

# Correlation Functions in Berkovits' Pure Spinor Formulation

Gautam Trivedi <sup>1</sup>

*Department of Physics*

*University of North Carolina, Chapel Hill, NC 27599-3255*

We use Berkovits' pure spinor quantization to compute various three-point tree correlation functions in position-space for the Type IIB superstring. We solve the constraint equations for the vertex operators and obtain explicit expressions for the graviton and axion components of the vertex operators. Using these operators we compute tree level correlation functions in flat space and discuss their extension to the  $AdS_5 \times S^5$  background.

---

<sup>1</sup> gautamt@physics.unc.edu

## 1. Introduction

In this paper we compute explicitly several tree level string correlation functions for the Type IIB superstring using the pure spinor formulation developed by Berkovits *et al.* [1-9]. This quantization gives us tools to evaluate string correlation functions in a manifestly supersymmetric and covariant manner. The formalism uses the usual ten-dimensional superspace coordinates  $x^m, \theta^\alpha, \bar{\theta}^{\bar{\alpha}}$  and introduces new worldsheet bosonic fields  $\lambda^\alpha, \bar{\lambda}^{\bar{\alpha}}$  which are spacetime spinors and satisfy the pure-spinor condition  $\lambda\gamma\lambda = 0$ . It also provides a nilpotent BRST charge  $Q$ , a Virasoro current with vanishing conformal anomaly and a ghost current. Physical vertex operators for massless fields have conformal weight zero and are states of ghost number 1 in the cohomology of  $Q$ . Recently this formalism has been used to obtain vertex operators for some of the massive fields of the open superstring [9]. It has also been applied to construct a worldsheet action for the superstring in a Ramond-Ramond plane wave background [10].

In sect. 2 of this paper we review the main components of the pure spinor formalism. We will explicitly write down the vertex operators for the physical states using their constraint equations. In sect. 3 we use the vertex operators and the prescription for integration over the zero modes of  $\theta$ s and  $\lambda$ s [1] to compute several flat space string correlation functions in position space. Finally, in sect.4 we discuss an extension of these calculations to the  $AdS_5 \times S^5$  background. We find the string amplitudes calculated using this procedure to be equal to the field theory expressions, in accordance with the non-renormalisation theorems for the super-symmetric three-point functions. We expect  $\alpha'$  corrections to first appear in the four point string tree amplitudes. Berkovits and Vallilo [2] have given a formal proof of the equivalence of the superstring amplitudes in their formulation with the Ramond-Neveu-Schwarz (RNS) quantization, at least in flat space. We present an explicit calculation in order to elucidate the computation of flat space correlation functions in the new quantization and to facilitate the extension to curved backgrounds.

## 2. Review of Components

We start by listing the worldsheet fields in the formulation. There are the usual ten-dimensional superspace coordinates  $x^m, \theta^\alpha, \bar{\theta}^{\bar{\alpha}}$  where  $1 \leq \alpha, \bar{\alpha} \leq 16$ . In addition to these there are worldsheet bosons  $\lambda^\alpha, \bar{\lambda}^{\bar{\alpha}}$  which satisfy the following condition

$$\lambda^\alpha \gamma_{\alpha\beta}^m \lambda^\beta = 0, \quad \bar{\lambda}^{\bar{\alpha}} \gamma_{\bar{\alpha}\bar{\beta}}^m \bar{\lambda}^{\bar{\beta}} = 0, \quad 0 \leq m \leq 9. \quad (2.1)$$

Here  $\gamma^m$  are the off-diagonal components of the  $32 \times 32$  ten-dimensional  $\gamma$  matrices in a Weyl representation (see Appendix A).

The massless vertex operator expanded in powers of  $\theta$  and  $\bar{\theta}$  is

$$\begin{aligned}
V(x, \theta, \bar{\theta}) = \lambda^\alpha \bar{\lambda}^{\bar{\alpha}} [ & h_{mn} \gamma_{\alpha\beta}^m \gamma_{\bar{\alpha}\bar{\beta}}^n \theta^\beta \bar{\theta}^{\bar{\beta}} \\
& \bar{\psi}_m^{\bar{\beta}} \gamma_{\alpha\beta}^m \gamma_{qrs\bar{\alpha}\bar{\beta}} \gamma_{\rho\bar{\sigma}}^{qrs} \theta^\beta \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} + \psi_m^\beta \gamma_{qrs\alpha\beta} \gamma_{\rho\sigma}^{qrs} \gamma_{\bar{\alpha}\bar{\beta}}^m \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\beta}} \\
& + F^{\beta\bar{\beta}} \gamma_{mnp\alpha\beta} \gamma_{\rho\sigma}^{mnp} \gamma_{qrs\bar{\alpha}\bar{\beta}} \gamma_{\rho\bar{\sigma}}^{qrs} \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} + \dots ].
\end{aligned} \tag{2.2}$$

Here  $F^{\beta\bar{\beta}}$  corresponds to the Ramond-Ramond field strengths and can be expanded as

$$F^{\beta\bar{\beta}} = C_m \gamma^{m\beta\bar{\beta}} + H_{mnp} \gamma^{mnp\beta\bar{\beta}} + F_{mnpqr} \gamma^{mnpqr\beta\bar{\beta}}. \tag{2.3}$$

where  $C_m \equiv \partial_m \Phi$ , for example, corresponds to the field strength for the axion and  $(\dots)$  correspond to auxilliary terms with higher powers of  $\theta(\bar{\theta})$ . One now defines the BRST operator as follows [1]

$$\begin{aligned}
Q &= \oint dz \lambda^\alpha d_\alpha \\
&\text{where}
\end{aligned} \tag{2.4}$$

$$d_\alpha = p_\alpha - \frac{1}{2} \gamma_{\alpha\beta}^m \theta^\beta \partial x_m + \frac{1}{8} \gamma_{\alpha\beta}^m \gamma_{m\rho\sigma} \theta^\beta \theta^\rho \partial \theta^\sigma.$$

where  $p_\alpha$  are the conjugate momenta for the  $\theta^\alpha$ s, with similar expressions for  $\bar{Q}$ . The constraint equations for the vertex operator are

$$[Q, V(z, \bar{z})] = 0, \quad [\bar{Q}, V(z, \bar{z})] = 0. \tag{2.5}$$

In this paper we consider the vertex operators for the graviton and the axion explicitly. Equations (2.5) imply that the simple poles for the OPEs between  $Q, \bar{Q}$  and  $V(z, \bar{z})$  vanish. This leads to following differential equations for  $V(z, \bar{z})$ .

$$\begin{aligned}
\lambda^\alpha \bar{\lambda}^{\bar{\alpha}} D_\alpha \bar{D}_{\bar{\alpha}} V &= 0 \\
D_\alpha &= \frac{\partial}{\partial \theta^\alpha} + \gamma_{\alpha\beta}^m \theta^\beta \frac{\partial}{\partial x^m}, \quad \bar{D}_{\bar{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\bar{\alpha}}} + \gamma_{\bar{\alpha}\bar{\beta}}^m \bar{\theta}^{\bar{\beta}} \frac{\partial}{\partial x^m}.
\end{aligned} \tag{2.6}$$

Using (2.6) we see that the terms with odd powers of  $\theta(\bar{\theta})$  are related to each other and so are the even powers. Next, we pick out the graviton and the axion vertex operators.

$$\begin{aligned}
V_{\text{graviton}} &= \lambda^\alpha \bar{\lambda}^{\bar{\alpha}} [ h_{mn}(x) \gamma_{\alpha\beta}^m \gamma_{\bar{\alpha}\bar{\beta}}^n \theta^\beta \bar{\theta}^{\bar{\beta}} \\
&\quad - \frac{1}{2} \partial_{\bar{m}} h_{m\bar{n}}(x) \gamma_{\alpha\beta}^m \gamma_{\bar{p}\bar{\alpha}\bar{\beta}} \gamma_{\rho\bar{\sigma}}^{\bar{m}\bar{n}\bar{p}} \theta^\beta \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \\
&\quad - \frac{1}{2} \partial_m h_{n\bar{m}}(x) \gamma_{p\alpha\beta} \gamma_{\rho\sigma}^{mnp} \gamma_{\bar{\alpha}\bar{\beta}}^{\bar{m}} \theta^\beta \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\beta}} \\
&\quad + \frac{1}{4} \partial_m \partial_{\bar{m}} h_{n\bar{n}}(x) \gamma_{p\alpha\beta} \gamma_{\rho\sigma}^{mnp} \gamma_{\bar{p}\bar{\alpha}\bar{\beta}} \gamma_{\rho\bar{\sigma}}^{\bar{m}\bar{n}\bar{p}} \theta^\beta \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \\
&\quad + \dots ].
\end{aligned} \tag{2.7}$$

where  $h_{mn}$  is symmetric and traceless and satisfies  $\partial^p \partial_p h_{mn} = 0$  and  $\partial^m h_{mn} = 0$  and  $(\dots)$  correspond to terms that have higher powers of  $\theta(\bar{\theta})$  and do not contribute to tree amplitudes. The vertex operator for the axion is

$$V_{axion} = \lambda^\alpha \bar{\lambda}^{\bar{\alpha}} [\partial_q \Phi(x) \gamma^{q\kappa\bar{\kappa}} \gamma_{mnp\alpha\kappa} \gamma_{\rho\sigma}^{mnp} \gamma_{\bar{m}\bar{n}\bar{p}\bar{\alpha}\bar{\kappa}} \gamma_{\bar{\rho}\bar{\sigma}}^{\bar{m}\bar{n}\bar{p}} \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} + \dots]. \quad (2.8)$$

With results from the appendix this can be written in a more convenient form as

$$V_{axion} = \frac{1}{16} \lambda^\alpha \bar{\lambda}^{\bar{\alpha}} [\partial_q \Phi(x) \gamma^{q\kappa\bar{\kappa}} \gamma_{m\alpha\rho} \gamma_{\kappa\sigma}^m \gamma_{\bar{m}\bar{\alpha}\bar{\rho}} \gamma_{\bar{\kappa}\bar{\sigma}}^{\bar{m}} \theta^\rho \theta^\sigma \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} + \dots]. \quad (2.9)$$

In the next section we use these vertex operators to calculate the three-graviton and the two-axion one-graviton correlation functions in flat space.

### 3. Correlation functions

#### 3.1. Three-graviton tree amplitude in flat space

The three-graviton amplitude is as follows

$$A_{ggg} = \langle 0 | V_{graviton}(z_1, \bar{z}_1) V_{graviton}(z_2, \bar{z}_2) V_{graviton}(z_3, \bar{z}_3) | 0 \rangle. \quad (3.1)$$

Using (2.7) we will find two different types of terms that will contribute to the amplitude. These will have the form  $h_{mn} h_{pq} \partial_r \partial_s h_{tu}$  and  $h_{mn} \partial_r h_{pq} \partial_s h_{tu}$ . There will be three terms of the first kind and six of the second. We begin by looking at a term of the first kind.

$$\begin{aligned} Term1 &\equiv \frac{1}{4} \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} h_{mn}(x) h_{pq}(x) \partial_r \partial_s h_{tu}(x) \\ &\quad \times \gamma_{\alpha\rho}^m \gamma_{\beta\sigma}^p \gamma_{w\gamma\delta} \gamma_{\kappa\tau}^{rtw} \gamma_{\bar{\alpha}\bar{\rho}}^n \gamma_{\bar{\beta}\bar{\sigma}}^q \gamma_{v\bar{\gamma}\bar{\delta}} \gamma_{\bar{\kappa}\bar{\tau}}^{suw} \\ &\quad \times \theta^\rho \theta^\sigma \theta^\delta \theta^\kappa \theta^\tau \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\delta}} \bar{\theta}^{\bar{\kappa}} \bar{\theta}^{\bar{\tau}} \rangle \\ &= \frac{1}{4} \langle h_{mn}(x) h_{pq}(x) \partial_r \partial_s h_{tu}(x) \rangle_X \\ &\quad \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} \gamma_{\alpha\rho}^m \gamma_{\beta\sigma}^p \gamma_{w\gamma\delta} \gamma_{\kappa\tau}^{rtw} \gamma_{\bar{\alpha}\bar{\rho}}^n \gamma_{\bar{\beta}\bar{\sigma}}^q \gamma_{v\bar{\gamma}\bar{\delta}} \gamma_{\bar{\kappa}\bar{\tau}}^{suw} \\ &\quad \times \theta^\rho \theta^\sigma \theta^\delta \theta^\kappa \theta^\tau \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\delta}} \bar{\theta}^{\bar{\kappa}} \bar{\theta}^{\bar{\tau}} \rangle. \end{aligned} \quad (3.2)$$

Since  $\lambda$  and  $\theta$  have conformal weight zero, only their zero modes will survive the bracket. In accordance with Berkovits' prescription [1-3], we are keeping only terms containing five  $\theta$ s and three  $\lambda$ s. These are the only ones that will contribute to the amplitude because there

is only one state  $(\lambda\gamma\theta)(\lambda\gamma\theta)(\lambda\gamma\theta)(\theta\gamma\theta)|0\rangle$  in the cohomology of  $Q$  with ghost number three. We can show that

$$\langle \lambda^\alpha \lambda^\beta \lambda^\gamma \theta^\rho \theta^\sigma \theta^\tau \theta^\omega \theta^\kappa \rangle = T_{\alpha_1 \beta_1 \gamma_1}^{\alpha \beta \gamma} \gamma_q^{\alpha_1 [\rho \gamma_r^{\beta_1 \sigma} \gamma_s^{\gamma_1 \tau} \gamma^{qr s \omega \kappa}]} \quad (3.3)$$

where  $[ ]$  stands for antisymmetrization over the indices  $\rho, \sigma, \tau, \omega$  and  $\kappa$ , with no overall normalization factor. One can obtain a similar expression for  $\langle \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\tau}} \bar{\theta}^{\bar{\omega}} \bar{\theta}^{\bar{\kappa}} \rangle$ .

Here  $T_{\alpha_1 \beta_1 \gamma_1}^{\alpha \beta \gamma}$  is defined [1] as

$$T_{\alpha_1 \beta_1 \gamma_1}^{\alpha \beta \gamma} = \frac{N}{4032} [\delta_{\alpha_1}^{(\alpha} \delta_{\beta_1}^{\beta} \delta_{\gamma_1}^{\gamma)} - \frac{1}{40} \gamma_m^{(\alpha \beta} \delta_{(\alpha_1}^m \gamma_{\beta_1 \gamma_1)}^{\gamma)}]. \quad (3.4)$$

The brackets  $( )$  corresponds to symmetrization over the enclosed indices with no overall normalization, and  $T_{\alpha \beta \gamma}^{\alpha \beta \gamma} = N$ . Using (3.3) we evaluate the following useful terms.

$$\begin{aligned} \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \gamma_{m\alpha\rho} \gamma_{n\beta\tau} \gamma_{p\gamma\sigma} \gamma_{\kappa\delta}^{mnp} \theta^\rho \theta^\tau \theta^\sigma \theta^\kappa \theta^\delta \rangle &= 2880. \\ \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \gamma_{\alpha\rho}^t \gamma_{n\beta\tau} \gamma_{p\gamma\sigma} \gamma_{\kappa\delta}^{mnp} \theta^\rho \theta^\tau \theta^\sigma \theta^\kappa \theta^\delta \rangle &= 288 \eta^{mt}. \\ \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \gamma_{\alpha\rho}^t \gamma_{\beta\tau}^u \gamma_{p\gamma\sigma} \gamma_{\kappa\delta}^{mnp} \theta^\rho \theta^\tau \theta^\sigma \theta^\kappa \theta^\delta \rangle &= 32 \eta^{t[m} \eta^{n]u}. \\ \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \gamma_{\alpha\rho}^t \gamma_{\beta\tau}^u \gamma_{\gamma\sigma}^v \gamma_{\kappa\delta}^{mnp} \theta^\rho \theta^\tau \theta^\sigma \theta^\kappa \theta^\delta \rangle &= 4(\eta^{tm} \eta^{nu} \eta^{pv} + \eta^{tn} \eta^{up} \eta^{vm} + \eta^{tp} \eta^{um} \eta^{vn} \\ &\quad - \eta^{tn} \eta^{um} \eta^{vp} - \eta^{tp} \eta^{un} \eta^{vm} - \eta^{tm} \eta^{up} \eta^{vn}). \end{aligned} \quad (3.5)$$

Note that the normalization used in (3.5) corresponds to  $N = \frac{1}{304}$ , which follows from a long but straightforward calculation of traces over various combinations of  $\gamma$ -matrices.

Using the third term from (3.5) in (3.2) we find

$$\begin{aligned} Term1 &= 256(\eta^{mr} \eta^{tp} \eta^{ns} \eta^{uq} + \eta^{mt} \eta^{rp} \eta^{nu} \eta^{sq} - \eta^{mt} \eta^{rp} \eta^{ns} \eta^{uq} - \eta^{mr} \eta^{tp} \eta^{nu} \eta^{sq}) \\ &\quad \times \langle h_{mn}(x) h_{pq}(x) \partial_r \partial_s h_{tu}(x) \rangle_X \\ &= 256 \langle 2h_{mn}(x) h_{pq}(x) \partial^m \partial^n h^{pq}(x) + 2h_{mn}(x) \partial^n h_{pq}(x) \partial^p h^{mq}(x) \rangle_X \end{aligned} \quad (3.6)$$

Similarly we now evaluate a term of the second kind.

$$\begin{aligned} Term2 &\equiv -\frac{1}{4} \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} h_{mn}(x) \partial_p h_{qr}(x) \partial_s h_{tu}(x) \\ &\quad \times \gamma_{\alpha\rho}^m \gamma_{\bar{\alpha}\bar{\rho}}^n \gamma_{\beta\sigma}^q \gamma_{\bar{\beta}\bar{\sigma}} \gamma_{\gamma\tau}^{prv} \gamma_{\bar{\gamma}\bar{\tau}}^{stw} \gamma_{\kappa\delta}^u \gamma_{\bar{\kappa}\bar{\delta}} \\ &\quad \times \theta^\rho \theta^\sigma \theta^\tau \theta^\kappa \theta^\delta \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\tau}} \bar{\theta}^{\bar{\kappa}} \bar{\theta}^{\bar{\delta}} \rangle \\ &= -256(\eta^{ms} \eta^{tq} \eta^{np} \eta^{ru} + \eta^{mt} \eta^{sq} \eta^{nr} \eta^{pu} - \eta^{mt} \eta^{sq} \eta^{np} \eta^{ru} - \eta^{ms} \eta^{tq} \eta^{nr} \eta^{pu}) \\ &\quad \times \langle h_{mn}(x) \partial_p h_{qr}(x) \partial_s h_{tu}(x) \rangle_X \\ &= 256 \langle h_{mn}(x) h_{pq}(x) \partial^m \partial^n h^{pq}(x) + 3h_{mn}(x) \partial^n h_{pq}(x) \partial^p h^{mq}(x) \rangle_X \end{aligned} \quad (3.7)$$

Combining (3.6) and (3.7) we compute the amplitude to be

$$\begin{aligned}
A_{ggg} &= 3Term1 + 6Term2 \\
&= 3072 \int d^{10}x h_{mn}(x) h_{pq}(x) \partial^m \partial^n h^{pq}(x) + 2h_{mn}(x) \partial^n h_{pq}(x) \partial^p h^{mq}(x). \tag{3.8}
\end{aligned}$$

This is proportional to the field theory result for the three-graviton amplitude in a flat background on shell in the  $\partial^m h_{mn} = 0$  gauge.

### 3.2. Two-axion one-graviton tree amplitude in flat space

Using (2.7) and (2.9) we can compute the 2-axion, 1-graviton scattering amplitude as

$$\begin{aligned}
A_{aag} &= \langle V_{graviton}(z_1, \bar{z}_1) V_{axion}(z_2, \bar{z}_2) V_{axion}(z_3, \bar{z}_3) \rangle \\
A_{aag} &= \frac{1}{256} \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} h_{mn}(x) \partial_p \Phi(x) \partial_q \Phi(x) \gamma_{\alpha\rho}^m \gamma_{\bar{\alpha}\bar{\rho}}^n \\
&\quad \gamma^{p\kappa\bar{\kappa}} \gamma_{r\beta\sigma} \gamma_{\kappa\tau}^r \gamma_{s\bar{\beta}\bar{\sigma}} \gamma_{\bar{\kappa}\bar{\tau}}^s \gamma^{q\eta\bar{\eta}} \gamma_{t\gamma\xi} \gamma_{\eta\pi}^t \gamma_{u\bar{\gamma}\bar{\xi}} \gamma_{\bar{\eta}\bar{\pi}}^u \\
&\quad \theta^\rho \theta^\sigma \theta^\tau \theta^\xi \theta^\pi \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\tau}} \bar{\theta}^{\bar{\xi}} \bar{\theta}^{\bar{\pi}} \rangle \tag{3.9} \\
&= \frac{1}{256} \langle h_{mn}(x) \partial_p \Phi(x) \partial_q \Phi(x) \rangle_X \\
&\quad \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} \gamma_{\alpha\rho}^m \gamma_{\bar{\alpha}\bar{\rho}}^n \gamma^{p\kappa\bar{\kappa}} \gamma_{r\beta\sigma} \gamma_{\kappa\tau}^r \gamma_{s\bar{\beta}\bar{\sigma}} \gamma_{\bar{\kappa}\bar{\tau}}^s \gamma^{q\eta\bar{\eta}} \gamma_{t\gamma\xi} \gamma_{\eta\pi}^t \gamma_{u\bar{\gamma}\bar{\xi}} \gamma_{\bar{\eta}\bar{\pi}}^u \\
&\quad \theta^\rho \theta^\sigma \theta^\tau \theta^\xi \theta^\pi \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\tau}} \bar{\theta}^{\bar{\xi}} \bar{\theta}^{\bar{\pi}} \rangle .
\end{aligned}$$

We now define

$$\begin{aligned}
A^{mnpq} &= \langle \lambda^\alpha \lambda^\beta \lambda^\gamma \bar{\lambda}^{\bar{\alpha}} \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\gamma}} \gamma_{\alpha\rho}^m \gamma_{\bar{\alpha}\bar{\rho}}^n \gamma^{p\kappa\bar{\kappa}} \gamma_{r\beta\sigma} \gamma_{\kappa\tau}^r \gamma_{s\bar{\beta}\bar{\sigma}} \gamma_{\bar{\kappa}\bar{\tau}}^s \gamma^{q\eta\bar{\eta}} \gamma_{t\gamma\xi} \gamma_{\eta\pi}^t \gamma_{u\bar{\gamma}\bar{\xi}} \gamma_{\bar{\eta}\bar{\pi}}^u \\
&\quad \theta^\rho \theta^\sigma \theta^\tau \theta^\xi \theta^\pi \bar{\theta}^{\bar{\rho}} \bar{\theta}^{\bar{\sigma}} \bar{\theta}^{\bar{\tau}} \bar{\theta}^{\bar{\xi}} \bar{\theta}^{\bar{\pi}} \rangle \tag{3.10}
\end{aligned}$$

We see that  $A^{mnpq} = A^{mnpq}$ . Also, since  $h_{mn}$  is traceless, the only component of  $A^{mnpq}$  that survives must be  $C\delta^{m(p}\delta^{q)n}$ , where  $C$  is some constant. It follows that

$$A_{aag} = C \int d^{10}x h_{mn}(x) \partial^m \Phi(x) \partial^n \Phi(x) \tag{3.11}$$

To calculate  $C$  requires another long but straightforward trace calculation.

#### 4. Graviton two-axion amplitude in $AdS_5 \times S^5$

In this section we will discuss correlation functions on the  $AdS_5 \times S^5$  background. We will consider the two-axion one-graviton amplitude  $A_{aag}$ . For  $AdS_5 \times S^5$ , we introduce the curved space gamma matrices which satisfy  $\gamma^{m\alpha\beta}\gamma_{\beta\gamma}^n + \gamma^{n\alpha\beta}\gamma_{\beta\gamma}^m = 2\bar{g}^{mn}\delta_{\gamma}^{\alpha}$ , where  $\bar{g}^{mn}$  is the  $AdS_5 \times S^5$  background metric. Similarly for the barred indices we have  $\gamma^{m\bar{\alpha}\bar{\beta}}\gamma_{\bar{\beta}\bar{\gamma}}^n + \gamma^{n\bar{\alpha}\bar{\beta}}\gamma_{\bar{\beta}\bar{\gamma}}^m = 2\bar{g}^{mn}\delta_{\bar{\gamma}}^{\bar{\alpha}}$ . In this case, following [1], we can convert between the barred and unbarred spinor indices using  $\delta^{\alpha\bar{\alpha}} = (\gamma^{01234})^{\alpha\bar{\alpha}}$ , *i.e.*  $G^{\alpha} = \delta^{\alpha\bar{\alpha}}G_{\bar{\alpha}}$  and  $G^{\bar{\alpha}} = \delta^{\alpha\bar{\alpha}}G_{\alpha}$ . Here [01234] are the  $AdS_5$  directions. Since  $\delta^{\alpha\bar{\alpha}}$  is an orthogonal matrix, contracting over two barred indices is the same as contracting over two unbarred indices, *i.e.*  $G_{\alpha}G^{\alpha} = G_{\bar{\alpha}}G^{\bar{\alpha}}$ .

We now need to compute (3.9) using the curved space  $\gamma$ -matrices<sup>2</sup>. Our rule for summing over barred indices implies that the computation is similar to taking the trace flat space, but now the flat metric  $\eta^{mn}$  is replaced by  $\bar{g}^{mn}$ . Furthermore, we can promote the partial derivatives to covariant derivatives since they only act on the scalar axion. Therefore, we suggest that the two-axion one-graviton string tree amplitude for Type IIB strings on  $AdS_5 \times S^5$  can be obtained by covariantizing the flat space result and will have the form

$$A_{aag} = C' \int d^{10}x \sqrt{\bar{g}} \bar{g}_{mp}\bar{g}_{nq} h^{mn} \bar{D}^p \Phi \bar{D}^q \Phi. \quad (4.1)$$

In order to calculate more general amplitudes on the  $AdS_5 \times S^5$  background, one would need to derive the invariant derivatives from the string constraint equations. One would also need to establish the auxilliary terms in the vertex operators, using either the constraint equations or symmetry arguments.

#### Appendix A. Gamma matrices.

This appendix describes the  $\gamma$ -matrices used in this paper. In the Weyl representation, the ten-dimensional gamma matrices are defined as

$$\gamma^{mA}{}_B = \begin{pmatrix} 0 & \gamma^{m\alpha\bar{\beta}} \\ \gamma_{\bar{\alpha}}^{m,\beta} & 0 \end{pmatrix} \quad (A.1)$$

where  $1 \leq A, B \leq 32$  and  $1 \leq \alpha, \bar{\alpha}, \beta, \bar{\beta} \leq 16$ . The ten-dimensional charge conjugation matrix is

---

<sup>2</sup> In principle, to establish (3.9) for the curved background, one also has to know the OPEs for  $Q$  and  $\bar{Q}$ . For this particular amplitude however, we do not need the auxilliary terms.

$$C^{AB} = \begin{pmatrix} 0 & C^{\alpha\bar{\beta}} \\ C^{\bar{\alpha}\beta} & 0 \end{pmatrix} \quad (\text{A.2})$$

where

$$C_{AB}^{-1} = \begin{pmatrix} 0 & C_{\alpha\bar{\beta}}^{-1} \\ C_{\bar{\alpha}\beta}^{-1} & 0 \end{pmatrix}. \quad (\text{A.3})$$

The charge conjugation matrix and its inverse can be used to raise and lower the indices on the gamma matrices as  $\gamma^{mAB} = \gamma^{mA}{}_D C^{DB}$  and  $\gamma_{AB}^m = C_{AD}^{-1} \gamma^{mD}{}_B$ . As in [1] the  $\gamma^{m\alpha\beta}$  and  $\gamma_{\alpha\beta}^m$  are the off-diagonal elements of the ten-dimensional  $32 \times 32$  Dirac-matrices and satisfy  $\gamma^m \gamma^n + \gamma^n \gamma^m = 2\eta^{mn}$  in flat space.

The matrix  $\gamma^{\alpha_1 \alpha_2 \dots \alpha_N}$  denotes the completely antisymmetric product of  $N$  gamma matrices. Specifically,

$$\begin{aligned} \gamma_{\alpha\beta}^m &= \gamma_{\beta\alpha}^m \\ \gamma_{\alpha\beta}^{mnp} &= \frac{1}{3!} \gamma^{[m} \gamma^n \gamma^{p]} = -\gamma_{\beta\alpha}^{mnp} \\ \gamma_{\alpha\beta}^{mnpqr} &= \frac{1}{5!} \gamma^{[m} \gamma^n \gamma^p \gamma^q \gamma^{r]} = \gamma_{\beta\alpha}^{mnpqr}. \end{aligned} \quad (\text{A.4})$$

where one can show that  $\gamma^m, \gamma^{mnp}$  and  $\gamma^{mnp}$  form a complete basis for expansion of any matrix  $A_{\alpha\beta}$ . Some useful results are

$$\gamma_{m(\alpha\beta} \gamma_{\gamma)\delta}^m = 0 \quad (\text{A.5})$$

$$\gamma_{mnp[\alpha\beta} \gamma_{\gamma]\delta}^{mnp} = 0 \quad (\text{A.6})$$

In order to show (A.6) we start by first showing a more general result

$$\zeta^\alpha \gamma_{\alpha\rho}^m \theta^\rho \eta^\beta \gamma_{m\beta\sigma} \theta^\sigma = \frac{1}{16} \zeta^\alpha \gamma_{\alpha\beta}^{mnp} \eta^\beta \theta^\rho \gamma_{mnp\rho\sigma} \theta^\sigma \quad (\text{A.7})$$

We note that

$$\begin{aligned} \theta^\rho \theta^\sigma &= -\theta^\sigma \theta^\rho \\ \Rightarrow \theta^\rho \theta^\sigma &= C^{pqr} \gamma_{pqr}^{\rho\sigma} \\ \Rightarrow \text{L.H.S of (A.7)} &= \zeta^\alpha \eta^\beta \gamma_{\alpha\rho}^m \gamma_{m\sigma\beta} C^{pqr} \gamma_{pqr}^{\rho\sigma} \end{aligned} \quad (\text{A.8})$$

Let us now look at the following term:

$$\begin{aligned} \gamma_{\alpha\rho}^m \gamma_{pqr}^{\rho\sigma} \gamma_{m\sigma\beta} &= -\gamma_{\beta\rho}^m \gamma_{pqr}^{\rho\sigma} \gamma_{m\sigma\alpha} \\ \Rightarrow \gamma_{\alpha\rho}^m \gamma_{pqr}^{\rho\sigma} \gamma_{m\sigma\beta} &= A_{pqr}^{\ell mn} \gamma_{\ell mn\alpha\beta} \end{aligned} \quad (\text{A.9})$$



One can now evaluate  $A_{pqr}^{stu}$  by tracing over the two side of (A.9) using a  $\gamma^{stu\alpha\beta}$ . The result one obtains is the following:

$$A_{pqr}^{stu} = -\delta_{[p}^s \delta_q^t \delta_{r]}^u \quad (\text{A.10})$$

Similarly one can show that

$$C^{pqr} = \frac{-1}{96} \gamma_{\rho\sigma}^{pqr} \theta^\rho \theta^\sigma \quad (\text{A.11})$$

Substituting (A.10) and (A.11) in (A.8) we get

$$\begin{aligned} \text{L.H.S of (A.7)} &= \frac{1}{16} \zeta^\alpha \eta^\beta \gamma_{pqr\rho\sigma} \theta^\rho \theta^\sigma \gamma_{\alpha\beta}^{pqr} \\ &= \text{R.H.S of (A.7)} \end{aligned} \quad (\text{A.12})$$

(A.6) then follows from (A.7) if we choose  $\eta = \theta$  and the use the fact that  $\gamma_{\alpha\beta}^m = \gamma_{\beta\alpha}^m$ .

**Acknowledgements:** We thank Nathan Berkovits and Louise Dolan for conversations. GT is partially supported by the U.S. Department of Energy, Grant No. DE-FG02-97ER-41036/Task A.

## References

- [1] N. Berkovits, “Super-Poincare Covariant Quantization of the Superstring”, JHEP 0004: 018, 2000; hep-th/0001035.
- [2] N. Berkovits, B.C. Vallilo, “Consistency of Super-Poincare Covariant Superstring Tree Amplitudes”, JHEP 0007: 015, 2000; hep-th/0004171.
- [3] N. Berkovits, “Cohomology in the Pure Spinor Formalism for the Superstring”, JHEP 0009: 046, 2000; hep-th/0006003.
- [4] N. Berkovits, “Covariant Quantization of the Superstring”, Presented at Strings '00, Ann Arbor, Michigan, Int. J. Mod. Phys. A16:801, 2001; hep-th/0008145.
- [5] N. Berkovits and O. Chandia, “Superstring Vertex Operators in an  $AdS_5 \times S^5$  Background”, Nucl. Phys. B596: 185, 2001; hep-th/0009168.
- [6] N. Berkovits, “Relating the RNS and Pure Spinor Formalisms for the Superstring”, JHEP 0108: 026, 2001; hep-th/0104247.
- [7] N. Berkovits and O. Chandia, “Lorentz Invariance of the Pure Spinor BRST Cohomology for the Superstring”, Phys. Lett. B514: 394, 2001; hep-th/0105149.
- [8] N. Berkovits, P. Howe, “Ten-Dimensional Supergravity Constraints from the Pure Spinor Formalism for the Superstring”; hep-th/0112160.
- [9] N. Berkovits, O. Chandia, “Massive Superstring Vertex Operator in D=10 Super-space”, hep-th/0204121.
- [10] N. Berkovits, “Conformal Field Theory for the Superstring in a Ramond-Ramond Plane Wave Background”; hep-th/0203248.