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ZIP Load Modeling for Single and Aggregate Loads and CVR Factor Estimation

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ZIP load Modeling for Single and Aggregate Loads and CVR Factor Estimation

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Abstract—ZIP load modeling has been used in various power system applications. The aggregate load modeling is common practice in utility companies. However, little research has been done on the theoretical formulation of the aggregate load. This paper formulates the aggregate ZIP load model using the single ZIP load model. The factors that may affect aggregate ZIP load estimation are studied. Common ZIP parameter estimation methods including least squares method, optimization method and neural network method have been used in this paper to estimate ZIP parameters. The case studies are based on the IEEE 13-bus and 34-bus system built in OpenDSS. The ZIP parameter estimation is also performed using field data, and the conservation voltage reduction (CVR) factor is computed based on the estimated ZIP load model.

Keywords— Least squares, load model, neural network, optimization, parameter estimation, ZIP load model

I. INTRODUCTION

Constant impedance, constant current, constant power (ZIP) load modeling has been widely used in steady-state and dynamic studies [1]. The ZIP load model is used in the optimal power flow study [2], feeder load forecasting [3], and voltage stability studies [4]. The ZIP load parameters are also useful for examining the benefits of conservation through voltage reduction (CVR) and voltage and var optimization (VVO) program [5][6][7]. Accurate load modeling can also benefit accurate fault location applications [8]. Therefore, it is crucial to have accurate ZIP load model parameters.

Various estimation methods have been used to estimate ZIP parameters. An optimization method was proposed in [9] to estimate ZIP parameters. The algorithm is tested on a 2-bus system. The least squares method was proposed in [10] to determine the ZIP coefficients for residential appliances. In [11], the time-varying ZIP parameters were estimated using a neural network method with sliding window using the single-phase data at a transformer. All the three estimation methods mentioned above are used and compared in this paper for ZIP parameters estimation.

The individual load models are used in power system distribution system analysis such as integrating distributed energy resources with improved power quality and reliability. The household appliances and industrial equipment were tested in the laboratory by varying the voltage to find individual ZIP parameters in [10]. In [14], a method for estimating individual ZIP load model by analyzing the load switching events across the distribution feeder is proposed. In [15], the authors developed a ZIP load model for a residential house appliances.

The feeder-level aggregate load model is also very important as it is used in various distribution and transmission system analyses. In [13], the authors proposed a hybrid learning algorithm that combines the genetic algorithm (GA) and the nonlinear Levenberg–Marquardt (L-M) algorithm for the aggregate ZIP load model estimation. In [12], the household and feeder level ZIP parameters are obtained by aggregating the appliance-level ZIP parameters.

However, little research has focused on the theoretical formulation for the aggregate ZIP load and how different factors can contribute to the estimation results. The measurement noise, load unbalance, voltage unbalance, and the load connection types are examined in detail in this article.

In Section II, the estimation for single load and different estimation methods will be discussed. In Section III, the estimation for aggregate loads with detailed examples with different connections will be provided. In Section IV, case studies for ZIP parameter estimation based on the IEEE 13-bus and 34-bus systems will be provided. Section V provides the conclusion.

II. ZIP LOAD MODEL AND PARAMETER ESTIMATION FOR A SINGLE LOAD

A single load can have two possible connection types. One is star connected, i.e., being connected between a phase and the neutral. The other is delta connected, i.e., being connected between two different phases.

For a single load, the ZIP load model is described by (1) and (2), where $|V|$, P, and Q are the voltage magnitude, real power, reactive power, as measured. P_0 , Q_0, V_0 are the base real power, base reactive power, and base voltage. a, b, and c are the ZIP parameters and their sum is equal to 1. Subscripts to the ZIP parameter p and q indicate values for real and reactive power, respectively.

$$P = P_0\left(a_p\left(\frac{|V|}{V_0}\right)^2 + b_p\left(\frac{|V|}{V_0}\right) + c_p\right) \tag{1}$$

$$Q = Q_0\left(a_q\left(\frac{|V|}{V_0}\right)^2 + b_q\left(\frac{|V|}{V_0}\right) + c_p\right) \tag{2}$$

For a star-connected single load, in (1-2), V represents the phase to neutral voltage, and P and Q represent the phase real and reactive power.

For a delta-connected single load, in (1-2), V should be the voltage difference between the two phases, and P and Q are the sum of the real and reactive power measured at each phase. For example, suppose that a load is connected between phase B and C. Equation (1) may be more explicitly written as

$$P_B + P_C = P_0\left(a_p\left(\frac{|V_{BC}|}{\sqrt{3}V_0}\right)^2 + b_p\left(\frac{|V_{BC}|}{\sqrt{3}V_0}\right) + c_p\right) \tag{3}$$

$$V_{BC} = V_B - V_C \tag{4}$$

where, P_B and P_C are phase B and C real power, $\sqrt{3}V_0$ denotes the base voltage of the delta load, and V_{BC} is phase B to C voltage.
However, in practical applications, the phase angle of voltage may not always be available, and only phase voltage magnitude is measured. Then, the average voltage magnitude of phase B and C is often used in the ZIP load model, i.e., $|V_{BC_{avg}}|$ in . For balanced cases, $V_c = V_B\angle -120°$, we have

$$|V_{BC_{avg}}| = \frac{|V_B| + |V_c|}{2V_0} = \frac{|V_B|}{V_0} \tag{5}$$

which is the same as $\frac{|V_{BC}|}{\sqrt{3}V_0}$. Therefore, using average magnitude will get identical results as using line-to-line voltage in the ZIP model for balanced voltage cases. Different results may be yielded for unbalanced cases, which will be further explained later.

In the following sections, the three commonly used estimation methods for the ZIP load model are discussed. A window size n is selected for each estimation, i.e., n sets of measurements are used for each estimation. It is assumed that the ZIP load model parameters remain constant, i.e., a, b, c, P_0, and Q_0 stay unchanged during the period of taking the n sets of measurements, although ZIP parameters may change over time.

A. *Least Squares Method*

The least squares method has been described in [10], [16][14], [18][16] to estimate ZIP parameters based on the matrix equation $Z = Hx$. In summary, we have

$$Z = [P_1, P_2, \dots, P_n]^T \tag{6}$$

$$H = \begin{bmatrix} \left(\frac{|V_1|}{V_0}\right)^2 & \frac{|V_1|}{V_0} & 1 \\ \left(\frac{|V_2|}{V_0}\right)^2 & \frac{|V_2|}{V_0} & 1 \\ \vdots & \vdots & \vdots \\ \left(\frac{|V_n|}{V_0}\right)^2 & \frac{|V_n|}{V_0} & 1 \end{bmatrix} \tag{7}$$

$$x = \begin{bmatrix} a_pP_0 \\ b_pP_0 \\ c_pP_0 \end{bmatrix} \tag{8}$$

where, Z consists of measured real power, H is composed of voltage measurements, and x is the unknown vector. The solution of the least squares method is given by

$$x = (H^T H)^{-1} H^T Z \tag{9}$$

Once x is obtained, P_0 is calculated as the sum of x, which implies that the sum of a_p, b_p and c_p is one. The ZIP parameters a_p, b_p and c_p are calculated by dividing x by P_0.

B. *Optimization Method*

The optimization method has been used in [9] to estimate ZIP parameters and works similarly to the theory of the least squares method. For a window size of n, (10) can be written based on the ZIP load model. f is the difference between the measured P and the estimated P (calculated by using the measured voltage and estimated ZIP parameters). The optimization method will produce the estimation results that minimize $f_1(x)^2 + f_2(x)^2 + \cdots + f_n(x)^2$. Constraints can be easily imposed including $a_p + b_p + c_p$=1 and $a_p, b_p,$ and c_p being non-negative for loads.

$$\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} - \begin{bmatrix} \left(\frac{|V_1|}{V_0}\right)^2 & \frac{|V_1|}{V_0} & 1 \\ \left(\frac{|V_2|}{V_0}\right)^2 & \frac{|V_2|}{V_0} & 1 \\ \vdots & \vdots & \vdots \\ \left(\frac{|V_n|}{V_0}\right)^2 & \frac{|V_n|}{V_0} & 1 \end{bmatrix} \begin{bmatrix} a_p P_0 \\ b_p P_0 \\ c_p P_0 \end{bmatrix} \tag{10}$$

C. *Extension to Least Square and Optimization Method*

For practical applications, if we only want to obtain an average set of ZIP parameters, then the following method can be used.

We assume every N_1 consecutive time points, e.g., 4 points, P_0 remains constant. Suppose we have a total of n sets of measurements, say 2 sets, each of which contains such N_1 points. Then the total number of measurements will be nN_1. We have

$$\begin{aligned} P_1 &= P_{01}\left[a_p\left(\frac{|V_1|}{V_0}\right)^2 + b_p\left(\frac{|V_1|}{V_0}\right) + c_p\right] \\ P_2 &= P_{02}\left[a_p\left(\frac{|V_2|}{V_0}\right)^2 + b_p\left(\frac{|V_2|}{V_0}\right) + c_p\right] \\ P_{N_1} &= P_{0N_1}\left[a_p\left(\frac{|V_{N_1}|}{V_0}\right)^2 + b_p\left(\frac{|V_{N_1}|}{V_0}\right) + c_p\right] \\ &\cdots \\ P_{(k-1)*N_1+i} &= P_{0k}\left[a_p\left(\frac{|V_{(k-1)*N_1+i}|}{V_0}\right)^2 + b_p\left(\frac{|V_{(k-1)*N_1+i}|}{V_0}\right) + c_p\right] \end{aligned} \tag{11}$$

Where, $a_p + b_p + c_p = 1$, $k = 1, \dots, n$ is measurement set index, and $i = 1, \dots, N_1$ are time points within each measurement set. The number of equations: nN_1 and the number of unknown variables is P_{0k}, a_p, b_p, c_p, a total of $n + 3$ unknowns. Then either the least squares or optimization method can be used to solve for the unknowns.

In another variant, the power can be written such that one ZIP parameter such as c_p is eliminated

$$P_{N_1} = P_{01}\left\{a_p\left[\left(\frac{|V_{N_1}|}{V_0}\right)^2 - 1\right] + b_p\left[\left(\frac{|V_{N_1}|}{V_0}\right) - 1\right] + 1\right\} \tag{12}$$

D. *Neural Network Method*

The neural network has been used in [11], [17] to estimate ZIP parameters. The structure of the proposed neural network is shown in Fig. 1. It has two inputs ($\left(\frac{V}{V_0}\right)^2$, $\frac{V}{V_0}$) and one ouput P. The measured voltage and power are used as training data to train the neural network. The neural network will be trained for every n sets of measurements. After training, the weights and bias can be extracted from the trained neural network, from which the ZIP parameters can be obtained.

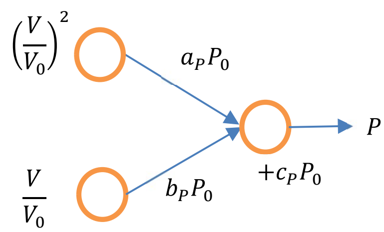


Fig. 1. The structure of the proposed neural network for ZIP load modeling.

E. Measurement Noises

In this section, the ZIP parameter estimation is studied under the effect of measurement noises. The ZIP parameters and base power are kept constant: $a_p = 0.25, b_p = 0.15, c_p = 0.60, P_0 = 110$. The noise is added to both power and voltage measurements and is defined as normally distributed random numbers with the mean $\mu = 0$ and the standard deviation $\sigma = 0.01\%$ of the measurement magnitude. The base voltage is $120\,V$. The estimation is performed using the least squares method. With such small measurement noises, the estimation error can be substantial as shown in Fig. 2.

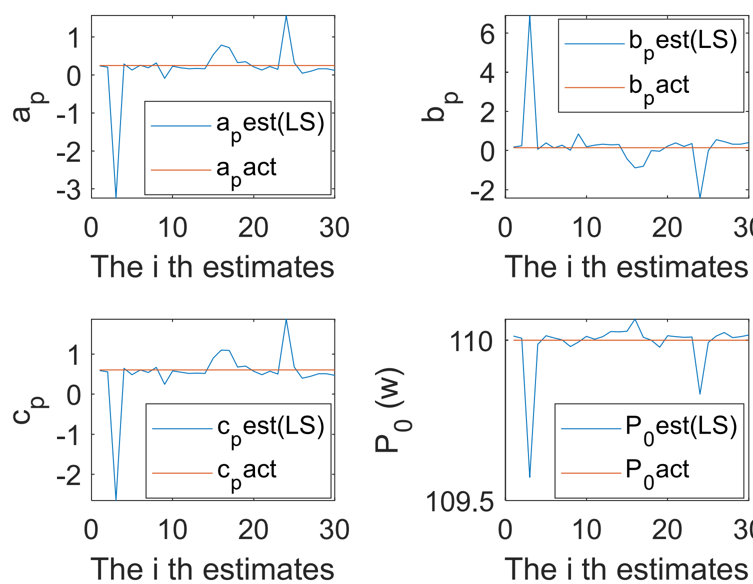


Fig. 2. The ZIP estimation results for a single load with measurement noise (blue: estimated values; red: actual values).

For example, for the 3rd estimate, the voltage and power measurement without noises are: $(114.6535, 114.3669, 117.1774, 116.7569)\,V$ and $(106.8690,\ 106.7042, 108.3334, 108.0877)\,W$. The estimated ZIP parameters and P_0 are: $(0.2497, 0.1506, 0.5997)$ and $109.9998\,W$, which are very close to the actual ZIP parameters. The mean squared error (MSE) is 5.61×10^{-8}. Then, the voltage and power measurement with noises are: $(114.6320, 114.3859, 117.1775, 116.7657)\,V$ and $(106.8867, 106.6926, 108.3361, 108.0999)\,W$. The estimated ZIP parameters and P_0 are: $(-3.2461, 6.8988, -2.6527)$ and $109.5718\,W$. The MSE is 2.24×10^{-4}. If we use the actual ZIP and base power value $(0.25, 0.15, 0.60, 110)$, the MSE is calculated as 3.67×10^{-4}. Therefore, although the estimated values deviate from true values, they indeed yield smaller MSE than true values do. Generally speaking, the larger the voltage variation within the measurement set is, the less sensitive the estimation is to the measurement noises.

III. ZIP LOAD MODEL AND PARAMETER ESTIMATION FOR AGGREGATE LOAD

In a power system, the measurement at a single load might not always be available and there is a need of having a ZIP load model for several loads. Based on the modeling for a single load, we can also derive the ZIP load model for several loads connected together, i.e., aggregate loads.

This section presents ZIP load model and parameter estimation for aggregate loads. Subsection A discusses a single delta load continuing from Section II-A.

A. ZIP Load Model for a Single Delta Load

This section studies a single load connected between two phases. In this case, the load is connected between phase B and C, as shown in Fig. 3. Suppose the load has base power $P_0 = 100kW$, base line to line voltage $V_{0LL} = 4.16kV$, ZIP parameters $a_p = 0.2$, $a_p = 0.5$, and $c_p = 0.3$.

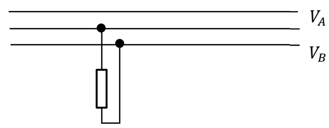


Fig. 3. The circuit diagram of a single delta load.

The following subsections study the ZIP estimation for single delta under the balanced and different unbalanced voltage conditions. For each subsection, the ZIP parameters of the single delta load are estimated using the average voltage. The line-to-line voltage has also been used in estimation, but the results are not presented in each subsection since it always gives the accurate results. All the estimation results are produced by using the least squares method.

1) Case 1: Balanced Voltage

In this section, the ZIP parameter estimation for a single delta load is studied under balanced voltage condition. The voltage and power data used for estimation are shown in TABLE I. .

TABLE I. VOLTAGE AND POWER DATA FOR THE SINGLE DELTA LOAD UNDER THE BALANCED VOLTAGE CONDITION

$\|V_B\|$ (kV)	$\angle V_B$ (degree)	$\|V_C\|$ (kV)	$\angle V_C$ (degree)	P (kW)
2.2800	-120	2.2800	120	95.4882
2.4480	-120	2.4480	120	101.7395
2.5680	-120	2.5680	120	106.3245
2.6400	-120	2.6400	120	109.1235

If we use the line to line voltage magnitude, the estimated ZIP parameters are:$a_p = 0.2000$, $b_p = 0.5000$, and $c_p = 0.3000$. If we use the average voltage magnitudes, the estimated ZIP parameters are: $a_p = 0.1998$, $b_p = 0.5000$, and $c_p = 0.3002$. The results show that using average voltage for estimation can provide good estimation accuracy.

2) Case 2: Unbalanced Voltage

a) Unbalanced Voltage Magnitude

This section studies the effect of the unbalanced voltage magnitude on ZIP parameter estimation for a single delta load.

The estimated ZIP are: $a_p = 0.1993$, $b_p = 0.5012$, and $c_p = 0.2995$.

TABLE II. VOLTAGE AND POWER DATA FOR THE SINGLE DELTA LOAD UNDER THE UNBALANCED VOLTAGE MAGNITUDE CONDITION

$\|V_B\|$ (kV)	$\angle V_B$ (degree)	$\|V_C\|$ (kV)	$\angle V_C$ (degree)	P (kW)	Voltage unbalance
2.2572	-120	2.2800	120	95.0714	0.50%
2.3998	-120	2.4480	120	100.8314	1.00%
2.5423	-120	2.5680	120	105.8296	0.50%
2.5898	-120	2.6400	120	108.1460	0.96%

b) Unbalanced Voltage Angle

The effect of unbalanced voltage angle on ZIP parameter estimation is studied in this case. The four sets of measurements used for estimation are shown in TABLE III. The estimated $a_p = 0.4171$, $b_p = 0.0488$, and $c_p = 0.5341$. TABLE IV. shows another set of measurements with voltage angle unbalance, the estimated $a_p = 0.2020$, $b_p = 0.5005$, and $c_p = 0.2975$.

In the first case, the voltage angle unbalance is relatively small compared to the second case. But the unbalance is varying and the estimation results deviate a lot from the theoretical values. For the second case, as long as the voltage angle unbalance is invariant, the estimation results are promising despite the fact that the voltage angle unbalance is large.

TABLE III. VOLTAGE AND POWER DATA FOR THE SINGLE DELTA LOAD UNDER THE UNBALANCED VOLTAGE ANGLE CONDITION ONE

$\|V_B\|$ (kV)	$\angle V_B$ (degree)	$\|V_C\|$ (kV)	$\angle V_C$ (degree)	P (kW)
2.2800	-119	2.2800	120	95.9062
2.4480	-119.3	2.4480	120	102.0643
2.5680	-119.5	2.5680	120	106.5736
2.6400	-119.2	2.6400	120	109.5377

TABLE IV. VOLTAGE AND POWER DATA FOR THE SINGLE DELTA LOAD UNDER THE UNBALANCED VOLTAGE ANGLE CONDITION TWO

$\|V_B\|$ (kV)	$\angle V_B$ (degree)	$\|V_C\|$ (kV)	$\angle V_C$ (degree)	P (kW)
2.2800	-118	2.2800	120	95.9062
2.4480	-118	2.4480	120	102.1105
2.5680	-118	2.5680	120	106.7717
2.6400	-118	2.6400	120	109.4862

B. *Aggregate ZIP Load Model for Three-Phase Star Load*

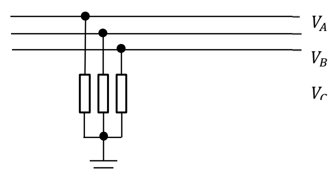


Fig. 4. The circuit diagram for a three-phase star load.

In this section, the aggregate ZIP load model for the three-phase star connected load is discussed. Fig. 4 shows the circuit diagram. The individual ZIP load models for the real power of three single loads connected at phase A, B, and C can be described by (13)–(15)

$$P_A = P_{0_A}\left(a_{p_A}\left(\frac{|V_A|}{V_0}\right)^2 + b_{p_A}\left(\frac{|V_A|}{V_0}\right) + c_{p_A}\right) \tag{13}$$

$$P_B = P_{0_B}\left(a_{p_B}\left(\frac{|V_B|}{V_0}\right)^2 + b_{p_B}\left(\frac{|V_B|}{V_0}\right) + c_{p_B}\right) \tag{14}$$

$$P_C = P_{0_C}\left(a_{p_C}\left(\frac{|V_C|}{V_0}\right)^2 + b_{p_C}\left(\frac{|V_C|}{V_0}\right) + c_{p_C}\right) \tag{15}$$

There is no way to obtain an analytical form of ZIP load model for an aggregate ZIP load like a single ZIP load model unless the three-phase voltages are balanced. So, if $|V_A| = |V_B| = |V_C| = V$, then we can obtain (16) by summing (13)–(15).

$$P_{Agg} = P_{0_{Agg}}\left(a_{p_{Agg}}\left(\frac{V}{V_0}\right)^2 + b_{p_{Agg}}\frac{V}{V_0} + c_{p_{Agg}}\right) \tag{16}$$

where

$$P_{Agg} = P_A + P_B + P_C \tag{17}$$

$$P_{0_{Agg}} = P_{0_A} + P_{0_B} + P_{0_C} \tag{18}$$

$$a_{p_{Agg}} = \frac{P_{0_A}a_{p_A} + P_{0_B}a_{p_B} + P_{0_C}a_{p_C}}{P_{0_A} + P_{0_B} + P_{0_C}} \tag{19}$$

$$b_{p_{Agg}} = \frac{P_{0_A}b_{p_A} + P_{0_B}b_{p_B} + P_{0_C}a_{p_C}}{P_{0_A} + P_{0_B} + P_{0_C}} \tag{20}$$

$$c_{p_{Agg}} = \frac{P_{0_A}c_{p_A} + P_{0_B}c_{p_B} + P_{0_C}c_{p_C}}{P_{0_A} + P_{0_B} + P_{0_C}} \tag{21}$$

The aggregate ZIP parameters are the weighted averages of the individual ZIP parameters of each load while the weights are the base power of each load.

For the unbalanced system, the average three-phase voltage magnitude can be used:

$$V = \frac{|V_A| + |V_B| + |V_C|}{3} \quad (22)$$

Then we studied the ZIP parameter estimation for the three-phase star load with four different configurations. The single loads at phase A, B, and C can have same or different base power (P_0) and same or different ZIP parameters, which make up the four configurations. For each configuration, the ZIP parameter estimation is performed using the balanced and unbalanced voltage data shown in TABLE V. The voltage unbalance [19] is calculated by

$$unbalance\ \% = \frac{max\ deviation\ from\ avg.\ V}{avg.V} \quad (23)$$

The estimation results are shown in TABLE V. – TABLE IX. The fourth row of every table is for the calculated theoretical ZIP parameters based on (19)–(21). It is shown that the ZIP parameters can be estimated accurately under balanced voltages condition for all four configurations. However, under the condition of the unbalanced voltage, the ZIP parameters can only be estimated without much deviation for the three-phase loads with the same P_0 and same ZIP parameters configuration.

TABLE V. THREE-PHASE BALANCED AND UNBALANCED VOLTAGES

Balanced Voltages Case			Unbalanced Voltages Case			
$\|V_A\|$ (kV)	$\|V_B\|$ (kV)	$\|V_C\|$ (kV)	$\|V_A\|$ (kV)	$\|V_B\|$ (kV)	$\|V_C\|$ (kV)	Voltage unbalance
2.3040	2.3040	2.3040	2.3040	2.2560	2.3520	2.08%
2.4240	2.4240	2.4240	2.4240	2.3520	2.4720	2.64%
2.5200	2.5200	2.5200	2.5200	2.4480	2.5920	2.85%
2.6160	2.6160	2.6160	2.6160	2.5680	2.6400	1.53%

TABLE VI. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE STAR LOAD (SAME P_0 AND SAME ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A	0.4	0.2	0.4	120
Load connected at phase B	0.4	0.2	0.4	120
Load connected at phase C	0.4	0.2	0.4	120
Calculated Aggregate ZIP	0.4	0.2	0.4	360
Estimated Aggregate ZIP (balanced voltages case)	0.4	0.2	0.4	360
Estimated Aggregate ZIP (unbalanced voltages case)	0.36	0.27	0.37	360.07

TABLE VII. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE STAR LOAD (SAME P_0 AND DIFFERENT ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A	0.4	0.2	0.4	120
Load connected at phase B	0.6	0.3	0.1	120
Load connected at phase C	0.2	0.3	0.5	120
Calculated Aggregate ZIP	0.40	0.27	0.33	360
Estimated Aggregate ZIP (balanced voltages case)	0.40	0.27	0.33	360
Estimated Aggregate ZIP (unbalanced voltages case)	1.14	-1.25	1.10	357.37

TABLE VIII. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE STAR LOAD (DIFFERENT P_0 AND SAME ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A	0.4	0.2	0.4	120
Load connected at phase B	0.4	0.2	0.4	150
Load connected at phase C	0.4	0.2	0.4	80
Calculated Aggregate ZIP	0.4	0.2	0.4	350
Estimated Aggregate ZIP (balanced voltages case)	0.4	0.2	0.4	350
Estimated Aggregate ZIP (unbalanced voltages case)	0.94	-0.90	0.96	348.12

TABLE IX. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE STAR LOAD (DIFFERENT P_0 AND DIFFERENT ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A	0.4	0.2	0.4	120
Load connected at phase B	0.6	0.3	0.1	150
Load connected at phase C	0.2	0.3	0.5	80
Calculated Aggregate ZIP	0.44	0.27	0.29	350
Estimated Aggregate ZIP (balanced voltages case)	0.44	0.27	0.29	350
Estimated Aggregate ZIP (unbalanced voltages case)	1.80	-2.52	1.71	345.32

C. Aggregate ZIP Load Model for Three-Phase Delta Load

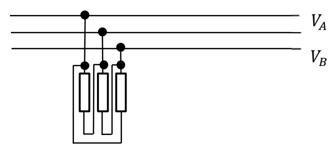


Fig. 5. The circuit diagram for the three-phase delta load.

This section presents the aggregate ZIP load model for the real power of the three-phase aggregate delta loads. The individual ZIP load model of the three loads can be expressed as in (24)–(26)

$$P_{AB} = P_{0_{AB}}\left(a_{p_{AB}}\left(\frac{|V_{AB}|}{\sqrt{3}V_0}\right)^2 + b_{p_{AB}}\left(\frac{|V_{AB}|}{\sqrt{3}V_0}\right) + c_{p_{AB}}\right) \tag{24}$$

$$P_{BC} = P_{0_{BC}}\left(a_{p_{BC}}\left(\frac{|V_{BC}|}{\sqrt{3}V_0}\right)^2 + b_{p_{BC}}\left(\frac{|V_{BC}|}{\sqrt{3}V_0}\right) + c_{p_{BC}}\right) \tag{25}$$

$$P_{AC} = P_{0_{AC}}\left(a_{p_{AC}}\left(\frac{|V_{AC}|}{\sqrt{3}V_0}\right)^2 + b_{p_{AC}}\left(\frac{|V_{AC}|}{\sqrt{3}V_0}\right) + c_{p_{AC}}\right) \tag{26}$$

If the three-phase voltages are balanced, i.e., $|V_A| = |V_B| = |V_C| = V$, then (24)–(26) can be rewritten as (27)–(29).

$$P_{AB} = P_{0_{AB}}\left(a_{p_{AB}}\left(\frac{V}{V_0}\right)^2 + b_{p_{AB}}\left(\frac{V}{V_0}\right) + c_{p_{AB}}\right) \tag{27}$$

$$P_{BC} = P_{0_{BC}}\left(a_{p_{BC}}\left(\frac{V}{V_0}\right)^2 + b_{p_{BC}}\left(\frac{V}{V_0}\right) + c_{p_{BC}}\right) \tag{28}$$

$$P_{AC} = P_{0_{AC}}\left(a_{p_{AC}}\left(\frac{V}{V_0}\right)^2 + b_{p_{AC}}\left(\frac{V}{V_0}\right) + c_{p_{AC}}\right) \tag{29}$$

Then an analytical form of aggregate ZIP load model can be obtained by summing (27)-(29)

$$P_{Agg} = P_{0_{Agg}}\left(a_{p_{Agg}}\left(\frac{V}{V_0}\right)^2 + b_{p_{Agg}}\frac{V}{V_0} + c_{p_{Agg}}\right) \tag{30}$$

where

$$P_{Agg} = P_{AB} + P_{BC} + P_{AC} \tag{31}$$

$$P_{0_{Agg}} = P_{0_{AB}} + P_{0_{BC}} + P_{0_{AC}} \tag{32}$$

$$a_{p_{Agg}} = \frac{P_{0_{AB}}a_{p_{AB}} + P_{0_{BC}}a_{p_{BC}} + P_{0_{AC}}a_{p_{AC}}}{P_{0_{AB}} + P_{0_{BC}} + P_{0_{AC}}} \tag{33}$$

$$b_{p_{Agg}} = \frac{P_{0_{AB}}a_{p_{AB}} + P_{0_{BC}}a_{p_{BC}} + P_{0_{AC}}a_{p_{AC}}}{P_{0_{AB}} + P_{0_{BC}} + P_{0_{AC}}} \tag{34}$$

$$c_{p_{Agg}} = \frac{P_{0_{AB}}a_{p_{AB}} + P_{0_{BC}}a_{p_{BC}} + P_{0_{AC}}a_{p_{AC}}}{P_{0_{AB}} + P_{0_{BC}} + P_{0_{AC}}} \tag{35}$$

For the case of unbalanced voltages, we can use the three-phase average voltage as the voltage variable for the ZIP load model for the three-phase delta load.

The ZIP parameter estimation for the three-phase delta load is also studied for the four different configurations under balanced and unbalanced voltages conditions using the voltage data in TABLE V. The results are shown in TABLE X. –TABLE XIII. From the results, we have similar observations: the voltage unbalance can cause the estimated ZIP parameters to deviate from the theoretical ZIP parameters, and the deviation is minimum for the three-phase load with the same base power and same ZIP parameters configuration.

TABLE X. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE DELTA LOAD (SAME P_0 AND SAME ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A and B	0.4	0.2	0.4	120
Load connected at phase B and C	0.4	0.2	0.4	120
Load connected at phase A and C	0.4	0.2	0.4	120
Calculated Aggregate ZIP	0.4	0.2	0.4	360
Estimated Aggregate ZIP (balanced voltages case)	0.40	0.20	0.40	359.73
Estimated Aggregate ZIP (unbalanced voltages case)	0.38	0.24	0.38	359.77

TABLE XI. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE DELTA LOAD (SAME P_0 AND DIFFERENT ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A and B	0.4	0.2	0.4	120
Load connected at phase B and C	0.6	0.3	0.1	120
Load connected at phase A and C	0.2	0.3	0.5	120
Calculated Aggregate ZIP	0.40	0.27	0.33	360
Estimated Aggregate ZIP (balanced voltages case)	0.40	0.27	0.33	359.71
Estimated Aggregate ZIP (unbalanced voltages case)	0.53	0.00	0.47	359.20

TABLE XII. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE DELTA LOAD (DIFFERENT P_0 AND SAME ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A and B	0.4	0.2	0.4	120
Load connected at phase B and C	0.4	0.2	0.4	150
Load connected at phase A and C	0.4	0.2	0.4	80
Calculated Aggregate ZIP	0.4	0.2	0.4	350
Estimated Aggregate ZIP (balanced voltages case)	0.40	0.20	0.40	349.74
Estimated Aggregate ZIP (unbalanced voltages case)	0.55	-0.10	0.55	349.19

TABLE XIII. THE ZIP PARAMETERS OF EACH LOAD AND THE ESTIMATION RESULTS FOR THE THREE-PHASE DELTA LOAD (DIFFERENT P_0 AND SAME ZIP PARAMETERS)

	a_p	b_p	c_p	$P_0(kW)$
Load connected at phase A and B	0.4	0.2	0.4	120
Load connected at phase B and C	0.6	0.3	0.1	150
Load connected at phase A and C	0.2	0.3	0.5	80
Calculated Aggregate ZIP	0.44	0.27	0.29	350
Estimated Aggregate ZIP (balanced voltages case)	0.44	0.27	0.30	349.70
Estimated Aggregate ZIP (unbalanced voltages case)	0.69	-0.24	0.55	348.76

D. Aggregate ZIP Load Model for Multiple Three-Phase Star Loads

This section studies the aggregate ZIP load model for multiple three-phase star-connected balanced loads in power systems. Suppose there are n three-phase balanced star loads connected together. Assume that $|V_A| = |V_B| = |V_C| = V$. The real power of each three-phase load are P_i $(i = 1, 2, \dots, n)$; the ZIP parameters and base power of the ZIP model for three each load are P_{0_i} and $(a_{p_i}, b_{p_i}, c_{p_i})$. Each three-phase star-connected loads can be expressed as

$$P_i = P_{0_i}\left(a_{p_i}\left(\frac{|V|}{V_0}\right)^2 + b_{p_i}\frac{|V|}{V_0} + c_{p_i}\right) \quad (36)$$

Then the aggregate ZIP load model for the multiple three-phase loads can be expressed as

$$P_{Agg} = P_{0_{Agg}}\left(a_{p_{Agg}}\left(\frac{|V|}{V_0}\right)^2 + b_{p_{Agg}}\frac{|V|}{V_0} + c_{p_{Agg}}\right) \quad (37)$$

where

$$P_{Agg} = \sum_{i=1}^{n} P_i \quad (38)$$

$$P_{0_{Agg}} = \sum_{i=1}^{n} P_{0_i} \quad (39)$$

$$a_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_i} a_{p_i}}{\sum_{i=1}^{n} P_{0_i}} \quad (40)$$

$$b_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_i} b_{p_i}}{\sum_{i=1}^{n} P_{0_i}} \quad (41)$$

$$c_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_i} c_{p_i}}{\sum_{i=1}^{n} P_{0_i}} \quad (42)$$

If the ZIP parameters of all loads are the same, then the aggregate ZIP parameters will also be the same as the ZIP parameters of each load despite what the base power of each load is and how the base power of each load changes with time.

If all loads do not have the same ZIP parameters, the aggregate ZIP parameters will only keep constant if the base power of each load are constant or the ratio between the base powers of all loads are constant. For unbalanced system, we can use the three-phase average voltage as the voltage variable for the ZIP load model.

E. *Aggregate ZIP Load Model for Multiple Three-Phase Delta Loads*

The aggregate ZIP load model for multiple three-phase delta connected loads is exactly the same as the aggregate ZIP load model for multiple three-phase star loads as given by (37) under the assumption that the system voltages are balanced.

F. *Aggregate ZIP Load Model for Multiple Three-Phase Star and Delta Loads*

The aggregate ZIP load model for multiple three-phase star and delta loads can also be described by (37) since the aggregate ZIP load model for both multiple star loads and multiple delta loads can be described by (37).

G. *Aggregate ZIP Load Model for Multiple Star Loads*

In this case, the aggregate ZIP load model for multiple three-phase and single-phase star connected loads are studied. Fig. 6 shows the circuit diagram.

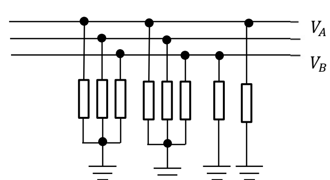


Fig. 6. The circuit diagram of three-phase and single-phase star loads.

First, the aggregate ZIP load model can be obtained for each phase. Suppose that n single loads are connected to phase A and the voltage drop on the line is ignored. Thus all loads will have the same voltage, and the general form of load i ($i = 1, 2, \dots, n$) connected to Phase A can be described by (43)

$$P_{A_i} = P_{0_{A_i}}\left(a_{p_{A_i}}\left(\frac{|V_A|}{V_0}\right)^2 + b_{p_{A_i}}\frac{|V_A|}{V_0} + c_{p_{A_i}}\right) \quad (43)$$

Then the aggregate ZIP load model can be obtained

$$P_{A_{Agg}} = P_{0_{A_{Agg}}} \left(a_{p_{A_{Agg}}} \left(\frac{|V_A|}{V_0} \right)^2 + b_{p_{A_{Agg}}} \frac{|V_A|}{V_0} + c_{p_{A_{Agg}}} \right) \tag{44}$$

where

$$P_{A_{Agg}} = \sum_{i=1}^{n} P_{A_i} \tag{45}$$

$$P_{0_{A_{Agg}}} = \sum_{i=1}^{n} P_{0_{A_i}} \tag{46}$$

$$a_{p_{A_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} a_{p_{A_i}}}{\sum_{i=1}^{n} P_{0_{A_i}}} \tag{47}$$

$$b_{p_{A_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} b_{p_{A_i}}}{\sum_{i=1}^{n} P_{0_{A_i}}} \tag{48}$$

$$c_{p_{A_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} c_{p_{A_i}}}{\sum_{i=1}^{n} P_{0_{A_i}}} \tag{49}$$

The aggregate ZIP load model can be derived similarly for the loads connected at phase B to neutral and phase C to neutral. Suppose $V_A = V_B = V_C = V$ and there are m loads and k loads connected at Phase B and C, respectively, the ZIP load model for multiple star aggregate loads can be represented by (50)

$$P_{Agg} = P_{0_{Agg}} \left(a_{p_{Agg}} \left(\frac{V}{V_0} \right)^2 + b_{p_{Agg}} \frac{V}{V_0} + c_{p_{Agg}} \right) \tag{50}$$

where

$$P_{Agg} = \sum_{i=1}^{n} P_{A_i} + \sum_{i=1}^{m} P_{B_i} + \sum_{i=1}^{k} P_{C_i} \tag{51}$$

$$P_{0_{Agg}} = \sum_{i=1}^{n} P_{0_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} \tag{52}$$

$$a_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} a_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} a_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} a_{p_{C_i}}}{P_{0_{Agg}}} \tag{53}$$

$$b_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} b_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} b_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} b_{p_{C_i}}}{P_{0_{Agg}}} \tag{54}$$

$$c_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} c_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} c_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} c_{p_{C_i}}}{P_{0_{Agg}}} \tag{55}$$

The aggregate base power is the sum of the base power of each load, and the aggregate ZIP parameters are the weighted averages of the ZIP parameters of each load while the weight are the base power of each load. If all loads have the same ZIP parameters, the aggregate ZIP load model will have the same ZIP parameters regardless of the base power of each load.

H. Aggregate ZIP Load Model for Multiple Delta Loads

In this case, the aggregate ZIP load model for multiple delta-connected loads (three-phase and single delta loads) are studied. The circuit diagram is shown in Fig. 7.

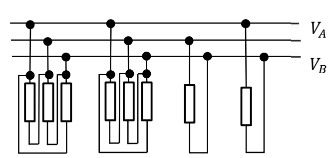


Fig. 7. The circuit diagram of three-phase delta loads.

First, the aggregate ZIP load model can be obtained for every two phases. Suppose that n single delta loads are connected between Phase A and B. Thus, all loads will have the same line-to-line voltage and the general form of load i ($i = 1, 2, \dots, n$) connected to Phase A and B can be described by (56)

$$P_{AB_i} = P_{0_{AB_i}} \left(a_{p_{AB_i}} \left(\frac{|V_{AB}|}{\sqrt{3}V_0} \right)^2 + b_{p_{AB_i}} \frac{|V_{AB}|}{\sqrt{3}V_0} + c_{p_{AB_i}} \right) \tag{56}$$

Then the aggregate ZIP load model can be obtained

$$P_{AB_{Agg}} = P_{0_{AB_{Agg}}} \left(a_{p_{AB_{Agg}}} \left(\frac{|V_{AB}|}{\sqrt{3}V_0} \right)^2 + b_{p_{AB_{Agg}}} \frac{|V_{AB}|}{\sqrt{3}V_0} + c_{p_{AB_{Agg}}} \right) \tag{57}$$

where

$$P_{AB_{Agg}} = \sum_{i=1}^{n} P_{AB_i} \tag{58}$$

$$P_{0_{AB_{Agg}}} = \sum_{i=1}^{n} P_{0_{AB_i}} \tag{59}$$

$$a_{p_{AB_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} a_{p_{AB_i}}}{\sum_{i=1}^{n} P_{0_{AB_i}}} \tag{60}$$

$$b_{p_{AB_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} b_{p_{AB_i}}}{\sum_{i=1}^{n} P_{0_{AB_i}}} \tag{61}$$

$$c_{p_{AB_{Agg}}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} c_{p_{AB_i}}}{\sum_{i=1}^{n} P_{0_{AB_i}}} \tag{62}$$

The aggregate ZIP load model can be derived similarly for the loads connected at phase BC and phase AC loads. Then if we can assume $|V_A| = |V_B| = |V_C| = V$ (consequently $\frac{|V_{AB}|}{\sqrt{3}V_0} = \frac{V}{V_0}$) the aggregate ZIP load model for all three phases can be obtained by summing phase AB, BC, and AC loads. If there are m and k loads connected at phase BC and phase AC, the aggregate ZIP load model for all three-phase can be expressed as

$$P_{Agg} = P_{0_{Agg}} \left(a_{p_{Agg}} \left(\frac{V}{V_0} \right)^2 + b_{p_{Agg}} \frac{V}{V_0} + c_{p_{Agg}} \right) \tag{63}$$

where

$$P_{Agg} = \sum_{i=1}^{n} P_{AB_i} + \sum_{i=1}^{m} P_{BC_i} + \sum_{i=1}^{k} P_{AC_i} \tag{64}$$

$$P_{0_{Agg}} = \sum_{i=1}^{n} P_{0_{AB_i}} + \sum_{i=1}^{m} P_{0_{BC_i}} + \sum_{i=1}^{k} P_{0_{AC_i}} \tag{65}$$

$$a_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} a_{p_{AB_i}} + \sum_{i=1}^{m} P_{0_{BC_i}} a_{p_{BC_i}} + \sum_{i=1}^{k} P_{0_{AC_i}} a_{p_{AC_i}}}{P_{0_{Agg}}} \tag{66}$$

$$b_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} b_{p_{AB_i}} + \sum_{i=1}^{m} P_{0_{BC_i}} b_{p_{BC_i}} + \sum_{i=1}^{k} P_{0_{AC_i}} b_{p_{AC_i}}}{P_{0_{Agg}}} \tag{67}$$

$$c_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{AB_i}} c_{p_{AB_i}} + \sum_{i=1}^{m} P_{0_{BC_i}} c_{p_{BC_i}} + \sum_{i=1}^{k} P_{0_{AC_i}} c_{p_{AC_i}}}{P_{0_{Agg}}} \tag{68}$$

The aggregate base power is the sum of the base power of each load, and the aggregate ZIP parameters are the weighted averages of the ZIP parameters of each load while the weight is the base power of each load. If all loads have the same ZIP parameters, the aggregate ZIP load model will have the same ZIP parameters despite what base power each load has.

I. *Aggregate ZIP Load Model for Multiple Star and Delta Loads*

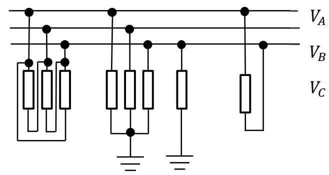


Fig. 8. The circuit diagram of three-phase star and delta loads.

The circuit diagram of star and delta connected loads are shown in Fig. 8. As we can learn from previous derivation from Section G and H, the analytical form of the aggregate ZIP load model can only be obtained if we assume the three-phase voltages are balanced. Based this assumption, we can express the aggregate ZIP load model as

$$P_{Agg} = P_{0_{Agg}}\left(a_{p_{Agg}}\left(\frac{V}{V_0}\right)^2 + b_{p_{Agg}}\frac{V}{V_0} + c_{p_{Agg}}\right) \tag{69}$$

where

$$P_{Agg} = \sum_{i=1}^{n} P_{A_i} + \sum_{i=1}^{m} P_{B_i} + \sum_{i=1}^{k} P_{C_i} + \sum_{i=1}^{n2} P_{AB_i} + \sum_{i=1}^{m2} P_{BC_i} + \sum_{i=1}^{k2} P_{AC_i} \tag{70}$$

$$P_{0_{Agg}} = \sum_{i=1}^{n} P_{0_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} + \sum_{i=1}^{n2} P_{0_{AB_i}} + \sum_{i=1}^{m2} P_{0_{BC_i}} + \sum_{i=1}^{k2} P_{0_{AC_i}} \tag{71}$$

$$a_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} a_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} a_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} a_{p_{C_i}}}{P_{0_{Agg}}} + \frac{\sum_{i=1}^{n2} P_{0_{AB_i}} a_{p_{AB_i}} + \sum_{i=1}^{m2} P_{0_{BC_i}} a_{p_{BC_i}} + \sum_{i=1}^{k2} P_{0_{AC_i}} a_{p_{AC_i}}}{P_{0_{Agg}}} \tag{72}$$

$$b_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} b_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} b_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} b_{p_{C_i}}}{P_{0_{Agg}}} + \frac{\sum_{i=1}^{n2} P_{0_{AB_i}} b_{p_{AB_i}} + \sum_{i=1}^{m2} P_{0_{BC_i}} b_{p_{BC_i}} + \sum_{i=1}^{k2} P_{0_{AC_i}} b_{p_{AC_i}}}{P_{0_{Agg}}} \tag{73}$$

$$c_{p_{Agg}} = \frac{\sum_{i=1}^{n} P_{0_{A_i}} c_{p_{A_i}} + \sum_{i=1}^{m} P_{0_{B_i}} c_{p_{B_i}} + \sum_{i=1}^{k} P_{0_{C_i}} c_{p_{C_i}}}{P_{0_{Agg}}} + \frac{\sum_{i=1}^{n2} P_{0_{AB_i}} c_{p_{AB_i}} + \sum_{i=1}^{m2} P_{0_{BC_i}} c_{p_{BC_i}} + \sum_{i=1}^{k2} P_{0_{AC_i}} c_{p_{AC_i}}}{P_{0_{Agg}}} \tag{74}$$

For the circuit with three-phase balanced voltages, the aggregate ZIP parameters are the weighted averages of the ZIP parameters of each load while the weight is the base power of each load; the weight of each load solely depends on the base power of that load no matter what the connection type (delta or wye) is. If all loads have the same ZIP parameters, the aggregate ZIP load model will have the same ZIP parameters despite what base power each load has.

J. *Aggregate ZIP Load Model for Reactive Power*

Since Equation (1) and (2) are completely analogous, all the previous discussions regarding ZIP load model for real power are applicable to ZIP model for reactive power.

Special attention is given to reactive power compensating devices as follows, when calculating the aggregate ZIP for a load together with the reactive power compensating device.

1) *Reactive Power Compensating Device Modeled as Constant Q*

In this scenario, the reactive compensating device is modeled as a constant Q source. An example will be a power electronics based reactive power compensator that is capable of regulating its reactive power output.

Suppose there is a single load connected between phase B and neutral; the load has base power $Q_{0_1} = 100\text{kVar}$, ZIP parameter $a_{q_1} = 0.2$, $b_{q_1} = 0.5$, and $c_{q_1} = 0.3$. A constant Q source is connected in parallel to the load with $Q_{0_2} = -30$ kVar, $a_{q_2} = 0$, $b_{q_2} = 0.0$, and $c_{q_2} = 1.0$. The aggregate ZIP parameters are found to be $a_q = 0.2857$, $b_q = 0.7143$, and $c_q = 0$. If $Q_{0_2} = -120$ kVar, $a_{q_2} = 0.0$, $b_{q_2} = 0.0$, and $c_{q_2} = 1.0$, The aggregate ZIP parameters are found to be $a_q = -1$, $b_q = -2.5$, and $c_q = 4.5$.

The examples manifest that the aggregate ZIP parameters can be negative for reactive power when there is a large capacitor bank connected with loads.

2) *Reactive Power Compensating Device Modeled as Constant Impedance*

In this scenario, the reactive compensating device is modeled as a constant impedance load. An example will be a regular capacitor bank, whose reactive power is proportional to the square of the terminal voltage.

Suppose there is a single load connected between phase B and neutral. Suppose the load has base power $Q_{0_1} = 100\text{kVar}$, ZIP parameter $a_{q_1} = 0.2$, $b_{q_1} = 0.5$, and $c_{q_1} = 0.3$. A Q source is connected in parallel to the load with $Q_{0_2} = -30$ kVar, $a_{q_2} = 1.0$, $b_{q_2} = 0.0$, and $c_{q_2} = 0.0$. The aggregate ZIP parameters are found to be: $a_q = 0.2857$, $b_q = 0.7143$, and $c_q = 0$. If $Q_{02} = -120$ kVar, $a_{q2} = 1.0$, $b_{q2} = 0.0$, and $c_{q2} = 0.0$, The combined ZIP parameters are found to be: $a_q = 5$, $b_q = 2.5$, and $c_q = -1.5$.

K. *Aggregate ZIP Load Model for a Load Combined with Real Power Source*

This section discusses calculation for the aggregate ZIP parameters for a load together with the real power injecting device such as a PV generator. The real power injecting device is modeled as a constant P.

Suppose there is a single load connected between phase B and neutral. Suppose the load has base power $P_{01} = 100\text{kW}$, ZIP parameter $a_{p1} = 0.2$, $b_{p1} = 0.5$, and $c_{p1} = 0.3$. A constant P source is connected in parallel to the load with $P_{02} = -40\ kW$, $a_{p2} = 0.0$, $b_{p2} = 0.0$, and $c_{p2} = 1.0$. The combined ZIP parameters are found to be: $a_p = 0.3333$, $b_p = 0.8333$, and $c_p = -0.1666$. If the real power source has $P_{02} = -120\ kW$, $a_{p2} = 0.0$, $b_{p2} = 0.0$, and $c_{p2} = 1.0$. The combined ZIP parameters are found to be: $a_p = -1$, $b_p = -2.5$, and $c_p = 4.5$.

IV. CASE STUDIES

We choose the IEEE 13 and 34 bus system for the case studies. The configuration of the 13-bus system is displayed in Fig. 9.

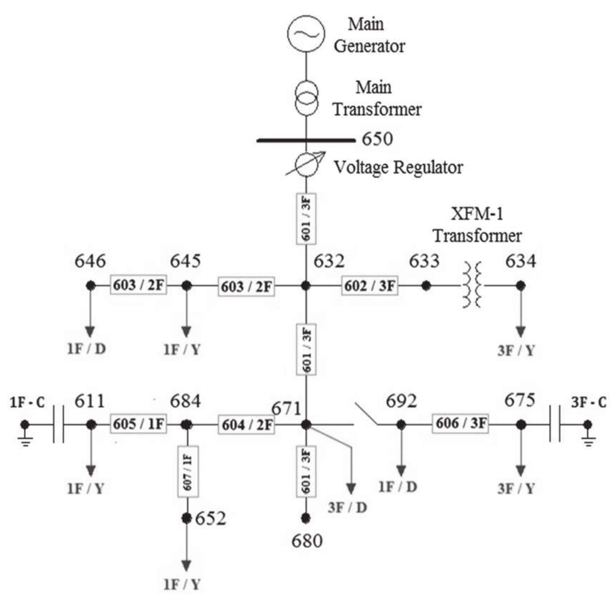


Fig. 9. Configuration of IEEE 13-bus test feeder. [20]

The ZIP parameters of loads are predefined in the OpenDSS simulation program. Thus the estimation performance can not only be assessed by the estimation error for the power but also be evaluated by comparing the estimated ZIP parameters with the preset ZIP parameters.

The measurements data used in estimation are generated using OpenDSS Simulation. The base power P_0 and Q_0 are changing hourly while the ZIP load model coefficients (a, b, c) are kept constant. The window size of each estimation n is set to 4. The measurements are taken every 15 minutes for a day.

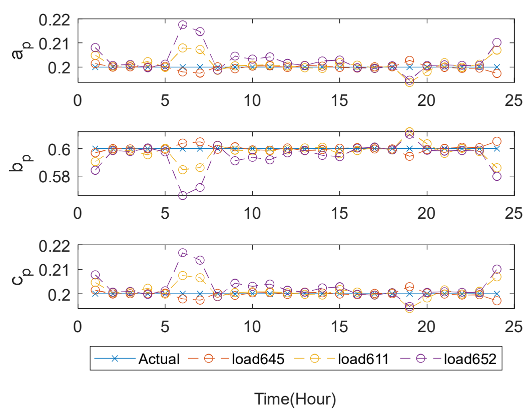
Some cases will only show the estimation results for the ZIP load model for P since the results for Q are similar.

A. IEEE 13-Bus Test Feeder

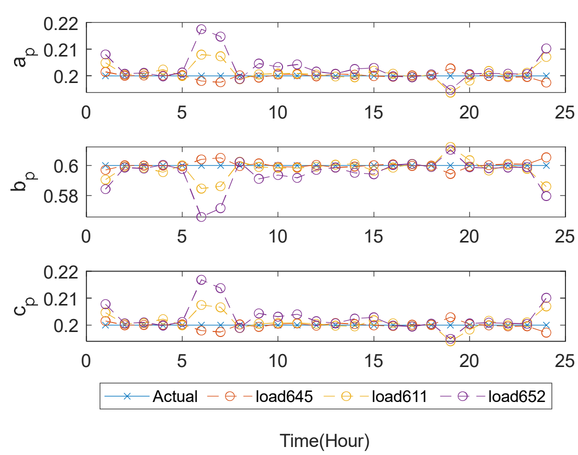
1) Single Load

a) Single Star Load

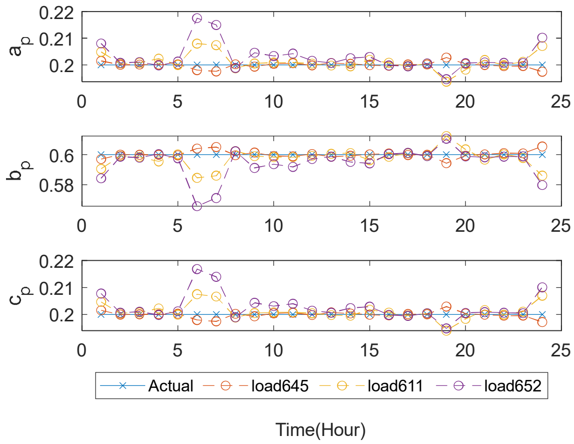
In this case, the single star loads in the IEEE 13-bus system, i.e., loads at bus 645, 611, 652, which are connected between one phase and neutral, are studied. The estimated ZIP parameters for real power using the three different methods are shown in Fig. 10. We can see that all three methods can estimate the ZIP parameters accurately. Based on the estimated ZIP parameters, the estimated power is computed. The estimated power is then compared with the measured power to compute the absolute percentage error. The errors are also shown in Fig. 10. The estimation results for reactive power are similar and thus are not shown here.



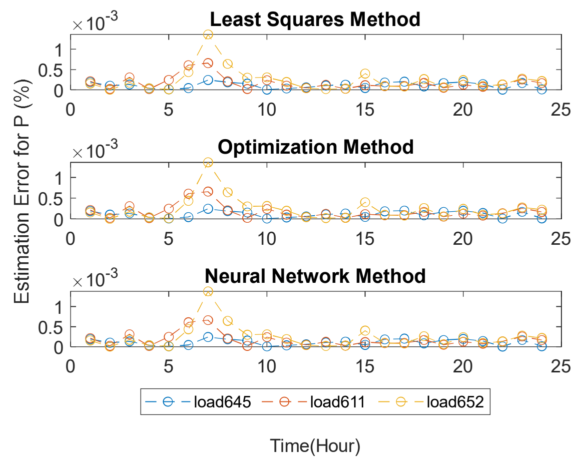
(a) Least squares method



(b) Optimization method



(c) Neural network method

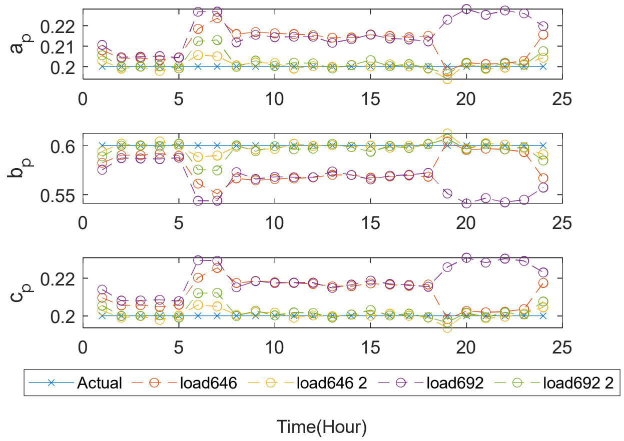


(d) Estimation error

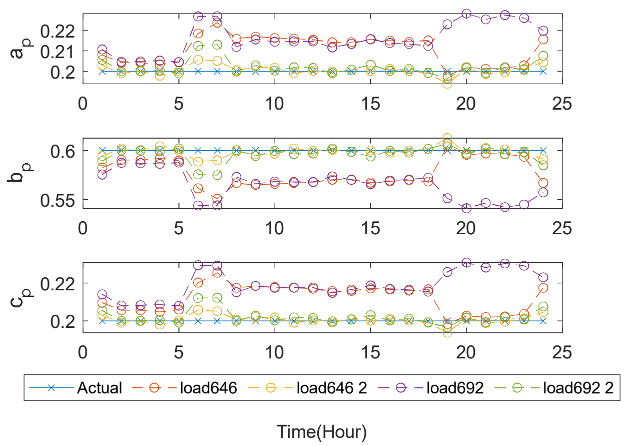
Fig. 10. ZIP load model estimation results for the real power of the single star loads.

b) Single Delta Load

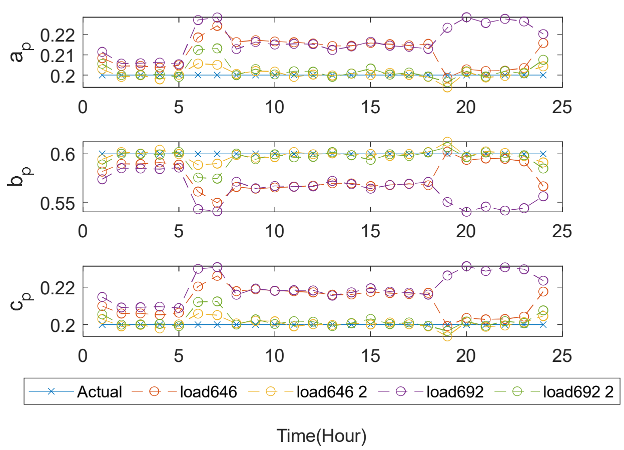
In this case, the single delta loads connected between 2 different cases are studied. They are the loads at bus 646 and 692. In Fig. 11, lines “load 646” and “load 692” are estimated using the average voltage of the 2 phases while “load 646 2” and “load 692 2” are estimated using the line-to-line voltage. From the figure, we can see that the estimation results using line-to-line voltage are more accurate than using the average phase voltage.



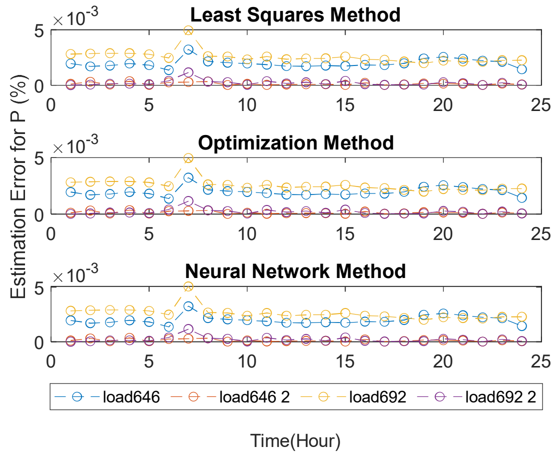
(a) Least squares method



(b) Optimization method



(c) Neural network method



(d) Estimation error

Fig. 11. ZIP load model estimation results for the real power of the phase-to-phase loads.

2) *Aggregate Load*

a) *Three-Phase Star Load*

In this case, the ZIP load model for the three-phase star loads at bus 634, 670, and 675 are estimated using the average phase voltage and the sum of three-phase power.

The base real and reactive power of all three loads at each phase are shown in TABLE XIV. and TABLE XV. respectively. Among the three three-phase star loads, load 634 is the most balanced load while load 670 is most unbalanced load.

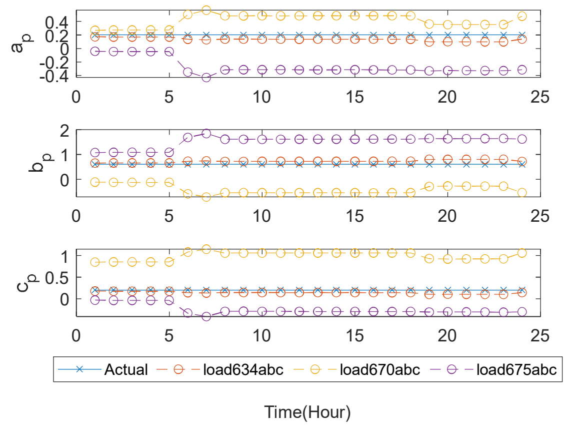
TABLE XIV. THE BASE REAL POWER OF THE THREE-PHASE STAR LOADS OF IEEE 13-BUS SYSTEM

Load	Phase A $P_0(kW)$	Phase B $P_0(kW)$	Phase C $P_0(kW)$
634	160	120	120
670	17	66	117
675	485	68	290

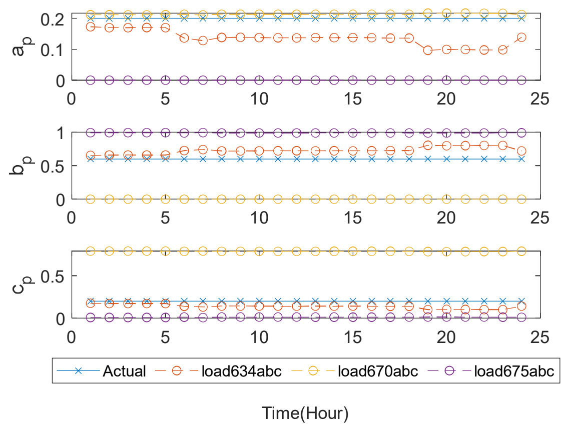
TABLE XV. THE BASE REACTIVE POWER OF THE THREE-PHASE STAR LOADS OF IEEE 13-BUS SYSTEM

Load	Phase A $Q_0\ (kVar)$	Phase B $Q_0\ (kVar)$	Phase C $Q_0\ (kVar)$
634	110	90	90
670	10	38	68
675	190	60	212

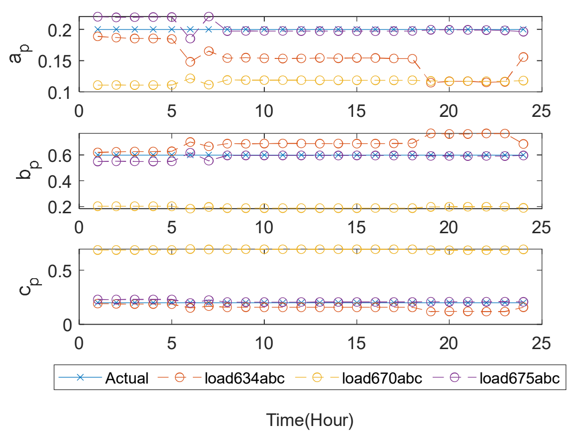
The estimation results using the three methods are shown in Fig. 12. As expected, we can see that the estimated results differ from the calculated results due to load unbalanced. For bus 634, the estimated results are close to the calculated results, as it has the smallest load unbalance.



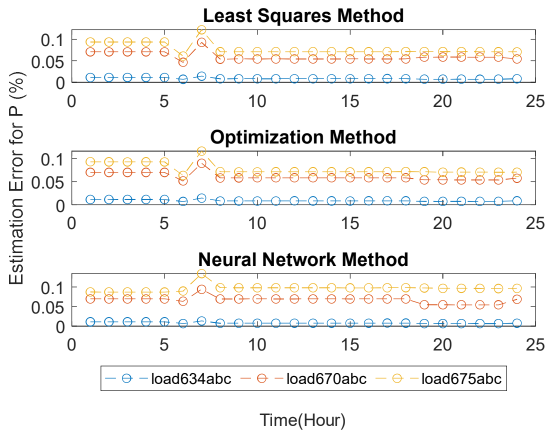
(a) Least squares method



(b) Optimization method



(c) Neural network method

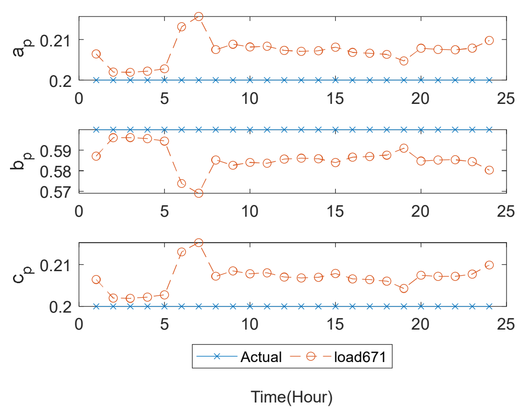


(d) Estimation error

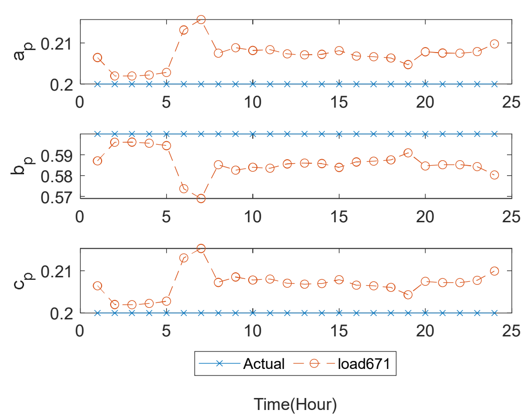
Fig. 12. ZIP load model estimation results for the real power of the three-phase Y-connected loads.

b) Three-Phase Delta Load

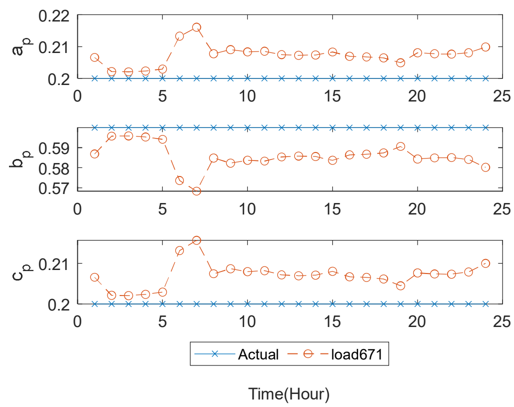
In this case, the load at bus 671, which is the only three-phase Δ-connected load in the IEEE 13-bus system, is studied. The estimation results are shown in Fig. 13. The actual ZIP parameters are computed using (30). All the three methods can estimate the aggregate ZIP parameters accurately.



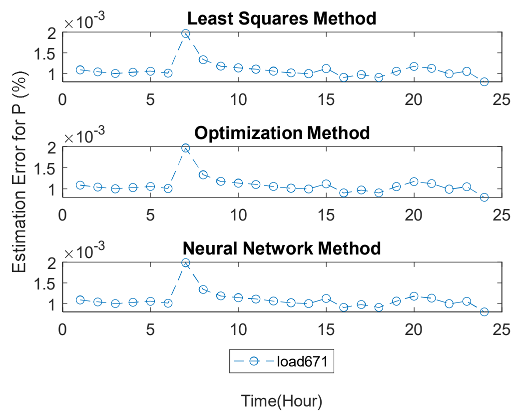
(a) Least squares method



(b) Optimization method



(c) Neural network method



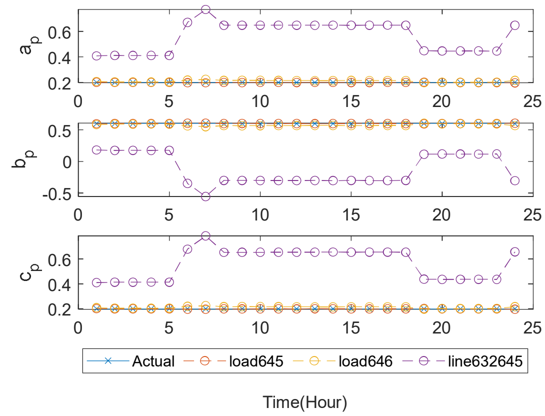
(d) Estimation error

Fig. 13. ZIP load model estimation results for the real power of the three-phase Δ-connected loads.

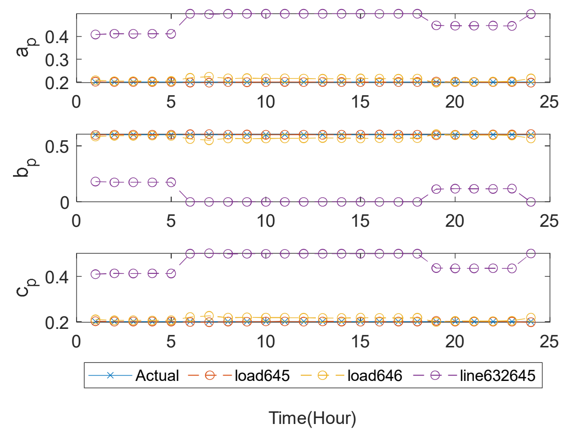
c) Multiple Loads 1

In this case, the aggregate load models for the loads at bus 645 and 646 are studied. Load 645 is a single star load, which is connected between phase B and neutral, while load 646 is single delta load, which is connected between phase B and C. The aggregate load is estimated using the average voltage of phase B and C at bus 632 and the power from bus 632 to 645. The (a_p, b_p, c_p) for 645 and 646 are both set to $(0.2, 0.6, 0.2)$, with their P_0 set to $170kW$ and $230kW$.

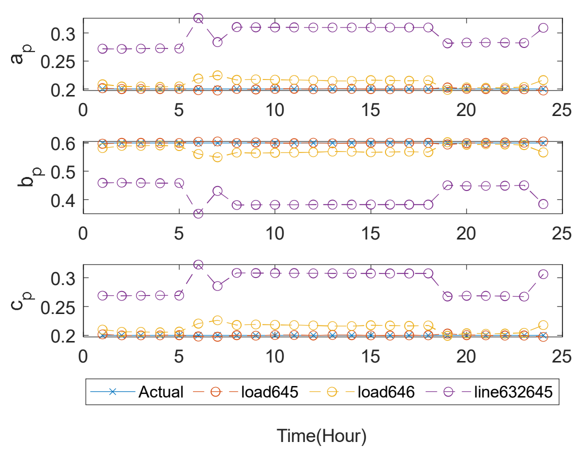
The estimation results are shown in Fig. 14. From the results, we can see that the estimation for each load is accurate while the estimated aggregate ZIP parameters deviate from our expectation. The reason is that this aggregate load consists of both phase-to-neutral and phase-to-phase loads, which means the ZIP load model for the first load is a function of phase B and C voltage while the second load only depends on phase B. Thus using the phase B and C average voltage as the voltage for the aggregate ZIP load model is inappropriate. But we also cannot choose only phase B voltage as the voltage for the aggregate ZIP load model as the load at bus 646 is also dependent on phase C.



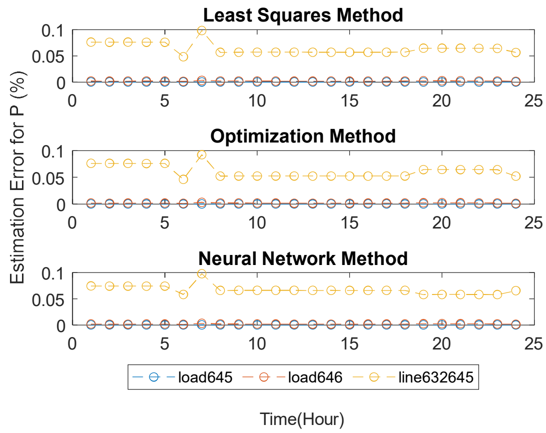
(a) Least squares method



(b) Optimization method



(c) Neural network method

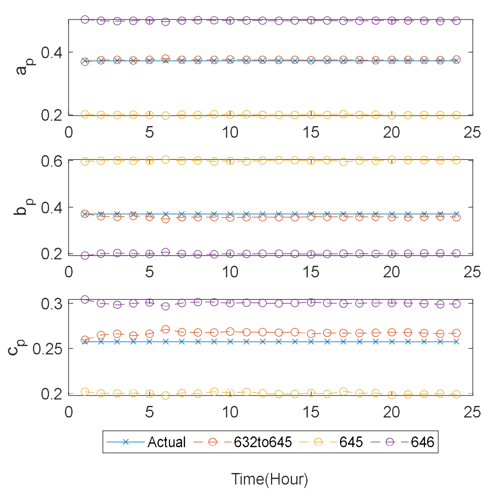


(d) Estimation error

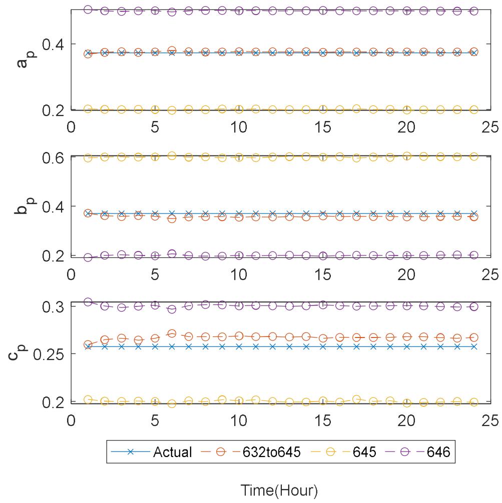
Fig. 14. ZIP load model estimation results for the real power of the Aggregate Load .

d) Multiple Loads 2

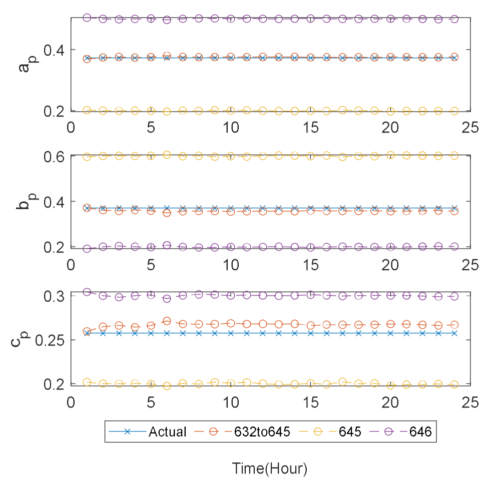
In this case, we changed the load at 646 from phase-to-phase (BC) load to phase-B-to-neutral load. The purpose of this case study is to demonstrate that the aggregate load can be estimated more accurately when they are all phase-to-neutral connected especially at the same phase. The load at bus 645 is also a phase-B-to-neutral load. The (a_p, b_p, c_p) for 645 and 646 are set to $(0.2, 0.6, 0.2)$ and $(0.5, 0.2, 0.3)$, respectively. The loads have their P_0 set to 170 kW and 230 kW, respectively. For the easiness of demonstration, both P_0 will change hourly but according to the same set of multipliers. In this way, the aggregate (a_p, b_p, c_p) can remain at $(0.3725, 0.37, 0.2575)$. However, if P_{0_1} and P_{0_2} vary based on different multipliers, then the aggregate (a_p, b_p, c_p) will not be static. The estimation results are shown in Fig. 15. As expected, the aggregate ZIP parameters as well as the ZIP parameters of each load can be estimated accurately. The estimation results for each load are based on the voltage and power measured at each load. The aggregate ZIP parameters are estimated using the phase B power flowing from bus 632 to bus 645 and the phase B voltage at bus 632.



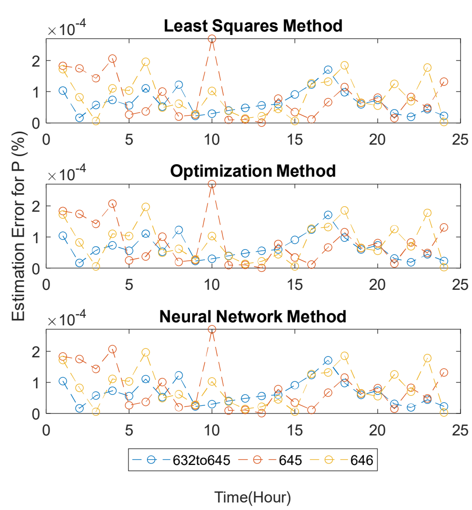
(a) Least squares method



(b) Optimization method



(c) Neural network method



(d) Estimation error

Fig. 15. ZIP load model estimation results for the real power of the case aggregate load.

B. IEEE 34-Bus Test Feeder

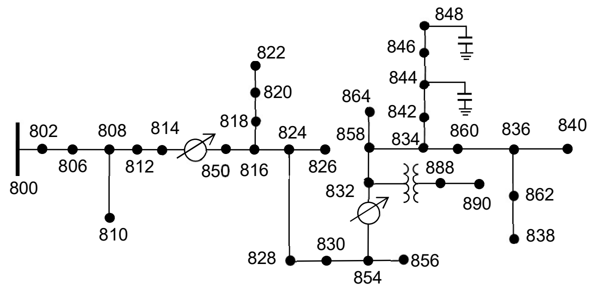


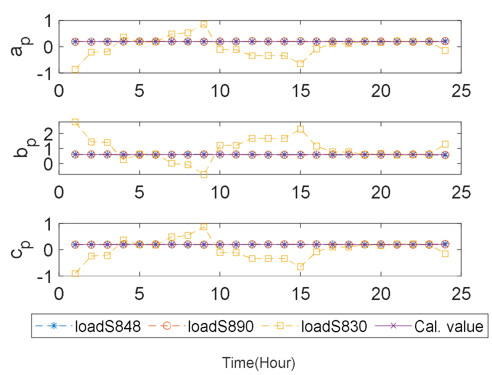
Fig. 16. Configuration of IEEE 34-bus test feeder.

The case studies are also performed based on the IEEE 34-bus system [21]. The distributed load along the lines in the system are modeled as a load located at the middle of those lines. Since there is just one three-phase delta load in IEEE 13-bus system, the ZIP parameter estimation for the three-phase delta load are also studied in the IEEE 34-bus system. The configuration of the IEEE 34-bus system is shown in Fig. 16.

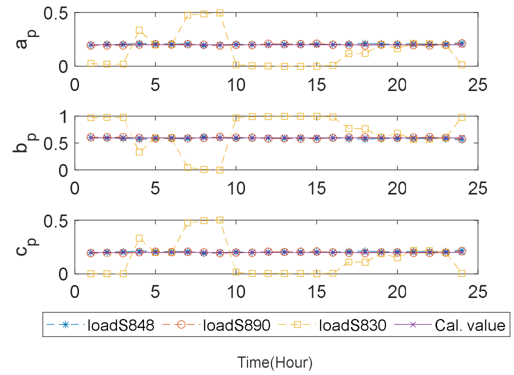
1) Aggregate Load

a) Three-Phase Delta Load

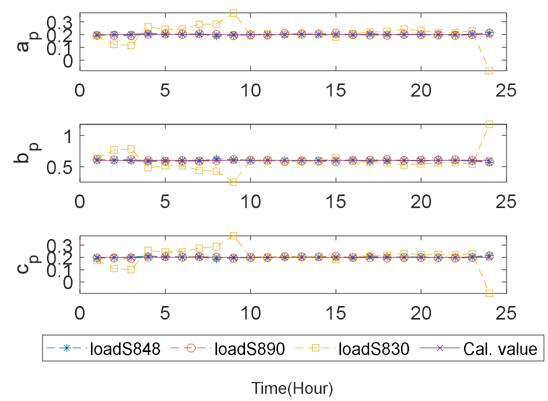
The ZIP parameter estimation for the three-phase delta loads in the 34-bus system, i.e., load 848, 890, and 830, is studied in this section. The ZIP parameter estimation results for real power are shown in Fig. 17. The estimation results for reactive power are similar and thus are not shown. The ZIP parameters of load 848 and 890 can be estimated accurately while the results for load S830 deviate from the theoretical calculated ZIP parameters. The average voltage magnitude unbalance for load 848, 890, and 830 in the 24 hours are 1.66%, 1.54%, and 1.64%. Voltage unbalance is not the only reason for the deviation of estimated ZIP parameters. The other reason is the load unbalance. Among the three loads, loads 848 and 890 are balanced while load 830 is unbalanced as shown in TABLE XVI. and TABLE XVII. The results are consistent with our findings in Section III.C.



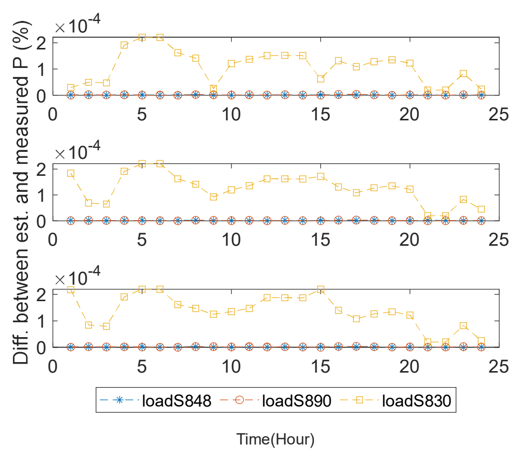
(a) Least squares method



(b) Optimization method



(c) Neural network method



(d) Estimation error

Fig. 17. ZIP load model estimation results for the real power of load 848, 890, and 890.

TABLE XVI. THE BASE REAL POWER OF THE THREE-PHASE DELTA LOADS OF IEEE 34-BUS SYSTEM

Load	Phase A $P_0(kW)$	Phase B $P_0(kW)$	Phase C $P_0(kW)$
848	20	20	20
890	150	150	150
830	10	10	25

TABLE XVII. THE BASE REACTIVE POWER OF THE THREE-PHASE DELTA LOADS OF IEEE 34-BUS SYSTEM

Load	Phase A $Q_0\ (kVar)$	Phase B $Q_0\ (kVar)$	Phase C $Q_0\ (kVar)$
848	16	16	16
890	75	75	75
830	5	5	10

C. CVR Study Based on Field Data

In this section, the ZIP parameters and CVR factor are estimated using optimization method based on field data provided by Louisville Gas & Electric and Kentucky Utilities. Field captured three-phase voltages as well as real and reactive power, with a temporal resolution of 5 minutes, was provided for a 2-day period in February 2021. The data was collected from a representative urban circuit of moderate length in Louisville, KY, with approximately 260 customers (95% residential, 5% commercial). ZIP parameters are estimated for every 3 data points resulting in time-varying ZIP estimates over the two-day span. The mean ZIP parameters are then computed as the average of the individual ZIP parameters during the 48 hours. The mean ZIP parameters were found to be $a_p = 0.3948$, $b_p = 0.0660$, and $C_p = 0.5392$ for the circuit studied.

Then, based on the individual ZIP parameters and the mean ZIP parameters, the estimated power is computed. The estimated power is compared with the actual field-measured power to calculate the mean absolute percentage error (MAPE). The estimated energy is calculated by summing the estimated power. It is compared to the actual measured energy (calculated by summing the measured power). The results are shown in TABLE XVIII.

TABLE XVIII. MAPE FOR POWER AND ERROR FOR ENERGY

	MAPE for power (%)	***Error for energy (%)***
Using individual ZIP	1.10	7.50×10^{-4}
Using mean ZIP	3.44	0.14

To estimate a CVR factor for this feeder, the feeder head voltage is reduced by a specific percentage, and the real power is estimated. The estimated power under 1%, 2%, and 3% voltage reduction is computed based on the estimated individual and mean ZIP parameters as shown in Fig. 18 and Fig. 19.

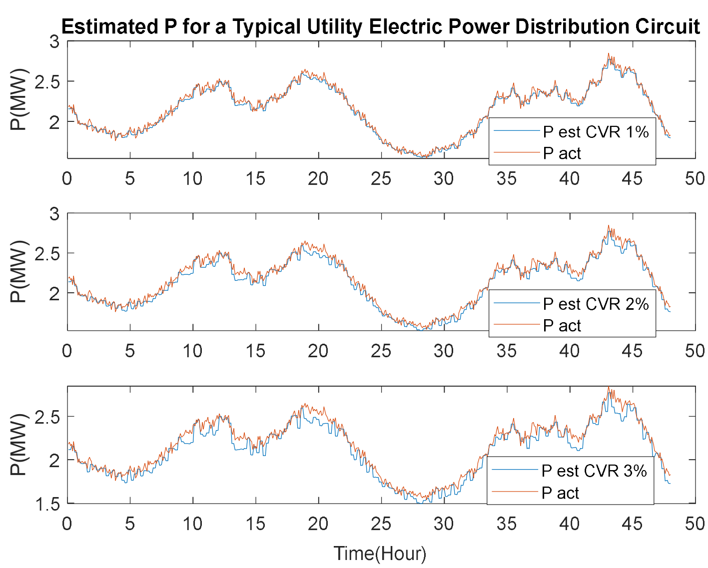


Fig. 18. Estimated power based on the individual ZIP parameters under different levels of voltage reduction.

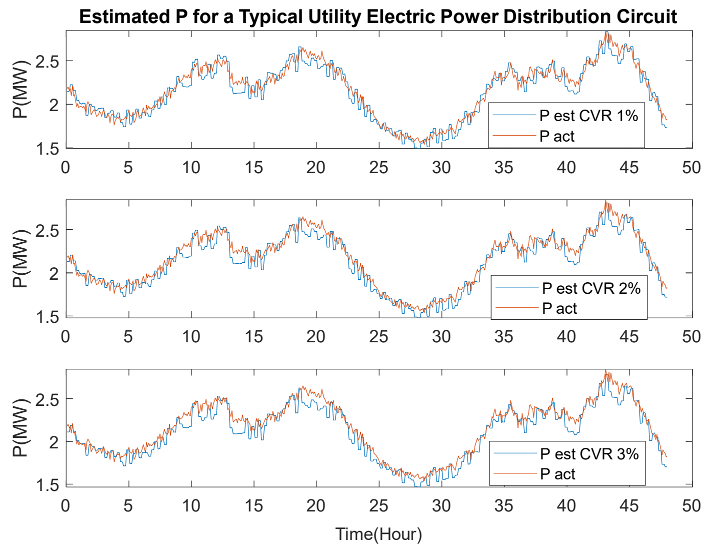


Fig. 19. Estimated power based on the mean ZIP parameters under different levels of voltage reduction.

The CVR factor is then calculated based on

$$\text{CVR factor} = \frac{\%Energy\ Consumption\ Reduction}{\%Voltage\ Reduction} \tag{75}$$

The computed CVR factors under different levels of voltage reduction are shown in TABLE XIX. .

TABLE XIX. CVR FACTORS UNDER DIFFERENT LEVELS OF VOLTAGE REDUCTION

	1% voltage reduction	***2% voltage reduction***	***3% voltage reduction***
Using individual ZIP	0.8616	0.8577	0.8537
Using mean ZIP	0.7544	0.8185	0.8371

V. CONCLUSION

This paper provides a comprehensive theoretical formulation for aggregate ZIP load model. The effects of load combination and voltage unbalance on ZIP parameter estimation are presented. General estimation accuracy for different load characteristics with voltage unbalance using total power and average voltage of relevant phases to estimate aggregate ZIP parameters is described in TABLE XX. The three estimation methods, i.e., least squares, optimization and neural network methods, exhibit similar estimation accuracy based on the case studies. The CVR factor of a feeder can be calculated through simulation studies based on the estimated ZIP parameters by computing consumed power while varying voltage magnitude.

TABLE XX. EFFECT OF LOAD COMBINATION AND VOLTAGE UNBALANCE ON ZIP PARAMETER ESTIMATION

Quantities for different phases	Balanced Voltages	Unbalanced Voltages
Same P_0, Same ZIP	Accurate	Small errors
Same P_0, Different ZIP	Accurate	Large errors
Different P_0, Same ZIP	Accurate	Large errors
Different P_0, Different ZIP	Accurate	Largest errors

ACKNOWLEDGEMENT

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