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Transitioning to an Active Learning Environment for Calculus at the University of Florida

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TRANSITIONING TO AN ACTIVE LEARNING ENVIRONMENT FOR CALCULUS AT THE UNIVERSITY OF FLORIDA

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1. Introduction

In the fall of 2017 the University of Florida launched Faculty 500 (`https://faculty500.hr.ufl.edu/`), an ambitious plan to add 500 new faculty to campus in a two-year span. One stated goal of the program was to reduce class sizes, which in the case of the math department had grown as large as 650 in Calculus I. While our students were generally well-served by the existing excellent cadre of permanent lecturers, no one really believed that this format was the best way to teach the material. Hence the department's proposal for hiring in the first year of the expansion included adding several new lecturers who would be dedicated to teaching Calculus I in smaller sections. While we were at it, we would also transition those classes to an active learning format and renovate three classrooms for that purpose. In the end, the department hired five new lecturers, each of whom would teach three 64-student sections per semester.

Luckily, we were not starting from nothing. Flipped sections of calculus had been piloted, in classes as large as 100, by one of the authors (Knudson) using content videos initially developed for UF's online university. He developed some in-class materials and experimented with standards-based grading as well. While this was a good base it was not nearly sufficient for a large-scale rollout; thus, the new team needed to expand this collection of materials and work together on exams and other assessments. And they had to hit the ground running–there was only one week between their start dates and the beginning of the fall semester.

In this note we will describe the first iteration of this project, which took place during the fall 2018 term. After introducing the team (Section 2) we will outline our approach in Section 3, which involved dividing students into traditional lecture sections and flipped classes for comparison purposes. In Section 4 we present some data, and in Section 5 we discuss our plans moving forward.

2. The Team

The project team consisted of the following members of the Department of Mathematics at UF.

- Kevin Knudson, professor and chair of the department;
- Scott Keeran, lecturer and coordinator of Calculus I;
- Darryl Chamberlain, lecturer and coordinator of Basic College Algebra;
- Amy Grady, newly hired lecturer;
- Ian Manly, newly hired lecturer;

- Melissa Shabazz, newly hired lecturer;
- Corey Stone, newly hired lecturer;
- Alexander York, newly hired lecturer.

Knudson and Keeran have been at the university for several years and have taught large lecture sections of Calculus I many times. Chamberlain was hired in 2017 to overhaul the university's college algebra course, but has also taught Calculus I at UF. His research is in collegiate mathematics education and his expertise in that area has been useful, especially for data analysis. The new lecturers have varying levels of experience: Shabazz and Stone each had some postdoctoral experience at large state universities, while Grady, Manly, and York were new Ph.D. graduates from large state schools.

3. The Plan

Prior to the fall 2018 term, Calculus I was taught in large lectures ranging in size from approximately 250 to 650, the latter consisting entirely of students majoring in engineering. These lectures were coordinated by an experienced lecturer, who kept the various instructors on the same schedule, wrote the common exams, and developed the course notes for students. approximately 60 discussion sections, taught by graduate teaching assistants, were attached to these lectures. Total enrollment in the course during a typical fall semester was approximately 1,800 students.

The plan was to divide the students into two cohorts. Approximately 900 students would be placed into two large lectures of 300 and 600, and the remaining 900 students would be placed into 15 active learning sections of 64 students each. All classes would meet for three 50-minute periods per week with a fourth 50-minute discussion period in classes of 32 students. Students registered for their classes during summer orientation and had full knowledge of which class format they were getting. During the fall drop/add period they were free to switch classes if they so desired.

The five newly-hired lecturers, along with Keeran, decided that every class would have the same grading scheme but they each would have individualized weightings on graded assignment groups. For the flipped classrooms these assignment groups were all the same. These included lecture quizzes, participation, discussion quizzes (TAs mainly handled these), online homework, midterm exams, and final exam. The weighting on the midterm exams and the final exam was the same across all the flipped classes.

3.1. **Student profile.** The University of Florida is the state's flagship, an AAU institution rated among the top 10 public universities by U.S. News and World Report (`https://bit.ly/2qEkhCD`). As such, the incoming first-year student body is quite competitive. The class of 2022 boasts an average high school core GPA of 4.4, an average SAT score of 1360, and an average ACT score of 30. The enrolled class is 6.8% African-American, 10% Asian, and 20.8% Hispanic. A complete profile of the class is available at `https://bit.ly/2XDDCA3`.

By show of hands, a large majority (in excess of 90%) of students in Calculus I had taken some flavor of the course in high school. Placement into the course was based on obtaining a score of 75 or greater on the ALEKS placement exam. Roughly one-third of the students in the course were engineering majors with the

remainder representing a broad array of STEM disciplines. Approximately 1,000 students had declared a pre-health emphasis.

3.2. **Course flow.** The lecturers adopted the system of course flow that had been developed by Knudson in the previous flipped calculus offerings. Prior to each class meeting students were expected to watch a lecture video associated with the day's topics and complete an online lecture quiz based on that information. In class, students would complete an activity or assignment to reinforce and expand their skills in the new topic(s). After class students were assigned online homework due at a later date. In discussion students were quizzed on certain topics, usually the previous week's material. The midterm exams covered certain portions of the class content and the final was cumulative. The course covered standard Calculus I content in the traditional order. This includes limits, differentiation, applications of differentiation, antiderivatives, definite integrals, and the Fundamental Theorem of Calculus. Each of these topics was broken into 5-9 video lectures, yielding a total of 32 lectures plus 4 additional videos of precalculus review material.

The lecture quizzes used in the flipped courses had been constructed previously, but each lecturer was free to modify them. Each lecture quiz corresponded to one of the 32 total lectures and consisted of 2-3 questions covering basic definitions and examples from the lecture. These seemed to work well as a method to ensure that students at least glanced at the material before class.

The lecturers decided to split the work of creating in-class assignments by each week. These assignments consisted of worksheets for each individual student or student group to complete. This rotation worked well for creating content. However, the variety of activities was limited and certainly could be improved upon in future semesters. Each worksheet consisted of basic concept checks and simple examples to work through with problems gradually increasing in difficulty and complexity as students worked through the assignments. As these were not meant to be a hard check on the students' ability to use the concepts from the topics, the lecturers were able to insert particularly challenging or thoughtful application problems students normally would not see in a traditional calculus course. We observed that these challenges seemed to enhance student interest in the course. This also gave students for whom Calculus I is their last mathematics course a reason to want to learn material and see it related to their field of study. Two sample lecture activities from the course are included in Appendix B.

3.3. **Discussion sections.** All students in Calculus I have a weekly discussion section. The enrollment in these sections is capped at 32 students and the sections are taught by TAs. The purpose of the sections is to allow the students the opportunity to meet in smaller class sizes and to ask questions concerning course material, homework problems, test questions, etc. In general there is a graded quiz or assignment in these discussions which is used to encourage attendance, participation, and to ensure that students are keeping current with the class. Each TA is observed by one of the lecturers at least once during the semester to make sure they are performing their duties competently and that the students are getting consistent treatment of the material.

3.4. **Classrooms.** The mathematics building at UF has three classrooms designed specifically for active learning; these are utilized for the flipped calculus courses. Each room contains a document camera, a projector connected to the document

camera and in-room computer, and a collection of 32 two-seat movable tables for a total of 64 students. Two of the rooms utilize television screens on all four walls to allow projection of information along with a collection of dry-erase boards for the instructor and students. The third room has a wrap-around whiteboard along all of the walls as well as two projector screens at the front of the room. It is natural to set up the tables into groups of four by placing two tables together. This allows up to sixteen groups of four students. The lecturers mostly agree that four students is a logical number to reduce off-topic interaction but still allow enough socialization to help keep students engaged. The layout of the rooms also allows the instructors to include a variety of group activities and exercises. The students are able to work at their tables in groups, on the boards with dry-erase markers, work in larger or smaller groups, and present material or solutions on the document camera or boards. The lecturers also feel that groups of four made it easier to interact with students when they request help or clarification. Addressing these smaller groups limits interruptions of the entire class.

3.5. **Learning assistants.** To assist the instructor with the activities, each section of the course had three to four undergraduate learning assistants who attended each class period. Their role was typically to answer student questions and address any difficulties they had in completing the day's assignment. The summer before the course began, Knudson solicited applications from undergraduate math and engineering majors to become learning assistants. More than 150 applications were received for approximately 60 positions, so the students were selected based on performance in mathematics courses, prior teaching/tutoring experience, and a very brief statement of interest. Once selected the students could elect to receive course credit for serving as assistants, though most chose not to do so to avoid incurring additional tuition charges. After being assigned to a section of the course, they were contacted by the instructor and were briefed on their responsibilities.

The majority of the learning assistants performed their duties admirably. They were on time, worked well with the students in the course, and reviewed the assignment ahead of class to be prepared for questions. These learning assistants were of great help to their section, as it is difficult for a single instructor to handle all of the student questions. However, some of the learning assistants skipped many of the class periods, while others did not review the assignment before class. The learning assistants who were unprepared were often of little help, as they did not know how to complete the assignment. This may be attributed to the fact that the learning assistants did not need to sign up for a course to volunteer and had no repercussions for missing periods or being unprepared. Fortunately, these assistants were in the minority, so overall the learning assistants were more of an asset than a liability.

3.6. **Course materials.** To substantially reduce costs to students, the course utilized the open source calculus text provided by the OpenStax project (`https://openstax.org/details/books/calculus-volume-1`). In addition, we implemented an online homework system called Xronos. The development of the Xronos system was spearheaded by Knudson and is based on the Ximera Project at the Ohio State University (NSF Grant DUE-1245433, `https://ximera.osu.edu/`). This is an open source platform in which instructors write modules in LaTeX that then get converted to interactive HTML5. Xronos is provided free of charge to students. For

each of the major concepts of the course such as the derivatives of trig functions, u-substitution, and so forth, a set of homework problems (usually 10 to 20 questions) was devised (27 sets in all to match the 27 lectures of calculus material covered in the flipped classrooms) to give students practice in computation and to reinforce theoretical concepts. The homework assignments were available through Canvas, were opened after the corresponding concepts have been covered in class, and students generally had three to seven days to complete each problem set. Students were given an unlimited number of attempts to answer each question.

3.7. **Exams.** Both the flipped and standard lecture classes developed the concepts of calculus in the "traditional" order given in most texts and spent similar amounts of time on each topic. There were four common tests for the semester with three midterms consisting of multiple choice (70 points) and free response (35 points) questions and the final cumulative exam being all multiple choice questions (110 points). The lecturers agreed upon a list of topics for each exam and then individually chose topics from the list for which they developed test questions. The questions were reviewed in a group meeting and modified if necessary in order to gain a consensus. Once the questions were approved, the final version of each exam was constructed. There were two different versions of each exam. The multiple choice questions were graded by computer and the free response questions were graded by the course TAs using a common grading rubric. The design of exam items commonly asked students to recall and apply a procedure, or apply their understanding to make some conclusion about the mathematical situation [7].

4. Data Analysis

4.1. **Exam Scores.** Since the students all took the same exams we are able to compare their performance directly. Table 1 shows the results of the final examination, which consisted entirely of multiple-choice questions. The distributions of grades are nearly identical across the two formats.

The three mid-term examinations consisted of both multiple choice and free response questions. Table 2 shows the average scores of students across the two formats along with the median total free response scores for each of the three exams. Again, we see little difference among the two groups. There were three questions with a margin of at least half a point:

  (1) Exam 1, Question 4 was a question about the Intermediate Value Theorem, in which students were asked to identify an interval on which a cubic polynomial had at least one root.
  (2) Exam 1, Question 5 asked students to identify horizontal asymptotes for a function such as $f(x) = x/\sqrt{9x^2 + 1}$.
  (3) Exam 3, Question 3 consisted of several parts aimed at having students sketch a graph of a rational function. The function and its first and second derivatives were provided to eliminate errors stemming from incorrect differentiation of the initial function. Students were asked to identify asymptotes, critical points, intervals of increase/decrease, inflection points and intervals of concavity, and local extrema.

It is interesting to note that in all three of these cases the traditional lecture students performed better. Of course there were other questions on which the flipped students did better as a group, but not by as wide a margin.

Final Exam Scores-Traditional
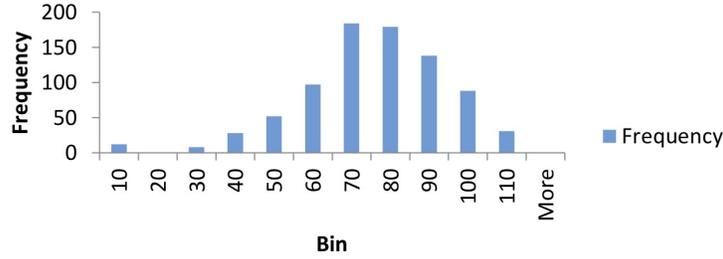
Final Exam Scores - Flipped
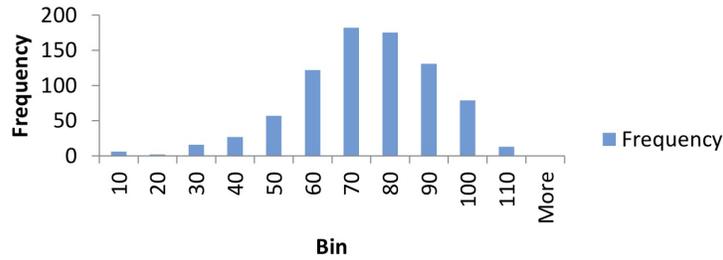
TABLE 1. Final exam grade distributions

	E1Q1	E1Q2	E1Q3	E1Q4	E1Q5
Value	7	7	7	7	7
Flipped (μ, σ)	6.38, 1.60	5.58, 2.27	5.81, 1.89	4.62, 2.29	4.55, 2.48
Traditional (μ, σ)	6.13, 1.57	5.45, 2.07	5.67, 1.83	5.27, 2.27	5.31, 2.23

	E2Q1	E2Q2	E2Q3	E2Q4	E2Q5
Value	7	7	7	7	7
Flipped (μ, σ)	5.95, 1.30	6.06, 1.58	4.82, 2.05	5.68, 1.68	5.72, 1.83
Traditional (μ, σ)	5.95, 1.37	6.14, 1.39	5.15, 1.98	5.86, 1.68	6.00, 1.67

	E3Q1	E3Q2	E3Q3
Value	7	7	21
Flipped (μ, σ)	6.23, 1.46	6.06, 1.73	17.31, 3.99
Traditional (μ, σ)	6.12, 1.43	5.97, 1.82	18.11, 3.81

	E1 median	E2 median	E3 median
Flipped	29	30	31
Traditional	30	31	32

TABLE 2. Mid-term examination free response results

	Flipped	Traditional
Total enrollment	861	902
Withdraws	37	77
Percentage	4.30%	8.50%

TABLE 3. Withdrawal data

4.2. **Course Grades.** Since the instructors of the flipped and traditional sections used slightly different grading schemes, a direct comparison of final grades is not useful. The grades tended to skew higher overall in the flipped sections, but this can be attributed to those instructors choosing to drop some test scores. The numbers of students earning non-passing grades (less than C) was essentially the same across the two formats. One meaningful comparison to consider is the drop rate for the course, shown in Table 3. Roughly half as many students dropped the flipped course as opposed to the traditional lecture. We posit that this is due to the smaller class size and greater engagement with faculty in the active learning sections, but we do not have any direct evidence to substantiate this.

The upshot is that the academic performance of the students was effectively the same across the two formats. This is perhaps unsurprising since the whole group consists primarily of first-semester first-year students, most of whom had taken calculus in high school and were generally well-prepared. The one discernible benefit to the active learning classes, which we anticipated beforehand [3, 6, 1], was the decreased drop rate.

4.3. **Student Affect.** In addition to academic performance, the team was interested in investigating whether there were other differences between students for the two types of courses, such as students' self-perceptions, confidences, attitudes, and beliefs. We refer to these collectively as *student affect*. Indeed, it is well-known that students' emotions and motivations are critical when assessing student success in a course [4]. To assess this in our students we utilized the Collegiate Active Learning Calculus Survey (CALCS), developed at the University of Nebraska-Lincoln [5]. This is a 5-point Likert scale survey with 33 affect questions. This survey was administered at the beginning and at the end of the semester to explore whether there was a difference in students' affect between the two course formats [2].

Survey data was then cleaned so that the team could run multiple chi-squared tests to determine whether the course format (traditional/flipped) was independent of student affect along each question. First, all data collected by students who opted out of having their data included in the analysis was removed. Then, any incomplete surveys were removed, especially those who did not indicate their section. Finally, a question was included in the survey to ensure students were providing thoughtful responses - all students who answered this question "incorrectly" had their responses removed. In total, 1443 complete student responses were collected at the beginning of the semester and 1262 complete student responses were collected at the end of the semester. The Bonferroni correction was used to compensate for the large number of categories being tested, and thus categories were determined to be "independent" with a chi-squared value of greater than 17.54 (significance level of $\alpha = 0.05/33$). Chi-squared values are presented in Tables 4 and 5 to concisely summarize the results. Mean values for each item are also included to succinctly illustrate rough response rates.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Trad. Avg	3.27	3.08	2.72	2.90	2.46	4.13	4.03	2.44	2.27	3.71
Flip Avg	3.27	3.14	2.79	2.96	2.48	4.13	4.04	2.40	2.23	3.77
χ^2	0.36	2.40	2.35	1.09	4.37	2.14	0.88	1.54	5.99	1.98

	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
Trad. Avg	3.42	4.02	3.98	3.72	2.13	2.31	3.31	2.86	1.95	3.97
Flip Avg	3.49	3.97	4.01	3.93	2.34	2.33	3.36	2.81	1.88	4.05
χ^2	2.83	4.16	0.69	24.32	22.72	0.95	3.13	7.83	4.73	5.43

	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30
Trad. Avg	3.64	3.82	3.91	3.20	3.86	4.00	3.85	2.80	3.66	3.35
Flip Avg	3.68	3.62	3.99	3.26	3.88	4.00	3.76	2.78	3.80	3.4
χ^2	2.29	18.89	6.23	4.28	2.08	NA	7.07	1.54	9.09	4.58

	Q31	Q32	Q33	Q34
Trad. Avg	2.49	3.77	2.40	3.70
Flip Avg	2.61	3.84	2.60	3.68
χ^2	4.79	3.07	24.14	3.28

TABLE 4. Pre-Semester Survey item averages with 798 Traditional and 645 Flipped responses. Independence determined by $\chi^2 \geq 17.54$.

Chi-squared tests were run on each of the 33 survey items to check for any initial selection bias as any large difference between the two class averages could not be explained by the structure itself. Of the 33 items, 29 items were "Independent" of class format. The other four questions were:

(14) To understand math I discuss it with other students ($\chi^2 = 24.32$);
(15) I do not spend more than five minutes on a math problem before giving up or seeking help from someone else ($\chi^2 = 22.72$);
(22) When a math problem arises that I can't immediately solve, I stick with it until I have made progress toward a solution ($\chi^2 = 18.89$); and
(33) If I get stuck on a math problem, there is little chance I'll figure it out on my own ($\chi^2 = 24.14$).

Flipped students' average response was higher than the traditional students' average response on items 14, 15, and 33. Of these questions, only the first would suggest there may be an affective selection bias. This would suggest there is some other unexamined factor more prevalent in the flipped class (but not necessarily related to it). Overall, it does not appear there is a strong selection bias between students of each class format.

Chi-squared tests were then run on the post-semester survey data, with 30 items determined "Independent" of class format. The remaining 3 questions were:

(4) I can learn from hearing other people's mathematical thinking, even if their thinking is not correct ($\chi^2 = 22.75$);
(14) To understand math I discuss it with other students ($\chi^2 = 31.21$); and
(29) I enjoy figuring out math problems with other people ($\chi^2 = 18.77$).

Question 4 asked about students learning from others mathematical thinking. This was a common occurrence in the flipped classroom and not in the traditional

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Trad. Avg	3.10	3.02	2.75	2.91	2.40	3.89	3.91	2.60	2.40	3.23
Flip Avg	3.02	3.13	2.79	3.14	2.43	3.95	3.98	2.56	2.31	3.41
χ^2	5.87	8.82	8.25	22.75	0.95	2.87	3.56	3.14	3.01	10.05

	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
Trad. Avg	3.26	3.96	3.85	3.66	2.29	2.30	3.40	2.95	2.02	3.83
Flip Avg	3.36	3.91	3.92	3.91	2.47	2.33	3.35	2.90	2.01	3.88
χ^2	6.64	2.93	10.40	31.21	16.98	5.38	7.82	2.49	2.13	1.85

	Q21	Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30
Trad. Avg	3.53	3.67	3.97	3.22	3.80	4.00	3.86	2.92	3.49	3.32
Flip Avg	3.61	3.64	3.94	3.25	3.77	4.00	3.76	2.79	3.73	3.34
χ^2	2.67	5.46	4.82	2.36	9.30	NA	6.41	13.49	18.77	3.45

	Q31	Q32	Q33	Q34
Trad. Avg	2.55	3.79	2.40	3.61
Flip Avg	2.62	3.83	2.53	3.71
χ^2	1.27	12.53	7.07	8.11

TABLE 5. Post-Semester Survey item averages with 709 Traditional and 553 Flipped responses. Independence determined by $\chi^2 \geq 17.54$.

	Pre Flip	Pre Trad	Post Flip	Post Trad
Strongly Agree	5.1%	4.5%	6.7%	4.1%
Agree	32.9%	31.6%	41.7%	32.0%
Neutral	22.5%	22.4%	18.8%	22.8%
Disagree	31.8%	32.6%	24.6%	33.3%
Strongly Disagree	7.8%	8.9%	8.2%	7.8%

TABLE 6. Distribution of responses for question 4.

classroom. The flipped course structure appeared to change a significant number of students' perception of their ability to learn from other, potentially incorrect, thinking as flipped responses for "Agree" increased by 8.8%. Moreover, the traditional course seemed to hold their initial perceptions and remained relatively the same before and after the course ended. This statistically significant increase can be attributed to the course structure itself. Complete response rates for question 4 are provided in 6.

Question 14 asked whether students discussed math with other students to understand it. This was the most pronounced chi-squared value and was evident as a difference between flipped and traditional sections at the beginning of the course. The most striking difference in response rates was the percentage of students who strongly agreed with the statement: 25.2% in flipped and 15.5% in traditional. Again, the structure of the flipped classroom encouraged students to work in groups to discuss mathematical ideas. These response rates suggest the flipped classroom fostered a stronger collaborative environment. Complete response rates for question 14 are provided in 7.

	Pre Flip	Pre Trad	Post Flip	Post Trad
Strongly Agree	23.3%	18.5%	25.2%	15.5%
Agree	54.0%	50.8%	52.0%	51.9%
Neutral	15.8%	17.4%	13.8%	17.6%
Disagree	6.0%	10.4%	6.9%	13.0%
Strongly Disagree	0.9%	2.9%	2.4%	2.0%

TABLE 7. Distribution of responses for question 14.

	Pre Flip	Pre Trad	Post Flip	Post Trad
Strongly Agree	20.3%	17.8%	18.5%	12.7%
Agree	50.9%	47.7%	51.1%	47.0%
Neutral	18.9%	20.1%	17.9%	20.9%
Disagree	8.1%	11.0%	10.0%	15.2%
Strongly Disagree	1.9%	3.4%	2.7%	4.2%

TABLE 8. Distribution of responses for question 29.

Question 29 asked whether students enjoyed working on math with other students. The most striking difference in response rates was the change in students who strongly agreed with the statement: $20.3\% \to 18.5\%$ for flipped and $17.8\% \to 12.7\%$ for traditional. This is a surprising result when paired with question 14, as it suggests that students may discuss math with other students to understand it but not necessarily *enjoy* working on math with other students. Regardless, it appears that the flipped structure provided a softer decrease in students' enjoyment working on math with others. Complete response rates for question 29 are provided in 8.

Overall, the flipped classroom appeared to have a positive effect or reduced the negative effect Calculus I has on students when it came to the social aspects of the classroom. The social aspect of learning can affect student learning even in upper-level mathematics courses [8] and so should be considered when evaluating the success of students in a course.

5. Conclusions and Further Steps

The structure of the flipped course was built upon a foundation of guiding the students through the material. In a traditional setting, student learning is often broken into chunks by exams where the students study material and prepare for a short amount of time prior to the examination. This is then followed by a large gap waiting for the next examination during which the students tend to not study or prepare. Homework is used to help alleviate these concerns but part of the structure of this particular flipped calculus class was designed to keep students familiar with the material on a more regular basis. The progression, lecture video $\to$ lecture quiz $\to$ in-class assignment $\to$ homework assignment $\to$ exam, is meant as a way to break the study-exam cycle into smaller chunks by assessing the student's learning more regularly. The in-class assignment creates a space where the instructors can ask challenging questions that require the students to form connections with previous topics in the semester. For example, after learning how to take the derivatives of trigonometric functions in Lecture 14, students were asked to use the derivative of $f(x) = x - \cos(x)$ combined with the Intermediate Value Theorem from Lecture

7 (part of the previous unit) to show the function has only one real root. This gives the student the opportunity to take more ownership in the concepts than they might in a traditional lecture when these types of problems are completed as a class, and the lecturers felt that the students were more aware of each concept when exam time came and how individual topics interacted with each other.

The design of the flipped course allows the students to have more face time with their instructors on a daily basis. The flipped classroom instructors and their learning assistants are able to engage each student individually; this is not possible in the large lecture setting. This helps to create an environment where the students feel comfortable asking their instructors and learning assistants for help, and allows the instructors and learning assistants to address any misunderstandings that the students may have the day the material is covered. The time that the instructors and their learning assistants have to work with each group individually allows them to tailor their guidance to what the students in that particular group are struggling with without hampering other students' learning. The increased face time with the students benefits the instructors as well as the students, as it provides feedback to the instructor on what specific concepts the students have not grasped. The instructors can then discuss the concept in more depth, and tailor the future in-class assignments to help the students work through the concepts they are struggling with.

Ultimately, the design of the course was meant to move students away from the usual pitfalls of a traditional lecture environment and give them a reason to keep up to date with the material. The lecturers felt that the pace of the course forced students to interact more, both with each other and the material, and kept them more aware of each topic and engaged in the course. The ability to enlarge the scope of the course by including more information and challenging applications benefited the students who require such insights to be engaged and allowed students to see real-world uses of the topics they explore. Also, by interacting more one-on-one with the students, those who may have fallen behind or need more explanation can receive help during class time without disrupting the learning of their peers. This connection that students develop with their instructor may also push more students to seek help or guidance about the course from their instructor outside of class time more than those who would feel less comfortable seeking help in a traditional lecture environment. However, there are definite cons to such a structure. By having students work in groups in class, some students can rely too heavily on their partners or the instructor and fall behind in utilization of the material and not realize their mistake until examinations are upon them. Also, unless the instructor makes it a priority to include step-by-step written mathematical notation, which is prevalent throughout a traditional lecture where a student may take notes, the students' mathematical writing ability can suffer in the flipped environment. All in all, the plan was to design a course to keep engagement high and to maximize learning; the lecturers feel that this did happen with the students.

We do have some plans for improving the course moving forward, along with some additional questions to investigate.

(1) We would like to expand the in-class assignment banks. This is not limited to quantity; rather, we plan to develop more interactive and engaging assignments. This could also include introducing games and other activities

(piloted a bit in spring 2019 by some of the instructors), as well as using more props for demonstrations.

(2) The lecture videos are not interactive. Rather, students watch the video and then take a separate online lecture quiz to ensure they have watched it. We plan to examine ways to make the quiz embedded into the lecture, thereby giving students an incentive to watch the video and actively listen to it.

(3) The learning assistants could use more training and guidance. While most of them were capable and ably assisted the instructors, there were some gaps in their preparation and pedagogical skills that we need to address. One simple thing we will do in the future: have the learning assistants work out the solutions to the daily activities in advance. This is one of those obvious things that we missed the first time around.

(4) We plan to expand the library of videos and other supplemental materials for students. The department recently purchased a lightboard and the team will begin work on creating more example videos for inclusion into the course site.

(5) The varied grading schemes made it impossible to compare final course grades. We plan to develop ways around this in the future.

(6) There are a number of additional questions we might ask about the data. One instructor theorizes that the students who most benefit from the flipped class are those who are not as well-prepared or who struggle with the material. We plan to investigate this idea. In [3] the authors noted a statistical difference in the bottom third of students' grades, so it could be interesting to check for this as well.

(7) In the pre-semester chi-squared tests, we noted there seemed to be an unexamined factor that caused four questions to show a dependence on the class structure. While we treated the course structure as a homogeneous collection of students, there are various subcollections of students that may have been more strongly affected than others (e.g., low-income, minority, and particular majors). We would need to examine each subcategory individually to determine whether their trend is the same as the overall trend.

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Appendix A. List of Survey Items

(1) A significant problem in learning math is being able to memorize all the information I need to know.
(2) I think about the math in my everyday life.
(3) After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic.
(4) I can learn from hearing other people's mathematical thinking, even if their thinking is not correct.
(5) Knowledge in math consists of many disconnected topics.
(6) I am not satisfied until I understand why something works the way it does.
(7) I cannot learn mathematics if the teacher does not explain things well in class.
(8) I don't understand how some people can spend so much time on math and seem to enjoy it.
(9) I do not expect mathematical approaches to help my understanding of ideas; they are just for doing calculations.
(10) When a question is left unanswered in math class, I continue to think about it afterward.
(11) I study math to learn knowledge that will be useful in my life outside of school.
(12) If I get stuck on a math problem on my first try, I usually try to figure out a different way that works.
(13) Nearly everyone is capable of understanding mathematics if they work at it.
(14) To understand math I discuss it with other students.
(15) I do not spend more than five minutes on a math problem before giving up or seeking help from someone else.
(16) If I don't remember a particular mathematical approach needed to solve a problem on an exam, there's nothing I can do to come up with it on my own.
(17) Understanding math basically means being able to communicate your reasoning with others.
(18) If I want to apply a mathematical approach used for solving one math problem to another problem, the problems must look very similar.
(19) In doing a math problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem.
(20) In math, it is important for me to make sense out of mathematical approaches before I can use them correctly.
(21) I enjoy solving math problems.
(22) When a math problem arises that I can't immediately solve, I stick with it until I have made progress toward a solution.

(23) Mathematical formulas express meaningful relationships among variables.
(24) Learning math changes my ideas about how the world works.
(25) Reasoning skills used to understand math can be helpful to me in my everyday life.
(26) We use this statement to discard the survey of people who are not reading the questions. Please select agree (not strongly agree) for this question to preserve your answers.
(27) I can usually figure out a way to solve math problems.
(28) College mathematics has little relation to what I experience in the real world.
(29) I enjoy figuring out math problems with other people.
(30) There are times I solve a math problem more than one way to help my understanding.
(31) To understand math, I sometimes think about my personal experiences and relate them to the topic being analyzed.
(32) When I solve a math problem, I think about which mathematical ideas apply to the problem.
(33) If I get stuck on a math problem, there is little chance I'll figure it out on my own.
(34) When studying math, I relate the important information to what I already know rather than just memorizing it the way it is presented.

Appendix B. Lecture Activities

Sample Lecture Activity 1

MAC 2311   Lecture 21 Problems Part 1   October 26 2018

Name: _____   **Section:** _____

1. Write the statement of Rolle's Theorem.

List the three conditions a function must satisfy in order to use Rolle's Theorem on an interval $[a, b]$

(a)

(b)

(c)

List the conclusion of Rolle's Theorem.

2. Verify that $f(x) = 5 - 12x + 3x^2$ satisfies the three conditions (hypotheses) of Rolle's Theorem on the interval $[1, 3]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

MAC 2311   Lecture 21 Problems Part 1   October 26 2018

3. Write the statement of the Mean Value Theorem.

List the two conditions a function must satisfy in order to use the Mean Value Theorem on an interval $[a, b]$

(a)

(b)

List the conclusion of the Mean Value Theorem

4. Verify that $f(x) = \frac{1}{x}$ satisfies the conditions (hypotheses) of the Mean Value Theorem on the interval $[1, 3]$, and find the numbers c that satisfy the conclusion of the Mean Value Theorem.

MAC 2311   Lecture 21 Problems Part 1   October 26 2018

5. Show that $x^3 + e^x$ has exactly one real root.

(a) First show that there exists at least one root. (Hint: Use the Intermediate Value Theorem)

(b) Now use Rolle's Theorem to prove that there cannot be more than one real root. (Hint: First assume there is more than one)

6. Let $f(x)$ be a continuous and differentiable function on the interval $[-7, 0]$ such that $f(-7) = -3$ and $f'(x) \leq 2$. What is the largest possible value for $f(0)$?

MAC 2311   Lecture 21 Problems Part 1   October 26 2018

7. Suppose you are headed to Disney World for the weekend and you are driving the last stretch of the trip on the Florida Turnpike where the speed limit is 70mph. You stop to get gas at Okahumpka Service Plaza and as you reenter the highway at 8:30am your speedometer reads 65mph. Then 27 miles later when you take the exit towards Disney at 8:50am your speedometer reads 68mph. Are you guilty of speeding? Justify your answer.

8. **Theory**: A number a is called a **fixed point** of a function $f(x)$ if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x, then $f(x)$ has at most one fixed point.

Sample Lecture Activity 2

Name: _____ Section:_____

1. What is the definition of an **antiderivative** for a function $f(x)$?

2. **Warmup**: By reversing our basic derivative rules for common functions, we get antiderivative rules for most of the common functions. State the (most general) antiderivative $F(x)$ of each for the following functions. (Here, $a \neq 0$ is a constant and n is also a constant).

(a) $f(x) = x^n$, $n \neq -1$ (b) $f(x) = \dfrac{1}{x}$

(c) $f(x) = e^{ax}$ (d) $f(x) = \cos(ax)$

(e) $f(x) = \sin(ax)$ (f) $f(x) = \sec^2(x)$

(g) $f(x) = \sec(x)\tan(x)$ (h) $f(x) = \dfrac{1}{1+x^2}$

(i) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ (j) $f(x) = \dfrac{1}{x\sqrt{x^2-1}}$

3. For each of the functions below, find the *unique* antiderivative $F(x)$ that satisfies the given initial condition.

(a) $f(x) = \cos(x) + x^5$, $F(0) = 1$

(b) $f(x) = 3x^2 - 4x + 2$, $F(0) = 2,235,141$

(c) $f(x) = \dfrac{2}{x} + \dfrac{1}{1+x^2} - e^{6x}$, $F(1) = 0$

(d) $f(x) = \dfrac{2x}{x^2+1}$, $F(0) = 1$. (Hint: Notice that the numerator of $f(x)$ is the derivative of the denominator, i.e., $f(x) = \dfrac{g'(x)}{g(x)}$ for $g(x) = x^2 + 1$. What kind of function $F(x)$ would give a derivative of the form $\dfrac{g'(x)}{g(x)}$?)

4. Sheeraz is walking on the moon. He jumps off of a cliff 20 meters high at an initial velocity of 2 meters per second. Fortunately, the acceleration due to gravity on the moon is -1.6 meters per second (only 1/6 that of Earth's!).

(a) Find a formula for $v(t)$, Sheeraz's velocity at a given time t. (Velocity is the antiderivative of acceleration).

(b) After 3 seconds what is Sheeraz's velocity?

(c) Find a formula for $r(t)$, Sheeraz's position at a given time t.

(d) At what time does Sheeraz land on the ground? (You may use a calculator to find this value)

5. A company estimates that the marginal cost (in dollars per item) of producing x items is $1.92 - 0.002x$. If the cost of producting one item is \$562, find the cost of producing 100 items.

6. **Challenge**: Consider the situation of someone throwing a ball through the air. If the only force acting on the ball is the force of gravity, then this type of motion is called **projectile motion**. The ball moves in a parabolic arc and so has two different positions, its horizontal position and its vertical position (relative to the starting position).

Suppose that the initial velocity of the ball in the horizontal direction is $v_x(0) = 20$ feet per second and in the vertical direction is $v_y(0) = 16$ feet per second. The acceleration due to gravity only affects the ball vertically and is $a_y(t) = -32$ feet per second (that is $a_x(t) = 0$).

(a) Find a formula for the position of the ball in both the horizontal direction $s_x(t)$ and in the vertical direction $s_y(t)$.

(b) At what time does the ball hit the ground after being thrown?

(c) How far does the ball travel horizontally before it hits the ground?

(d) How high does the ball travel vertically? (Hint: $s_y(t)$ is a parabola, use the t value corresponding to its vertex)

(e) Note that if we graph the position of the ball the graph would consist of the points $(s_x(t), s_y(t))$. Graph the position of the ball (Note that time starts at $t = 0$ and ends when the ball hits the ground)

7. **Challenge**: Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of 10 meters per second and its downward acceleration is

$$a(t) = \begin{cases} 9 - 0.9t & \text{if } 0 \leq t \leq 10 \\ 0 & \text{if } t > 10 \end{cases}$$

If the raindrop is initially 500 meters above the ground, how long does it take to fall?

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