Topology Control for Maintaining Network Connectivity and Maximizing Network Capacity Under the Physical Model

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***Abstract*—In this paper we study the issue of topology control under the physical Signal-to-Interference-Noise-Ratio (SINR) model, with the objective of maximizing network capacity. We show that existing graph-model-based topology control captures interference inadequately under the physical SINR model, and as a result, the interference in the topology thus induced is high and the network capacity attained is low. Towards bridging this gap, we propose a centralized approach, called *Spatial Reuse Maximizer* (MaxSR), that combines a power control algorithm T4P with a topology control algorithm P4T. T4P optimizes the assignment of transmit power given a fixed topology, where by optimality we mean that the transmit power is so assigned that it minimizes the average interference degree (defined as the number of interferencing nodes that may interfere with the on-going transmission on a link) in the topology. P4T, on the other hand, constructs, based on the power assignment made in T4P, a new topology by deriving a spanning tree that gives the minimal interference degree. By alternately invoking the two algorithms, the power assignment quickly converges to an operational point that maximizes the network capacity. We formally prove the convergence of MaxSR. We also show via simulation that the topology induced by MaxSR outperforms that derived from existing topology control algorithms by 50%-110% in terms of maximizing the network capacity.**

I. INTRODUCTION

Topology control and management – how to determine the transmit power of each node so as to maintain network connectivity, mitigate interference, improve spatial reuse, while consuming the minimum possible power – is one of the most important issues in wireless multi-hop networks [1]. Instead of transmitting using the maximum possible power, wireless nodes collaboratively determine their transmit power and define the topology by the neighbor relation under certain criteria.

A common notion of neighbors adopted in most topology control algorithms [2], [3], [4], [5], [6], perhaps except those in [7], [8], is that two nodes are considered neighbors and a wireless link exists between them in the corresponding communication graph, if their distance is within the transmission range (as determined by the transmit power, the path loss model, and the receiver sensitivity). Algorithms that adopt this notion are collectively called graph-model-based topology control. Under this notion, topology control aims to keep the node degree in the communication graph low, subject to the network connectivity requirement. This is based on the common assertion that a low node degree usually implies low interference.

We claim that this assertion no longer holds under the physical *Signal-to-Interference-Noise-Ratio (SINR)* model. This is because under the physical model, whether the interference — the sum of all the signals of concurrent, competing transmissions received at the receiver — affects the transmission activity of interest depends on the SINR at the receiver, which in turn depends on the transmit power of all the transmitters and their relative positions to the receiver of interest. The node degree under the graph model, however, does not adequately capture interference. In particular, a transmission of interest may fail because other concurrent transmissions cause the SINR at the receiver to fall below the minimal SINR required for the receiver to decode the symbols correctly. This could occur even if competing transmitters are outside the transmission range of the receiver.

There are two undesirable consequences as a result of the inadequacy of graph-model-based topology control under the physical model. First, because the node degree does not capture interference adequately, the interference in the resulting topology may be high, rendering low network capacity. Second, a wireless link that exists in the communication graph may not in practice exist under the physical model, because of high interference (and consequently low SINR). As a result, the network connectivity may not even be sustained.

In this paper, first we formally argue that a node with a small node degree in the communication graph may suffer from high interference. Then, we define the interference graph that faithfully captures interference under the physical model. An interesting question is whether or not there exists a power assignment that enables the communication graph of the topology to represent its interference graph as well. We formally prove that such a power assignment exists *only if* the topology satisfies a certain criterion. Unfortunately, most of the topologies generated by existing graph-model-based topology control do not satisfy this criterion.

In order to mitigate interference, improve network capacity, while maintaining network connectivity, we propose a centralized approach, called **S**patial **R**euse **Max**imizer (**MaxSR**), that consists of two component algorithms: **T4P** and **P4T**. Conceptually, given the topology induced by certain topology control algorithm, each node may, instead of using the minimal possible power to reach its farthest neighbor (as defined in the communication graph), increase its transmit power in order to increase the SINR at the receiver and better tolerate interference. On the other hand, if every node transmits with high power, it contributes more to the interference as perceived by other nodes. **MaxSR** seeks to strike a balance between increasing the SINR and controlling the interference as perceived by others to an acceptable level. Specifically, **T4P** optimizes assignment of the transmit power given a fixed topology, where by optimality we mean that the transmit power is so assigned that it minimizes the average interference degree (defined as the number of nodes that will interfere with transmission on a link), and (ii) **P4T** constructs, based on the power assignment made in **T4P**, a new topology by deriving a spanning tree that gives the minimal interference degree. By alternately invoking the two algorithms, the power assignment quickly converges to an operational point that maximizes network capacity. We formally prove the convergence of **MaxSR**, and show via simulation that the topology induced by **MaxSR** outperforms that derived from existing topology control algorithms by 50-110% in terms of maximizing network capacity.

The remainder of the paper is organized as follows. We first introduce in Section II the notation and the assumptions made throughout this paper. Then we formally argue that a small node degree does not necessarily imply low interference in Section III. Following that, we investigate in Section IV the issue of whether or not a feasible power assignment exists that enables the communication graph to represent the interference graph as well. After obtaining a negative answer, we devise in Section V a new topology control algorithm, called **MaxSR**, that alternatively invokes **T4P** and **P4T** until the power assignment converges to an optimal operational point. We also formally prove its convergence there. We present in Section VI simulation results. Finally, we provide an overview of related work in Section VII, and conclude the paper in Section VIII with a list of future research agendas.

II. PHYSICAL INTERFERENCE MODEL

In this section, we first give the notation used and the assumptions made throughout in the paper. Then we explicitly define interference under the physical model.

A. Notation and Assumptions

We envision a wireless network as a set of nodes V located in the Euclidean plane. All nodes are stationary or have low mobility. Let (X, Y) denote the Euclidean coordinates, $v \in V$ the shorthand of $v(x, y)$, $x \in X$ and $y \in Y$, and $d_{ij} = d(v_i, v_j)$ the Euclidean distance between two nodes v_i and v_j. Every node v_i is configured with a transmit power $p_t(i)$ and P_t denotes the transmit power assignment $\{p_t(1), p_t(2), ..., p_t(n)\}$, where $n = |V|$.

The large-scale path loss model is used to describe how signals attenuate along the transmission path. Let g_{ij} be the channel gain from node v_i to node v_j (which is usually assumed to be a constant independent of the distance), then the received power can be expressed as

$$p_r(i,j) = \frac{g_{i,j} \cdot p_t(i)}{d_{i,j}^{\alpha}},$$

where α is the path loss exponent. The value of α typically ranges between 2 and 4, depending on which propagation model is used (e.g. $\alpha = 2$ for the free space model and $\alpha = 4$ for the two-ray ground model).

Whether a transmission succeeds or not is determined by two factors: namely the *receive sensitivity* and the *signal to interference and noise ratio* ($SINR$). Specifically, let RX_{min} be the threshold for the receiver to decode the received signal correctly, and β the $SINR$ threshold. A signal can be successfully received and decoded only if the following two constraints are satisfied:

$$p_r(i,j) = \frac{g_{i,j} \cdot p_t(i)}{d_{i,j}^{\alpha}} \geq RX_{min}, \tag{1}$$

and

$$SINR_{i,j} = \frac{g_{i,j} \cdot p_t(i) \cdot d_{i,j}^{-\alpha}}{N + I_j} \geq \beta, \tag{2}$$

where N denotes the noise power, and I_j the interference perceived at receiver v_j and contributed by other concurrent transmissions. We will elaborate on I_j in Section II-B. Eq. (1) also defines the minimal power required to reach a receiver at a distance of $d_{i,j}$ away. In this paper, we assume that all nodes are homogeneous, i.e., they have the same maximum power level P_{max}, SINR threshold β, and receiver sensitivity RX_{min}.

Definition 1. A link (i, j) is said to exist (i.e., node v_i can send packets to node v_j that is $d_{i,j}$ away, without consideration of interference) if and only if

$$p_t(i) \geq \frac{d_{i,j}^{\alpha} RX_{min}}{g_{i,j}}.$$

We also define an edge as a bi-directional link. That is, an $edge_{i,j}$ exists if and only if $p_t(i) \geq d_{i,j}^{\alpha} RX_{min}/g_{i,j}$ and $p_t(j) \geq d_{i,j}^{\alpha} RX_{min}/g_{j,i}$.

Given all the definitions, the communication graph of a network is represented by a graph $G = (V, E)$, where E is a set of undirected edges. Note that following the definition of an edge given in *Definition 1*, E is actually determined by the power assignment P_t. In other words, given a power assignment P_t, E is induced according to *Definition 1*. This is the graphic model used in conventional topology control. Note that the same model is also used in [9] [4] and [2].

B. Interference Model

As mentioned in Section I, mitigating interference is one of the major objectives of topology control. However, most existing topology control algorithms characterize interference with the node degree, and argue that a low node degree implies low interference. While this is an appropriate assumption under the graphic model, this may not be valid under the physical model. Before delving into the analysis, we first define interference under the physical model.

Recall that in Section II-A, the constraint in Eq. (1) is used to define the existence of a communication link. We now use Eq. (2) to define the interference in terms of the *interference degree*.

Definition 2. Interfering node: A node $v_k \in V$ is said to be an interfering node for link (v_i, v_j) if

$$\frac{p_t(i)d_{i,j}^{-\alpha}}{N + p_t(k)d_{k,j}^{-\alpha}} < \beta. \tag{3}$$

The physical meaning of the above definition is that if node v_k transmits with power $p_t(k)$, then the transmission on link $(v_i,\ v_j)$ can not proceed simultaneously, i.e., the receiver v_j is unable to decode the received signal due to the violation of the SINR constraint. The transmission activity which node v_k is engaged will either be blocked or collide with the transmission activity on (v_i, v_j).

Definition 3. The *interference degree* of a link (v_i, v_j) is defined as the number of interfering nodes for (v_i, v_j). Let $\hat{V}_I(v_i, v_j)$ denote the set of $v \in V$ containing all interfering nodes of (v_i, v_j), then the interference degree $D_I(v_i, v_j) \triangleq |\hat{V}_I(v_i, v_j)|$.

A link with a high interference degree implies multiple nodes can interfere with its transmission activity, causing channel competition and/or collision. This is undesirable because both channel competition and collision degrade the network capacity (i.e., the number of bytes that can be simultaneously transported by the network). Indeed it is the interfering nodes (rather than the communication neighbors) that substantially affect the throughput capacity under the physical model. Hence, the interference degree is a better index than the node degree in quantifying the interference. In Section III, we will show that the interference degree does not necessarily relate to the node degree.

Given the definition of the interference degree, we are in a position to define the *link interference graph* which is the counterpart of the communication graph under the physical model.

Definition 4. A *link interference graph* represents the interference of a link (v_i, v_j) as $G_I(V_I(v_i, v_j), E_I(v_i, v_j))$, where $V_I(v_i, v_j) = \hat{V}_I(v_i, v_j) \cup v_i \cup v_j$ and $E_I(link_{i,j})$ is the set of edges such that $(w, v_j) \in E_I(v_i, v_j)$, $w \in V_I(v_i, v_j) \setminus \{v_j\}$.

III. INTERFERENCE UNDER THE PHYSICAL MODEL

In this section we show that a small node degree does not directly relate to low interference under the physical model. Hence, the topology rendered by conventional topology control algorithms may not be capacity-efficient. Moveover, we show that the interference can be reduced by adequate power adjustment.

As mentioned in Section II-A, the topology is a graph induced by the transmit power assignment. Most existing topology control algorithms produce topologies by simply assigning the minimum possible power so as to ensure edges exist for *network connectivity*. Figure 1 gives an example that shows that this type of power assignment does not serve the purpose of mitigating interference under the physical model. Consider a link (i, j) in Figure 1 (a) and compare its interference degree against node j's degree. The node degree of j is 2. Let $\beta = 10$, $\alpha = 4$ and $N = 0$, and each node be configured with the minimal power so that it can communicate with its farthest neighbor (i.e., Eq. (1) holds). Under this configuration, the transmission activities of all the other nodes (A, B, C, D or E) transmitting lead to $SINR_{i,j} = 1/0.6^4 = 7.7 < 10$. That is, by Definition 2. all the other nodes are the interfering nodes to link (i, j), rendering the link interference graph of link (i, j) in Figure 1(b). Although the node degree of j is only two, link (i, j) has six interfering nodes, i.e., the transmission activity on link (i, j) may have to compete for channel access with 5 other potential transmissions. As a result, the attainable link capacity is much lower than it is expected to be. Such high interference, induced by graph-model-based topology control (and its associated power assignment), is obviously undesirable.

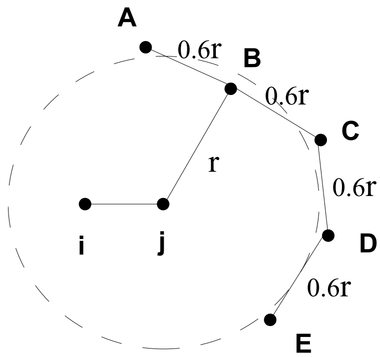
The above example also demonstrates that the interference degree dose not necessarily relate to the node degree. As a matter of fact, the interference degree is affected by several parameters such as β, N, α and p_t. Among them, N and α are environmentally determined and not controllable. β is a controllable parameter, and in the interest of Shannon's capacity, should be set to a reasonable large value. In this paper we thus focus on adjusting the transmit power p_t.

Now we show, by using the same example, that adjusting the transmit power (with the physical SINR model in mind) can indeed mitigate the interference. If the transmit power of node i is raised to 1.5 times of that in Figure 1. Even if any other node transmits concurrently with node i, $SINR_{i,j}$ now increases to $1.5/0.6^4 = 11.5$. This implies, instead of using the minimum power to maintain network connectivity, an adequate power level can substantially reduce the effect of concurrently transmitting nodes and thus improve the link capacity. Note also that a similar observation is also made by Moscibroda *et al.* in [10]. Note that the above example considers only peer interference. If the cumulative interference (i.e., interference contributed by multiple, concurrent transmissions) is considered, the interference in the topology induced by graph-model-based topology control will become even more severe.

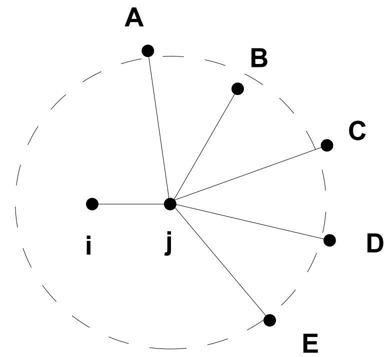
The inadequacy of graph-model-based topology control is rooted at the fact that the underlying communication topology it induces does not capture the interference appropriately under the physical model. An interesting question is then whether or not there exists a power assignment that enables the communication graph to represent the corresponding interference graph as well. We will address this question in Section IV.

IV. POWER CONTROL IN KNOWN TOPOLOGIES

In this section, we seek the answer to the following question: given a communication topology, is it possible to find a



(a) Network topology



(b) Link interference graph

Fig. 1. A low-node-degree topology does not necessarily imply low interference

power assignment such that the communication graph of the topology is identical to the physical-model-based interference graph? The rationale for enabling the communication graph to represent the interference graph is because the topology rendered by some of topology control algorithms exhibits several desirable properties such as bi-connectivity [9] and low node degree [4], [2]. If we can find a power assignment to enable the communication graph to represent the interference graph, we can invoke the new power assignment procedure after the topology is generated. All the desirable properties are preserved, and yet the adverse effects caused by interference are mitigated. We first formulate the problem as an optimization problem, and then investigate the feasibility of this problem.

A. Problem Statement

We first define what we mean by the communication graph of a topology representing its interference graph.

Definition 5: Under the physical model, the communication graph of a topology $G(V,E)$ is said to represent its interference graph, if and only if for every $edge_{i,j} \in E$, both $G_I(v_i,v_j)$ and $G_I(v_j,v_i)$ are the subgraphs of G.

Let $G'(V,E')$ be the complement of G. By Definition 5, the power assignment $P_t = \{p_t(1), p_t(2), ..., p_t(n)\}$ must satisfy the following constraints: for each pair of neighbors v_i and v_j in G,

- An $edge_{i,j} \in G$ exists.
- Any $edge'_{k,j} \in G'$ does not exist in $G_I(v_i,v_j)$.

The first constraint implies that the power assignment $p_t(i)$ and $p_t(j)$ guarantees the communication capability between v_i and v_j if $edge_{i,j} \in G$, i.e., $p_t(i) \geq d^{\alpha}_{i,j} RX_{min}/g_{i,j}$ and $p_t(j) \geq d^{\alpha}_{i,j} RX_{min}/g_{j,i}$. Without loss of generality, we assume that the channel gain is $g_{i,j} = 1 \; \forall i,j$. The first constraint can then be expressed as

$$p_t(i) \geq d^{\alpha}_{i,j} RX_{min}, \quad ; p_t(j) \geq d^{\alpha}_{i,j} RX_{min}. \tag{4}$$

The second constraint implies that, if $edge_{k,j}$ does not exist in G, the transmit power $p_t(k)$ of node v_k should not be large enough to enable v_k to become an interfering node of link (v_i,v_j) (with node v_i having the transmit power $p_t(i)$), i.e.,

$$\frac{p_t(i) d^{-\alpha}_{i,j}}{N + p_t(k) d^{-\alpha}_{k,j}} \geq \beta. \tag{5}$$

The above inequality implies that from the perspective of the transmission activity $v_i \rightarrow v_j$, v_k's transmission can simultaneously take place without impairing v_i's transmission. Thus $edge_{k,j}$ does not exist in $G_I(v_i,v_j)$. Eq. (5) can be rewritten as

$$\beta d^{\alpha}_{i,j} p_t(k) - d^{\alpha}_{k,j} p_t(i) \leq -\beta N d^{\alpha}_{i,j} d^{\alpha}_{k,j}. \tag{6}$$

With the two sets of constraints, we can formulate the problem as a linear programming with respect to $p_t(i), i = 1, ..., n$:

$$\texttt{minimize} \sum_{i}^{n} p_t(i)$$

subject to

$$\begin{array}{rcl} p_t(i) & \leq & p_{max} \\ p_t(i) & \geq & d^{\alpha}_{i,j} RX_{min}, \forall edge_{i,j} \in G \quad (7) \\ \beta\, d^{\alpha}_{i,j} p_t(k) - d^{\alpha}_{k,j} p_t(i) & \leq & -\beta\, N\, d^{\alpha}_{i,j} d^{\alpha}_{k,j} \\ \texttt{if} \quad edge_{i,j} & \in & G \quad \texttt{and} \quad edge'_{k,j} \in G' \end{array}$$

If the above linear program has a solution, it gives a feasible power assignment that enables a given communication graph to represent the interference graph.

B. Feasibility of the Problem

To study the feasibility of the linear program formulated, we use the communication graph induced by a representative topology control algorithm – *local minimal spanning tree (LMST)* [4] and its extensions [6] and [5]. LMST is chosen because as reported in [4], the node degree in its resulting topology is proved to be bounded by six. Moreover, as shown in the simulation study in [4], the average node degree in the resulting topology is comparatively lower than several other algorithms.

A total of 20 topologies are generated by exercising *LMST* in 20 random networks. Each network has 20 nodes which are uniformly placed in a rectangle area of 400×400 m^2. We first assign to each node the minimal possible power so that Eq. (1) holds for every link in the resulting topology. Based on this assignment and *Definition 3*, we can compute the interference degree for each link with respect to different values of β. Figure 2 shows the average interference degrees v.s. the average node degree. As anticipated, the minimal

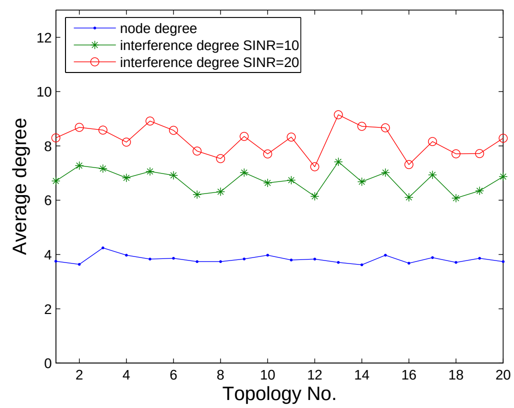


Fig. 2. Average interference degree v.s. average node degree

power assignment cannot ensure that the interference degree remains small in the interference graph under the physical model (Section III). The gap between the node degree and interference degree is surprisingly large. Moreover, the two average degrees are not linearly related to each other.

Now we investigate whether or not there exists a feasible power assignment to the the linear program given in Section IV-A. By solving the linear program on each topology induced by LMST, we found that no feasible solution exists for most of the cases, suggesting that the domain of p_t defined by the constraints is likely to be infeasible. (Solutions exist for some of the topologies when the number of nodes is no more than 6.) Moreover, most of the infeasibility is caused by the violation of Eq. (6).

To further understand under what condition Eq. (6) is violated, we consider a simple scenario shown in Figure 3. The network has a total of four nodes: 1, 2, 3 and 4. The solid lines mark the links present in the topology (e.g., link $(1, 2)$ and link $(3, 4)$), while the dotted lines indicate the links not present in the topology (e.g., link $(1, 4)$ and link $(3, 2)$). Let the distance between nodes 1 and 2, between nodes 3 and 4, between 1 and 4, and between 3 and 4 be respectively denoted as a_1, a_2, b_1 and b_2. Now we consider link $(1, 2)$ first. If node 3 is not an interfering node to this link, then by Eq. (6), we have

$$\beta a_1^\alpha p_t(3) \leq b_2^\alpha p_t(1). \tag{8}$$

Similarly, by considering link $(3, 4)$, we have

$$\beta a_2^\alpha p_t(1) \leq b_1^\alpha p_t(3). \tag{9}$$

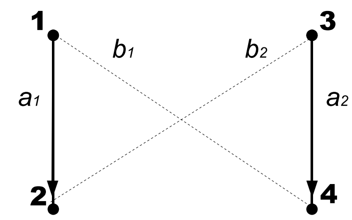


Fig. 3. A case of infeasibility

Eqs. (8) and (9) hold at the same time if and only if the following inequality holds

$$\frac{SINR_{min}^2 a_1^\alpha a_2^\alpha}{b_1^\alpha b_2^\alpha} \leq 1. \tag{10}$$

Otherwise, the power assignments $p_t(1)$ and $p_t(3)$ contradict with each other. Note that this particular topology can be a subgraph of a larger topology. Hence any power assignment for such subgraph should satisfy the constraint given by Eq. (10); otherwise the power assignment for the whole topology will be infeasible under the physical model. Now we generalize this feasibility constraint.

Definition 5: An *alternating cycle* C_a in a topology $G = (V, C)$ is a cycle that alternates between edges in G and edges in G'.

For example, $1 \to 2 \to 3 \to 4 \to 1$ is an alternating cycle in Figure 3. Let the length of an edge in G be denoted as a_i and that in the complement topology G' be denoted as b_i. The feasibility constraint can be stated as follows.

***Theorem** 1:* Any power assignment for a topology is infeasible under the physical model if there exists an alternating cycle in G such that

$$SINR_{min}^m \prod_{i \in C_a \bigcap E} a_i > \prod_{j \in C_a \bigcap E'} b_j,$$

Unfortunately, none of the existing topology control algorithms can ensure that the resulting topology satisfies this constraint. In our experiments, the probability that a power assignment for the resulting topology is feasible diminishes with the increase in the number of nodes (when $n > 6$, the probability is almost zero). This suggests that it is not likely to find power assignments to a topology induced by graph-model-based topology control to represent the corresponding interference graph. Therefore, as far as mitigating interference (and hence improving network capacity) is concerned, most existing topology control algorithms do not perform well under the physical model. In the next section we will propose a novel algorithm that combine topology control and power control to mitigate interference and improve network capacity.

V. TOPOLOGY CONTROL TO MAXIMIZE SPATIAL REUSE

In this section, we propose a novel algorithm to maximize spatial reuse and improve network capacity. The approach is composed of two component algorithms: (i) **T4P** that computes a power assignment that maximizes spatial reuse with a fixed topology, and (ii) **P4T** that generates a topology that maximizes spatial reuse with a fixed power assignment. By alternately invoking the two component algorithms, both the topology and the power assignment converge to a point that globally maximizes the network capacity.

A. Spatial Reuse Metric

Conceptually, spatial reuse is referred to the capability of a network to accommodate concurrent transmissions. Although a number of studies have been carried out on spatial reuse, there have not been explicit metrics defined to characterize the level spatial reuse. Most topology control algorithms use interference as an implicit metric, based on the intuition that low interference implies high spatial reuse. Although the intuition is correct, we show in Section IV that graph-model-based topology control inadequately captures interference under the physical model. Indeed, the interference degree, rather than the node degree, affects the link capacity. From a link's point of view, if there are less interfering nodes in its vicinity, it will have more chances to access the channel. From the network's point of view, if every link has a small number of interfering nodes, then the network will be able to accommodate more concurrent transmissions. Based on the above observation, we use the *average interference degree* as the metric for spatial reuse. It is obtained by taking all interference degree over all nodes in the network.

B. Topology to Power assignment: T4P

Under the physical model, whether some other concurrent transmission interferes an ongoing transmission of interest depends on several factors. If the transmit power is high, the ongoing transmission may tolerate interference better because of a higher SINR. On the other hand, if every node transmits with high power, the interference is likely high, depending on the relative positions of competing transmitters to the receiver of interest. In Section II, we have defined an interfering node in Eq. (3). Let the left hand side of Eq. (3) be defined as $\beta_k(i,j)$. Then we define an indicator function to denote whether a node k is an interfering node to link (v_i, v_j)

$$I(\beta_k(i,j)) = \begin{cases} 1, & \beta_k(i,j) < \beta \\ 0, & \beta_k(i,j) \geq \beta \end{cases} \tag{11}$$

Locally minimizing the *interference degree* may cause high interference to others. Hence all the nodes within the interference range must cooperate to achieve some level of global optimality. As such, we formulate the T4P problem as an optimization problem:

$$\texttt{minimize} \sum_{link(i,j)\in T} \sum_{k\neq i,j} I(\beta_k(i,j))$$

subject to

$$P_{min} \quad \leq P_t \leq \quad P_{max}.$$

The above problem is an integer program because of the existence of indicator functions. Fortunately, as indicated in [11], the hard SINR requirement can be "softened" by the sigmoid function. The sigmoid function is a continuous function expressed as

$$sig(x) = \frac{1}{1+e^{-a(x-b)}}. \tag{12}$$

When x is greater than the threshold b, $sig(x)$ will quickly rise up to 1, and when x is less than the threshold b, $sig(x)$ will quickly drop down to 0. The parameter a determines how quickly the sigmoid function changes near the threshold. Figure 4 gives two example sigmoid functions. We approximate

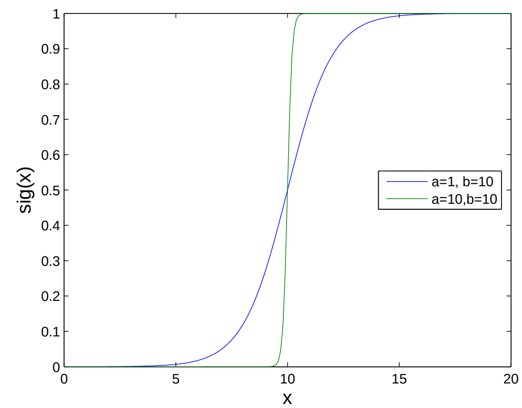


Fig. 4. Sigmoid function

the integer program by replacing the indicator function with the sigmoid function:

$$\texttt{minimize} \sum_{link(i,j)\in T} \sum_{k\neq i,j} sig(\beta_k(i,j))$$

subject to

$$P_{min} \quad \leq P_t \leq \quad P_{max}. \tag{13}$$

where we set the parameter $b = \beta$. The problem can then be solved by using a *sequential quadratic programming (SQP)* method [12], [13].

In summary, **T4P** finds an optimal power assignment given a fixed topology as follows.

Algorithm 1 Topology to Power: T4P

Require: Topology(V, E)

Solve the optimization problem (13) with the SQP method

Ensure: Power Assignment P_t

C. Power assignment to Topology: P4T

The above algorithm **T4P** determines an optimal power assignment with a given topology. However, the input topology may not be optimal in terms of maximizing network capacity. If different topologies (induced by different topology control algorithms for the same network) are used as input to **T4P**, different power assignments result. It is obviously undesirable to test out all possible topologies for optimality.

To address this problem, we devise another component algorithm **P4T**, which generates an optimal connected topology, given a fixed power assignment. The algorithm is similar to the minimum spanning tree algorithm, except that we attempt to find the spanning tree that gives the minimal interference degree. The pseudo code of **P4T** is given below. Specifically,

Algorithm 2 Power to Topology: P4T

Require: Power assignment $\{p_t(1), p_t(2), ..., p_t(n)\}$
  for all node pairs u, w such that $distance(u, w) \leq$ transmission range **do**
    compute its interference degree by Eq. (3)
  end for
  sort edges in the non-decreasing order of interference degree, and let $\tilde{e}_1, \tilde{e}_2, ...$ be the resulting sequence of edges
  initialize n clusters, one per node, $E = \emptyset$ and $i = 1$
  while the number of cluster > 1 **do**
    for $\tilde{e}_i(u, w)$
    if cluster(u) $\neq$ cluster(w) **then**
      merge cluster(u) and cluster(w)
      E=$E \bigcup \{\tilde{e}_i\}$
    end if
    i=$i + 1$
  end while
Ensure: Topology $T(V, E)$

given a power assignment, we compute (by Eq. (3)) the interference degree for every pair of nodes whose distance is less than the maximum transmission range (i.e., the $d_{i,j}$ value that makes the equality in Eq. (1) hold). The interference degree calculated is considered as the weight of the edge $edge_{i,j}$. Initially, each node forms a one-node cluster. Edges are selected in the non-decreasing order of their weights. If the node pair of the selected edge is in different clusters, then the two clusters are merged. The above step is repeated until there is one cluster. Note that **P4T** not only gives a topology but also implicitly gives P_{min} that ensures network connectivity. It can be used as the lower bound for the optimization problem in **T4P**. In Section V-D, we will prove that the topology induced by **P4T** is optimal in terms of minimizing the interference degree.

D. Spatial Reuse Maximizer

So far we have devised two algorithms: (i) **T4P** gives a power assignment such that the interference degree given a fixed topology is minimized, and (ii) **P4T** derives, given a fixed power assignment, a spanning tree that gives the minimal interference degree. To optimize both P_t and T, we propose an **MaxSR**. It works by alternatively invoking **T4P** and **P4T** until the power assignment converges to a point. Formally we present **MaxSR** below. Now we prove MaxSR does converge with the following lemma and theorem.

Lemma 1: Algorithm **P4T** gives an connected topology that minimizes the interference degree with a fixed power assignment.

Algorithm 3 SpatialReuseMaximizer

Require: Node set V and their coordinates $\{X, Y\}$
  let ϵ be a small value
  let $D(T, P_t)$ be the sum of interference degree with given T and P_t
  initialize Δ = 1, $T = T(P_{max})$ and P_t=**T4P**(T)
  while $\Delta > \epsilon$ **do**
    $D_{old} = D(T, P_t)$
    T=**P4T**(P_t)
    P_t=**T4P**(T)
    $\Delta = ||D_{old} - D(T, P_t)||$
  end while
Ensure: Power assignment P_t

The proof of lemma1 is similar to *Theorem III* in [9], which proves that a minimum cost spanning tree algorithm gives an optimum connected graph that minimizes the transmit power. The only difference is that **P4T** intends to find a spanning tree that gives the minimal interference degree. Hence we can prove Lemma 1 following the same line of argument in [9] except that we replace the edge weight of distance by the edge weight of interference degree.

Theorem 2: **MaxSR** converges to an optimal point.

Proof: Let $D(P_t^{(n)}, T^{(n)})$ be the sum of interference degree after the n-th iteration. Because **T4P** intends to minimize the sum of interference degree in a fixed topology, after $(n + 1)$-th running **T4P**, we must have

$$D(P_t^{(n+1)}, T^{(n)}) \leq D(P_t^{(n)}, T^{(n)}).$$

Similarly, by Lemma 1, we have

$$D(P_t^{(n+1)}, T^{(n+1)}) \leq D(P_t^{(n+1)}, T^{(n)}).$$

Consequently, $D(P_t^{(n)}, T^{(n)})$ is a monotonic non-increasing function in n. Since P_t has a lower bound, $D(P_t^{(n)}, T^{(n)})$ should also be bounded in a connected graph. Thus $D(P_t^{(n)}, T^{(n)})$ converges, and we conclude that algorithm **MaxSR** converges. ∎

According to our experiments, Figure 5 illustrates the convergence speed of **MaxSR** versus the network size, where $\epsilon = 0.02$. The observation is that the number of iterations is independent of the network size and **MaxSR** normally converges within 10 iterations. But note that the running time of **T4P** and **P4T** should depend on the number of nodes.

VI. Simulation Study

In this section, we carry out a simulation study to evaluate the performance of **MaxSR** and compare it against three schemes: **MaxPow** (i.e., all nodes transmit with their maximum transmit power), **LMST** [4] and **CBTC**($5\pi/6$) [2].

Metrics That Are of Interest: In the simulation study, we are primarily interested in the following metrics:

- *Interference Degree*: Given a power assignment, the interference degree can be computed for each link.

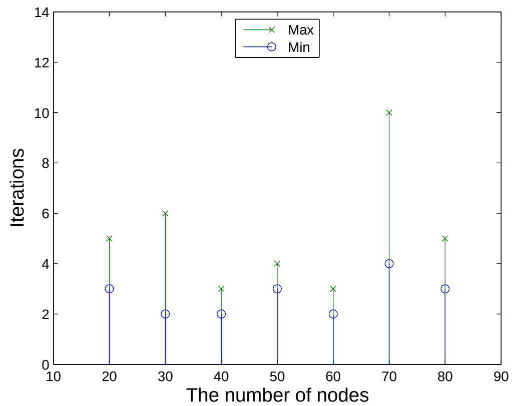


Fig. 5. convergence speed v.s. the network size, where $\epsilon = 0.02$

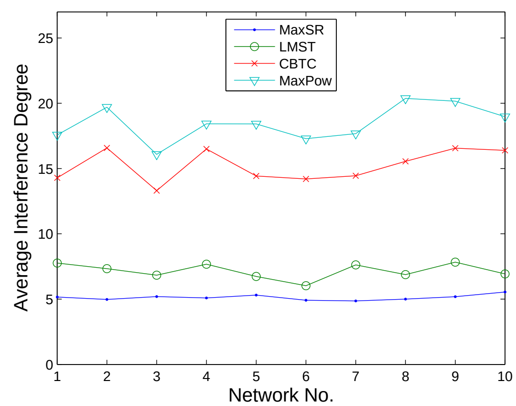


Fig. 6. Average interference degree under different algorithms: 10 random networks each with 40 nodes randomly placed in 500m×500m area

- *Network Connectivity*: Connectivity is perhaps the most important criterion for topology control. In our study, we quantify the level of connectivity under the physical model by the number of disconnected flows during the simulation time.
- *Throughput Capacity*: As discussed in Section V-A, interference degree is a good metric for characterizing spatial reuse and hence network the capacity improvement. We evaluate the performance of various algorithms with respect to network capacity by keeping track of the saturated throughput in random networks.

a) Computation Result: First we give the computation result of **MaxSR** against three schemes: **MaxPow**, **LMST** and **CBTC**, with respect to the average interference degree. A total of 10 networks are generated randomly, and for each network a total of 40 nodes are uniformly placed in a rectangle area of 500×500 m^2. For each network, **MaxSR** derives both the topology and the power assignment; **MaxPow** assigns the maximum transmit power to each node and the topology is induced by the power; while **LMST** and **CBTC** derive the topology and induce the power assignment by assigning the minimum power so as to maintain the derived topology.

Based on the topology and the power assignment derived/induced, we then compute the interference degree for each link and take the average over all links. Figure 6 gives the average interference degree under the various algorithms. Not surprisingly **MaxPow** has the largest average interference degree, cofirming the intuition that large power gives rise to high interference. Based on the minimum spanning tree algorithm, **LMST** gives perhaps the minimum interference among all conventional topology control algorithms. **MaxSR**, on the other hand, gives the minimum average interference degree among all the algorithms.

b) Simulation Setup: We leverage J-sim [14] to carry out the simulation study for the following reasons: (i) *ns-2* does not take into account of the effect of accumulative interference; and (ii) *ns-2* computes the interference range, assuming that all nodes use a common transmit power, whereas topology control algorithms assign different levels of transmit power to different nodes.

In our simulation study, we consider IEEE 802.11-based networks. Table I shows the system parameters used in the simulation. Again a total of 10 networks are generated randomly, and for each network a total of 40 nodes are uniformly placed in a rectangle area of 500×500 m^2. A total of 20 sorce-destination pairs are specified. In order to decouple the effect of routing protocols from topology control, we consider the saturated throughput of one-hop flows, i.e., a source and its corresponding destination are so chosen that they are neighbors of each other.

TABLE I
SIMULATION PARAMETERS

RXThreshold	3.6e-10	Traffic pattern	CBR
Inter-arrival time	4e-4	Trans. protocol	UDP
CPThreshold	20dB	Routing protocol	AODV
Packet payload	512 bytes	Slot time	20 μs
PHY header	24 bytes	CW_{min}	31
ACK frame	38 bytes	CW_{max}	1023
DATA bit rate	6 Mbps	Retry limit	7
PHY bit rate	1 Mbps	Max txpower	0.2818
α	4	hr,ht	1.0m

Performance Evaluation: Although we have decoupled the effect of routing protocols from topology control, we have to consider the effect of the carrier sense threshold in IEEE 803.11-based networks. This is because the network capacity depends also on the setting of the carrier sense threshold. On the one hand, if the carrier sense threshold is too small, spatial reuse cannot be fully exploited and the network may encounter the exposed node problem. On the other hand, if the carrier sense threshold is too large, interference becomes severe and the network may encounter hidden node problem. Thus, we will run simulation with different carrier sense thresholds and observe its effect on the network connectivity and capacity.

Figure 7 gives the simulation result of the aggregate throughput v.s. the carrier sense threshold under various algorithms. As anticipated, **MaxSR** achieves the highest aggregate throughput except when the carrier sense threshold is small

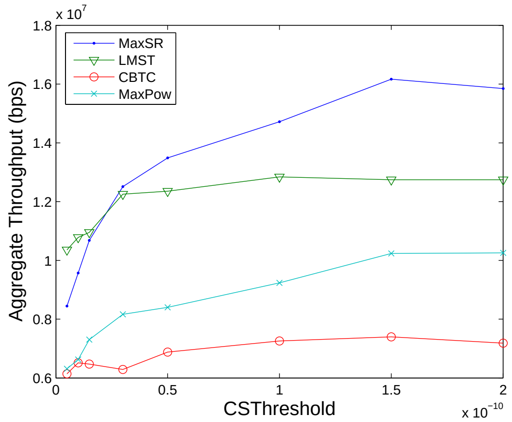


Fig. 7. Aggregate throughput v.s. carrier sense threshold

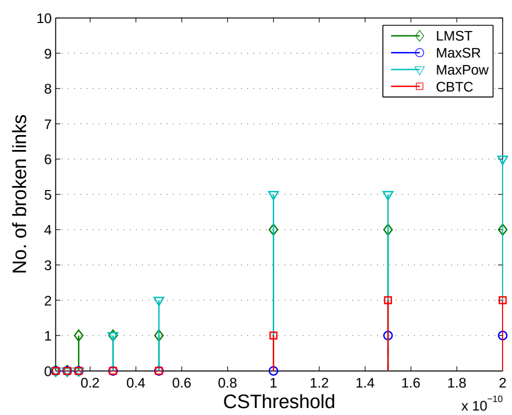


Fig. 8. The number of broken links v.s. carrier sense threshold

(under which case spatial reuse is constrained by the carrier sense threshold). It outperforms **LMST** by 50%, **CBTC** by 110% and **MaxPow** by 102% in terms of maximizing network capacity.

Another interesting observation is that that the aggregate throughput increases as carrier sense threshold increases. This is because increasing the carrier sense threshold mitigates the effect of the exposed terminal problem and achieve better spatial reuse. However, the increase in the aggregate throughput levels off when the carrier sense threshold increase beyond the point at which the the maximum capacity achieved by the specifc network topology. If the carrier sense threshold is further increased, the network starts to experience the hidden terminal problem. Although the hidden node problem does not affect aggregate throughput dramatically, it may cause severe unfairness and partition the network. Figure 8 gives the number of broken links v.s. the carrier sense threshold. When the carrier sense threshold is too large, several links fail under the physical model, due to severe interference. **MaxSR** nevertheless still gives the best network connectivity.

VII. Related Work

We categorize related work into the following three categories:

Topology control/management under the protocol model: The issue of power control has been studied in the context of *topology maintenance*, where the objective is to preserve network connectivity, reduce power consumption, and mitigate MAC-level interference [2], [3], [4], [5], [6]. Rodoplu *et al.* [3] introduced the notion of *relay region* and *enclosure* for the purpose of power control. A two-phase distributed protocol was then devised to find the minimum power topology for a static network. In the first phase, each node i executes local search to find the enclosure graph. In the second phase, each node runs the distributed Bellman-Ford shortest path algorithm upon the enclosure graph, using the power consumption as the cost metric.

CBTC(α) is a two-phase algorithm in which each node finds the minimum power p such that transmitting with p ensures that it can reach some node in every cone of degree α. The algorithm has been analytically shown to preserve the network connectivity if $\alpha < 5\pi/6$. It has also ensured that every link between nodes is bi-directional.

Li and Hou [4] devised a *Local Minimum Spanning Tree (LMST)* algorithm and its variations [5], [6] for topology control and management. In LMST, each node builds its local minimum spanning tree *independently* with the use of locally collected information, and only keeps on-tree nodes that are one-hop away as its neighbors in the final topology. They have proved analytically that (1) if every node exercises LMST, then the network connectivity is preserved; (2) the node degree of any node in the resulting topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links).

As mentioned in Section I, topologies derived under these graph-model based topology control algorithms may not capture interference adequately under the physical SINR model. As a result, interference may be outrageously high in the topology induced by graph-model based algorithms, rendering sub-optimal network capacity.

Control of transmit power for capacity improvement: Use of power control for the purpose of spatial reuse and capacity improvement has been treated in the *COMPOW* protocol [15], the *PCMA* protocol [16], the *PCDC* protocol [17], the *POWMAC* protocol [18], and the *PRC* protocol [19]. Narayanaswamy *et al.* [15] developed a power control protocol, called *COMPOW*. In *COMPOW* each node runs several routing daemons in parallel, one for each power level. Each routing daemon maintains its own routing table by exchanging control messages at the specified power level. By comparing the entries in different routing tables, each node can determine the smallest common power that ensures the maximal number of nodes are connected.

Monks *et al.* [16] propose *PCMA* in which the receiver advertises its interference margin that it can tolerate on an out-of-band channel and the transmitter selects its transmit power that does not disrupt any ongoing transmissions. Muqattash and Krunz also propose *PCDC* and *POWMAC* in [17], [18] respectively. The *PCDC* protocol constructs the network topology by overhearing RTS and CTS packets, and the computed interference margin is announced on an out-of-band channel. The *POWMAC* protocol, on the other hand, uses a single channel for exchanging the interference margin information.

Kim *et al.* [19] studied the relationship between physical carrier sense and Shannon capacity, and showed that the achievable network capacity only depends on the ratio of the transmit power to the carrier sense threshold. They then propose a decentralized power and rate control algorithm, called *PRC*, to enable each node to adjust, based on its signal interference level, its transmit power and data rate. The transmit power is so determined that the transmitter can sustain a high data rate, while keeping the adverse interference effect on the other neighboring concurrent transmissions minimal.

All the efforts reported in this category focus more on devising practical power control protocols, and have not formally established optimality in the course of algorithm/protocol construction.

Joint topology control and scheduling under the physical SINR model: Moscibroda, Wattenhofer, and Zolliner [8] are the first to consider topology control under the physical model. They focus on reducing the schedule length in topology-controlled networks. They proved that if the signals are transmitted with correctly assigned transmission power levels, the number of time slots required to successfully schedule all links is proportional to the squared logarithm of the network size. They also devised a centralized algorithm for approaching the theoretical upper bound. In a similar problem setting, Brar, Blough, and Santi [20] presented a computationally efficient, centralized heuristic for computing a feasible schedule under the physical SINR model. They did not explicitly consider topology control, although whether or not communication succeeds is determined based on the SINR model. In some sense, **MaxSR** complements the above two efforts. Recall that **MaxSR** aims to improve network capacity without assuming any specific scheduling policy. Instead of attempting to reduce the schedule length, we focus on deriving a network topology, along with its power assignment, to maximize the network capacity.

VIII. CONCLUSION

In this paper, we investigate the issue of topology control under the physical SINR model, with the objective of maximizing network capacity. We show that existing graph-model-based topology control captures interference inadequately under the physical model. In order to address the problem, we introduce a new metric for spatial reuse, called the interference degree. It measures the actual interference under the physical model. To mitigate interference and improve spatial reuse, we then propose a centralized approach **MaxSR** that combine a power control algorithm **T4P** with a topology control algorithm **P4T**. We also show via simulation that the topology derived by **MaxSR** outperforms that induced from existing topology control algorithms by 50-110% in terms of maximizing the network capacity.

We have identified several avenues for future research. We will design, based on the insight shed from the study reported in this paper, a decentralized version of **MaxSR** that maximizes spatial reuse. We would also like to investigate how to combine **MaxSR** with a scheduling policy (such as that proposed in [20]) so as to maximize network capacity in both the *spatial* and *temporal* domains.

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