UNIVERSITY OF TURIN

Department of Philosophy and Education Sciences
Master of Arts in Philosophy

FINAL DISSERTATION

The normativity of logic in a psychologistic framework

Three approaches

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Academic Year 2020/2021

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Abstract

Contemporary psychologism has been amended for most of the objections by its opponents over a century ago. However, some authors still raise doubts about its ability to account for some peculiar properties of logic. In particular, it is argued that the psychological universality of patterns of inferential behavior is not sufficient to account for the normativity of logic. In this paper, I deal with the issue and offer three alternative solutions that do not rely on mere empirical universality. I will use the works of Laurence Jonathan Cohen, Diego Marconi and Marcello D'Agostino, adapting them for the purpose of defending logical psychologism. I will therefore argue that, although more refined work on the subject is needed, contemporary psychologism has the key resources to retain its place in the philosophical debate on the foundations of logic.

1. Introduction

1.1. Three approaches to "save" psychologism

Since its baptism, the term "psychologism" assumed a negative and pejorative connotation, a characteristic that can still be ascribed to it nowadays (see Notturno 1985, pp. 10-19). Particularly, *logical psychologism* (LP, henceforth) has been subjected to many critiques over time, starting with those advanced by the most famous anti-psychologists, Frege (1884) and Husserl (1900). Nonetheless, thanks to the emergence of cognitive sciences[1] with their *universalistic* aims, the new variants of psychologism seem to have solved most of their theoretical problems[2]. Not all of them, though. For example, Pelletier, Elio and Hanson (2008) argue that they are still unable to account for the normativity of logical laws:

> *Universality is not normativity*: Even if everyone agreed with one another that would be merely a description and not an explanation of their normativity. A description of the views of everyone cannot justify why we all ought to have these views concerning truth and validity. There can be no justification for the claim that anyone should reason in the way s/he does — it is merely a description of how everyone reasons. There can be no notion of good or bad reasoning (pp. 26-27).

It is not hard to see the reason why LP fails to fulfil the normativity task according to these authors: regardless of the universality of its findings, psychology is a *descriptive* science whereas logic is a *normative* discipline. Since Hume's times (1739-40), the famous *is-ought* divide is part of the philosophical orthodoxy: it is not

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[1] See Thagard 2020.

[2] A list of the issues concerning LP have been systematized in a few taxonomies by Pelletier et al. 2008, Kusch 1995, pp. 58-61, and Jacquette 2003, pp. 9-16.

possible to derive what *ought* to be from what *is*, in our case, (logical) *norms* from (psychological) *descriptions*.

In the present work, by contrast, I will argue that LP theories *are* able to account for the normativity of logic. I aim to show that Pelletier and colleagues' objection, while pertinent, is not conclusive: the universality of inferential patterns of behavior does not represent a unique possibility for an LP advocate to achieve her goal. To show it, I will expose three different ways to "bridge" the descriptive side proper of psychology explanations with the normative side of logical laws. This will be possible thanks to additional assumptions, either with an epistemological or axiological nature, that will make possible such a connection avoiding the *naturalistic fallacy*.

To start, in the second section, I will expose the theory of Laurence Jonathan Cohen, a ultra-rationalist British philosopher who puts human intuitions as the basis for every possible logical norm: as a result whatever is the set of rules we choose as a normative standard to guide our reasoning, it is one and the same with the set of rules composing the human *logical competence* to be described by psychology. Here we do not need to link anything: the switch from norms to descriptions is just a choice of perspective in studying intuitions. In section three, I will expose instead an original proposal taking inspiration from a few works of Diego Marconi, who followed in turn the late Wittgenstein for what concerns the normativity of meaning. I will take advantage of his intuitions to transpose them, *mutatis mutandis*, onto the laws of logic. In this approach the role of the community will become prominent: logic, as well as language, is first and foremost a communication tool, so there is a need to standardize such a tool to make communication effective. The *principle of conformism* will therefore turn what would be otherwise simple (psychological) widespread patterns of behavior into real norms. Finally, in the fourth section, I will transpose into psychologistic terms the work of Marcello D'Agostino, who, aiming to make logical prescriptive models more appropriate to realistic agents, provides a semantics (accompanied by a natural deduction system) for so-called *k*-depth logics (logics at increasing depths). The underlying idea is that even rationality among resource-bounded agents comes in degrees. The normativity of logic is then postulated as an *optimum* to direct every possible rational agent and not only the actual ones.

1.2. Background assumptions

Before entering the discussion, however, a few remarks are crucial. First, since it is not possible to find a common definition in literature (Kusch 1995, pp. 4-5), I shall

provide a maximally general idea of what we are facing in talking of logical psychologism. I will readapt the definition offered by Pelletier, Elio and Hanson in their *Is Logic all in our Heads? From Naturalism to Psychologism* (2008)[3] as follows:

> *(LP): logical psychologism means that some important aspects of logic rely upon, or are constituted by, facts and issues of human psychology.*

As it is easy to figure out, LP must be understood today as the common thread of different theories for the foundation of logic rather than a unique general doctrine[4]. In the present work, however, I will *not* make any attempt to pair specific LP theories (see footnote 4) with the three approaches to the normativity issue I am going to treat. The sole issue I will deal with is the possibility to assume norms for (logical) reasoning starting from inferential patterns of behavior shaped by our cognitive faculties. For the same reason, I will *not* engage with the choice of the best or the most suitable *theory of reasoning* among the many available in the landscape of today's psychology. This does not exclude the possibility that each of the three approaches I am going to outline below could be more appropriately suited by a specific LP kind, which in turn would be better founded if based on a specific theory of reasoning. Yet, this is not one among the purposes of this brief essay. There will be exceptions only where I think it could be useful, but even these must be considered little more than suggestions; the ways to characterize each of these approaches are many and still to be explored.

A second necessary specification regards the normativity of logic in itself. A huge amount of literature treats the topic (see Steinberger 2020)[5]. The normativity of logic aims to constrain – at least *prima facie* – reasoning, characterizing so *theoretical* phenomena more than practical ones. Logic is widely considered to be part of so-called *Standard Picture of Rationality* (SPR)[6], jointly with the theory of probability and the expected-utility theory, the group of theories that are supposed to govern reasoning in

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[3] "[…] psychologism, in any field […], means that some important aspects of the realm of study […] rely upon, or are constituted by, facts and issues of human psychology" (Pelletier et al., 2008, p. 2).

[4] Following again Pelletier and colleagues, it is possible to subsume all the contemporary attempts to found logic on psychology under three main approaches: (PD) *Psychological Descriptivism*: Identify logic with some description of the observable performance of people's behavior in the realm; (CA) *Cognitive Architectures*: Argue that whatever might be discovered about logic is a function of the human cognitive machinery that is doing the discovering, and that this machinery itself has been shaped by the world in which it must operate; (IC) *The Ideal Cognizer*: Identify logic with what an ideal cognizer would cognize about it (Pelletier et Al., 2008, pp. 7-8).

[5] Preliminarily, it might be useful to briefly define the terms "norm" and "normativity": a *norm* is a standard that determines a partition between what is correct and what is not. This distinction is intrinsically connected with the epistemological notion of justification. Moreover, norms aim to represent a guide for rational agents' choices, actions and reasoning. *Normativity* is then the property of something (a habit, a convention, a rule, a method, a system, etc.) to constitute a standard for the behavior of rational agents (see Lalumera 2010, pp. 22-24).

[6] The set of theories has been labelled with this name by Edward Stein (1996).

a rational way. A whole series of additional issues could be raised up to better circumscribe the notion. For example: is logic normative only for reasoning, for the thought as a whole or for practical activities as well? Is it only normative or also constitutive for reasoning? Moreover: there are many formal *logics* today. Which one should be taken as the normative standard among them? Does psychologism conduct us to choose one among the many?

In the rest of the text I will *not* deal with these issues and I will *not* take any specific position on them. In fact, even though a few among the aforementioned positions could fit better specific kinds of psychologism, none of them is in principle hostile to LP. Particularly, for what concerns the choice among different formal systems I consider that there is some space for a logical pluralism[7] in each of the proposed approaches. The set of inferential rules that can be caught starting from our patterns of behavior can be drawn by means of many different epistemological *desiderata*. In reality, it is not necessary at all for the purpose that a supporter of psychologism carries on, to choose a formal system: it could be well possible to consider an *open set* of inferential rules as the best normative standard possible for human reasoning.

Now, with these specifications as a background for the entire argumentation, in the next three sections we will enter one by one each of the approaches I outlined above.

2. *Cohen's argument for normative standards*

One first possible way to solve the gap between psychology descriptions of humans' mental processes and the normative role of logic consists in denying the very existence of such a gap. It is what Laurence Jonathan Cohen has done, particularly in his 1981's *Can Human Irrationality Be Experimentally Demonstrated?*, where he takes sides against a substantial tradition in cognitive psychology, according to which the observation of systematic errors in reasoning tasks is a clue to a certain amount of human irrationality. Taking inspiration from the works by Nelson Goodman (1954) and Noam Chomsky (1965), the British philosopher elaborates a sophisticated but controversial theory.

Cohen (1981) is divided in two main parts, the first establishing an original theory concerning the role of human *intuitions* in justifying rational norms for

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[7] See Priest 2006.

everyday reasoning, the second being a review of many cognitive psychology findings with the aim of denying the possibility to experimentally demonstrate any form of *human irrationality*. For the purpose of the present work, the relevant issues emerge in the first section of Cohen's paper, on which we will focus. Some remarks on the second part will also be presented to the extent that they can impact on LP defense.

Cohen critically addresses certain experimental studies of human rationality (especially within the fields of deductive and probabilistic reasoning), that rely on well-defined normative standards as a benchmark for reasoning. It is widely supported that *classical logic* and the *theory of probability,* respectively, are the relevant standards in such fields (see Stein 1996). Cohen's critique relies on two main thesis, which we shall consider thoroughly (Cohen 1981, p. 317):

> (I) People's intuitions are the basis of normative criteria for the evaluation of reasoning;
> (¬FC) No faulty competence can be ascribed to humans (by means of any experimental result).

2.1. Intuitions as the basis for reasoning norms

In this first section, I am offering my reconstruction of Cohen's argument for (I). The structure of the argument can be outlined as follows:

$$(GT \land IJ) \to I$$
$$GT$$
$$\underline{IJ}$$
$$I$$

Cohen's starting point is a series of assumptions in favor of (I), two positive claims and a negative one, that can easily be enumerated:

> (GT): Goodman's thesis;
> (IJ): Intuitions underlie judgements;
> (¬A): There are not alternative accounts of normativity.

Let's briefly enter each of them.

[GT] - The core of the issue is here the *justification* of inference rules. Nelson Goodman is credited by Cohen as the first one having the insight of a connection between inference rules and people's judgements (1981, p. 317). In a few widely influential pages from *Fact, Fiction and Forecast* (1954), Goodman was questioning about the justification of deductive rules: while we justify an argument in conformity with general rules of deduction, how do we justify the validity of the rules themselves?

Principle of deductive inference are justified by their conformity with accepted deductive practice. Their validity depends upon accordance with the particular deductive inferences we actually make and sanction. If a rule yields inacceptable inferences, we drop it as invalid. Justification of general rules thus derives from judgements rejecting or accepting particular deductive inferences (p. 66).

Here is what Goodman calls a *virtuous circle*: rules and inferences "are justified by being brought into agreement with each other. A rule is amended if it yields an inference we are unwilling to accept; an inference is rejected if it violates a rule we are unwilling to amend" (p.67). Cohen takes this statement as a starting point but adds a few remarks that seem quite important for an LP account of logic.

[IJ] - In the first place, he redefines the term *judgement* as used by Goodman in connection with the term *intuition* and explains: "an intuition that *p* is here just an immediate and untutored inclination, without evidence of inference, to judge that *p*" (Cohen 1981, p. 318). Notice that there is no mention here of superior cognitive abilities or to introspection. What Cohen seems to be referring to is a *psychological disposition* to a certain mental behavior (judging that *p*, in this case).

[¬A] – In Cohen's view, (I) receives further support observing the alternative accounts to justify deductive rules. In his opinion, there are no other ways than that of intuitions to make the work. He considers the only two accounts assumed as competitors and raises a few objections that he regards as decisive for their failure. They are the *empirical-inductive* strategy by Stich (1975) and any *metamathematical* account that aims to justify rules in formal terms[8]. In my reconstruction of Cohen's argument, however, I do not consider ¬*A* among the premises to derive (I) for two reasons: being a negative statement it is more appropriate to see it as a clue for (I) rather than a real premise; moreover, an LP defender could accept (I) even when other accounts to justify deductive rules were available. It is not a necessary assumption for a psychologistic account of logic.

[I] – From the conjunction of (GT) and (IJ), (I) is meant to follow and our normative standard must be defined from our intuitions about deductive and inductive rules. Clearly, once we picked up a sufficient amount of intuitions, the resulting collection calls for a systematization before it acquires a normative status. How to do

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[8] The first strategy, said *empirical-inductive,* is exemplified with the work of Stich (1975),who offers, in the wake of Quine, an account of logic treated as a part of a whole, namely science in general. This strategy fails, according to Cohen, due to the several problems affecting every *holistic system* of the kind. The second considered strategy is so-called *metamathematical* account: Cohen considers here "any system in which rules of derivation are specified in formal terms". In fact, Cohen complains the absence of good reasons to suppose that logical particles of natural language can be mapped onto those of the formal system. He refers one among the classical troubles of material implication, which conducts to several paradoxes as a cause for his dissatisfaction (see 1981, pp. 318-319)

that? Unlike other kind of hunches, "on indisputably normative issues – on issues about how people *may* or *ought* to think or behave, as distinct from how they *do* – we cannot expect a major point at stake to be settled by observation" (Cohen 1981, p. 320). Contrary, the way to proceed is the following: a) we must formulate *general hypotheses* by a process of abstraction, idealization and precisification starting from people's intuitions on the subject; b) we must *check our hypotheses* against intuitions in concrete individual cases (1981, Section I.3). Cohen sees this as the only manner to effectively obtain a *coherent* set of rules as a normative standard for everyday reasoning.

So far the argument fits perfectly an LP theory and, if taken to be sound, it would be sufficient to offer an account for the normativity of logic relying on cognitive-psychological studies of human behavior. It adheres to that kind of LP we called *Psychological Descriptivism* (see footnote 4): we must observe the *commonalities* among human communities in accepting or, on the contrary, amending rules and rejecting inferences. The resulting set of deductive rules assumes a normative role *in virtue of* their acceptance by (alleged) rational agents. This is not the final aim of Cohen's paper, however. In the subsequent parts of his work, he assumes a few more controversial views. Let's consider them briefly.

2.2. A theory of competence as normative standard for reasoning

The basis for our next step is the assumption of a famous distinction introduced by Noam Chomsky (1965) concerning humans' linguistic abilities: the distinction between *competence* and *performance*. Chomsky wrote in his *Aspects of the Theory of Syntax*:

> We thus make a fundamental distinction between *competence* (the speaker-hearer's knowledge of his language) and *performance* (the actual use of language in concrete situations) (pp. 3-4).

For convenience, let's call this "Chomsky's distinction" (CD). Cohen takes it for granted and transposes it into the discussed topic. Once we established a normative theory relying on intuitions in the way we described above, such intuitions might become the subject matter of a subsequent *factual* theory: "it will describe a competence that normal human beings have" (1981, p. 321). According to Cohen, though, the resulting theory "is a contribute to the psychology of cognition and it is a by-product of the logical and philosophical analysis of norms rather than something that experimentally oriented psychologists need to devote effort to constructing"

(p.321). Given this premise, he derives one of the most controversial and debated passage of his theory:

> In other words, where you accept that a normative theory has to be based ultimately on the data of human intuition, you are committed to the acceptance of human rationality as a matter of fact in that area, in the sense it must be correct to ascribe to normal human beings a cognitive competence – however often faulted in performance - that corresponds point by point with the normative theory […]. To ascribe a cognitive competence, in this sense, within a given community is to characterise the content of a culturally or genetically inherited ability which, under ideal conditions, every member of the community would exercise in appropriate circumstances. It states what people can do, rather than what they will do […] (p. 321).

The consequence of establishing such an equivalence between a normative standard and human competence is the *impossibility* to ascribe any form of irrationality to humans, namely (¬FC) in our systematization of Cohen's argument. Certainly, people can err in reasoning, but in Cohen's theory such errors can at best be imputable to *poor performance*, without any relevance for their competence, exactly as it happened in Chomskyan linguistics (Chomsky 1965, p. 3).

Once again, we can represent this part of Cohen's argument as follows:

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\begin{array}{l}
(I \wedge CD) \rightarrow \neg FC \\
I \\
\underline{CD} \\
\neg FC
\end{array}
$$

What follows in his paper is then a review of many decades of cognitive psychology literature as an attempt to reinterpret specific findings – which are normally considered to be clues to the existence of reasoning biases – in the light of his different framework of rationality. As Cohen explains, if we accept the conclusion of the aforementioned argument (¬FC), we need to search for different explanations of the alleged irrational answers that participants express in psychological tests. A few classical findings about systematic reasoning errors (as *confirmation bias*, *conjunction fallacy*, *base rate neglect*, etc.) are now re-categorized either as simple *performance errors* or as the result of some *mistaken interpretation* of the experiment itself (1981, Section II).

2.3. *The adherence of Cohen's theory with LP*

Letting the controversy aside for a while, we must assess Cohen's overall argument from a psychologistic point of view. I shall recall the definition of the *Psychological Descriptivism* offered by Pelletier, Elio and Hanson of one of the

contemporary LP forms (see footnote 4): "Identify logic with some description of the observable performance of people's behavior in the realm" (2008, p. 7). As a matter of fact, Cohen states that the *only* possible basis for the achieving of a coherent set of inference rules (a logical system) is the collection of human intuitions on the subject matter, as Goodman had already put forward. Furthermore, given how Cohen defined intuitions (see IJ), the *only* way to catch and collect them is to observe people's behavior (performances) in logical tasks. Accordingly, Cohen's theory perfectly expresses a form of Psychological Descriptivism.

Moreover, I shall here bring to light a slight but relevant epistemological difference between Goodman and Cohen's characterizations of the basis for the justification of inference rules. Goodman refers it to the conformity with accepted *deductive practices*, leaving only implicitly room to interpret judgements as the expression of intuitions[9]. Cohen makes this interpretation explicit (IJ) and switches the focus from judgements to the *(psychological) inclinations* that underlie them[10]. Whatever this specification aims to clarify, it is relevant from an LP point of view, since it undoubtedly collocates justification within a psychologistic framework.

How does such a theory deal with the key objection that considers *universality* insufficient to found normativity? According to Pelletier and colleagues, even if every single human being had the same intuition within the same logical context, the universality of reasoning behavior would not be sufficient to account for the normativity of logic, due to the gap between descriptions and norms. Yet, Cohen's theory does not need to bridge any gap between the descriptiveness of psychology and the normativity of logic, for such gap has been ruled out. Saying that humans have the inclination to judge according to an inference rule of the form $\Gamma \vdash \alpha$ in appropriate circumstances assigns in the first place a normative role to the rule $\Gamma \vdash \alpha$, assuming it is sound in those circumstances. At the same time, it permits to enumerate the ability to infer according to $\Gamma \vdash \alpha$ within the reach of human competence. Intuitions thus play a double and parallel role: on the *epistemological* side they warrant a set of rules to have a normative role, on the *factual* side they describe an overall competence to infer according to those very rules. Admittedly, "this factual theory of competence will be

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[9] Which is anyway the commonly accepted interpretation of this Goodman's passage (see Cohnitz and Rossberg, 2020, Section 5.2).

[10] We perhaps could interpret such a qualification as the anteroom for the Chomskyan competence/performance distinction (CD) at the factual level. It seems acceptable to collocate the single judgements on the performance side, reserving to the underlying intuitions the role of qualify competence.

just as idealised as the normative theory from which it derives" (Cohen 1981, p.321). Descriptive and normative theories here have the same content point-by-point, which means there is no gap to bridge between them. We could make this thesis clearer as follows:

> An inference rule is a norm for reasoning *if and only if* humans have the competence to use that rule in appropriate circumstances.

Therefore, if we take Cohen's argument to be correct, the objection against the ability of LP to account for the normativity of logic is neutralized.

2.4. Objections to Cohen's theory and their impact on LP

As already mentioned above, these arguments have often been described as controversial. Of course, since so-called "rationality wars" are still ongoing, a concrete possibility is to support a psychologistic account of logic in the manner Cohen does, accepting every implied consequence. Nevertheless, it is useful to show that an LP advocate *does not need* to embrace Cohen's whole theory. Much of the complaints against Cohen arise when, in the second section of his work, he refuses the outcomes of great part of the cognitive psychology experiments along decades. If we accept the argument of the first section – he claims – then many classical findings concerning human irrationality turn out to be wrong: we should search for other explanations of the faulty *performances*. Later he will reinforce the already strong thesis:

> However, nothing in the existing literature on cognitive reasoning, or in any possible future results of human experimental enquiry, could have bleak implications for human rationality, in the sense of implications that establish a faulty competence (p. 330).

Hence, he offers a new systematization in four categories that should cover exhaustively the entire experimental production on the topic. Here they are: a) *studies of cognitive illusions*, b) *tests of intelligence or education*, c) *misapplications of appropriate normative theory*, d) *applications of inappropriate normative theory*. For each of them, Cohen proposes a different account (CA) for the relative findings coherent with his premise for such section (¬FC).

I will not enter such additional argument that goes beyond the scope of the present work. I mean to show, however, that many opponents to Cohen's theory end up refusing his whole argument just because they reject the accuracy and truthfulness of this Cohen's literature review. In other words, given the conditional ¬FC → CA, that Cohen himself puts forward (1981, p. 325), if one can demonstrate that cognitive psychologists are right about their findings, that is ¬CA (Cohen's accounts of these

findings are wrong), by *modus tollens* she can refuse Cohen's argument, because its conclusion (¬FC) is false. To illustrate, Crupi (2016) argues that the charges brought by Cohen against a specific finding by Tversky and Kahneman (1983), so-called *conjunction fallacy*, fail. Taking seriously Cohen's challenge, it is possible to check his hypotheses about troubles in the experiment with additional control tests. Crupi points out that, at least for what concerns the conjunction fallacy, we are dealing with well-founded tests that demonstrate reliably how human competence can be faulty while facing specific tasks. It should be noted that similar works have also been done with tasks regarding more properly logic (Vindrola 2017).

One could then easily be tempted to reject *tout court* the British philosopher's theory. Yet, the point I shall bring to light here is that even an advocate of psychologism could disagree under many respects with Cohen's argument without giving up her support for LP. Cohen's way to account for the normativity of logic showing an alleged equivalence between the descriptive theory of competence and the normative standard for everyday reasoning is a strong claim that is not necessarily needed. What is relevant for LP purposes is to preserve (I) as a central claim. Accordingly, it is possible to refuse other assumptions or conclusions that the British philosopher supports in his theory. Let's summarize one more time Cohen's overall line of thought:

First argument (Section I.1)	Second argument (Section I.4)	Third Argument (Section II)
(GT ∧ IJ) → I	(I ∧ CD) → ¬FC	¬FC → CA
GT	I	¬FC
IJ	CD	CA
I	¬FC	

Where:

(GT): Goodman's thesis; (IJ): Intuitions underlie judgements; (I) People's intuitions are the basis of normative criteria for the evaluation of reasoning; (CD): Chomky's competence/performance distinction; (¬FC) No faulty competence can be ascribed to humans by means of any experimental result; (CA) Cohen's alternative accounts for cognitive psychology findings.

Apart from denying CA, whose corresponding strategy we just mentioned, a few additional ways to reject Cohen's theory while retaining LP can be found and supported with good reasons. In what follows I will consider briefly three options:

1) ¬ (¬ FC → CA);
2) ¬ CD;
3) ¬ ((I ∧ CD*) → ¬FC)

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$[\neg (\neg FC \rightarrow CA)]$ - Starting with the first case, one can claim that even conceding Cohen's account of human competence (infallible within a specific field), this is not sufficient to fully exclude irrationality of human beings. For example, it is possible that even if people are in principle well-equipped to deal with every logical task, for other reasons they are systematically misled so that their performances in a few specific tasks are normally wrong. Thus, even conceding that human rationality is bounded[11] (a factor that accounts well for performance errors), it is hard to consider rational an agent who possesses all the competence to solve every possible task in a field but lacks the ability to use such competence, or an agent who systematically fails in dealing with those tasks[12].

$[\neg CD]$ – CD represents another critical assumption. Various authors emphasized the problematic nature of the Chomskyan distinction, both in the original connotation (see Frixione 2001, Kaufer 1979, Rosenberg 1988) and in the specific transposition that Cohen suggests (see Evans & Pollard 1981, Glucksberg 1981). Some authors defined the way in which Chomsky employs the distinction to be *ad hoc* with the aim of preserving his own theory. It has been objected that observations contradicting the theory are dismissed as performance errors, while those that confirm it are taken as being part of people's competence. Sampson reports an example:

> A clear case, discussed by Reich (1969 - and cf. Sampson 1975:77-79), concerns the phenomenon of "self-embedding": certain syntactic constructions that are normal enough when used singly produce bizarre, scarcely comprehensible utterances when iterated two or three times in a nested fashion, as in for example, "The boy that the cat that my wife found scratched was angry". It happens that Chomsky's grammatical theory predicts that such a sentence should be "good", since each individual construction it contains is normal, and the theory makes grammaticality depend exclusively on the properties of individual constructions rather than on the global patterns into which constructions are organized. Accordingly, Chomsky treats the unacceptability of such sentences as a matter of performance rather than competence. Yet it appears that the unacceptability of self-embedded sentences is systematic and is not predictable on the basis of any principles known independently of linguistic theory; the only basis for

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[11] As Herbert Simon (1957) theorized and Cohen affirms to support (Cohen 1981, p. 322).

[12] Notice that I am not alluding to the fact that a wide reflective equilibrium could generate contradictions whereas the narrow reflective equilibrium would not. Even remaining into the boundaries of the latter, as Cohen wants, it is possible that some mechanism other than logical competence interferes with it impeding its right use. It is what dual-process theories in cognitive psychology suggest as explanation for systematic errors in reasoning (see e.g. Evans 2008). It is not only a matter of bad performances due to unideal circumstances, but a very "bug" of cognition: there are specific tasks in which so-called system 1 (which are fast and resource frugal) systematically interferes with system 2 (which are analytic, consequentially driven, and effortful). Now, even without assign such a position to any psychologist in particular, what I am suggesting here is that, within such a framework, Cohen's competence would concern only system 2, since he consider competence to be strongly topic-related (and logic is supplied by system 2). Thus, even admitting an infallible competence of such a system, the overall behavioral answer of a person, could be still subjected to many other interferences which impede the correct answer most of the times. It would be hard in this case to maintain the perfect rationality of such a person observing her systematic errors in specific tasks.

assigning the phenomenon to performance rather than competence is that this preserves the theory from refutation (1981, p. 350).

[¬ ((I ∧ CD*) → ¬FC)] - However, an additional natural objection against Cohen's way to adopt (CD) might probably emerge looking better at the relationship between intuitions and judgements as they appear in his theory. We pointed out explicitly from the beginning that Cohen defines intuitions as (psychological) *inclinations* to specific judgements in given circumstances. Since he himself declined ways to observe intuitions like introspection or the resorting to other higher cognitive faculties, the only remaining possibility is to collect a significant and statistically relevant amount of people's judgements in order to later generalize about the underlying intuitions. That is: all we can directly observe are people's performances in given circumstances – what is provided by psychologists through experimental tests. As Cohen argues, we can then establish the competence identifying the inclinations (intuitions) behind them. Now, in absence of other well-defined methodology, all we can do for the purpose is to identify *systematic answers* and take them to be the effect of such an alleged psychological inclination – once again, what psychologists do in explaining experimental tests outcomes. Yet, it goes without saying that in this case we should take to be inclinations also those that underlie statistically relevant answers that finally turn out to be wrong. In other terms, in absence of different epistemological criteria (which Cohen does not provide), we should consider the competence as the collection of *all* the inclinations to judgements. Hence, if we want to exclude CD to be *ad hoc* distinction, we must accept all its consequences, so that its right characterization (CD*) would conduct to define human competence as being *faulty*, having the ability to solve some tasks but not others.

It is worth pointing out one more time that each of these objections can impact Cohen's overall theory without affecting (I), the only crucial thesis for an LP advocate, as we said. Anyway, since here we are exposing a specific way to account for the normativity of logic whithin a psychologistic framework, I must highlight that Cohen's theory provides such an account by means of its more controversial position, namely the equivalence between a theory of competence and a normative standard for everyday reasoning. Yet, an LP advocate might well refrain from such an ultra-rationalist theory and search for other ways to bridge the gap between the descriptive nature of psychology and the normativity of logic. Two possible ways will be exposed in the next sections.

3. *Normativity into a community view*

A different way to account for the normativity of logic from a psychologistic point of view does not need to collapse the differences between the descriptive and normative theories involved. In fact, it is still possible to search for a "bridge" between the two sides so that normativity involves other factors than universality of reasoning behavior, considered to be insufficient by itself. In what follows, I will expose a hybrid strategy that takes inspiration from a Wittgensteinian tradition, combining it with a renewed view about the mind and the mental processes coming from contemporary cognitive psychology. In doing that, I will borrow a semantic tool concerning language proposed by Diego Marconi to argue subsequently that it can be applied, *mutatis mutandis*, to logic as well.

3.1. *The hyperconformistic systems*

In a 2019's seminar held at University of Bergamo[13], Diego Marconi sketched an original proposal about language that takes place within the Wittgensteinian tradition. His main thesis is that language must be considered a (*quasi*) *hyperconformistic system* made of conditioned norms behind which a principle of conformism holds. The entire theoretical building is based on two pillars concerning semantics, belonging to the thought of the late Wittgenstein, and an additional assumption concerning communication intended as the main function of language. I will expose each of them step by step.

The Wittgensteinian observations can be found in his *Philosophical Investigations*:

> *[MU]: Meaning is use*: "For a large class of cases—though not for all—in which we employ the word "meaning" it can be defined thus: the meaning of a word is its use in the language" (1953, §43)[14];
> *[MN]: Meaning is normative*: the idea is that, in a similar way to what happens in a game, even in speaking a language and using its words we are committed in following some rules[15].

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[13] https://youtu.be/3IsHJkjjwHk

[14] It is appropriate to highlight how Wittgenstein specifies that his definition does not hold for every possible case, remarking the character of his new philosophical method, more similar to an "album" of philosophical observations than to a systematic theoretical proposal. The present observation is what can be called a *grammatical observation*, which is the tool utilized all along the *Philosophical Investigations*. Actually, in a few later observation, concerning the grammatical connotation of the term "understand", Wittgenstein himself differentiate between "meaning" and "use", being the first immediate whereas the other is extended over time (1953, §139). On the contrary, many authors, starting from this paragraph, formulated a real "use theory of meaning" that would be, in their opinion, incompatible with a normative account of language (see, in example, Wikforss 2001, Hattiangadi 2006).

[15] Even in this case, it is necessary to highlight the observational character of the Wittgenstein's insight, while a proper theory of the normativity of meaning is due to to Saul Kripke (1982).

The two observations are joined together by saying that *meaning-statements* of the kind "'moon' *means* the MOON" have a double characterization:

a) they *describe* the use of a word made by speakers into a linguistic community (in this case the use of the word "moon" in referring to the only satellite of planet Earth);

b) they *prescribe* that specific use as the only correct for that word (that is, in this case it is forbidden to use the word "moon" referring to something other than the Moon).

However, the idea that the two assumptions can fit together has always been criticized by many philosophers. This is a relevant point for the purpose of the present work, since the trouble must be traced back to Hume's famous *is-ought* principle, according to which norms cannot be derived from facts (that is precisely what a psychologistic account for normativity aims to do). *Prima facie*, this is exactly the case of such a genre of semantics. The use of a word (or an expression), indeed, is nothing but a *descriptive* instance of a series of facts, namely the linguistic behaviors of speakers. Following Hume, then, any *normative* implication should be denied to meaning intended as use. Nonetheless, Marconi defends the idea that meaning-statements are at the same time both descriptive and normative. The Italian philosopher proposes then the aforementioned Wittgensteinian observations in a slight different version, namely:

> [MMU]: *Meaning is generated by use*: For a large class of cases—though not for all—in which we employ the word "meaning" it can be defined thus: the meaning of a word is *generated* by its use in the language;
> [MMN]: *Meaning is both descriptive and normative.*

Thus: how to account for the normativity of meaning within a theory of use? A first possible way out has been offered by Paul Horwich, in his book *Meaning* (1998, Chapter 8), where meaning is said not to be *intrinsically* normative but to have normative *implications*. What Horwich aims to show is that the use of a word can be caught with an empirical generalization called *law of use*, a mere factual description (1998, p. 47). Therefore, if meaning is use, it cannot show any normative property in itself. A law of use of the form

1) X is called 'Moon' iff X is the only satellite of planet Earth

does nothing but describe the circumstances in which speakers use the word "Moon". In Horwich's theory of meaning, the laws of use are exactly what define the meaning of a word.

Accordingly, from (1), it is possible to say that:

2) 'Moon' *means* the MOON.

However, even if they are not intrinsically normative, meaning-statements have normative implications, in the form of conditioned norms:

3) If 'Moon' *means* the MOON, then it *ought* to be used only in referring to the MOON.

Given the characterization of the meaning in terms of laws of use, it is quite evident that conditionals of the form of (3) have a descriptive antecedent and a normative consequent. These kinds of norms are what govern the language, according to Horwich but seem a clear violation of *is-ought* principle. Nonetheless, many of the conditionals that we assess as being *true* in our everyday practices have a similar structure – says the British philosopher –, yet, we would not be ready to get rid of them because of the Humean principle.

A clarification comes from Marconi's argument: conditional norms of this kind do not run into the naturalistic fallacy *because*, under a deeper exam, they result to be the contraction of a broader argument that includes a normative premise. For example, a conditioned norm like

4) If John killed Bill, then he *must* be punished

is in fact the contraction of the broader argument:

5) John killed Bill;   (fact)
6) Killing is a crime;   (fact)
7) Crimes *must* be punished;   (norm)
8) John *must* be punished.   (norm)

The conclusion (8) has normative strength in virtue of the assumption of a norm (7). Therefore, a conditioned norm like (4) results to be *true* because is the *syncopation* of a broader *valid* and *sound* argument (5-8).

With this elucidation in mind, Diego Marconi can offer a characterization of language in terms of conditional norms of this sort. The main idea is to define a system in which laws of use rise to the rank of norms. More precisely, a *hyperconformistic system* is a system in which:

(i)   Every customary practice is a norm;
(ii)   Every non-customary practice is forbidden;
(iii)   Nothing else is a norm.

Here customary practices are those concerning linguistic behavior, describable by Horwich's laws of use. In turn, a law of use assigns the meaning to a word. Hence, language is dotted with thousands norms of the form of (3)[16]. In example:

9) If 'cat' *means* CAT, then it *must* be used only in referring to cats;
10) If 'guitar' *means* GUITAR, then it *must* be used only in referring to guitars;

and prohibitions

11) If 'cat' *does not mean* GUITAR, then it *must not* be used in referring to guitars;
12) If 'guitar' *does not mean* CAT, then it *must not* be used in referring to cats.

Now, as already mentioned this kind of norms hold in virtue of a tacit normative assumption in the underlying argument. Here it is appropriate to consider the additional assumption with the aim to confer order and generality to this theoretical characterization of language. Apart from specific authors who denied it (e.g. Chomsky), it is widely recognized by thinkers of various disciplines that communication is the main purpose of language. A second commonly accepted idea linked to it is the fact that communication requires the *uniformity* of meaning (Marconi 1997b, pp. 86-90): it would be enormously difficult to communicate without any agreement on what words and expressions mean.

Therefore, when it comes to search for those tacit normative premises that underlie conditional norms, Marconi supports the idea that in the case of language we are dealing with a unique (normative) *principle of conformism*. It has a pragmatic value and expresses the desirability of a certain conformity to customs in order to facilitate the communication among people: it is easier to communicate if we all call 'moon' the MOON than if we referred the same word to different celestial bodies. Bilgrami expressed this concept with a hypothetical imperative:

"I ought to use words as other do, *if I want to be easily understood*" (1992, p. 110).

In the case of a hyperconformistic system, instead, it assumes the form of a categorical imperative (since it is tacitly assumed that we all want to be easily understood):

[PCONF]: "Every linguistic expression must be used (always and by everyone) as it is customary used" (Marconi 2019).

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[16] It is just worth to mention a couple of additional remarks to the extent in which they will be useful for what follows in the text: a) even in a *quasi*-hyperconformistic system, meanings are not *always* generated by use: there are specific cases in which the attribution of the meaning to a word is deferred to an authority on the subject (e.g., to scientists for what concern *species*); b) the use of a word can change over time, since it can be replaced from a new one, becoming deviant or obsolete. This would amount to generate a new meaning for that word.

All in all, a system like language is defined by a principle of conformism which drives people to adopt the customary uses of words that they already find in the linguistic practice of the members of their own community. The laws of use about such linguistic practice have therefore the double role of generating *meanings* and *norms*. In short, meaning-statements have both a descriptive and a normative side.

Thus, we are here in that ideal condition that an LP supporter is searching for: descriptions can generate norms. For the purpose, however, it is necessary to test two additional hypotheses: 1) is a theoretical tool like hyperconformistic system suitable to describe logic as well?; 2) is the descriptive side of the affair coherently referable to psychology findings? I will respectively address these questions on paragraphs 3.3 and 3.4. However, before arguing about it, let's briefly have a look at the theoretical ground on which Marconi's building rests, namely the main observations contained in Wittgenstein's *Philosophical Investigations*. I am going to do that exclusively to the extent that it can be useful for our topic, deliberately avoiding to enter in complex debates concerning the exegesis of such a work.

3.2. *Some remarks on the Wittgensteinian characterization of language*

As already mentioned, the basic assumptions for the entire theoretical building of Marconi's semantics come from two Wittgensteinian insights about the nature of meaning: *meaning is use* and *meaning is normative*. These are strictly intertwined thesis that find their expression in another key notion of the *Philosophical Investigations*: the *language-game*. Language is not a uniform system described by some essence other than grammar itself (Andronico 1997, p.243), but the resulting of many different activities[17] (Voltolini 1998, Section 2.1)*. Game* and *language* are both *open concepts*, which extensions are not well defined by any shared essence among members; nonetheless all of us know what they are referring to. This is possible thanks to the *family resemblances* that games (and linguistic practices) share. No game has all the characteristics ascribable to the concept "game" but every activity that we call with this name shares a certain amount of them (Biletzki & Matar 2020, Section 3.4). Something analogous happens with language. "Here the term *"language-game"* is meant to bring into prominence the fact that the *speaking* of language is part of an activity, of a form of life" (Wittgenstein 1953, §23). It clearly emerges the

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[17] "Review the multiplicity of language-games in the following examples, and in others: Giving orders, and obeying them – Describing the appearance of an object, or giving its measurements – Constructing an object from a description (a drawing) – Reporting an event - Speculating about an event" (Wittgenstein 1953, §23).

praxeological approach that Wittgenstein gradually assumes in his late thought: what we can observe in reality is exclusively the variety of *uses* of the words and expressions which people do, so that there are several ways to speak a language, each of them governed by specific *rules* which constitute and define that language-game (see Wittgenstein 1953, §§185-242, see also Messeri 1997, pp. 151-192). Rules are *constitutive* of a game as well as of language, since both requires a normative standard for their practicing[18]. Looking closer, thus, the practice of *denoting* is just a particular case among several language-games (a "denotational game"). What permits to play a game is so-called process of *following-a-rule*. The use of a word must be compliant with the grammatical rules that govern such a use (Voltolini 1998, p. 99).

Thus: which are those rules that we are committed to follow in language? Wittgenstein solves the problem pointing to the relative activities that take place into an epistemic community, called *form of life* (Andronico 1997, pp. 241-254). Both the normative status of meaning and the general following-a-rule task reach their justification on the "bedrock" on which the "spade is turned" (Wittgenstein 1953, §217), namely into a *paradigmatic application* of the rule.

Here one finds so-called *community view*: the community has the double role of referee and player in language-game activities. On the one hand, the criteria that define a language make it essentially *public*: the possibility of any linguistic practice requires by definition an intersubjective dimension (Wittgenstein 1953, §202). The *condition of existence* of a language is the availability of a public standard of correctness endorsed by a community. On the other hand, the correctness standard itself is the result of the emerging within a community of certain *paradigmatic applications* of rules, the ones that will go to assume a normative role for language (Voltolini 1998, p.41). Both the applications and the constitution of the rules arise together with the practice of the members of a community without, in Wittgenstein's theory, any need to invoke ideal or mental entities staying *behind* or *below* the rule itself (Voltolini 1998, p. 42)[19]. But:

> The application has priority of existence over the rule because the instances of the rule application enter into existence and are what they are independently of the rule itself; and it is vice versa that the rule in its determinateness that enters into existence, and comes

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[18] We are not in presence of categorical imperatives however: rules are not constrictive – nobody is forced to play a game – but if we decide to play a particular game – let's say poker – we cannot play it without follow poker rules. If we do otherwise, Wittgenstein says, we are playing another game (Messeri 1997, Section 3, Voltolini 1998, pp. 41-42).

[19] "Wittgenstein does not intend to argue that the rule is not a certain rule. He recalls, however, that the rule is determined precisely in the course of its use and that the use of the rule is always in progress and never completed […]. The rule determines its use but does not predetermine it" (Messeri 1997, p. 168).

to be what it is, as the instances of application succeed and in force of them (Messeri 1997, p 168).

Now, it is pretty obvious that, to the extent that it is at least in principle possible for individuals to act according to different applications of the rule (see the example of the recalcitrant student – Wittgenstein 1953, §185), it is necessary to assume one among many as being *paradigmatic* for that rule, the one that assumes a *normative role*. It is notable how Wittgenstein does not presuppose an arbitrary resolution among the members of a specific community, he rather clarifies that their symbolic reaction shows a spontaneous convergence on that particular application, therefore identified as the paradigmatic one. This is nothing but a matter of fact and can be expressed as being characteristic of a *form of life* (§241).

How the abovementioned notions can give support to a psychologistic account of logic? How can an LP advocate take advantage from such Wittgensteinian insights to solve the problem of the normativity of logic? In the next section I will show how to transpose the hyperconformistic system theorized by Diego Marconi into a logical system, using the Wittgensteinian notions of following-a-rule and language-game. In the last part of this section, I will mention briefly how an LP defender could define a *form of life* in psychological terms.

3.3. *Logic as a quasi-hyperconformistic system*

In this section, I am going to characterize logic as a hyperconformistic system in Marconi's sense. As a first step, let us see how in Wittgenstein's *Philosophical Investigations* logic itself is taken to be a language-game among others. The issues about the meaning of logical constants and the inference rules can be subsumed respectively under the general notions of *meaning as use* and *following-a-rule*. In such a framework, logical constants receive their meaning in virtue of their *use* that people express in their logical behaviors, that is, in those linguistic practices related to logical tasks. Furthermore, even the choice of inference rules obey to the most general process of assigning a normative role to the *paradigmatic application* of a rule in the way we described[20]. All in all, within the late Wittgenstein's approach, logic turns out to be a language-game governed by those inference rules which assumed a normative role into a certain community in virtue of its representing a form of life. Indeed, it is this mere *fact,* which entails the embodying of a common nature and the sharing of a certain

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[20] For an in-depth discussion of the theme of following a rule in logic and mathematics, see Frascolla 1997, Section 3.

education, that allows people's behavior to converge towards the same paradigmatic application and makes it a *norm*. Otherwise said, from a mere empirical fact – people's natural behavior – we build our normative standards in logic.

Let's give now a characterization of this approach to logic in Marconi's terms. The starting point for the construction of the hyperconformistic system is once again the collection of uses. In order to formulate so-called *laws of use* to characterize logic as a hyperfconformistic system, we must focus on that linguistic practice which reveal the role of logic, namely argumentation.

In their ordinary lives, people speak and behave in accordance with some arguments. Normally, arguments are the linguistic expression of some individual's *propositional attitudes*, a fact that can be (and is often) asserted by the defender of psychologism, who affirms that there are *necessarily* certain psychological-cognitive states behind logical inferences (against Wittgenstein's opinion). In other words, *only if* people *know* (*believe* or *are informed*) that certain propositions are true when others are true, arguments and consequent rational behavior are given. However, without committing to a theory about it, in this context we must understand the arguments as the outcomes of situations in which people are *naturally* inclined to assert a certain conclusion, given certain premises[21]. This act of connecting in such a way premises and conclusion is nothing but the act of *inferring*. In Wittgensteinian terms, we could say that being part of a specific form of life like humankind will naturally lead to certain logical performances that are describable by means of widespread argumentative forms.

Of course, there are specific clues in linguistic practices to detect the presence of an argument and an inferential act. For example[22]:

> Hitchcock (2007) defines an argument as "a claim-reason complex" consisting of (1) an act of concluding, (2) one or more acts of premising (which assert propositions in favour of the conclusion), and (3) a stated or implicit inference word that indicates that the conclusion follows from the premises (Groarke 2020).

Those "stated or implicit inference" words of the point 3 are said *inference indicators* (they are normally words like "thus", "hence", "because", "since", and so on).

Now, depending on how the argument is made, it can be said that a certain inferential rule has been delineated. We therefore admit to collect, starting from the

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[21] Andrea Iacona affirms, roughly said, that people is arguing when submit certain propositions (the premises) as the reasons why another proposition (the conclusion) is true (2005, pp. 27-39).
[22] See also Iacona 2005, Chapter 1.

patterns of inferential behavior of people, all the possible arguments a_1, a_2, a_n, b_1, b_2, b_n, c_1, c_2, c_n, ..., and group them abstracting all their structures in an ideal set S: {A, B, C, ...}. One can therefore indicate such structures, or argumentative forms, as being *paradigmatic applications* of specific rules, so outlining a set of inferential rules starting from observations of an empirical nature (our set S). Since the paradigmatic applications are nothing but people's uses of rules, the *laws of use* of inference rules can be expressed as follows:

13) The argument *A* is the *paradigmatic application* of the *modus ponens* rule iff *A* has $\alpha \rightarrow \beta$ and α as premises and β as conclusion;
14) The argument *B* is the *paradigmatic application* of the *modus tollens* rule iff *B* has $\alpha \rightarrow \beta$ and $\neg \beta$ as premises and $\neg \alpha$ as conclusion.

The following step is to *define* the inference rules thanks to their paradigmatic application[23]:

15) *Modus ponens* is that inference rule that permits to conclude β whenever the premises are $\alpha \rightarrow \beta$ and α (from 13);
16) *Modus tollens* is that inference rule that permits to conclude $\neg \alpha$ whenever the premises are $\alpha \rightarrow \beta$ and $\neg \beta$ (from 14).

As clearly emerged from the previous sections, the communitarian context is a characteristic feature of this specific way to account for the normativity of logic. Hence, it is not inappropriate to point out that, if conformism about meanings was a necessary condition for a good communication, this is *a fortiori* true for what concerns argumentation and reasoning[24]. Therefore, Bilgrami's hypothetical imperative could be paraphrased in this way:

"I ought to use *inference rules* as others do, *if I want to be easily understood*".

The *principle of conformism* that warrants these norms for logic can then be reformulated as follows:

[LogPCONF]: "Every logical rule must be applied (always and by everyone) as it is customary applied"

Subsequently, in a hyperconformistic system, paradigmatic applications generate norms:

17) If *modus ponens* is paradigmatically applied by argument *A*, then it *must* be applied only in that way (from 13 + *LogPCONF*);
18) If *modus tollens* is paradigmatically applied by argument *B*, then it *must* be applied only in that way (from 14 + *LogPCONF*).

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[23] I shall recall here that, in the late Wittgenstein's philosophy the application has priority of existence over the rule (see above, pp. 21-22).
[24] A few authors theorized that reasoning emerged during evolutionary times thanks to argumentative skills, because they permit an easier communication and a public organization (see, for example, Mercier & Sperber 2011). Iacona supports the idea that arguments show the research of an agreement among people (Iacona 2005, *Introduction*).

A couple of remarks are helpful in order to conclude this section. I titled this paragraph "logic as a *quasi*-hyperconformistic system". There are two reasons for the term *quasi* (see footnote 16). In the first place, as well as for language, even for logic it is possible to *defer* part of the decisions about norms: in the language, experts can define the meaning of certain words regardless of the customary uses if useful for specific aims (for example, as scientists do in outlining a taxonomy of species). Similarly, in logic experts can choose a different application of a rule as a paradigm where customary uses could be problematic, incoherent or uncertain, that is to preserve only valid arguments. Secondly, unlike language, where uses vary over time with a certain frequency (and this was a central point in Marconi's discussion), human form of life seems to guarantee a greater continuity about logical behaviors so that the analogy with uses in language could seem inappropriate: in fact, within this framework (involving the *form of life* concept) the only difference that we can appreciate is essentially quantitative and not a matter of principle. Therefore, with the right clarifications, I mantain that a (*quasi*) hyperconformistic system can well represent an account for the normativity of logic in a Wittgensteinian framework.

3.4. Wittgenstein's antimentalism vs. pscyhologism

So far we showed how, using observations inspired by the philosophy of the late Wittgenstein, it is possible to derive norms from facts and build an entire system with this method. In this approach, however, descriptions and norms do not coincide or overlap: they are two epistemological separated dimensions. As we saw, a principle of conformism is needed to connect them, a principle that instantiates a value, so belonging to the normative side. If universality of behaviors is not sufficient in itself to account for the normativity of logic, the LP advocate does not have to worry about, since the norms are here generated from facts *in virtue* of the hypothetical imperative that pushes us to conformism, given our own nature.

The idea that norms emerge from natural behaviors fits well with the grounds of a psychologistic account of logic but, of course, a supporter of psychologism cannot be satisfied of such a claim alone: she needs to found such behaviors on psychology. Notoriously, in Wittgenstein's philosophy this possibility have to deal with many questions, given his well-known antimentalism. The antifoundationalist metaphilosophy of the Austrian philosopher had the very purpose to get rid of "awkward" ideal or mental entities. There are different reasons for this aversion that

we will not treat analytically, but we can trace two main points sustaining Wittgenstein's arguments against mentalism:

- A certain diffidence and dissatisfaction for the psychology as a science;
- The conviction that mental entities do not have any explanatory power in accounting linguistic (or logical) behaviors of human beings.

Nonetheless, I believe a today's defender of psychologism to have the necessary arguments for rebutting Wittgenstein's skepticism. Firstly, she could demonstrate the greatest effectiveness in methods and explanations of the renewed psychology: the assertion that it would not be possible to study the subjective in an objective way must considered to be false today. Cognitive psychology starts from trans-subjective invariances, so that mental processes and entities are no longer considered neither idiosyncratic nor private. They are in fact considered invariable and objective in the sense of being subject to *causal regularity* (Sacchi 2013, pp. 178-179). For what concerns the second point, it might be argued that, even if coherent and defensible, the Wittgensteinian position is incomplete and dissatisfactory. He says, for example: "I have not said *why* mathematicians do not quarrel, but only *that* they do not" (1953, §II). In this case we are in front of so-called "missing explanation" (Casati 1997, p. 209): if we are dealing with *regularities* about behaviors and answers to certain tasks − e.g., in following-a-rule − we should avoid to consider them as the result of a "miracle"[25] and search for an explanation of such regularities in our best theories on the subject, which are cognitive sciences nowadays. An LP advocate, in doing so, would pull the "bedrock on which the spade is turned" one layer below, on the psychological ground[26].

Altogether, what a today's psychologism defender might argue is that if − as Wittgenstein supports − it is the *form of life* to determine our natural convergence towards certain logical behaviors, cognitive sciences can define in psychological terms such a form of life, describing how humankind mental abilities and events *cause* such a convergence. Notice that the LP defender is not automatically committed to claim that our psychological abilities predetermine behavior *uniquely*, but only that they

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[25] I am deliberately alluding to the "no miracle" argument by Hilary Putnam (1975).

[26] Wittgenstein could object that, even if these regularities were explained by psychology and fill that explanatory gap, this would be just an empirical matter without any role in justification of those customary behaviors, since *conceptually* the term "understanding" does not involve any mental activity, being entirely rooted in the observable performance. However, even this observation could be refused today from a philosophical point of view, since it could be objected that even the concept of "understanding" is qualitatively changed from Wittgenstein's times (see Searle 1980, Cole 2020, Marconi 1997b, pp. 158-165).

dictate their conditions of possibility[27]. In other words, she can argue that the applications of the inference rules implemented by the members of a community are nothing but public acts based on psychological mechanisms. The communitarian context assumes value, regarding the normativity of logic, for pragmatic reasons, that is, the effectiveness of communication and the improving of public activities that involve reasoning. All in all, for an LP advocate, the norms of logic are nothing but paradigms chosen from public performances that ultimately have a psychological foundation.

4. *Depth-bounded logic and rational agents*

Our third solution takes its cue from two well-known theories in the rationality debate and proposes a convergence between them through the work of Marcello D'Agostino (2010, 2013, 2015). On the one hand, there is the well-established idea that rational human agents enjoy limited cognitive resources, or a form of *bounded rationality*[28]. The second idea comes from a well-known theory of reasoning according to which man would be equipped with a *mental logic*, that is, inferential rules would be an innate baggage proper to human cognition[29]. These two traditions could seem to be in contrast concerning the role of logic in human reasoning, the first requiring a normative standard to guide humans beyond their natural cognitive limits, whereas the second seems to describe a normatively self-sufficient deductive system embodied in human minds. I will propose here that the depth-bounded logics by Marcello D'Agostino might represent the link between the two, since he assumes that:

    i.   individual rational agents have limited and different inferential capabilities, measurable in terms of "depth";

    ii.   that they can be corrected and improved by higher-depth logical models.

The supporter of psychologism can well argue that such inferential abilities are described by a minimal mental logic shared by everyone and that, anyway, an external

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[27] Clearly, a certain *deviance* is allowed and the performing of a certain act rather than another may perhaps be part of a probabilistic characterization. This fact, above all, agrees well with the possibility of cases such as those of the "recalcitrant pupil" and offers some support to the need to establish norms starting from customary behavior.

[28] It is about memory capacity, computational skills, performance timing, etc. It is a line of research that has Herbert Simon (1957) as a founder, but that has found experimental confirmations in every line of investigation belonging to the cognitive sciences (see Wheeler 2020).

[29] This theory has a long tradition as well and has its roots in a series of works of a purely logical nature, such as those of Gerard Gentzen (1969), and, also, in the progressive development of Artificial Intelligence (Newell & Simon 1972).

normative standard is needed to level upwards the differences among the capacities of rational agents.

4.1. Mental logic theory

The hypothesis according to which reasoning is a process permitted by the presence of any innate logic in human mind comes from afar[30]. The general idea underlying every mental logic theory is that individuals are equipped with a *cognitive architecture* highly specialized to perform deductive tasks. They contain innately a certain numbers of inference rules: how much of them and which ones we can recognize is a matter of empirical research. The entire process is a kind of *syntactic* treatment of sentences without any specific role of meanings and contexts. The human mind would be able to recognize and manipulate those specific representations that are *propositions* and perform a calculus: this is how reasoning is understood according to such a theory.

For example, Lance Rips with his *The Psychology of Proof: Deductive Reasoning in Human Thinking* (1994) proposes one of the most discussed and cited variants of this theory. It can be summarized in a few main claims:

i)   Deduction is a central aspect of cognition. Accordingly, a lot of different kinds of problems are treated and transformed in an adequate deductive form in order to be solved or to lead to a conclusion (pp. 11-14, 94-99, Chap. 8);

ii) The psychological deductive process that takes place in reasoning and solving problems assumes the form of a *mental proof*, with the production of a certain necessary number of sentences which constitute either a premise or what follows from one (or more) of the preceding ones through the use of the system's rules (Chap. 4);

iii) The deductive apparatus of the human mind is equipped with a series of these inference rules – Rips lists about fifty of them (Chap 5-7)[31].

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[30] In XX century the first thinker to import this idea into psychology was Jean Piaget (1953). Other two contributing elements can be traced back to Gentzen's *natural deduction*, that "intended first to set up a formal system which comes as close as possible to actual reasoning" (Gentzen 1969, p. 68), and to the development of a specific field of Artificial Intelligence, called *Automated Reasoning* (Portoraro 2019) or, in a more specific domain, *Automated Theorem Proving* (Davis 2001). Many authors worked along this research strand in psychology during the decades; among others, a few important works on the matter are: Newell and Simon (1972), the early Johnson-Laird (1975), Osherson (1976) and Braine et al. (1984). Altogether, it could be said that the theory of mental logic has been orthodoxy in reasoning psychology for a long time and has only recently been challenged and, presumably, overcome by more up-to-date rival theories.

[31] Inference rules concern the use of connectives and quantifiers, and they are basically utilized in three ways from the system: forwards, backwards and in either directions (the forwards rules permit to derive conclusions from premises, the backwards rules permits to rise back from a conclusion to its premises.

Rips' theory follows mostly the tradition of the strand, except for the additional thesis according to which human cognitive apparatus operates a transformation of almost each reasoning task in a deductive form. However, the core idea is that humans possess innately the rules to perform inferences and reasoning. Mental logic theories lend quite naturally themselves to shape a logical psychologism, more precisely, to what Pelletier, Elio and Hanson define *Cognitive Architectures (Psychologism)* (see footnote 4): "Identify the science with whatever is given by some 'faculty' or 'module' within this architecture that is common across people" (2008, p. 8).

Yet, if human minds are so well equipped with logical tools, it is hard to give explanation of the errors that people are prone to commit. Rips rebuts saying that errors depend on the number of rules that are necessary to solve the task or to infer a conclusion: the more rules an argument requires, the more difficult the performance is. With the increasing of the difficulties, also the percentage of errors rises up. Nonetheless, it remains to be explained how such a perfectly designed deductive machine can run into *systematic errors* even in simple logical tasks. Apparently, there are two main paths to go over the impasse: either choose an alternative theory of deductive reasoning, or scale down the alleged cognitive power of such a machine. In the next sections, with the works of Marcello D'Agostino, we will offer an example of the latter solution.

4.2. Depth-bounded rational agents

D'Agostino (2010, 2013, 2015) aims to give an informational view of classical logic "that stems from a kind of informational semantics whereby the meaning of a logical operator is specified solely in terms of the information that is *actually* possessed by an agent" (2015, p. 1). He approaches the topic starting from different perspectives, but there is a common central idea behind them: classical logic can hardly have a direct normative role for realistic agents, who are in fact characterized with limited resources. If one pretends the classical conception of logical consequence to determine the conclusions to draw in everyday reasoning, she stumbles upon a series of apparently insoluble paradoxes. The most notorious among them takes place in *epistemic logic* (but it can be replicated in doxastic logic and information logic) and is known as the *problem of logical omniscience*.

Let $\Box_i$ express any of the propositional attitudes at issue[32], referred to the agent i. Then, the "logical omniscience" assumption can be expresses by saying that, for any finite set Γ of sentences,

(1) If $\Box_i A$ for all $A \in \Gamma$ and $\Gamma \vdash B$, then $\Box_i B$,

where $\vdash$ stands for the relation of logical consequence. Observe that, letting $\Gamma = \emptyset$, it immediately follows from (1) that any rational agent i is supposed to be aware of the truth of all classical tautologies […] (D'Agostino 2010, pp. 1-2).

D'Agostino claims that this problem arises due to the *received view* about logic, which looked at it as being *analytic* and *informationally trivial* (or uninformative, non-ampliative)[33]. However, it is widely recognized that this is not the way in which a real agent uses logic in everyday reasoning. The fundamental question to answer is then:

(FQ) do we *actually* possess the information that the conclusion of a valid inference is true *whenever* we possess the information that its premises are true? (D'Agostino 2013, p. 6).

There are two main reasons to answer "no" to this question: the first is strictly connected with the properties of formal systems and addresses the solution towards a different characterization of semantics (informational), the second requires a better look at realistic agents. I shall briefly treat the latter aspect now, while I will expose D'Agostino's semantic solution on next section.

When "realistic agents" are called into question, we are referring to intrinsic limits that characterize their inferential abilities. It is noteworthy that these limits hold even from a computational point of view in the area around which D'Agostino moves, that is, Artificial Intelligence: there are factors such as the time of calculation and the memory required that determine the *tractability* of a problem, namely the fact that a problem that could be solved in theory takes too many resources to be solved in practice. Obviously, the intractability of a problem increases when it comes to be solved by humans, given the additional limits due to cognitive factors, to practical needs and, moreover, as D'Agostino addresses the topic, to the lack of *actual information*. Dwelling for a moment on the latter aspect, Jaacko Hintikka defines

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[32] Propositional attitudes like "knowing that A", "believing that A", "being informed that A".

[33] Notice that the problem arises even in a semantic characterization *à la* Kripke. "The latter is carried out in terms of structures of the form $(S; \tau; R_1;...;Rn)$, where S is a set of possible worlds, τ is a function that associates with each possible world s an assignment $\tau(s)$ of one of the two truth values (0 and 1) to *each* atomic sentence of the language, and each R_i is the "accessibility" relation for the agent i. Intuitively, if s is the actual world and $sR_i t$, then i would regard t as "possible". Then, a forcing relation $\vDash$ is introduced to define truth for the complex sentences of the language, starting from the initial assignment to the atomic sentences. The forcing relation incorporates the usual semantics of classical propositional logic and defines the truth of $\Box_i A$ as "A is true in all the worlds that i regards as possible". In this framework, given that the notion of truth in a possible world is an extension to the modal language of the classical truth-conditional semantics for the standard logical operators, (1) appears to be both compelling and, at the same time, counter-intuitive" (D'Agostino 2010, p. 2).

actual information as the ones "we actually possess (as distinguished from the information we in some sense have potentially available to us) and with which we can in fact operate" (1973, p. 229).

Thus, in referring logic to realistic agents' reasoning we have to deal with two different but concomitant issues: their limited resources and the possible lack of actual information. On one side, if we want logic to have a normative role in our real life, we must begin to consider that reasoning has costs. As Christopher Cherniak put it:

> Until recently, philosophy has uncritically accepted highly idealized conceptions of rationality. But cognition, computation and information have costs; they do not just subsist in some immaterial effluvium (1986, p. 3).

Additionally, we must take into account that in everyday reasoning, real agents often find themselves making hypotheses to get over the missing information (and draw a conclusion) or, in other words, everyday reasoning has to deal with indeterminacy of information. Therefore, the answer to the fundamental question (FQ) that we asked above seems to be "no" even starting from a close examination of rational agents.

D'Agostino calls such (necessary) hypotheses to conclude an inference given certain circumstances *virtual information* (2010, p. 7), to contrast the *actual information* at our disposal. Virtual information has a double contrasting effect in human reasoning: it permits to effectively reason in practice, and it increases the complexity of the inferring process, making some inferences out of rational agents' range of abilities.

In the informational characterization of logic, virtual information intervenes each time we are not informed about the truth or falsity of a proposition, that is, when we are facing *indeterminacy* of information about a proposition. For example, consider the following case:

(i) We are informed that "tomorrow either the train will normally circulate or a replacing shuttle will cover the same route" – that is, we are informed that the disjunction "train or shuttle" is true – let's refer to this actual information as $v(T \lor S)$;

(ii) We are informed that "if the train will circulate, we will arrive at destination" – namely, $v(T \to D)$;

(iii) We are informed that "if the shuttle will circulate, we will arrive at destination" – namely, $v(S \to D)$;

The obvious conclusion is that:

(iv)   We are informed that "tomorrow we will arrive at destination" – namely, v(D).

Now, this simple example reveals how we do use virtual information to derive a conclusion: particularly, we are not (actually) informed neither that the train will normally circulate tomorrow nor that the shuttle will offer the replacement service. However, in deriving (D), we virtually assume both these states as being true, we observe that in both the cases we can derive the same conclusion, therefore we can infer the conclusion regardless of which one will be really true (in the derivation we can discharge the assumptions T and S that we previously made). Now, this simple inference would be inadmissible if we should base on the sole actual information.

Nonetheless, virtual information may have a side-effect depending on how large is the so-called *virtual space* which we are forced to include in our reasoning. Of course, potentially, there could be infinite virtual assumptions in an inference process, but it is pretty easy to see how a realistic agent could not take charge of such a task. This is a crucial point in D'Agostino's theory background. The deductive inference process does not always present the same cost, even in computational terms. Since (realistic) rational agents do not always have the same cognitive abilities, it is important to model the normativity of logic by relating the *complexity* of the cognitive task and the resources available to rational agents.

D'Agostino characterizes the problem in terms of *depth*: specifically, he offers a depth-bounded logics model in which the complexity of the inference determines the necessary depth to adopt on the formal side for its resolution and, at the same time, an ideal measure of the cognitive ability of a rational agent who is able to deal with such an inference. The depth-bounded logics develop itself along a range starting with a 0-depth logic, that is, a logic which does not make use of virtual information, and increases *discretely* depending on how many k virtual assumptions are necessary to solve the inference. The limit as k approaches to infinity on this ideal line is represented by classical logic. Correspondingly, on the ideal line that measures the cognitive capacities of rational agents[34], an agent who would be able only to handle actual information is said the *minimal agent*, while, on the opposite side, the agent who would be able to solve every possible inference as classical logic in principle does, is said the *logically omniscient agent*. Naturally, *realistic agents* will be placed in the middle along the range, being characterized by bounded logical capacities at various depths.

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[34] "Idealization may well be a matter of degree" (D'Agostino 2010, footnote 2).

We might call them *k*-depth rational agents. On the next paragraph, I will illustrate the informational semantics formulated by D'Agostino to confer increasing degrees of strength to the logical consequence nexus (the classical one is the arrival point), with the background assumptions that rationality itself is a matter of degrees and that realistic agents approximate those logics in practice (2013, p. 20).

4.3. The semantics for Depth-Bounded logics

Taking a step back, we mentioned that the answer to what we called the "fundamental question" (FQ) is "no" even for intrinsic properties of formal systems. It is well known Church's theorem[35] (1936) about the undecidability of first-order logic, however D'Agostino puts out a similar problem for what concerns propositional logic:

> […] despite the existence of decision procedures for classical propositional logic, the widely believed conjecture that $P \neq NP$, makes it highly improbable that there exists a feasible one. So, again, there is no guarantee that we are in a position to *feasibly* recognize that the conclusion of a valid propositional inference is true in all informational situations in which we recognize that its premises are true. Therefore, if we construe the notion of "actually possessing" a piece of information as having access to it in practice, and not only in principle, a positive answer to (FQ, *A/N*) sounds highly counterintuitive even in the restricted domain of propositional logic (2015, pp. 1-2).

What we need, it is submitted, is to outline "an 'informational view' of classical propositional logic that naturally yields a sequence of tractable approximations" (p. 2). As already mentioned above, indeterminacy of sentences is an integral part of everyday reasoning: we are not always informed about the truth of the sentences we need to involve in our inferences. Therefore, the first step that such a semantics must take is to translate the classical notion of truth and falsity into the related *informational truth* and *informational falsity*. This information must be practically available to agents, who should be in the position to hold the information that *A* is true (or false) and operate with it.

Where does the indeterminacy come from in such a system? It is easy to see: while informational truth-values respect the Principle of Non-Contradiction – nobody, indeed, can at the same time hold the information that *A* is true and that *A* is false –, they break with the Principle of Bivalence: "it may well be that for a given *A*, we neither hold the information that *A* is true, nor do we hold the information that *A* is false" (D'Agostino 2015, p. 3). Consequently, the logical inference here can no longer
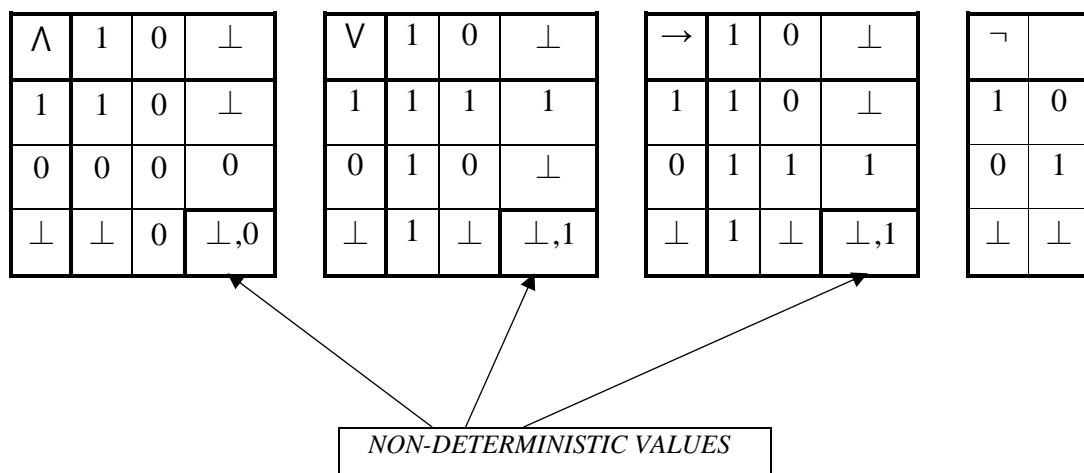
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[35] Church's theorem states that the set of valid formulas of first-order logic is not effectively decidable, namely, there is not an effective method for deciding which formulas among them are valid.

be understood as a truth-transmission device but, we might say, an informational-truth-transmission device, which necessarily has to take charge of indeterminate values. The way to fix such issue is to add the *informationally indeterminate* value, denoted by "⊥". Logical operators will be re-defined with the additional third value that sentences can assume. However, since D'Agostino wishes his semantics to offer a normative role in theory of rationalities, he excluded deterministic tables[36] because they cannot account for some of our intuitive judgements. Quoting Quine:

> Conjunction has its blind spot […] when neither component commands assent or dissent. There is no direct way of mastering this quarter. In some such cases the conjunction commands dissent and in others it commands nothing. This sector is mastered only later, in theory-lade ways. Where the components are "it is a mouse" and "it is chipmunk", and neither is affirmed nor denied, the conjunction will still be denied. But where the components are "it is a mouse" and "it is in the kitchen", and neither is affirmed nor denied, the conjunction will perhaps be left in abeyance […]. Alternation, like conjunction, has its blind quarter where neither component commands assent or dissent. We might assent to alternation of "it is a mouse" and "it is a chipmunk" or we might abstain (1974, p. 77).

Quine's observation leads to *non-deterministic* tables of informational values for the logical operators.

Here they are:

∧	1	0	⊥
1	1	0	⊥
0	0	0	0
⊥	⊥	0	⊥,0

∨	1	0	⊥
1	1	1	1
0	1	0	⊥
⊥	1	⊥	⊥,1

→	1	0	⊥
1	1	0	⊥
0	1	1	1
⊥	1	⊥	⊥,1

¬	
1	0
0	1
⊥	⊥

NON-DETERMINISTIC VALUES

What is relevant here is that the aforementioned 3ND-valuation describes "a minimal information state that is closed under the implicit information that depends only on the informational meaning of the logical operators". Therefore, this is information that we *actually possess*: "we have […] a *natural* and *feasible* procedure to decide, for every formula *A*, whether the information that *A* is true, or the

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[36] Kleene (1952, §64) had already tried to take charge of the indeterminacy of truth with apposite 3-valued tables but keeping the determinism of logical consequence. This is criticized by D'Agostino as insufficient to account for our intuitive judgments.

information that *A* is false, or neither of them actually belongs to our information state" (2015, pp. 4-5). The implicit information available under the closure of such a characterization of logical operators has a fork: either we dispose of *actual information*, or we need *virtual information*.

As we mentioned above, the depth-logics lower level, namely the *0-depth logic*, models those inferences that do not require virtual information, those inferences whose conclusion can be drawn without making hypotheses, thanks to the actual information we dispose. Let's expose briefly its semantics.

Firstly we need a characterization of formulae in informational terms: let be "*A, B, C, ...*" variables ranging over (unsigned) formulae, the *signed formulae* (*S-formulae*) *T A* and *F A* mean that "*A* is informationally true" (we actually possess the information that *A* is true), respectively that "*A* is informationally false". A 3ND-valuation *V satisfies* an S-formula *T A* if $V(A) = 1$ and a S-formula *F A* if $V(A) = 0$. Signed formulae will be denoted by "$\varphi, \psi, \theta, ...$", while "*X, Y, Z, ...*" will indicate sets of S-formulae.

> Definition 1: for every set *X* of S-formulae and every S-formulae φ, we say that:
> - φ is a *0-depth consequence* of *X* if *V* satisfies φ for every 3ND-valuation *V* such that *V* satisfies all the S-formulae in *X*;
> - *X* is *0-depth inconsistent* if there is no 3ND-valuation *V* such that *V* satisfies all the S-formulae in *X*.

The symbol $\vDash_0$ denotes the 0-depth consequence relation. The *unsigned formulae* will be the formulae without a 3ND-valuation, that will be denoted by variables "*A, B, C, ...*", or taking the unsigned part of φ by means of the symbol φ^u. Similarly, sets of unsigned formulae can be denoted by "$\Gamma, \Delta, \Lambda, \ldots$", or by X^u, taking the unsigned part of *X*, namely the set $\{\varphi^u \mid \varphi \in X\}$. Also unsigned formulae can be treated as 0-depth consequences:

> Definition 2: For every set Γ of unsigned formulae and every unsigned formula *A*, we say that:
> - *A* is 0-depth consequence of a set Γ if *T* $\Gamma \vDash_0$ *T A*;
> - Γ is 0-depth inconsistent if *T* Γ is 0-depth inconsistent.

Complexively,

> The 0-depth consequence relation $\vDash_0$ is a subsystem of classical propositional logic obtained by replacing the notion of 'possible world' with our weaker notion of information state (described by a 3ND-valuation). It is not difficult to show that the relation $\vDash_0$ is a *Tarskian consequence relation*, that is, it satisfies reflexivity, monotonicity and cut (2015, p. 5).

So far, we dealt only with actual information; yet, implicit information includes virtual information as well. Thus, now we will introduce the necessary semantics for

taking charge also of virtual information into depth-bounded logics. In the previous section we expressed the need for virtual information with the "train or shuttle" example. Now, let's give a formal evaluation of a different case:

$$\frac{A \lor B \quad \neg A \lor B}{B}$$

3ND-tables cannot justify such an inference, since we might have that $V(A) = V(B) = \bot$ and $V(A \lor B) = V(\neg A \lor B) = 1$. In order to achieve such a conclusion (B) we need to introduce a refinement of V by means of *virtual information*. The refinement call into question the Principle of Bivalence: indeed, what we are doing is to redefine the indeterminate value $\bot$ of A, with a determinate one. The only two determinate values are T ($V(A){=}1$) and F ($V(A){=}0$). Therefore:
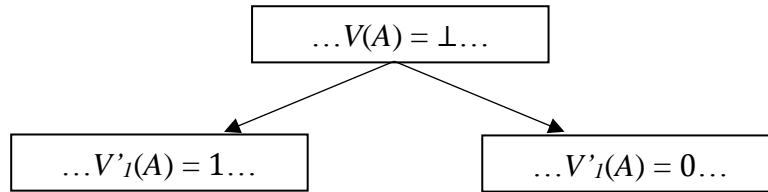
Now, using the 3ND-tables, it is easy to see that $V'(B) = 1$ for every such a refinement of V. Thus, we can say that we cannot solve this inference with the 0-depth logic, since we need to introduce once some virtual information, that is, we need a *k*-depth logic with profundity 1 ($k = 1$) to conclude B. I report below how D'Agostino characterizes the semantics of *k*-depth logics.

> The notion of *k*-depth consequence depends not only on the depth at which the use of virtual information is allowed, but also on the subset of Form($\mathcal{L}$)[37] on which the introduction of virtual information is allowed. […] this subset […] can be simply defined as a function *f* of the set $\Gamma \cup \{A\}$ consisting of the premises Γ and of the conclusion *A* of the given inference. Let $\mathcal{F}$ be the set of all operations *f* on the finite subsets of Form($\mathcal{L}$)[38]:
>
> Definition 3: For all X, φ, and for all $f \in \mathcal{F}$,
> - $X \models_0^f \varphi$ if and only if $X \models_0 \varphi$[39];
> - $X \models_{k+1}^f \varphi$ if and only if $X \cup \{T\ A\} \models_k^f \varphi$ and $X \cup \{F\ A\} \models_k^f \varphi$ for some $A \in f(X^u \cup \{\varphi^u\})$.
>
> When $X \models_0^f \varphi$ ($X \models_0^f$) we say that φ is a *k-depth consequence* of X (X is *k-depth inconsistent*) over the *f*-bounded virtual space.

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[37] Form($\mathcal{L}$) denotes the set of all the formulae of a Boolean language $\mathcal{L}$ with all the standards logical operators.

[38] Such that: (i) for all Δ, at(Δ) $\subseteq$ *f*(Δ), (ii) *f*(Δ) is closed under sub-formulae, that is, sub(*f*(Δ)) = *f*(Δ), (iii) |*f*(Δ)| $\leq$ p(|Δ|) for some fixed polynomial *p*, where we denote by |Λ| the number of occurrences of symbols in Λ (the *size* of Λ).

[39] Notice that the above definition covers also the case of k-depth inconsistency by assuming $X \models_0^f$ as equivalent to $X \models_0^f \varphi$ for all φ.

The transition from $\vDash_k^f$ to $\vDash_{k+1}^f$ corresponds to an increase in the depth at which the nested use of virtual information (restricted to formulae in the virtual space defined by f) is allowed.

Then, it is not difficult to show that:

Proposition 3.1. *For every f, the relation $\vDash_\infty^f = \cup_{k \in \mathbb{N}} \vDash_k^f$ is the consequence relation of classical propositional logic.*

Given a formal semantics for every k-depth logic, with k ranging from 0 to ∞, on the next paragraph we will try to join together such a formal side with the cognitive ability of realistic agents in dealing with inferences of different depths.

4.4. K-depth logics as normative for rational agents

The k-depth consequence, unlike the classical consequence, is not defined univocally, but in relation with the complexity of the inference, characterized in turn with the depth at which the use of virtual information takes place. This depth is also, as we exposed above, the measure that distinguishes the ability of realistic agents along an ideal line going from the minimal agent to the logically omniscient one.

Taking a step back, a psychologism based on the mental logic theory, as the one formulated by Lance Rips, must worry about two theoretical concerns on logic and deduction:

i) It must define logic with the inference rules possessed by humans in their cognitive architectures;

ii) It must retain the inference process ensured by a mental proof.

Now, as we have very briefly mentioned above, a mental logic of that kind is perhaps too demanding and unable to give account of the systematic logical errors made by reasoners in ordinary life. Nevertheless, also taking for fulfilled those two tasks, that is, adhering to such a psychologism, a more realistic mental logic could be assumed as represented by the set of rules introduced by D'Agostino in its proof-theoretical characterization of his k-depth logics.

These rules "are presented in terms of *S*-formulae, to highlight their correspondence with the informational semantics of the previous sections" (2015, p. 7). Such a set of rules is called *intelim*, acronym for *introduction-elimination*; indeed, except for the PB rule that will be presented later, the only rules necessary for the proof into k-depth logics are rules for introduction and elimination of the four standard logical operators: ¬, ∧, ∨, →. At the 0-depth logic level, *S*-formulae can assume two informational values, *T* or *F*, thus, *intelim* rules must consider only these two cases, so

that an exhaustive list will take into account introduction and elimination for each of the possible *S*-formulae (see 2015, p. 8, for the exhaustive list).
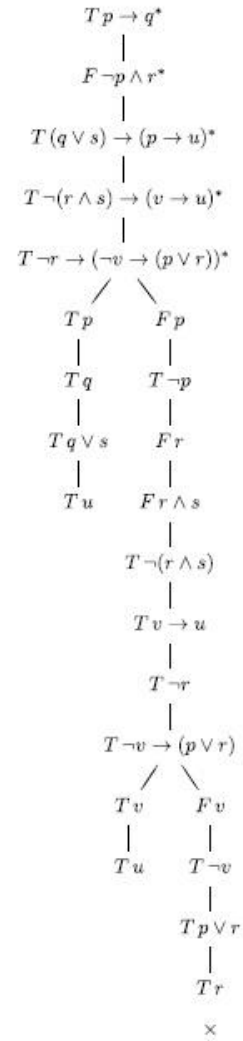
> Their soundness can be immediately verified by inspection of the 3-ND tables. For example, if an agent *x* actually possesses the information that $A \vee B$ is true (the value of $A \vee B$ is 1) and *x* actually possesses the information that *A* is false (the value of *A* is 0), then *x* actually possesses also the information that *B* is true, since the other possible two values are ruled out by the table for $\vee$. It turns out that the intelim rules are also complete for the 0-depth logic […] (D'Agostino 2015, p.7).

On the contrary, *intelim* rules are not complete for Boolean logics and subsequently for *k*-depth logics. What is required is the addition of the PB (Principle of Bivalence) branching rules: whenever we lack some information about a premise, that is, we neither are informed that *A* is true, nor that *A* is false (the value of *A* is $\perp$), then we need virtual information and we introduce two different hypotheses by means of a PB rule; we then go on towards a conclusion on both the branches until one is conclusive. Such a process is easier to be grasped by means of an *intelim* tree (of depth 2 on the right side).

D'Agostino is concerned at this point to define his non-standard proof-theory for *k*-depth logics (similarly for what happened at the semantic level, both $\vdash_0$ and $\vdash_k$), but we will not enter into the detail of this discussion. What is pretty clear, also looking at the *intelim* tree is how the inference process can be very hardly carried out when a huge amount of virtual information is required, so that correspondingly, since realistic agents are resource-bounded, "the capability of correctly recognizing inconsistency or logical entailment is a matter of degree" (2013, p. 4).

The depth indicated with *k* in such a system is a useful tool even to provide more realistic prescriptive models for rational agents. The starting problem, I shall recall, was the unrealistic assumption that classical logic (even limiting to the propositional system) could represent a good model of rationality for realistic agents. Now, with an infinite hierarchy of approximating models to classical logic, we can define in turn infinite prescriptive models to depth-related realistic agents.

As a matter of fact, reasoners fail systematically in carrying on certain deductive tasks. The assumption that frames the whole D'Agostino's work, that realistic "agents may be endowed with unequal deductive power […], but share the same understanding

$$
\begin{array}{c}
T\,p \to q^* \\
| \\
F\,\neg p \wedge r^* \\
| \\
T\,(q \vee s) \to (p \to u)^* \\
| \\
T\,\neg(r \wedge s) \to (v \to u)^* \\
| \\
T\,\neg r \to (\neg v \to (p \vee r))^* \\
\diagup \quad \diagdown \\
\begin{array}{cc}
T\,p & F\,p \\
| & | \\
T\,q & T\,\neg p \\
| & | \\
T\,q \vee s & F\,r \\
| & | \\
T\,u & F\,r \wedge s \\
& | \\
& T\,\neg(r \wedge s) \\
& | \\
& T\,v \to u \\
& | \\
& T\,\neg r \\
& | \\
& T\,\neg v \to (p \vee r) \\
& \diagup \quad \diagdown \\
& \begin{array}{cc}
T\,v & F\,v \\
| & | \\
T\,u & T\,\neg v \\
& | \\
& T\,p \vee r \\
& | \\
& T\,r \\
& \times
\end{array}
\end{array}
\end{array}
$$

of the logical operators" would also permit to consider with a finer-grain exam the systematic errors issue. Actually, one thing is to say that one or more agents with cognitive depth k fail an inference with depth $k + j$ (with $j \geq 1$), one thing is to say that the same agents fail an inference of depth $\leq k$. From a psychological point of view, the first case corresponds to what was expected in realistic agents, while the second would denote a problem in the way agents use their own resources. Anyway, whatever explanation psychology could offer of the systematic errors, from a normative perspective D'Agostino's solution represents a way out in the following manner: since the complexity of an inference determines the depth k of the relative logic to deal with it, that very k-depth logic will be the prescriptive model[40] for any agent with cognitive abilities of depth $\leq k$. That is, that very model is the *necessary* guide for any agent with lower cognitive abilities to the needed to solve the inference.

Moreover, one step further is possible. Naturally, any k-depth logic is at the same time able to manage every inference of depth $k - j$ (with $1 < j < k$), so that each logic along the hierarchy line from k to ∞ is a prescriptive model for an agent of cognitive abilities $\leq k$. This progression can also be made formal by means of the induction principle as follows:

> *Basis*: 1-depth logic is a prescriptive model for agents with cognitive abilities ≤ 1;
> *Step*: if k-depth logic is a prescriptive model for agents with cognitive abilities $\leq k$,
> then $(k+1)$-depth logic is a prescriptive model for agents with cognitive abilities $\leq k$.

The natural end point, then, is the only model that could be prescriptive for every possible agent whatever her cognitive depth is. This model, as D'Agostino explained several times, is represented by classical logic (see Proposition 3.1). As better explained in footnote 40, we reserve to classical logic model the term "normative", as an ideal standard which could guide every possible rational agent, even if it is hardly useful for realistic agents with limited resources. However, what once again we were aiming to show is how the normativity of logic can be accounted independently of the universality of patterns of inferential behavior. In the present framework, the normativity of logic results as the ideal top of a hierarchy of infinite prescriptive models for realistic agents with the aim of enclosing below any possible inferential

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[40] We are here terming these models as *prescriptive* following a well-reputed tradition in psychology of reasoning, where often the term "normative" is reserved to an ideal standard that would permit to solve every possible task in the field, but that would be hardly useful for a bounded-rational agent. Prescriptive models, on the contrary, take into accounts such limits and furnish a realistic guide for humans (see, i.e., Stanovich & West 2000, p. 648). This distinction fits well the D'Agostino's theory, since the hierarchy of prescriptive models has an arrival point in classical logic that could be considered that ideal normative standard for every possible rational agent.

prescription in the logical field, whatever the level of depth and difficulty. This represents in effect a third way for an LP defender to account for the normativity of logic without renouncing to its psychological foundation.

5. *Conclusions*

According to some critics, contemporary psychologism, even though amended from its old problems, is unable to account for the normativity of logical laws. The *universalistic* descriptions of cognitive psychology are not sufficient as a basis to define normativity: trying to found logic on psychology is simply committing the *naturalistic fallacy* of Humean inspiration. Whatever psychological basis an LP theory takes for this purpose, it will not be sufficient to account for logical norms for reasoning.

In the present work, however, I showed how such a critique is inconclusive: the defender of psychologism has many other cards to play. I illustrated three alternative ways out from the *impasse* that permit to solve the normativity of logic question. These approaches are not based on the universality of patterns of inferential behavior, even when such a uniformity is described by cognitive psychology. The first one denies the gap itself between the descriptive and the normative dimensions: logical intuitions are in themselves normative. The inferential competence of humans and the normative standard of logic are nothing but one and the same set of rules considered from two different points of view, factual and epistemological respectively. The other two approaches, although the many differences, refer to a value dimension: it is useful (good, desirable) to improve and standardize as much as possible the inferential abilities of rational agents. The one inspired to Marconi's works, in his late-Wittgensteinian spirit, assigns to logic an important role into communitarian contexts. The step to the normative dimension here is ensured by the benefits of an effective communication, which make sense to establish norms based on the most commonly spread inferential practices. Finally, the third approach, based on D'Agostino's papers, looks rather to individuals, who needs to be guided in logical tasks in order to go beyond their own limited resources shaped by psychology. Once again there is a value assumption (the usefulness of an improved ability of individual reasoning) as a steppingstone towards normativity. The normativity of logic emerges then as an ideal standard at the top of an infinite hierarchy of more realistic prescriptive models for rational agents.

I must recall that we focused here exclusively on the ways to connect norms to psychological inferential behaviors, but, of course, much work has still to be done if one wants psychologistic theories of logic to be made more effective and competitive. For example, to make maximally coherent and exhaustive an LP theory, the supporter should indicate if there is a specific *theory of reasoning* that suits better its doctrine, if such a logical psychologism leads to a ready-made formal system, or, on the contrary, if it will define a different set of rules which does not overlap any existing one. In this second case, perhaps additional epistemological principle might be required: how to choose the rules? How to fix eventual conflicts? What to prioritize: coherence (leading closer to the ideal perfect reasoner) or efficiency (getting closer to the practical needs of realistic agents)?

These and other questions need to be addressed before for an LP theory to be fully convincing. Nonetheless, I believe what has been argued in the present work as being a good basis to construct a stable building. All in all, I retain the initial objection from which we started to remain elusive: LP theories continue to deserve a place in the contemporary philosophical debate.

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