# Flutter stability analysis of an aircraft wing as a function of damping ratio

*Undergraduate Research Project*
*Independent Study*

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# ABSTRACT

This report provides insight into flutter stability analysis as a function of damping ratio. Flutter, an unstable self-excited vibration in which the structure extracts energy from the air stream and often results in catastrophic structural failure, is analyzed as dynamic instability, which may eventually result in stall or buffeting conditions or classical bending and torsion coupling actions. These couplings occur when the aerodynamic forces associated with motion in two modes of vibration cause the modes to couple in an unfavorable manner. The characteristic equation will be analyzed and derived in order to study stability and its relationship with the damping ratios of the aircraft wing. Analytical analysis of different mode shapes will be carried out by means of applying Newton's equation of motion and time-dependent boundary conditions. Underdamped, critically damped, over-damped and flutter diagrams will be obtained for different damping ratios. Furthermore, these diagrams will be compared with analytical derivations. Those results indicate stability dependence on damping ratio as it is basically a free vibration problem. Comparison between different models shows accurate results. Further research is focused on considering and proving detailed results on experimental data on lab.

**Keywords:** Flutter, damping ratio, MATLAB.

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# CHAPTER 1

# 1. INTRODUCTION

Contents



## 1.1. MOTIVATION

Aeroelasticity phenomenon is a combination of physical phenomena which include iteration between inertia, elastic and aerodynamic forces. There are grounds for believing that aeroelasticity has an impact on stability and control, -and thus, on flight mechanics-, structural vibrations and static aeroelasticity. In that manner, **Collar diagram** provides an useful tool which allows to relate inertial, elastic and aerodynamic forces forming a triangle. Therefore, we can conclude that stability and control is the addition of dynamics and aerodynamic phenomena; structural vibrations is the result of the coupling of dynamics and solid mechanics; and finally, static aeroelasticity, is composed by steady flow aerodynamics and solid mechanics.

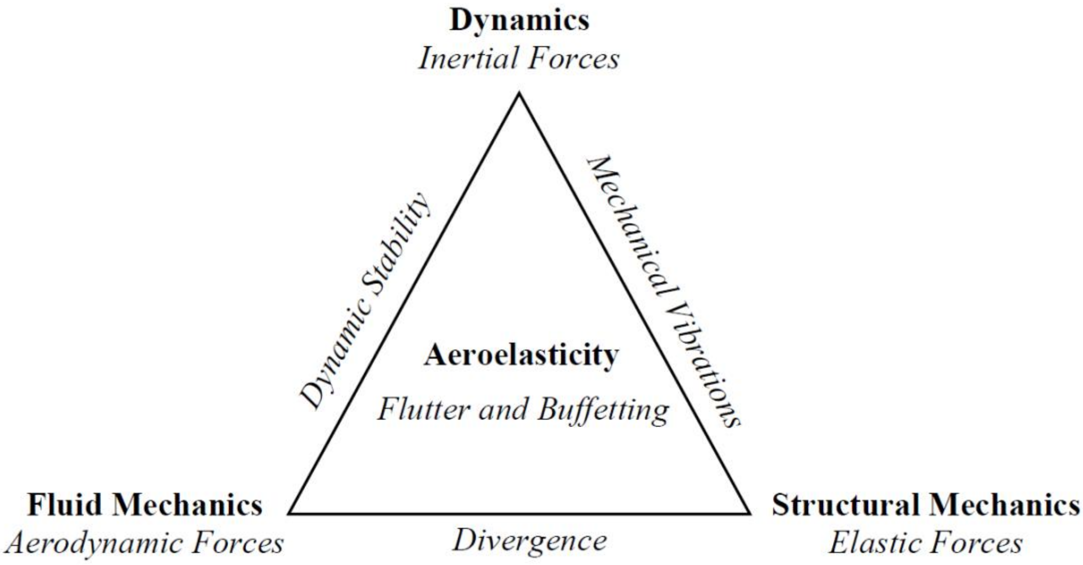



FIGURE 1. COLLAR AEROELASTICITY DIAGRAM

Firstly, in order to provide an accurate approach to flutter stability analysis, we consider **STATIC AEROELASTICITY** as essential for design criteria as it may result on structural failure, e.g., aircraft wing flutter, where inertia forces are neglected. It **constitutes the first engineering design criteria** that every design must fulfill as we want to prevent our design from divergence, which may result in <u>torsional divergence of the aircraft wing.</u>

Secondly, **DYNAMIC AEROELASTICITY** must also be taken into study, as it considered as the response to external disturbances. The first phenomenon regarding dynamic response is **flutter or dynamic instability**, which may be produced by <u>stall of buffeting conditions</u>, or by <u>classical bending and torsion coupling actions</u>. Further detail will be provided in the following chapter regarding this phenomenon as it is the focus of this report. Other condition included in dynamic aeroelasticity is **force response**, defined as a response to external aerodynamic excitation loading that may be coupled with the natural frequencies leading to resonance phenomenon on: <u>HPT/LPT gas turbines</u>, due to the perturbation of the potential field, wakes, vortex eddies or shockwaves; and aircraft wing, HTP and VTP, due to atmospheric turbulence or gust.

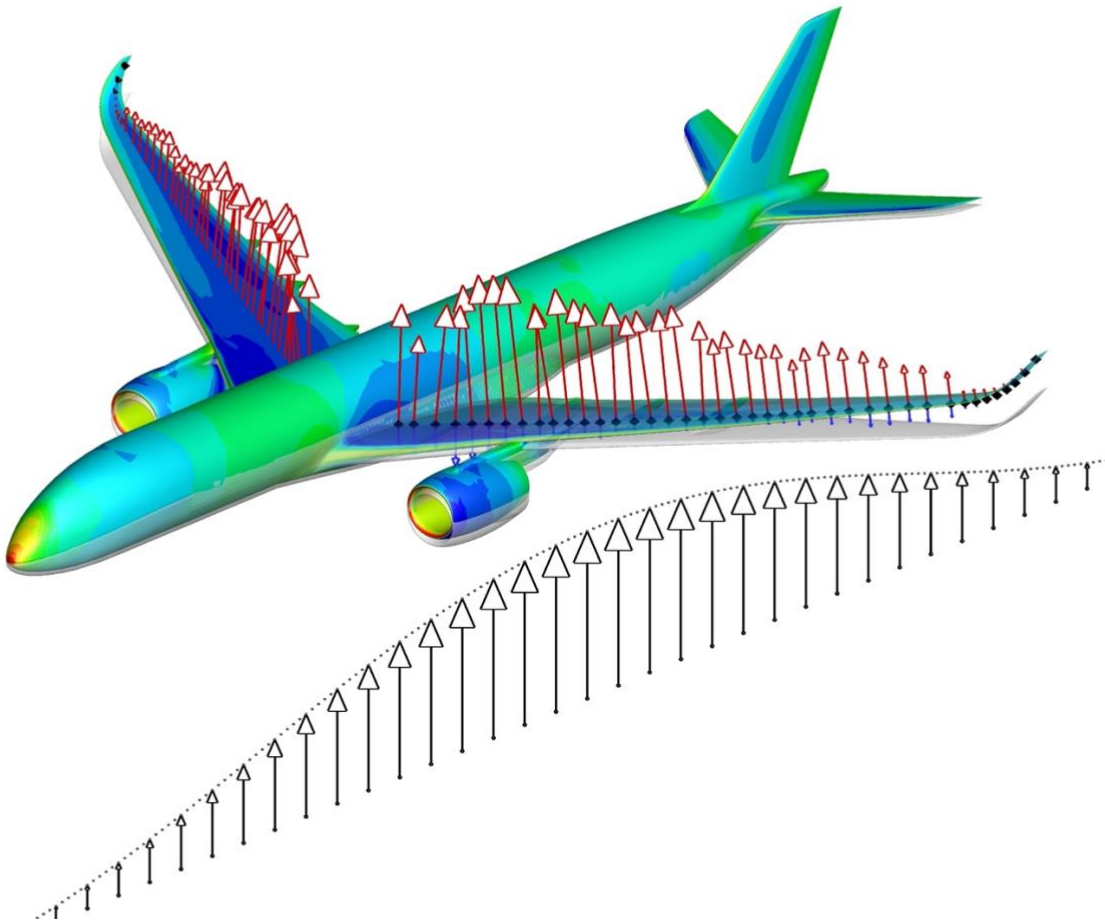


FIGURE 2. GUST SIMULATION ON AN AIRCRAFT WING

Moreover, **non-synchronous vibration**, which are linked to aerodynamic instabilities produced by vortex shedding, are part of dynamic aeroelasticity, resulting on Vortex Induced Vibrations as aerodynamic disturbances occurring near the stability limit; and boundary layer detachment. NSV can occur in the fan, compressor, and turbine stages of the turbomachine. In the case of a gas turbine blades, these disturbances propagate around the circumference and excite blade vibration modes. Moreover, we need to hardly consider this phenomenon when the distribution of the disturbances is altered by the blade oscillation, which leads to the development of notable fluid-structure interaction with safety-critical amplitudes. In this case, the importance of unsteady effects produced by pulsating flow might be estimated by a Strouhal number, which is the ratio of the characteristic time, -time for fluid particles to be transported through the turbine components-; and residence time, -time scale of the unsteadiness of pulsating flow-.

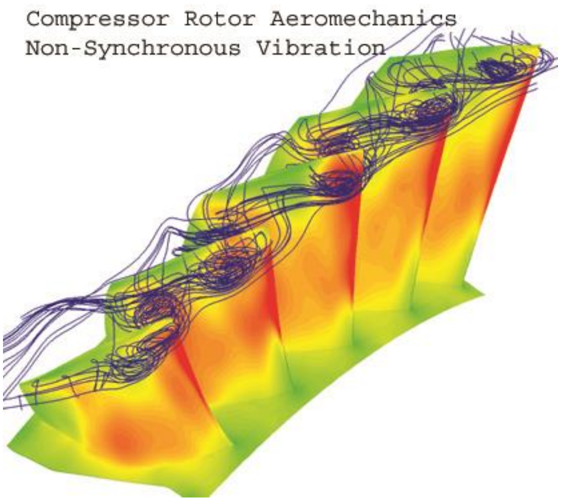



**FIGURE 3. NON-SINCHRONOUS VIBRATION ON COMPRESSOR ROTOR**

This figure represents the pressure contours with the streamlines near tip region showing the "tornado" vortices with the vortex axes normal to the blade suction surfaces. It can be observed that the forcing oscillation induced by the traveling tornado vortices excites the non-synchronous vibration.

## 1.2. OBJECTIVES

The objectives of this mechanical design report are:

- To evaluate damping ratio as key element for flutter stability analysis on an aircraft wing.
- To analytically obtain the derivation of natural frequencies and mode shapes of the one degree of freedom system.
- To obtain different scenarios diagrams as a function of damping ratio and define representative points of the solution.
- To obtain the mode shapes plots of the system and its temporal evolution for different critical damping ratio values.
- To compare both solutions and provide accurate and solid conclusions on flutter stability analysis.

# 1.3. AEROELASTICITY

## 1.3.1. DYNAMIC AEROELASTICITY

Firstly, **dynamic aeroelasticity** must be taken into study, as it considered as the interaction of inertial and aerodynamic forces:

### 1.3.1.1. FLUTTER

According to **Collar, 1978; Garrick and Reid, 1981**, flutter is considered one of the most important of all the aeroelastic phenomena and is the most difficult to predict. It is an **unstable self-excited vibration** in which the structure extracts energy from the air stream and often results in catastrophic structural failure (Wright & Cooper, 2008). This phenomenon, understood as a dynamic instability, may be produced by **stall or buffeting** conditions.

Furthermore, **classical** bending and torsion **coupling actions** occurs when the aerodynamic forces associated with motion in two modes of vibration cause the modes to couple in an unfavorable manner. Flutter is categorized into at least five different areas, each with its own characteristic modes of motion: classical flutter – wing bending & torsion; control surface flutter – surface rotation and wing bending; empennage flutter – fuselage torsion and tail torsion; stall flutter – wing torsion; and, finally, body freedom flutter – wing bending and fuselage pitch. It may occur at:

A. WING. HTP. VTP.

Flutter is a dynamic instability occurring in flight, at a speed called the **flutter speed**, where the elasticity of the structure plays an essential part in the instability of fixed surfaces, such as the wing or the stabilizer, as well as on control surfaces such as the aileron or the elevator.

**FIGURE 4. WING FLUTTER**

B. TURBOMACHINERY. HTP & LPT.

Blade flutter is the self-excited vibration of blades due to the interaction of structural-dynamic and aerodynamic forces. This constitutes a problem for turbomachinery due to phase differences between the blades when they are vibrating. **Campbell diagram** demonstrates that it occurs at frequencies very close to a blade mode frequency because the structural forces dominate and the aerodynamic forces do not significantly affect the vibration frequency.

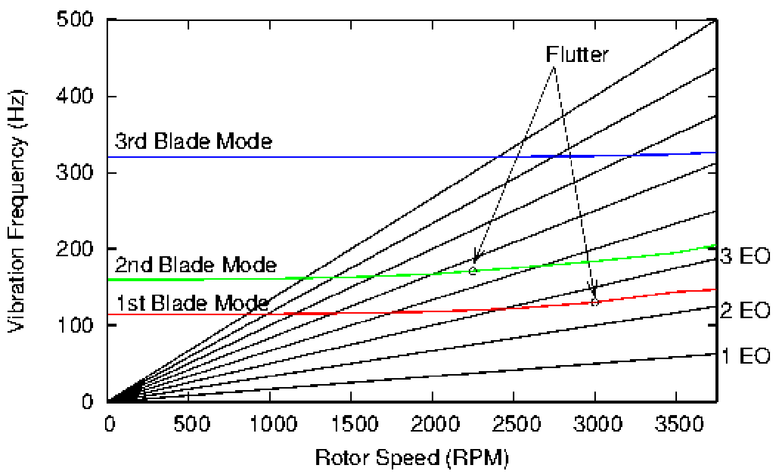



**FIGURE 5. CAMPBELL DIAGRAM**

C. ACOUSTIC FLUTTER

Acoustic flutter may occur in compressors and fans as a result in **acoustic propagation and reflection** with the effect of the neighboring blade row close to resonance of acoustic modes.

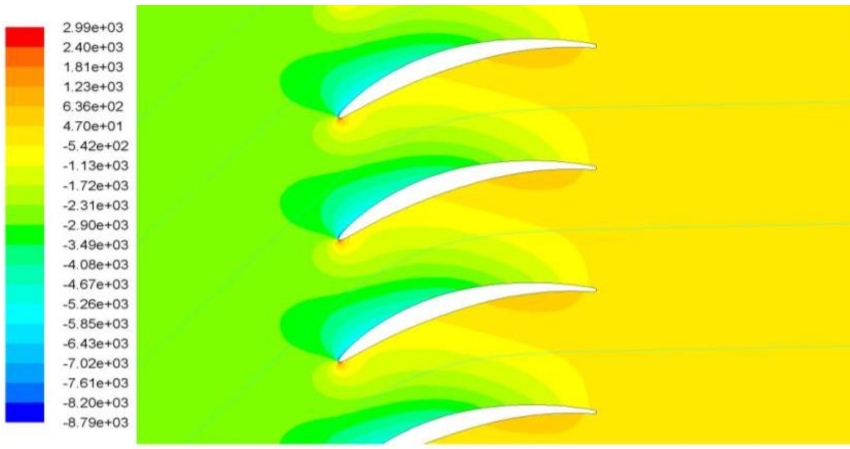



**FIGURE 6. ACOUSTIC FLUTTER**

D. CHOKE FLUTTER

Choke flutter appears when a strong shockwave chokes the blade to blade channel. The steady flow is subsonic upstream and downstream of the blade row and supersonic in the blade-to-blade channel. A strong shockwave chokes the channel from the suction side to the pressure side.

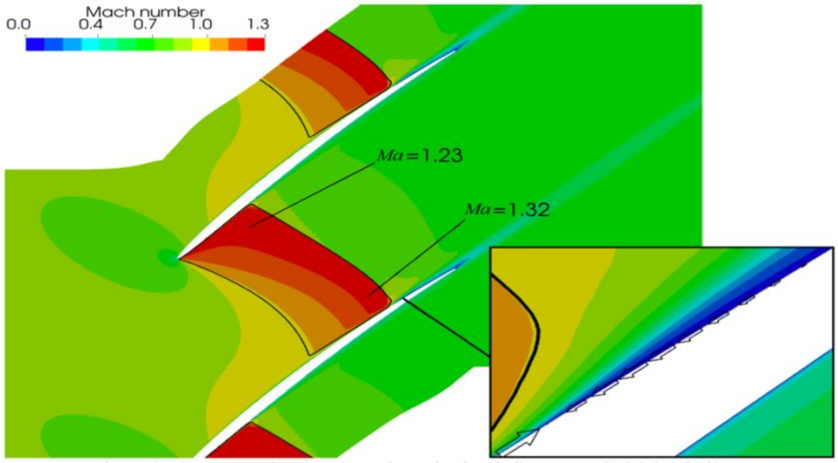



**FIGURE 7. CHOKE FLUTTER.**

Also, other phenomena, -as Force Response & Non-Synchronous Vibrations-, belong to Dynamic Aeroelasticity:

1.3.1.2. NON-SYNCHRONOUS VIBRATIONS

Non-Synchronous Vibrations can occur in the fan, compressor, and turbine stages of the turbomachine. In the case of a gas turbine blades, these disturbances propagate around the circumference and excite blade vibration modes. Moreover, we need to hardly consider this phenomenon when the distribution of the disturbances is altered by the blade oscillation, which leads to the development of notable fluid-structure interaction with safety-critical amplitudes. Vortex formation and vortex shedding occurs behind airfoils at certain angles of attack. These vortices create an unsteady periodic force on the structure, leading to structural vibrations. These Vortex Induced Vibrations are a real problem when the vortex shedding frequency equals the natural frequency of the structure, leading to resonance and possibly failure of the structure.

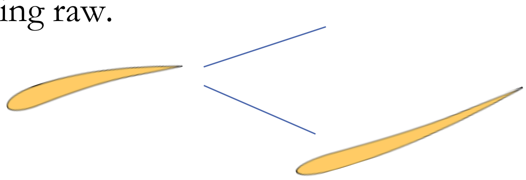
#### 1.3.1.3. FORCE RESPONSE

**Finally,** this report aims to provide insight into **Force response** aeroelastic phenomenon that occurs as a response to external aerodynamic nature excitation loading on HPT/LPT gas turbines and also due to atmospheric turbulence or gust on aircraft wing, HTP and VTP. This last topic will be studied throughout the report. The excitation load frequency may be coupled with the natural vibration frequency. For this given situation in which the external excitation frequency coincides with the natural vibration frequency, **resonance** will appear.

A. TURBOMACHINERY.

Furthermore, external forcing forces that may lead into dynamic stability force response can be analyzed due to:

- **Wakes**, from viscous nature that are initialized at the trailing impact on the leading edge of the following raw.



- **Perturbation of potential field,** as the components are moving, those wakes perturbate the potential field.
- **Vortex eddies,** at the trailing edge whack the leading edge of the following rotor-stator raw.

- **Shockwaves,** may appear under certain conditions impacting on different components.

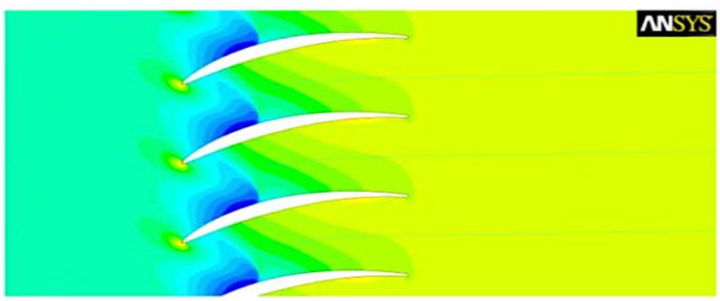



FIGURE 8. FORCE RESPONSE DUE TO SHOCKWAVES.

B. WING. HTP. VTP.

Main potential source for external response on aircraft wing and control surfaces is the **gust**. Gust, in meteorology, a sudden increase in wing speed above the average wind speed. More specifically, wind speed must temporarily peak above 16 knots (about 30 km per hour) after accelerating by at least 9–10 knots (about 17–19 km per hour) to qualify as a gust.

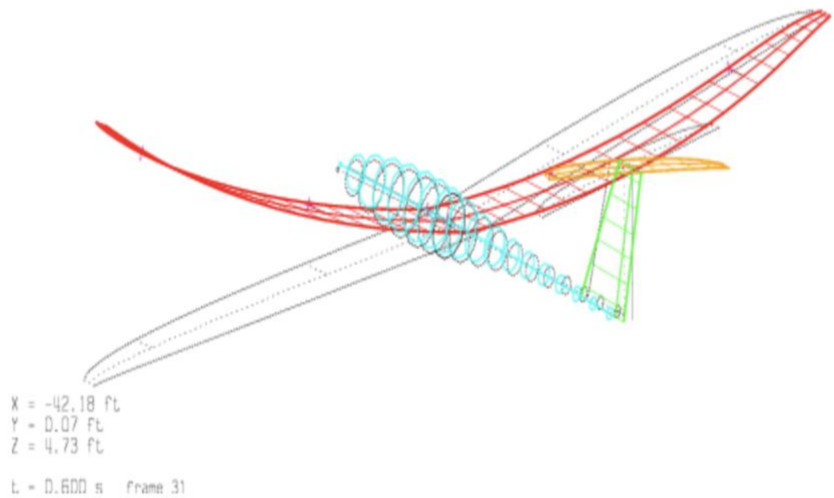



**FIGURE 9. COMPUTATIONAL GUST SIMULATION ON AIRCRAFT WING**

## 1.4. INSTABILITY

According to **Sisto, F., 1978. "A modern course in Aeroelasticity" E.H. Dowel,** *"the most dramatic physical phenomenon in the field of aeroelasticity is flutter, a dynamic instability which often leads to catastrophic structural failure"*. In this case, we can relate it with stability. Firstly, instability definition will be defined to avoid confusion on differentiating resonance & instability.

Instability is a growing motion with the distinguish features: on the one hand, it is self-excited, -caused by the motion itself, as no forced is applied onto the system-. On the other hand, it grows exponentially, leading to a linear failure due to an increase on amplitude.

Therefore, we can differentiate between **linear & nonlinear failure**:

- Linear System: Destructive failure as instability grows exponentially due to the fact that stresses are higher than those of the design envelope as amplitude is high. Small oscillations appear compared with the characteristic length.

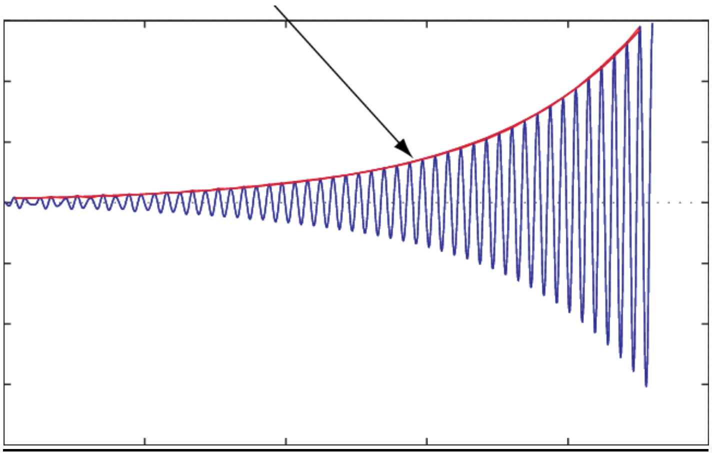


FIGURE 10. INSTABILITY LINEAR SYSTEM

- Non-Linear System: Motion grows exponentially at the beginning. As nonlinearities become significant, Limit-Cycle Oscillations of large amplitude that promotes excessive vibrations & fatigue appear.

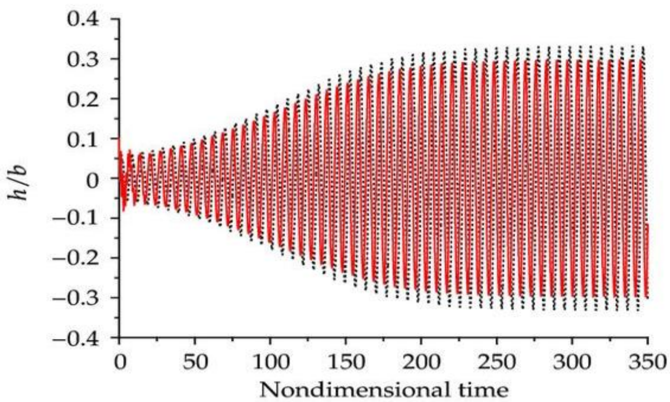



FIGURE 11. INSTABILITY NON-LINEAR SYSTEM

Thus, after having defined instability it is quite convenient to remark that resonance requires external excitation and can only grow linearly. In that case, natural frequencies cannot coincide with the external excitation frequency as it is a self-induced vibration.

### 1.4.1. FLUTTER CLASSIFICATION

Instabilities can be divided by means of non-linear stabilities in which an **unstable self-excited vibration** allows the structure extracts energy from the Von Karman vortices and often results in catastrophic structural failure (Wright & Cooper, 2008). This phenomenon, understood as a dynamic instability, may be produced by fluid sloshing, mechanical hysteresis, etc., resulting in **stall flutter, Vortex Induced Vibrations or buffeting** conditions

Furthermore, mode- couple instabilities may appear as **classical** bending and torsion **coupling actions**, which occurs when the aerodynamic forces associated with motion in two modes of vibration cause the modes to couple in an unfavorable manner. Flutter is categorized into at least five different areas, each with its own characteristic modes of motion: classical flutter – wing bending & torsion; control surface flutter – surface rotation and wing bending; empennage flutter – fuselage torsion and tail torsion; stall flutter – wing torsion; and, finally, body freedom flutter – wing bending and fuselage pitch.

## 1.5. VIBRATIONS

According to **S. Graham Kelly on Mechanical Vibrations: Theory & Applications,** vibrations are oscillations of a mechanical or structural system about an equilibrium position. (Kelly, 2011) Vibrations are classified by its nature as free vibrations and forced vibrations. If the vibrations are initiated by an initial energy present in the system and no other source is present, the resulting vibrations are called free vibrations. If the vibrations are caused by an external force or motion, the vibrations are called forced vibrations. If the external input is periodic, the vibrations are harmonic. Furthermore, essential data to describe vibration motion is needed, regarding the number of degrees of freedom necessary for their modeling and the boundary conditions used in the modeling. Vibrations of systems that have a finite number of degrees of freedom are called discrete systems. In this report, we will focus our research on one degree of freedom self-induced vibration as a simplification of an aircraft wing.

### 1.5.1. AEROELASTIC MODEL: FREE VIBRATION DAMPED SYSTEM

**Free vibrations** can be defined as a system which no external force is causing the motion, and that the motion is primarily the result of initial conditions, such as an initial displacement of the mass element of the system from an equilibrium position and/or an initial velocity.

In this one degree of freedom case, the **aircraft wing** will be represented as a mass attached to a spring, being the last one a **physical model** simplification for storing kinetic energy that the wing may absorbed. In order to provide a real approximation to the physical model, it is assumed that there is a force acting on the mass in a direction opposite to the motion. This force is also proportional to the motion, -the faster an object moves, the higher the friction-. So, **damping force** of viscous nature is introduced.

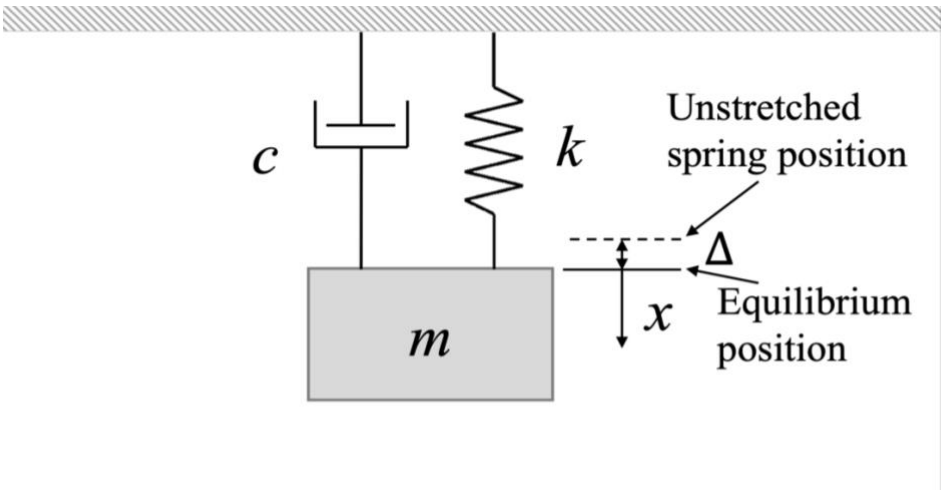



FIGURE 12. FREE VIBRATION DAMPED SPRING–MASS SYSTEM

Thus, applying Newton's second law Newton's second law, representing the free vibration 1DOF damped spring-mass system, we can obtain the differential equation for damped motion of an aircraft wing simplification expressed as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

where **m** and **k** are the mass and stiffness matrices, respectively; **c**, is the damping force, and **x** is the *n*-dimensional column vector of generalized coordinates. To solve the differential equation, we first need to solve the characteristic equation:

$$m\lambda^2 + c\lambda + k = 0 \rightarrow Assuming\ exponential\ form\colon x(t) = Ce^{\lambda t}$$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

Therefore, the behavior of the system exclusively depends on the factor inside the square root. The critical damping coefficient $c_{cr}$ is defined as:

$$c_{cr}{}^2 - 4mk = 0\,; \quad c_{cr} = 2\sqrt{mk} = 2m\omega$$

For calculus convenience, we may express this equation in terms of:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

Thus, damping ratio and the undamped natural frequency can be defined:

$$\zeta = \frac{c}{2\sqrt{mk}} = \frac{c}{c_{cr}}\,; \qquad \omega_n = \sqrt{\frac{k}{m}}$$

We can rearrange the terms of the equation in ordee to obtain:

$$\lambda = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Therefore, different scenarios will eventually take place as a function of the damping ratio.

i) $\zeta > 1$ (Overdamped motion, the roots of the characteristic equation are two real values).
ii) $\zeta = 1$ (Critically damped, the roots of the characteristic equation are two real & similar values).
iii) $0 < \zeta < 1$ (Underdamped motion, the roots of the characteristic equation are complex conjugates).
iv) $\zeta < 0$ (Flutter response, self-excitation instability).

A. OVERDAMPED

Overdamped motion occurs when the damping ratio ζ is bigger than one. In this case, the roots to the characteristic equation are two real values. Therefore, solutions to the equation can be stated as:

$$\lambda_1 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1}$$

$$\lambda_2 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1}$$

Applying the general solution to the characteristic equation, we eventually obtain the following expression.

$$X(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$X(t) = C_1 e^{\left(-\zeta w_n + w_n\sqrt{\zeta^2-1}\right)t} + C_2 e^{\left(-\zeta w_n - w_n\sqrt{\zeta^2-1}\right)t}$$

Boundary conditions can be defined in terms of initial displacement and initial velocity at t=0:

$$\begin{cases} X(0) = 0 \\ \dot{X}(0) = 0 \end{cases}$$

Diferenciating the displacement equation with respect time, we obtain:

$$\dot{X}(t) = C_1\left(-\zeta w_n + w_n\sqrt{\zeta^2 - 1}\right)e^{\left(-\zeta w_n + w_n\sqrt{\zeta^2-1}\right)t} + C_2\left(-\zeta w_n - w_n\sqrt{\zeta^2 - 1}\right)e^{\left(-\zeta w_n - w_n\sqrt{\zeta^2-1}\right)t}$$

In order to solve the constants, we may substitute the boundary conditions into displacement and velocity equations. It can be easily seen that:

$$X_0 = C_1 + C_2$$

$$0 = C_1\left(-\zeta \mathrm{w}_n + \mathrm{w}_n\sqrt{\zeta^2 - 1}\right) + C_2\left(-\zeta \mathrm{w}_n - \mathrm{w}_n\sqrt{\zeta^2 - 1}\right)$$

Therefore, replacing one equation into the other, we can find out that:

$$0 = C_1\left(-\zeta \mathrm{w}_n + \mathrm{w}_n\sqrt{\zeta^2 - 1}\right) + (X_0 - C_1)\left(-\zeta \mathrm{w}_n - \mathrm{w}_n\sqrt{\zeta^2 - 1}\right)$$

$$0 = 2\, C_1\sqrt{\zeta^2 - 1} + X_0\left(-\zeta - \sqrt{\zeta^2 - 1}\right)$$

$$C_1 = \frac{X_0\left(-\zeta - \sqrt{\zeta^2 - 1}\right)}{2\sqrt{\zeta^2 - 1}}$$

$$C_2 = \frac{X_0\left(-\zeta + \sqrt{\zeta^2 - 1}\right)}{2\sqrt{\zeta^2 - 1}}$$

The overdamped equation for motion will be analyzed. It can be observed that both real roots for the characteristic equation are real and negative. This has a physical impact on the system. In this case, as the damping higher than one, the restoring force is high enough to avoid the system to oscillate. Therefore, a free force overdamped harmonic oscillator does not oscillate. Since both exponents are negative, every solution in this case tends to the equilibrium.

B. CRITICALLY DAMPED

Critically damped motion occurs when the damping ratio ζ is equal to one. It means that the damping coefficient is equal to the critical damping coefficient. In this case, the roots to the characteristic equation are two similar and real values. Therefore, solutions to the equation can be stated as:

$$\lambda_1 = -\zeta \mathrm{w}_n$$

$$\lambda_2 = -\zeta \mathrm{w}_n$$

Applying the general solution to the characteristic equation, we eventually obtain the following expression.

$$X(t) = C_1 e^{\lambda_1 t} + C_2 t\, e^{\lambda_2 t}$$

$$X(t) = C_1 e^{(-\zeta \mathrm{w}_n)t} + C_2 t e^{(-\zeta \mathrm{w}_n)t}$$

Boundary conditions can be defined in terms of initial displacement and initial velocity at t=0:

$$\begin{cases} X(0) = 0 \\ \dot{X}(0) = 0 \end{cases}$$

Differentiating the displacement equation with respect time, we obtain:

$$\dot{X}(t) = C_2 e^{(-\zeta \mathrm{w}_n)t} - \zeta \mathrm{w}_n (C_1 + C_2 t) e^{(-\zeta \mathrm{w}_n)t}$$

In order to solve the constants, we may substitute the boundary conditions into displacement and velocity equations. It can be easily seen that:

$$X_0 = C_1$$

$$0 = C_2 - \mathrm{w}_n C_1$$

Therefore, replacing one equation into the other, we can find out that:

$$C_1 = X_0$$

$$C_2 = \mathrm{w}_n X_0$$

As in the overdamped case, the critically damped equation for motion will be analyzed. There are grounds for believing that critically damped systems do not oscillate. As in overdamped analysis, the solutions to the characteristic equations are real and negative. In this case are equal, what allows the system to behave in a curious manner. For a given mass and spring constant, we can observe how the decay of a critically damped system compared with these of the overdamped system is faster. Some engineering applications involve this state in order to approximate a stable result.

C. UNDERDAMPED

Underdamped motion occurs when the damping ratio ζ is less than one and greater than zero. In this case, it means that the critical damping coefficient plays an important role. The roots to the characteristic equation are two complex conjugates that can be easily defined by:

$$\lambda_1 = -\zeta \mathrm{w}_n + \mathrm{w}_n\sqrt{\zeta^2 - 1}\,\mathrm{i}$$

$$\lambda_2 = -\zeta \mathrm{w}_n - \mathrm{w}_n\sqrt{\zeta^2 - 1}\,i$$

$$\mathrm{w}_d = \mathrm{w}_n\sqrt{\zeta^2 - 1}$$

Where $W_d$ can be defined as the damped angular frequency of the system which is not periodic. Applying the basic general solutions to the characteristic equation, we eventually obtain the following expression.

$$X(t) = C_1 e^{\lambda_1 t} + C_2\, e^{\lambda_2 t}$$

By linear combination of the basic general solutions, we can easily obtain:

$$X(t) = C_1 \cos(\mathrm{w}_d t)\, e^{(-\zeta \mathrm{w}_n)t} + C_2 \sin(\mathrm{w}_d t)\, e^{(-\zeta \mathrm{w}_n)t}$$

$$X(t) = [C_1 \cos(\mathrm{w}_d t) + C_2 \sin(\mathrm{w}_d t)]\, e^{(-\zeta \mathrm{w}_n)t}$$

$$X(t) = A\, e^{(-\zeta \mathrm{w}_n)t}\, [\sin(\mathrm{w}_d t + \phi)]$$

Where amplitude A and phase ϕ can be defined as:

$$A = \sqrt{{X_0}^2 + \left(\frac{\zeta \mathrm{w}_n X_0}{\mathrm{w}_d}\right)}$$

$$\phi = \tan^{-1}\left(\frac{\mathrm{w}_d}{\zeta \mathrm{w}_n}\right)$$

The underdamped equation for motion will be analyzed. When the damping ratio is equal to zero, the response of the system is a sinusoid and energy is dissipating. Increasing the damping ratio results in an oscillation, -sin(ωdt − φ)-, although the amplitude is constantly decreasing up to equilibrium state along the time. The second term of the equation is a negative exponential factor, showing a decay in amplitude. When $t \rightarrow \infty$, the exponential function goes asymptotically to 0, so x(t) also goes asymptotically to its equilibrium position x = 0.

# CHAPTER 2

## 2. ANALYTICAL APPROACH

Contents



### 2.1. MATLAB PROGRAM

MATLAB can be used to determining the natural frequencies and mode shapes of the system and use them to formulate a mathematical model for flutter stability analysis as a function of the damping ratio. The main mission of this code is to obtain the mode shapes for different damping ratios and frequencies.

```
%This MATLAB CODE is property of Daniel de la Peña Jiménez

% In order to proceed, I have divided the following code into:
% 1. Function variables declaration
% 2. Function declaration for different damping ratios
%
%
%===============================================================
% CONFIDENTIAL -2020/21 - CONFIDENTIAL
%===============================================================

%FUNCTION VARIABLES DECLARATION:

chi=0; %damping ratio
wn=1; %natural frequency
xo=1; %initial displacement
t=linspace(0,10,10000); %time domain


%FUNCTION DECLARATION FOR DIFFERENT DAMPING RATIOS:

    if chi > 1 %OVERDAMPED
    a1=(((-chi+sqrt((chi^2)-1))*wn*xo))/(2*wn*sqrt((chi^2)- 1));
    a2=(((chi+sqrt((chi^2)-1)*wn*xo)))/(2*wn*sqrt((chi^2)-1));
    xt=(exp(-chi.*wn.*t)).*((a1.*exp(-wn.*(sqrt((chi^2)-
1)).*t))+a2.*(exp(wn.*(sqrt((chi^2)-1)).*t)));
    Xdot=xt/xo;
    plot(t,Xdot, 'r-')
    title('OVERDAMPED RESPONSE','Fontsize',20);
    subtitle('Author: Daniel de la Peña Jiménez','Color','blue',
'Fontsize',18);
    grid on
    ylabel('x(t)/x0', 'Fontsize',18)
    xlabel ('Wnt', 'Fontsize',18)

elseif chi == 1 %CRISTICALLY DAMPED
    a1=xo;
    a2=wn*xo;
    xt=(a1+a2.*t).*(exp(-wn.*t));
    Xdot=xt/xo;
    plot(t,Xdot, 'b-')
    grid on
```

```
    title('CRITICALLY DAMPED RESPONSE', 'Fontsize',20);
    subtitle('Author: Daniel de la Peña Jiménez','Color','blue',
'Fontsize',18);
    ylabel('x(t)/x0', 'Fontsize',18)
    xlabel ('Wnt', 'Fontsize',18)

else %UNDERDAMPED
    wd=wn*sqrt(1-chi^2);
    A=(1/wd)*(sqrt(((chi*wn*xo)^2)+((xo*wd)^2)));
    phase=atan((xo*wd)/(chi*wn*xo));
    xt=A*sin(t*wd+phase).*exp(-chi*t*wn);
    Xdot=xt/xo;
    plot(t,Xdot, 'm-')
    title('UNDERDAMPED RESPONSE', 'Fontsize',20);
    subtitle('Author: Daniel de la Peña Jiménez','Color','blue',
'Fontsize',18);
    grid on
    ylabel('x(t)/x0', 'Fontsize',18)
    xlabel ('Wnt', 'Fontsize',18)
    end
```

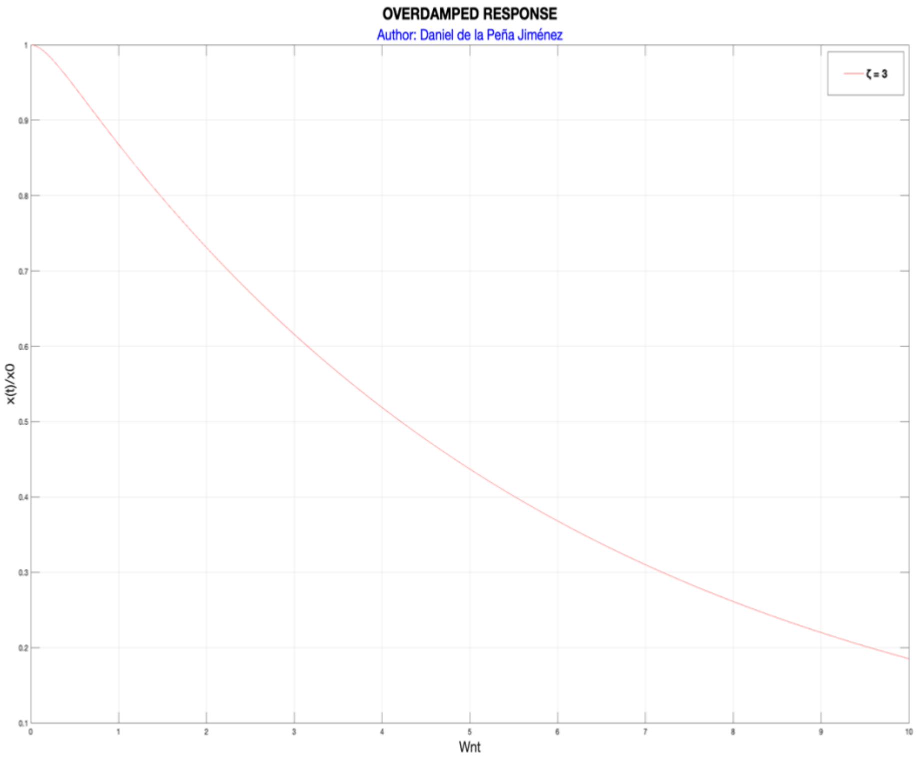



**FIGURE 13. OVERDAMPED RESPONSE**

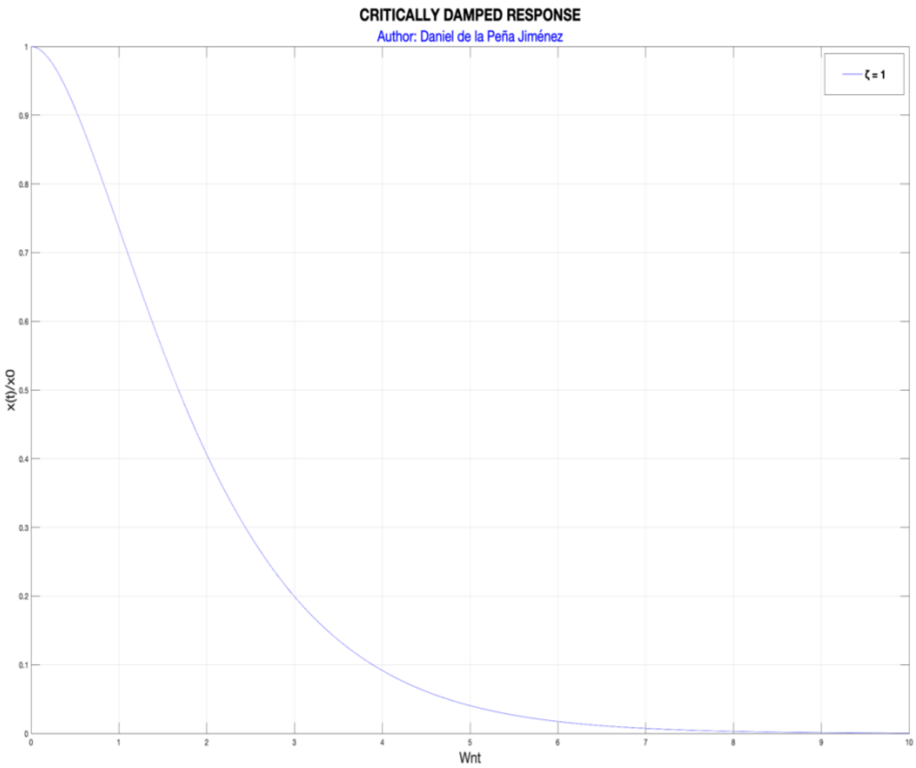



**FIGURE 14. CRITICALLY DAMPED RESPONSE**

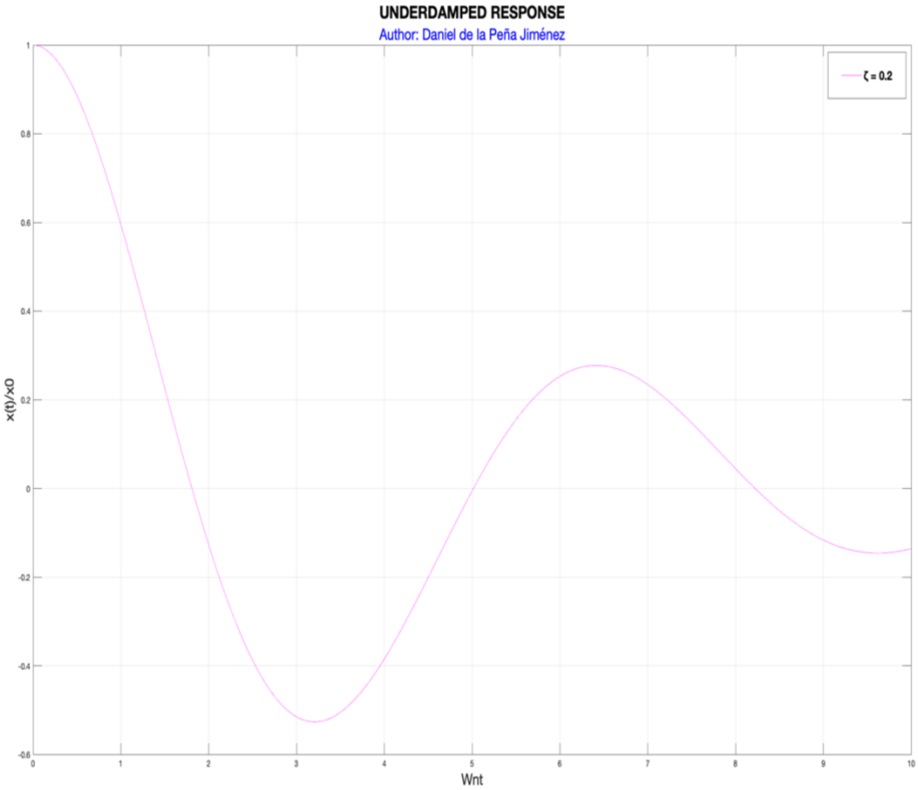



**FIGURE 15. UNDERDAMPED RESPONSE Z=0.2**

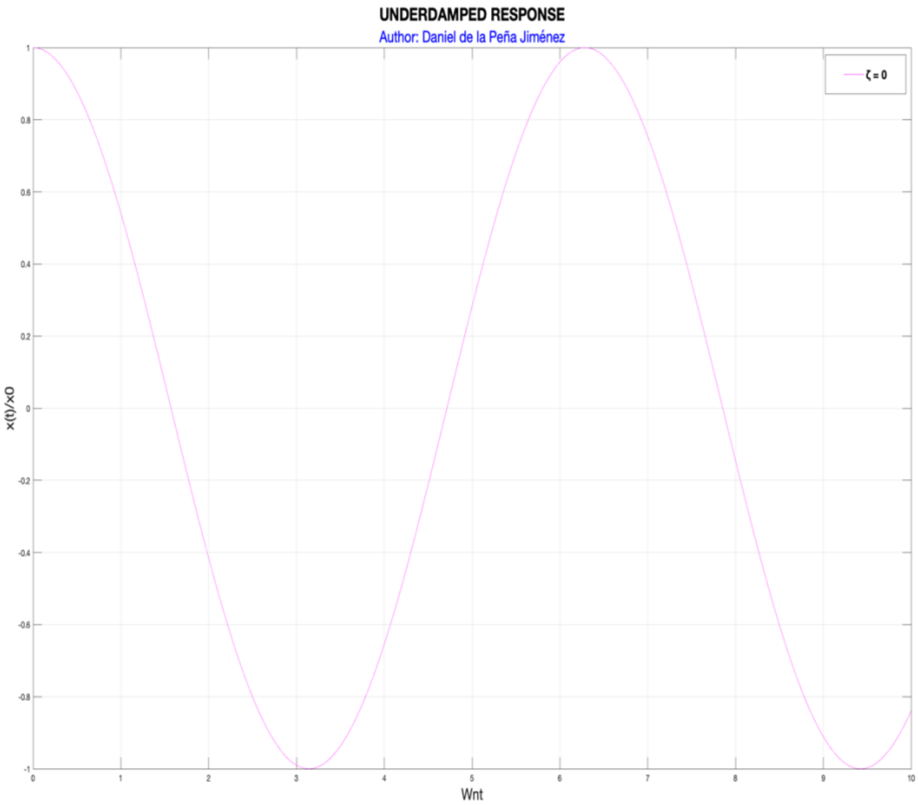



**FIGURE 16. UNDERDAMPED RESPONSE Z=0**

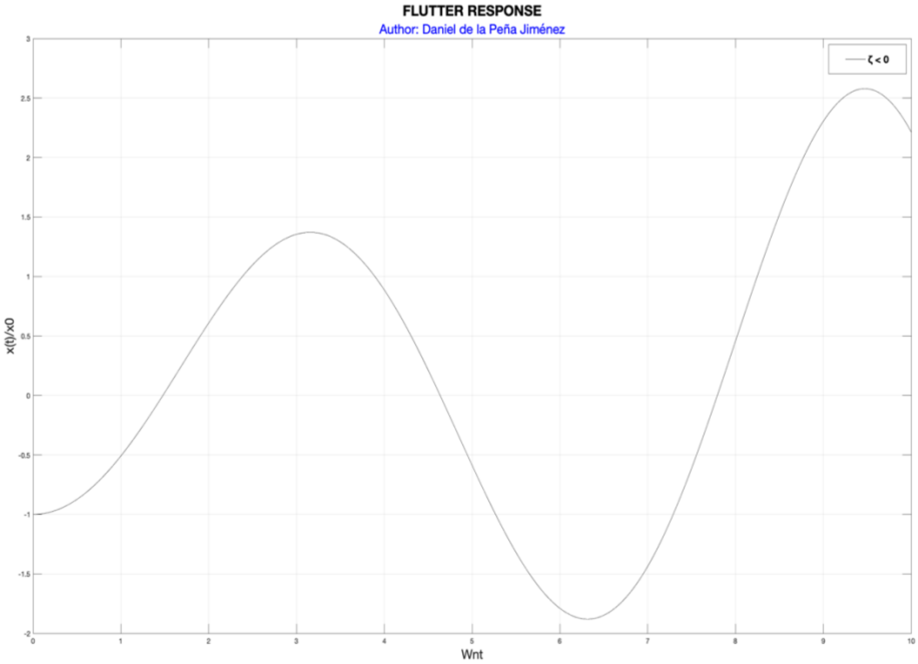



**FIGURE 17. FLUTTER RESPONSE**

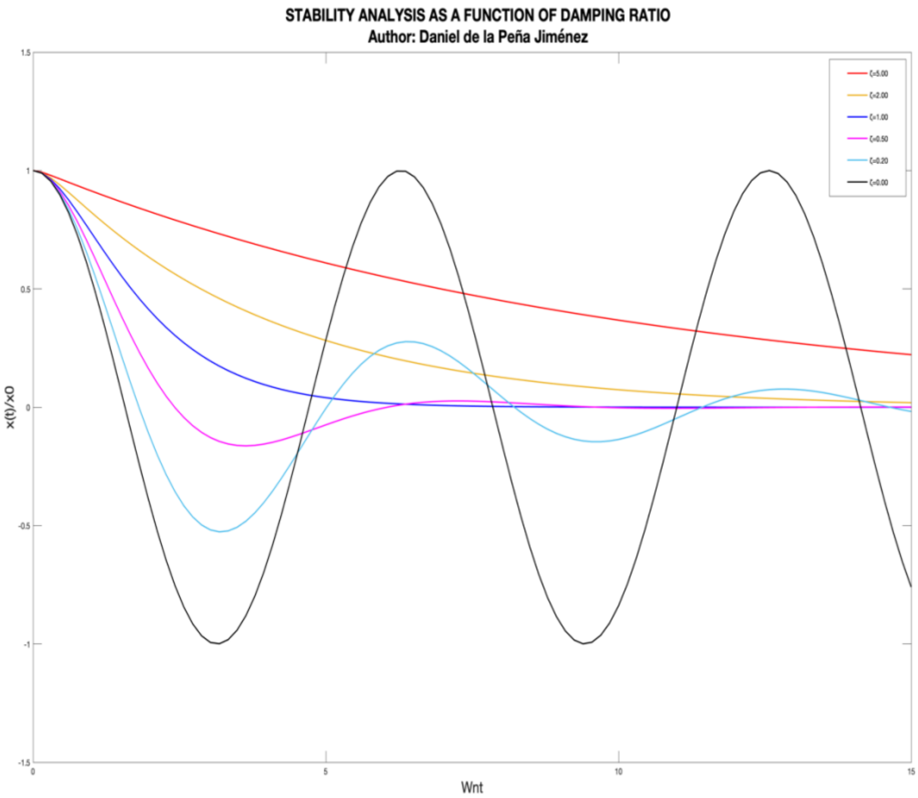



**FIGURE 18. STABILITY ANALYSIS RESUTLS**

## 2.2. DISCUSSION OF THE RESULTS

Analyzing this phenomenon, it can be observed that for overdamped systems, both real roots for the characteristic equation are real and negative. This has a physical impact on the system. In this case, as the damping higher than one, the restoring force is high enough to avoid the system to oscillate. Therefore, a free force overdamped harmonic oscillator does not oscillate. Since both exponents are negative, every solution in this case tends to the equilibrium. Also, a similar graph is obtain involving critically damped response, but with a difference on the decay. As in overdamped analysis, the solutions to the characteristic equations are real and negative. In this case are equal, what allows the system to behave in a curious manner. Also, there are grounds for believing that critically damped systems do not oscillate. For a given mass and spring constant, we can observe how the decay of a critically damped system compared with these of the overdamped system is faster. Some engineering applications involve this state in order to approximate a stable result. Differences can be obtained when analyzing underdamped motion. When the damping ratio is equal to zero, the response of the system is a sinusoid and energy is being dissipated. Increasing the damping ratio results in an oscillation, -sin($\omega dt - \varphi$)-, although the amplitude is constantly decreasing up to equilibrium state along the time. The second term of the equation is a negative exponential factor, showing a decay in amplitude. When $t \rightarrow \infty$, the exponential function goes asymptotically to 0, so x(t) also goes asymptotically to its equilibrium position x = 0. Thus, accurate analytical results for stability analysis as a function of the damping ratio show reasonable agreement.

# CHAPTER 3

# 3. CONCLUSIONS

Contents



## 3.1. CONCLUSION & FURTHER WORK

This report has provided insight into flutter stability analysis as a function of different damping ratios. Flutter, an unstable self-excited vibration in which the structure extracts energy from the air stream and often results in catastrophic structural failure, has been related with dynamic instability, which may eventually result in stall or buffeting conditions or classical bending and torsion coupling actions. These coupling occurs when the aerodynamic forces associated with motion in two modes of vibration cause the modes to couple in an unfavorable manner. The characteristic equation has been analyzed and derived in order to study stability and its relationship with the damping ratios of the aircraft wing. Analytical analysis of different mode shapes has been carried out by means of applying Newton's equation of motion and time dependent boundary conditions. Underdamped, critically damped, overdamped and flutter diagrams were obtained using MATLAB computational software for different damping ratios. Furthermore, these results have been compared with analytical derivations. Those results indicate stability dependence on damping ratio as it is basically a free vibration problem. Comparison between different models shows accurate results. As future engineers, we must tighten studies to research about dynamic aeroelasticity problems and its solution applied on flutter. Further research will be to consider and prove detail results on experimental data on lab acquiring technical knowledge.

# 4. REFERENCES


1. Mechanical and Graphic Design Subject. Universidad Europea de Madrid. Degree in Aerospace Engineering of Aircraft. Almudena Vega Coso.
2. Kelly, G. S. (2021). *Fundamentals of Mechanical Vibrations* (2nd Ed). McGraw Hill.
3. Herrin, D. W. (2007). *Vibro-Acoustic Design in Mechanical Systems*. Department of Mechanical Engineering.
4. Brown University. (n.d.). *Introduction to Dynamic Aeroelasticity*. School of Engineering. Brown University. https://www.brown.edu/Departments/Engineering/Courses/En4/Notes/vibrations_forced/vibrations_forced.htm
5. Dowel, E. H., & Sisto, F. (1978). *"A modern course in Aeroelasticity."* Sisto, F.