μ I calculate the content of complex scalars by investigating in $SU(4)$ the $\mathbf{6} \equiv (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}, \alpha^{-a_3}, \alpha^{-a_2}, \alpha^{-a_1})$ with $a_1 = A_1 + A_2, a_2 = A_2 + A_3, a_3 = A_3 + A_1$ where all quantities are defined (mod 12).

Finally I identify the nodes (as C, W or H) on the dodecahedral quiver such that the complex scalars

$$\Sigma_{i=1}^{i=3}\Sigma_{\alpha=1}^{\alpha=12} \left(N_\alpha, \bar{N}_{\alpha \pm a_i} \right) \tag{4}$$

are adequate to allow the required symmetry breaking to the $SU(3)^3$ diagonal subgroup, and the chiral fermions

$$\Sigma_{\mu=1}^{\mu=4}\Sigma_{\alpha=1}^{\alpha=12} \left(N_\alpha, \bar{N}_{\alpha + A_\mu} \right) \tag{5}$$

can accommodate the three generations of quarks and leptons.

It is not trivial to accomplish all of these requirements so let me demonstrate by an explicit example.

For the embedding I take $A_\mu = (1, 2, 3, 6)$ and for the quiver nodes take the ordering:

$$-C - W - H - C - W^4 - H^4 - \tag{6}$$

with the two ends of (6) identified.

The scalars follow from $a_i = (3, 4, 5)$ and the scalars in Eq.(4)

$$\Sigma_{i=1}^{i=3}\Sigma_{\alpha=1}^{\alpha=12} \left(3_\alpha, \bar{3}_{\alpha \pm a_i} \right) \tag{7}$$

are sufficient to break to all diagonal subgroups as

$$SU(3)_C \times SU(3)_W \times SU(3)_H \tag{8}$$

The fermions follow from A_μ in Eq.(5) as

$$\Sigma_{\mu=1}^{\mu=4}\Sigma_{\alpha=1}^{\alpha=12} \left(3_\alpha, \bar{3}_{\alpha + A_\mu} \right) \tag{9}$$

and the particular dodecahedral quiver in (6) gives rise to exactly *three* chiral generations which transform under (8) as

$$3[(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})] \tag{10}$$

I note that anomaly freedom of the underlying superstring dictates that only the combination of states in Eq.(10) can survive. Thus, it is sufficient to examine one of the terms, say $(3, \bar{3}, 1)$. By drawing the quiver diagram indicated by Eq.(6) with the twelve nodes on a "clock-face" and using $A_\mu = (1, 2, 3, 6)$ in Eq.(5) I find five $(3, \bar{3}, 1)$'s and two $(\bar{3}, 3, 1)$'s implying three chiral families as stated in Eq.(10).

After further symmetry breaking at scale M to $SU(3)_C \times SU(2)_L \times U(1)_Y$ the surviving chiral fermions are the quarks and leptons of the SM. The appearance of three families depends on both the identification of modes in (6) and on the embedding of $\Gamma \subset SU(4)$. The embedding must simultaneously give adequate scalars whose VEVs can break the symmetry spontaneously to (8). All of this is achieved successfully by the choices made. The three gauge couplings evolve according to Eqs.(1,2,3) for $M_Z \leq \mu \leq M$. For $\mu \geq M$ the (equal) gauge couplings of $SU(3)^{12}$ do not run if, as conjectured in [8,9] there is a conformal fixed point at $\mu = M$.

The basis of the conjecture in [8,9] is the proposed duality of Maldacena [13] which shows that in the $N \rightarrow \infty$ limit $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory, as well as orbifolded versions with $\mathcal{N} = 2, 1$ and 0 [14,15] become conformally invariant. It was known long ago [16] that the $\mathcal{N} = 4$ theory is conformally invariant for all finite $N \geq 2$. This led to the conjecture in [8] that the $\mathcal{N} = 0$ theories might be conformally invariant, at least in some case(s), for finite N. It should be emphasized that this conjecture cannot be checked purely within a perturbative framework [17]. I assume that the local $U(1)$'s which arise in this scenario and which would lead to $U(N)$ gauge groups are non-dynamical, as suggested by Witten [18], leaving $SU(N)$'s.

This is a non-gravitational theory with conformal invariance when $\mu > M$ and where the Planck mass it taken to be infinitely large. The ubiquitous question is: What about gravity which breaks conformal symmetry in the ultraviolet (UV)? This is a question about the holographic principle for flat spacetime.

From the phenomenological viewpoint the equal couplings of $SU(3)^{12}$ can, instead of remaining constant at energies $\mu > M$, decrease smoothly by asymptotic freedom to a conformal fixed point as $\mu \rightarrow \infty$. This possibility is less restrictive and may fit in better with the AdS/CFT correspondence.

The desert resides in the unexplored domain of the orders of magnitude in energy scale between 4 TeV and the gravitational scale, M_{Planck}.

As for experimental tests of such a TeV GUT, the situation at energies below 4 TeV is predicted to be the standard model with a Higgs boson still to be discovered at a mass predicted by radiative corrections [19] to be below 267 GeV at 99% confidence level.

There are many particles predicted at $\simeq 4$ TeV beyond those of the minimal standard model. They include as spin-0 scalars the states of Eq.(7). and as spin-1/2 fermions the states of Eq.(9), Also predicted are gauge bosons to fill out the gauge groups of (8), and in the same energy region the gauge bosons to fill out all of $SU(3)^{12}$. All these extra particles are necessitated by the conformality constraints of [8,9] to lie close to the conformal fixed point.

One important issue is whether this proliferation of states at ~ 4 TeV is compatible with precision electroweak data in hand. This has been studied in the related model of [12] in a recent article [20]. Those results are not easily translated to the present model but it is possible that such an analysis including limits on flavor-changing neutral currents could rule out the entire framework.

As alternative to $SU(3)^{12}$ another approach to TeV unification has as its group at ~ 4 TeV $SU(6)^3$ where one $SU(6)$ breaks diagonally to color while the other two $SU(6)$'s each break to $SU(3)_{k=5}$ where level $k = 5$ characterizes irregular embedding [21]. The triangular quiver $-C-W-H-$ with ends identified and $A_\mu = (\alpha, \alpha, \alpha, 1)$, $\alpha = \exp(2\pi i/3)$, preserves $\mathcal{N} = 1$ supersymmetry. I have chosen to describe the $\mathcal{N} = 0$ $SU(3)^{12}$ model in the text mainly because the symmetry breaking to the standard model is more transparent.

The TeV unification fits $\sin^2\theta$ and α_3, predicts three families, and partially resolves the GUT hierarchy. If such unification holds in Nature there is a very rich level of physics one order of magnitude above presently accessible energy.

Is a hierarchy problem resolved in the present theory? In the non-gravitational limit $M_{Planck} \to \infty$ I have, above the weak scale, the new unification scale ~ 4 TeV. Thus, although not totally resolved, the GUT hierarchy is ameliorated. The gravitational hierarchy problem is not addressed.

My final remark is on the non-appearance of relevant deformations which break conformal invariance above 4 TeV. This is an assumption I make by analogy to several other systems in Nature with a large scaling region, *e.g.* superfluid helium where there is a comparable non-appearance of relevant operators over many orders of magnitude in scale size.

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