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What the Heck is Logic?

Logics-as-formalizations, A Nihilistic Approach

By

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Abstract

Logic is about reasoning, or so the story goes. This thesis looks at the concept of logic, what it is, and what claims of correctness of logics amount to. The concept of logic is not a settled matter, and has not been throughout the history of it as a notion. Tools from conceptual analysis aid in this historical venture. Once the unsettledness of logic is established we see the repercussions in current debates in the philosophy of logic. Much of the battle over the 'one true logic' is conceptually talking past each other. The theory of logics-as-formalizations is presented as a conceptually open theory of logic which is Carnapian in flavour and grounding. Rudolf Carnap's notions surrounding 'external' and 'pseudo-questions' about linguistic frameworks apply to formalizations, thus logics, as well. An account of what formalizations are, a more structured sub-set of modelling, is given to ground the claim that logics are formalizations. Finally, a novel account of correctness, the COFE framework, is developed which allows the notions of logical monism, pluralism and nihilism to be more precisely formulated than they currently are in the discourse.

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This thesis is dedicated to my parents and sisters. For, without their love support I would not have been able to get to where I am today.

Author's declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

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Chapter 1

Introduction

The concept of logic is unsettled. The tongue in cheek title of this thesis, “What the Heck is Logic?", refers to this unsettled nature of logic. Logic is about reasoning[1], but in what domain or context? This lack of clarity has led to a cluttered conceptual landscape in the philosophy of logic. The problem has seeped into discussions of logic and correctness. Examining the concept of logic is more than just conceptual house-keeping, it will bring clarity to debates in the discourse, and our theories of logic. A theory of logic needs to address the concept of logic, either by stipulation or by conceptual openness, in order to secure the foundations for following work. The aim of this work is to do precisely this: give a conceptually open account of logic and address the question of correctness and logic in turn.

1.1 A methodological aside

Doing philosophy of X, whatever X is (some domain or concept) there is a common methodology and issues which will have to be addressed, regardless of what X is.[2] They range from the conceptual, ontic, and epistemic to the methodological. What are the primitive concepts, and objects, of X. How do these these things relate? Are these concepts clear, or are there multiple notions of X at play?

The ontic status of X will be important, as it may impact our notion of Xing. Epistemically, the vector to which we gain knowledge of X will be important in the discussion of the philosophy of X. Particularly if the epistemic-ontic gap is wide. Regardless of what the X is, issues of these sorts will crop up; such is the nature of philosophy.

[1] This is not a wholly uncontentious claim, as hinted at in the abstract; there are some who refute this conception of logic as well, and it will be addressed directly in section 6.2.3.1.

[2] In the month preceding submission, when teaching a seminar on the philosophy of mathematics I discovered that Stewart Shapiro also uses the phrase ‘philosophy of X’ in his introduction to *Thinking about Mathematics*, though the approaches are not the same.

There may also be questions of the methodology of X, if it is discipline, e.g. science. When doing philosophy of X, one should be careful in such arenas not to legislate to those who do X, unless there are strong reasons to do so (Kouri 2016, p. 31). The notion of those who do X leads to another notion, when X is a domain of discourse, rather than a concept or object. If the aim is to do philosophy of X, and the theory of X can account for more of the activities of those who X, there is a sociological advantage to the theory. There are, obviously, also unique questions that apply only to certain Xs.

One of these unique questions in the philosophy of logic: what is the correct logic, or logics? This thesis will take on the conceptual matters and this unique question.

1.2 Dimensions of disagreement

Upon first blush, it may seem like there is a 'generic problem of pinning down logic'. This is not the case. There is a multitude of different ways in which disagreement over the concept of logic can be had; this highlights the unsettled nature of logic. I will briefly explore this space to give a flavour for how expansive it is, as well as indicate where further discussion can be expected. The differences explored in the coming pages are anything but exhaustive.

First we need to clear up a possible conflation between the concept, and study, of 'logic' and the class of mathematical structures called logics. The area of concern for this work is the former. Logics, as mathematical structures, are formal languages which have certain properties: soundness, completeness, etc. By themselves they are just another area of mathematical study. We often use these, however, in the study of reasoning, inference, and other such things, i.e within the study of 'logic' as a discipline. The debates invoked, and discussed, in the pages which follow are primarily concerned with this notion of logic, despite the discourse often being preoccupied with matters that have to do with the structures. This will often be invoked as the difference between the theoretical and technical layers. One way this is highlighted in the literature is by the distinction of the study of logics as pure or applied. The study of a logic for the sake of investigating it, its properties, and its relation to other systems is the pure study of logic, i.e. it makes no appeal to things external to it (Priest 2006a, p. 164). John MacFarlane unpacks the word 'formal' in three ways, the first being "[l]ogic can be treated purely syntactically, without reference to the meanings of expressions" (J. MacFarlane 2000, p. 31). The notion of pure logic will be taken to be this syntactic structural approach. Contrast this with the use of logic to spell out what a universal, or good, inference is, e.g. the inference rule *modus ponens*.[3] Here there is a notion of the logic being applied to the concept of inference, reasoning, etc. This is sometimes referred to as the distinction between pure and applied logics, in much the same way those terms are used in mathematics.

While this distinction helps clear up some confusion, it still puts the focus on the technical layer, rather than the theoretical. Having said that much of the disagreements have been such focussed.

Thomas Hofweber offers four main conceptions of logic in the discourse.

[3] *Modus ponens* states that if we have a true conditional statement (if... then) and the antecedent is true, then the consequent must be true: $R \to W, R \vdash W$.

1. the study of artificial languages
2. the study of formally valid inferences and logical consequence
3. the study of logical truths
4. the study of the general features, or form, of judgements

The first is an example of the pure study of logics. The second focuses on argument patterns, validity and 'what follows from what' (roughly logical consequence). The third presumes a notion of analyticity for a class of truths, which we discuss below. The fourth is a Kantian concern wrapped in the idea that logic is about what the central cognitive feature of the rational mind being that of judgements, thus seeing logic as the study of those features, and the form of judgements. These distinctions are interrelated and are not always clear cut, but are a good starting point as the discourse has followed these distinctions. The actual picture is more complex, as disagreements can occur across many vectors.

A fundamental way we can disagree on the concept of logic is the subject matter of logic itself. That is, we could disagree on what the bounds, or scope, of logic are. Is logic about the world, reasoning, or is it bound to language(s). Taking one of these stances will greatly change the nature of one's study of logic and to what arguments and debates they may find relevant. One can take a universalist or a particularist stance on logic. Many think the laws of logic to be universal, in all circumstances, if *modus ponens* is a rule, when we have a conditional and it's antecedent being true, the consequent is necessarily true. A particularist denies that validity is so rule-bound, instead seeing validity as a property of particular inferences, instead of rules (Payette and Wyatt 2018, p. 280). One can also view logic as universal language or calculus; this is the van Heijenoort-Hintikka distinction (Hintikka and Sandu 1994a, p. 279). On the former view, logic is universal, one is limited in what they can say about it. This is because there is no external place to look at the language and describe it. The truths of logic, then, are the truths of the world. When logic is viewed as a calculus (mathematical structure), we do not run into this problem. In mathematics we have a metalanguage in which to discuss structures and their properties. These views are very different in their conceptual pictures of logic.[4]

Along the same vein, one can be an antiexceptionalist about logic, or not. Antiexceptionalism about logic is taking the stance that logic is not special or in any way different from the rest of the sciences, in both theories and methods (Hjortland 2017, p. 632). This is pushing back against the notions of logic being analytic and *a priori*; its truths being knowable in virtue of its constituent parts, rather than via knowledge about the world. These notions of logical truths being *a priori* and analytic lead to debates about what the bearers of truth are.

On the more technical front, disagreements happen with respect to the logical constants. For example, first-order logic traditionally uses the connectives: negation 'not', conjunction 'and', disjunction 'or', conditional 'if... then' and, biconditional 'if and only if'. We can define the biconditional in terms of

[4]This is further discussed in section 2.4.2.

two conjoined conditional statements, and some systems do not have the biconditional as primitive. Technically, we only need the negation and one of the other connectives to define the rest.

This is fine from a pure logic point of view, but not straightforward from the position of ‘logic’. If logic is about natural language, then we have external grounds to claim whether the conditional, or biconditional, is primitive in logic. There is also the system of first-order logic with identity, where ‘=’ is added to the vocabulary, this helps with expressivity and with certain technical problems, but some may balk at its inclusion because of their underlying concept of logic.[5] These formal properties of logics are a contested area as well. This is sometimes put as the logico-formal boundary, or roughly ‘what makes something a logic’. Properties like soundness and completeness, or a recursively enumerable consequence relation are often invoked as demarcators. This seems fine from a purely mathematical point of view, as they seem to pick out class of mathematical structures. However, if one’s concept of logic requires such properties this becomes a theoretical, as well as a technical concern.

There are two types of technical study of logical systems: proof theory and model theory. The model theorists claim that model theory is intuitively primary, as it captures directly the meaning of the expressions being examined. The proof theorists see their area as primary as it captures the inferential patterns which constitute what our words mean (Peregrin 2008, p. 263). While they are two technical disciplines, their application to logic comes from different areas of concern, and possibly different conceptions, of ‘logic’.

There are different views on the nature of logical investigations. Some see logic as a prescriptive enterprise, it tells us how we ought to reason. Others see it as a descriptive one, logic clearly states the ways we reason. Most fall in the middle, ascribing both characteristics to the notion of logic. This has led to extensive discussion on the normativity of logic (Field 2009), (Kouri Kissel and Shapiro 2017), (Steinberger 2017a). Close to this debate is the difference between logic being about validity, or about good reasoning. These, again, are very different conceptual grounds at the foundation of our investigations.[6]

The reader need not worry if the above comes off more as a list of jargon to look up, rather what is needed is an appreciation that the notion of ’disagreeing about logic’ is multi-faceted and can mean many different things. The present work does not require much more than a familiarity with these topics and the use of logic, or other formalisms, in philosophy.

1.3 Structure of the work

In the first chapter, I establish the claim that the concept of logic is unsettled, and has been throughout history. The conceptual tools of polysemy, Friedrich Waismann’s open-textured concepts, and Rudolf Carnap’s explication are used in this historical and conceptual analysis (Waismann 1968), (Carnap

[5] For an interesting discussion on the lack of true logical primitives see (Varzi 2002).

[6] This brief discussion of the dimension of logical disagreement is not total, nor does it provide the detail owed by the fruitful debates on these lines. Interested readers would benefit from the following as starting points (Eklund 2012), (Gómez-Torrente 2019), (Hofweber 2018), (Russell 2019), and (Steinberger 2017b).

1962). Logic is a *multiply-open-textured concept*. In the second chapter I take this and examine the logical monism debate; the debate over what the 'one true logic' is, in much of the debate it is unclear whether these claims are really rivals, given the different concepts of logic at play within them.

The third chapter is a deeper dive into Carnap's ideas of the limits of what can, and can't, be discussed, due to the boundaries of our linguistic frameworks. That is, questions an only be asked externally to our frameworks if they are pragmatic; otherwise they are pseudo-questions, as there can be no fact of the matter to settle them, one way or another. Chapter four explores the nature of formalization, highlighting that it should be seen as a process, and the products of formalization should not be our sole objects of analysis. Formalization is a sub-set of modelling, with some structural requirements over and above modelling. It is is a process which takes some input, the concepts, properties and relations between them, and produces an output, a formal system. The properties of this process are as important for the analysis of formalization as the properties of the resulting formal system. This is an under-explored area in the literature. The nature of formalization is important as in chapter five the answer to the question "What is Logic?" is established. Here I present my theory of logic, logics are formalizations (LaF). That is, they are formalizations of reasoning within some target domain. LaF is a conceptually open theory, it does not dictate the correct concept of logic. It points out that the concept of logic at hand will inform the purpose of logic, which in turn informs the applications of logics, the formalization of this conceptual domain. The latter half of the chapter explores more novel logics, and approaches to representation, to show how the LaF framework accounts for them through its conceptual openness, and thus also has a sociological advantage as a theory of logic.

Chapter six reintroduces the notion of correctness. There are two key moves in this chapter. First, the notion of correctness is disambiguated, between the ontic notion of correct and the pragmatic notion of 'good'. The second is that a claim of correctness contains at least four sub-claims: conceptual, ontic, formal, and epistemic claims (COFE). The COFE framework is introduced, with both of these insights in hand, such that the various positions in the discourse that are currently known as pluralisms may not be so, given the notion of correctness which they appeal to.

Lastly, in the following pages you will see that I have stopped using the first-person 'I' in favour of 'we.' This is done consciously to bring the reader along on the dialectic narrative. I trust this will not be too off-putting to readers sensitive to these matters.

CHAPTER 2

THE CONCEPT OF LOGIC THROUGHOUT HISTORY

2.1 Introduction

The concept of logic is unsettled. In this chapter we will use some tools of conceptual analysis to show that this is the case, and has been so throughout history. The beginning decades of the twentieth century saw a boom in the use, and support, of *formal* methods and the newly christened first-order logic that represented the core of this formal approach, this has sometimes been called the beginning of the third 'Golden Age' of logic (Malpass 2017, p. 3). Not only did we see an expansive approach to the use of formal methods across philosophy, mathematics and science, but it also played a central role in foundational concerns. The efforts of Frege, Russell, Hilbert and others to settle the question of mathematics within a logical, and 'universally true', context were ultimately crushed by the famous incompleteness results of Gödel. The idea of formalised languages being the optimal end-state of analytical pursuits continued to build steam that has rolled through to the present day.

In any introductory logic course you will see the repercussions of decades of embracing this point-of-view. However, the idea of the primacy of formalised languages was not always the view of those working in, and with, logic. While we can trace a clear path through history that supports the roughly Fregean theory of logic, this doesn't reflect all views of logic as a concept, let alone an object of study. Here we must tread carefully otherwise we will fall into the trap of confirmation bias. That is, only seeing the prevailing narrative of logic, and thus missing the other concepts treated as 'logic' that have existed simultaneously (Ziche 2011, p. 243).

The notion of an 'unsettled core' of logic is one that has also been criticised elsewhere. Owen Griffiths notes that the notion of a settled core of logical consequence is at the heart of the logical pluralism of J. C. Beall and Greg Restall (Griffiths 2013, p. 171). Beall and Restall identify possible candidates of the settled core of logical consequence as necessity, formality and normativity, while also highlighting as potential candidates axiomatisability and lack of existential commitment, though they themselves push some doubt towards both. The core features of logical consequence are the ones central to the tradition, the former group is in the core because there is consensus, the state of the debate has (more or less) settled on these, while the latter two are still being discussed. Griffiths points out that all the tests for whether a feature is part of the settled core are empirical, not conceptual. That is, the tests put forward have been based on looking at various components/properties and determining if they line-up with the 'tradition' of logic to determine membership in the core, rather than justifying said membership (Griffiths 2013, p. 172).

There has been a recent trend in analytic philosophy to engage with *conceptual engineering* in order to glean insight into our philosophical pursuits, (Novaes and Reck 2017), (Tanswell 2018).

> In any inquiry, whether scientific or practical, we use concepts to frame questions about reality... It is clear that many great leaps in human insight and understanding have been associated with the forging of "better" concepts, which has enabled us to ask "better" questions. (ConceptLab 2016)

A reasonable question one might have is why should one look at the history of the concept, if one is not pursuing historiography and, if one is primarily concerned with the current concept at hand. Firstly, the concept has changed over time, and noting those changes helps inform our view of the current concept. Concepts are often not clearly defined, despite the fact they are often assumed to be in the common knowledge, so by studying the growth (and change) of the concept we can get a better handle of what 'logic' has meant, what it currently means, and what it does not mean. Secondly, if the concept itself is not settled then examining the history will help identify the parallel conceptual meanings which may be at play today.

To properly understand the progression of the concept of logic, we first discuss the nature of concepts themselves. In section two we will go over the notions of vagueness, ambiguity, and polysemy to understand the different ways a concept can be unsettled. We then introduce two ways of looking at concepts *dynamically*, open texture and explication.

With these tools of conceptual engineering in hand, we will briefly discuss the highlights of the history of logic, from a conceptual perspective. Section three will cover the pre-modern era of logic, roughly from Aristotle to Kant. Section four takes on the modern era of logic viewing the various concepts of logic through the primary distinction of logic as a universal language and logic as a calculus. We aim to show that logic, historically and currently, is a series of overlapping concepts which are themselves changing over time, rather than a continuously distinct concept. In our conceptual terminology, the concept of logic is multiply open-textured.

2.2 Concepts

The central claim of this chapter is that there are multiple concepts of logic, at different points in history and concurrently, even today. This is what we mean by 'unsettled'. Before we present any historical evidence to support this we need to be clear about the relationship between concepts and meaning. We first disambiguate between the notions of vagueness, ambiguity and polysemy before discussing Waismann's open texture and Carnap's explication and how they can fruitfully be used together to describe the change of a concept over time.

2.2.1 Vagueness, ambiguity and polysemy

If the concept of logic, or our use of the term 'logic', is a floating target, the first place to look is with the usual linguistic suspects: vagueness and ambiguity. In order to understand what the concept is, and how that might affect our discourse, we need to figure out just what is going on.

Vagueness and ambiguity both have to do with an uncertainty of the meaning, or the borders of the meaning of a term, or concept. The prototypical example of ambiguity is that of the word 'bank'. Consider the sentence "Your mother went to the bank." It could be the case that your mother went to a financial institution (bank) or that she went to the river's edge (bank). The use of the term is ambiguous between the two meanings, both are linguistically plausible. Now consider the two meanings of the word 'aunt', it could mean your father's sister, or your mother's sister. The two meanings can be turned into one 'parent's sister'. Here we don't have ambiguity, as the meaning is clear, 'parent's sister'. This is an example of under-specifying, leading to a sense-general sentence, not ambiguity. The meaning is clear, but it doesn't specify some detail which leads to an uncertainty of the meaning. In linguistics this type of sense-generality is called vagueness (Tuggy 1993, p. 273).

W. V. O. Quine devised a test for whether something is vague or ambiguous. Can the sentence "φ and not φ" be true? Thus, as "This is a bank, but it is not a bank" can be true, it is ambiguous, but "I have an aunt, but I don't have an aunt" can not be true, so it is vague (Tuggy 1993, p. 273). There are other sorts of tests, and the results do not always match up. For example, Quine's test will not work under a dialetheist approach to vagueness, but this gives a flavour for the differences between the terms and the space they occupy.

There is another sort of uncertainty of meaning, more familiar to philosophers. It is a vagueness which presumes an ordering. This is the grounds of the Sorites paradox. Consider a heap of sand. If we remove a grain of sand at a time, at what point will the heap stop being a heap. This vagueness is of unclear boundaries. Baldness is also vague in this way. Pinning down vagueness is a notoriously difficult affair, but it has to do with lack of precision in the meaning, with borderline cases: cases which are neither clearly in nor clearly out of the extension of the term or concept, like bald and heap. This notion of an ordering captures certain vague terms, and misses out on others. Consider the case of 'religion', there is a set of properties where if none are present a social practice is definitely not a religion, and if the practice has them all then it is certainly a religion. There can still be borderline cases in the middle, without an underlying continuum, where no possible line of inquiry could settle whether it is a religion or not. Borrowing from Pelletier and Berkeley, these two types of vagueness can be called *sorites vagueness* and *family resemblance vagueness* (R. Audi and P. Audi 1999, p. 947).

It is clear, even on first blush, that logic is not a concept that has sorites vagueness. Similarly, it doesn't seem like we are concerned with borderline cases when speaking of logic, except perhaps in establishing a clear set of formal properties which logics have, but that is a problem of a technical, not a conceptual variety. This leads us to the sense in which 'logic' does have an ambiguous usage. When we speak of logic we can mean the concept of logic, or a member of a type of mathematical structure we call 'logics'. Often, as the latter is used precisely because of its relation to the former, we can comfortably use them interchangeably, or ambiguously. Because we are concerned with the concept of logic and how that affects our discourse we will need to keep this distinction clear as we move forward.

There is yet a third option, *polysemy*. Polysemy is when a term has multiple interrelated meanings, in contrast to ambiguity which has more than one term, though they appear the same. Polysemy is a sort of halfway point between vagueness and ambiguity (Tuggy 1993, p. 277). The term is the same, but its use doesn't immediately pick out a clear meaning. Consider the verb 'to paint'. We can use the sentence 'she is painting' and it is unclear whether we mean painting with oils on canvas, watercolours on paper, or the walls of a room (Tuggy 1993, p. 238).

Wittgenstein describes a similar phenomena known as *family resemblance*. Where there is a cluster of ideas that are commonly supposed to be of the same type, but in fact they are connected by a series of overlapping similarities. One of the most famous examples is that of 'games'. There is a category of things we call games, but within that we have many different sub-types: card games, ball games, board games, competitive games, solitaire games, cooperative games, etc.

> And we can go through the many, many other groups of games in the same way; we can see how similarities crop up and disappear.
> And the result of this examination is: we see a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities. (Wittgenstein 2009, p. xcv)

Wittgenstein's notion of family resemblance is about not having a clear set of properties that pick out the entire extension of the concept, as with games. If we try to delineate all the properties of games, there will be some things we clearly see as games that do not have all those properties. Polysemy is related, as it is a cluster of meanings of the same term that grew out of each other, though here it is not a lack of clarity on specific properties, as is the case with family resemblance. Polysemy can be viewed as a cluster of concepts, which are related (historically and conceptually) and thus exhibit a family resemblance.

Our claim, supported by the discussion in the following pages, is that logic is polysemous. The concept has changed over time and currently has a cluster of meanings associated with it. This is similar to the claim put forward by Stewart Shapiro, though he aims at 'validity' and 'logical consequence' as cluster concepts (Shapiro 2014, p. 15). We aim for the central concept, but the polysemous nature of the one naturally leads to the polysemy of the others.

2.2.2 Open texture and explication

We are not only interested in the nature of the concept of logic and its current meanings now, we are also interested in how we got to this cluster of meanings of logic in the first place. For that, our conceptual tool-kit needs more than a static analysis, we are interested in the changes and precissifying the concept has undergone. Vagueness, ambiguity, and polysemy are terms which describe the relation between words and meaning as they are now, but we are also interested in how concepts change over time, both in terms of direct analysis and the process itself. We will discuss the dynamics tools of Waismann's open texture and Carnap's explication, briefly presenting both before combining them, along with polysemy, such that we have enough conceptual tools to accurately describe the status of the concept of logic.

2.2.2.1 Open texture

The notion of open texture concepts is useful to understand how the domain of a concept is not static, and changes over time. While Waismann himself did not provide a direct definition of open texture, he did clearly spell it out with examples. The most illuminating of which is of the cat next door:

> What, for instance, should I say when that creature later on grew to gigantic size? Or if it showed some queer behaviour usually not to be found with cats, say, if, under certain conditions, it could be revived from death whereas normal cats could not? Shall I, in such a case, say that a new species has come into being? Or that it was a cat with extraordinary properties? (Waismann 1968, pp. 121–122)

So a concept is open-textured if there are dimensions of the concept that are not currently settled, and cannot be until we encounter something that makes us question our current boundaries of the concept. Waismann notes that most of our concepts, especially the empirical ones, are open-textured (Waismann 1968, p. 121).

There are two important distinctions to open texture concepts. Firstly, that the concept is only delimited in some possible directions, and not others. That is, we do not have exhaustive definitions of our concepts–we can never be one hundred percent sure that some unforeseen situation which will challenge our definition cannot arise. Secondly, though perhaps less directly relevant to the concept of logic, is the idea of the "essential incompleteness of empirical descriptions".[1] Shapiro gives an interpretation of open texture, but it parts ways from Waismann's. The main difference is that Waismann's open texture is about the potential to expand concepts to new, or larger domains, while Shapiro's interpretation of open texture is more closely related to vagueness and is about sharpening concepts (Tanswell 2018, p. 884). Tanswell provides definitions for the two interpretations of open texture.

[1] In reference to seeing the cat next door, Waismann asks if that is enough to conclude that it is a cat, "[o]r must I, in addition to it, touch the cat, pat him and induce him to purr? And supposing that I had done all these things, can I then be absolutely certain that my statement was true? (Waismann 1968, p. 121)

> **Open Texture (OT1)** A concept or term displays open texture iff there are possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept or not.
>
> **Open Texture (OT2)** A concept or term displays open texture iff there are cases for which a competent, rational agent may acceptably assert either that the concept applies or that it disapplies. (Tanswell 2018, pp. 884–885)

Following Tanswell, we see Shapiro's interpretation (OT2), as much closer to vagueness and language usage rather than to concepts in general, as (OT1) is. There is no fact of the matter whether something currently falls under the domain, there are borderline cases where we may be warranted to make the claim on either side of the fence and it is about a competent agent's acceptable assertions on the matter. With (OT1) the application of an open-textured concept is about whether the term currently applies. As the concept changes over time, the open-texturedness is revealed. Consider the concept 'mother'. This is not a vague concept but through the progress of technology the open-texturedness becomes apparent.[2] Now we can differentiate between the mother who produces the ovum, the mother who carries the foetus to term, and the mother who raises the baby after its birth (Blackburn 2005, p. 261). With these distinctions one cannot point to the 'real' mother as the term has not been developed to yield a decision in any direction given the new circumstances. This is a sort of dynamic vagueness, it is the progression of the concept which causes the inability to know either way. Or perhaps we should say that when the open-texturedness of the concept is revealed we are left with an ambiguity between differing versions of the concept without the ability to pick out which version is the operative one, as it is not yet decided. We will take (OT1) as our working definition of open texture.

Now let us combine the notions of polysemy and open texture concepts. If a concept is polysemous then it is a cluster of interrelated concepts. As most, if not all, concepts are open-textured, and the members of the cluster are interrelated conceptually and historically, then each member of the cluster is probably also open-textured. So, for each meaning of the polysemous term there is an open question as to whether a possible object is outside the standard domain for which there is no fact of the matter whether they fall under the concept or not. Let us call this *multiply open texture* (MOT).[3]

We now have enough in our conceptual toolkit to describe concepts which are clustered and whose extension may shift in the future. We do not have a good explanation of those shifts. For this we turn first to Carnap and his theory of explication.

2.2.2.2 Explication

In the latter part of his life Rudolf Carnap developed his philosophy of (the methodology of) science around the notion of explication. Explication was introduced as part of Carnap's greater project on

[2] Pun, sadly, very much intended.

[3] We could have opted for the more conceptually accurate *polysemously open texture*, but instead chose the more phonetically clean option.

the logic of probability and inductive logic; given the reliance of science on empirical observations. Explication is a process of conceptual development that he described thusly:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an explication for, the earlier concept. (Carnap 1988, pp. 7–8)

> By the procedure of *explication* we mean the transformation of an inexact, prescientific concept, the *explicandum*, into a new concept, the *explicatum*. Although the explicandum cannot be given in exact terms, it should be made as clear as possible by informal explanations and examples. The task of *explication* consists of transforming a given more or less inexact concept into an exact one or, rather, in replacing the first by the second. (Carnap 1962, p. 3)

Explication is a theory about how concepts are refined and precisssified towards the end goal of greater understanding. Certain concepts can reach an exactness through explication. Consider the concept ‘two’, it is a clearly defined concept that has not changed much throughout human history.[4]

This does not mean that the end-state of all explication is some formal definition of the concept; not all concepts will be so lucky. Informal explanation and examples are the highlighted ways in which this refinement operates. He believed that very few concepts would end in a clear, fully-precisified, formal state; this is a theory of the sciences, or the empirical world, after all. It is also not a ‘one-and-done’ process, rather a constant evolution of the concept, “with each iteration of the explication process, we obtain an increase in exactness. But at each step, something is ‘transformed’ as well, resulting both in gains and in losses” (Novaes and Reck 2017, p. 202). That is, as the concept is refined there will be things that previously fell under the explicandum, but do not fit with the explicatum (and vice versa). Explication is a description of the process of concepts changing to become more precise, or rigorous. Carnap’s canonical example is the relation between ‘warmer’ and temperature. Warm is a qualitative explicandum, and ‘warmer than’ is comparative. We can think of temperature as an explicandum for ‘warmer than’. Temperature is a regimented and measurable concept so we can use it to make both qualitative and comparative statements (Carnap 1962, p. 13). Not all concepts will result in formal or quantitative states, but that shouldn’t hinder us to seek more refined qualitative explicans.

One thing that Carnap did not address directly was that explication is not a necessarily a uniform thing across communities, or humanity in general. We do not conceptually reason in a vacuum. As explication is an iterative process of concept refinement, we often do it in parallel with others in our fields (and in other fields as well). Ideally this would yield similar results, but we are not guaranteed that this will be the case.

[4] This is a touch of an oversimplification as the nature of numbers themselves, and the properties of their concepts have been in various stages of debate since at least Kant, but the gloss does little damage here.

Here we introduce the idea of *parallel explications* to capture this notion. A concept can be explicated in different ways, or directions, at the same time precisely because the concept has different meanings within a discourse, or community of individuals. If we have a polysemous concept, then as individuals explicate, they will do so on the terms of the meaning they take the concept to represent, and the difference in meaning will remain, even as the concept itself is refined. As we work to refine and replace a concept, one individual, or group, may choose one way, and another a different way. Until these groups get together and explicitly hash out what they each consider the concept to be and then come to a consensus, we will retain these parallel explications. Explication tells us how concepts can be refined towards being more useful, precise, and rigorous, but if the starting point is different, then there is no guarantee that these refinements will coincide.

Putting these two notions together, we see that while a concept is going through (iterations of) explication it will (tend to) be open-textured. If a concept is closed-textured, then for all objects outside of the standard domain there is a clear fact of the matter whether they fall under the concept, or not. If this is the case, then there are no matters, with respect to the domain, which would trigger further explication. If it is open-textured then the opposite is true. There can be objects, that we currently do not know about, which when we encounter them there is no clear fact of the matter if they fall under the concept or not. There could still be explication, which would further change the domain of the concept. Given that we can have multiple explications of a concept, in either the short or long term, this means that there is an ambiguity in the usage of the term. We are unsure if by 'logic' one is trying to pick out one explicated concept of logic, or a different parallel explication of logic. If logic is MOT, then further explication of the concept will not necessarily lead to being able to decide membership of a particular object, until we fix which concept of logic we are currently using.

If an open-textured concept goes through parallel explication then we will end up with an ambiguity between two (or more) open-textured concepts that are referred to using the same term. It is polysemous, so if we are not careful we will step into ambiguous usage. It is possible that the explications may yield a closed-textured concept, i.e. the explication cycle is finished, but this seems unlikely for a concept that has already split, and not rejoined.

We can see that open-textured concepts are concepts which will never have a fully settled domain, so they will always be part of some (potential) explication cycle. There is no platonic explicatum out there, which is the end-state of explication, for these concepts.

Returning to logic, we need to keep clear the distinction between 'logic' as a concept versus as an object of study. That is, we need to analyse the broader concept of logic, which is the target of study in philosophy of logic as compared to a specific formal system, say Frege's first-order logic. We can now state our claim; that logic is a polysemous concept which has had substantial changes in the 'standard domain of application' through various rounds of explication throughout history. These explications, however, have not been universal narrowings of the target phenomena of logic, rather they are parallel explications within their philosophical contexts.

In the end we conclude that all of these rounds of explication have not yielded just one concept of logic, but rather we are in a situation of overlapping concepts which bear a *family resemblance* to each other; logic is MOT. With this conceptual toolkit in hand it remains to be shown that logic fits these conceptual parameters, this is the aim of sections 2.3 and 2.4. First let us consider some of the ways the concept of logic can differ.

2.3 The concept of logic throughout history

We start our historical analysis with a brief discussion of the precursors to Aristotle, showing the foundation that led to his explicit treatment of logic as discipline and as a formal system, his *syllogistics*. We then proceed through the Stoics, medieval logicians and, finally, to Kant to show how the concept of logic has not been historically settled.

2.3.1 Pre-modern logic

The time preceding Aristotle showed an increase in clear, systematic, inferential reasoning. Zeno of Elea (500 B.C.E.), reasoned for the illusion of motion via a series of paradoxes to support the claims of his teacher Parmenides. His paradoxes were all attempts to show something (motion) didn't exist via contradiction, a proto-*reductio ad absurdum* approach. While this was clearly an approach using a specific inferential method, there is little evidence to suggest that Zeno paid the method itself much mind.

> [S]imply arguing logically is not sufficient to make one a logician. What is required in addition is an appreciation of the notion of valid inference in abstraction from any particular argument; and this, it seems, did not properly arrive before Aristotle. (Malpass 2017, p. 3)

The dialectical practices, of which Plato's dialogues are considered emblematic, of ancient Greek discourse represent the earliest documented attempts at developing the idea of the *deductive argument*. While these dialogues were not limited to deductive inferences, both psychological and rhetorical aspects were part of the mix, they represented a regimentation of general dialectical practices. In them Socrates shows his opponents that their position on the topic is flawed (Novaes and Reck 2017, p. 179).

The dialogues represent a nascent logical theory. Through the act of dialectical back-and-forthing arguments are made, questions asked, fallacies and inconsistencies shown. Here is the regimentation of concepts, and the use of regimentation in order to expose faulty arguments. Though not labelled 'logic' as such, here we see a hardening of the act of arguing, or reasoning through discourse if one prefers, that would find itself labelled as such under Aristotle.

2.3.1.1 Aristotle

The first person, in the Western philosophic tradition, to take on the nature and structure of arguments as a whole, rather than speaking of specific arguments, was Aristotle. With him the very notion of logic as well as the pervading views of logic for most of the following two millennia were set.

Not only does he introduce a more rigorous set of rules for argumentation but also he explicitly is studying the act, and nature, of argumentation. Here truly logic was born.

In the *Topics* Aristotle introduces the idea of a *deduction* as “an argument in which, certain things being laid down, something other than these necessarily comes about through them” (Aristotle 1984, p. 381). With the idea of a deduction in hand, he discusses regimenting dialectical discourse. Philosophical inquiry can be split into three categories, ethics, natural science, and logic. The subject matter of logic is the study of patterns of reasoning; matters of fact about the world are for the natural sciences. It is what is general about these patterns that is important to the logician. The key idea here is that of *necessitation*, the patterns show that regardless of what terms we are using, if the right pattern comes up, then we can conclude necessarily based on this pattern. These patterns are the *syllogisms*, with the familiar constituent premises and conclusions. The key to Aristotle’s logic was the simple technique of using letters to stand in for terms, the method of abstraction, which allowed one to grasp the logical form of the underlying argument without being concerned with the specific meaning of the terms (Rini 2017, p. 33).

Logic is the study of reasoning abstracted from individual arguments. Aristotle saw that there were certain structures of argumentation which guaranteed the truth of the conclusions necessarily. Put another way, with syllogisms, given the truth of the premises the conclusion cannot fail to be true–more commonly known today as *logical validity*. This insight of Aristotle’s remains the cornerstone of what we believe logic to be today, and he developed his *formal system* of the syllogisms to capture and develop this insight. We see that the formal system is one of abstraction, using term variables, and what is necessarily true, given the structure of the arguments.

Aristotle’s logical works were ordered and put together by Peripatetics in a volume called the *Organon*, or ‘tool’, which is the standard collection to this day. This titling was because the common thread throughout the works was that logic was not necessarily a specific part of philosophy, rather it represented an instrument of philosophy (Shields 2016). From the beginning, logic was seen as a tool to evaluate arguments in order to ensure the validity of conclusions. While a fruitful pursuit of inquiry in and of itself, logic is still a tool of the philosopher. The study of logic was in order to develop a tool for securing truth generally, not factually (Rini 2017, p. 35).

Here is a brief overview of the Aristotle’s syllogistic system, to give the unfamiliar reader an understanding of how the system works. A syllogism consists of three sentences, two premises and a conclusion. These sentences can be of four different types, based on the quantifier which occurs in them: *all (A)*, *no (E)*, *some are (I)*, and *some are not (O)*. Consider the following argument:

$$\frac{\begin{array}{c}\text{All men are mortal}\\ \text{All Greeks are men}\end{array}}{\text{All Greeks are mortal}}$$

The argument itself is self-evident, it is clear that the conclusion follows from the premises. Aristotle realised that there was something about the structure, rather than the content, which makes it so. His system is an examination of the various possible forms; two premise, one conclusion combinations of A, E, I, O and whether the conclusion necessarily follows or not. He does this by identifying four syllogisms, the perfect deductions, which are self-evident, and then proceeds to prove (through various conversion principles) or provide contradictions for the other possible syllogisms. The syllogisms abstract from specific terms, e.g. men, mortal, Greeks and replaces them with term variables, the meaning of the terms not being the object of study, rather the structure of the arguments themselves. The above example abstracts into one of the perfect syllogisms, *Barbara (AAA)*.[5]

$$\frac{\begin{array}{c}\text{All } A\text{s are } B\text{s}\\ \text{All } C\text{s are } A\text{s}\end{array}}{\text{All } Cs \text{ are } B\text{s}}$$

While the *Organon* is regarded as one complete work, it is in fact a collection of treatises that treat logic and deduction in different ways. As Hao Wang notes, *Categories* has a flavour of Kant's transcendental logic, *On Interpretation* touches on theories of truth and meaning, the *Prior Analytics* has to do with correct inferences, and some notions of modal logic, the *Posterior Analytics* suggest a study of scientific methods and induction, and the *Topics* can be seen as a treatment of the art of thinking. There isn't a clear way to formulate a unifying task of all of the *Organon* (Wang 1994, p. 262). Logic has been polysemous from the start.

2.3.1.2 The Stoics

The Stoics were the dominant force of Greek philosophy for the three centuries after the death of Aristotle, but they were not merely carrying the *syllogistic* torch. For them philosophy itself consists of three parts: physics, ethics and logic. That is, knowing the world, living according to the natural order, and distinguishing the true from the false (Ierodiakonou 2017, p. 51). This is similar to Aristotle's science, ethics and logic, though these concepts do not overlap completely.

This third rail of philosophy was *logos*, which covers reason in all forms of speech, was further divided into *rhetoric*, the art of speaking well in whole, continuous speeches and dialectic, conducting discussions by means of short questions and answers, aiming to discover what is true, and what is false. Dialectic was split into two areas of study as well: significations and utterances.

What resulted was a logical system for a entirely different class of argument than Aristotle's syllogistics. A logical system which is based on propositions, not terms. The purpose of logic was the establishment of a true and stable understanding of the world, which is essential for human beings to live a well-reasoned happy life (Ierodiakonou 2017, p. 51).

What is important to us is that we see a formal treatment of logic based on propositions, but also that the very conception, and role, of logic for the Stoics does not line-up with that of Aristotle's

[5] The other three are *Celarent*, *Darii* and *Ferio*. The names are a mnemonic shorthand, established by medieval philosophers, for the quantifier order in the sentences. The ordering of the vowels in *Barbara* is AAA, and so we get the syllogism with the 'all' quantifier in each line.

syllogism.[6] This change in the scope of logic, a widened explicatum, results in a very different formal system. Here, there are propositions which are either true or false, as well as *assertibles*, which are more complicated, with their own time and tense. It is not simply that the Stoics designed their formal system with different base elements, but rather it was designed with a different *purpose* in mind, as the target phenomena, logic, was a fundamentally different concept. Logic was not only about dialectical argumentation, as it was for Aristotle.

The Stoics explicated Aristotle's notion of *logos* into a wider ranging notion, which included rhetoric and dialectic. As we know that there wasn't a stable notion, of *logic*, coming from the *Organon*, we can say that one of these conceptual elements was explicated by the Stoics. What is important, for our purposes, is that the concept underwent a change for this influential school of thought, but there were still followers of Aristotle's teaching at the time. There were multiple conceptions of logic during this period, not just an explication from one to another.

2.3.1.3 Euclid of Alexandria

Euclid was a mathematician and not a logician; he did have a large impact on formal methodology. *The Elements* is one of the most important works in the history of mathematics. This is not only because of it collected and presented much of mathematics, and geometry in particular, but because of the way in which it did so. This is the earliest example of the axiomatic method.

> The deductive method cannot be considered an achievement of recent times. Already in the *Elements* of the Greek mathematician Euclid (about 300 BC) we find a presentation of geometry which leaves nothing much to be desired from the standpoint of the methodological principles stated above. For 2200 years, mathematicians have seen in Euclid's work the ideal and prototype of scientific exactitude. (A. Tarski and J. Tarski 1994, p. 120)

In these words Alfred Tarski highlights the importance of the approach and formal rigour Euclid used in order to outline truths of mathematics in the Elements. From a small set of axioms, he derived mathematical truths, or facts. Axioms are statements that represent the bedrock of a particular subject. That is, they are so basic (or fundamental) that they cannot be proved, but they must be assumed to be the case. We can then prove other mathematical truths by showing that they necessarily follow from the axioms, or from statements which themselves follow from the axioms.

This approach to mathematical knowledge has become the cornerstone of mathematical rigour. It also represents a different kind of formal system than previously, one that restricts mathematical knowledge to that which we can distinctly show comes from our basic assumptions. This would have a foundational impact on both logic and mathematics.

[6] We discriminate here between the general followers of Aristotelian philosophy, the Peripatetic school, versus just the formal logic which endured into the 19th century BCE.

2.3.1.4 Medieval logicians

While the medieval period is not known for its logical achievements, scholars focussed on the application of logic, specifically on the linguistic side of things.

> In the Middle Ages, logic was a linguistic science; it arose from a desire to understand how language is used (properly) in order to assist in textual exegesis. As a result, from an early period the study of logic was closely connected to the study of grammar (indeed, these two studies, along with rhetoric, made up the trivium, the branch of learning that was the core of the undergraduate's curriculum in the early universities). (Uckelman 2017, p. 72)

Much like the Stoics, here the matter of logic is bound up with that of natural language, and thus once again rhetoric is the permanent playmate of logic. Within the narrower field of logic the medieval logicians moved away from pure syllogistics to the study of consequence. Pinning down the exact notion of logical consequence is an exercise that will take some time. For the present purposes it can be thought of as the 'following-from' relationship. In modern parlance we have the following three relationships:

> *Implication*: When the consequent must be the case when the antecedent is true (in a conditional).
>
> *Entailment*: When a proposition must be the case if another proposition is true.
>
> *Inference*: Moving from the truth of one proposition (premise) to the truth of another (conclusion) in an argument.

The medieval logicians used the term *consequentia*, or 'follows with', to cover all of the above relationships. Although there is some degree of conceptual conflation going on, from the modern perspective, given these concepts have since been explicated apart, the key is that logic became the study of *bene consequentia*, rather than the study of Aristotle's formal system, the syllogistic, as it had been for centuries previous. The result of this focus were treatises which defined different notions of consequence and what makes a good consequence, and rules of inference which preserve this goodness. With the shift to consequence as the centre of logical work, these results closely mirror modern propositional logic (Uckelman 2017, p. 78).

The medieval logicians embraced part of the Stoic concept of logic, but explicated it into the realm of language and consequence. Even at this point in time the concept remained polysemous. There emerged two schools of thought on the nature of consequence: the English and Continental traditions. For the English tradition there must be a strong connection between antecedent and consequent.

> A consequence said to be formally valid is one of which if it is understood to be as is adequately signified through the antecedent then it is understood to be just as is adequately signified through the consequent. For if someone understands you to be a man then he understands you to be an animal. (Strode 1517)

This is important as here we are not relying on the mere form of the sentences. Semantic content is baked into the very notion of consequence. It isn't enough that a consequent is coincidentally always true when the antecedent is as well, there most be something more which binds the two propositions together.[7]

The Continental tradition, on the other hand, looks much closer to modern definitions, pointing at necessary truth, without the added caveat of some sort of relation. Here is Buridan:

> Some proposition is antecedent to another which is related to it in such a way that it is impossible for things to be in whatever way the first signifies them to be without their being in whatever way the other signifies them to be, when these propositions are formed together. (Uckelman 2017, p. 80)

Here we note two things. Firstly, we do not have the relational requirement, as with the English tradition, just that if the antecedent is the case there is no way that the consequent will not also be the case. Recall that for the medieval logician consequence is more than material implication, it includes the relationship between propositions and of arguments in general. This relationship relates to the final sentence of the definition: 'when these propositions are formed together'. As we are dealing with linguistic acts, and not abstract truths in general, if this requirement were not present then we would not get the truth of any consequent which was not uttered, even if it must be the case when the antecedent is true. Propositions can only have truth values when they are uttered, thus regardless of the consequence relation, if both propositions are not uttered there can be no relation between, or truth-value of, either.

With the medieval logicians we see a return to parts of Aristotle's notion of logic being about spoken arguments, and a move away from the Stoic tradition. We also see that this conception of logic has been explicated to highlight the role of *consequence* in logic and argumentation. Even in the stretch of time between ancient and medieval the concept of logic has not been static and underwent significant explications.

2.3.2 Kant's psychologism

Immanuel Kant left an indelible mark on Western philosophy. While he didn't directly yield large changes in the prevailing logical system, still Aristotle's syllogistics, he did fundamentally alter logic's place in philosophy.

Kant didn't propose any formal changes to logic, but put forth a conceptual shift in what logic is. For him, logic was not about ideal forms of argumentation. Logic is about the ideal internal reasoning of an agent; in Kant's critical philosophy logic moves from external discourse to a purely internal one. Both his general and transcendental logics are reasoning that occur within mind of an individual agent.

[7]Though dormant for quite sometime, we have seen a resurgence in this point of view with relevance logic and its proponents.

While his formalization of logic has been widely viewed as naive, or at least too narrow, his idea of its form(s) changed the conception of what logic is, and is prevalent to this day.

Kant thought that the world is made up of both concepts and objects, but that the two were fundamentally different categories, so we need two separate logics to reason in, general logic and transcendental logic. General logic looks familiar to us today, it is analytic *a priori* and general in the sense of topic-neutral or universal. Kant claimed that both mathematics and transcendental logic were synthetic *a priori*. For Kant, analytic statements were of the subject-predicate form, e.g. "All S's are P's". An analytic statement is one where the predicate concept, P, is contained within the subject concept S. Synthetic statements are all those that aren't analytic. Kant thought that general logic is ontically unrestricted, while transcendental logic is ontically restricted. This is because we are committed to the existence of specific categories of objects. General logic only deals in universals (forms) and so not with the intersection of form and matter, hence the need for transcendental logic (Tiles 2004, p. 102). Transcendental logic and mathematics rely on the intuition due to their synthetic nature.

Kant's theories highlighted three important features of logic. The distinction between concept and object, the primacy of the proposition as the unit of logical analysis,[8] and the idea of using logic to investigate the structure of logical systems, not just the validity of specific inferences (Tiles 2004, p. 85).

This internalisation affected the relation of logic to other fields. Pure general logic is uniquely distinguished from transcendental and special logics, mathematics and the special sciences in that it completely abstracts away semantic content (John MacFarlane 2002, p. 28). The heart of the human mind, for Kant, is the innate capacity for judgement, the cognitive faculty of the mind that yields all mental representations. This is Kant's psychologism.

This shift to the mental realm displays an explication to the open texture of logic, when confronted with the notion that reasoning wasn't just about language, as others in the past did, Kant explicated the concept to redefine the domain to, what he considered, the correct realm, the mind of rational human animals.

2.4 Towards modern logic

In this section we will map the progression of ideas that take us to the modern concepts of logic. We will discuss two major schools of thought on logic described by Jean van Heijenoort as *logic as calculus* and *logic as language*. We will see this split of the concepts of logic with the two founders of modern logic, C. S. Peirce and Gottlob Frege. Peirce and Frege independently introduced quantification and bound variables, the hallmarks of mathematical logic, with their formal systems. They reached these formal ideas from very different conceptual grounds, as we shall see in the following pages.

Irving Anellis describes seven features of the so-called 'Fregean revolution' which replaced Aristotelian logic with the modern mathematical logic.[9]

[8] This is distinct from the linguistic notion of proposition which the medieval logicians used.

[9] These features were gleaned form various writings and correspondences of the eminent historiagrapher of logic Jean van Heijenoort and most of them appear in (Van Heijenoort 1967).

1. A propositional calculus with a truth-functional definition of connectives, especially the conditional.
2. Decomposition of propositions into function and argument instead of into subject and predicate.
3. A quantification theory, based on a system of axioms and inference rules.
4. Definitions of infinite sequence and natural number in terms of logical notions (that is the logicization of mathematics).
5. Presentation and clarification of the concept of a formal system.
6. Relevance and use of logic for philosophical investigations (especially for philosophy of language).
7. Separating singular propositions, such as “Socrates is mortal” from universal propositions such as “All Greeks are mortal.” (Anellis 2012, p. 343)

Anellis notes that these, fairly uncontroversial, characteristics apply to the formal systems of both Frege and Peirce. We note however, that with the exception of the sixth feature, relevance for philosophical investigations, all of the above characteristics are based on technical features of the revolution and there is no mention of the conceptual shift that came along with the modern conception of logic. In the following pages we see that the two candidate systems of Perice and Frege are birthed from very different philosophies of logic. The fact that they both satisfy these requirements despite their different conceptual approaches is evidence of this narrow look at the technical features.

2.4.1 Alternative discoveries

Paul Ziche discusses the work on logic during this time, by Frege, Peirce, and others. Ziche’s claim is that the general narrative of the development of modern mathematical logic is too simple, too narrow. He introduces two ideas for the discussion of concept development: *Multiple Discoveries (MD)* and *Alternative Discoveries (AD)*. Multiple discoveries are when the same phenomena is discovered simultaneously by multiple people, this may be any stock element of science: theories, laws, observations. The standard example, from Thomas Kuhn, is the principle of energy conservation. Completely independently, and using entirely different approaches, both James Joule and Robert Mayer arrived virtually simultaneously at the conclusion that is known as the principle of energy conservation (Fowler 2002). Multiple discoveries are often discussed by historians, rather than the ‘discoverers’ themselves. This seems like a good description of the development of first-order logic by Peirce and Frege, when cast against the feature list of Anellis.

Alternative discoveries are a different case, as they are explicit claims of discovery by the protagonists themselves. The alternative claims tend to emphasise the uniqueness, and thus the differences, of their alternative and the others. This is important when we are speaking of conceptual, rather than technical, or empirical, matters, as we cannot simply adjudicate between accounts by appealing to the external. The competing claims have to be viewed as stages of an open-ended process which would contribute to a diversified conception of rationality, rather than an integrated picture (Ziche 2011, p. 243). This open-ended branching concept maps well to our idea of MOT. That is, alternative discoveries come from parallel explications. As they don't end in the unity of a concept the concept will be polysemous, either remaining so or becoming polysemous as a result. Ziche identifies the following characteristics of modern (mathematical) logic:

i. It is based on mathematics (especially the developments of 19th century mathematics)

ii. It incoporates a theory of relations (going beyond syllogistics)

iii. It aims to develop a scientific discipline that is foundational within the sciences. (Ziche 2011, p. 245)

We previously criticised the criteria of Anellis for leaning too heavily on the contextual, while ignoring the conceptual changes of logic– a curious case for a 'revolution.' The first of Ziche's criteria is implicit in Anellis' criteria, the majority of which are technical (or formal if one prefers). The second is not explicit, however the discovery of quantification is very much related to the logic of relations. The third criterion is one of the view of logic's role within the sciences, left out by Anellis but important when we consider the conceptual expansion involved. Aristotle's commentary on the importance of syllogistics in the *Prior Analytics*, and his acknowledgement of induction as part of logic and reasoning, would seem to satisfy this, at least partially. There is a question of what would count as 'foundational', but as our aim is to highlight the conceptual differences between these discoveries perhaps a broader interpretation of foundational makes sense here.

Ziche uses these criteria to show that there were others working at around the same time who pursued an expansion of logic that follows the above, though they veer significantly from that of Frege. Here we discuss one as illustration, that of Wilhelm Otswald. Otswald saw logic as the first and most general science, more fundamental than mathematics. For him logic was about the problem of concept formation and the task of creating new types of languages, for absolute clarity in communication. With the new algebraic mathematics Otswald saw a general framework that could be applied to structuring concept formation (Ziche 2011, pp. 247–248).

This description is very similar to that of Frege and his aim for mathematics. The primary difference being Otswald doesn't go so far as to say that he is aiming for the logical structure of of thought, rather that we can be perspicuous in our communication of scientific (mathematic) discourse by way of languages which use these new mathematical innovations. Otswald aimed at the same set of mathematical tools that both Peirce and Frege use, but his reason to do so is somewhere in the middle.

Common to all the protagonists was seeing the developments in algebra and the theory of manifolds as proof that a new logic could be developed, and with it (perhaps) a re-assessment of the foundations of mathematics.

2.4.2 The van Heijenoort-Hintikka distinction

In his historical work on logic Jean van Heijenoort drew a sharp distinction between two traditions: Logic as a calculus, *calculus ratiocinator*, and the universality of logic, *lingua characteristica*.[10]

Logic as a calculus can be said to have been born out of Boole's algebraic work on logic. The universality of logic stems from Leibniz', then Frege's, quest for the universal language of rational thought. In this view there is a plurality of methods and languages, and that they are re-interpretable according to context (or purposes) at hand. As such they allow for many universes of discourse as well as modal and intensional considerations. Hintikka points out that this view is the heart of the model-theoretic tradition in logic, and points to Peirce's work on modal logic, via his Gamma graphs, as evidence of Peirce belonging to this camp (Hintikka 1997, p. 149).

In contrast, the universalist position holds that there is one logic 'to rule them all' and as such there is only one universe of discourse, mainly the actual universe. This is Leibniz's language of thought, which also means that our thoughts are bound by the rules, or expressiveness, of such a logic. This tradition is often associated with proof-theory, as the study of proofs are done within the formal system and do not shift the domain of discourse in the process. Hintikka suggests that the historical primacy of the Frege-first view is a symptom of the larger prejudice amongst logicians against the model-theoretic and betrays the bias towards the universalist tradition (Hintikka 1997, p. 143). We will go through the specifics of these views, and how the conceptions of logic are still open within them.

2.4.3 Logic as a universal language

The logic as language tradition started with Leibniz's search for the universal language to express mathematics in. The tradition continued with Frege's ground-breaking, though ultimately doomed, logicist program, and still has a large impact on how we see logic today. The logic as a universal language concept was very much a response to Kant's psychologism. Relegating logic, and mathematics, to the boundaries of the rational mind was seen as a demotion and logic and mathematics are about more than that, there is a realist claim for both at the centre of this conception of logic.

2.4.3.1 Leibniz

Leibniz perceived that algebra, geometry and logic all shared a key element, they used formal reasoning patterns which enabled one to draw conclusions about all instances of reasoning covered by their formal statements, or equations. Once we become aware of these patterns, this could be done in a mechanistic manner. As logic is topic-neutral, it is the most general discipline, and so he spent most of his life trying to capture the rules of logic in a mathematical structure, the universal language of mathematics (Maat 2017, p. 103).

[10] Here we use the cumbersome sounding 'universality of logic' to steer clear of confusion with 'universal logic' a more recent field looking at the common features of all logical systems.

For Leibniz, the key property of logic was its formal and systematic nature. The big Leibnizian move was seeing the gains for mathematics such a thing could have, this 'universal mathematics'. He pursued the goal of mathematising logic in order to secure this. Syllogistics were only one part of this universal language, and he attempted to secure a theoretical foundation for this logical system.

Unfortunately his contemporaries did not share this view and much of his logical writings were only discovered, and thus discussed, posthumously. Using the clarity of mathematics, Leibniz aimed to arithmetise logical inference. That is, he pursued the goal of formalising logical inference using mathematical tools such that he could gain the power of purely formal systems in examining inference. Logic was the most general form of study of patterns of reasoning, and that there was a language out there which was a total universal form of this.

2.4.3.2 Frege

The basis of the modern conception of logic was birthed in Gottlob Frege's 1879 *Begriffsschrift*. Frege's work set the stage both conceptually, and technically, with the help of predecessors De Morgan, Boole, Peirce to name a few. Here was (one of) the first formal treatments of logic that married propositions and quantification; the two branches stemming from Aristotle and the Stoics respectively. All seven of Anellis' features, unsurprisingly, come out of Frege's work, hence the moniker 'Fregean revolution'.

For Frege there is no separation of mathematics and logic, indeed the truths of the former are merely truths of the latter in disguise. Frege's main goal was to show that arithmetic and analysis were purely analytic. Following on from Leibniz, Frege's aimed to develop the universal language of thought. We can see this right in the subtitle of his landmark work *A formula language of pure thought, modelled upon that of arithmetic*. The language of pure thought is precisely the language of pure reason. As such, Frege was firmly an anti-psychologist. He saw this universal as normative, how we ought to think and reason; rather than how we actually do. There is no space for psychology in logic; Kant had led us astray.

This idea that mathematics is reducible to logic is called *logicism*. Logicism comes in several flavours, and have been applied to various areas of mathematics, including the discipline as a whole, but there are two core principles common to them: all of the objects in the subject matter of these branches of mathematics are logical objects, and logic is capable of defining the primitive concepts of these areas of mathematics, thus the first principles will be results of logic alone (Walsh 2014, p. 86).

Specifically, numbers are logical objects. Though they lack spatio-temporal properties, they are fully objective, and as such are the right kind of objects for us to issue judgements about them. The notion of judgement is central to Frege's view, stemming from Kant, as the act of judgement is the act that generates concepts– including the inferential relations of thought (Pedriali 2017, p. 188).

His project was two-fold. Firstly, to provide a foundation of mathematics in the laws of logic. That is, to show that arithmetical truths could be analysed in purely logical means and thus show that arithmetic lay firmly on logical foundations. The second task was an epistemological one, about why we are justified in taking mathematical statements as true.

For his second project the answer was clear: proof. We get our justification for our mathematical statements by showing proofs that show that they are true. Not just any proof will do, however, the proofs must be gap-free. There can be no intuitive leaps, nor ambiguous steps. The manner to which these gap-free proofs can be provided is his logic, his universal language of pure reason. This language is not to be taken as a model of the mental processes mathematicians go through when reasoning, and proving. This language brings forth the justificatory structure of the given theorem (Pedriali 2017, p. 189). It shows the justificatory grounding of the mathematical theorem. While we see Frege's logic as the progenitor of modern logic it would be incorrect to say that they were equivalent. This is for both conceptual and mathematical reasons.

Conceptually, Frege was a universalist. There is one ever-reaching logic and the newly invented quantifiers ranged over the entirety of existence. The task at hand, as he saw it, was the Leibnizian idea that there is some set of universal inferences, and relations between them, and attempt to provide a formal system which captured precisely this structure of truths.

This universality is what led Frege, amongst others, towards proof theory–the study, and development of, deduction systems for the purposes of proving theorems. This universality was also a limitation on his concept of logic. For Frege the ontological furniture of the universe divides into objects and functions. In his system, as a result, the quantifiers that bind variables range over all objects in the universe; his universe is the universe. It is fixed (Van Heijenoort 1967, p. 325).

There are two major repercussions of this view. The first is that every function has to be defined for all objects, so '+' isn't just defined between the numbers but also between the Moon and 1. The second is that there is no way to speak outside of the system (Van Heijenoort 1967, p. 325). Frege did not engage with any meta-issues regarding the formal system, as that is not a coherent notion under his conception of logic. This is what led to his failure to understand Hilbert's axiomatic theories for different branches of mathematics. The axioms allow one to get a better understanding of the nature of mathematical concepts by looking at the corresponding formal systems and their metalogical properties (Hintikka and Sandu 1994a, p. 280).

As Frege's aim was for a universal system, the notion of interpretations examining models, or specific interpretations, would be a restriction in the face of the goal of the project. This does show the stark difference between what is considered modern logic, and the philosophy of logic which Frege held. Frege's system comes along with an interpretation already baked in, something he saw as a clear substantial improvement over the work of Boole and others (Van Heijenoort 1967, p. 325). Modern notions of logic hold logical consequence at the heart of logic, and this concept is defined in terms of truth in all models. This is a fundamental difference between what we consider logic and what the 'father of logic' held as the nature of logic (Pedriali 2017, p. 186).

In the end Frege's logicist project was doomed, and he admitted as much when confronted with Russell's paradox which showed that he could not give a fully logical grounding for mathematics as his Basic Law V led to an incurable contradiction.[11] This did not spell the end of his conception of logic,

[11] We go through a rough sketch of Russell's paradox in chapter six, section 6.3.1.

and there are neo-logicists working to revive the project (Hale and Wright 2001). The property of the universality of logic is alive and well, and informs much of the discourse of logic, as we shall see in the coming chapters.

While it is clear that the momumental shift of both the accepted concept and the practice of logic was at the watershed of the *Begriffschit*, it is also clear that Frege's concept of logic has not moved forward through the decades as his technical innovations have.

2.4.3.3 Russell and Gödel

Bertrand Russell continued Frege's project, after collapsing it with his paradox. Russell was engaging in a development of a foundation of mathematics, which was his type theory. He avoided his own paradox by differentiating between sets and classes, as different types, and stipulating that one could not refer to one's own type, but only the levels below. He explicated the general notion of class into more precise notions of set and class.[12]

The universality of logic remained, however. Its objects remained the actual objects of the universe. This lead to Russell needing to stipulate the axiom of infinity, in order to retain mathematics beyond the finitely many objects which exist in the universe. In the *Principia Mathematica* there was no discussion of metalogical properties (Van Heijenoort 1967, p. 326). Russell explicated logic along a different path than Frege's logicism, while retaining the universality of logic.

Kurt Gödel shared Frege's beliefs that logic is primarily a theory about concepts and that set theory is a part of logic (Wang 1994, p. 269). He noted that there are two fundamentally different versions of Russell's paradox. There are two ways we can construct a set. We can start with a collection of objects and conceive of them together to form a set, the *extensional* concept of set. Or, a set can be conceived of as the extension of a concept or property such that all and only the objects which fall under that concept are in the set, the *intensional* concept of set (Wang 1994, p. 267). Frege assumed implicitly that every set is an extension of some concept, thus set theory formed a theory of concepts. It is this assumption that lies at the heart of the problem of Basic Law V, which leads to Russell's paradox.[13]

Gödel observed that Russell's paradox is no longer a concern for set theory, as it is based on the extensional concept of set and we have the set/class distinction. Russell's paradox is still a problem for a theory of concepts, as that needs the intensional conception of set, which will lead us to back to the paradox. Frege overlooked the fact that in general the extension of a concept is quantitatively underdetermined (Wang 1994, p. 268). Wang described Gödel's concept of logic as matching Frege's with the exception of following Cantor's assumption that sets are primitive objects in the same way the general objects of logic are. Thus, "we have a fairly well-developed set theory but no mature theory of concepts" (Wang 1994, p. 270).

[12] In fact he defined an infinite number of types, each with an infinite set of levels between in order to achieve this distinction.

[13] We note that our notion of open texture inhabits this same space, and is somewhat confirmed by Russell's paradox. If a concept is open-textured, as most concepts are, then the extension of the concept is not a fixed thing, by its nature.

2.4.3.4 Logical realists

The idea of logic as universal is undergoing a revival with the logical realists. Logical realists take a realist position on the ontic nature of logic. The rules of logic are out there knit into the fabric of the universe, they are part of the privileged structure of reality (McSweeney 2019, p. 1). Because of this metaphysical grounding of logic the logical realists also make a claim that there is ‘one true logic’, there is a single logic that is objectively correct. We bring this up to show a current conception of logic that is based on the universality of logic, and foreshadow the discussion of correctness in the coming chapters.

2.4.4 Logic as a calculus

The logic as a calculus view is an instrumentalist view on logic, or logics. In contrast with the universal language, this view doesn’t ascribe meaning, as Frege wanted of his system. This was Frege’s complaint with Boole’s logic, it was abstract and the propositions remained unanalysed (Van Heijenoort 1967, p. 325). Another way of saying this is that there is no ontological weight to logic as a calculus. This is a very different notion of the topic-neutrality of logic. Whereas the logic as a universal language conception gets topic-neutrality from its complete universality, logic as a calculus remains topic-neutral by avoiding meaning entirely. This is in the spirit of Aristotle’s initial insight of abstraction; the use of the letters in the place of terms.

There are two important distinctions for the logic as a calculus view. Firstly, as they are not universalists, they can set the domain of discourse, and speak outside of the system, about the system. Here we can speak of metalogical characteristics, like completeness, consistency, independence of axioms etc. (Van Heijenoort 1967, p. 326). The second is its relation to mathematics. If logic is a calculus, then it has the same status as other mathematical structures. Logic is part of mathematics, and not the other way around. There will be no foundational grounding of mathematics in logic here.

2.4.4.1 Peirce

Charles Saunders Peirce developed his first-order logical system as part of his greater philosophical program. Peirce was not simply aiming to create a more rigorous formal system of logic, but also to extend the horizons of logic itself. This expansion of logic was very much tied to his other philosophical programs of semiotics and pragmaticism.

From his study of the medieval logicians, Peirce saw logic as part of the *trivium*, along with Grammar and Rhetoric. Though the terms his used for this three-way distinction changed over the years, to mirror his conceptual shifts, logic (from the medieval dialectical) remained constant within (Ohstrom 2017, p. 165). This knowledge of the history of logic left Peirce with the idea that logic was not a static enterprise and that a new logical system could clearly be developed. Peirce criticized Kant and his table of categories, not because of their dependence on formal logic but he believed the formal logic which Kant used was poor and not expressive enough.

Peirce, heavily influenced by his father, viewed mathematics as a core discipline and sought to use mathematics to expand logic. Logic was very much a subordinate to mathematics in Peirce's eye. The recent developments in mathematics were the inspiration for him to try and develop a broader, more rigorous, formal logic. His initial foray into logic was to precisely "inquire whether it cannot be extended over the whole realm of formal logic instead of being restricted to that simplest and least useful part of the subject, the logic of absolute terms, which, when he wrote, was the only formal logic known" (Peirce 1870, p. 318).

Peirce saw that formal logic had a potential larger scope in the theory of relatives. That is, deductive reasoning involves the relations between objects and with newer mathematical advances, like the work of De Morgan on relations, Boole's system could be extended to include these relations.

For example, up until this point, formal logics used terms with the aim of picking out properties. Aristotle's syllogisms abstracted terms such as 'Greeks', 'mortals', and 'Socrates' replacing them with term variables. Note that terms could pick out a specific object, Socrates, or all the objects that had some property, Greeks. However these properties only extend to that group of objects, there is no way to talk about more than one group of objects that are related by some property. That is, we do not have the expressibility in logic, yet, to say something like "Socrates taught Plato". This is the theory of relations which Peirce wanted to extend formal logic to include. For Peirce this move to the logic of relatives was an expansion of formal logic, not merely an extension of it (Hintikka 1997, p. 149).

The other major technical achievement that earmarks modern logic is quantification. For Peirce quantification came out of his work on relatives. Quantification was a part of his endeavour to expand the scope of formal logic, beyond that of Aristotle's logic. In his 1870 paper, *Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus*, he first presents his theory of the logic of relatives, including his treatment of quantification and bound variables. However, this presentation of his theory was not a full axiomatised formal system, we would have to wait until the 1885 *On the Algebra of Logic: A Contribution to the Philosophy of Notations* paper for that.[14] With this theory of relations one can also express the more complicated sentence "Every parent is older than their children".

Peirce's view of deductive logic as part of a greater area of philosophy is key to understanding his conception of logic. Formal logic is about deduction, the study of inferences given the facts, or propositions, which we already know. Induction is the act of reasoning from what we have evidence for, but cannot conclude necessarily. This is one of our primary modes of reasoning in our everyday lives. Indeed most would describe induction as the heart of scientific discovery. Peirce went further, showing that there is, in fact, a third mode of reasoning *abduction* which is the logic of scientific discovery. This is a return to the wider concept of logic of Aristotle's, along with an explication of induction, splitting it into the two different modes of reasoning.

Peirce viewed mathematics as the pinnacle of deductive reasoning. His father defined mathematics as the science which draws necessary conclusions. Peirce followed this up with deductive logic is the

[14] This six year gap between the publication of this paper and Frege's *Begriffsschrift* is what ultimately yields the 'initial discovery' to Frege, though Peirce had no knowledge of the work of Frege at the time, and neither Frege of Peirce's work.

science of drawing necessary conclusions. Put another way, mathematics is the practice of deduction while logic is the study and analysis of deduction. As such, logic does not provide any insight or justification for deductive reasoning. Deduction is firstly mathematical in nature, not logical. He claimed that logic was the theory of all reasoning, while mathematics is the practice of a specific type of reasoning (Haack 1993, p. 45). The logic he is speaking about is the overarching subject matter that includes both induction and abduction, rather than the more narrow deductive logic we have been speaking about since Aristotle. Deductive logic is the logic of mathematics, whereas inductive and abductive logic are the logics of the sciences.

Logic is a normative, not a descriptive, science. Logical necessity is one of non-empirical facts, not the necessity of thinking. Psychology is in need of logic, and not the other way around. Peirce's pragmaticism and methodological pluralism led to the idea that in logic we don't fix in advance what may, in the future, be considered to fall within the scope of logic (Haack 1993, p. 45). Here we see that Peirce's view of logic was an open-textured view.

2.4.4.2 Brouwer and intuitionism

L. E. J. Brouwer wrote his dissertation "On the foundations of mathematics." Here he took an anti-universalist approach, denying the logic as universal language interpretation. Mathematics is independent of mathematics, and logic depends on mathematics. Because of the use of logical methods, mathematics has been led astray in some areas (Wang 1994, p. 271).

For Brouwer the words of a mathematical demonstration accompany a mathematical construction, so Frege and Russell have it exactly backwards. As they are dealing with languages, without an accompanying mathematical system, there is no possible notion of a contradiction. Based on this idea of mathematical construction Brouwer critiqued the law of excluded middle, that every proposition is either true or false. On his interpretation, the law of excluded middle claims that every proposition is either provable or disprovable constructively (Wang 1994, p. 272). Moreover the law of excluded middle allows for proofs which are not constructive, e.g. proof by contradiction. As these are not considered mathematical, in the relevant sense, we should reject all proofs of this sort, and the inference rules which lead to them. This also marked a return of logic to the domain of the mind, Kant's intuition. Hence the name intuitionism. This also can be interpreted as putting logic into the realm of knowledge rather than objective truth. Intuitionistic logic is based on these ideas of mathematical construction, though Brouwer would not have understood the need to provide such a formal *linguistic system*.

Here we see a concept of logic which rejects universality and is dependent on mathematics, as with Peirce, but a fundamentally different overall conception of logic itself, though both are under the logic as calculus umbrella with their rejection of universalism. We will say more about intuitionism in the next chapter.

2.4.4.3 Model theory

The model-theoretic approach to logic is grounded in the intensional concept of a set. By viewing sets more abstractly, and not conceptually, the idea of varying the domain of discourse becomes straightforward. This was the same underlying conception that drive Hilbert's foundational program, through axiomitising various branches of mathematics. When we can talk about a system, or mathematical structure, from the outside we can then analyse the properties it has, and the difference structures have between them. Notions of the soundness, when a system proves something it is actually the case, completeness, when a system can prove everything it can express, can then be discussed and analysed.

Landmark results have followed from this approach. By thinking about validity in terms of domains Löwenheim proved his theorem that if a well-formed formula is valid in a denumerable domain, then it is valid in every domain (Van Heijenoort 1967, p. 327).[15] Gödel's landmark incompleteness results of Peano arithmetic also require such an external view of logic. His results devastating Hilbert's foundational program in a similar way as Russell's paradox did Frege's logicism.

With the distinction between logic as a universal language and logic as a calculus we see that the modern concept of logic is polysemous, as is its relation to mathematics. Within those categories there are different conceptions of logic, some explicating on the previous, but also some which are separate even under their respective umbrellas. For Frege, logic is the universal language of pure reason, it provides the justification and structure of mathematical knowledge. From the Peircean perspective, mathematics remains primary, while logic is the study and analysis of necessary thought, rather than the practice of it. Moreover, deductive logic is just one part of logic, the study of reasoning, and that just one part of the study of language, with the other elements of the *trivium*. The intutionists share the rejection of the universal language, while not embracing the overall Peircean picture. Gödel represents a view that is somewhere in the middle, his concept of logic is Fregean, but it is an explication of Frege's which changes its relationship with mathematics while retaining the notion that logic is about concepts. The modern conception of logic is MOT.

2.5 Conclusion

From this brief historical trip it should now be evident to the reader that 'what logic is,' is not a clear nor a static thing. Starting with Aristotle's *logos*, logic has been the study of necessary reasoning, ideal debate, the internal (ideal) reasoning of an agent and the basis for mathematical reasoning.

There have been multiple paths of development using differing concepts logic at the same time. Both historically and currently logic has referred to different things, it is an open-textured concept. The various notions of logic are overlapping concepts which bear a family resemblance to each other but remain distinct, logic is polysemous.

[15] Denumerable just means that it can be put in a one-to-one correspondence to the natural numbers.

The term logic has shifted over different concepts throughout history. The very concept of logic since Aristotle started to hold different ideas for different people. Whether logic is a derivative of language, or apart from it entirely but accessed from it, is a difference that is still unclear to this day. Some take it to be the one way, others not, but the discussion of what one thinks is rarely undertaken.

We see a similar story with the relationship between logic and mathematics. Even though we have a fairly uniform view of logic thanks to the Fregean revolution, upon closer examination this uniformity dissolves. Both Peirce and Frege came up with formal systems that are roughly equivalent to first-order logic, and since that time we have adopted a story that puts this formal system front and centre. However even at this mathematical confluence, the underlying concept of logic that these two thinkers held was vastly different. This differing idea of logic also leads to a wholly different idea of the relationship between mathematics and logic, that has been represented within logic and philosophy to this day. The above discussion of the history of the concept of logic, although brief is served up as evidence that the concept of logic has not been settled throughout history and remains unsettled today. We see that logic has been developed at various times by multiple groups and individuals for multiple ends and the concept has changed via that development. Logic has undergone parallel explication throughout its history and we have no evidence which indicate that these explications have converged, which would yield a singular concept.

We have seen that the concept of logic has taken large leaps forward, such as with the advent of the first-order calculus, and so it was obviously an open-textured concept. Given that we still have multiple concepts of logic at play today, we know that we are dealing with a MOT concept; there is an ambiguity between more than one concept when the term ‘logic’ is used. Any discussion of the nature of logic, thus, needs to be wary if all sides are in fact talking about the same concept and not merely using the same term, on pain of a vacuous debate.

CHAPTER 3

THE LOGICAL MONISM DEBATE

3.1 Introduction

The debate over the ‘one true logic’ can be deflated, not eliminated. This deflation comes on the heels of the conceptual analysis of the previous chapter: logic is a multiply open-textured concept. The idea of the ‘one true logic’ is about correctness, what is the correct logic, and thus what are the correct forms of reasoning.

The discussion of correctness and logic has come to the fore in recent years. There are many views on this, which can be split into three general types. *Logical monism* is the idea that there is one exactly one correct logic, and the monists usually champion a formal system to fill that role. *Logical pluralism* is the idea that there are multiple correct logics, with the pluralists offering at least two formal systems as exemplary of this plurality of correctness. *Logical nihilism* is the view that there are no correct logics, and the focus is on showing that there are gaps for any logical system, or inference rule, where it is not universal. Notice that all three approaches stake their claims on technical grounds, and not theoretical ones. How different theories fall out of these views depends on exactly what is meant by both *correct* and *logic*. In this chapter we will focus on the claims of the logical monists, how they interpret correctness and logic and how this doesn’t easily fit with the idea of logic being multiple open-textured (MOT).

There are two types of debate to be had, and thus two sorts of monist positions one could take. The primary mode of debate in the literature is about some candidate formal system, or another, being correct and all others failing in some way. That is, that this formal system represents the *one true logic*. However, above this discussion there is the debate of the *one true concept of logic*. The aim of this chapter is to show that while much of the discussion has been of the first type, there is a disconnect between these claims and those of the second type that deflates much of the discourse. The various monist schools of thought actually are appealing to different concepts of logic, and so are ‘talking past’ each other. The battle of correctness is being fought on the wrong level and on the wrong ground.

The monist replies to date have focussed on the specific champion formal systems and why their structure is the most appropriate. They do this without clarifying what exactly the ‘logic’ that their formalization of choice captures, and so it is not only hard to compare the different logics but also even given the idea of one true logic, whether theirs is clearly the champion.

In section two we will discuss the basic notions, of correctness and the purpose of logic, that are needed to frame the discussion of the monist positions. Then, in section three, we will present the monist positions of intuitionistic, relevance and paraconsistent logicians. These won’t be exhaustive of all who champion these formal systems, rather it will give a taste of how certain views interact at the conceptual level. An example of the replies to logical pluralism is examined to show the base assumptions that rest on their account as well as what it seems their conceptual notion of logic may be. The fourth section will address the deflation, not elimination, of the monist debate. We contrast against two other deflationist moves, Matti Eklund’s, and Quine’s meaning variance, we then offer our conceptual deflation. We then introduce a rough set of desiderata for a clear claim of correctness, based on the concept of logic being MOT, as well as the theoretical technical gap involved with monists claims. This gap has led to a ‘missing claim’ regarding the right type of mathematical structure to capture logic, whatever the concept at hand. Without this fact it is unclear whether there is, or can be, one one universal formal system to rule them all.

3.2 Theories of correctness

There are a multitude of views on the existence and number of correct logics. These views can be sorted in roughly three categories: logical monism, pluralism and nihilism. While these claims may seem straightforward we must be careful. What exactly is meant by a ‘correct’ logic?

This seems to be a conceptual claim, rather than a technical one. It might be puzzling, as Gillian Russell points out, that much of the discourse on logic is about “*logics*– classical, modal, relevant, tense, abelia, intuitionistic, counterfactual, paraconsistent and so on– and of arguments which are valid and invalid in particular logics” (Russell 2008, p. 593). We must be careful of conflating logic with ‘logic’. That is, conflating the formal, or mathematical, structures we use in logic with the concept of logic itself. While some may claim that they are co-extensive this is a claim which must be directly made and supported, rather than assumed to obviously be the case.

> In fact, there are different kinds of ‘which is the right logic?’ questions. One type of question - the vertical question, we may call it - concerns the scope of logic: is second-order logic really logic? what about modal logic? etc. Another type of question - the horizontal question - concerns which of many in some sense rival logics is the right logic, classical logic or intuitionistic logic, or fuzzy logic, or paraconsistent logic, etc. (Eklund 2012, p. 217)

We see this distinction as useful towards a further one. Just as we need to be clear not to conflate 'logic' with logics, we also need to make sure we don't put the technical before the theoretical. It is no use arguing whether such-and-such is a member of the class of logics, and correctly so, if we are not clear on what logic is. That is, if we do not know what the class of logics purports to pick out, then the membership in the class is not illuminating; it is a secondary claim. We need to build a clear foundation of the theoretical (horizontal) before building too much on the technical (vertical). This is the primary gap in the debate of correctness, as we shall see in the coming pages. We are primarily concerned with the concept of logic, we will mainly be discussing the theoretical layer, though the technical is still important.

There are a handful of concepts that need to be defined, which are critical to understanding the differences between the theories, keeping in mind the dimensions to which these theories can disagree from the previous chapter.

Much of the discourse on logic and correctness is about *logical consequence*, the relation between premises and conclusions that ascribes validity to the argument. It is often assumed that the logical consequence relation is the core of a logic. So, claims of correctness are ones about the classical consequence relation or the intuitionist consequence relation, etc. Most of these claims are done at the technical layer, some are explicit that the technical (fully) captures some 'pre-theoretic' notion of logical consequence, so we do have a pairing of both technical and theoretical claims. "An account of logical consequence is an account of what follows from what-of what claims follow from what claims (in a given language, whether it is formal or natural)" (Beall and Restall 2006, p. 3).

The *purpose* of logic is considered to be what is in the scope of a logician's study. *Correctness* can then be understood as whether something fits in the scope or not, usually in terms of logical consequence or a formal system. That is, purpose informs correctness. We cannot properly evaluate the former without considering the latter. Often correctness is discussed explicitly, while purpose remains inadequately addressed. Just as purpose informs correctness, so too does the concept of logic inform the purpose at hand.

This chapter isn't about claims of correctness in general, though we will tackle that question in chapter seven, rather it is specifically about claims of logical monism.

In order to make a claim that some logic, L, is the correct logic we must first settle what the underlying concept of logic is. At the conceptual level the claim that there is one correct logic (or set of rules to reason with) that governs all types of reasoning by any agent no matter the situation (or context) is called universalism. This universal set of rules was precisely what Frege was trying to capture: the 'universal language of rational thought' of Leibniz. So, the first step of the conceptual monist is to adopt this universalist stance and, ideally, defend it explicitly. The monist may not be just one of consequence, but also of the aims to which logic is directed. Let us keep this link in mind as we move through the various theories.

3.3 Monist views on logic

Logical monism is the theory that there is exactly one correct logic, that there is a correct way to reason and one is either right or wrong as a result. The monist position has initial intuitive force from the folk idea of correct reasoning, which seems to subscribe to the idea that there is *one* correct way to reason. We rarely hear, outside the halls of philosophy, talk of 'which logic' or 'logical rules' someone is flouting when they are described as 'illogical', for example. One is simply described as being logical, or not. When we reason in the sciences there is no discussion about types of reasoning that should be adopted, we simply reason the way everyone seems to have agreed is correct and move on. As such, this underlying idea that there is one correct logic out there seems almost (perhaps) obvious. Let us start with a clearer definition.

> Logical monism is the view that there is only one correct logic or, alternatively, the view that there is only one genuine consequence relation, only one right answer to the question on whether and why a given argument is valid, only one collection of valid inferences (or of logical truths), or only one right way of reasoning. (Estrada-González 2011, p. 111)

Even defining the idea of logical monism results in a branching definition. This is due to the fact that not only is the underlying concept of logic unsettled, as we saw in the previous chapter, but also that the core part of a logic, to which the rest is generated around, is unsettled. For some, the core of logic is logical consequence, for others it is the meaning of the connectives, for yet others still the set of valid inferences to which the logic yields, etc. So we need to be sure that, when we define correctness, we define it in terms of the core notion of logics, or risk talking past one another. This is not the largest worry though, as the usual suspects of candidate logics can be defined in terms of all these core notions. It is worth noting that this variance of core notion(s) is often due to the variance of the concept described above.

While the universalist stance is indeed appealing and intuitive, when we look at it from a conceptual level we see an immediate problem. The concept of logic is one that is MOT. There are multiple operative concepts of logic due to the fact that it has been parallelly explicated throughout history, and we do not have any evidence to suggest that these explications have merged somewhere in the background. If there is this ambiguity between what is meant by logic, then this ambiguity will bleed over to not only just the understood purpose of logic, but then also to what qualifies as a correct logic. If the purpose to which logic is supposed to be aimed it is different according to different people, then we should not be surprised when they disagree on which target system is 'right' for such a purpose.

For the monist, then, we first must establish whether they are a universalist, and what the concept of logic they are assuming explicitly is.[1]

Only then will an evaluation of a candidate formal system make much sense. Taking a look at the debates, however, we see that is just not the case. Monists come forward with their champion formal

[1] One can be a domain-relative monist, for example, i.e. there is one correct logic with respect to a certain domain. Here most of the discussion will be on the primary category of logical monism, those who do make the universal claim.

system already in hand and busy themselves arguing against the champions of others without actually stating the problem their formal system is supposed to be solving.

Most monists either argue against classical logic, as it is the *de facto* standard in philosophy, or they argue against some logical pluralist system that has been put forward. As Beall and Restall's logical pluralism[2] rekindled interest and debate into pluralism it is often the target for these types. Unfortunately this approach often involves showing the shortcomings of the attacked system and how the championed system does not fall prey to said shortcomings. That is, the discourse has been on the technical layer, resting on the assumption that there is already agreement on the concept of logic at hand. So we must pick through the arguments against the opponents to tease out what the underlying concept of logic might be.

The other consequence of this approach is that while we may be able to uncover what logic is thought to be by the monists, we do not get an argument for the universality of the logic at hand. Logical monism is a universalist position, or at least those that argue for monist positions claim that they are arguing about the one true logic, which is universal. Avoiding the direct discussion of what concept of logic they are using, the question as to whether that concept is clearly universal or could be universalised becomes opaque.

We will discuss three candidate systems: intuitionistic, relevance, and dialetheist logics from the point of view of some of their major philosophical proponents. The following discussion characterises major parts of the debate and shows where the other branches will also be tripped up. Classical logic has been the logical hegemon for almost its entire existence, most monist positions argue directly against its candidacy, or against pluralistic systems. For this reason we need not directly examine classical logic, as it will be discussed in turn anyway.[3] Note that these are presented in terms of the championed formal system, as that is how the discourse has developed, but these are not exhaustive of the conceptual views that are out there which end up championing certain formal systems, e.g. not all intuitionists will be characterised by the discussion of intuitionistic logic.

3.3.1 Intuitionistic logic

Intuitionsists show themselves as opponents of classical logic. They are concerned with mathematical reasoning. We will discuss two general types of intutionists, following the philosophies of L. E. J. Brouwer and Michael Dummett respectively.

Brouwer, and his followers, are concerned with mathematical knowledge. They think that mathematics depends on human thought, and so we are limited to mathematical truths which can be constructed. Brouwer thought that the language (symbols and equations) we use in mathematics were only descriptions of the mathematical activities that happen in the mind. The constructivism also stems from Kantian notions. Numbers are mental entities. They are constructed out of the sensation of time.

[2] Primarily presented and discussed in (Beall and Restall 2000), (Beall and Restall 2006).

[3] There are proponents who defend classical logic, Tim Williamson and W. V. O. Quine, but as the aim is to show that the debates shortcomings we need not engage with every tendril at this time.

As mathematical entities are constructed in the mind, this means that we must bar treating infinite collections as mathematical objects themselves (such as the set of natural numbers $\mathbb{N}$); they cannot be constructed (Haack 1974, p. 92).

Logic is secondary to mathematics. Mathematics is foundational and logic represents the collection of the rules of mathematics that we find *a posteriori* to be true (Haack 1974, p. 91). It is the combination of their psychologism and their constructivism that leads them to the problem with classical logic, not just the concept of logic that underlies it.

One of the proponents of classical logic, in Brouwer's day, was David Hilbert. Hilbert was engaged in a program of axiomatising all of mathematics, starting with geometry and arithmetic. For him truths in mathematics were bound in the axiomatic system which they were stated in. There was no external to appeal to, such as constructions in the mind, in order to establish mathematical facts. This difference is what led Brouwer to deny that axiomatics had any place within the foundations of mathematics, contra Hilbert.

For intuitionists, the core notion at the heart of logic shifts away from *truth* to *proof*. Propositions are not given a truth-value of true or false, rather they are only considered true when a proof is given for them; when we have direct evidence of the proposition. The truth of φ is if we have a (valid) proof of φ. The idea of falsity is replaced by the idea that φ can be disproved. If we do not have a proof, or construction, for some φ, and we don't have a construction of $\neg\varphi$, then φ does not have a truth-value— it is not yet proved or disproved. Intuitionistic reasoning is not *bivalent*, there are more than two possible truth-values. In classical logic, and thus classical mathematics, propositions are thought to be independently true or false. This is the mistake the constructivist mathematician seeks to avoid.

The formal logic known as intuitionistic logic is one that closely resembles classical logic. It has all the same logical connectives, more precisely it can be defined in exactly the same ways with the same range of connective definitions, and all of the classical deduction rules– save one. In intuitionistic logic the law of excluded middle (LEM) is not a valid inference rule.

(LEM): For all $\varphi, \vdash \varphi \vee \neg\varphi$

Roughly, this says that give any proposition (or sentence) either it is true or its negation is true; there is no other state between truth and falsehood. The problem they have with LEM is that by assuming that every proposition is settled in truth or falsehood we can reason in ways which assure us of something making a statement true, without actually constructing the object itself.

The way the law works in practice is that by knowing all things are either true or false, we can assume something is not the case, and show that leads to a contradiction, so if it is not false then it must be true, this is known as reasoning by *reductio ad absurdum*. The problem is that as we have not given positive evidence of this proposition, we cannot support that it itself is true, only that it cannot be false. Making the claim that $\neg\varphi$, is to say that you have a proof which shows that is impossible to prove φ. The denial of LEM means that a lot of what is normally considered mathematical knowledge is not actually mathematical knowledge, according to the intuitonsts. The proof techniques that are used to create *legitimate* mathematical knowledge do not include the use of LEM, or its equivalents.

The above does not describe all of the intuitionistic views, though most intuitionists who follow Brouwer fall into this camp. While Dummett is an intuitionist, his philosphical grounding is very different. He finds fault with the psychologism involved with Brouwer intuitionists. Instead he comes to intuitionism by a different route, *assertion*. Truth-conditions are the problem, and as classical logic, and LEM, are based on fundamental truth-conditions, they must go. Dummett is an anti-realist when it comes to truth (M. Dummett 1975, p. 106).

Given his anti-realist stance on truth, Dummett says we must focus on assertibility-conditions. Mathematics is no different than other languages. This is a general theory of meaning, even outside of mathematics we should replace truth with proof. The “meaning of an expression must be exhaustively manifested by the use of that expression” (M. A. Dummett 2000, p. 260). We understand a sentence via our awareness of the conditions under which a correct assertion can be made by uttering that sentence. Assertions are the primary mode of employment of sentences (M. A. Dummett 2000, p. 260). Proofs are gap-free, so in general they are needed to ensure exhaustive manifestation of our sentences. The notion of exhaustive manifestation is where Dummett gets his constructivism. A constructive approach denies both bivalence and LEM, which are grounded in truth. Assertability asks more, it is about the utterer of the sentence, not just the sentence itself. It is in this way the two camps of intuitionists are still connected. They have to do with the epistemic states, and constraints, of the agent, rather than pure facts about the world.

3.3.1.1 The exposed concept and debate

From the above discussion we see that the Brouwer intuitionistic monist logician is one who cares about a certain type of mathematical knowledge: constructions. Further they claim that these mathematical truths are the correct ones and the mathematical truths which do not follow from constructions, namely those which make use of LEM or are provable in classical logic but not in intuitionistic logic, are not actually mathematical truths. They are the results of using incorrect reasoning patterns.

This move away from truth to proof signals the move towards knowledge, or knowability, of propositions as opposed to propositions as independently factive. For Brouwer intuitionists this has been solely about mathematical knowledge, and mathematics is constructed in the human mind. Intuitionists logicians make a claim about what mathematical truths should be counted as true and what should not, based on if we can come to know them via the method of proof. This shift is more than just a refocussing of the laws at the core of reasoning; they are arguing about the correct way to deductively reason in the context of mathematical knowledge. They are no longer talking about the correct set of rules to reason from truths to truths– the subject itself has changed. For Brouwer this subject matter change comes about due to the ontological claim about the nature of mathematical knowledge, and logic being subordinate to mathematics. Mathematics is bound by human thought, and thus must be constructed. Brouwer does not go further to say that all reasoning is of the same sort as mathematical. He does not make the universalist move, explicitly. As mathematics is bound in thought, it too bounds reason.

The actual debate then is one limited to the discussion of the right way to reason in mathematics. The Brouwer intuitionist says that the only valid way to achieve mathematical fact is to show that something is the case, not merely to show that it cannot not be the case. For any logician out there worried about the universal rules of reasoning, that may be the case outside of mathematics, or the mind, they need not heed this end of the debate. Consider the logical realists, who think that the laws of logic are knit into the fabric of the universe. It is not clear that there is clash unless we make a further claim that the mind is somehow in tune with these laws, assuming that everyone believes that mathematics are inside our heads. For those who think that mathematics is governed by classical rules, and proofs by contradiction are just as valid as direct proofs, there is still a debate on our hands. As for the universalist classicalist, their major claim is unimportant, and the corollary with respect to mathematics is soley what they care to discuss– in virtue of the classical monist also being a universalist with respect to logic.

Logicism is the foundational doctrine that all mathematical truths are a type of logical truth; that mathematics is built upon logic. Frege subscribed to logicism. There is a weaker version of logicism which makes the claim that only the mathematical theorems themselves correspond to logical truths, rather than all mathematical facts. The intuitionist turns this upside-down. Logic is a part of mathematics, not the ground to which it is built upon. Here we must be clear that this is not just a claim of the mathematical structures called logic being part of mathematics, but the ‘logic’ itself as well. The intuitionist, by placing logic within mathematics is a reductionist. They reduce the scope and nature of logic, as a concept, to reasoning within mathematics.

Within mathematics they argue that LEM is a poor inferential rule and we should not use it to license any mathematical reasoning, due to the connection between mathematics and thought. Reasoning within natural languages, for example, is not under the purview of the logician, it is a different conversation.

Here we must pause, for a moment, to take a look at mathematics, specifically mathematical practice. Mathematicians, on average, tend to reason classically; they reason with LEM and the *reductio* strategy. When the reasoning patterns of mathematicians are examined they yield a roughly classical picture. The main exception of this is when the mathematician in question is working with an intuitionistic logic, or generally practising constructive mathematics, read as a sub-discipline of mathematics. This bring up the notion of legislation.

As we noted in chapter one, when we are doing philosophy of some X we should be wary that our philosophy doesn’t legislate against the activities of those who actually do X, without good grounds to do so. Following Kouri, we take it that we should not legislate to the sciences[4] “[l]egislating to these sciences comes down to telling the science they are wrong about their practices on philosophical grounds” (Kouri 2016, p. 31). If the science in question appears to be well-functioning, i.e. there is

[4] Where sciences is broadly construed, including fields like linguistics and theoretical physics.

no evidence to suggest that there is some flaw in the practice and results of the science, we should not presume a problem where there is not. Moreover, we as philosophers, are not the subject-matter experts, the scientists themselves are. Kouri claims that as a point of humility we ought not to do this; at the very least we ought not to do this if we can avoid it. We take this point as obvious as well. The intuitionist move is one of legislation. Mathematicians are doing it wrong in this specific way. The reason for the legislation is based on ontic views on the nature of mathematics and the resulting conception of logic. The argument must begin on these grounds.[5]

Dummett, however takes the intuitionistic argument to be more widely applicable. For him, the claim is a universalist one. The nature of meaning is such that use should exhaust meaning, and so we never have more than what we can show– in all discourse. The ontically weighty notion of truth is replaced with a 'truth' which is epistemically constrained. Its replacement is at the heart of language, not merely mathematical pursuits.

The actual debate for him, then is that the logic is about meaning, and meaning is fully exhausted by constructive claims. Here we have a debate with the universalist classicalist, not only about the fundamental rules of reasoning, but also target concept of reasoning as well. Reasoning, for Dummet, is about assertion, thus proof, and he thinks we need to re-conceive our central theory of meaning to reflect that. As such the meaning, and nature, of logic will also need to be re-conceived. Here the battle must first be fought on conceptual grounds, the formal system being a direct artefact of Dummet's theory of meaning.

For Dummett truth is a non-starter, meaning is not dependant on the truth or falsity of a sentence. Rather, what we should be focussing on is *assertion*. At the heart of meaning, and therefore language, is what we know. Intuitionism's shift of focus to proof is exactly the right template for this theory of meaning. A proof is a step-by-step movement from premises to conclusion, using rules which are deemed truth-preserving. Truth is reduced to what can be shown to be the case from what we know and rules of inference that have been deemed truth-preserving. The intuitionist here is claiming that LEM is not truth-preserving because it allows one to reason around the existence of things, it allows us to show that things must be the case because they cannot be false, without having to show the thing at all.

In both of the above cases we have a shift from direct fact-of-the-matter truth, and what we can come to know. If one wants to keep the concept of logic as 'what follows from what' generally, rather than in the case of human knowledge, whether in total or limited to mathematics, then one need not engage the intuitionistic monist, or at least we must first engage on this theoretical note to join them, for they are having a very different conversation.

[5]It very well may be the case that all practising mathematicians rely on unfounded metaphysical assumptions, in which case the legislation would be in order. The claim at hand is weaker, just that we should be wary of such moves and provide clear and good reasons for such legislation, when it is warranted.

3.3.2 Relevant logic

Nuel Belnap is a champion of relevant logic. He points to the paradoxes of the material implication as proof that there is something amiss with classical logic. These are a group of arguments which are valid in classical logic, but they go against our intuitions about conditional statements. They describe the ways in which the material implication, or conditional operator, is defined such that it does not line up with the natural language expression "if... then" which it is ostensibly supposed to model. Consider Mares' $M3$:

$$(\varphi \rightarrow \psi) \vee (\psi \rightarrow \varphi)$$

This statement is a logical truth, in every interpretation (truth-value) of φ and ψ the sentence is true, and this is the case for any φ and ψ. This is counter to our pre-formal notion of implication (Mares 2004, p. 8). Take φ to be "the moon is made of green cheese" and ψ to be "it is raining in Bristol". Then we have the statement, using the form of $M3$, that "Either, if the moon is made of green cheese, then it is raining, or if if it raining, then the moon is made of green cheese" is a true statement by the rules of first-order (classical) logic.

The key notion for the relevantist is that the conclusion and premises are relevant to each other in some way. A logic without this requirement is said to be incorrect because of this lack of relevancy. In the above language the propositions are about what the moon is made of, and whether or not it is raining, these are not connected in any relevant sort of way.

The formal logic known as relevance logic closely resembles classical logic. It has all the same logical connectives, or at least they can be defined similarly, and all of the classical deduction rules. The change here is that conclusions must be relevant to the premises from which they come. There are several different ways one can cash out this requirement, resulting in a small family of relevance logics. These also often differ on the theoretical cashing out of 'relevancy' of the various proponents at play.

A standard approach is to outlaw the the principle of explosion, *ex falso quod libet* (EFQ). This is the rule that from a contradiction we can derive anything. If EFQ were a valid inference rule then we would be able to derive conclusions from premises which have no relation to them.

(EFQ): $(\varphi \wedge \neg\varphi) \rightarrow \psi$

According to the pre-theoretic logical notions we are not justified in inferring any proposition from a logical falsehood, which is exactly what EFQ says (Mares 2004, p. 9). We must restrict our notion of deducibility such that ψ is deducible from φ only if φ is used in the derivation of ψ. That is, if φ is relevant to ψ. The standard way of doing this is to require that φ and ψ share at least one propositional variable, thus they are relevantly related (Haack 1978, p. 199).

Stephen Read, a proponent of relevance logic, describes the problem as the understanding of validity. While validity is indeed about truth-preservation, and the central notion of logic, the "esssential feature of validity is that it should *warrant* one in proceeding from the truth of the premises to that of the conclusion" (Read 2006, p. 208).

3.3.2.1 The exposed concept and debate

Read's discussion exposes a conceptual shift at play for relevant logicians of his type.[6] Here we are talking about what types of inferences should be used when some person (agent) is reasoning. We are talking about what you are entitled to believe given what you know, can know, or have come to know.

There are limits to truth-preservation precisely because of the limits of our knowledge. This concept of logic is fundamentally different from others, particularly those of his opponents. This is truth *in situ*, where the situation appears to be the reasoning of agents (humans) with imperfect information. Here it seems that epistemic and doxastic logics are prime candidates for the correct formal system, given this conception of logic. Once we begin the journey into modelling the beliefs of agent's with imperfect information it becomes less clear that simple first-order logics are the only candidate systems in town. The burgeoning field of formal epistemology seems to indicate that reasoning under these circumstances may best be modelled using a wider set of formal tools, such as those found in decision theory.

In his "Monism: The One True Logic", Stephen Read argues against the logical pluralism of Beall & Restall. He zeroes in on their notion of validity, which is the heart of their framework, particularly how it affects modal logic. They say that there are too many modal logics for them all to each hold of metaphysical necessity. "What is required, they say, is to specify what a logic is meant to do, and then there is scope for disagreement. If we want to capture metaphysical necessity, one modal logic is the right logic" (Read 2006, p. 198).

Read claims a difference between this sort of pluralism and Beall & Restall's because logics of this type, e.g. alethic modal logics, are supplementary logics. They are aimed at a different task than 'logic proper'. Here we see Read is trying to set the boundaries of logic, versus other formal enterprises. He makes a similar comment about moving to higher-order logics, and thus validities. Both higher-order and additional constants are fine to the monist, "these are all part of the one canon of validity for the monist" (Read 2006, p. 200). He is making a claim about the substructural nature of both modal and higher-order logics. We start with a standard logic, and extend it. In the case of alethic modal logics we add operators for necessity, $\Box$, and possibility, $\Diamond$, and for higher order logics we add quantifiers beyond those for just the objects.

This is a troublesome claim, as it hinges on such moves being built upon the same foundation as the base logic you are championing within the correct arena of 'logic', whatever that may be. The standard epistemic logic $S4$, for example, is one with classical implication. That is, it is constructed by starting with classical logic and adding a new modal operator, K, with its own axioms.[7] These axioms are all presented in conditional form, e.g. 4: $K(\varphi) \to K(K(\varphi))$. The various axioms for K all take a similar form, containing only one propositional variable φ, and we can construct different modal logics based on our choice of them. As they all only have to do with the one proposition, we might think that

[6] We remind the reader that this argument is not applicable for all relevant logicians, just the partition of them who follow the same conceptual moves as Read.

[7] This takes the place of $\Box$ described above.

the move to relevance logic is unproblematic because φ is definitely relevant to itself. Modal logics also have closure properties, however. If something logically follows from what is true at a world, then it is true in that world as well. Take the sentence $\varphi \wedge \psi$, if it is true at a world then, by logical closure, so are both of its conjuncts φ and ψ. What is true at a world of a modal logic based on relevant logic will be vastly different from what is true at one based on classical logic, due to the different meanings of the conditional, and the closure that will result from the conditional statements which are true at the world.

Read's choice of relevance logic as the one true logic is based on the idea of warranted, or entitled belief. We should only conclude things that follow from what we know directly, our conclusions must be relevant to our premises. In describing this situation, it becomes clear that the purpose of logic is to set out the normative rules of inference for people. Agents which have imperfect information and where some propositions will forever be unsettled. Logic is about how people should reason, and the rules of how we reason need to include our limitations as well. As it can never be settled, in our minds, whether φ or $\neg\varphi$, for all propositions φ, our formal logic needs to reflect this as it limits both what we can reason to, but also how we can reason within *partial worlds*, i.e. worlds where not every proposition is set to true or false.

This is a framework about what we can know, or believe in Read's words. However when we change gears to talk about metaphysical necessity, as in his example, those limitations need not apply. Indeed, if we ponder on why we need such limitations given our presented purpose of logic, it is very unclear why such limitations would be desirable when speaking of metaphysical necessity. Metaphysical necessity is about the way the world is, or must be, and is not limited to our observations of the world. When we refer to the way the world is, we refer to facts-of-the-matter. Somewhere out there either φ is the case, or it is not the case. Just because we do not know either way, and indeed might be unable to come to know it, that doesn't change the fact of φ. This is because of the epistemic-ontic gap. What we know, or don't know, doesn't affect the status of the facts of the world external to us. So limiting the base logic to relevant logic doesn't make sense for standard alethic modal logic. If Read allows this move, staying with a classical base logic for alethic modal logic, then he has lost his universalism, relevant logic is not the one true logic.

Returning to the Brouwer intuitionists, for example, the basis of intuitionism is rejecting that either φ or $\neg\varphi$ is the case, because of the limits of mathematics as part of human thought. Read's claim is that adding the modal operators to a logic is unproblematic as the monist has nothing to say about extensions to the base logic. However metaphysical necessity is not bound by human thought, under most metaphysical theories, so it is not clear at all that the intuitionist would support an intuitionist foundation to build the alethic modal logic on top of, without some external argument in addition. On the technical layer Read's may have a point, however it dissolves on the theoretical layer.

For relevant logicians like Read, as it is about reasoning *in situ*, again the rejection of the classical rules must be justified externally when moving out of the realm of reasoning agents, and into what necessarily follows from what metaphysically. At the very least, if one is to be a relevant alethic logician, one would have to give their philosophical justification for why metaphysical necessity is bound by similar constraints as reasoning agents. Logic, as presented by Read, is an epistemic, or doxastic, venture, and as such we need to be sure that our definition of validity reflects that.

3.3.2.2 Obeying the metalanguage requirement

In his discussion of Beall and Restall's logical pluralism, Read points out that they seem to be using classical logic in their metatheory, and this undoes their pluralism.

> The right response is to insist on doing one's semantics in the logic in which one believes. If Beall and Restall insist on doing semantics classically, they they are classical logicians for whom non-classical 'logics' are, if not just an intellectual amusement, then an exercise in applying logic so some particular activity... if one believes that... or especially, when applied to the semantic study of one's chosen account of validity. (Read 2006, p. 207)

The problems with this, for Read, are twofold. Firstly, the idea of one's chosen account of validity is tightly wrapped up in one's ultimate concept of logic and its purpose. As we've seen relevantists are concerned with warranted belief which is not the standard universalist, or classical, idea of logic. As such, we should not be surprised that the chosen account of validity does not match between them. Moreover, it is not immediately clear that these accounts of validity clash across the concepts of logic, and thus result in one's use of another logic as mere 'intellectual amusement.'

Read has another problem. He claims that if one has a champion logic then one must use that logic in their reasoning. The metatheory must engage with the rules of reasoning of the formal theory. It seems that he fails to do so, within the very paper which prescribes such a requirement.

The structure of the paper is as follows. There is a claim that logical pluralism is a good account of validity and correctness; this stems from the claim of validity of cases, V_{LP}. Read claims that there is an alternative concept of validity which should be adopted, V_{RL}. While he states V_{RL} he does not argue its case, rather it is claimed to be the one true logic based on the failure of the alternative. To be clear, the argument against pluralism is about how it does not entertain the monist positions on their own grounds, and so it paints a picture of the grounding of the relevant logician. However it stops short of giving external justification for why that ground is the case.

Thus we have a paper which takes the following form:

$$\frac{\begin{array}{c} V_{LP} \vee V_{RL} \\ \neg V_{LP} \end{array}}{V_{RL}}$$

This is an argument which uses disjunctive syllogism (DS). However DS is not a valid inference move within Read's relevance logic, as DS is "well-known to lead to the validity of EFQ in four easy moves" (Read 2006, p. 208). If we are to hold our logicians to account for carrying through the reasoning of their chosen system at the metatheoretic level, then it seems that Stephen Read is also a classicalist in relevant clothing.

3.3.3 Dialetheism

Dialetheism is the theory that there are true contradictions. A *dialethia* is any true sentence of the form: φ and it is not the case that φ (Priest 2006b, p. 4).

There are sentences which are both true and false. This should be separated from the idea that all contradictions are true, sometimes referred to as *trivialism*, as it is usually an extension of the idea that all sentences are true. The dialetheist wants to say something about specific contradictions, not a blanket statement about all sentences.

Dialetheism is a type of paraconsistency about logic. Any logic which doesn't have explosion is a paraconsistent logic. The principle of explosion is targeted as problematic by the relevant logicians, as discussed above, and so most relevant logics are paraconsistent. The dialetheists go further, the law of non-contradiction is in their sights as well. In his *Metaphysics* Aristotle argued that the law of non-contradiction cannot be denied coherently as it is built into assertion itself, it is a logically primitive: a law.

$$\textbf{(LNC):}\ \models \neg(\varphi \wedge \neg\varphi)$$

No proposition can be both true and false. Paraconsistent logics which reject EFQ, but not LNC, are called 'weakly paraconsistent', as they prevent explosion but do not allow true contradictions. Paraconsistent logics which deny LNC as well are called 'strongly paraconsistent' and include the dialetheists. Graham Priest doesn't deny the semantic validity of LNC, but denies that assertions need to presuppose LNC in order to be made. He appeals to the idea of the *content* of a sentence. A sentence can have content even if it does not logically 'rule out' anything. φ rules out $\neg\varphi$ classically. If dialetheism is true, then it does not do so. The content of a sentence is the information that it carries, so φ and ψ can have different content if φ carries different information than ψ. Priest's examples are "Pittsburgh is in Pennsylvania" and "The Australian Labor party is left wing". The two sentences imply different sentences than each other, so the content of φ is the set of sentences that it entails, similar for ψ. Formally we can describe the content of φ in a possible worlds model as the ordered pair $\langle W_1, W_2\rangle$, where W_1 is the set of worlds where φ holds, and W_2 where $\neg\varphi$ holds (Priest 2006b, p. 95).[8] This is a semantic reading of sentences, the idea of content is linked to the meaning of the sentences. Priest grounds his rejection of LNC based on the content of the sentences and so has shifted away from the topic-neutrality of logic.

By rejecting EFQ, contradictions don't lead to everything being true, so the dialetheist has a foot in the door to say something about certain contradictions without falling into triviality. With that safety net they can then do away with LNC. Recent support for dialetheism comes from dealing with paradoxes. There are two types of paradox: semantic and set-theoretic (or formal). The Liar paradox is an example of a semantic paradox, while Russell's paradox is an exemplar of the latter.[9] The Liar paradox is based

[8] This formal story relies on the logical closure property of the modal logic, described in the previous section.

[9] Both paradoxes can be given in formal or natural language terms, but the liar is paradigmatic of semantic notions, and Russell's paradox, as a formal paradox, dismantled Frege's logicist program in one fell-swoop. We will be discussing Russell's paradox in more detail in section 6.3.1.

on a sentence referring to its own truth-value:

(L_1): The sentence L_1 is false.

The paradox is one of self-reference, in that if L_1 is viewed as true one must conclude that it is false, and if viewed as false one must conclude that it is true. This is an old puzzle and various solutions have been offered throughout history. A common approach is to reject bivalence, that is reject the idea that every sentence is either true or it is false. By adding a third truth-value we can avoid the liar sentence. There are two ways to do this, embracing truth gaps or truth gluts. The glutty approach is to add a new truth-value for 'both true and false', the gappy approach is to add a new truth-value for 'neither true nor false'. These new values short-circuit the liar sentence, as we do not immediately flip from true to false, or false to true, due to the presence of a third truth value. Graham Priest, the foremost proponent of dialetheism, points out that the gappy approach can be used to form new liar sentences that cannot be evaded by the new concept (Priest 2006b, p. 15). Consider these two strengthened liars:

(L_2): The sentence L_2 is not true.

(L_3): The sentence L_3 is false or neither true nor false.

If sentence L_2 is true then it means that L_2 is either false or neither. If it false or neither it is true. A similar argument can be made for L_3. Priest points out that while we add a third value so that sentences are not totally divided into two categories: true and false. We end up being able to divide all sentences into true and *the rest*, which yields the same problem (Priest 2006b, p. 16).

These paradoxes somewhat support the dialetheist claim that there are some true contradictions. They rely on facts of natural language, or perhaps our thought processes. The paradoxical characteristics are easily expressed in ordinary natural language, e.g. English. Priest uses this as a critique of formal languages which lack the ability of self-reference. For Priest, it is the ability to handle these problems of self-reference, and other paradoxes, which show that dialetheism is the right conception of truth, and so his LP is the one true logic.

Priest succeeds in showing that the assumption of consistency as a clear and obvious virtue is not an uncontentious one and that its base assumption has left a gap where a clear justification should be. Priest's arguments are focussed on problems with the truth predicate and other corner cases. He unambiguously adopts a universalist position and has argued against pluralist positions (Priest 2006a, p. 208), (Priest, J. Woods, and Brown 2001).

Priest appeals to the fact that natural languages seem to allow such sentences as proof of *dialethia*. Here we run into the problem of legislation again. In order to be a monist one must believe that there is exactly one meaning to the logical connectives. Montague semantics is the foremost linguistic theory of meaning, many linguists think it is true. While it is descriptive, and not normative (prescriptive) it still models natural language as it is done. Montague semantics shows that there are many uses of the logical connectives which require more than a single meaning for the connective. Kouri gives the following example of dynamic conjunction in English.

> Tom and Mary got married and had a baby.
> Tom and Mary had a baby and got married. (Kouri 2016, p. 37)

Here we have an example of a conjunction which is not commutative, that is $A \wedge B$ does not mean the same thing as $B \wedge A$. Again we seem to be saying that despite linguistics being a well-functioning science, there is something wrong at the foundations of their practice and we will set them right. It is true that as linguistics is a non-normative science there is some wiggle room to be had, it still requires some explanation for why there is both one set of norms for all reasoning and that this formal system is a total encapsulation of said norms. For Priest it is worse as he is leaning on the unique features of natural language to give credence to his universal claim for his dialetheist logic, while still trying to hold on to the absolute single meaning of the connectives of his formal system in his other hand.

3.4 Deflating the debate

The logical monist debate is one about which logic is the one true logic. At the heart of these monist claims, then, is the claim of universalism. That there is one set of rules that governs all reasoning. Unfortunately we rarely get justification for this more basic claim as the various monist players are most concerned with their champion formal systems. The arguments are mostly made on the technical layer not the theoretical. There is minimal engagement with the concept of logic and how it affects the various moves. It is the aim of this section to deflate the monist debate, not to do away with it altogether. We will start with a brief discussion of two other deflationary arguments, before presenting ours, based on the fact that the concept of logic is MOT.

3.4.1 Other deflations

Matti Eklund introduces a notion of pluralism called *Multitude*: that there are different possible languages with different logics. He then presents a notion of deflating the debate around the one true logic.

> *Deflate*: All that the (horizontal) issue of which logic is the right logic amounts to is (a) which logic is the logic of the language we actually use, or (b) which logic is best to use for pragmatic reasons, whence the question of which logic is the right logic lacks the depth and significance otherwise accorded to it. (Eklund 2012, p. 218)

He then proceeds to sharpen the notion of Multitude, showing that it does not entail Deflate. We agree with his reasons and find the conclusion valid. Our deflationist move is different from Eklund's, however. While we find fruit in looking at the horizontal question, we primarily do so on conceptual grounds. The horizontal question is more related to conceptual matters than the vertical, or at least the conceptual bounds of the one will affect the other. From a conceptual perspective this notion of deflate is much too narrow. We are offered a deflation based on natural language or pragmatics. This does not follow from the MOT concept of logic.

A more famous deflationist move is one of *meaning-variance*, first put forward by Quine. When the dialetheist is arguing with the classicist, about LNC, what exactly is the disagreement about. Quine proposed meaning-variance as an explanation. The meaning of the connective is bound to the logic it is a part of. Take the sentence $\varphi \wedge \neg\varphi$. While the notation looks the same when written by a classical logician and a 'deviant' (non-classical) logician, doubtless they think they are speaking of the same thing, "surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form '$\varphi \wedge \neg\varphi$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject" (Quine 1986, p. 81).

When we change our logic we necessarily change the subject. There is no genuine disagreement, they are talking past each other. Meaning-variance leads to a sort of logical pluralism: there are two logics, one where LNC is valid, and another where it is not (Hjortland 2013, p. 2).[10] As they are not rivals they are both, in some sense, legitimate.

The problem with meaning-variance deflation is that it makes its claims on the technical layer. The debate is deflated precisely because of the relation between the formal system and the discourse. As logic is MOT, we need to be sure we engage our debates on the theoretical layer first, agreeing on our concept of logic, before we move onwards to notions of meaning and formal systems. We are proposing a structurally similar deflation, but on a deeper level a *conceptual variance* argument. Conceptual variance implies meaning variance, and so Quine's problem will still need to be dealt with. By not establishing the concept of logic at hand they end up talking past each other.

3.4.2 Conceptual deflation

Since the Fregean revolution of logic, where logic proceeded into a mature mathematical discipline and the mathematical structure of logic become clearer and more interesting, there has been a focus by logicians, and philosophers of logic, on these formal systems and their properties.

This focus has come at a premium. We are not clear as to what 'one true logic' refers and what problem it is purported to solve. Even the notion of correctness is unclear until we know this aim. Underlying this is also the question as to whether the monist is actually making a universalist claim, i.e. are they claiming that there is one set of rules of reasoning that is applicable, and correct, everywhere (in all contexts)? Or are they claiming some boundary to logic, and that this formal system captures all reasoning within said boundary?

If we are to evaluate any claim towards the candidacy for the one true logic, we first must know what the operative concept of logic at work is. The concept of logic is MOT; there are multiple different concepts of logic, as such there is an ambiguity when we use the term 'logic' as to which concept of logic is meant. As these concepts differ, so does the purpose of logic, and thus the correct target system.

[10] Hjortland uses the example of a classical logician (Williamson) arguing with a non-classical logician (Field) about LEM, rather than LNC as we used above. We continued our example from Quine and trust the shift obviously does not impact Hjortland's point, to which we are citing.

While one can study logics purely mathematically, as mathematical structures with certain properties, and thus not subscribe to any particular purpose, logical monists are not doing this. To make the claim that there is one true logic, and that candidate system L is indeed that system is to also make the normative claim: These are the sets of rules we should by reasoning with.

As this is a normative claim, the assumed purpose of logic is very important, and if two people are arguing across differing purposes, then the fact that they are supporting different candidate formal systems does not make up a debate.

Most monist claims are ones about specific formal logics being the correct logic to inhabit the space of the one true logic. The various rules which are included, and thus the classes of inferences which are licensed by it become the focus of either positive arguments for the candidate, or negative arguments against some other possible candidate. Missing from this approach is the clarity of what problem is being solved. By skipping over the conceptual debate, it becomes unclear whether they are, in fact, arguing over the same problem. Until the conceptual layer is settled it will continue to be unclear whether a true debate is occurring at the formal level.

Consider a classical monist, with respect to mathematics. If they are arguing about the class of mathematical truths and an intuitionist, such as Brouwer, argued against their claim, then we have a debate about which logic is the correct one for reasoning for establishing mathematical truths. However this is only part of the story. The intuitionist is making their claim of correctness on the basis of an understanding of mathematics as a product of human thought, and thus bound by it. This is a claim about the nature of mathematics, and of logic as a part of mathematics. The classicalist maybe a mathematical realist: mathematical objects exist in the universe just like physical objects exist. Moreover, most mathematical classicalists subscribe to logicism, that logic provides a foundation for which mathematics is built.

The classicalist and the intuitionist argue about the applicability and validity of LEM. They make these arguments based on what truths of mathematics come out of the application, or lack thereof, of this inference rule. For the mathematical intuitionist, it seems that they are saying that mathematics is bound by thought and the correct rules of reasoning in mathematics, as a result, should exclude LEM. They could be saying more than that, namely that all thought should thus be bounded under the same concerns. Dummett seems to be making such a claim, though some have interpreted his claims to be within natural language, rather than general thought (Rein 1985, p. 522). This is exemplified by the shift from truth *simpliciter*, to proof (and assertion) as the central notion of logic.

This debate seems like it is one that is straightforwardly about the correct logic to use in mathematics. But when we examine the operative notions of mathematics, and logic, of the various accounts we see a large conceptual divergence. Is it any wonder that these two groups have different ideas on what formal system best captures reasoning in mathematics?

Also consider a general classical monist, a universalist. Their claim is substantially different from the classicalist described above. This monist wants to say something about reasoning in natural language, and general reasoning in any and all contexts. The intuitionist is not concerned for most of those affairs, and so the debate is at best one-sided.

For the Read type relevant logician we seem to be talking about yet a different thing. By focussing on what is warranted to infer, or entitled belief, the relevant logician is showing that they do not care about facts of the world; they care about what we can say follows from what, given what we know. Logic is about the norms of reasoning of human-like agents. The danger for them is we have developed other formal tools to model such reasoning, and there is not (yet) a clear argument on why the correct mathematical structure for such a task is a logic, as compared to say a Bayseian system which is more popular in formal epistemology.

This is a broader problem for all claims of the one true logic, regardless of the concept of logic at hand. Why is a mathematical structure (formalism) of this type the right kind of structure to capture this concept, and purpose, of logic? By focussing on the technical layer, this ‘missing claim’ of the correct type of formal system is lost.

Graham Priest’s dialetheism is a formal system built to deal with a certain type of problem, paradoxes. Namely, but not limited to, those of self-reference. He gives a clear description on what he thinks logic is: “the canonical application of logic is to reasoning” (Priest 2006a, p. 165). He appears not to differentiate between mathematical and general reasoning. Because of his paraconsistent logic’s ability to deal with paradoxes he contends that it is the one true logic. While he gives a broader story then most, his account draws short in certain places. It is not clear whether he thinks that one true logic should be extended to reasoning in natural language, presumably so as he is making a universalist claim. If this is the case, then he must explain away the problem of singular meaning of the logical connectives. Ideally in a way that does not legislate away the usefulness and applicability of other systems, like Montague semantics or other similar linguistic systems, to model meaning in natural languages.

While it is clear that dialethic logics are very useful when dealing with paradoxes, Priest is oddly silent when it comes to the effect of the change to his logic on reasoning with non-paradoxical problems. With the introduction of third truth-value, and the change to the structural rules, the set of inferences in general is restricted, in order to deal with the puzzles of paradoxes. If one was not making a universalist claim this would not be a big deal, we would have a system suitably designed to deal with problems such as paradoxes and self-reference, and we could turn to some other system in cases where these issues do not crop up. The grounding of dialethia is given by natural language content, and thus it is not immediately clear why said grounds apply to alethic reasoning as a result. The monist relevant logician has an analogous problem with altheic reasoning, given their concept of logic as warranted inference. The greater claim of universality needs to be supported.

The monist debate is fought with formal systems, and logical laws, in hand but it seems that the disagreement which is actually going on goes much deeper. While it is true that their intuitions support the specific designs of their champion formal systems, thus it is easy to use those intuitions to make ones’ claim, this focus on the formal specifications does the discussion a disservice.

3.5 Desiderata

Here we gather together the tendrils of the above discussion to list the desiderata of a clear monist claim to correctness.

1. The concept of logic: What is logic?
2. The purpose of logic.
3. Support for the universalist claim.
4. Why is this type of mathematical structure appropriate for (1) & (2)?
5. Why is this formal system (the champion logic) the correct one for this role.

While there may be other important questions we submit that this is, at a minimum, a good start. Any claim of the one true logic must give a clear answer to these points.

Looking at the desiderata listed above we see that most monist claims, with perhaps the exception of Graham Priest's dialetheism, fail to engage until the bottom half of the list. This discussion has not been representative of all proponents of the various non-classical logics, rather focussing on main proponents and their theoretical grounding for picking such champions, it exposes the problem of focussing on the technical virtues versus the theoretical differences which is present throughout the discourse. This leads to a debate that is quite often conceptually talking past one another. What one considers the nature of logic to be, and in the narrower case what the relationship between logic and mathematics is, has a profound effect on ones ideas on what good reasoning is. We will use this desiderata to motivate our discussion in chapter seven.

By focussing the debate on which logical (formal) system is the right one, and which inference rules should, or should not, be used the debate has grown with a base assumption that everyone agrees that they are talking about the same thing. As logic is MOT, there is an ambiguity between at least two concepts of logic, we know that there is a discussion to be had about what one takes logic to be. By ignoring this part of the conversation arguments have grown out of what is assumed to be a common ground, but is in fact anything but.

Before we can visit the idea of which logic is correct, we need to be firm on our conceptual grounds. What is the concept of logic, and then what is a logic as a result. That is, what exactly is a formal logic supposed to be capturing? The idea of correctness is a normative one, and so understanding the purpose of logic, as one sees it is integral to being able to evaluate any correctness. One thing that is often taken for granted, and spotted implicitly in monist positions is the very idea of the universalism of logic. That there is one set of rules, or one type of reasoning, that governs every subject and every situation is not a straightforward one and it must be defended rather than assumed.

One can be a universalist without being a monist. That is, one could think that there is a universal logic out there, but that our formal systems are not the right type of thing to capture the entirety of reasoning, due to their structural limitations. For the monist then, an argument must be given to support the claim that the family of formal systems which we call logics are the right type of mathematical structure to entirely capture what we mean by logic.

We must note here, as well, that even if we lucked out and everyone had agreed on a base concept of logic, we still have yet to receive the arguments on why the monists think that a formal system, of this type, is the appropriate object that can totally capture this concept of logic. It may yet be the case that formal systems only capture a fragment of the concept of logic. This is the ‘missing claim’ from all monist positions highlighted by the fourth desideratum, rather it is a presumption that hasn’t been clearly supported thus far. The universal monist cannot reverse to this point, as they are committed to the universal application of logic. Unless, of course, they are willing to let go of their champion and make their claim purely on conceptual grounds.

We first must argue about our concepts and when (if) we are agreed upon the concepts at play, and the purposes/norms that result, then can we turn to the more accessible question of the right design of a formal system to capture this concept. The theoretical grounds must be established before we move to the technical arena.

Only after all of the above has been clearly dealt with can we proceed to present our champion formal system with any hopes of being able to clearly evaluate such a claim. By rushing through to this stage we have invoked a debate that involves conceptual clashes that remain un-addressed in favour of technical puzzles.

Chapter 4

Carnap, Connectives, and Correctness

4.1 Introduction

> The first stop on any tour of logical pluralism must be Rudolph Carnap's notion of logical tolerance. (Cook 2010, p. 497)

While there are many flavours of logical pluralism in the wild, most philosophers will claim the first instance of logical pluralism came from Carnap's *principle of tolerance*. The recent resurgence of interest in logical pluralism, kicked off by the work of Beall and Restall (Beall and Restall 2006), (Restall 2002), (Beall and Restall 2001), has caused a similar renaissance in modern Carnapian scholarship. In particular surrounding his principle of tolerance.

"*Pluralism* about a given subject, ... is the view that different accounts of the subject are equally correct, or equally good, or equally legitimate, or perhaps even true" (Shapiro 2014, p. 33). There are many different logical pluralisms, and attempts have been made to not only categorise them, but also do so with respect to the notion, or concept, to which the specific pluralisms pivot. Providing such a taxonomy is not the goal here, (Cook 2010), (Russell 2019), (Shapiro 2014) are good places to start, however some familiarity with these frameworks will be useful to the reader. These taxonomies of pluralisms go a long way to point to the various ways in which views of logic can be built, or based upon. This dovetails nicely with our idea that logic is a multiply-open texture concept, thus we should not be surprised that there are many ways in which the question of correct logic(s) might be approached as well.

The creation of new types of logical pluralism, and logical relativism, have come from many corners of philosophy. In that wave there have been attempts to revitalise this first pluralism of Carnap, and interpret it in the modern logical era. In the next chapter we will pursue such a program. First we must spend some time setting the groundwork for what Carnap's pluralism was and how it fit into his greater philosophical program. This discussion of Carnap's *Logical Syntax of Language* will inform the view of logics-as-formalizations as a form of logical nihilism presented in the chapters six and seven.

The recent work of Stewart Shapiro and Teresa Kouri Kissel, (Kouri Kissel and Shapiro 2017), on a pluralism based in logical instrumentalism is one such example. While they lean on the work of Carnap to promote their view on pluralism, there exists a fundamental misunderstanding of Carnap's view of correctness within.

A discussion of Kouri's "A New Interpretation of Carnap's Logical Pluralism" will help illuminate these important details of Carnap's philosophy of logic, as well clear up some misinterpretations of Carnap within Kouri's paper. In chapter six we will be presenting our own theory of logics, which will be Carnapian in character. The deeper understanding of Carnap's views of the boundaries of linguistic frameworks and our ability to ask questions of them will illuminate the same boundaries in our following view.

Kouri presents an alternative interpretation of Carnap's view where the logical connectives can, and do, share their meanings across languages (in certain contexts). She rejects the slogan that is meant to characterise Carnap's view on logics and languages: that a change in logic can only occur when there is a change in language, or connective meaning. This new interpretation depends on extending Carnap's linguistic frameworks to also include meta-linguistic frameworks which allow us to talk about the linguistic frameworks themselves. We will show that this interpretation misses the mark of Carnap's greater program, and while it does allow a richer playing field for the logician, it stops short of providing a fully meaningful conversation about meaning across languages.

Section two starts with a sketch of the greater Carnapian program for philosophy, then we go into some detail on his notions of languages and logics, and their close relations. In section three we introduce Kouri's 'slogan' that is supposed to characterise the interpretation of Carnap which is the target of her paper. Following that we discuss Carnap's ideas of internal and external questions, in section four, to make clear what types of philosophical inquiries he thinks are good and what are pseudo-questions; which types of question should be eradicated from philosophical discourse. With these in hand, we can properly address Kouri's 'new interpretation', in section five, and show where it seems to part ways strongly from Carnap. We round out with a discussion of the impacts of pragmatic choice on the choice of framework, meta or regular, and the limitations of translation. Given the limitations of the view, in the appendix, we offer an alternative 'old interpretation' of Carnap's view, from his own *Logical syntax of Language*, which achieves the same ends as Kouri's 'new interpretation'.

4.2 Carnap, the first pluralist

We will cover the general project of Carnap, as part of the logical positivist movement, before focussing on his theories of logical syntax and languages, or in modern parlance: formal languages, and his pluralism: the principle of tolerance.

Carnap's tolerance appears in the middle of a broader two-pronged reductive program, redefining the discipline of philosophy, (Carnap 1935), (Carnap 1937). He opens the program with the idea that philosophical problems are made up of components from three rough categories: Metaphysical, Psychological and Logical. It is the object of his program to fully reject the former, leave the second to psychology and reduce philosophy only to the realm of the latter. Areas such as ethics are about human thought and so should be pushed back to the psychologists. Metaphysics is concerned with questions which have no empirical or logical answers, they are pseudo-questions and so philosophers should not concern themselves with such matters. Further, logic was to be removed from the realm of dealing with meaning entirely and left only in the formal, the syntactic. Logic is reduced and clarified, while "[p]hilosophy is to be replaced by the logic of science– that is to say, by the logical analysis of the concepts and sentences of the sciences, for the logic of science is nothing other than the logical syntax of the language of science" (Carnap 1937, p. xii).

Carnap's tolerance emerged out of this greater project to not only redefine the borders of philosophy, but also bring logic closer to the sciences as well. The proposed shift of philosophy widens the scope of phenomena in which logic can be applied, as it is a tool to be used within the sciences and towards understanding in each individual field, rather than solely its own abstract field.

What counts as objective and truth-functional varies across the sciences (with their varying standards of evidence). By bringing logic to the sciences individually Carnap is advocating a plurality of useful, or applicable, formal languages, new logical syntaxes.

We must be careful here, Carnap is advocating a wider use of logic, but he is not saying that the scope of logic should be increased to encompass the whole of science. Logic is precisely the study of inference and reasoning. In fact, part of Carnap's reductive program is to reduce philosophy to only the study of formal inference and reasoning (Carnap 1935, p. 35). Semantic notions and their effect on reasoning in general are deemed outside of the scope of the logician and philosopher's work.

The logical syntax of a science has two consequence relations, one logical and one extra-logical. The logical consequence relation deals with the purely inferential parts of the language, while the extra-logical consequence relation is derived from the natural laws which govern the science in question (Carnap 1935, pp. 50–51). This may include semantic relations between propositions, and laws which govern the movement between them. Carnap is clear that the logician should pay no mind to these; the study of logic should be limited to inferential reasoning and formal systems. The semantic analysis should be left to the experts, the scientists. The logician and scientist work together towards a common goal, but the logician doesn't presume to know more about the details of the application than the expertise that she contributes, namely that of formal inference. Carnap recognised the importance of semantics, and later remarked on the relation between semantics and abstract entities in "Empiricism, Semantics, and Ontology" which we will discuss further in section 4.3.4.

It is under this backdrop of changing philosophy that Carnap introduces his theory of pluralism.

> *Principle of tolerance*: It is not our business to set up prohibitions, but to arrive at conventions.

> In logic, there are no morals. Everyone is at liberty to build his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap 1937, pp. 51–52)

The principle of tolerance is one aimed at building and using any syntactic formal system that one can. We should not limit the work of others, nor ourselves by being bound to any one formal system or another. Crucially, whenever we use a syntactical system for a purpose, we should be clear on our methodology and give our arguments based on the syntactic structures. Note that here Carnap says in the same breath logic and 'form of language'. For him the two ideas are intricately connected and so we first must clarify what he means by the two.

4.2.1 Languages and logics

> When we say that the object of logical syntax are languages, the word 'language' is to be understood as the *system of the rules* of speaking, as distinguished from the acts of speaking. Such a language-system consists of two kinds of rules, which we will call formation rules and transformation rules. (Carnap 1935, p. 41)

The rules of formation determine how sentences of the language can be formed out of its symbols, that is what counts as a well formed formula in the language. The rules of transformation "determine how given sentences may be transformed into others; in other words: how from given sentences may we infer others"(Carnap 1937, p. 43). The grouping of all such rules and the set of related premises and conclusions allow us to generate our familiar notion of logical consequence. A language, then, for Carnap is a formal system which contains the symbols, rules of formation and the rules of transformation. This is in direct contrast with our everyday notion of language which includes natural languages, such as English, as well as our more technical idea of a language as formal system without inference rules yet applied (Beall and Restall 2006, p. 3).

A language is called formal when there is no reference made to either the meaning of the symbols (e.g. the words) or the sense of the expressions (e.g. sentences)(Carnap 1935, p. 39). Logic is concerned with the formal treatment of sentences, it is solely dependant on the syntactical structure of the sentences (Carnap 1937, p. 1). Carnap is showing that the mathematical developments of recent history have enabled us to give a formal treatment of inference rules such that they are now part of syntax. The difference being that syntax was traditionally occupied with the rules of formation, but now we can give syntactical rules of transformation which capture inference patterns, hence 'logical syntax'.

He compares constructed languages, say classical logic, with natural languages, say English. Noting that giving the statement of the formal rules of formation and transformation "would be so complicated that it would hardly be feasible in practice...they must, of necessity, be still very complicated from the logical point of view owing to the fact that they are very conversational languages" (Carnap 1937, p. 2). Here we note the difference between the *object-language*, the formal system we are studying, and the language which we speak about the syntactical forms of the object-language the *syntax-language*, or the language of discourse if one prefers. Carnap's aim is towards constructed languages as object-languages aimed at making clear inferential reasoning at the syntactic level.

Carnap's focus is on the logical syntax of languages, meaning the formal theory of the linguistic forms of the theory, the system of the formal rules which govern the language together with the consequences which follow from these rules, the more familiar notion of *logical consequence*.

There are two forms of tolerance important to Carnap that are closely tied, linguistic and logical tolerance. They are exactly as they sound, tolerance of logic choice and tolerance of language choice, here language is read in the more standard reading of pre-inference syntax, or just the formation rules. Linguistic tolerance is central to the idea that verbal disputes are not theoretical disputes about the theoretical domain we are talking, rather they are practical conversations about the best ways to use words and concepts given our specific goals. Logical tolerance is often thought to be the source of Carnap's pluralism, “everyone is at liberty to build his own logic." As Carnap is concerned with formal languages which are created by stating the formation and transformation rules these two ideas are very close, married together.

Carnap wrote the *The Logical Syntax of Language* at a time where we did not have the depth of theories of meaning and logics as we do now, a pre-tarskian landscape.[1] This had a strong impact on the idea of language choice for Carnap. First we pick some expressions, and then we give rules of inference for them. Carnap was explicit that this choice could be arbitrary, and this would dictate the meaning assigned to the logical symbols (Carnap 1937, p. xv).

These rules give languages their meaning, thus there is no question of rightness of wrongness of the expressions, via tolerance. Thus to be tolerant of language choice is already to be tolerant about choice of logic; as languages, under Carnap's conception, come with their logics already ‘built-in’ (Russell 2019).

Supporting tolerance is the idea that we should not be fighting over the ‘correct’ meaning of the logical connectives, or the rules of transformation (logical consequence) rather we should approach from the other direction. Let us define the meaning of the logical constants in terms of the logical system chosen. No one meaning of a connective, or even its status as a connective, nor any rule of inference is sacrosanct.

There is no uniquely correct logic, instead there are various ways we can attempt to structure the language of science. Logic is the study of these formal ways. For Carnap, there is no correct way of doing this, asking after correctness is to misunderstand the task at hand. Rather, alternate proposals for how to structure the the logical syntax of science, are just that *alternates*. No evidence or theoretical argument can clearly make the case for one being the correct one, let alone the uniquely correct one. For there is no platonic ideal to which we are hoping to model with our formal logic of science. That is not to say that all frameworks are valid, we still have the criteria of efficiency, fruitfulness and simplicity

[1] Before Tarski's grand innovation of his semantics, adding an interpretation into the mathematical structure giving much more control over the formal system. Before this most logics followed Frege's original universalist move of having a built-in interpretation.

to measure our framework against. The closest that we can get is to ask the pragmatic question: When I attempt to apply this formal language to a certain application are the results useful?[2]

4.3 A slogan

Kouri begins her project with the following claim:

> There is one slogan that many people who hold that Carnap was a logical pluralist agree upon: Carnap's pluralism is one in which a change in logic can occur only when there is a corresponding change in connective meaning. In this way, it is claimed that a Carnapian pluralism requires language change any time there is logical change. (Kouri 2018, p. 1)

Kouri aims to show that this slogan is not a good one by providing a framework where we do have different meanings of the logical connectives within the same language. From our above discussion about Carnap's view on the right way of choosing, and creating, a language, and thus getting a language with a logic built-in, this slogan should not be so surprising. Because of his use of the term 'language' we are in danger of conflating this technical term with our standard usage of the word. If we read 'language' here as Carnap's term then the slogan directly follows as the language is composed of both the syntax and the syntactical presentation of the inference rules, the rules of transformation. If we read it with our standard notion then the slogan seems ill-fitted as given any set of formation rules there will be a plethora of transformation rules which we could apply to make our constructed language.

Looking to the literature we see an example of this in Beall and Restall's discussion of the differences between their logical pluralism and that of Carnap. The core of their discussion is the idea that tolerance goes much further than needed for logical pluralism. This is unsurprising given that they hold a notion of correctness and Carnap eschews the idea altogether. They characterise this theoretical difference with the following example:

> *Logical Pluralism*: $A, \neg A \vdash_C B$ but $A, \neg A \nvdash_R B$
> *Carnap*: $A, \neg_C A \vdash B$ but $A, \neg_R A \nvdash B$
> Any difference in logical consequence is due to a difference in languages. (Restall 2002, p. 4)

Here they are contrasting the argument from A and $\neg A$ to B, which is valid classically but not valid relevantly, as the principle of explosion is not valid in relevant logics.

According to their logical pluralism they are holding the language constant (or at least they can do so) while varying the consequence relation. So we see that only the consequence turnstiles have a subscript, C and R respectively. They proceed to explicate tolerance as having the logical connectives change, signified by the subscripts. Thus, any difference in logical consequence is due to a difference in language.

[2] This will be further explored in sections 4.3.4 and 4.3.5.

Recall that when Carnap uses the term ‘language’ he uses it to mean the language system which includes the symbols, rules of formation, and the rules of transformation, which maps to our modern definition of logic: a language and a consequence relation $(L, \Rightarrow)$. Carnap’s claim that there is no competition between logical syntaxes, logics, must be read as the combination of L and $\Rightarrow$, and not just about L. In modern terminology, the slogan should read: Any difference in logical consequence is due to a difference in logics. Which, for Beall and Restall, is defined in terms of logical consequence. That is, the slogan is unenlightening and bordering on tautological.

The given characterisation of Carnap’s pluralism seems unfair, or at least problematic. Here they seem to claim that the only thing that varies across the two arguments is the negation. But again we are faced with Carnap denying intra-lingual comparison, Beall and Restall amend their example in discussion stating that Carnap “probably also wants to say that... the A and B range over sentences in language C and... language R (Beall and Restall 2006, p. 11). However, this is still not enough. As we are focussing on consequence relations as the heart of logic, the role of consequence must be specified, and it is this entailment which they highlight in the case of logical pluralism, but it is absent in both characterisations of Carnap’s pluralism. Carnap’s ‘language’ is a formal system that includes both L and $\Rightarrow$. A more appropriate characterisation is:

$$A_C, \neg_C A_C \vdash_C B_C \text{ but } A_R, \neg_R A_R \nvdash_R B_R$$

This makes it clear that we only evaluate the entire structure, not some internal parts within. In Carnap’s tolerance we have no ability, nor do we wish for one, to compare A_C and A_R.[3]

[3]There is a similar problem in their characterisation of their own framework. A claim like the one made above, regarding the argument from A and $\neg A$ to B classically and relevantly can only be made when we have fixed the language. This language fixing has not been motivated except for it being a useful choice, if one wants to conduct such examinations. If we do not fix the language we revert to the exact characterisation provided of Carnap’s tolerance above.

Logical pluralism allows us to choose to work internally. Only once we have fixed the language can we then get something that resembles their characterisation above. An explicit reading of this is:

$$A_1, \neg_1 A_1 \vdash_C B \text{ but } A_1, \neg_1 A_1 \nvdash_R B_1$$

The core of their pluralism is that we can, in the post-Quinean landscape, choose to fix our language across logics so that we can compare results across the linguistic bridge. Different consequence relations do not have to speak to different arguments, they can speak to the same argument. This offers the ability of *cross-linguistic-evaluation*. “We can take a premise from language 1 and a premise from language 2, and see how they lead to a conclusion from language 3. Consequence relations, while they are used to evaluate connections between claims expressed in some language or other, cannot be restricted to just that language” (Restall 2002, p. 103).

It is not clear that this claim can be borne out. Missing from this description is the active consequence relation(s) at hand, but we can illustrate the problem without. Consider:

$$\frac{A_{L_1} \qquad B_{L_2}}{C_{L_3}}$$

Let $A_{L_1}, B_{L_2}, C_{L_3}$ be $(\varphi \rightarrow \psi)_{L_1}$, φ_{L_2} and ψ_{L_3} respectively. Firstly we are not guaranteed that φ_{L_1} and φ_{L_2} can be treated as equivalent. Second, here is a perceived correspondence of the meaning of the symbol $\rightarrow$ as a common notion of the conditional across the languages, such that we can apply any consequence relation across all three languages. This is just presupposing that they share the same meaning, or it appeals to some pre-theoretic notion, which threatens the notion of linguistic independence. The meaning of the connectives is given by the the transformation rules, i.e. logical consequence. We have no guarantee that all three languages even have the same set of logical connectives, except by stipulation. If L_2 and

Here is a real-world example of how the slogan can be misconstrued based on the different meanings of the term ‘language’, so the slogan has both a rightness and a wrongness to it, from a Carnapian perspective.

Kouri sees the Carnapian view as entailing two things. The first is in-line with Shapiro’s loose interpretation of the principle of tolerance, just that we should be tolerant, read: collegiate, of different logics. “As long as we can build a logic..., and provide applications for it, that logic is legitimate"(Kouri 2018, p. 2). Here we must be careful. While Carnap did say that any evaluation we have on a logic would be done on the pragmatic, or application, level he did not go so far as to indicate that this would threaten its status as a logic. Moreover, as language choice and logic choice are closely tied this goes the same way for languages. They are just constructed things and their legitimacy is guaranteed by the act of construction. Carnap’s anti-prohibition stance on logical syntax rejects these sort of necessary ties to application and the very notion of ‘legitimate’ as used here. The second entailment is that the meaning of the logical connectives are *given by* their inference rules. Meaning here, then, is solely syntactic and defined by our construction of the language.

Kouri notes that there is a possible conflation at work here. We must be careful to differentiate between two notions. That of building a language from rules, and that of building a language from rules which the builder knows to be distinct (at a fundamental level) from the rules of other languages (Kouri 2018, p. 2). The second is roughly ‘Do they mean the same thing?’ Carnap merely says that we should build our languages up from there rules, and so long as we do so we are free to do it with no prohibitions. This opens the door for people to ‘redevelop’ languages whenever they want, perhaps with different symbols, e.g. $\wedge$ instead of & for conjunction. The conclusion that Carnap clearly means the former notion carries through.

While Kouri’s epistemic point is interesting and seems a likely culprit in certain exchanges it does not get us as far as she hopes. “However, a builder cannot claim that her language is different from any other without first making some other assumptions about a meta-framework in which she is making that claim."(Kouri 2016, p. 2) Here Kouri is making a claim that the only way to compare systems is via a meta-framework, her new interpretation. This is not so. Carnap specifically says that we can, and do, talk about languages, it’s just that we do that in our meta-language and that we should do so on the pragmatic level, talking about the consequences of the theories in their totality.

Here is a reasonable way of tackling the problem of identical systems. Their consequences, when used in some application, are identical across the board, so they are very likely the same system. We do not get license so easily when trying to do the same for individual sentences, or connectives, as we cannot so easily pick just the segment of the language which has those things, the various transformation and formation rules have interplay which may result in differences in consequence despite simple sentences only containing said connectives seeming to have the same consequence, or transformation rules.

L_3 do not contain a definition of the conditional, $\rightarrow$, then it seems like we cannot even take the evaluative step. This flies in the face of the fact that we *could* do such an evaluation, provided that we fix the language, but the example is claiming that we precisely do not do so. We have stipulation, not justification. We will discuss the problems of translation in section 4.3.5.

Moreover, Carnap says that we shouldn't talk outside of languages about them. Questions of that sort are incoherent, discussed in the next section. Thus this alternate reading, which may be conflated by some with the simpler question is one that Carnap wants to reject as incoherent as it is about meaning

4.4 Internal and external questions

The central question that Kouri wants to account for is: "When do corresponding connectives in distinct linguistic frameworks have the same meaning?"[4] To understand the problems with asking such a question we first must discuss Carnap's approach to questions in general, which was most clearly developed in his "Empiricism, semantics, and ontology", where he further developed his philosophical program, with its anti-metaphysical goals, when discussing languages and abstract objects.

The first distinction is that of theoretical questions and non-theoretical questions. Theoretical questions represent the type of question that scientists and philosophers can, and should, be asking and answering. The non-theoretical questions are the type of questions that his program of changing philosophy, following the logical empiricists, is directly aimed at. For Carnap, there are two categories of questions: internal and external. Internal questions are questions which are asked within the linguistic framework, and thus follow the rules of that framework. All internal questions are theoretical as they are bound by the framework that they occur in. For example the question: when do the expressions φ and ψ mean the same thing, or are synonymous, in language L.[5] As an example, Carnap describes two people, one a realist about numbers and the other an anti-realist. The question at hand is 'Are there numbers?' he notes that the internal reading of the question is trivial. If the language in which we are asking 'are there numbers' in has number-terms in it then the answer must be 'yes', if it does not, then the answer is 'no'. The answer to these type of questions directly follows from the definition of the language. He uses this to suggest that it is doubtful that philosophers could mean this question, given how much time and effort has been put into answering it, if it were so trivial (Carnap 1950, p. 25).

External questions are ones asked outside of a linguistic framework, though they could be about some particular linguistic framework. There are two kinds of external questions: pragmatic and pseudo-questions. An example of a pragmatic question is whether a given language is useful for some purpose. These are legitimate questions which can be answered. Example properties that questions of the application of a language one could ask about are: efficiency, fruitfulness and simplicity (Carnap 1950, p. 31). Questions concerning these qualities are theoretical in nature, despite being external. These are pragmatic type questions, although they are asked in our language of discourse they are theoretical because they are directly about the choice, or use, of a linguistic framework. In this way they are about the frameworks. They are not binary yes-no questions, but ones of degree. This linguistic framework

[4] Here Kouri is using the term 'linguistic frameworks' instead of languages to differentiate between our everyday concept of languages, i.e. including natural languages, and Carnap's notion of constructed formal languages.

[5] To find the answer we look at the *content* of all sentences which have φ in them to see if they are *equipollent*, i.e. they are consequences of each other (Carnap 1937, p. 42).

is more useful for some application because of its simplicity and general accuracy, etc. Introducing a new system of number-entities to our language and then asking questions about whether certain entities exist within this framework are internal questions. External questions are those that concern the existence or reality of the system of (number) entities as a whole (Carnap 1950, p. 21).

Carnap warns that accepting a language along these grounds should not be conflated with accepting the objects of that language with the reality of the world (Carnap 1950, p. 24). Returning to the example, accepting a language with number-terms in it does not mean we accept that there are things called ‘number’ out there in the real world. Instead it is accepting that using a language with this type of word is useful for some purpose or another. In the other direction, even if you do not believe that there are things in the world that are numbers this does not mean you are forced to not use a language with number-terms.

Taking the acceptance of the language with number-terms to indicate something about the world, and the existence of numbers, is a misstep and an example of a pseudo-question, which are all other non-pragmatic external questions. Pseudo-questions are exactly the type of philosophical inquiry that we should avoid. They are questions which are asked without any relation to any linguistic framework, and this lack of tether to any language is what prevents them from being theoretical questions. If a statement is meant externally then it is non-cognitive, it lacks cognitive content (Carnap 1950, p. 26). This notion of cognitive content is a technical one.[6] Only statements of facts have cognitive content, while statements of values do not. This is a form of non-cognitivsm commonly associated with the logical positivists (Hofweber 2016, p. 19). Internal questions, in virtue of being asked in and being about, a linguistic framework, have cognitive content. Carnap supposes that perhaps the external question asker is not asking after the reality of the world, but rather a practical question of deciding on the structure of our language. This would make the question a useful, or pragmatic, one despite not having cognitive content.

The problem lies with questions that seem to be factual but are asked externally. If Carnap’s interpretation about the practical decision concerning the structure of the language we should use is resisted, then we are left with questions that seem to be about facts, ‘are there numbers?’, but which are asked externally so have no cognitive content. They cannot be answered, as they are outside of a linguistic framework and not about framework choice, and so they are illegitimate.

This fits into his greater program for philosophy, and logic, as a whole. Pursuing such questions is bad philosophy, according to Carnap, and these question are *incoherent*. Suppose we have two philosophers, one a realist about numbers, the other a subjective idealist. There is no evidence that can be given that would be relevant and satisfy both philosophers and, if found, would settle the matter, or at least make one more plausible than the other. “Therefore I feel compelled to regard the external question as a pseudo-question, until both parties to the controversy offer a common interpretation of the question as a cognitive question; this would involve an indication of possible evidence regarded as relevant by both sides" (Carnap 1950, pp. 24–25). In order to have cognitive content there must

[6] For a clear and detailed discussion of these notions read (Hofweber 2016, pp. 15–20).

be some fact of the matter that is settle-able in a manner accepted by all parties, otherwise it is a pseudo-question which is not worth pursuit as it is not answerable. Without this, in a sense it is a matter of orthodoxy and thus not a legitimate philosophical pursuit. The fact that the question is being asked outside of any linguistic framework shows that they have not succeeded in giving cognitive content to the external question, and thus there can be no such content for any answers that follow. The structure of questions like 'are there numbers?' is similar to that of theoretical, or internal, questions but they are non-theoretical, pseudo-questions, in disguise as they are not meant to be evaluated internally and are not pragmatic in nature (Carnap 1950, p. 25).

Kouri is correct in determining that the question 'When do corresponding connectives in distinct linguistic frameworks have the same meaning?', as it is formulated, it is a pseudo-question. There are two problems with this external question. The first, and more obvious, is that this is clearly a question of meaning which is external to, not one but, two linguistic frameworks. This type of question is a binary, yes-no, question; a matter of fact, but not an internal question. It is also not about the best fit or usefulness of the linguistic frameworks being used, that is it is not a value-based question that is pragmatic in nature. We can clearly mark it as a pseudo-question, not having cognitive content, and philosophically incoherent.

The second problem is the idea of 'corresponding connectives'. The very idea of a correspondence between the connectives seems to imply some relation between them, despite this being decidedly in the realm of external as well. We can, perhaps, suggest that the correspondence is one of mere pragmatic choice, but then we will get entangled in specific application choices in order to make sense of that. We will return to this problem in the next section.

A potential answer to the question of corresponding connectives that might immediately appeal is 'never'. It is not enough to say that they will never have the same meaning, as the question is not answerable, it is external and not pragmatic, thus a pseudo-question.

While Kouri comes to the same conclusion that Carnap would seem to, that 'never' is not the right solution, she gets their on an odd point. "Even if... we still have no postulated meta-linguistic framework in which to compare the rules and postulates in question. The answer to the question, then, cannot be 'never' " (Kouri 2018, p. 3). Here she has focused on the fact that the only coherent version of this would be an internal one, but seems to take the leap that this is an issue of language construction. What is stopping us is the lack of a framework to ask such a question, not the character of the question itself. We are in danger of begging the question in answering that it is the lack of such a framework that is preventing the question, which is unmotivated at this point.[7]

By positing a meta-framework the question ceases to be external, as it becomes internalised to the new framework. It is not a question of what is *really* true, but what is true with respect to this new framework (Kouri 2018, p. 4). What is missing from her discussion is this act of internalising changes

[7]It may seem that the preceding quote from Carnap referring to 'a common interpretation' goes to support a move towards such a framework, but, as we will see in the coming pages, the choice, or construction, of such a framework is not so easily done. To choose a framework presupposes a common ground, it is chosen on pragmatic grounds, and technical machinery cannot add cognitive content. Such a framework will bring the same problems which exist at the lower level along with them. The details of this are in the section 4.5.

the question as well; internalising has its costs, it is not just a new formalization. This is invoking the technical layer in order to deal with the theoretical.

The purported goal of the enterprise was to extend Carnap's idea of linguistic frameworks such that we could talk coherently about the meaning of two connectives in distinct languages and ask whether they are the same or not. Here we have a question of whether two connectives have the same meaning *with respect to a particular meta-framework which we have constructed for this precise purpose*.

Kouri motivates the original question by appealing to the idea of resolving meaning disputes of connectives between languages, and states that what is required for such a task is a meta-framework with the appropriate terminology and relations. But the meta-framework choice is pragmatic, and constructed, thus we choose the one which gives us the answer that we are looking for.

For Carnap these questions are incoherent and a diversion from good philosophy. His strong stance against metaphysics as a fruitful pursuit was precisely because it involves nothing but pseudo-questions, which should be avoided at all costs (Carnap 1935, pp. 21–22). Pseudo-questions are more than just hollow statements looking for a meaning to occupy them such that they gain meaning. Carnap wants us to avoid pseudo-questions, not because they are not in the right context but because they are not real questions, they have no cognitive content (Carnap 1950, p. 25). There seems to be a gap between Kouri's notion of pseudo-question and Carnap's. It is unclear why Carnap would want to pursue such a question as fruitful, regardless of the formal mechanisms that may be employed to make such an attempt. He does not just think that the connectives in distinctive frameworks don't have the same meanings, he thinks the question misses the point entirely.

4.4.1 Meta-linguistic frameworks

Kouri proposes a solution to the problem of the external question by the way of *meta-linguistic frameworks*.

> What, though, if we postulated the existence of such a meta-linguistic framework, one which was capable of talking about the two linguistic frameworks in question? This would, I will show, give us an opportunity to answer the question of when two distinct frameworks have the same logical terminology. Additionally, I will show that the answer to whether two object-frameworks have corresponding connectives with the same meanings in a meta-framework will vary depending on our pragmatic goals, and hence the meta-framework we select. (Kouri 2018, p. 5)

At the heart of Kouri's approach is the idea of embedding the question within a meta-linguistic framework, in order to revoke its pseudo-status and *internalise* the question. We should pause here and ask what the motivation of such a project is. Unfortunately Kouri does not go further than asking the 'what if' question, so we are left with the results of the project as its own recourse. As the project is an attempt to interpret Carnap we should be wary of the concentration on a pseudo-question in general.

Kouri suggests that if there were an over-arching linguistic framework from whose point of view we could evaluate the two languages, then we could answer a different question. When do two linguistic frameworks have logical terminology which means the same thing (with respect to the meta-framework)? This is no longer about what is really true, making it a theoretical, not external, question (Kouri 2018, p. 5).

Here we have a positive claim about the question. It seems the worry of corresponding connectives has been removed, though perhaps it is just buried in the move to logical terminology instead of connectives here. The key premise that allows this to be a theoretical question is that this is being asked within some meta-linguistic framework.

As Kouri notes, the move to the internalised version of the question isn't about *real* meaning. Before moving on we must ask what relation the two questions have to each other. By internalising the question to some meta-framework the question is being changed. To what degree does this change matter is central to the move.

Returning to Carnap's discussion of such questions we see that when people are asking 'Are there numbers?' they are not asking it in virtue of some system or another, they seem to be asking, or at least attempting to ask, something more than that. They are after an external sense of existence. With Kouri's original question 'When do corresponding connectives in distinct linguistic frameworks have the same meaning?' the motivation seems to be about the semantic meaning of the connectives in the two distinct frameworks, and not their syntactic reading with respect to some other framework.

By embedding the question, and thus making it an internal one it seems unlikely that the thrust of the question remains, given the move from semantic to syntactic meaning. Here we should ask what is to be gained by asking, and answering, the embedded question instead of the un-embedded question? And are they the same type of gain? Put another way, is this move desirable to those who are asking the original question?

There is an additional problem with the meta-linguistic frameworks. Kouri motivates her project by way of examples but does not give a clear characterisation of what a meta-linguistic framework is. In the appendix to this chapter, we show that one can use the syntactical tools already available in Carnap's "Logical Syntax of Language" to achieve this goal. By not giving a clear, read: syntactic, explanation of meta-linguistic frameworks, Kouri runs afoul of Carnap's principle of tolerance.

> Everyone is at liberty to build up his own logic, i.e. his own language. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical [as opposed to scientific] arguments. (Carnap 1937, pp. 51–52)

4.4.2 An example

The first example Kouri considers the two object frameworks of classical and intuitionistic logics. She proposes two meta-frameworks, positing that the only necessary information are the rules of the framework which determine sameness of meaning (Kouri 2018, pp. 5–6). The first meta-framework is the standard Gödel-Gentzen translation, T_1.

1. if φ is atomic, then $T_1(\varphi) = \neg\neg\varphi$
2. $T_1(\varphi \wedge \psi) = T_1(\varphi) \wedge T_1(\psi)$
3. $T_1(\varphi \vee \psi) = T_1(\varphi) \vee T_1(\psi)$
4. $T_1(\varphi \rightarrow \psi) = T_1(\varphi) \rightarrow T_1(\psi)$
5. $T_1(\neg\varphi) = \neg T_1(\varphi)$

Now consider T_2 which is defined as above except:

$2^\star$. $T_2(\varphi \wedge \psi) = \neg\neg(T_2(\varphi) \wedge T_2(\psi))$

Both meta-frameworks have the desired property that φ is provable classically if and only if $T(\varphi)$ is provable intuitionistically.

Here our translation is at the syntactic level and we can ask “do intuitionists and classicalists mean the same thing by ‘$\wedge$’?". If translations give us a relation of synonymy then as $\wedge$ is translated by T_1 as conjunction, and double negation of conjunction in T_2, then we have sameness of meaning with T_1, but difference of meaning with T_2. Depending on our purposes, one translation will be better than another, for the task at hand, as we can generate different relations for the ‘same meaning’ (Kouri 2018, p. 5).

4.4.3 Meta-framework disputes

> There is no ‘external’ perspective from which we can ask which logic is really correct, as any such question will have to be formulated within a language, and thus from within the perspective of some framework or other. (Cook 2010, p. 497)

While Cook is directly addressing the notion of logical correctness in the above quote, the point relates to all external notions, such as the meaning of connectives. The dispute over meaning is not one that can be answered as it is asked externally. However, with Kouri’s meta-frameworks in hand we need not worry as we are asking it with respect to a framework.

The spectre of pragmatic choice rises up. On the one hand, we are left with two frameworks which tell different stories about the meaning of connectives, which may be of use in our pragmatic choice of frameworks. On the other hand, we have no recourse in the case of disputes between meta-frameworks. As we are selecting our meta-framework based on our pragmatic concerns, when there is a dispute between such concerns we are at an impasse as there is no use of the machinery to settle the matter, much like we are in with respect to questions of the one true logic that Cook is referring to.

Returning to the example, we have T_1 and T_2, as potential candidates for meta-frameworks to answer the question as to whether the $\wedge$ means the same in intuitionistic and classical logic. Based on the syntactic fallout, It is clear that they have different answers to this question.

We can imagine a similar scenario to that of Carnap's number existence. There are two people submitting their meta-frameworks to compare intuitionsitic and classical logic. The first goes the Gödel-Gentzen route with T_1 and the second proposes T_2. They both have their pragmatic reasons to support their theories, for the given application at hand. It seems that there is no clear fact of the matter to determine things, one way or the other, and so we are at a pragmatic stalemate. "I cannot think of any possible evidence that would be regarded as relevant by both philosophers, and therefore, if actually found, would decide the controversy or at least make one of the opposite theses more probable than the other" (Carnap 1950, p. 37). At least if such evidence exists it does not turn on any of the machinery at hand and will just be left in the pragmatic discourse of the two philosophers. The machinery does not help us settle the matter of whether the two connectives are the same.

4.5 Pragmatic choice

> I will show that the answer to whether two object-frameworks have corresponding connectives with the same meanings in a meta-framework *will vary depending on our pragmatic goals* and hence the meta-framework we select.[8] (Kouri 2018, p. 4)

Ultimately, Kouri claims that "[s]ince rules determine connective meaning, this implies the connectives will sometimes share a meaning and sometimes not " (Kouri 2018, p. 7). That is, depending on your choice of meta-framework the connectives will be deemed the same, or not. This reduces the exercise to one of pragmatic choice of meta-framework as we saw above.

Returning to Carnap's notion of the triviality of internal questions we see that given any linguistic framework, which has been constructed in the appropriate way, the answer to whether connectives mean (syntactically) the same come easily from the given inference rules of the framework. By constructing our framework based on our pragmatic goals we guarantee the answer that we are looking for. The meta-framework is not doing the work here, our choice of framework, pragmatically driven as it is, does all of the lifting.

As the decision is ultimately being made on the pragmatic level, and then reflected in the syntax of the meta-framework in question, this casts substantial doubt that the core of the problem has really been reduced to the syntactic. Despite presenting the reformulated question in terms of a meta-framework, making it internal for that framework, the question has to be already decided when we construct, or choose, the meta-framework to use. It will then trivially follows from that framework.

The previous quote of Carnap, mildly elaborated in footnote 7, suggests that there is a way out for the two mathematicians, so long as they can find a 'common interpretation' in which to converse. The meta-linguistic frameworks appear to offer such a solution. There are problems with taking this approach, however. The idea of a common interpretation requires agreement from both parties as to the interpretation to be taken. Recall that the meta-framework dictates the answer, that is the framework we choose to embed into has to 'pre-choose' what the translation will be. We choose the framework

[8] Emphasis mine.

that gives us the answer that we are looking for. Given that this is precisely the area in which the mathematicians disagree, this means that said disagreement will carry forward into the choice of appropriate meta-framework. The dispute would have to be solved *before* we embed into the framework of choice. This should not be surprising, as adding more technical machinery cannot add cognitive content. In the instance of the question of whether there are numbers, adding more technical machinery to our discussion cannot secure facts about the world which were not already reachable. Internalising questions of this nature misses Carnap's point that these types of questions are not coherent, at least for a Carnapian philosopher. This is not to say that meta-linguistic frameworks are not useful in general, and likely for a wide variety of scenarios, just that the invocation of them to solve pseudo-questions is not a fruitful enterprise.

Here is a second example, adapted from a case from Shapiro (Shapiro 2014, pp. 127–133).

> Here, we have two mathematicians, one classical and one constructive, discussing some form of analysis. The classicist normally defines her connectives via truth conditions, and the constructive mathematician normally uses proof conditions. One might think that the only way to embed these two object-frameworks into a meta-framework is to do so in such a way that none of the acceptable translations translate the connectives homophonically. However, during their exchange, and for the purpose at hand, they never discuss the connectives, nor do they consider any results which are not acceptable to both of them. In this situation then, it makes sense to talk about them as though they are "speaking the same language", that is, as though they are both using connectives which mean the same. This is an example where rules which do not look the same may in fact imbue corresponding connectives with the same once embedded into a meta-framework. (Kouri 2018, p. 8)

This example is one of conversation, involving syntax, semantics and pragmatics. Carnap addresses the problem of formalising natural, or conversational, languages as a difficult and overly complex task precisely for these reasons. Taking Carnap's view to this situation it seems that they are discussing this form of analysis and discussing its consequences pragmatically as we can in our language of discourse. The use of conversational language is at odds with our regimented use of Carnap's term 'language' which motivated the shift to using the term linguistic framework in the first place.

Looking only at the formalism, it does seem like we are limiting our discourse to syntactic notions, as required for the question to be theoretical and not a pseudo-question. However, the formalization, or construction, of the meta-framework is a process that is not only involved, it is determinate of the syntactic results. This construction will be based in our pragmatic choice of meta-framework. The pragmatic choice of a framework is based on its usefulness for some purpose. In this case it is about whether the connectives in two distinct languages are the same. However, as we are building the syntactic framework to our specifications, we must be appealing to something beyond that to guide those specifications. We are ultimately left with the question, external to any linguistic framework, “do corresponding connectives in distinct linguistic frameworks have the same meaning?"

4.5.1 Translation and support

Kouri claims that the standard passages of Carnap’s pluralism show nothing that explicitly prevents comparison of two languages within a third. From Carnap we see that the original question has no cognitive content, so appealing to a lack of a specific prohibition in this case does not show agreement in any strong sense. It is true that asking the question with respect to a third language is an internal, thus theoretical, question, and not a pseudo-question, this is not her claim though. It is not given that questions of meaning are theoretical questions, they could be pseudo-questions. Claiming that all meaning questions are theoretical misses Carnap’s point. Further, as these meaning questions are being considered externally, as we are replying by invoking a meta-framework, this makes the questions clearly external and thus pseudo-questions.

“[M]eaning questions are external questions on his view, and can only be answered if we make them internal by asking them with respect to some framework" (Kouri 2018, p. 7). As Carnap describes external questions as ones which do not have cognitive content, it is not clear where the cognitive content is coming from if we are dealing with the same question. If this is a different question, as argued above, then the new internal question trivially falls out of our construction of our meta-framework. The pragmatic choices we make are the only available source for cognitive content, but the choice of one framework or another will be based on our end goals, and thus external ideas.

Kouri appeals to Carnap’s notions of translation as support noting that Carnap discusses translations between language in 61 of *The Logical Syntax of Language*. Using the following from Carnap Kouri concludes that translations must occur in a meta-language.

> The interpretation of the expressions of a language S_1 is thus given by means of a *translation* into a language S_2, the statements of the translation being effected in a syntax-language S_3; and it is possible for two of these, or even all three, to coincide.(Carnap 1937, p. 228)

The terminological shift from meta-framework to meta-language is curious, but it is troublesome regardless because it seems to describe something above the linguistic frameworks, and thus the shared meaning having some upward potency, but it is in fact just a third language with translations between, which are limited to the rule of translation, and thus the syntactic synonymy defined will only reflect certain pragmatic goals.

Aaron Guthrie introduces the diversity of translations before moving on to discuss formal translations.[9]

[9] Guthrie’s argument is that translations do not necessarily preserve meaning and they involve language dependence which would fly against the Kouri’s project.

> It might seem that translation is utterly straightforward; a translation preserves meaning! But there is more to translation than this. One might have a translation that preserves meaning, truth conditions, metaphor, extension, intension, hyper-intension (and so on). (Guthrie 2017, p. 16)

When we formalize a concept, what counts as a good translation is dependant on what we are formalising, and why. For logical consequence, the truth-conditions may not suffice in all cases. If we are formalizing an intensional concept, we may need to preserve intensional content (Guthrie 2017, p. 17). Translations are tricky beasts. In order to preserve what we think is relevant we must design our translation to carry that over to the new language. Carnap holds that what a translation must do is dependent on our theoretical goals.

Depending on what we are trying to do pragmatically we will choose a translation which carries over that relevant feature of the language. *Synonymy* means something different for each translation, namely it reflects sameness with respect to whatever is being preserved by the translation at hand.

Here we must pause, for what is of concern here is precisely meaning. Returning to the original question “when do corresponding connectives in distinct linguistic frameworks have the same meaning?” Presumably this is about the *entire* meaning of the connective. With this shared idea of the limitations of translations it seems like addressing this question via translations is unlikely to be fruitful. With the internalised version of the question the answer trivially follows from the meta-framework of choice. The choice driven by our pragmatic aims of constructing such a translation. “When do two linguistic frameworks have logical terminology which mean the same thing (with respect to some meta-framework)?” When we want them to.

4.6 Conclusion

Following the principle of tolerance we do not offer a prohibition on constructing such a system as Kouri proposes. Carnap would definitely support the idea of new language forms in general. It seems that can be done with the already present notions of translation on offer, but we must ask for syntactic arguments given the system and further evaluate it pragmatically along the purpose given. Here we seem to have let go of the very notion driving the original question ‘When do two connectives in distinct linguistic frameworks mean the same thing?’ Following Carnap we conclude that this is a pseudo-question and pursuing something past philosophy’s domain. Internalising the question changes it substantially and we are still left with all the work being done by our choice of meta-framework and translation. Pragmatically the new machinery gives us the same results as Carnap’s super-languages and our judgements of equivalency reduce to our chosen orthodoxy on the connectives at hand. Any dispute between meta-frameworks results in the same traps that Carnap warns us of when dealing with pseudo-questions.

Appendix: An old interpretation of Carnap

We have seen that the external question has shifted when embedded into a formal framework. Kouri claims it has gained cognitive content in virtue of the fact it is limited to the internal question of whether or not the connectives of the linguistic frameworks have the same meaning within the larger formal language.

Above we suggested that we are trapped, as Carnapians, without a clear notion of a meta-linguistic framework. Perhaps we are being too hasty. We can provide a syntactic system that addresses the goals of Kouri's meta-framework without leaving the early work of Carnap in the *Logical Syntax of Language*. The structures required to describe such comparisons between langauges are already available.

An obvious candidate for meta-linguistic frameworks are linguistic frameworks, which are built with some relation to our two distinct frameworks of concern. Here it turns out we need not go further than Carnap's *Logical syntax of Language* to supply the necessary syntactic machinery. In sections 61 and 62 he addresses translation, synonymy and the interpretation of languages. Below we will provide these definitions and show how they seem to play the appropriate role for Kouri's project. As Kouri wants the notion of translation to be open we must first start with more generic terms building up to notions of translations and synonymy.

With the deflation of scope of Kouri's meta-frameworks and the depth of Carnap's discussion on these matters it appears that Kouri's meta-frameworks are little more than a particular kind of language which contains, or can be built to contain, our frameworks at hand. Carnap provides the syntactic machinery to talk about syntactical synonymy between these linguistic frameworks.

Theorem 1. *If a sentence is derivable from other sentences, then it is also a* consequence *of them.*

$\mathfrak{R}_1$ is a class of expressions (finite string of symbols of the language).
The logical *content* of $\mathfrak{R}_1$ is the class of non-analytic[10] sentences which are a consequence of $\mathfrak{R}_1$.
Sentences (or classes of sentences) which have the same content are *equipollent*.
Let $\mathfrak{Q}_1$ be a syntactical correlation between syntactic objects, which can be across languages.
If $\mathfrak{Q}_1$ is between all sentential classes (or all sentences, or the expressions of an expressional class $\mathfrak{R}_1$, or all symbols) of some language, S_1, and another S_2 it is called a *transformance* of S_1 into S_2, provided that the consequence relation in S_1 is transformed into the consequence relation of S_2.
A transformance of S_1 into S_2 is called *reversible* when its converse is a transformance of S_1 into S_2.
Let $\mathfrak{Q}_1$ be a transformance of S_1 into S_2; if $\mathfrak{Q}_1$ is reversible, then $\mathfrak{Q}_1$ is a one-one relation. The converse is not universally true.
Let $\mathfrak{Q}_1$ be a transformance of S_1 into S_2; and let S_2 be a sublanguage of S_3. Then $\mathfrak{Q}_1$ is called a *translation* of S_1 into S_3.

[10]The class of sentences which is a consequence of the null class of sentences.

For Kouri's meta-linguistic framework we then define the two target languages with respect to some overarching language.

S_2 is called a *sub-language* of S_1 if the following conditions hold:

1. Every sentence of S_2 is a sentence of S_1.
2. If $\mathfrak{R}_2$ is a consequence-class[11] of $\mathfrak{R}_1$ in S_2, then it is likewise a consequence-class of $\mathfrak{R}_1$ in S_1.
3. S_2 is a *conservative sub-language* of S_1, if $\mathfrak{R}_2$ is a consequence-class of $\mathfrak{R}_1$ in S_1 and both $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are in S_2, then $\mathfrak{R}_2$ is also a consequence-class of $\mathfrak{R}_1$ in S_2.
4. If S_2 is a sub-language of S_1, but not S_1 of S_2, then S_2 is called a *proper sub-language* of S_1.

Let S_1 and S_2 be sub-languages of S_3; and let $\mathfrak{Q}_1$ be a translation of S_1 into S_2. If in this case, $\mathfrak{R}_1$ and $\mathfrak{R}_1[\mathfrak{Q}_1]$ are always equipollent[12] in S_3, we call $\mathfrak{Q}_1$ an *equipollent translation* in respect of S_3.
If $\mathfrak{Q}_1$ is a translation in respect of symbols or expressions such that for any expression, $\mathfrak{U}_1$, and its translation, $\mathfrak{Q}_1[\mathfrak{U}_1]$ are always synonymous is S_3, we call $\mathfrak{Q}_1$ a *synonymous translation* in respect of S_3 (Carnap 1937, p. 226).

Mapping the meta-framework

With all this machinery in place we can now describe Kouri's meta-framework within Carnap's logical syntax of language. There are two options for going forward. First we can take it that the meta-framework is of the same type as linguistic frameworks in general, then it will be some object language S_3. As the external question requires the two languages to be distinct, at the very least, then both S_1 and S_2 will be proper sub-languages of S_3, but not of each other.

If this is the case then we run into a problem. S_3, being a linguistic framework, has its own built-in logic. The rules of formation and transformation here will supercede, or at least we are stuck with its rules being determinate and not the rules of the distinct frameworks.

This may be too restrictive and we could, instead, take the meta-framework to be the rules governing the transformance $\mathfrak{Q}_1$ itself. In either case the distinctness clause will require the two languages to not be sub-languages of each other.

Kouri uses the following from Carnap to justify a multitude of possible transformances as viable: "Sometimes... it must depend upon a reversible transformance, or it must be equipollent in respect of a particular language, and so on" (Carnap 1937, p. 228). In this passage Carnap is speaking in terms of translations in general, and specifically about translations between natural languages here. Our natural understandings of the limits of translations preserving whole meaning in this circumstance should cause us to pause on wholly accepting this broad set of transformances.

[11] $\mathfrak{R}_2$ is called a consequence-class of $\mathfrak{R}_1$ if every sentence of $\mathfrak{R}_2$ is a consequence of $\mathfrak{R}_1$. $\mathfrak{S}_1$ is a consequence of the sentential class $\mathfrak{R}_1$ if $\mathfrak{S}_1$ belongs to every sentential class $\mathfrak{R}_i$ which satisfy (1) $\mathfrak{R}_1$ is a sub-class of $\mathfrak{R}_i$. (2) Every sentence which is a direct consequence of a sub-class of $\mathfrak{R}_i$ belongs to $\mathfrak{R}_i$.

[12] see fn. 5 above

This is where the choice comes into play. When defining our transformance, $\mathfrak{Q}_1$, we make a choice as to what syntactical objects are being correlated: all sentential classes, all sentences, the expressions of an expressions class $\mathfrak{R}_1$, or all symbols.[13] If we choose to translate at the level of symbols we will run afoul of the problem Kouri addreses with the example from Field (2009), translating between classical, intuitionist, and some paraconsistent logics, specifically the treatment of negation. “Sometimes, translating the connectives by the transitive translation... will serve our theoretical purposes... [s]ometimes, however, it will not, and the best available translation will not map two things we thought might mean the same onto each other" (Kouri 2016, p. 5).

As the meta-framework, in either case, is concerned with answering the question “When do the connectives in two distinct languages have the same meaning?" this puts some limitations on the possible candidates for $\mathfrak{Q}_1$. We are free to choose $\mathfrak{Q}_1$ to our pragmatic means. As we are discussing whether the connectives have the same meaning, our transformance $\mathfrak{Q}_1$ will have to be *isomorphic*, that is reversible. Similarly, as we are concerned about meaning here the translation will have to be equipollent, if not synonymous. If we choose a transformance that is not synonymous then we are choosing that the connectives will not have the same meaning, syntactically. This highlights the entire role that pragmatic choice has on the exercise.

[13]On page 225 of *The Logical Syntax of Language* Carnap gives a useful table showing what properties are preserved depending on the transformance being used.

Chapter 5

What is Formalization?

5.1 Introduction

The central claim to this dissertation is about what logics are; namely that they are formalizations. In order to establish this we must first be clear on what we mean by formalizations, which is the aim of the present chapter. From this we will be able to describe and explore the proposal of *logics-as-formalizations* in the next chapter.

Traditionally the focus has been on the notion, and properties, of *formal* and thus demarcating the logico-formal border becomes a core concern (J. MacFarlane 2000), (Iacona 2018), (Dutilh Novaes 2011). That is, exactly what constitutes a logic which leaves other, similar, formal systems out in the cold. We shall see that this does not tell the whole story and so will instead focus on *formalization*, as an activity or process. We will still find the discussion of formal, and the possible ways to demarcate the logical from the formal as useful to our discussion, and by the end of the chapter we will have a clearer picture on what such a border represents.[1]

In section two we will discuss the standard views on formalization, we will discuss axiomatic systems, and show that they are both too broad and too narrow to capture formalizations. We will then discuss recent commentary on the notion of 'formal' noting that this falls prey to product process ambiguity, the characteristics of formalization as a process when we just look at the resulting product: formal systems. We briefly discuss translation and how formalization cannot be read as simply a map between natural and artificial languages. In section three we engage with the idea of formalization as a process, looking first at the properties of the process and then what the input of the process is, concepts and their relations. Recent commentary has put formalization as a subset of Carnap's explication, we show that this mapping is unjustified and that it diffuses the paradox of adequate formalization. To do

[1] See (J. MacFarlane 2000, p. 7) for a good discussion on some reasons why one might care about demarcating the logic from the formal.

this we employ Sven Hansson's two-step model of formalization as well as the distinction between regimented language and specialised terminology. We end the section with the notion of different levels of formalization, concluding that the Hansson model is too permissive. We then move on, in section four, to discuss how formalization is a specific sub-type of the process of modelling before defining formalization and its relation to Kennedy's notion of *formalism-freeness* in section five.

Sören Stenlund differentiates two different notions of formalization that have existed since Frege. The first is that formalization is fundamentally a conceptual investigation of some domain of given concepts and conceptual relationships, not just the application of certain techniques. Logical analysis is the clarification of given concepts and their relationships, thus it is assumed that within that domain one can be absolutely right or wrong. This stems from Frege's universalism about logic. We call this the *ontic account* of formalization. The second view sees formalization as techniques for paraphrasing and transforming a conceptual domain into a formal structure; where there isn't a sense of absolute right or wrong, but rather its based on usefulness in applications (Stenlund 1994, p. 365). We call this the *pragmatic account* of formalization. We note that both accounts have formalization as the investigation of concepts and the relationships between them. What differs is whether the concepts are *given* or *approximated*.

Before continuing, we must first address what we mean by a formalization. We will start with a simple, and uncontroversial, example and then proceed to explicate the notion throughout the discussion.

Take the simple exercise in an introductory logic class. A sentence in a natural language, say English, and come up with a formalization of it in first-order logic.

> Formalize the following sentence into propositional logic:
> *If it is raining, then the grass is wet*
> $R \rightarrow W$

This seems simple enough, now we can ask questions about the nature of the sentence, $R \rightarrow W$, as a result of formalization, and as a formal object. We have a formalization in hand, and it is part of a well-known category. Propositional logic is part of a family of mathematical structures, or formal systems.

5.2 Standard views

Standard accounts on the nature of logic focus on the logico-formal border. As such they focus on what exactly is meant by formal, how logic exhibits these properties, and in what ways it is distinct from other formal systems because of these properties. We start the section off with a look at axiomatic systems as a candidate for the type of systems that we might want to focus on, for our greater project of explaining what logic is. This will prove to be both too limiting and too generous. We will then review recent views of formal, and the *de facto* standard account of formal system and show that they are lacking in that they miss out on the dynamic nature of formalization. It is a process, not just the end results, a formal system. Formalization, into artificial languages, is not just a mapping between natural and formal languages; it is not a translation. We finish the section with a brief discussion on the limits of translation, so the understanding of formalization as a simple mapping is too narrow for translation and that formalization is more complex than translation in general.

5.2.1 Formal systems

Defining precisely what formal systems are is not a straightforward task. Any definition of formal system will encompass logics. An obvious path would be to define them closer to our goal-posts of logic. This runs the risk of an ad-hoc definition of convenience, but let us see how far such a move can take us. We don't want to just stipulate that formal language means logical language; in fact these two do not coincide. This is the underlying assumption underneath the debate of the logico-formal border. This should be unsurprising, given our discussion in chapter two, our underlying concept of logic will inform a purpose, and that in turn an application, for logic. If there were no logico-formal border then we have removed the conceptual grounding of logic and entered the realm of pure logics. That is, we are only concerned with the formal properties of systems for the sake of studying them, and not for how they relate to some conception of logic.

If formal language were just logical language then we would be forced to exclude such systems like Euclid's *Elements* widely acknowledged as the first real treatment of mathematics in an axiomatic system. While it was subsequently proven that his attempt had flaws, surrounding the parallel postulate and superposition of figures, that is a question of success not one of whether it is a formal system itself (Wagner 1983, p. 63). There is a difference between what type something is and whether it is a 'good' example of that type.

Axiomatic systems start with some undefined (primitive) terms and also give a set of rules (postulates) that govern the relations between the terms, such that theorems can be derived from the initial postulates. This seems to map to Stenlund's notion that formalization is about concepts and their relationships.

As a work that was based on the success and form of *Elements*, Spinoza's *Ethics* serves as an additional example. Spinoza was impressed by the rigour and clarity of *Elements* and based his work on the same method, hoping to achieve the same level of rigour. The work is split into five parts, each starting with a list of primitive terms/properties, followed by a set of axioms/rules. The bulk of the book is deriving conclusions from this set starting point. Here is a brief sampling of the work: Part I - Concerning God: cause of itself, finite in its kind, substances, attributes, modes, God, free, and eternity. This is followed by seven axioms; the first of which is "All things which are, are in themselves or in other things" (Spinoza 2006, p. 4). This appears to not be a logical treatment of ethics, but it does seem to be a formal one.

Focussing on axiomatic systems has given some insight on to where a logico-formal border might be. It is both too tight and too wide of a net. If we look solely at axiomatic systems we will capture such things as Spinoza's *Ethics*. Axiomatics will also miss out on things we want to count in our definition. Consider Aristotle's syllogistics. Most easily described as the first system of formal logic, and the birth of the field of formal logic itself. If we restrict ourselves to the (modernly) familiar axiomatic systems we are stuck leaving syllogistics out in the cold. Returning to the difference between pure logics and logic in general, or a conceptually grounded notion of logic. At this point we should recall the difference between the study of pure logics and logic, that we pure logic in picks out a certain class of mathematical structures. This should not be conflated with our attempts to capture logics and formalizations in the general sense. Axiomatics do well to pick out the former sense, but fail to accurately do so in the latter. Syllogistics clearly is not a logic in the mathematical structure sense, but it would be odd to not call it a formalization, let alone one of logic or reasoning.

The key move of Aristotle's syllogistic system was the notion of *abstraction*. This is the same abstraction we employed in our first example with 'it is raining' and R. The semantic meaning of the terms are abstracted away, the syllogistic premises state the relation between the terms used. The syllogism use schematic letters to abstract away from the terms, so we have *Barbara* as the following schema:

$$\frac{\begin{array}{c}\text{All } A\text{s are } B\text{s}\\ \text{All } C\text{s are } A\text{s}\end{array}}{\text{All } C\text{s are } B\text{s}}$$

We can capture the idea of Greek men's mortality via the following instance of *Barbara*:

$$\frac{\begin{array}{c}\text{All men are mortal}\\ \text{All Greeks are men}\end{array}}{\text{All Greeks are mortal}}$$

Here we note that the meaning of the terms 'Greek', 'men', and 'mortal' play no role in the syllogism. The inference is grounded in the status of Barbara as a perfect syllogism; that is, it is a logical truth. What makes the conclusion follow are the relations between the terms in the premises, not the meaning.

The aim of the syllogistics, and Aristotle's entire project on deduction and logic in the *Organnon*, was to bring clarity to reasoning and point out patterns of necessary reasoning, based on the structure of the syllogisms (Aristotle 1984, p. 104). Focusing on axiomatics will leave out the first formalization of reasoning.

5.2.2 Formality

The standard approach in discussing logic, formal systems, and like matters is to centre on the notion of 'formal'. We will briefly look at three such views, and determine the scope of 'formal' too narrow. This is because they focus on the end result of formalization, formal systems, and fail to capture the important features of formalization as a process.

In his PhD dissertation John MacFarlane proposed a three-way distinction of in which logic can be formal.

> Three things: logic is said to be formal (or "topic-neutral")
> (1) in the sense that it provides constitutive norms for thought as such,
> (2) in the sense that it is indifferent to the particular identities of objects, and
> (3) in the sense that it abstracts entirely from the semantic content of thought. (J. MacFarlane 2000, p. ii)

MacFarlane makes the point that the tradition most interested in this demarcation, the Leibniz to Frege tradition, was interested in formal in the sense of (1), and that has virtually disappeared from the discourse, despite its previous centrality.

Looking at Aristotle's syllogistics we see that they pass through all three of these gates. One of Aristotle's primary aims was to provide a guide for good reasoning, constitutive norms. The second is a bit trickier, as one of the major shortfalls of the syllogistic system is that it abstracts both objects and properties into the same type: terms. That is, it lacks the ability to quantify over objects, which limits the expressive power of the system. It is indifferent to objects, but in a clunky manner. This clunkiness is about degrees of success, not passing through the gate. This same abstraction provides the third path, the semantic content of thought is replaced.

One of the most recent treatments of the notion of formal, and how it relates to logic, is from Andrea Iacona, in (Iacona 2016) and (Iacona 2018). Here he claims that there are two theoretical roles that logical form needs to fill: the logical role and the semantic role. The logical role is the formal explanation of logical relations, e.g. entailment, contradiction. The semantic role is the formulation of a compositional account of meaning (Iacona 2016, pp. 617–618). However, there are two notions of logical form: the syntactic and the truth-conditional. The syntactic notion is that logical form is determined by syntactic structure. The truth-conditional notion of logical form is that logical forms are determined by their truth-conditions. Neither notion can fill both roles, but the syntactic notion suits the semantic role, while the truth-conditional notion suits the logical role. Particularly this means that we must give up the *uniqueness thesis*.

> **(UT)**: There is a unique notion of logical form which fulfils both the logical role and the semantic role.

This distinction between roles maps to our notions of the technical and theorectical layers. We can safely talk about pure logics, and thus the logical role, without considering theoretical concerns about the concept of logic, and so we miss out on the semantic role; the account of meaning that is needed for us to use logic as a tool of analysis.

Given our analysis of syllogistics and MacFarlane's distinction, something appears to have gone awry. This is because none of the three distinctions cover the semantic role; (2) and (3) concern the logical role while (1) is concerned with the normative use of the system. Both of these gaps have to do with the use of a formal system. MacFarlane's (1) is about what the formal system can do for our reasoning. The semantic role, similarly, is about obtaining an account of meaning from our formal system.

Exploring the issue Catarina Dutilh-Novaes gives a taxonomy of notions of formality; providing eight different variations of formality, grouped into two clusters: pertaining to forms, and pertaining to rules. The first cluster is aimed at de-semantification. It is about the goals of moving to a formal representation. The second cluster is aimed at computability, roughly the properties of the resulting formal representation. These, naturally, are not co-extensive enterprises. One is process-oriented, while the other concerns properties of the results of creating a system of formal representation, i.e. formalization.

In these investigations of the logico-formal border we can, for any technical property P, find a candidate system which falls on one side of the border and another residing on the opposing side. This is because we don't easily have a way to distinguish between the pure logics and the other formal systems. We find that these attempts to identify logics versus formal systems encumbered due to this deficiency. They focus only on the ontic, and neglect the pragmatic nature of logic.

We see from these various views that there is an ambiguity on both the notion of formal and the problem it is supposed to solve. Iacona highlights the semantic role of formal objects which is tied to what they are used to formalize. Here we should note that our example is unfinished, we are missing the *correspondence scheme*[2] which maps the natural language sentences to the proposition letters (or predicates in the cases of first-order systems). Thus the full formalization is:

If it is raining, then the grass is wet
$R \rightarrow W$
R: It is raining.
W: The grass is wet.

The correspondence scheme acts as a bridge between the object and its formal counterpart, showing which original elements are mapped to their syntactic counterparts. The semantic content, in the sense of the meaning of the proposition, is also part of the correspondence scheme, though it does not survive to the end of the process. Without the correspondence scheme, there is no clear way of knowing whether the formalization was done correctly or not.[3] This example of formalization exemplifies the semantic role and MacFarlane's (1), it is about meaning and norms of reasoning. It also seems to inhabit the space of Dutilh-Novaes' first cluster of formal notions, as we are clearly engaging with the process of formalization, rather than just the properties of the results of that process.

In the philosophy of language there is a phenomena known as the product process distinction. This is an ambiguity when we use a term and it is unclear whether we are referring to the process or the end results. "In logic, for instance, the term 'inference' is understood as ambiguous as between the process of drawing an inference and the inference that results from that process" (Johnson 2009, p. 3).

[2] In logic classes it is often referred to as a key. Baumgarnter and Lampert call it a *realization* (Baumgartner and Lampert 2008).

[3] Peregrin & Svoboda raise the point that the assumption in the logic class is that there is a set of sentences that are classified as correct and others that are incorrect. Further that the teacher is unproblematically able to differentiate between these two, but that there has not been much study on what such criteria of correctness could be, as it is all implicitly assumed (Peregrin and Svoboda 2013, p. 2901).

The focus on the logico-formal border, in discussing the nature of logic and formalization falls prey to this ambiguity; we miss out on the features of the process by focussing on the end product.

When we examine our formal systems, e.g. syllogistics, we have to look at the properties of the system, the process which we engage with the system, as well as our aims for using formalizing in the first place. We are concerned with logics as formal systems related to the concept of logic; that is, their purpose and use.

5.2.3 Translation

We often use phrases like "translate the following into symbolic logic", rather than the direct use of 'formalize' which was used in the example. Similarly the correspondence scheme can evoke the same idea as it seems like a mapping from the one language (natural) to the other (formal). We should be careful of not reading too much into this similarity to translation as a functional description of the exercise, versus a useful term to convey the functional nature of the exercise to the budding student. There is a difference between the surface level of sentences and the logical (or conceptual) level. In a first-order language the surface grammatical structure perfectly maps the logical structure. This is not the case in English, or any natural language. This is known as the *misleading form thesis* (Brun 2003, p. 161). So, some logically different sentences in English will yield structurally different sentences in our first-order language. The adequacy of formalization could be interpreted as some form of sameness of meaning, as this is what we usually mean by translation. But, of course, the resulting formulas are in a schematic language, and thus do not have a fixed meaning that we could then re-compare to the natural language sentence we started with. Formalization, whether between natural languages and formal languages, or between two formal languages, preserves something, just not meaning (Guthrie 2017, pp. 16–17). One candidate is that our formalization into logic preserves truth-conditions. As we saw in our discussion in chapter four, this approach is limited, e.g. if we are formalizing an intensional concept, we will need to also preserve intensional content. Georg Brun points out that matching truth-conditions isn't the right goal for formalization. Consider when we are confronted with a sentence with unclear truth conditions, via ambiguity or vagueness etc. We do not then reach through the formalization to a sentence which also has unclear truth-conditions. Rather we settle the matter by formalizing the sentence (Brun 2014, p. 7).

John Etchemendy notes fluent speakers of a language have intuitions of both the logical properties of sentences and the structural (grammatical) properties of sentences, with formalization being the interplay between the two (Etchemendy 1983, p. 321). Brun brings up a second misconception related to this. It is not clearly the case that the logical form of a sentence is embedded and waiting to be discovered. To determine the logical form of a sentence is to distinguish between the logically relevant parts of the sentence and the logically irrelevant. When formalizing this is done against the backdrop of some logical system. The boundaries of this system may cause us to deviate from the 'normal' understanding of the sentence (Brun 2014, p. 8).

Formalization is more complicated than mere translation, though on the surface the two activities bear a resemblance. Translation is about a mapping between the two languages while formalization is a conceptually transformative process. The full output of a formalization is a more complicated object than simply a translated sentence; even when we are using familiar tools, e.g. the propositional calculus (Svoboda and Peregrin n.d., p. 4). The relationship between the input and the output highlights the importance of examining formalization as a process. Dutilh-Novaes' groupings of the desired properties of formalizing and the properties of the product, the formal systems, reflect this view of formalization as a process as well.

Formalization is a process, or an activity. In this case we are going from a sentence in the natural language English into a sentence of propositional logic.[4] The so-called 'pure' logics are exactly the ones cooked up by mathematical logicians for the sake of examining their properties, not in order to *capture* something external to the formalism. We are concerned with both the logical and semantic roles. That is, both the formal properties and the use of the formal system. We can safely put these pure formal systems to the side as they are not concerned with the use of formal systems, but rather the mathematical properties of formal systems in general. Moreover, we can learn more about the nature of logics by looking closer at the process of formalization itself.

5.3 Formalization as a process

In this section we will take a more holistic approach to study of formalization; namely we will look at it as a process, rather than just the product of that process. Capturing exactly what constitutes this activity is difficult, as we shall soon see, but it is clear at the outset that the process involves at least the following: idealisation and precisification, expressivity, and applicability. We then briefly discuss the targets of this process: concepts and their relations.

Following recent discussion, we will look at the relation between explication and formalization. Though they are similar, and related, activities, they are distinct in crucial ways. The story of explication is too coarse-grained to give a clear picture of formalizations. The mapping of formalization as a type of explication has led to concerns of a *paradox of adequate formalization* (Novaes and Reck 2017). By showing the differences between these two processes, we also show that this worry, while not unfounded, is less concerning than it is portrayed to be.

We will then adopt a Sven Hansson's two-step model of formalization, which focuses on idealisation, one of our key properties (Hansson 2000). This will help us distinguish between regimented language and specialised terminology. The former being important for formalization, while the latter for explication. Hansson's model leads to an interesting conundrum. The types of activities that could be described under it seem to capture much of analytic philosophy. We find the discussion of this model fruitful, but ultimately it is too widely scoped.

[4] We can assume classical logic given the background context of a question being put forward in an introduction to logic class.

5.3.1 Properties of formalization

Formalization is a process, or activity. It being a process implies going from some input to some output; it is also *targeted*. That is the formalization is done for some purpose or goal. Defining this activity is difficult, but we will develop the boundaries in the coming pages. Let us first turn to the necessary properties of formalization. It is uncontroversial, or at least wildly accepted, that the act of formalization has (at least) the following properties: *idealisation*, *precisification*, *expressivity* and *applicability*.

> It has been contended that no finite number of propositions could describe exhaustively all that is involved in a particular experience. In other words, it is impossible to formalize without residue the complete intuition at the moment. (Wang 1955, p. 231)

The residue that Hao Wang is referring to is left by idealisation. Idealisation is often mentioned in the discussion of formalization, or formal models (or even models in general). As with all our concepts we need to be clear as to exactly what we mean when we say formalization involves idealisation. It is the "deliberate simplifying of something complicated (a situation, a concept, etc.) with a view to achieving at least a partial understanding of that thing. It may involve a distortion of the original or it can simply mean a leaving aside of some of the components in a complex in order to focus better on the remaining ones" (McMullin 1985, p. 248). One can make the distinction between different types of idealisation.[5] MacMullin is referring to *Galiliean idealisation*. This is often contrasted with *Aristotelian idealisation*, which can be seen as the dual of Galiliean idealisation, it is the the suppression of known properties of the target phenomena. We will adopt these concepts using the terminology of John Woods & Alirio Rosales (John Woods and Rosales 2010, p. 3). They differentiate between *abstractions* (Galilean) and *idealisations* (Aristotelian). Where they differ is in the type of simplification that occurs. Idealisations are expressed by statements known to be false, while abstractions are achieved by suppressing what is known to be true.

Consider the *rational agent* used in economics who makes all of their decisions entirely according to maximizing their expected utility, or the agent modelled in deontic modal logics. In these systems the agent's beliefs are closed under logical consequence. That is, the agent immediately knows all the propositions that follow from their set of beliefs without needing any sort of introspection, e.g. The agent believes that P and acquires the additional belief that Q, this means that they just believe $(P \wedge Q)$, $(P \vee (P \wedge Q))$, etc.[6]

[5] Some mark the difference between idealisation as superlative, versus as simplification. Idealisation as superlative is when perfect properties are ascribed to an object in the model. This is the perfectly rational agent used in economics and epistemic logics. Contrast this with the idealisation of simplification, for example when we formalize a sentence in first order logic, like in our example above, we simplify the sentence to only the syntactic, simplifying away the semantics and pragmatics (Hansson 2000, pp. 164–165).

[6] Note that this is the same as our example of with Hansson's superlative idealisation. There can be confusion between the superlative and simplification. Consider in physics the 'billiard ball' models. There is an assumption in these that there is no friction on the table. This is done for reasons of simplification. However, the plane is assumed to be perfect, in the sense of not contributing friction to the scenario. We often choose superlative properties with the goal of simplification and so this framework doesn't clearly delineate between the two. For this reason we prefer the framework of Woods & Rosales.

Contrast this with moving from a sentence of natural language to the language of propositional logic, it suppresses the semantic relationships between propositions. For example, consider:

Mary is a bachelor and *Mary is male*.
$B \wedge M$
B: ... is a bachelor. M: ... is male.

While it is true in the natural language sentence that if the left conjunct is true there is no way the right conjunct could be false, as all bachelor's are necessarily male. But in the first-order language we do not restrict the possible valuations from having B true and M false. This is because we have abstracted away from the meaning and relationship between these propositions. The semantic content is not preserved by the formalization.

Another element involved in formalization is *precisification*. It is the act of making something more precise, i.e. removing ambiguity, vagueness and adding clarity. We can see this as an implicit goal of formalization. We are formalizing in order to refine our concepts and remove the polysemous nature of the terms we use. If our formalization is not taking us to a point of greater clarity then we would seem to be falling into the trap described by Hao Wang: "[w]e can compare many of the attempts to formalize with the use of an airplane to visit a friend living in the same town. Unless you simply love the airplane ride and want to use the visit as an excuse for having a good time in the air, the procedure would be quite pointless and extremely inconvenient" (Wang 1955, p. 233).

Idealisation, abstraction, and precisification are grouped together as they are both part of the design, or choice, of the formalization. We choose what we idealise, abstract and which concepts we want to precissify. Put another way, we aim to precissify but we try not to idealise or abstract too much from what we are formalizing.

The properties, of expressivity and applicability lie on the other end of the process and point to the formal system as a product and what we want from them. They are similar to Dutilh-Novaes' second cluster of formal. They are pragmatic properties of formalization. First we have the property of *expressivity*, it is similar to the notion of rigour. It is a tacit requirement we have of our formalizations. Whatever remaining parts of the McMullin's 'something complicated' that we have not idealised or abstracted away are, we want our formalizations to be expressive enough to capture, or express, them. Otherwise the idealisations and abstractions of the other parts would be for naught. There are limits to expressivity, too much and we end up with a cumbersome system, too little and we cannot say what we wish. "The right course of action is to be as rigorous and detailed as the occasion or the purpose requires" (Wang 1955, p. 230).

Lastly, recall that formalization is a process towards some goal. Thus, there is a property of *applicability* of formalizations, this is analogous to Carnap's 'fruitfulness'.[7] There is a tension between the above properties, however. The more idealised a formalization is, the less precise it can be about multiple concepts. Idealisations and abstractions involve choices on what concept, and their, properties

[7] See the discussion in chapter four.

the formalization will focus on. The more idealisations and abstractions, the less broad the application and less expressive the formalization.[8]

5.3.2 The target of formalization

We have stated that formalization should be viewed as a process, thus it has inputs and outputs. While we have been clear that the output is some form of formal system, broadly construed such that we do not exclude syllogistics, we have not yet spoken of the inputs. We rectify that in this section.

Formalization is about concepts and the relations between them. These are the objects that we formalize. This maps to both of Stenlund's ontic and pragmatic accounts, as well as Hao Wang's description below.

> Significant formalization of a concept involves analysis of the concept, not so much in the sense of analysis when we say that being a bachelor is being unmarried, but more in the sense that an analysis of the problem of squaring the circle is provided by the proof of its unsolvability. When formalization is performed at such a level, it does serve to clarify and explicate concepts. (Wang 1955, p. 229)

So formalization is an act of conceptual analysis within a specific domain, or context. We also see this in Hilbert's discussion of mathematical problems and the notion of truth in formal system. "If contradictory attributes be assigned to a concept, I say, that mathematically the concept does not exist. But if it can be proved that the attributes assigned to the concept can never lead to a contradiction by the application of a finite number of logical processes, I say that the mathematical existence of the concept...is thereby proved" (Hilbert 1902, p. 448). For Hilbert, the truth of something in a formal system is enough to establish the existence of a concept.

Consider decision theory as an example. Here we are formalizing our concepts of subjective probability and subjective desirability, or utility (Jeffrey 1990, p. xi). While this might not be agreed upon by all, the general consensus on formalization, and logic, is that it is about concepts and their relations.[9] The target of formalization is concepts and the relations between them.

5.3.3 Explication

The recent resurgence in discussing formalization has focussed on formalization as a form of explication. Explication is about conceptual refinement in philosophical discourse, for 'logical analysis'. Focussing on just the explication that ends in a formal object, as done in (Novaes and Reck 2017), does not tell the story correctly. We will give a brief overview of Carnap's explication before turning to the paradox of adequate formalization, showing not only that explication is a different conceptual process than formalization, but also that the paradox is not as strong as is claimed.

[8]We note that a possible confusion can result from our terminology, here we mean the property of expressivity, and not the notion of expressive power of a formal language.

[9]See our historical discussion of Kant and Frege in chapter two.

Carnap's discussions of explication fall into two perspectives. The first deals with explication as procedures, while the second focuses on the result of explication, specifically the characteristics that pick out adequate formalizations (Brun 2016, p. 1215). This is analogous to our above discussion of the distinction between formalization as a process and the properties of the products of formalization; our two groupings of properties also highlight the two perspectives. The former involving idealisation, abstraction, and precisification; while the latter focuses on expressivity and applicability. Recall Carnap's description:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an explication for, the earlier concept. (Carnap 1988, pp. 7–8)

Explication is the refinement or replacement of a concept for the express purpose of use in philosophy. Carnap uses the term *logical analysis* as the term for the narrowed scope of philosophy that he prescribes. Key to this process is that it is *iterative* in nature. Concepts go through explication over time as we make them more exact and crucially this might *never* result in a concrete, or closed-texture concept. Explication casts a wider net than formalization, describing the process of moving from vague, informal concepts to more precise accounts generally (Novaes and Reck 2017, p. 196).

Carnap described a minimal set of properties of explication, discussing the balance between them, though he specifically didn't want to make the claim that this was the whole set of desirable properties of a successful explication. These properties are: *similarity to the explicandum*, *exactness*, *fruitfulness* and *simplicity* (Carnap 1962, p. 5). We turn to Carnap's own example to make the idea of explication clear.

> When we compare the explicandum Fish with the explicatum Piscis, we see that they do not even approximately coincide [...]. What was [the zoologists'] motive for [...] artificially constructing the new concept Piscis far remote from any concept in the prescientific language? The reason was that [they] realized the fact that the concept Piscis promised to be much more fruitful than any concept more similar to Fish. A scientific concept is the more fruitful the more it can be brought into connection with other concepts on the basis of observed facts; in other words, the more it can be used for the formulation of laws (Carnap 1962, p. 6).

Explication is a process that precissifies, idealises and abstracts so that our applications can be more expressive within a field, or domain of discourse.

5.3.3.1 The paradox of adequate formalization

In their article “Carnapian explication, formalisms as cognitive tools, and the paradox of adequate formalization", Catarina Dutilh-Novaes and Erick Reck point out and discuss the tension between two properties of formalizations; leading to the titular paradox. They start out with the initial assumption that formalizations are a special category of Carnap’s explication.

The paradox of adequate formalization is the tension between fruitfulness and similarity. This tension can be seen is the same as the tension, mentioned above, between idealisation and abstraction with applicability. Idealisation & abstraction are the flip-side of similarity as they pick out what has been changed, while similarity highlights what has not been. Fruitfulness and applicability are synonymous for our purposes here. They describe the paradox thusly “[o]n the one hand, a particular formalization has to be sufficiently similar to its target phenomenon to be rightly described as a formalization of that target phenomenon, and also to be applicable to the same, or at least closely related, purposes. On the other hand, the formalization will be more useful insofar as it says something about the target phenomenon which prior, informal conceptualizations of it did not reveal. In other words, an adequate formalization is one that is faithful to the target phenomenon and reveals something new about it; there is an obvious tension between these two desiderata" (Novaes and Reck 2017, p. 211).

We must note while the tension itself may be obvious, the degree to which the two properties interact is not clear. It is not obvious that large changes in fruitfulness will have a resulting large loss in similarity. The degree of change in the one doesn’t imply the same degree of change in the other.

The inadequacy of natural languages has been a staple since Frege wrote the *Begriffsschrift*, and it carried through the Vienna Circle and the logical empiricists. The key failing of natural, or informal, languages are their lack of *exactness*. The move to formal languages is said to assuage this problem by moving to a concrete language with clear definitions of the concepts. This is the desired property of precisification of formalization. The calls for exactness often are bound to the desire of expressivity. That is, the informal nature of natural languages does not allow us to express the core concepts, or the distinctions between them, adequately (Novaes and Reck 2017, p. 200). On first reading it may seem that Carnap’s explication is focussed on exactness as a primary goal of explication. Dutilh-Novaes and Reck argue that exactness is subsumed by the goal of fruitfulness or, in our parlance, applicability (Novaes and Reck 2017, p. 202). Thus our two pragmatic properties, or goals, of formalizations are paired, with the one in the service of the other. As the central aim of explication, this confirms that explication is a pragmatic enterprise.

Recall that explication is iterative. It is not a one-off process resulting in an exact concept which replaces an inexact one. Explication is a process of degrees, we move from an inexact concept to a relatively more exact one. This process can be iterated, with the explicandum becoming more exact with each iteration. Most concepts we deal with are open-textured and thus no matter how many iterations of explication they undergo will never be fully exact. Dutilh-Novaes and Reck discuss the change of logic itself, throughout history, as illustrative of this iterated process.

We started first with the dialectical practices of debate described in Plato’s dialogues. These are the explicandum of the explication that leads us to Aristotle's game of questions and answers in the *Topics*. This game is then further explicated into the definition of *syllogismos*, or a deductive argument. Further to this definition the explication takes us to the formal system presented in the *Prior Analytics*. This too has been further explicated by being formalized in new systems with modern notation (Novaes and Reck 2017, p. 202). There is no end to this cycle of iterations as logic is an open-textured concept.

While these explications have resulted in more exactness, each step involves a transformation; in Dutilh-Novaes and Reck's terms, there are both gains and losses. Here we see that these are from idealisation and abstraction. The first explication to the definition of *syllogismos* gives clarity to the concept, with the sacrifice of all arguments that are not necessary truth-preserving, despite regular use in dialectical practices. The move to the syllogistic formal system reigns in the objects of study to only those arguments that can be presented in a categorical form, which are limited to two premise one conclusion arguments (Novaes and Reck 2017, p. 202).

Fruitfulness is the heart of Carnap's explication. Explication is a pragmatic enterprise. For Carnap, an explication is fruitful when the explicatum gives knowledge that the explicandum alone could not do, or it would be very difficult to do so. Returning to Carnap's example of *picis*, the concept reveals properties of this class of animals that the concept of fish cannot so reveal. Dutilh-Novaes and Reck correctly point out that this shows that there is a necessary mismatch between the explicandum and the explicatum. The ability to gain new knowledge from the explicatum is the exact source for this gap, and indeed it is the goal of fruitful explication. Thus, similarity is apparently a doomed property. So, they conclude, explication is doomed to be a paradoxical enterprise.

This paradox of formalization is an offshoot of Moore's paradox of analysis: "[i]f the verbal expression representing the analysandum has the same meaning as the expression representing the analysans, the analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect" (Langford and Schilpp 1943, p. 319). Thus no analysis can both be correct and informative.

Here we see the an argument of the same structure as that of the paradox of formalization, but raised a level of abstraction. We need to be careful that we do not conflate similarity and sameness, as the paradox of analysis is grounded in the notion of sameness being at odds with the two properties. It is sameness which requires correctness and prevents informativeness. With similarity we do not have such a problem. Dutilh-Novaes and Reck acknowledge this difference but appeal to Carnap's lack of clarity on similarity as evidence that their paradox still has teeth. They refer to the potential options of similarity of extensional, intentional terms or some sort of 'continuity-of-purpose' (Novaes and Reck 2017, p. 213). By taking explication as a primarily pragmatic enterprise we have the option of viewing similarity under the lens of applicability. That is, we can take a more holistic approach to similarity. If the resultant concept is to be useful, then there must be some similarity between the explicandum and the explicatum. Without such a similarity it is hard to say how we could end up with a useful explicatum. For instance, if *picis* picked out the set of ground-burrowing rodents it would not be a useful explication and thus not be adopted. While we agree with Dutilh-Novaes and Reck that there is work to be done on what types of things may count as similarity within explications, it does not seem that this definitional worry is enough to think that the tension between similarity and fruitfulness is justified in being a paradox akin to that of analysis. Paradox implies unsolvable, as in the problem of sameness in Moore's paradox, tension implies balance, which we have here with fruitfulness and similarity.

The tension of adequate formalization will be something we will have to be cognizant of, but there is no reason to think the degrees of change in the one will result in a similar sized loss in the other. The tension around applicability and similarlity is important to our purposes as we see both explication and formalization as primarily pragmatic enterprises; the process is a means to a specific explanatory ends.[10]

5.3.4 The two-step model and regimented language

There are multiple discussions of formalization that highlight it as a two-step process (Hansson 2000), (Brun 2003), and (Svoboda and Peregrin n.d.). They share structural similarities, starting with a notion of *regimentation* as the first step, and *systemitization* as the second. We will use Hansson's two-step idealisation theory of formalization for our discussion as it highlights one of our important properties. "Formalization in philosophy typically results from an idealisation in two steps, first from common language to a regimented philosophical language and then from regimented into mathematical or logical language" (Hansson 2000, p. 164). Here Hansson does not differentiate between idealisation and abstraction, but they both play the same role in the model.

The first step is Quine's idea of *regimentation*, paraphrasing sentences into 'canonical notation'. For Quine this is the language of classical first-order logic. But the general sense of the move from the sentence into a paraphrase which helps remove things like ambiguity and vagueness (Quine 2013, Chapter 5). The move of regimentation is adopted though in a looser sense, i.e. regimentation as paraphrase is meant, not directly into first-order logic. Returning to our example:

> *If it is raining outside, then the grass will be wet.*
> If Raining, then Grass-is-wet.
> $R \rightarrow W$

The second line represents the idea of regimentation, where we move to a hybrid language which exposes the formal structure that we want to aim for in the final formal system.

Here we must be clear on what we mean by regimented language. One candidate explanation is that it is akin to specialised terminology in other disciplines. We will show how that mapping does

[10] This argument is similar to the problem of subject-change, first brought up by Strawson (Strawson 1963). Here he levels the claim that as explications are primarily concept replacements, they end up changing the subject instead of clarifying and analysing our actual concepts. Carnap does leave some room for explication to be refinement in some cases, but we need not lean too heavily on this. Firstly, clarification is an enterprise that by definition will leave some assumed candidates of the concept out when they used to be in, otherwise no additional clarity is added. The worry is that the concept changes seems ill-founded. Secondly, explication is a process, and within the process are stages which take into account the goals of similarity and exactness. Only a brute notion of replacement yields a candidate for Strawson's worry.

not stand, and this discussion will show the disconnect between the processes of explication and formalization.

Specialised terms are terms that denote a more precise concept within a field, context, or discipline, than in everyday usage. An example is the term “unemployed” in labour economics which means the portion of the labour force who does not have a job but is actively looking for work, compared to “not in the labour force” which covers those who do not have a job, nor are they looking for one. The everyday usage combines the two groups as roughly unemployed meaning “people who do not have jobs”. The meaning of a specialised term can change over time through explication. Explication gets the story right with how the specialised terms come about in the first place. Recall that explication aims to respect some uses of a term but also stipulates for/against others, e.g. *picis* and fish. While it is clear in zoology that the new term is to be taken forward in discourse, it doesn't necessarily mean that in other contexts, e.g. everyday discourse, the previous term fish would not still be useful, or used. Specialised terms are explicated within their context; there is an explication path.

Consider Quine's example of the ordered pair in set theory (Quine 2013, p. 237). The pair $\langle x, y\rangle$ is defined as the set $\{\{x\}, \{x, y\}\}$. This definition does not capture all the properties of an ordered pair, rather it does just the essential character needed for the context. The essential character here is that ordered pairs are themselves identical if their parts are identical: $\langle x, y\rangle = \langle u, v\rangle$ iff $x = u$ & $y = v$. While the above set theoretic definition captures this property it brings along other properties which are not built in to the idea of ordered pairs. That is, it is an implication of this definition that x is a member of a member of $\langle x, y\rangle$, though that clearly isn't the case before we moved to set theory. We could also represent the ordered pair as $\langle y, x\rangle$ instead, it is only by mathematical convention that we stick to the original representation. An explication of a concept is brought forward for certain purposes in certain contexts. It's adequacy as a new rendition of the concept depends on these purposes and contexts (Quine 2013, pp. 239–239).

Specialised terminology is used, introduced, settled, and agreed upon within a context, or discourse. In the *picis* example it is within the field of zoology in which we see the explication take place. Carnap was clear that not all concepts would, or could, end in a mathematical, or in our terms closed-textured, explicandum. Most concepts will iterate over time and never result in such a clear ‘end-state’, they are open-textured. Specialised terminology is used to denote concepts that are specific to some context, or field of study. As they are specific to these contexts it should be unsurprising that the overall concept is open-textured, as the explication only acts within the context. This can even be true of specialised terminology which is closed *within* its context; the overall concept remains open because of its cross-contextual nature.[11] If the understanding of a concept undergoes a dramatic shift then it is clearly an explication of replacement. Kuhn's theory of conceptual changes through paradigm shifts are a clear example of large-scale, and multiple, conceptual changes via replacement. Carnap himself is unclear if there really are explications of refinement, rather than replacements with very similar definitions.

[11] To be precise, the concept itself can be closed-textured as well, the claim here is that the closed-texture of a concept which is bound to a specific context does not necessarily imply that the overall concept be closed as well.

Regimented language, as Hansson's first-step, is a much more narrow move. Regimented language is when we attempt to refine a concept for usage within a specific (philosophical) context, as we (generally) cannot appeal to empirical considerations, as we can with the sciences. The justification for these terms lies with their capability to clarify the corresponding non-philosophical concepts (Hansson 2000, p. 164). This seems to be in-line with our discussion of explication being context-bound, so far so good. There are differences between regimented language and specialised terminology however, and this will strain the explanatory power of formalization as explication.

An example from philosophy is the use of 'permitted' in deontic contexts. While permitted means something in everyday usage, when used in the specialised philosophical context it means something more narrow, more precise. The move from everyday to specialised terms is a textbook example of explication at play. Hansson notes that such a move involves idealisation. In everyday usage when someone is 'permitted' to perform an action it means that one is able to perform the action or not perform it, he calls this *bilateral permission*. Contrast this with the philosophical usage, 'permitted$_{phil}$', where being permitted to perform an action is compatible with having to perform the action, *unilateral permission*. The usage of unilateral permission in our formal system is convenient as one can define bilateral permission in terms of it, though there is no available definition in the other direction (Hansson 2000, p. 165). Thus, permitted$_{phil}$ is an idealisation of permitted. Here, unlike with specialised terminology we don't a clear case of explication. The general concept of permitted has not been precissified, but we have a new idealised definition based on wanting a more easy to use formalization.

There are a couple of key differences between regimented language and specialised terminology worth discussing, however. There is the issue of individual versus community explication, discussed in chapter two. Specialised terminology is relative to a field of study, or a context. It is generally agreed upon and a stable concept within the context, or at least will move towards stability over time, through discourse. The iterations of explication take place within the discourse of the community remaining relatively stable until new evidence comes in to challenge that definition, and then is explicated in the community to some new definition. Specialised terminology is used within a context to indicate a precise meaning of a concept within that context, and unless otherwise indicated it is assumed that the generally accepted meaning of the concept within that context is being used. Regimented language, on the other hand, can be introduced within a single paper in order to further the aims of that one thesis. It is not generally understood to be a stable conceptual explication across the discipline, unless it ends up being accepted and used throughout the community, or context and even then the idealisations and abstractions are not guaranteed to be part of the newly acquired concept. That is, regimented language can become specialised terminology but it does not automatically become so.

One could respond that the explanation above of regimented language being distinct from specialised terminology is just that the description of the regimented language is given at the individual introductory moment while the specialised terminology is described when it has already proliferated throughout the context. While we can identify 'first-usages' of specialised terminology and it may seem very similar to our regimented language there is still the difference of goal. The aim of introducing a new explication of a concept for specialised terminology is to add clarity to the discourse via the stipulations involved. Fish becomes *picis*, and the zoologists have a generally easier time discussing/classifying etc. With regimented language the aim of the idealisations is not always that of clarifying the concept, often these are idealisations (abstractions) of convenience. A limiting of the concept so that it is in some way easier to deal with, despite that idealisation and abstraction, perhaps, containing parts of the essential characteristics that the explicator does not necessarily think should be generally done away with for the concept, versus their purposes at hand.

As in the example above, regimented language can involve idealisations of convenience. It is not the case that the philosophers believe that permission should only be unilateral, and thus they adopt permitted$_{phil}$, rather it makes the resulting formalization that they are aiming towards easier to work with. Key to adopting permitted$_{phil}$ is that they can define bilateral permission from it, retaining the scope of the everyday usage, if need be. This differs from specialised terminology as here we are 'over-explicating', that is the explicandum is more narrow in regimented language as it is being used for a specific purpose, as part of the activity of formalization. We call such moves *artificial explication* as it is an explication like activity but its aims are not 'conceptually permanent' in the same way standard explication is aimed. The resulting explicandum is narrower than, or not in line with, the general understanding of the concept, including by the formalizer. Regimented language is part of the process of formalization. The artificial explication is done explicitly in order to complete the formalization.

The second-step of formalization is the move from regimented language into a formal object, this too involves idealisation (Hansson 2000, p. 164). Formalization is often viewed as a strict superset of mathematization (Griesemer 2013, p. 298). The key to this step is the move to a system which relates the underlying concepts. We are not merely re-regimenting our language we are moving from open informal discourse, to discourse in a well-described domain, that is the heart of the second-step. We hesitate to commit to such a narrow range off the bat, for reasons that will become apparent throughout the discussion.

Returning to Hansson's example the move is from permitted$_{phil}$ to the deontic predicate P, in some sufficiently rich deontic language. Here we have entered a mathematical (formal) language. The meaning of P is fully abstracted away except for the relevant modelled relations defined in the language. The meaning of P, and thus the design/requirements of the formal language, is determined by modelling the regimented term permitted$_{phil}$. Due to the regimenting step being part of, and purposed towards, the formalization P and permitted$_{phil}$ are much closer concepts than permitted and permitted$_{phil}$; most of the idealisation occurs in the first step (Hansson 2000, p. 165). Often in formal philosophy the two-steps are combined, leaving the regimented step implicit or minimally described in the prose. It is important to note that even in these cases there is both the conceptual idealisation as well as the abstracting idealisation of moving to a formal model. They are both explications of replacement and involve idealisations that may not line up with the general understanding of the concept, this is the tension between simplicity and applicability.[12] These two steps are very different however, so treating

[12]This is noted as a trade-off by Hansson with respect to philosophical and scientific model-making, however his discussion

them as the same may be too simplistic an approach. That the paradox of adequate formalization based on Carnap's goals doesn't quite line up with the tensions of formalization discussed above should be unsurprising given the difference between the two processes. Hansson adds to the discussion of this tension, but his focus on idealisation misses out on precisification and the other goals of explication.

Here we see that explication is ill-equipped to describe the formalization as a whole. Formalizations are not just focussed on a single concept. The formal system that results, or is used, is a formalization of multiple concepts and the relations between those concepts. Explication as a framework to explain the changes of concepts, and their meanings, over time is too singular in its approach to explain the relational part of formalization. We will return to this idea in the discussion below.

5.3.5 Levels of formalization

We have a set of properties which we know formalizations have, and we have certain target systems, which have these properties, that we want to include in our definition, e.g. syllogistics. Let us start with the uncontroversial, a mathematical structure formalizing belief, and continue from there. Let's take the following as our example that we are trying to formalize. "John lives in a garden flat in Bristol. The weather in Bristol has been typical, and John does not have anything in the garden; he barely uses it. So, John believes that if it rains then the grass will be wet."

Consider the standard epistemic modal logic.[13] We have the model $\mathfrak{M} = \langle \mathbf{W}, \mathbf{R}, \mathbf{V} \rangle$. A set of possible worlds $\mathbf{W}$, a binary relation $\mathbf{R}$ and a valuation function $\mathbf{V}$. We can say our agent knows that "if it is raining then the grass is wet" if it is the case that $K(R \rightarrow W)$. That is $R \rightarrow W$ is true at all accessible worlds, via the binary relation $\mathbf{R}$. This precissifies the concept of knowledge while abstracting the propositions in the usual way. The uncontroversial passes our minimal test.

Taking a step back from agent knowledge, consider the first-order propositional version of our example. The sentence $R \rightarrow W$ with the correspondence scheme R: It is raining, W: John believes that the grass is wet. Again this easily fits our definition in an unsurprising way; we have precisification and abstraction with some structural (relational) properties as well.

Now consider the general sentence "If it is raining, then John believes the grass is wet". This is a precisification of our original description. It can be considered an abstraction as there are no umbrellas or other such coverings in this world, at least as far as John believes. While we are no longer dealing with symbols, this stage seems to map to regimentation. We started with some description of things and moved to a clearer view of what we think is salient in that description. We have isolated the various concepts we want to make clear and abstracted/idealised the propositions as well. The current description is unclear as to how much regimentation is going on as we have left it open what concept, for example, *believes* picks out and whether it is narrower than the usage in the initial description. This should not put us off however, as most regimentation is done implicitly, especially if it is on the way to a formalization (Hansson 2000, p. 164). Now we have a problem, of sorts. In much of

is not one about explication, though he uses similar terms to make the point (Hansson 2000, p. 164).

[13]The technical details of this example need not be fully apprehended to follow the reasoning being motivated.

contemporary philosophy we make regimented claims in premise-conclusion form in order to clearly show the argument structure. This doesn't mean that most philosophy is done with direct display of arguments as clearly indicated premises-conclusion, rather that it is the aim of analytic philosophy to give arguments.

Let us return to our original prose, it seems like we do not really have an instance of precisification, abstraction or idealisation present. So prose seems external from formalization. Consider Phillipa Foot's famous trolley problem:

> The real culprit being unknown, the judge sees himself as able to prevent the bloodshed only by framing some innocent person and having him executed... To make the parallel as close as possible it may rather be supposed that he is the driver of a runaway tram which he can only steer from one narrow track on to another; five men are working on one track and one man on the other; anyone on the track he enters is bound to be killed... The question is why we should say, without hesitation, that the driver should steer for the less occupied track while most of us would be appalled at the idea that the innocent man could be framed. (Foot 1967, p. 3)

Here we have a description in prose that aims to precissify our intuitions on harm and the 'double effect'. The tram case is an idealisation, in that it posits only two choices in order to make the conceptual interplay clear. It seems that prose is not so clearly out of the picture when it comes to thought experiments. We will leave thought experiments asides, as there may be grist to mill with respect to how to properly view them, it is clear that this discussion will not get us closer to formalizations and logics.[14] We consider thought experiments as exemplary of the problem of casting the net too wide, as these properties of formalization can be broadly construed to encompass things such as thought experiments.

We can take the same approach as Carnap does in his discussion of languages; we could just as easily choose the placement of a match on a piece of paper instead of the ink-strokes of a pen to symbolically represent our terms (Carnap 1937, p. 6). In that example the form of representation is non-standard but as they operate in the same role and follow the same rules there is no functional difference. In much the same way we could define our the primitives and rules of a formal system and in our usage of them stick to the full prose definitions in all occasions. The resultant system would look like a long brick of prose, but it would functionally be a formalization, in this example an axiomatization. What this highlights is that the general level of presentation does not matter as much as the process that is undertaken to get there.

With thought experiments possibly in the running, as well as regimented argumentation as formalized. This would make virtually all of analytic philosophy formal, just at different levels. We seem to have defined ourself into a situation which no longer maps to the phenomena to which we are trying to explicate. Is this not a bridge too far?

[14] For an interesting discussion on this topic we direct the reader to (Arfini 2016).

Regimentation may be a combination of precisification and idealisation, but that doesn't make all instances of it formalization. Recall that the first step of formalization is regimentation; it may be sufficient with respect to precision and abstraction/idealisation, but that is only part of the story. If we stopped after this first step we would be able to more deftly wield the concepts at hand, but we would still be relying on intuition, or our natural sense of deduction, to make our claims. This seems like it tells the general analytic philosophy story well. This notion of regimentation is more in line with Quine's discussion in chapter five of *Word and Object*. Here we have regimentation as the goal step, not as a part-way move towards some other purpose, as described by Hansson's framework of formalization. Quine's regimentation seems not to be 'artificial' in the same way, as a result. This seems like a characteristic of non-empirical concept moulding and perhaps can cleanly be explained by explication as Carnap initially described. It is only the regimentation as a step towards formalization that concerns us, so the net may not be too broad on these grounds. That is, the second-step of systemitization is key to formalization, though it shares philosophical elements of regimentation with broader philosophical activities.

While it was clear that specialised terminology is well-represented by explication, and it is similar to regimentation there is a gap in the explanatory power of explication with respect to regimentation. This is because of the aims of regimentation, and formalization, being not necessarily permanent changes in a concept, even within a context it is artificial, as well as the fact that regimentation is part of a more complicated process: formalization.

Formalization is a process that takes an input, a set of concepts and their relations, regiments it, then mathematises it producing a system. Accounts of formalization will describe such a system as *representative* of the concepts, or discourse, once we have finished idealising and abstracting. This sounds very close to the idea of modelling.

5.4 Modelling

The close relationship between formalizations and modelling is well-known; they are sometimes treated as synonymous. Here we will explore the the differences between the two to get a better sense of where logics fall in the greater picture. Stewart Shapiro and Roy Cook have offered the logic-as-modelling theory as an answer to the general question that we ask in this work: What is logic? The most extensive treatments are in (Shapiro 2014), and Cook's dissertation (Cook 2000). What is missed in these approaches is a clear idea of what modelling picks out. It is our aim to do such a picking-out, and show how modelling and formalization differ.

Models are representations of some target. This can be a representation of a selected part of the world, 'the target system' or it can represent a theory by interpreting the rules of that theory (Godfrey-Smith 2006, p. 726).[15] There are different ways of of delineating model-types, here we are only interested with respect to formalization.

[15] This is an inclusive 'or', models can play both representative roles.

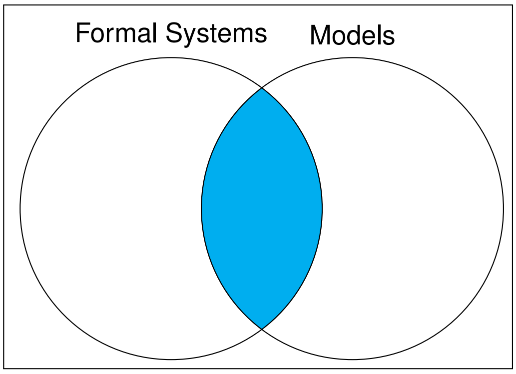

Figure 5.1: The standard view

One of the main problems with delineating formalization and modelling is a familiar one. We have a clear picture of formal systems and models, but that is only part of the story. As we highlighted in section 5.4.3, the key to understanding formalization is viewing it as a process, and not focussing on a common class of the outputs of formalization. We find the analogous problem with modelling and formalizations, they both suffer from product process ambiguity. For clarity let us consider the standard view, depicted in figure 5.1.

Consider the class of mathematical structures that are constructed by mathematicians just to investigate them, the structures of pure mathematics. That is, they are not inspired by some prior concept they are defined purely from the activity and then the properties of these structures are investigated. These are formal systems and thus in the left-hand circle. They are not models of anything, however, so they are not part of the right-hand circle, and thus not the intersection. There is no target to which the formal system is aimed to 'capture'.

Conversely we can think of other types of models which are not formal systems, e.g. scale models. However, as we do not want to get embroiled in the complex question of what counts as a model we can also find examples which are clearly models in science, but which we would refrain from calling a formal system. We can think of the Bohr model of the atom, which is representative and useful, though clearly idealised away from its target, or the Lotka-Volterra model of predator-prey which represents the relationship between predator and prey populations via a pair of nonlinear differential equations.[16]

$$\frac{dx}{dt} = \alpha x - \beta x y$$

$$\frac{dy}{dt} = \delta x y - \gamma y$$

Where x is the population of the prey, y is the population of the predator, t is time, and $\frac{dx}{d}$ and $\frac{dy}{dt}$ the growth rates of the prey and predator species respectively.

[16] Thanks to Antonis Antoniou for first pointing out this example.

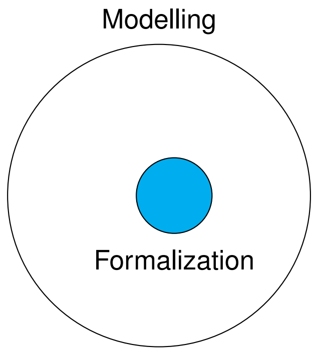

Figure 5.2: The process view

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This set of equations models the dynamics of biological systems with respect to the interaction between the two populations. They contain a number of idealised/abstracted assumptions that are not reflected in the real world, e.g. the prey has limitless appetite and ample food supply. Here we have a set of concepts being represented by mathematical objects. Under standard notions of formality this would not be considered a formal system, but it is being used as a representative system. The general category of predictive/stochastic models lands in the white of the right-hand circle.

Our usual suspects, the logics of some sort or another, fall into the coloured centre. While there may be some quibbles on these characterisations, e.g. we have not given clear criteria of formal systems; we know that this is already a partial story. This discussion helps draw the relevant outlines to help us along the way. Following from above, the missing characterisation is that formalization and modelling, are processes and viewing just the results of the process misses out on key properties of the process that should be taken in to account. The relevant distinction is not between objects of certain types, rather it is in the outputs of certain activities, namely formalization and modelling. Both are activities which start with some target input and result in an output. The picture changes thusly in figure 5.2.

The claim here is that modelling is the wider class of activities taking an input, either real-world or theoretical, and producing some representative output. The activity of modelling involves idealisation, abstraction, and precisification to various degrees as well.

Formalizations are a narrower type of modelling. Formalization is a class of activities that take a set of target concepts and makes explicit the relationships between said concepts. Models, on the other hand, need not be at this conceptual level. The stochastic biological models would clearly fall, once again, into the wider white circle. The relationships between the concepts need not be given by a set of formal rules, as we saw earlier the axiomatic net is too small. The relation between concepts can be a set of axioms, a deductive system, or any other representation that preserves their relations. This leaves space for both proof theoretic and model theoretic formalizations. Thus, logics seem to comfortably fit into the blue inner circle of formalizations. This approach leaves the entire category of pure formal systems out, as they aren't aimed at representing any specific target.

Returning to syllogistics we see that it is aimed at represent reasoning. The primitive vocabulary are the terms (semantic content abstracted away), the universal and the particular with their denials, and the perfect and imperfect proofs. So we have a set of concepts and a system which maps the relations between the concepts. Our net seems to catch this system of formal logic left out by a pure axiomatic definition of formalization.

Returning to the Lotka-Volterra equations, we seem to have the primitive concepts of prey populations and predator populations and we have the relationship between them described by the two differential equations. Does this make the Lotka-Volterra equations a formalization that represents predator-prey population dynamics? To answer that question we need a firm grasp on what we mean by formalizations, specifically what is meant by 'the relationship between the concepts.'

5.5 Formalization defined

We now have all the pieces we need to build a definition of formalization that captures all the moving parts discussed above.

The two-step model of Hansson was useful to show how we formalize and that abstraction and idealisation take place at both steps. However, it seems an insufficient model to differentiate from other modelling activities. We move to John Baldwin's framework to better characterize the process of formalization. Baldwin is dealing with mathematical formalizations in his paper, but the building blocks will be useful. A topic is a collection of concepts and the relations between. Baldwin sees the relation between intuitive conceptions in some area and some formal system describing that area as of central concern; hence the focus on formalization not formal system, much as we described above (Baldwin 2013, p. 88).

Definition 5.1. We see a *full formalization* as involving the following components:

1. Vocabulary: specification of primitive notions.
2. Logic:
   (a) Specify a class of well-formed formulas.
   (b) Specify truth of a formula from this class in a structure.
   (c) Specify the notion of a formal deduction for these sentences.
3. Axioms: Specify the basic properties of the situation in question by sentences of the logic. (Baldwin 2013, p. 88)

The first step is to list the intuitive concepts which are the subject of the formalization, or in Hansson's language regimentation, this is the primitive vocabulary (1). He groups three components together as they are all formal properties. This maps to our second-step of systematization. The class of well-formed formulas is the syntax of the formalization (2a). The specification of truth of a formula is the semantics, it is here we retain a relationship with the target concepts (2b). The notion of a formal deduction tells us how, and in what ways, we can infer using the formalization (2c). The axioms are separate category as they pick out the actual subject area, they are our initial assumptions (3). While this captures the regimentation step it leaves out the key to formalization as a process, the input or *target*. We add this to the top of the list to get our definition.

Definition 5.2. We see a *full formalization* as involving the following components:

1. Target: a set of concepts, their properties and relations, which are undergoing the formalization, often characterized by (2).

2. Vocabulary: specification of primitive notions.

3. Formal Components[17]

   (a) Specify a class of well-formed formulas.

   (b) Specify truth of a formula from this class in a structure.

   (c) Specify the notion of a formal deduction for these sentences.

4. Axioms: Specify the basic properties of the situation in question by sentences of the logic.

Baldwin pairs this definition of formalization with Kennedy's notion of *formalism freeness*, which she extracted from Gödel's 1946 Princeton Bicentennial Lecture (Kennedy 2013).[18] Formalism free is a matter of degrees, it involves the supression of some components of Definition 5.1, with the exception of clause 1, the vocabulary, which is sacrosanct (Baldwin 2013, p. 95). A definition is formalism free when it is given semantically without any distinction between syntax and semantics. That is, the most extreme version has (1) and some form of (2b).

We note that this more precise definition of formalization lines up with our claim above that non-conceptual modelling, such as stochastic biological models, will fall outside of our purview in investigating formalization. Our discussion in section 5.3.5 reflects the idea of formalism freeness. Formalization is a spectrum of approaches which share this common core and can also have the other properties listed in Definition 5.1.

> As a simple example, consider 'Are there infinitely many primes?'. At the most basic level, one might take this to mean 'prime natural numbers'. But it is not clear whether these are numbers in the structure $(\mathbb{N}, \times, 1)$, $(\mathbb{N}, \times, 1, <)$, $(\mathbb{N}, +, \times, 0, 1)$, or many other choices including $(\mathbb{N}, |, 1, S, <)$; see Arana (2011) for a number of variants. A natural response from a mathematician might be, 'In what ring?' (Baldwin 2013, p. 89)

[17]Here we have moved away from Baldwin's use of the term 'logic' for these components. By 'Logic' he is referred to the class of mathematical structures known as logic, and not necessarily the notion of logic that we are trying to explicate, thus we have chosen the more neutral 'formal'.

[18]Gödel uses the term *formalism independent* in the lecture.

Our notion of the natural numbers can be expressed in a variety of mathematical structures, each of which may have different components of Definition 5.1.

The notion of 'computability' is formalism free due to the equivalence between Turing machines, Markov algorithms, Church's λ-definable functions, and Gödel characterizations (Kennedy 2013, p. 362). As Baldwin notes, only the Gödel characterization as δ^0_1-definable functions in number theory fully matches the definition of formalization above (Baldwin 2013, p. 95). The other equivalent notions of computability are missing some parts of Definition 5.1, though are clearly apprehensible in their contexts.

Though there is direct reference to axioms, or rules, in component (3) we should be clear that we still retain the difference between formalizations and axiomatic systems from above. This distinction is also held by Kennedy and Baldwin."Euclidean geometry was given initially as an axiomatic system, but it was formalized - in a preliminary sense - in Hilbert's 1899 *Grundlagen der Geome* (Kennedy 2013, p. 373). Axioms are not required, nor disbarred.

This shouldn't seem too foreign. In his discussion of logical truth Gomez-Torrente lists several distinct phenomena called 'formalization': a completely specified set of artificial symbols, a completely precise grammar for the formulae construed out of the artificial symbols and the postulation of a deductive calculus with a very clear specification of axioms and rules of inference. All three of these phenomena, are present in Frege's landmark work (Gómez-Torrente 2019). While Gomez-torrente views these as three distinct phenomena. Using Baldwin's framework we can see them all as constitutive of a formalization, with a different set of artificial symbols and the truth-conditions being necessary. In this way we see that the apparently competing notions of formalization in other works are facets of the same definition. Any formalization we encounter which is lacking any number of these phenomena is just somewhere along the formalism-free spectrum. For Hansson's model, the various parts of (2) and (3) represent the second-step of formalization; that is, it all exists in the suppression class of components of formalization. Similarly our idea of levels of formalization fits well with formalism freeness. It also gives insight in the previously unanswered question of the notions of prose and sentential framing of argumentation. At a minimum we need (1) a topic, (2) a specification of primitive notions, and (2b) a formal specification of truth from this class. Whether or not things like thought experiments fall under this umbrella or outside of it relies on our interpretation of these notions. This is an argument for another day, though we just note one can make use of this framework to add clarity to the argument from either side.

Returning to syllogistics, using Definition 5.2 we have a topic, reasoning (1), a clear notion of a vocabulary, the terms (2). We have a class of well-formed formulas, the syllogisms (3a). There is a notion of truth, 'the true' (3b), and a notion of deduction, the permitted moves of the syllogisms. This notion of deduction is informal, and spread across the books of the *Organon*, so it doesn't seem, or at least it is unclear, that the syllogisms have (3c).There is also not an explicit set of axioms (4), though we have something close. The four perfect deductions, which Aristotle saw as primitive, could be seen as axiom-like, but again this is a loose sense of axioms. Syllogistics are partially formalism free, but the most formalized system of any of its time or before.

The Lotka-Volterra equations have a clear vocabulary (2): the variables, rates and parameters in the equations. These seem to, as claimed above, pick out some primitive concepts within the topic of predator-prey population relations (1). We have a pair of non-linear differential equations, but they do not seem to line up with a formal notion of well-formed formulas (3a). There isn't an explicit syntax within the system, though there is a restrictions on what values the vocabulary can take on, e.g. non-negative populations. The notion of a formal deduction (3c) could be read as implicit in the greater mathematical area. That is, we know how to mathematically solve differential equations through calculus, but again this is not a formal sense of deduction in the system. It is clear that there is no set of axioms given in the standard presentation of the Lotka-Volterra equations, i.e. we are missing component (4). The trickiest component is (2b) the truth of a formula from this class in the structure. It is also a necessary element for a formalization, as Baldwin notes. A formalization is at the far end of the formalism free spectrum when it is comprised of only (2) and (3b), and (1) with our addition. The standard presentation of Lotka-Volterra equations does not address this notion of truth explicitly. There is perhaps an argument of implicit notions of truth of formulas in well-trodden mathematical formalizations, such as with calculus. We shall stop short of such an argument and point out that if such a semantics was made explicit, and clear arguments towards the implicit, or greater mathematical, notions of (3a) and (3c), then it would seem to be a formalization. That is, this representative mathematical structure is *formalizable*, though not a formalization itself. This underlines Baldwin's claim that the semantics are the key component, with the others subordinate.

5.5.1 Pragmatics and the paradox

Recall the paradox of adequate formalization. It describes the tension between fruitfulness and similarity. Both explication and formalization are pragmatic processes, that is they are oriented towards some result. Formalization has a target phenomena, set of concepts, it is aimed at and the result is a system, broadly construed, which relates the set of concepts to each other in a representative manner. This system needs to have a clear vocabulary, set of concepts, and a semantics.

Similarity is associated with the target of the formalization, and what 'essential properties' of the concepts the resulting system will capture. The degree, and form, of the similarity relies on the pragmatic aims of the formalization. What are the important aspects of the primitive concepts being modelled? To what aim are we hoping to apply the resulting system? This is the second half of the 'paradox', fruitfulness. We are formalizing for a reason, and how well we accomplish that is how fruitful, or applicable, the system ends up being. Discussed this way, we see that the paradox is less tense than advertised. These are two sides of the pragmatic coin, and need not be seen as permanent adversaries.

5.6 Conclusion

We now have a clear idea of what formalization is and how it relates to explication and modelling. Explication, as a framework for singular conceptual change is ill-equipped for explaining the conceptual changes involved with formalization. Formalization is a process that takes place over two-steps, and may not lead to the general concepts explication. Our definition of formalization fits with both the ontic and pragmatic accounts.

In establishing the vocabulary for a formalization there are certain idealisations/abstractions that are done for the sake of the final system and do not reflect a 'permanent' conceptual narrowing as in standard explications, they are artificial explications. We contrast this type of conceptual narrowing through targeted regimentation with that of specialised terminology which easily fits into the explication paradigm.

The process of formalization is a sub-type of the process of modelling. Both are target-bound and result in a representational system. Formalizations, at a minimum, need to have the base concepts as vocabulary and a semantics that grounds the model. These are models that represent concepts and the relations between them. While there is a tension between what is idealised/abstracted away from the target phenomena and the usefulness of the resulting system, when we view the process of formalization as mainly a pragmatic enterprise we see that this falls short of a paradox. With a clear idea of formalization in mind we can now move to explaining the main thesis of this work, logics-as-formalizations in the next chapter.

CHAPTER 6

LOGICS AS FORMALIZATIONS

6.1 Introduction

We now have all the requisite parts to state our answer to the overall question of this work: What is logic? The answer is straightforward: Logics are formalizations. Moreover, they formalize a notion of inference, 'what follows from what', within target domains. This is a process oriented view, as they are formalizations, not formalisms simpliciter. The concept of logic informs the purpose, and thus the target for the formalization. This means that whether or not something counts as a logic will depend on the underlying concept of logic at hand, pushing the debates from the technical to the theoretical first. From the previous chapter we have a clear notion of what formalization is; it is a structured modelling process that includes, as a minimum, a target, a vocabulary, and a semantics. In this chapter we will present the logics-as-formalization theory (LaF), which is a conceptually open theory of logics.

In section two we will go over these details, using first-order and second-order logic as examples. We will then clarify the difference between logics and formalizations, on both theoretical and technical grounds. This includes a discussion of Gil Harman's work on the disconnect between logic and reasoning, and how that squares nicely with the LaF theory. We will end the section with a discussion of Rudolf Carnap and how the LaF theory of logic lines up with Carnap's general theory of logic. With the clarified theory in hand, in section three, we will discuss the development of alternate logical programs, paraconsistent logics, dynamic logics, and diagrammatic logics to show the scope of logic under LaF, and how this framework exemplifies a both conceptual openness and a 'sociological advantage', alluded to in chapter one.

6.2 Logics-as-formalization

In this section we will present the LaF theory of logic. Most of the heavy lifting has been done in the previous chapters. We will start with a reiteration of our definition of formalization, and a brief aside on the difference between constructions and formalizations.

The theory is called logics-as-formalization and not just 'logic' for two reasons. The first highlights the inherent plurality of the approach; there are many logics. The second reflects the conceptual openness of the theory. The concept of logic is open-textured and polysemous, it has meant different things throughout history and simultaneously. The plurality in the moniker reflects that even when we speak of logic conceptually, there is a plurality of concepts that it could be and that are discussed.

This is a process-oriented view, so what counts as a logic, or not, will be determined by the initial concept that is taken, which then goes under formalization. This highlights that the debates have to take place on the theoretical level first, then the technical. If there is a disagreement around a formal system being a logic, or a specific rule being logical, we first must make sure that the 'logic' in question is the same, otherwise we run the risk of talking past each other. This is a reflection of what we found in chapters two and three. If a common interpretation cannot be found between the disputants then the debate cannot fruitfully be held, let alone on the technical layer.

When we say that logics are formalizations we are stating that there is a necessary gap between the target phenomena and what is represented; in more familiar terms there is a gap between the model and what is modelled. This gap is necessary, as when we formalize we have to make choices as to what components and properties of the target the formalization will represent, and how it will attempt to do so. These will incur debts of idealisation and abstraction which are paid out by the gap between the result and the target.

The term logic is often used to refer to the result of the formalization, rather than the target which is being formalized. The former is sometimes referred to as the pre-theoretic notion of logical consequence. The nature of this depends on our theoretical notions, our concept of logic, and provides the target for formalization. There are some, the logical realists, who think that the rules of logic are knit into the fabric of the universe. Often they point to classical logic as the one true logic. The LaF theory highlights the difference between classical logic and this ontically heavy notion of logic. We can tell a similar story with logic as natural language inference. There are rules within natural language that govern how we reason within said languages. When we apply a logic (here a formal system not the conceptual notion) to natural languages we are attempting to represent those inference-guiding rules, but we are still representing them not dealing with them 'directly'.

Choosing a formalization relies on identifying the concepts, properties, and relations that are important for our analysis of the target. This is informed by what we are going to do with the formalization: the purpose of our application. We need to ensure that the idealisations and abstractions will not impact these core concepts, properties and relations in ways that will impact the analysis. As we are using these formalizations for a purpose the efficiency, fruitfulness, and simplicity of the use of the formalization may be among the decisive factors, paraphrasing Carnap (Carnap 1950, p. 23).

This theory may look similar to a more familiar theory, that of logic-as-modelling (LaM), (Shapiro 2014), (Cook 2000). This is no accident as LaF can be viewed as an explication, or precissification, of LaM. Stewart Shapiro describes the LaM theory of logic as one where logics are mathematical models, and so which features are focused on, idealised and ignored will depend on the purpose we are to use it (Shapiro 2014, p. 93). This is in-line with our view of the process of formalization as well.

Where our theory differs from Shapiro's is that we give a more detailed notion of formalizations. That is, modelling casts too wide a net, and we can be more precise while still capturing all logics by focussing on formalizations, a sub-set of modelling. We noted the difference between idealisations and abstractions; accepting falsehoods and ignoring truths in our formalizations The approach that Shapiro recommends of which parts of our target phenomena to focus on, and which are safe to either idealise or abstract around, matches a general set of norms of fruitful modelling and formalization. That is, by following them the we are not threatening our analysis, where not following them would lead to a sub-optimal analysis.

The LaF theory differs from LaM in another way. LaM is an instrumental approach which says nothing about the conceptual notion of logic, merely that logics are models. LaF goes further, being explicitly conceptually open. Once we have the conceptual grounds to logic then whatever we choose to formalize this notion is a logic, under that concept of logic. Similarly, other things will not be seen as logics under that conception. Whether it is a good formalization or not is a separate and, following Carnap, pragmatic issue. We will discuss this in more detail in the next chapter.

Recall our definition of formalizations, adding the criteria of the 'target' of the formalization to Baldwin's original definition (Baldwin 2013, p. 88).

Definition 6.1. We see a *full formalization* as involving the following components:

1. Target: a set of concepts, their properties and relations, which are undergoing the formalization, often characterized by (2).

2. Vocabulary: specification of primitive notions.

3. Formal Components:

   (a) Specify a class of well-formed formulas.

   (b) Specify truth of a formula from this class in a structure.

   (c) Specify the notion of a formal deduction for these sentences.

4. Axioms: Specify the basic properties of the situation in question by sentences of the formal system.

With Definition 6.1, we also see that there are minimal structural qualities of formalization, that are not required of all models. It is clear that the LaMers will agree to the inclusion of the target phenomena (1), however as they do not produce a clear notion of what a model is, it is unclear if Baldwin's minimal criteria of a vocabulary (2) and a semantics (3b) would be too restrictive for their purposes. As we don't think that all formalizations are logics, it is possible that additional structural requirements might emerge for logics, in which case we are providing a more formal precissification. We will discuss this possibility below.

In the previous chapter we carried through our discussion focussing on the first formalization of logic, Aristotle's syllogistics as our ongoing example for the various properties involved in the process of formalization. Syllogistics have (1), (2), and (3ab), with (3c) and (4) being missing, or at least unclear. This lack of clarity is because the ideas of syllogistics were spread across multiple books. Writing out the rules which specify the basic properties of the situation and giving a notion of formal deduction is possible, but it is unclear whether that would be a reconstruction of the idea without the grounding there to support such a presentation. It is still clearly a formalization, so we need not dwell on this matter.

A formalization that has all of these components is said to be 'full'. Paired with this definition is the Gödel-Kennedy notion of *formalism-freeness*; that formalizations need not have all elements of Definition 6.1. Formalizations could have all the components, but they are all not necessary for formalization. Different modes of construction can lead to the same set of mathematical truths, if this is the case then they are, in Gödel's sense, formalism independent.[1] The minimum criteria of a formalization is vocabulary (2) and the semantics (3b), and (1). We added (1) to differentiate between construction of a formal system and a formal system from formalization, evading the process-product ambiguity discussed in the previous chapter. That is there are two processes which result in a formal system: formalization and construction. Construction is the process of creating a mathematical structure for the purpose of analysing the structure itself. Formalization is a process which takes a target and models it with, at least, (2) and (3b). Without (1) we would not be able to differentiate between formal systems created from construction, which lack (1), and those created by formalization. The different processes also help distinguish the mathematical, or formal, investigation of the system at hand with the formalization of *something* into the system in order to analyse it.

The distinction between logics from construction and logics from formalization is similar to Graham Priest's pure and applied logics. The difference here is that we have focussed on the process that led to the logics, whereas Priest presents it as first the construction of a pure logic and then the application of a pure logic by interpreting it in some way or other. That is, for Priest, logics are first constructed and then we can apply them to some purpose. This application "then becomes a *theory* of how the domain in which it is interpreted behaves" (Priest 2006a, p. 195). This idea of application is more in line with the second type of formalization, formalizing the sentence of natural language. That is, with the formal system already in hand we apply it to the purpose of formalizing a sentence e.g. 'If it is raining then the grass is wet.' Our slightly different view allows for the development of logics *for a purpose*, rather than just applying a known construction to a domain. It highlights the process that starts with the target application and ends with the formal system. We can still accommodate Priest-style applications of formal systems in just the same way we did the formalization of the sentence. We prefer to use the

[1] See section 5.5 for a more in-depth discussion of this.

term 'pure' to indicate the purely formal study, or use, of the formal system in question, rather than its initial impetus. Many formal systems were first purely motivated, and then applications were found after the fact. Our distinction reflects this.

Consider *group theory*; it is the mathematical study of algebraic structures called groups. Taken as just an area of pure mathematical study, it is not caught by the net of formalization, nor logic as a result. Recall MacFarlane's first notion of formality, "treated purely syntactically, without reference to the meanings of expressions" (J. MacFarlane 2000, p. 31). As there is no target phenomena in this purely syntactical approach, this sort of study is left with mathematics and not in logic, or as any sort of other formalization. There is a conceptual wrinkle here. If one was an ontic realist with respect to mathematics, say believing that mathematical objects exist in the platonic realm, then there would be target objects to which the formal system refers. In this case, because of the ontic commitments, the difference between constructed and formalized systems narrows, or perhaps even disappears. Let us remark that even under these circumstances the formal system is still a representation of the objects in the platonic realm, so there is a notion of formalization at play, even under these strong ontic commitments. The wrinkle continues, as there is no notion of idealisation, or abstraction that comes along with these representations. The maths perfectly represents the platonic objects. It is this continued notion of 'purity' which, despite the representative notion that comes along with the ontic commitment, pushes things back into the pure realm, though we recognize this extra-step is unique and shows that the borders are not wholly clear.[2]

Priest uses this idea to motivate his notion of *theoretical pluralism*: given a fixed application, there may be multiple applied logics which constitute theories of behaviour within the domain, and thus there can be disputes about which theory is right (Priest 2006a, p. 196). In our parlance we cash out theoretical pluralism as the dispute as to which formalization is correct given the target domain. Note that correctness here is a pragmatic notion, which we will expand on below and in detail in the following chapter. Priest notes that we can apply a logic to many purposes so in order to make sense of our theoretical pluralism we need to focus on the *canonical application of logic*.

As we have seen, the purpose of logic is important for our current discussion. A logic can be the result of a formalization of many targets, so we need to clarify what the target is for logics (1). As we are looking exclusively at logics from formalization, rather than construction, we know that there is a target as well. To clarify what such a target might be, let us look towards the most usual of suspects: first-order-logic.

6.2.1 First-order logic

First we note that there are multiple ways of building first-order logic, from a technical stand-point. We can construct first-order logic through natural deduction, Hilbert-style systems, the sequent calculus, etc. All of these constructive methods lead to a formal deductive system which is both sound and

[2] This same move can be done with other mathematical realists, as the representation mapping is believed to be identical, or non-existent.

complete, properties we will touch on later. These formal systems syntactically show when a formula (of the system) is a logical consequence of another. These consequences are equivalent across the formal systems.[3] Here we see that, despite a variety of formal apparatuses, the 'truths' and properties that come out of the systems are equivalent. In the Gödelian sense first-order logic is formalism independent.

One might worry that our working definition of formalization leans towards model-theoretic accounts of logic, versus proof-theoretic ones. This will only be an issue if one is looking at the pure study of these formal systems. A purely proof-theoretic study, i.e. the study of the mathematical structure of some proof system and its results, falls out of our scope of analysis. This is due to our distinction between 'logic' (the concept) and logics (the mathematical structures). If we are dealing with the study of a formal system on its own rights, then this is not a formalization at all, but neither is it within our scope. Note that a model theorist who researches for the sake of interesting, and novel, properties of various model-theoretic structures falls out of our scope for the same reasons. If we were to include pure approaches to these formal systems it could only be under the conception of 'logic' as solely the mathematical study of certain structures. This seems undesirable, but also probably not what those who undertake this study would claim. We keep the distinction between the area of formal (mathematical) study of logics and the conceptually loaded 'logic' separate. Paraphrasing Carnap, when one asks the questions like 'what is logic?' it doesn't seem like they are asking merely what structure is that, rather they are asking something else.

Proof theorists work within mathematical logic and philosophy; as alluded to in chapter one they can have conceptual reasons for seeing proof theory as primary. It captures the inferential patterns that are constitutive of what our words mean. Peregrin claims that *the* logic of inference is intuitionistic logic, based on its proof theoretic strength (Peregrin 2008, p. 264). Here we have no issues, as there is a target phenomena, inference, and the presence of formal machinery which maps between the relevant concepts, properties, and relations between them for this target. Keeping the distinction between the pure mathematical study and the more conceptually weighty discipline continues to be the clear way forward.

There is another way in which first-order logics are differentiated; whether or not the identity relation is included in the language or not. While there is disagreement upon whether identity should be viewed as a base element of logic, or if we should view it as an extension, read: first-order logic with identity, this discussion can be run in parallel. No matter what side of the fence you are, on our arguments can proceed. We can safely avoid the need to choose one side or the other without worrying that this may jeopardize our notion that first-order logic has formalism-freeness.

Returning to Definition 6.1, it seems clear that first-order logic is a formalization and not a more general type of model. To check we just need to see if it has, at a minimum, components (1), (2), and (3b). We must be careful at this point. There are two distinct ways of viewing first-order logic as a

[3] This notion of equivalence can be read as pragmatic as well as syntactic, this distinction between will be important in the next chapter. The functional role of these systems when applied to any set of sentences to evaluate the consequences is equivalent.

formalization. We have to distinguish between the formalization that results in first-order logic and formalizing something into the formal system of first-order logic. The formalization which results in the creation of first-order logic is an attempt to formalize the basic concepts of logic, or reasoning, itself.

Contrast this with formalizing some sentence, or argument, from ordinary language, say “If it is raining, then the grass is wet" into a first-order logic, $R \rightarrow W$.[4] That is, *using* the first-order logic to formalize the sentence.

Both of these formalizations share most of the required criteria, as the latter is using the former. There is a clear specification of primitives, a vocabulary (2). There is a class of well-formed formulas, usually given inductively on atomic sentences (3a). Depending on our construction of choice we will have different formal components that address the (3c), but they will have a specification of formal deduction, indeed that is the entire point. It is similar for (4), the axioms, no matter our formalization we have axioms, in that we have specification of the basic properties of the situation in question, or the initial conditions. So, it would seem that we have a more structured type of formalization than the minimum. Just as there are different ways of specifying first-order logic syntactically, there are different ways of giving the semantics, (3b). But again, we need not worry about the specifics, as there will always be some semantics. Formalism-freeness is a spectrum notion, first-order logic is more structured, in that it seems to require more components, and thus is less free than other formalizations.

Looking at the use of first-order logic to formalize some sentence we see that most of the above holds. We must add more to (3b), however. Recall the correspondence scheme, or key, discussed in the previous chapter. This is the mapping from Predicates to natural language. For our example sentence we had:

$R \rightarrow W$
R: It is raining.
W: The grass is wet.

This is the semantic bridge between the natural and formal languages, for the formalization. Our earlier insight, that including this was necessary for the formalization is supported by Kennedy's point that the semantics are essential to the formalization, because the correspondence scheme is part of (3b) for any formalization of an argument of natural language. Along the same lines we also add the specific predicate letters as part of the vocabulary of the formalization of the natural language sentences.

In the case of a formalization of a sentence this is pretty clear, the target is the natural language sentence, e.g. “If it is raining then the grass is wet.” With the formalization of the sentence into first-order logic, it seems clear that we are speaking of some form of natural language inference, though this does not necessarily have to be so. If we take the Fregean concept of logic as the universal laws which prescribe the way we ought to think then the sentence which we are formalizing might be a sentence

[4]For simplicity we have reused the example from the previous chapter. For clarity R and W are predicates with arity of 0, hence no objects are associated with them, the sentence is still one of first-order logic, and not a sentence in propositional logic.

in that language and not in the logically imperfect natural languages (Gottleb Frege and Furth 1966, p. 12).

For the case of the general formalization of first-order logic, the target is less clear. The target of logic, as a formalization of a system, is dependent on the purpose of logic underlying the exercise. As we saw in chapter two the concept of logic is MOT. The concept of logic informs its purpose.

This means that the purposes of formalizing a logic are also varied, and thus the target being formalized is not universal. As an example, we see the notion of conjunction is in the primitive vocabulary of first-order logic, usually written as $\wedge$. If we think logic is about natural language, then the resulting formal symbol, and its accompanying deduction rules, are an idealisation/abstraction of the word ‘and’. If we think logic is about the universal language of thought then $\wedge$ represents something more esoteric than that, perhaps the pure concept of combination, or agglomeration.

All we need at present is that there is a target of formalization. If it is a formalization of a sentence, or argument, then it is of Priest’s applied logics and the domain of interpretation gives us our target. If it is the total formalization that leads us to the logic itself then it is informed by the concept of logic the formalizer has, and this same concept leads us to the purported canonical purpose. That is, so long as there is a clear concept of logic which informs the purpose of logic, and the formalization follows this purpose, we have a logic which corresponds to that concept. The arguments around the canonical purpose are derived from the polysemous and open-textured nature of the concept of logic. We can safely avoid them as although they differ fundamentally they still lead to a target. We will return to the idea of the purpose of logic as part of our clarifying discussion below in section 6.2.3.1. Any of these formalizations of first-order logic will lead to the satisfaction of (4) as well. We also clear the trap laid out in chapter three, the missing claim of ‘why this type of mathematical structure is the correct one to capture logic?’ does not apply here. It is clear that whatever the aims are, logic is a formalization. First-order logic passes through our definitional gate.

6.2.2 Second-order logic

Our discussion of second-order logic will serve two purposes, firstly it will elucidate the above discussion, and secondly it will establish what it means for second-order logic to be a logic under the LaF theory. Under certain concepts of logic, the formal system of second-order logic will come out as an appropriate formalization of said concepts of logic, and under others it may not.

Second-order logic is an extension of first-order logic where we have quantifiers for not only objects but also properties. The existential quantifier in first-order logic is “there is an object in the domain.” The second-order existential quantifier is “there is a property of object in the domain.” With this addition we increase the expressive power of the formal language.

The ‘logic-hood’ of second-order logic is contested; some say it should be considered logic, others not. For our approach we see that the same elements in first-order logic are present here. There is a clear vocabulary (2), and a semantics (3b) so it meets the minimal criteria of a formalization. So long as we aren’t viewing it purely as a construction we see that there will be a target as well, meeting our additional criteria (1). As it is an extension of first-order logic we see that second-order logic will also have the remaining components of Definition 6.1.

There are two types of arguments against the logic-hood of second-order logic: technical and theoretical. As the theoretical layer informs the technical we should search for the underpinning theoretical reasons of any technical requirement. The technical arguments usually point out desirable properties of a formal system which second-order logic lacks. The main contender for this is that there is no deductive system for second-order logic that is sound, complete and effective; second-order logic lacks a complete proof theory. This was one of Quine's arguments against second-order logic's logic-hood (Quine 1986, p. 90). Properties, such as soundness, completeness and a complete proof theory, are properties of the class of mathematical structures known as logics. So Quine's claim can be stated as the mathematical structures called 'logics' are those that have complete proof theories. If we are curious as to delineating a subset of very useful/interesting members of this class then perhaps this approach would be fruitful. If we wanted to study the constructions themselves then perhaps narrowing our focus to the subset that does have a complete proof theory could be an interesting avenue of pursuit. This is probably an uncharitable reading. What we need are arguments for what logic is, and why, given that, these technical properties are necessary to formalize 'logic'.

This technical argument isn't so straightforward. Leon Henkin provided a non-standard semantics for second-order logic, one which restricts the domains of the second-order variables. He proved that with Henkin semantics second-order logic was both complete and compact. This, essentially, reduced second-order logic to first-order logic, and the comprehension axioms of set theory (Enderton 2015). Even if we thought that rejecting second-order logic on technical grounds was appropriate, we need to be clear as to what form we are rejecting, based on what 'missing' properties. As we think that syllogistics are a clear examples of a formalization of logic, and it definitely is lacking such properties, it isn't even in the class of mathematical structures, the technical route doesn't seem promising.

On the theoretical path there are two ways of interpreting the trouble of second-order logic. One could be suspicious of second-order logic as a whole (using the full, non-Henkin, semantics), or one could be suspicious of second-order quantification itself (Turner 2015, p. 468). The former worry is encapsulated by Quine's claim that second-order logic is just set-theory in sheep's clothing, while the latter is what Turner calls 'ontological guilt.'[5]

The argument from ontological guilt is roughly that something funny is going on with second-order quantification. The standard presentation of this worry is that if we allow our quantifiers to range over properties, or predicates, then that ontologically commits us to the existence of these things. If we don't think properties/predicates are real things, the logic must not be correct as it will overcommit us. Turner notes that those that take on the Fregean concept of logic will have no problem with arguments of this sort as there is no problem with a system having ontological commitment to things that are distinctively logical. So this is a trouble only for a specific understanding of the concept of logic being the grounding

[5] We will just briefly touch on these issues, in order to show how they are not troublesome, regardless as to which side of the argument one is on, for the logics-as-formalization view. For more details and a clear presentation of the arguments see (Turner 2015).

of mathematics. The worry is that it does too much, and ascribes properties where there aren't any. This is just an idealisation. Under our theory we can see that the worriers, for their concept of logic, are disputing that second-order logic is a good model/formalization of logic because there is an idealisation that affects ontological commitments. If minimal ontological commitment is important to one's concept of logic, then it should be identified as a key property and thus not idealised in a troublesome way when one formalizes.

The other theoretical worry, is that second-order logic is set-theory in sheep's clothing, again from Quine. This is an argument against the full semantics of second-order logic. With this semantics we can form the sentences (CH) and (NCH), such that if the continuum hypothesis is true, then (CH) is true on all second-order models, and if it is not true then (NCH) is true on all second-order models (Turner 2015, p. 468). Thus it treats either (CH) or (NCH) as a logical truth. The worry is that no consequence relation should do this, either because it means that the logic isn't topic-neutral, or for normative reasons (Turner 2015, p. 478).

This seems to be a concern about its usefulness when applied to this specific concept of logic at hand. Topic-neutrality and normative concerns are knock-on properties depending on the concept of logic being assumed. Just like the previous theoretical concern, we are left with the idea that this is not a useful formalization for the purposes at hand, or at least if it is useful it will need further justification. We can understand the limitations of the concerns surrounding second-order logic by viewing the debate in terms of the logical concept and what valuable properties follow from the concept which should not be formalized away. The dispute seems to be based on a specific concept of logic and a narrow reading which regards the formal properties of the system as having ontic import. "At best we end in a dialectical stalemate: the friend of second-order logic certain she has the innocent interpretation she needs, and the foe certain she does not" (Turner 2015, p. 477).

We have seen that familiar logics fit nicely in the LaF theory, the key to this understanding is the target of the formalization, which will be informed by the concept of logic at hand. These logics have more structural properties than formalizations need. LaF has a clear conception of logics, they are formalizations, and as such are targeted. So long as our use of first-order, second-order logics are used as formalizations related to the concept of logic, they are logics. For those, like Gödel and Quine, who have technical concerns stemming from their conceptions of logic, or perhaps informing them, higher-order logics will not be candidates for lack of these properties. Under this concept of logic, the lack of a proof theory of second-order logic can be read as a fundamental mismatch for what makes something a logic, or it can also be read as a pragmatically poor system to formalize this concept because of the lack of this property. Both conceptions and arguments can be read under LaF, but LaF highlights where this disagreement starts, rather than fighting on the abstract technical notion of complete proof theories, without the clear theoretical grounding needed to support such disagreement. The stalemate may not be able to be broken, but if it can be it will do so based on the underlying concept of logic that has led to these two positions. Second-order logic cannot be ejected wholesale, as logic is MOT.

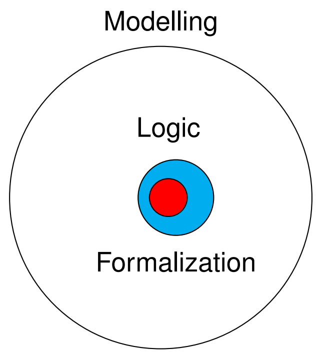

Figure 6.1: The updated process view

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We will now turn to a discussion of the differences between logics and formalizations, to better understand the differences between the two, and the bounds of the target of logic.

6.2.3 Logics vs formalizations

In the previous chapter we discussed formalizations, with modelling as a contrast class. Formalization is a modelling process, taking some target and describing the basic concepts and the relations between those concepts, usually in the form of rules, in some representative system. The target exists in a domain or context, and so does the resulting formalization. We will discuss the differences between logics and formalizations, first on theoretical and then on technical grounds, establishing that logics are formalizations of inference, within a context (domain).

Much of the debate on the logico-formal border seems to be caught in the separation of the class of mathematical structures called 'logics' and class of formal systems. We have already discussed that syllogistics fit with our established notion of formalization, despite not being a member of the usually referred to class of mathematical structures. The claim is that all logics are formalizations. However, not all formalizations are logics. That is, logics are a strict-subset of formalizations. Adding logics to our diagram from chapter five; logics are the small red circle in figure 6.1.

There are two routes we can take to narrow our current idea of formalizations to only logics. The theoretical route involves looking at the target of logics. Given our discussion above on the relation between the target and the underlying concept of logic this should be unsurprising. The technical route would be to look at the properties of logics and see if they necessitate other components of our Definition 6.1. We find the theoretical route to be more fruitful, and it may also influence the technical, so we will start on that path.

6.2.3.1 Theoretical differences

Formalization is an abstract process, it takes some input (target) and abstracts and idealises to yield some formal system which represents the input. This, of course, can be done well or poorly. The target of formalizations can be anything, we do not think the same of logics. Logic is an area of study bound by base concepts; 'logic' obviously is the main one, but there are others. Concepts, such as inference, implication, reasoning, follows-from, will be important depending on the concept of logic at hand. If we have a fixed concept of logic then we can find both the purpose of logic and thus the targets of the formalization. As we want our theory of logic to be conceptually open, this strategy is not available to us. Instead we have to look at the commonalities between the various concepts of logic. That is, we have to engage with the major question of this work, what is logic?

Broadly, logic is described as 'the science of reasoning'. This, of course, doesn't tell us much. Some refine this to "formal logic is the science of deduction" (Beall and Restall 2006, p. 23). Others broadly describe it as about inference. While most take these terms to be co-extensive, this is not uncontentious.

Gil Harman, in *A Change in View*, makes the claim that logic is not about inference nor reasoning, nor can it be. He makes the distinction between theoretical and practical reasoning. Theoretical reasoning is what we traditionally due with systems like logics, it is deductive. Theoretical reasoning affects our beliefs. Practical reasoning is a different thing entirely. Practical reasoning is about *reasoned changes in view*, it is the reasoning *in situ* of actual agents. It affects our intentions and plans. This distinction carries forward to show a difference between what he calls 'inference', the reasoning steps/process of actual agents, and implication, the consequence relation (or equivalent) in logical systems. Similarly, reasoning and argument come apart. Without this connection between the two it follows that "rules of deduction are rules of deductive argument; they are not rules of inference and reasoning" (Harman 1986, p. 5).

He makes a similar observation to the unsettled nature of logic, much the same as we established in chapter one. "Historically the term 'logic' has been used in [two] ways. Current usage favours restriction of term 'logic' to the theory of implication. The theory of reasoning is best called 'the theory of reasoning' or 'methodology' " (Harman 2002, p. 171). The parallels to our theories do not stop there.

Harman points out that the idealisations involved with logic are totally disconnected from what he calls inference, reasoned change in view. If logic is to guide reasoning it has to capture the following principles.

> *Logical Implication Principle* The fact that one's view logically implies P can be a reason to accept P.
> *Logical Inconsistency Principle* Logical inconsistency is to be avoided. (Harman 1986, p. 11)

He shows that these are incompatible for reasoning. One problem is that we have practical limitations to our beliefs, and taking the logical closure of some set of propositions will result in 'clutter'.

For example if we believe P, then by logical closure we believe $(P\&P)$, $(P\&(P\&P))$, and so on. Attempts to patch this problem, by changing the principles, inevitably bump into the problems of the idealisations in logic.

From the LaF perspective this can be read as a gap between the formalization and its usefulness. That is, Harman seeks to understand reasoned changes in views. Inference is about non-ideal agents reasoning with imperfect information *in situ*. There are other formal tools out there which are aimed at this target; decision theory being one. Hartry Field responded to Harman with such a proposition, offering a probabilistic account of Harman's problem cases and principles (Field 2009, pp. 252–9). Let us pause and assess the situation between Harman and LaF, before addressing decision theory, and like enterprises, with respect to LaF.

The heart of Harman's argument is that there is a conceptual conflation between implication and inference. They are two different things, which are decidedly not co-extensive. His opponents contend that there is no such conflation at play. This can be read as that implication and inference are co-extensive, these concepts do not clash in the way that Harman skillfully argues, or it can be read that they disagree on the concept of inference to which Harman appeal: ; inference isn't just about practical reasoning. If this is the case they seem to be at the same place we started at with logic. They are in danger of talking past each other. In which case the argument should be over the concept of inference, in much the same way we recommended the logic debate in chapter three. Indeed this seems very much what happened in the responses between Harman and Field to each other (Field 2009), (Harman 2009). Consider the logical realists, those that believe the laws of logic are external and part of the universe. They clearly have a different conception of logic, and inference, so it would seem the appeals to the limitations of actual agents would not work against their view.

The idea of reasoning *in situ* is not unfamiliar to us. Recall the Stephen Read type of relevant logician from chapter three. Here the 'correct' account of validity is that it should *warrant* one in proceeding from the truth of the premises to the truth of the conclusion (Read 2006, p. 10). Though their concepts of logic are not quite the same, there is an overlap. Harman's points about idealisations, and logic, limiting the use of his notion of inference seems to land against the Read approach to warranted belief. The conceptual difference around inference is enough to still move that debate back to clear up that baggage first. While there are similarities, to truly work out the disagreement the concepts of logic and inference at play need to be agreed upon. We are not claiming that Harman is simply speaking past everyone else, rather that his conception of inference is different, and we should engage on those grounds. In this case it seems we need to move our debate one layer up, to fix our notion of inference, before exploring the nature of logic and inference, and then the things beyond.

Harman's approach of conceptual clarity on inference and implication is in line with our overall concept-first approach. If one thinks logic encompasses practical, as well as theoretical, reasoning, they need to engage Harman on these grounds. Another reason we think that Harman would embrace the LaF theory is that he also points out the 'missing claim' from our desiderata in chapter three. The missing claim is that once we have a concept of logic in hand, we still have to establish why the class of mathematical structures we are using is a good one. “It may be a mistake to expect principles of reasoning to take the form of a logic" (Harman 1986, p. 6). This highlights the, potential, disagreement between Harman and Read discussed above. As LaF is conceptually open, it is not at odds with Harman’s approach. It is a process-oriented approach aimed at pushing technical debates back to the theoretical grounds to ensure that the base concept of logic, or indeed inference in this case, is agreed upon, before worrying about formal matters. This is Carnap’s common interpretation.

There are two possible scopes of interpreting the idea of logic as the science of reasoning, whether theoretical, practical, or both. The narrow reading is that logic is the formalization of reasoning itself as the topic, or context. From our discussion in chapter two we know that the concept of logic has shifted and split over time and so we should not be surprised that there is more than one concept of logic out there, even given this narrow reading. The wide scope, on the other hand, is that logics are reasoning in C, where C is some context or domain. Our discussion in chapter three of the conceptual differences that ground the various monist views suggest such a context variance is indeed at play. This might initially sound like an inherently pluralistic approach. However this scoping is perfectly compatible with a monist position. The monist can make either of two claims. The first is that no matter what context C is, the correct logic will turn out to be their champion logic. This is the idea of a universally applicable logic. Alternatively, one could make a claim as to what the *canonical* application of logic is and then offer the system of choice as the correct one (Priest 2006a, p. 196).

Let us consider the narrow view first, and see how far that gets us. Formalizations are about a specific context. They are models of the primitive concepts and the relations between said concepts, and they include (at a minimum) some semantics/specification of truth. Logics are formalizations about reasoning. That is, they are not bound to a particular type of mathematical structure; this is how we kept syllogistics in the fold.

One might worry that we have opened Pandora’s box of representation with such a broad definition of logic. Consider the case of Decision Theory; it is not considered to be a part of logic, rather it is its own area of study. It is the study of the reasoning that underlies an agent’s choices (Lindley 1977, p. 52). There are both normative and descriptive branches of decision theory. There are primitive notions (2), e.g. utilities, preferences. As we are dealing with mathematical representation of the concepts, much like the Lotka-Volterra equations discussed in the previous chapter, we have a specified class of well-formed formulas, (3a). We have a set of axioms that describe the basic situation and the relation between the concepts, e.g. the von Neumann-Morgenstern axioms. The question is do we have a clear notion of the truth of a formula in decision theory. Decision theory does indeed have a formal semantics (3b), it is clearly a formalization.

One approach to differentiate between logics and decision theory, despite that they both are trying to represent reasoning, is to point out their difference in ‘purity’. Logics are about deduction. While decision theory, by focusing on choice and what grounds the reasoning of an agent, is primarily concerned with abduction. Decision theory operates in a different paradigm, and thus mathematical basis. The differences between the mathematics involved does not get us very far, as we need to keep the differentiation between the class of mathematical structures called logics, and the conceptual logic apart. Once we have uncoupled the concept of logic from the type of mathematical structure, it is no longer so straightforward to view decision theoretic modelling of reasoning as an obvious step beyond.

Decision theory is about the deliberation of agents under imperfect information, and circumstances they can neither predict nor control (Jeffrey 1990). Perhaps this is difference enough. We have a slight problem, however.

Warranted belief is about an agent's limited knowledge, reasoning under imperfect information. If the concepts we are wielding are those of reasoning and what underlies the ability to conclude things one way or another: what we are warranted, then it seems that an argument against decision theory will be equally strong against this type of relevant logic.

This is problematic on the reading of logic as the science, or study of, reasoning. We could appeal to the idea that logic is about truth, while things like decision theory are not. Logical consequence is necessarily truth-preserving.

This approach does not get us as far as we may like. Consider epistemic logics. These logics have knowledge, not general truth, as the grounding of the propositions. Here the consequence relation will have truth-preservation on the semantic level but not the conceptual, or ontic, true. In this area we also have non-monotonic logics, used to model defeasible inference. Monotonicity is the property that if a set of premises entail another premise, then those any addition to that set of premises will still entail that other premise. In symbols:

(6.1) $$\text{Monotonicity: If } \Gamma \models \varphi \text{ then } \Gamma, \psi \models \varphi$$

So we seem to have a gap between formal necessary truth preservation and ontic necessary truth preservation. Epistemic logics are about the inferences we can make with what we know, non-monotonic logics are about defeasible inference. Neither of which are truth in the world.

This leads us to our greater general claim. Often when logic, and logical consequence, is described informally it is described as the study of 'what follows from what'. We will adopt this slogan. This also reflects the prominence of logical consequence in the discourse. Rather than focus on reasoning, which can be split up in conceptually committed ways, we will move back a step. All concepts of logic are about what follows from what, this is the common core to the MOT concept of logic. This matches well with our conceptually open approach. The concept of logic at hand then informs how we unpack 'what follows from what'. For Harman's concept of logic the 'what follows from what' will pick out theoretical, but not practical, reasoning. For Read, we suspect, it picks out both. The logical realists will see a universal scope on this notion. For simplicity we will call this notion a loose idea of inference. That is, we do not mean to imply a conceptually heavy notion of inference that would be at odds of, say, Harman. Rather, we acknowledge this inconvenience but adopt the term rather than having to invoke a slogan at every instance. Where this may be misconstrued we will re-invoke the slogan to be clear on the loose notion of inference being used.

Logic, then, is about inference. No matter the concept of logic at hand it is about inference, whether it is the inference rules of the universe, natural language or some other sort. The different concepts of logic ground inference in different ways. If the primitives of reasoning with logic are about inference, we dodge the problem of needing to ground logic in truth, and having the potential troubles of what to do about relevance logic and decision theory. According to LaF, a logic is any formalization aimed at modelling inference in some context, dependant on the concept of logic. Logics deal with the concepts, and the relations between them, of inference. Thus, if one's concept of logic is broad enough, epistemic logics and decision theory are logical systems. If one's concept of logic is more narrow, or just differently scoped, they will not be.

This may seem too permissive to some, it may seem likely to over-generate formal systems into the class logics. LaF is not an 'anything goes' theory. Rather, it is aimed at conceptual and theoretical clarity. We first fix the concept of logic, and then derive both the purpose and target in order to get the useful sets of formal systems we could use to formalize said target for that purpose. This pushes disagreements about correctness and logic-hood to either the pragmatic concern, is this actually a good system to capture that target, or the conceptual grounds, where fruitful discourse should start. This distinction is fundamentally different from the logic-formal boundary. The latter aims to draw strong borders based on formal properties. As we see logics fundamentally tied to the conceptual task at hand, that approach seems ill-fitting. In LaF we aim for clear boundaries based on theoretical grounds, while acknowledging that they are shifting due to the fact that logic is MOT.

This also means that other formal systems, like inductive and abductive logics, can also be part of a class of logics. We don't have to rely on things like structural family resemblance to make sense of them as logics. LaF is a view on logics in virtue of the instability of 'logic'.

6.2.3.2 Technical differences

The above theoretical discussion highlighted that there is a narrowness to logics, as compared to formalizations. This had to do with the scope of the target (1). Logics are about inference, while formalizations are not so inference-bound. We now return to Definition 6.1 and look to see if there is an accompanying narrowing of the structural freeness of formalizations.

Logics are formalizations, by that we know that they must at least have a vocabulary (2), a semantics (3b) and a target (1). The remaining components are the syntax (3a), a notion of formal deduction (3c) and the axioms (4). As we are inference-bound we will need a syntax of some sort, so logics require (3a). We are modelling inference in a context with our formal system so a notion of deduction within the system is needed, otherwise we seem to have missed what we are formalizing. As these are formal components we must be careful, we could have a formalization of inference in a domain that does not itself have a formal notion of deduction within, a sort of blackbox approach. Baldwin was speaking specifically about formalizations of mathematical areas, and (3a) and (3b) are basic, but (3c) need not exist (Baldwin 2013, p. 94). A notion of deduction is needed for a logic, but it need not be formal. Because we are interested in analysing inference within the target area, most logics will end up with some form of (3c), but it is not required.

The last component are the axioms (4). In our discussion of Aristotle's syllogistics we considered a loose reading of axioms as a set of rules which pick out the actual subject area. Under this loose reading syllogistics do have (4), the perfect deductions serve the purpose of being the primitive rules of the deductive system. For pragmatic reasons most logics will have axioms, setting the stage from the target, but they need not necessarily have those. Logics likely will have all of the components of Definition 6.1, but they need not. They do require, over and above formalizations, a syntax so (3a) is necessary. We now have a clear idea of what logics are, as formalizations. They represent reasoning within a context or domain and they have more structural requirements than a formalization, with syntax. More often than not they will have a formal deduction (3c), and a set of axioms, in the loose sense, due to the nature of their target.

6.2.4 Carnap

This section is a brief departure from the details of the LaF view. We will see that the core of LaF is in line with both the early and later Carnap's thoughts on logic and its scope. We will discuss the early work of Carnap on logical syntax followed by the later Carnap's view on languages and external questions. The limits of formalization, and thus logic, under LaF are Carnapian.

> The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, that impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities. (Carnap 1937, p. xv)

This famous passage from Carnap describes his approach to logic, which is wide in scope. It is encapsulated by his principle of tolerance: "*In logic, there are no morals*, Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him, is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments" (Carnap 1937, p. 52). Indeed our position may be more tolerant, as we do not restrict the underlying concept of logic, we just claim that this concept will inform the purpose and application of the logic. The principle of tolerance is one of openness to the uses of logics. We embrace this tolerance under LaF, as a prohibition on formalizations does not make sense. One might want to analyse different aspects of the target than another, and so a different logic should be chosen for that pragmatic reason. A clear presentation of the formalization, particularly the idealisations and abstractions is also wholly welcomed.

Key to Carnap's philosophy was his rejection of the notion of the 'one true logic', and from that the preoccupation with classical logic. In *The Logical Syntax of Language*, Carnap presented the idea of logical syntax as the study of 'languages', which are systems of the rules of speaking. These contain rules of formation and rules of transformation. The rules of formation determine how sentences of the language can be formed out of its symbols; the well-formed formulas. These may be seen as the same type of rules as grammatical syntax, though they do not involve the meaning of the words, just their type and order. For example, in Latin there is a grammatical rule which sets the gender of substantives which designate town and country to the feminine. The rules of transformation “determine how given sentences may be transformed into others; in other words: how from given sentences may we *infer* others" (Carnap 1935, p. 43). The grouping of all such rules, in a given system $\mathbf{L}$, is called the direct consequence of $\mathbf{L}$. If a class of premises, P, are connected to a sentence, C, by way of a chain of sentences, and every sentence in the chain is a direct consequence of some of its predecessors we call the sentence C a consequence of P. Using the concept of consequence and the rules of transformation we can generate the more familiar logical consequence relation. We are on familiar formal seas.

This view on (formal) languages has a clear (vocabulary), the rules of formation provide the class of well-formed formulas (3a), and the rules of transformation provide (3c). We are confirmed in our suspicions that formalizations in general, and logics in particular, do not necessarily need axioms (4). Carnap's languages do not necessarily need these given his definition. We have a rough equivalence between Carnap's conecption of logical syntaxes and our logics as formalizations.

This may seem like too quick a move against axioms, in favour of our reading of them as not necessary. Carnap states that part of his Language-I does take the same form as the construction of of any other descriptive axiomatic system, namely the construction of descriptive syntax (Carnap 1937, p. 76). Firstly this claim is made of a particular language of Carnap's design, Language-I, aimed at showing the flexibility of formal approaches, and he doesn't claim that this is a property that all languages need, so axiomatics remain in the middle ground. To understand what they are doing we need to be clear on what is meant by descriptive syntax.

Pure syntax refers to the possible arrangement of things without reference to the nature of the things or to the question of whether these elements are realised anywhere. Descriptive syntax is concerned with the syntactical properties of empirically given expressions, like physical geometry, compared to the pure syntax of pure mathematical geometry (Carnap 1937, p. 7). This empirical connection is important to his overall project. Carnap claimed the logician should only be concerned with syntax, but it wasn't the case that he thought there was no place for semantics. He valued the idea of the experts doing what they were experts in. The philosopher, an expert in formal languages, would work with the scientist, the expert in the target area, to create a language that had both formal and physical properties. He differentiated between these as L-terms and P-terms and similarly for the rules of the language. We see that even the early Carnap left space for semantics, but he thought that the logician might not be the right person to provide those rules. Similarly descriptive syntax takes on part of the role of deduction (3c), “[s]entences of descriptive syntax may, for instance, state that the fourth and seventh sentences of a particular treatise contradict one another" (Carnap 1937, p. 7). Descriptive syntax gives us the tools we need to describe the basic situation in questions by sentences of the logic. More often than not we will need such a basic situation in order to formalize our logical target. Here is where LaF departs from Carnap, under our notion of formalization we do not strictly need something that plays the role of descriptive syntax, though any formalization is welcome to use such a tool.

The main claim that Carnap makes of logics is that they are tools of analysis to be judged by the resulting outcomes, not their specific designs on their own. This judgement occurs at the theoretical layer, being dependant on the purpose of which the logic is being applied. The subject is concerned with how we can reason in any setting (application) represented by an appropriate formal language (logic). Carnap is clear that the study of logic should remain limited to *inferential reasoning*, which is where we part ways with him. Our conceptually open approach means that our scope will be much wider than what he means by inferential reasoning, under certain concepts of logic. However, the Carnapian concept of logic fits will within the LaF framework.

As logic is a tool to be used, we have a pragmatic notion of usefulness at hand. A logical language is to be judged on its 'fit' with the application at hand, or to be more precise the quality of the conclusions we can generate from it. He does have a certain class of mathematical structure in mind, namely structures that can be given by syntactical rules of formation and transformation.

The later Carnap had a more open approach to semantics.[6] While semantics take more of a front-and-centre role in this later work there are still restrictions. Questions can only be coherently asked from within some linguistic standpoint, external questions are pseudo-questions (Carnap 1950, p. 250). We can only ask the pragmatic question "which framework best suits our current purposes?" from the outside. LaF follows suit, as logics are formalizations and they necessarily have idealisations and abstractions within. If we were to compare across formalizations we would not be guaranteed any notion of 'sameness', equivalence, or synonymy between them. We can, of course, comment on the efficacy of one system given a specific application as compared to another.

It is the problem of external questions and the conceptual openness of the principle of tolerance that shows that LaF is Carnapian in both flavour and grounding.

6.3 Scope and the sociological advantage

In chapter one we discussed the questions of doing philosophy of X. There we pointed out the sociological advantage of any philosophy of X if it encompasses all, or most, of the actual practice of those who are in the business of X-ing. In this section we will discuss various different areas of logic, and how they developed, with an eye to show that they fit nicely within the LaF theory, showing its large sociological advantage as a theory of logic, as well as the conceptual advantage it has due to the conceptual openness at the heart of the theory.

We will start with a discussion of logical paradoxes and paraconsistent logics, then discuss the *dynamic turn* of Amsterdam logicians, and finally discuss the growth and status of diagrammatic logics. These newer forms of logic inhabit different concepts of logic and modes of representation, both of which fit well within the LaF theory. The concept of logic is multiply open-textured, and as logics are formalizations, the mode of representation is not fixed, let alone to sentential representation.

[6] See the discussion in chapter four.

6.3.1 Paraconsistency and the Paradoxes

In logic there has been much work devoted to paradoxes, historically Russell's paradox had a huge impact, dismantling Frege's logicist project (Coffa 1993, p. 116). Here is a rough sketch. Frege grounded his notion of the extension of a concept, from which he could generate the natural numbers, in Basic Law V.

$$\{x|F(x)\} = \{y|G(y)\} \leftrightarrow \forall x(F(x) \leftrightarrow G(x))$$

Unpacking this, $\{x|F(x)\}$ is the set of objects which satisfy the concept F. So, Basic Law V states that the set of objects that satisfy F is identical to those that satisfy G if and only if for all objects if it is in F then it it is in G, and if it is in G, then it is in F. This seems straightforward, we seem to be saying the same thing in our maths as we are saying in our logic.

Russell asked what if we defined a concept R which says it is the concept of concepts which are not in their own extension. Is R a member of itself? If it is, then it can't be. If it isn't then it has to be. This notion of concept extension leads to a contradiction. Various solutions have been provided, to heal this problem, but as Frege was trying to ground mathematics in logic his program was doomed.

Graham Priest's solution is to adopt a *dialethic* approach to truth. There can be true contradictions. In classical logic from a contradiction we can derive anything, this is called the principle of explosion: $\varphi, \neg\varphi \vdash \psi$. Paraconsistent logics, like Priest's LP, do not have this rule, so contradictions are not so disastrous for inference. For Priest the paradoxes are *bona fide* sound arguments, and dialethism is right account of truth to evaluate them (Priest 2006b, p. 10).

There is an alternate view of the paradoxes, nicely summed up by Hao Wang, "that we take the paradoxes too seriously largely because of our preoccupation with formalization and our lack of flexibility" (Wang 1955, p. 238). Instead of dissecting the argument, looking for the troublesome step that leads to the contradiction, we could simply acknowledge that the individual steps work but when we combine them we get an undesired results and endeavour to no longer put those steps together moving forward. This might explain the indifference of working mathematicians to the paradoxes. This approach only becomes problematic when we try to employ it in formal languages, as in them we break arguments up into individual steps which if they are valid they are valid anywhere. Wang offers two solutions: give up the attempt of any exact characterization of all valid arguments, or list all the warranted and unwarranted specific arguments. The latter would be quite unwieldy, Wang prefers the former (Wang 1955, p. 239).

We offer a third option, in light of the work of Priest. Following Wang, we view the paradoxes as mere artefacts of the formalization and not larger problems for truth and reasoning. As logics are formalizations they come with necessary idealisations and abstractions. The paradoxes could be a direct result of these. This does not make them uninteresting phenomena, it is a limitation of the formalization at hand. We note that Priest might disagree with part of the above, the liar paradox was originally a problem within natural language and he is grounding things on a strong idea of the truth.[7]

[7] We see these two paradoxes as emblematic of the two settings, but there are mathematical formulations of the liar paradox, and there is natural language counterpart of Russell's paradox: there is a barber who shaves all those who don't

If we are interested in looking at paradoxes, then they are our target phenomena, and we should formalize appropriately. This, it seems, is exactly what Graham Priest has done with his paraconsistent logic. This lines up perfectly with the LaF theory of logics. LaF can accommodate his view, as he has a concept of logic and truth and LP is the formalization of those conceptions.[8] With LaF we see no need to continue fighting between classical logic and LP, they both have their place in the logician's toolkit, depending on what we are trying to formalize. Each has its positives and negatives. When we aren't dealing with contradictions classical logic is easier to use, for reasons of simplicity and familiarity. When we are concerned about sentences like the liar, we have a useful logic to pull out of our toolkit.

6.3.2 Logical Dynamics

> “Argument" is a piece of a proof, but also an activity one can engage in, and so on. Logical systems as they stand are product-oriented, but Logical Dynamics says that both sides of the duality should be studied to get the complete picture (Van Benthem 2011, p. 6).

The dynamic turn was borne out of the idea that reasoning is a process which involves more than just inference. There are activities involved with our arguments, namely communication and observation; how we acquire the premises themselves before we take the inferential step. A standard example is the café scene.

> *Three friends are in a café and ordered a wine, beer and a water from a waiter. A second waiter arrives with the drinks, and the following chain of events occur.*
> *The waiter asks, “Who has the wine?", receives the answer and places it accordingly. Similar steps occur for the beer. The waiter then places the glass of water in front of the third person, without asking a question.* (Van Benthem 2011, p. 9)

This last step is an inference that can be shown as:

$$\frac{(((A \vee B) \vee C) \qquad \neg A \qquad \neg B}{C}$$

“[T]o me, there is a unity to this scenario which gets torn when we just emphasize the final *inference*. The waiter first obtains the relevant information by *communication* and perhaps *observation*, and then, once enough data have accumulated, he infers an explicit solution" (Van Benthem 2008, p. 186).

What van Benthem is highlighting is that the interesting phenomena here is not just the final (deductive) inference. The first premise is the basic setup of the scenario $((A \vee B) \vee C)$, obtained through observation. The asking of questions, and getting answers is what gives us the second and third premises, $\neg A, \neg B$. By using classical logic to model the situation we only formalize the final inference, and not the entire story. Instead of focussing on the final deductive inference, a dynamic

shave themselves. Now we ask, “does the barber shave himself?"

[8] It is probable that Priest would still reject the necessary gap, from idealisations and abstractions, that come along with LaF, butf it would be a disagreement with a conceptual debt to which we have happily committed.

model would represent all of the *information updates* involved in the scenario. That is we focus on all the updates and all types of inference.

The aim of logical dynamics is to model all of the flows of information which lead up to the inference being made. Limiting ourselves to the study of only one type of information flow is a needless narrowing. The project aims to widen the scope of logic by recognising, and modelling, these other flows. For Johan van Benthem logic is about *rational agency* (Van Benthem 2008, p. 187). Within this new paradigm communication is the main activity in which an agent gains new information. It is a short leap, with this conception of logic, to the idea of multi-agency in logic. Communication, questions, and answers are all acitivities of information flow between one agent and another.[9]

The scope of logic is widened beyond just deductive inference. It also clearly is aimed at knowledge rather than truth. The main difference between static logics, e.g. the epistemic logics mentioned above, and dynamic ones is whether one models the stimulus or just the results. In order to model these other information flows we must have a more expressive language. This is done primarily by taking a logic, say a modal epistemic logic, and adding dynamic operators to make a *dynamic epistemic logic*. These operators change the accessibility relation in the epistemic logic, thus changing the epistemic state of the agent. For example if our agent does not know whether φ is the case or not then we would model that in epistemic logic with an accessibility relation that connects the current world to worlds where φ is true and worlds where $\neg\varphi$ is true. If the agent learns that φ is the case, we can model that with the dynamic update $[\varphi]$. What this does is changes the accessibility relation and 'cuts' all the links to the worlds where $\neg\varphi$. So, after the update,in all accessible worlds, from the actual world, φ is the case: the agent knows that φ.

Van Benthem points out that there are some who are determined to state that logic is about consequence relations, and thus this new system is no longer a logic. His response is that he is happy to give up the label logic, but thinks there is interesting work to be done in the area. He notes that the idea that logic is about consequence relations may have been true at some point in history but that it doesn't seem to be the case now (Van Benthem 2008, p. 205). This reflects our claims in chapter two, the concept of logic, and thus its targets, has changed throughout its history and often different concepts are held at the same point in time. The nature of logic is dynamic, not static, as it is MOT.

These detractors point out that these new logics are doing more than just logic, they model other phenomena than inference, so it would seem that they may contain logic, but are something beyond. Indeed that is how they are standardly constructed. Let us consider the dynamic turn with respect to our general notion of logic being about inference, or what follows from what. The dynamic logician is advocating a change in the target of logic, they want to analyse more than the last inferential step. What they are adding is the ability to model the steps that lead the agent towards said inferential step. In the terms of our slogan, they want to model both the *'what follows'* and the *'from what'*. This does not seem to be huge step away from the core of logic, just a different perspective.

[9] Van Benthem notes that this can be viewed as a return to form for logic, as Plato's *Dialogues* represent a clear set of rules for argumentation between two parties, as we noted in chapter two (Van Benthem 2008, p. 187).

The dynamic turn feels right at home with the LaF theory. When we formalize we identify the properties of the target phenomena that are of interest, or important, with the application we aim to use the formalization. A successful formalization is one which is fruitful in its application. Making the previous steps explicit and part of the formalization seems like the right approach, given the dynamists interest in the entire process that leads to the inference being made. Or, put another way, the dynamic logician is also interested in the second 'what' in our slogan definition of inference: what follows from what. Here how we come about the initial premises matter, and their conception of logic includes all information flows as equally weighty, thus they all need to be formalized, and not abstracted away. The openness of looking at inference, and information flows, makes the dynamist seems well suited to join the LaF camp, based on the conceptually open stance on logic of the theory. Our theory clearly captures the activities of logicians of different stripes, we have more evidence of the sociological advantage of LaF. Van Benthem sees his project in a compatible light with LaF.

> I would say that the discipline of 'logic' is best described, not by any subject matter plus border patrols, but as that activity which is successfully performed by logicians using logical notions and tools, *wherever* those take them. (Van Benthem 2008, p. 189)

6.3.3 Diagrams, consequence, and Maps

In this section we will introduce the notion of linguistic bias and briefly discuss two arguments against diagrammatic reasoning: limited expressive power and the possibility to mislead and show that they fall short. We will address these with two systems, Shin's Venn-II and Mumma's Eu. We will then present Etchemendy's concept of logical consequence and the ramifications for non-sentential systems before concluding that all of these approaches are at home with LaF, for sociological and representational reasons.

Consider when one learns modal logic, the accessibility relations are almost always represented by node diagrams to show which worlds are accessible to which other ones. The use of diagrams is more fruitful for the teaching of the concept of abstract relations between abstract objects. We teach with Venn diagrams in logic and set theory classes. In set theory the diagrams elucidate the concepts of sets and set membership, in logic we can use them to represent the basic propositions. In figure 6.2 the shaded regions shows the conjunction of A and B, or the intersection of the two sets.

Venn diagrams are useful but this is where most logicians usually leaves things. Within logic and mathematics, diagrams are relegated to mere heuristic tools to aid in the cognition of abstract concepts. Once the concept is sufficiently apprehended, or the topic is developed beyond the scope of the heuristic, the diagrammatic representation is abandoned for the sentential form. Proofs are considered from a 'logical' standpoint, and so are represented by a sequence of sentences. Diagrams cannot represent arguments *rigorously* (Mumma 2010, p. 255). Recent work has called this assumption into question, labelling it the linguistic bias (Shin 1994, p. 1), (Barwise and Hammer 1996, p. 73). Historically this wasn't the case with Euler's diagrams having a place in mathematical reasoning, representing syllogisms. These were supplanted by Venn diagrams, designed as an improvement on Euler's. C. S.

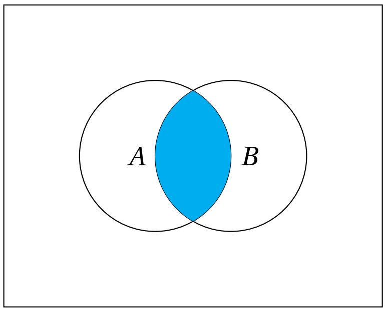
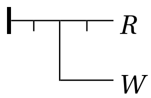
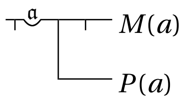


Figure 6.2: A simple venn diagram

Peirce continued this improvement seeking to increase the expressive power of Venn's diagrams. Both of the founders of modern (first-order) logic, Frege and Peirce produced their logics with diagrammatic elements. Peirce developed his existential graphs as a better way to express relations, now that they were part of logic. Frege used a two-dimensional notation to express the logical connectives and the new quantifiers (Gottlob Frege 1879, p. 14). The Fregean notation for the sentence "It is raining and the grass is wet" is represented as follows.



While the following expresses "some person is mortal", with P as person, M as mortal, and a as the object variable.



6.3.3.1 Objections to diagrammatic reasoning

Despite the historical diagrammatic presentation, we do not use this form of reasoning standardly. This linguistic bias has historical roots. Shin attributes it to two main factors, the limitations of diagrammatic representation and the possibility of the misuse of diagrams (Shin 1994, p. 3). The first is easily dealt with as there are limitations to linguistic representation as well. Not everything can be expressed in first-order logic. Spatial relations are much more easily expressed in diagrams versus sentences. The second factor is influenced by our answer to the first. The limitations of the expressive power of diagrams leads to a distrust. This distrust is unfounded.

One of the primary objections to the expressive power of diagrams is that they are limited on what they can represent. Consider a cube, we can represent it on a two-dimensional surface, but only with the use of illustrative heuristics to imply depth. The surfaces of a cube do not cross, but the two-dimensional representation has it that they do. If we move to four-dimensions we no longer have heuristics to represent this diagrammatically.[10]

Expressive power is not a one-way street however. Linguistic representation also has its limits. We know that not all sentence can be expressed in first-order logic, hence the move to second-order and other logics. The fact that we use diagrams as visual heuristics shows that the expressive power, at least in the understanding, of purely sentential representation is limited as well.

Peirce developed his existential graphs as a better way to represent relations and quantification. The existential graphs are essentially a syntax for logic using topological graph theory. The alpha graphs are equivalent to propositional logic, the beta graphs are equivalent to first-order logic with identity and the gamma graphs to a modal logic. This equivalence calls into doubt that there is such a large gap between the expressive powers of diagrams versus sentences. There are positives and negatives for both forms of representation, some contexts will suit one type of representation, and others will suit different forms of representation.

Most diagrammatic reasoning is not purely pictorial, diagrams are multi-modal, they have both linguistic and pictorial components. Our original example of Venn diagrams has both circles and labels. Current work in diagrammatic reasoning embraces this fact and the systems that they use are called *heterogeneous systems* to reflect this multi-modality. We will now consider two diagrammatic systems, based on the perceived short-comings of Venn and Euclid. These new systems, via their representational forms, patch the holes in the previous systems, showing the strength of diagrammatic reasoning, which is encompassed by LaF.

C. S. Peirce attempted to improve upon Venn diagrams, adding lines and the symbols o and x. Shin continued this work and modified Peirce's work to provide a formal system that was clearer to visually interpret and also sound and complete (Shin 1994, p. 132). She wanted the system to be more fruitful and more usable. Shin provided a diagrammatic syntax and semantics for her modified system Venn-I. She proved that if we manipulate Venn diagrams following the transformation rules that the diagrammatic method wouldn't mislead; dispelling the second worry through rigour. Venn-I has expressive limitations, with respect to disjunctives. Shin developed Venn-II to accommodate these limitations, again providing syntax, semantics and soundness and completeness proofs. Venn-II is equivalent to a monadic first-order logic (Shin 1994, p. 141). We see that both of the worries for diagrammatic reasoning were not borne out. Shin's languages were as expressive and diagrammatic reasoning provides different insights and inferences than linguistic reasoning (Allwein and Barwise 1996, p. vii).

Although historically *The Elements* was seen as the pinacle of rigorous reasoning, in the past century this view has been inverted, due to the gaps in Euclid's proofs (Mumma 2010, p. 256). The main problems with *The Elements* arise because some of his proofs rely on diagrams and not the underlying theorems. This is viewed as a deductive gap in his reasoning, dethroning *The Elements* as a paradigmatic example of mathematical rigour. There was, however, a consistency in his use of

[10]Thanks to Jessica Olsen for this simple, and elegant, example.

diagrams as proof. Ken Manders analysed the proofs and found a regularity in the way that Euclid used diagrams in these problematic proofs. Manders defined the notions of exact and co-exact properties to highlight this regularity. Exact properties are those that are affected by the slightest variation in a diagram, say the length of a line segment. Co-exact properties are those that are unaffected by some range of every continuous variation of the diagram (Mumma 2010, p. 264). Manders showed that Euclid's proofs which rely on his diagrams only did so through co-exact properties.

In his PhD thesis, and subsequent papers, John Mumma produced a diagrammatic formal system, **Eu** which showed how this dependence on only co-exact properties was consistent and could be formalised in a heterogeneous system (Mumma 2010), (Avigad, Dean, and Mumma 2009).[11] **Eu** successfully formalised the consistent reasoning that underlie the gaps of reasoning present in *The Elements*. This was achieved through a heterogeneous formal system. The expressive power of both sentence and diagrams combined to represent the missing links from Euclid's arguments. In **Eu**, diagrams represent the various components of a complex mathematical system, allowing the mathematician to consider all these components in a single place and focus on the relevant parts to the proof at hand. The formal system allows us to see the consistency at the core of *The Elements*, while staying true to Euclid's proofs and justification for them, despite his presentation lacking the mathematics to rigorously do so. Diagrammatic logics can be expressive and are not intrinsically misleading. So long as we design the systems clearly, and consistently use them according to their design, they pose no more of a threat to mislead than the misuse of sentential systems. They can also lead to different insights and add clarity to interesting historical problems in mathematics, at least. Not limiting our toolkit unnecessarily will allow us to have more fruitful and usable formalizations.

6.3.3.2 Consequence and representation

Looking at the target of logical formalizations as a plurality of pre-theoretic domains is a view of logical consequence held by John Etchemendy. We will briefly present his theory and its underpinning of representation before discussing how this wider notion of representation easily fits within LaF.

Etchemendy's theory of logical consequence is based on the idea of heterogeneous representation. For him any language gives rise to a consequence relation. Moreover this is the case for any well-defined system of representation, whether linguistic or not (Etchemendy 2008, p. 283). This means that there are consequence relations for every type of chart, graph etc. He uses the example of a map of an area San Francisco and the claim "The Old San Francisco Mint is at the corner of Mission and Fifth Streets." From this he asks whether the following sentences are true.

1. The Old Mint is south of Chinatown.

2. The Old Mint is east of Golden Gate park.

[11] The possibility of such a system, not its details, are important here, though the reader is encouraged to pursue them in the cited works.

The provided map contains both Chinatown and the intersection of Mission and Fifth streets, but does not contain Golden Gate Park. Thus, only the first claim is a consequence of the address and the map. Etchemendy points out that the traditional Tarskian picture of consequence cannot readily deal with this, as it is unclear what would function as the logical constants in this form of reasoning. Yet, this form of reasoning is more prevalent than that of first-order logic in everyday life (Etchemendy 2008, p. 285). He calls this a representational account of model theory and logic. This is because the set-theoretic structures are seen as “full-fledged representations: models of the world" (Etchemendy 2008, p. 187).

Etchemendy claims that there are consequence relations all over, wherever we have well-defined systems and we reason about them. We can correspond this with the previous ideas of pre-theoretic consequence and formalizable. These different consequence relations of Etchemendy’s are informal, in the technical sense. They are clearly inference-bearing, so our target phenomena is within the domain of logic. There is an informal logic to maps, charts etc. But there is no logic, yet. With Etchemendy’s wide concept of logic, a target is formalizable if we can make a logic that models it. Etchemendy’s criteria of a well-defined system of representation and ours of inference-bearing give us exactly that for these types of system. Etchemendy’s belief that there are consequence relations in any representational system is only restricted under LaF in that, for us, the concepts, properties, and relations, being formalized must be inference-bearing within the context. Given his discussion, and examples, this seems to be the basis of his claim of the plethora of consequence relations.

Etchemendy’s view is very much in-line with both LaM and LaF; it adds the clarity of representational freedom. As logics are just formalizations, that is a specific structure of model, there is no need to relegate ourselves to a single mode of representation. The LaF theory of logic embraces representational freedom.

> “[It] is equally unimportant from the syntactical point of view, that, for instance, the symbol ‘and’ should be specifically a thing consisting of printers’ ink. If we agreed always to place a match upon the paper instead of that particular symbol, the formal structure of the language would remain unchanged." (Carnap 1937)

Carnap’s comment here serves double purposes. it reaffirms Carnap’s affinity with our approach, and that our choice of representation is just that, a choice. It helps in another way as well. We can wonder why we don’t use matchsticks instead of symbols in ink? The general answer is two-fold, doubly pragmatic. It would be very cumbersome and prone to a loss of information, due to e.g. jostling, to use matchsticks. Using matchsticks, in various orientations, would also be hard to interpret, especially when compared to symbols in ink. Our choice of representation is informed by usability, as well as fruitfulness.

Let us reconsider the case of the map. We could formalize the consequence relation of maps in a purely sentential formal system, however it would not be very coherent or usable. If we, instead, used a heterogeneous system, adding the requisite formal elements, we would have a map logic that would be more useful for its application. Just as we need to identify the core concepts and properties that we wish to formalize, in order to assure ourselves that we don’t idealise and abstract in ways that will impact our analysis, we too can identify the most usable fruitful modes of representation of these concepts and properties to aid said analysis.

The LaF theory captures the actual activities of the diagrammatic logician, as well as agreeing with their call to reject the linguistic bias and pursue logics with different, and new, representational modes. We can be content sailing on Carnap's 'boundless ocean' having crossed the Strait of Representation.

6.4 Conclusion

This chapter presented the overall thesis of the work: Logics are formalizations. LaF is a process-oriented view, focusing on initial input, informed by the concept of logic at hand, and the output. Logics formalize inference (what follows from what) within domains (contexts). The concept of logic informs this target, and the process of formalization comes with idealisations and abstractions. This concept of logic may preclude 'standard' notions, such as inference, from mapping over. This is fine, as the aim is to clear technical debates to ensure that the theoretical grounds are shared, to ensure a fruitful discourse. If there is a disagreement, we roll back and ensure the concepts are shared. This is Carnap's common interpretation. We discussed how different views of first-order and second-order logic fit into this logical theory. LaF is a theory that is Carnapian in flavour, and can be seen as similar in claim, but distinct in aim the LaM view of Cook and Shapiro; there is no 'one true logic' here. We also showed how new and innovative work of logicians, in paraconsistency, logical dynamics, and diagrammatic logics, fit well in the LaF picture. Thus LaF is not only conceptually open but has a sociological advantage of describing the work of actual logicians. It is clear that LaF is a pluralistic approach, but what type of logical pluralism, if any, is it? That is the central concern of the next chapter.

Chapter 7

Logical Nihilism

7.1 Introduction

> Logical nihilism is ... the limit on a spectrum which contains logical monism... and logical pluralism. (Russell 2017, p. 125)

Logical Nihilism, broadly construed, is the claim that *there is no correct logic*. We contrast this with the logical monist claim that *there is (exactly) one correct logic*, or the logical pluralist claim *there is more than one correct logic*. In order to understand the distinctions between monists, pluralists and others we need to be clear on what claims are being made when one says there is a correct logic, or logics.

We aim to not only understand such claims of correctness but present a novel account of correctness which will allow us to formulate the notions of logical monism, pluralism and nihilism more precisely than they currently are in the literature, clearing up confusions in the discourse along the way.

The central concern of section one is the notion of correctness. A claim of correctness can be unpacked into four sub-claims: the *conceptual, ontic, formal, and epistemic*. These are based on the desiderata we established in chapter three. In section two we will critique three existing accounts of logical nihilism, drawing on the account of logical correctness developed in section one. Section three introduces the COFE[1] framework of logical theories which is based on disambiguating claims of correctness versus pragmatic goodness and highlighting what level of sub-claim logical theories pivot on. With this framework in hand, we will repopulate the landscape of logical theories and show how certain popular pluralistic views are more in-line with logical nihilism than pluralisms once the distinction is properly understood; logics-as-formalization (LaF) framework being one of them. We will also give some evidence that the other nihilistic views would be amenable to both the COFE framework,

[1] Pronounced ‘coffee’.

and the LaF view as well. We close the paper off with a discussion of the 'ontic backdoor' for those compelled by LaF but uncomfortable with the ontic weight of logical nihilism.

7.2 Claims of correctness

In this section we will do some table-setting on the areas where correctness applies, and where it doesn't, before moving on to discuss the various sub-claims involved in making a claim of correctness of a logic. These sub-claims are: conceptual, ontic, formal and epistemic in nature.

Recall that logics, and formal systems in general, are brought about via two distinct processes: *construction* and *formalization*. When we construct a formal system we do it for the sake of studying that system or mathematical structure. Contrast this with formalization which is a process that starts with a target to represent, and some purpose of application in mind. This is sometimes described as the difference between pure and applied logics. The notion of correctness in the latter case makes sense; it can be interpreted as a correctness of either representation of the target, or correctness with respect to some purpose or application. In the former case it is not clear that there is any place for the notion of correctness. Graham Priest similarly notes that there can be no sense of rivalry in pure logics, which are just well-defined mathematical structures, as rivalry can only occur when there is an application being required of the logic(s) (Priest 2006a, p. 164).

To elucidate the difference between contructed formal systems, and those formalized, or applied, to a purpose, consider the case of geometries. Up until the 19th century Euclidean geometry was the only thing called 'geometry'. Of Euclid's initial postulates the parallel postulate was the least intuitive, and there was no independent proof of it.

In the 1820s Lobachevsky and Bolyai both created geometries which did not include this postulate; Bolyai by leaving it entirely out, and Lobachevsky by adding its negation instead. Through his work Lobachevsky showed a corredpondence between the trigonometry that resulted from his new geometry and standard trigonometry, ultimately arguing that any contradiction that could be found in his geometry would have a parallel in Euclidean geometry (Torretti 2016). Since then there have been applications for Lobachevsky geometry and this widened the scope for the development of an expansive set of non-euclidean geometries, however the initial inception of these geometries was pure, not applied.

These new geometries represented new, and novel, areas of mathematics to investigate. The idea of wrongness, or incorrectness, simply doesn't make sense for non-euclidean geometries until you say something like "this is the geometry of the real world". Otherwise they are just areas of mathematics one can investigate.[2] So, we are just concerned with logics as formalized, or applied, and can safely leave discussion of logics as constructions asides; logics as construction are not within the scope of any claims of correctness of logic.

[2] Torretti also notes that "after these nineteenth-century developments, philosophers who dream of a completely certain knowledge of right and wrong secured by logical inference from self-evident principles can no longer propose Euclidean geometry as an instance in which a similar goal has proved attainable." (Torretti 2016) Another example of the weakening of axiomatics as the grounding for formal systems discussed in chapter four.

To understand what logical nihilism entails we need to know what logical monism and pluralism are claiming. That is, we need to know what a claim of correctness entails. Logical monists and pluralists both make claims of correctness, though they are often stated in terms of the *one true logic* (OTL) like the claim below.

(7.1) Classical logic is the one true logic.

This is because logic is thought to be about validity universally: what follows from what in all areas. An argument is valid if it is impossible for the premises to be true but the conclusion false. The different ways we can read the terms in this definition give rise to differing views on validity, and subsequently logic and the true, or correct, logic. 'Such and such is the OTL' is the claim that it gets validity correct. We might think we have a line as to what the purpose at hand is, but as the term valid is just as widely interpreted and scoped as logic we see that the correctness and one true logic are basically interchangeable, and are indeed treated as so in the literature.

Before we can make clear claims of correctness we need to know what it is we are being correct about. While claims like (7.1) seem straightforward, they actually require other foundational assumptions in order to get off the ground. Claims of correctness of a logic involve four sub-claims. They are: conceptual, ontic, formal and 'epistemic'[3] claims. We will refer to the entire group as the COFE claims, which rely on each other. This reliance is one of conceptual priority, in order to understand an ontic claim, we first must understand the underlying conceptual claim.

The first sub-claim is the *conceptual* claim. Recall that the concept of logic at hand informs the purpose one purports logic to have. The concept is about the nature of logic, while the purpose is about the application of some formal system in light of said nature. Purpose being the boundary of correctness. Graham Priest notes that different mathematical systems can be used for different applications, or purposes, but they can have a *canonical purpose* (Priest 2006a, p. 165). In our above discussion of geometries, then, we can say the canonical purpose of geometry is physical geometry, the spatial relations of the real world. Thus, under that purpose, we have a notion of whether a geometry is correct or not. Priest offers the canonical application of logic as reasoning. While this reflects general definitions of logic, and our discussions previously in this work, it is conceptually vague. For instance, two candidates for the conceptual purpose of logic are natural language inference and logical realism. Both of these concepts of logic seem to fit Priests general descriptor, but it isn't clear or immediately obvious that they necessarily coincide. Logical realism is the theory that logic charts the rules of the world itself (Rush 2014, p. 13). The rules of logic are written into the fabric of the universe. Frege's realism is based in the idea of logic as a universal medium, and so it has a direct correspondence with the world. The aim of his logic was to spell out this universal medium. He thought that his *Begriffsschrift* was closely related to natural languages, but with all the ambiguities and unclarities removed. Frege's

[3] *Epistemic* is in quotes as it is an epistemic claim of sorts but not of the same category as the others, this will make sense in the following pages.

entire project of developing his formal logic was based on the idea that natural language was not able to be as precise in its very nature (Hintikka and Sandu 1994b, p. 280).

In order to move forward, we must first fix the concept of logic at hand, e.g. logical realism, natural language inference, etc. With that in hand then we can make our next claim. The *ontic* claim, towards correctness, is that there is such a thing that satisfies this purpose, and it exists.

To illustrate this, let's take the concept of logic to be about natural language inference. The ontic claim would then be that there is a system of rules of inference that govern natural language inference; there is a clear idea of what follows from what in natural language inference. Now consider the logical realist. From this concept there can either be a positive or a negative ontic claim. The positive claim is, given that logic is the rules of reasoning that exist in the ether, that logic does indeed exist. The rules of reasoning are out there in the fabric of reality. The negative claim would be that whilst logic is about the rules of reasoning of the universe, there are in fact no such rules to be found. This then could not be part of a claim of correctness as the formal and epistemic claims would fizzle without a positive foundation. This would be our first example of a logical nihilist. A nihilist at this level would stop and not need to move further and make the other claims. Those who claim correctness of a logic, however, move onward.

The third claim is *formal*; it is the same as the missing claim of the monist highlighted in chapter three. Put simply, it is the claim that a mathematical structure of a certain type fully captures the logic that exists, via the positive ontic claim. Up until now we have just established that there is a specific thing, that is logic, which exists; in order for some formal system to be correct for its (canonical) purpose it must be of the appropriate class of systems. Returning to our example of geometry, if the canonical purpose is the physical/spatial relations of the universe (conceptual), and we think there is such a thing (ontic), then the claim is that a specific class of mathematical structures could be correct for this purpose. Basic decimal arithmetic fails to be the right type of structure for many reasons, e.g. not having spatial concepts at all. A negative claim here would be that there is no mathematical (formal) structure which can totally capture the spatial relations of the universe.

The final sub-claim we call 'epistemic'. Given the above, that mathematical systems of this type are appropriate to fully capture the existing logic which satisfies the purpose, the claim is roughly that 'I know that it is this particular formal system.' Some might baulk at a strong epistemic claim, and instead e.g. want to make this claim doxastically, hence the presented weakness to the epistemic claim. We put it in in scare quotes because it is a different sort of claim than the others. It is meant to be read weakly, as we do not want to restrict views based on a strong epistemic requirement. For example, if we required a strong epistemic claim, many logical realists would fail to pass this gate. It is a claim about the known, rather than the knowable. The epistemic claim is that of championing a token formal system of the type established in the previous claim. It is simply the last step in order to establish a claim of correctness and mostly reflects the surface level of the initial claim of correctness, like in claim (7.1).

Without all four of these sub-claims a successful claim of correctness cannot get off of the ground. It is unclear what is being said without addressing these four issues, so the claim can not easily be understood or evaluated. It is important to keep these layers in mind as we look at the treatment of the various theories of logic.

We note that the four sub-claims fall into natural pairings, which correspond to the two layers of discourse we highlighted in chapter three: the theoretical and technical layers. Take the conceptual and ontic sub-claims. The ontic claim directly falls out of a clear conceptual claim about logic. These are on the theoretical layer. As an example, if logic is about natural language inference, then a positive ontic claim seems inevitable. The formal and epistemic are similarly paired. As most claims of correctness are made via a champion logic (epistemic), the type of structure (formal) is presumed. The classical monist says it is the one true logic under the presumption that mathematical structures of logics are the right type. We will, following chapter two, call this the technical layer where it will not be an issue.

7.2.1 Principle of generality

The principle of generality comes up in the monist-pluralist debate, as a response to the pluralists' claim of correctness. Logical pluralists make the claim that certain logics are correct for different cases, contexts, languages, logical constants etc. The monist response to this is to appeal to the principle of generality. Consider the logical pluralism of J. C. Beall & Greg Restall. Their pluralism is based on the *Generalised Tarski Thesis*.

> **GTT**: An argument is valid$_x$ iff in every case$_x$ in which the premises are true, the conclusion is true (Beall and Restall 2006, p. 29).

There are multiple acceptable extensions of *case* which pick out different senses of *valid*. Cases can be read as possible worlds, mathematical constructions, situations etc. Thus, the correct logic will vary between classical logic, intuitionistic logic, and relevance logic respectively. So, validity shifts with the *case* at hand.

The monist then responds, appealing to the principle of generality, that a logic must hold for all cases. The pluralist replies with the following argument.[4]

$$\frac{\begin{array}{c}\text{To be a logic, it must hold in complete generality}\\ \text{No logics hold in complete generality}\end{array}}{\text{There are no laws of logic}}$$

The pluralist's intent is not an argument for logical nihilism. Both parties are operating under the assumption that logical nihilism is an absurd possibility. Thus, there must be a problem with the principle of generality, or with the pluralist stance. Following Russell, once we make way for logical nihilism as a non-absurd theory, this becomes a substantive argument for it, provided by the debate of its opponents.

Before moving on, we should caution on the reading of the principle of generality. There are two main ways of reading this principle, the pre-conceptual and the post-conceptual. That is, we can invoke

[4] This is adapted from the presentation of the argument by Gillian Russell, she presents it in terms of logical laws, for our purposes we paraphrase without changing the argument by writing logics instead (Russell 2018, p. 308).

the principle of generality before we establish the concept of logic that we are using, and thus the canonical application, or we can invoke it afterwards. On the one hand, if we invoke it afterwards, then the generality is bound by the purpose at hand. Thus, if logic is about natural language inference then the principle of generality is bound by *all cases in natural language* and is silent on any other case. Meaning that any argument from, say, mathematical reasoning, does not apply against said generality. On the other hand, if we invoke it before establishing the concept of logic then there are no such boundaries. This is at quite the cost however. If there is no established notion of logic then it seems that the principle of generality is doomed to be untenable. We can easily come up with toy examples for any logic, or logical law, as there is nothing to ground against such moves. This is because we have moved away from applied logics and into the territory of pure logics, which can have no rivalries.

Consider Prior's *tonk* connective, introduced in "The Runabout Inference-Ticket". The meaning of *tonk* is given by the inference rule that given any statement P we can infer any statement, Q, by first joining them by *tonk* and then eliminating *tonk* leaving us with Q.[5] Here is Prior's example:[6]

$$\frac{\begin{array}{c}\text{2 and 2 are 4}\\ \text{(2 and 2 are 4) } \textit{tonk} \text{ (2 and 2 are 5)}\end{array}}{\text{2 and 2 are 5}}$$

A language with *tonk* in it can take us from any true proposition to the truth of any other true proposition. This seems an undesirable property for a logic to have, but there is nothing inherently *incorrect* with it. What makes *tonk* problematic is that there is no correspondence with our natural language, or the way we think reasoning works, etc. Without a settled notion of what logic is about there does not seem to be any reason why such a logic shouldn't be on the table, except perhaps that it is uninteresting, or not very useful. Applying the principle of generality would rid of us tonk-logics, but we can ask the same questions of non-tonk logics leaving us with nary a logic to be found, so the argument goes.

The next step is then to show that logical nihilism is not absurd. We will first deal with an initial response to logical nihilism, the undermining argument, and then treat three views on logical nihilism in turn. We will discuss the views, presenting the main arguments to give the reader a flavour for the picture being drawn, following up with a critique given the table-setting we have already done.

7.3 Current nihilistic views

In this section we will start with a general argument levied against logical nihilists and then cover the three views of logical nihilism that currently are in the literature, those of Gillian Russell, Curtis Franks, and A. J. Cotnoir. They each approach logical nihilism from different angles but, as we shall see, they

[5] To be clear to the formally inclined, the *tonk* connective has the disjunction introduction rule paired with the conjunction elimination rule (Prior 1960, p. 39).

[6] The argument is presented with emphasis and mild stylistic paraphrasing.

do not fully, or clearly, engage at the theoretical layer, and so the views are lacking in certain key ways. These are similar to the ways highlighted and discussed of the monists in chapter three.

A. J. Cotnoir identifies two type of logical nihilism, which will be useful in the following discussion. We paraphrase for generality using *logic* to stand in for the fixed concept of logic at hand.[7]

LN1 There's *no* logical consequence relation that correctly represents *logic*; formal logics are inadequate to capture informal inference.

LN2 There are no logical constraints on *logic*; there are always counterexamples to any purportedly valid forms. (Cotnoir 2018, p. 303)

If we assume logic is about natural language inference, **LN1** yields a negative response to the formal claim, mathematical structures of the type logic do not, and cannot, fully capture the notion of logic that is in natural language. On the other hand we have **LN2** which aims deeper. If there are no logical constraints on natural language inference, then this is a denial at the ontic level, similar to the one pointed out in the previous section. The reason both focus on formal properties is straightforward: logics, and our choice of them, exist on the formal level. This does not mean that they will share the commitments on the lower levels, and we should aim to be clear on where they sit on these levels to properly understand and contrast them.

7.3.1 The undermining argument

The undermining argument is a first response to the claim that there is no correct logic; as such, all of the logical nihilists deal with it before even presenting the details of their views. The argument is straightforward: 'In your case of promoting logical nihilism you must use logic, thus you undermine yourself.'

All of the responses take the same general form. Arguments for logical nihilism are not undermining, as they are disputing the generality of logic, or logical rules. That is, it is perfectly consistent with logical nihilism to say that particular, and indeed most, instances of certain argument forms are unproblematic, without accepting that they are general (universal). Taking **LN1** the response is clearer. The claim is that no formal language perfectly captures the standards of good inference of natural language, and that is compatible with the argument forms being valid in some regimented language for a restricted purpose (Cotnoir 2018, p. 304). Classical logic might preserve natural language validity in many cases, but it could fail somewhere else, thus negating the claim of generality. Russell supports this with the analogy of the law of excluded middle and the intuitionists. The intuitionists deny the generality of the law of excluded middle because of problems with infinity, but they accept with no problems the use of it when not dealing with statements of infinite collections (Russell 2017, p. 127). The logical nihilist is making a similar move with respect to other laws, if they are a **LN2** nihilist, or logics themselves if they are **LN1**. Although they are presented as two options, the picture is more complicated. Given that a

[7] Cotnoir assumes logic is about natural language inference, and thus presents **LN1** and **LN2** directly in terms of natural language inference, which we will address in the discussion of his framework.

claim of correctness contains four sub-claims, one could be a nihilist at any of the levels. An epistemic logical nihilist, for example, may deny (or be agnostic about) the inference rules that apply to a domain, but claim that we cannot know about, or justifiably believe, said rules. This sort of nihilist will have a tougher time dealing with the undermining argument, due to their epistemic stance.

Here we note the structural similarity of the undermining argument to an argument against the logical pluralists, namely the problem of the metalogic. If there is more than one correct logic, but you are making your argument towards that goal using one of those logics, then surely that is the correct logic overriding the others. Graham Priest and Stephen Read bring such arguments against the pluralism of Beall and Restall (Priest 2006a, p. 203), (Read 2006, p. 193).

The response here is similar: the objection is based on a misunderstanding of the framework. Beall and Restall reply that "[t]he mere fact that a single logic has a particular role to play (in a particular context) does not mean that other things are not also logic... The pluralist claim is that, given a body of informal reasoning (that is, reasoning not produced in a particular system of logic), you can use different consequence relations in order to analyse the reasoning. As to which relation we wish our own reasoning to be evaluated by, we are happy to say: any and all (admissible) ones!" (Beall and Restall 2006, p. 99) There is a misunderstanding of implied hierarchy by the monist of the pluralist's position.

Similarly the nihilist can point to a misunderstanding on the part of their detractors. The undermining argument is based on over-interpreting the denial of the principle of generality. The argument seems to assume that the nihilist is denying the existence/availability of any logic, or argument. The denial of the principle of generality, however, merely claims that there is no universal logic, or set of argument forms. There can be very useful arguments which work in most contexts most of the time. And so the undermining argument fails to go through when this conflation is cleared up.

Now we will briefly discuss the available views of logical nihilism in the literature, starting with that of Gillian Russell, then moving on to Curtis Franks and then A. J. Cotnoir.

7.3.2 Russell

Russell's approach to logical nihilism is to focus on *logical laws*, making it an example **LN2**. The Russellian nihilist gives up on all laws, not just the controversial ones, such as the law of excluded middle, even *modus ponens*, the law of non-contradiction, and conjunction introduction are not spared. There are no universal laws of reasoning. She starts with the argument from generality discussed above, which we paraphrased to be about logics generally, but here it is in Russell's original form:

$$\frac{\text{To be a law of logic, it must hold in complete generality} \qquad \text{No laws of logic hold in complete generality}}{\text{There are no laws of logic}}$$

We should first note that by focussing on logical laws we are focussing on the formal, and either not engaging with, or presupposing the theoretical layer.

To understand this claim we need to understand what a logical law is. Russell discusses various candidate options of what a logical law is, but ultimately settles on laws being of the following form $\Gamma \models \varphi$ (Russell 2018, p. 311).[8]

We can now interpret the claim of logical nihilism as no matter what Γ and φ are $\Gamma \not\models \varphi$ (Russell 2017, p. 126). Another way of understanding this is that the extension of the logical consequence relation is empty. To properly understand that claim we need an idea of what the turnstile means. Russell offers three candidates, Beall & Restall's cases, Tarskian interpretations, and Williamson's universalist view. She adopts the standard Tarskian approach but the arguments don't pivot on this move. So, $\Gamma \models \varphi$ iff whatever (syntactically appropriate) interpretation is given to the non-logical expressions in Γ and φ, if every member of Γ is true, then so is φ. We can now state her logical nihilism as the view that for every argument of the form $\Gamma \models \varphi$, there are interpretations of the non-logical expressions in Γ and φ which would make every member of Γ true but, φ not true (Russell 2018, p. 312).

We will discuss two of Russell's positive arguments logical nihilism, the arguments from truth-values and from context-sensitivity.

7.3.2.1 Argument from truth-values

The first positive argument that Russell offers in favour of logical nihilism starts with the following observation.

Consider the argument form of affirming the consequent (AC).

(AC) $$\varphi \to \psi, \psi \models \varphi$$

This is normally considered as an invalid argument, as it can be the case that the premises are true, but the conclusion is false. For example when φ is false and ψ is true, that would make both premises true but the conclusion false. This is why affirming the consequent is not considered a valid law, or argument form. Russell invites us to think of the following scenario, if we artificially restrict the substitution class of sentences to only true sentences. Then, as both φ and ψ must be true, there is no interpretation, no matter what values we plug in to φ and ψ, where the premises will be true but the conclusion false. In this scenario, the way to avoid thinking of affirming the consequent as a logical law is to make sure that our interpretations include both the values for true and false; Russell notes that this is well motivated as there are many sentences which are indeed false (Russell 2018, p. 313).

She then asks if this lifting of restrictions is enough, take the law of excluded middle (LEM): $\models \varphi \vee \neg\varphi$. Given that the previous case, of affirming the consequent, was problematic because we did not have access to enough truth-values, we needed both true and false, we can then ask whether LEM only appears like a logical law due to a restricted set of truth-values. "Many philosophers think that

[8]Russell also considers laws which 'state connections between different claims of logical consequence, e.g. If $\Gamma \models \psi$ then $\Gamma, \varphi \models \psi$ (thinning). There seems to be an assumption about laws in the meta-language versus the object language not being a problem. This is problematic given the definition of logical nihilism at play is in the object language.

some sentences are neither true nor false, perhaps for reasons of reference failure, or vagueness, or because they concern future contingents" (Russell 2018, p. 314). Russell points out that sentences of this type still feature in arguments where we would want to assess for validity. So we can add this new truth-value, *N* (neither), to our interpretations. For the sake of argument, she assumes the Strong Kleene interpretation of the connectives that reflect the new truth-value. This leads to the a counterexample of LEM, an interpretation on which it is not true (Russell 2018, p. 314).

φ	φ	$\vee$	$\neg\varphi$
T	T	T	T
F	F	T	T
N	N	N	N

While Russell continues with her argumentation, we should pause here to note the candidacy of LEM as a law. Recall Russell's definition of a logical law as $\Gamma \models \varphi$, and the claim of logical nihilism as for any set of premises Γ and conclusion φ whatsoever, there is an interpretation in which every member of Γ is true, but φ is not. Here we have no premises so Γ must be $\varnothing$, to be more precise LEM is $\varnothing \models \varphi \vee \neg\varphi$. For the nihilistic claim to go through we need it to be the case that empty set is true in the last interpretation that gives us $\varphi \vee \neg\varphi$ as N. It is not clear why this must be the case. We could just stipulate that the empty set reads out as vacuously true, but we could equally stipulate it false, or more plausibly N. As Russell has not given any conceptual grounding we don't have anything that would lead us to the choice of the truth-value of the emptyset as true, without begging the question. Without this clarity it seems that any statement of the form $\varnothing \models \beta$ will come out as a logical law, as there is no instance of true premises and a false conclusion, including contradictions. It might seem that this is an odd problem to bring up as logical theorems are given in the form $\models \phi$ and we don't ask this question of the status of the empty set. This is because we define tautologies via a negative existential claim. There is no interpretation where the premises are true but the conclusion is false. So the status of the empty set is not needed. Russell's claim of logical nihilism is a positive existential claim, there is an interpretation where the premises are true but the conclusion is false, and so we need that interpretation to have true premises. Despite this problem, let us continue following Russell's line of argumentation.

Strong Kleene logicians are not nihilists, as they still have logical laws, e.g. *modus ponens* (MP) and disjunctive syllogism (DS). Just as some logicians are motivated into thinking there can be sentences/propositions with neither truth value, there are some which think that they can have *both*, namely the dialetheists. They are primarily motivated against DS, or the law of explosion: from any contradiction you can derive anything $\varphi, \neg\varphi \models \psi$. Adding *both* to our set of truth-values yields counterexamples to both DS and MP, with the conclusion being *false* and the premises *both*. Again we have to pause to unpack the claim of a counterexample being present. A counterexample to a logical law has to take the form of an interpretation where the premises, Γ, are true and the conclusion φ as not true. We clearly have the latter as the interpretations at hand have φ as false. The truth-value of Γ is *both*, however. We note here that both is interpreted as 'true and false' in the metalanguage, but in the object language it just represents another possible truth-value. As the concept of nihilism has been defined in terms of truth in the object language, it is not immediately clear that either of these will count as counterexamples given the present definition. The premises are not strictly true, as needed to be counted as counterexamples.[9]

Adding *neither* and *both* leaves us with the logic of first degree entailment, which still has some universal inference rules and thus is weak but not nihilism. Rather than show counters to every rule of first-degree entailment, she aims at the most widely accepted rules: conjunction introduction $\wedge E$ and identity ID.

7.3.2.2 Argument from context-sensitivity

The argument from context-sensitivity stems from the straightforward observation that natural language is context sensitive.[10] Logicians can deal with context-sensitivity in one of three ways: i) they can ignore it ii) stipulate a fixed context iii) if interested turn context-sensitive elements into logical constants. Given that context-sensitivity is widespread, both the ignoring and converting routes to constants seem like suboptimal paths, for expressive and practical reasons respectively. Russell takes the second path, invoking a special kind of context-sensitivity which is sensitive to the linguistic context in which it is in (Russell 2018, p. 315). Consider the sentence SOLO, which is always true so long as it appears alone as soon as it is part of a larger construction it becomes false, in the following argument.

$$\frac{\begin{array}{c}\text{SOLO}\\ \text{Snow is white}\end{array}}{\text{SOLO} \wedge \text{Snow is white}}$$

Here we have an argument which is of the standard form of $\wedge E$, valid in all the logics discussed so far, but where the premises are true and the conclusion false, as the left conjunct (SOLO) will be false rendering the entire conjunction also false.. This is a clear counterexample to $\wedge E$'s status as a logical law. Using the linguistically context-sensitive sentence PREM, which is true so long as it is in the premise of an argument but false otherwise, we can generate a counterexample to the law of identity. Here we don't bump into the problem from the definition of logical nihilism, but the argument from context-sensitivity finds trouble from a different corner.

7.3.2.3 Problems with Russell's view

As we noted at the outset Russell glosses over the conceptual and ontic levels, and argues firmly from the the formal level. It is this initial move that will cause troubles for her.

[9]Both of these problems are discussed with respect to *interpretations*, but as *cases* and the universalist view also revolve around nihilism being defined in terms of an instance (of the appropriate type) true premises and false conclusion the arguments will go through in the same manner for those views on the turnstile.

[10]Here is the first, and only, nod to what Russell takes to be the coneptual and ontic levels; that it is at least heavily informed by natural language.

Without fixing the conceptual level we end up with both floating goalposts and a lack of conceptual boundaries. The general approach of Russell's dialectic is to invoke any technical (formal) move that a logician may take as universal, presumably under the guise of the fight against the principle of generality.

As we saw above, until we fix the concept of logic, and thus the purpose at hand, the principle of generality is trivially false; we can concoct any sort of toy example that will be formally true if we are not bound in application/representation of a purpose. Without this notion it is not clear what work these example are doing, if any. We are in danger of just tonking around.

Russell does invoke natural language in her introduction of context-sensitivity, though it seems it is part of the dialectic of generality. If we suppose that it is a stronger claim that logic is about, or at least informed by, natural language, then we are restricted from the views of the intuitionists and Frege himself, who created his logic precisely because of the deficiencies of natural language, because of issues such as vagueness. Additionally if we are looking to natural language as our conceptual motivation for context-sensitivity then we need to continue to do so when we invoke the notion of linguistic context sensitivity. If there is no 'real-world' analogue to these notions of SOLO and PREM, then it sits as a mere technical artefact that we *could* introduce into a formal system.

Returning to the argument from truth-values we can continue the line of argumentation and ask: why should we restrict our set of truth-values to true, false, neither and both? Why not a fifth value of *both and neither*? Recall that the motivation for moving to neither and both was that some logicians appear to make such arguments. Presumably one could make such an argument for 'gappy-gluts' and then we would be motivated to move to a five-valued logic.[11] Similarly we could invoke the view that takes the truth-value of the empty set to be neither true nor false. With that in hand it would seem that we would be forced to then suddenly have some counterexamples disappear and new logical laws emerge as a result, such as LEM.

By focussing on logical laws, on the formal level, any claim is allowed via the generality requirement. This approach leads to a misreading of the moves she invokes as it glosses over the conceptual differences and ontic grounding the various logicians propose in favour of their frameworks. The monist's claim is deeper than generality and the classical logical monist, for example, would block the move to both Strong Kleene and First-degree entailment on the grounds that logic is strictly bivalent. Similarly, while both the Strong Kleene and the dialethiest deny the principle of bivalence, they specifically argue for the one third value against the other. In fact, the formal languages of the two are the same, it is only the interpretation of the third-value in the metalanguage which shifts between the views. It is not clear who would buy both moves except those who are already proponents of first-degree entailment. All of these logicians have conceptual reasons for their positive and negative claims. By ignoring this layer it is not clear what exactly the task at hand is. This line of argumentation presupposes the global notion of the principle of generality, which is both clearly false and also

[11] Such a possibility may seem far fetched but all we need is to state that it is possible that there is a sentence which is neither true-or-false, and also both true-and false. Given the lack of conceptual grounding of logic the current argument is taking place, this seems unproblematic to assert.

conceptually uninteresting. By not engaging with the theoretical layer there is no space for rivalry between logics, as we are dealing with constructed logics, not logics from formalization, or application.

As the various logicians who support the moves that are taken have different concepts of logic in mind when they make their individual arguments, we must conclude that the floating conceptual goalposts are intended, and thus so to the lack of conceptual boundaries which follow.

Combining our problems with the two arguments, we can see that absent a clear notion of what logic is (canonically) about there doesn't seem to be a clear problem with the following. Consider the following linguistically context-sensitive sentence TPFC. TPFC is the sentence which makes any sentence it is a part of that is a premise true (by adopting the appropriate truth-value) and when any sentence it is a part of is a conclusion it makes that sentence false. Now we can concoct a counterexample to any purported law of logic along the same lines as presented above. The conceptual grounding is the same as SOLO and PREM, as it is just in the bare notion of natural language being context-sensitive and the idea of linguistic context-sensitivity.

Russell uses these lines of argumentation along with Lakatosian lemma incorporation to argue against logical nihilism, but for the abandonment of generality.[12] For the example of LEM she notes that classical logic gives a proof of LEM given the assumption of bivalence. When using Strong-Kleene we have a 'counterexample' with the sentences being assigned neither. Russell concludes that the rule we should embrace then is:

(7.2) $$\text{For all bivalent } \varphi, \models \varphi \vee \neg\varphi.$$

Now we have incorporated the violated lemma into a logical law such that the counterexample no longer applies. Here we have a conceptual clash however. The intuitionists are proponents of the principle of bivalence, but are against the law of excluded middle as a general law, as they do not believe it should apply to infinite collections. It seems that Russell has disbarred this objection from even being able to be levied. By ignoring the conceptual motivations there is no space for the disagreement to be voiced. We seem to be meta-monster barring. Even if we grant the disagreement we are left with two options. It seems as Lakatosians we must incorporate a lemma to diffuse the argument. We can grant the problem of infinite collections wholesale and end up with something along the lines of

(7.3) $$\text{For all finite collections and bivalent } \varphi, \models \varphi \vee \neg\varphi$$

But this is just granting the intuitionist stance over and above the classicalist, which seems counter to the open approach espoused by Russell. Our other option is to more clearly shift the context and say that for classicalists we keep the original statement, and for intuitionists we adopt this new one. At

[12]Lakatos introduces a class learning geometrey with two key students, Gamma and Delta, as well as the teacher. A counterexample is found to a theorem the class was using. Gamma suggests they disallow that which proves the theorem wrong, monster-barring, in an effort to preserve the theorem. Delta takes the opposite approach, there is a counterexample so the rule should be thrown out entirely. Lakatos' teacher offers a middle ground solution: incorporate a lemma that acknowledges the troublesome counterexamples, limiting the theorem to those that are not problematic, explicitly acknowledging the problematic class of counterexamples while preserving the mathematical knowledge contained in the theorem.

this stage we seem to be evading the problem and there is no space for rivalry between logics. The argument is given while sticking to the formal level, but the theoretical layer is needed.

Ultimately, Russell advocates for the abandonment of generality, but throws out logical nihilism based on a reading of logical laws which is too narrow. The final line of argumentation is on conceptual grounds, but it does not have the previous grounding to make sense of such a move.

This argument is against global nihilism, but rather than advocate against nihilism once the concept is fixed, it ends up being an argument for it. Once a clear concept of logic is established, we see how lemma incorporation makes sense. The previous moves were based on a global sense of what some logicians do technically, while eliding the conceptual grounds for the formal moves made. Russell ends up presenting the strawman logical nihilism and attains a hollow victory due to the lack of conceptual boundaries on logic and the resulting 'anything-goes' approach that follows. This shows a large problem for anyone taking the **LN2** approach. The theoretical layer must be clearly established before formal matters, like logical laws, can be discussed fruitfully. Otherwise one ends up playing in the field of constructed logics, an area where correctness is not a coherent type of claim.

7.3.3 Franks

Curtis Franks' logical nihilism piece comes at the issue from two different directions, structural interest and the presuppositions involved in the idea of correctness. His approach is more historical and discursive, and so does not easily fit into either of our nihilistic categories, **LN1** and **LN2**.

The first direction concerns what we can learn by looking at the structural differences between logics, and how small formal changes can result in large logical ones. He uses Gentzen's observation, as an example, that the sequent calculus of classical propositional calculus can be modified into the intuitionistic sequent calculus by simply, and only, disallowing multiple-clause succedents (Franks 2015, p. 149). We can safely discard this line of argumentation as it about the study of mathematical structures and what we can learn about them as structures; its about pure logics and constructions. Perhaps more clearly, this is about the interesting relationships between mathematical structures of a certain type and thus has no bearing on any discussion of correctness. This is exemplified by his claim that "[t]he logician is loath to choose between classical and intuitionistic logic because the phenomena of greatest interest are the relationships between these logical systems" (Franks 2015, p. 154). These are the words of a mathematical logician interested in both the structures and the relations between the structures, without recourse to anything external to them.

The second direction is to question the point which unites the monists and pluralists "the idea that one thing a logical investigation might do is adhere to a relation of consequence that is 'out there in the world,' legislating norms of rational inference, or persisting some otherwise independently of our logical investigations themselves" (Franks 2015, p. 148). This is a rejection at the ontic level.

Franks makes his point by pointing to the presupposition that is present in the idea of correctness. That many simply assume that there must be patterns of reasoning that differ in kind to others, and that typically our minds are already made up with regards to the psychological and ontological circumstances which make up this difference (Franks 2015, p. 159). He asks why should anyone assume that there is a commonality to the reasoning which we employ outside of the fact that we so employ them.

Franks has rightly targeted that claims of correctness can only result from assumptions on the theoretical layer. We are driven by this presupposition looking for an extensional definition which we can apply to it, but we can only succeed in convincing those who already have such a presupposition. That is, it only makes sense to those who have already made a positive ontic claim. Franks also points out that the conception, and form of logic, has changed over time, and there is no reason to think it has stopped now, the approach mirrors the discussion in chapter two (Franks 2015, p. 159).

Despite coming from the point of view of mathematical logic, and thus being less concerned with application and correctness we can take this latter set of arguments on board as there is an overlap between his claims of interestingness and pragmatics. Franks seems to be speaking against a general conception of correctness, as he is ultimately concerned with logics as structures and so it seems his nihilism should be unsurprising. This set of points about the evolving nature of the field and presuppositions are ones which are equally applicable to more focussed, and applied, notions of logic and so should be taken on board regardless of their initial intent.

> The richness of logic comes into view only when we stop looking for such an essence and focus instead on the accumulation of applications and conceptual changes that have made current logical investigations possible. The study of logic might be the best practical antidote to the view of it that we have inherited. (Franks 2015, p. 164)

7.3.4 Cotnoir

In his paper "Logical Nihilism", A. J. Cotnoir acknowledges the gap between pure and applied logics, with respect to the question of correctness. He identifies, following Beall and Restall's pluralism, this gap as logic being a tool of analysis of the inferential relationships between premises and conclusions in arguments that we actually employ. That is, logic is about natural language inference. He leaves space for the normative nature of logic to balance out this descriptive goal, pointing to the minimum notion of correctness being bound in some way by actual inferential practice. So, this time we do have a fixed concept of logic and one that has a clear positive answer to the ontic question as well.

His focus is on his **LN1**: there is *no* logical consequence relation that correctly represents natural language inference; formal logics are inadequate to capture informal inference. He notes that this merely claims that there is no formal theory which perfectly captures the standard of natural language inference. There is a gap between the ontic and the formal; so he is giving a negative formal claim paired with his positive ontic claim.

We will discuss two of his claims which showcase this gap, though he gives more. These are arguments from expressive limitations: semantic closure and vagueness.

7.3.4.1 Expressive limitation argument

One of Cotnoir's arguments from expressive limitation of logic versus natural language is that of semantic closure. The argument is based on the (alleged) fact that there is a fundamental difference between formal and natural languages. Natural languages are able to represent their own semantics; all the truths of English can be stated in English. Formal languages cannot be supplemented with semantic notions without then becoming trivial. The argument is as follows.

$$\frac{\begin{array}{c}\text{Natural languages are semantically closed}\\ \text{No formal language is semantically closed}\end{array}}{\text{So, no formal language is adequate to natural languages}}$$

There are many purported solutions to this problem but the more successful ones seem to lead to such a weakening of the logical consequence relation that it no longer resembles the patterns of natural language (Cotnoir 2018, p. 313). By trying to solve this problem we end up losing most of the expressivity of the logic that made it seem that it could capture natural language inference in the first place. So, formal and natural languages are fundamentally different and this difference means the aim to fully capture natural language inference within a formal language is doomed to fail.

The second of Cotnoir's arguments is the problem of vagueness. He invokes Dummett and Eklund's claims that the use of vague predicates in natural language is inherently inconsistent. If there can be no coherent logic of vague expressions then we have a straightforward argument for logical nihilism, for natural language inference (Cotnoir 2018, p. 317). Put another way, it is due to the precision of formal languages that we are at loggerheads with capturing natural language inference. Vagueness in its nature is a type of imprecission, so only imprecise languages will be able to deal with vague predicates. We again are confronted with the spectre of Frege, who saw this same problem as his motivation for creating his logic.

7.3.4.2 Problems with Cotnoir's view

Cotnoir's arguments for the gap between formal and natural language inference are compelling.[13] Where we find fault in this is slightly ironic given our complaints on the previous candidates, Cotnoir's nihilism is one that only takes into account natural language inference. For those who think the concept is otherwise, e.g. Frege, his nihilism has nothing to say to their monism or pluralism. The previous two views seemed to avoid the theoretical layer and only involve themselves on the technical, this led to a gap as the latter is dependant on the former. Here Cotnoir has swung on the pendulum too far (for our tastes), and has addressed the theoretical in a very narrow fashion. While there is nothing wrong with this move inherently, the concept must be fixed and the ontic question addressed, it would be advantageous to be able to accommodate a variety of views with a framework of logic. Cotnoir's approach gives us a good starting point to present our view of logical nihilism.

[13] The detailed presentation of which is worth seeking out in (Cotnoir 2018).

7.4 The COFE framework of logic

The main goal of this section is to establish a flexible framework of logical theories, given the four sub-claims involved in claims of correctness. The first step of this is to clarify our notion of correctness, making a distinction between the ontic and the pragmatic. Following that we will present the COFE framework of logical monism pluralism and nihilism, building on that of Cotnoir, and then proceed to fit the familiar theories of logic within that framework including logic-as-formalization (LAF) presented in the preceding chapter.

7.4.1 Clarifying correctness

We started this chapter with a quote referring to the spectrum of views to which logical nihilism is a part of, this spectrum is one of *correctness*. This notion seems to clearly accommodate the three types of theory: logical monism, logical pluralism, and logical nihilism. The aim of such a spectrum is to carve up the landscape of theories of logic in to categories based on their stance on the cardinality of their claims of correct logics. We have yet to really pin down what correctness means, that is the main aim of this section.

The picture, sadly, is not so clear as the spectrum makes it out to be. The logical pluralism literature is a polluted landscape with many different views which share little in common, but all are classed as logical pluralisms. One of the reasons for this, is that people are making their pluralist claims at different levels inolved with correctness. There are multiple ways of being a logical pluralist and each way interprets the initial statement in different ways. Broadly these pivot on what *logic* and *correct* mean in the above statements.[14] These different ways of pivoting to pluralism imply similar views of monism, pluralism, and nihilism, a plurality of spectra if you will.

The most important factor, for the matter at hand, is the notion of correctness that underlies the different views. We want to differentiate between two key, and distinct, notions of correctness. Correctness, in this context, is polysemous, however the discussions often treats it as if it had one clear universal meaning that everyone is using. The first meaning of correctness is the correctness of the monists. The second is the notion of correctness espoused by the self-described pragmatic pluralists, e.g. Cook, Shapiro, Kouri, and of LaF.

The correctness of the logical monist is an ontic correctness, as discussed in section 7.2. That is, given the conception of logic the monist has, they claim that such a logic exists and that it can be captured by a formal system of the requisite type, with the various monists champion logics given as the candidate for the correct logic. Let us return to the logical realists. Here we clearly have positive claims at the theoretical layer. A classical monist realist then is claiming that classical logic is the of right type and the correct token to capture the logic that is out there in the world.

Some may object to the use of ontic as a descriptor, as it could be seem to imply a realist nature to logic that may not be assented to. Here we are just claiming that there is an underlying ontic claim

[14] See (Cook 2000), (Russell 2019), (Shapiro 2014) for good surveys of the ways to be pluralists about logic.

of existence that is needed for a claim of this type of correctness. Borrowing from the moral realists, we view the ontic commitment of realists as *stance independent*. That is, there exist facts about logic, whatever the concept of logic at hand is, which are not dependent on the beliefs, or attitudes, of people (Shafer-Landau 1994, p. 335).[15]

Take someone similar to Cotnoir, who thinks that logic is about natural language inference, and who also makes a claim of correctness. This still contains the ontic claim that there is an underlying logic to natural language inference, though this is hardly surprising. It is still an ontological claim, but one which is contingent on how natural language inference works, develops, and possibly the natural language at hand.

For the second type of correctness the aim is different. Correct here means that it is useful, insightful, fruitful, etc. It is not a ontic correctness, nor does it imply that there is something to which we are being correct against. That is, regardless of the conceptual and ontic levels this type of claim of correctness is not one in which the formal system totally captures the notion of logic, whether it exists or not. The aim of formal logics are fundamentally different as compared to those of the first type.

Matti Eklund goes through the different ways of being a logical pluralist in "Making sense of logical pluralism", noting the various ways can interact resulting in (at least) eight varieties of pluralism (Eklund 2017, p. 6). One of these ways he calls *goodness pluralism*, it is "the view that even given some particular purpose, perhaps some canonical purpose, different languages serve that purpose equally well (Eklund 2017, p. 5). This notion of goodness is a pragmatic one, and maps directly to our second notion of correctness. As we want to distinguish between claims of the former from the latter let us adopt the moniker 'goodness' for the pragmatic notion of correct giving us terminological clarity between the two.

With goodness and correctness in hand we see that they are very different claims being made. Until now, theories of logic which speak of correctness could be speaking of goodness or correctness. The conflating of the two leads to disputes on the technical layer that do not make sense because of the fundamental difference on the theoretical layer. Put another way, the disputes must start deeper in order to fruitfully continue.

7.4.2 The framework

Instead of leaving it ambiguous as to whether theories are using correctness or goodness, we propose healing the fracture by redrawing the lines along only correctness and clearing up the ambiguity as we go along. This is more than just conceptual house-keeping, although that is reason enough to go through the exercise. By clearing up the conflation of the two notions of correctness we are creating a good framework for fruitful debate. The COFE framework places the sub-claims involved in the various views in the spotlight, so that we can properly evaluate them and see on what level they clash, or if they are actually just passing ships in the debates.

[15] Thanks to Geoff Keeling for pointing out this distinction.

We will briefly discuss how the major players fit into this revised framework. This approach will not affect the monist theories, for the above discussed reasons. Similarly on the other end of the spectrum claimed nihilists will likely remain so, the clarity lies in the middle ground, with the pluralists. We will discuss the pluralist theories of Rudolf Carnap, Beall & Restall, and logic-as-modelling (LaM).

Additionally we leave open the core concept of logic, only noting that in order to make a coherent statement about correctness we must first fix the concept at hand. It is in this way that we fundamentally differ from the logical nihilism of Cotnoir. As logic is MOT, different concepts will be used at the same point in time, as well as shift over time. We see no need to artificially limit the discussion surrounding logic and correctness to a single conception of logic, indeed we see that as detrimental. The advantage of this approach is in-line with the sociological advantage outlined in the previous chapter. By leaving the door open for all concepts of logic, the framework can accommodate any theory of logic and give clarity on where it sits with respect to correctness and what the commitments are at at the different levels of the framework.

We want to be able to float conceptually – pivot ontically; then, and only then, should we bother with technical matters when considering the questions of the nature and correctness of logic(s).

As we know from the discourse in metaphysics, we can be global nihilists or local nihilists. That is we could be nihilists about some domains, and not others. In the same way we can be nihilists under certain conceptions of logic, and not under others. This is why it is crucial to make our conceptual claims clear at the outset. With this framework in hand, we can repopulate the landscape of theories of logic, placing the various theories in their respective camps, monisms, pluralisms, and nihilisms according to the fixed notion of correctness.

7.4.3 Repopulating the landscape

Let us start with the uncontroversial first, the logical monists. As discussed above, and in chapters two and three, there are monists who agree on the theoretical layer, but not on the technical, but there are (more often) monists who disagree on the theoretical, as well as the technical. Our discussion of the nature of correctness shows that the the logical realists and the monists about natural language inference will be untouched by the new framework, as intended. They have a clear notion of the concept of logic, a positive ontic claim and a single logic (token) of the right type labelled as correct. Their argument is fundamentally at the conceptual level, but they both remain monists.

The same reasoning will show that the Frege's universal language of rational thought, and Brouwer's logic as part of the constructive enterprise of the human mind, will remain clearly as monists. Again this is because they are making ontic claims, so the notion of correctness from their perspective remains unchanged.

7.4.3.1 Carnap

Throughout this work we have been appealing to the work of Carnap. The theory of formalization is built upon the bones of his explication, the theory of logic on his tolerance. In the previous chapter we briefly discussed how amenable Carnap would be to the logics-as-formalization approach. A question to be asked is where in the COFE framework would Carnap's pluralism sit. Recall the principle of tolerance and a related passage some paragraphs later.

> It is not our business to set up prohibitions, but to arrive at conventions.
>
> *In logic, there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap 1937, pp. 51–52)

Supporting this principle is the idea that we should not be fighting over the 'correct' meaning of the logical connectives, or the rules of transformation (logical consequence), rather we should approach from the other direction. There is no uniquely correct logic; no external concept out there to which we strive to capture with our formalisms. Instead there are various ways we can attempt to structure the language of science. Here we find the conceptual claim. Carnap's tolerance was part his greater philosophical program. It was a two-pronged program set to redefine the discipline of philosophy itself. "*[P]hilosophy is to be replaced by the logic of science*– that is to say, by the logical analysis of the concepts and sentences of the sciences, for the *logic of science* is nothing other than the logical syntax of the language of science" (Carnap 1937, p. xii). So, logic is bound up in the analysis and language of science; no surprise given the empiricism at hand.

Logic is the study of these formal ways. For Carnap, there is no one correct way to do this, looking for that is to misunderstand the task at hand. Rather, the alternate proposals for how to structure the logic of science, which is the only place where we can interact with meaning according to the greater reductionist project, are just that: alternates. No evidence or theoretical argument can clearly make the case for one being the correct one, for there is no platonic ideal to which we are hoping to model with our formal language of science.

The notion of correctness of which Carnap is concerned is a pragmatic one. What framework fits our current purposes. Following the later Carnap there can only be talk of competition between logics (linguistic frameworks) when some application is selected. And then only on pragmatic grounds, based on the fruitfulness of the application (Carnap 1950, p. 25).

Given his treatment and concept of logic it is clear that there is a negative ontic claim, formal languages are more instrumental under Carnap's conception. Using the COFE framework we see why any question of correctness to the Carnapian is a non-starter. The conceptual claim and the negative ontic one stop us before we can even ask what type or which token is correct. There is no space for correctness as we know there is nothing to be correct about. With our clear notion of correctness, thanks to the COFE framework, we see that Carnap was a logical nihilist, not the first pluralist as he is often described.

7.4.3.2 Harman

As discussed in the previous chapter, Harman differentiates between theoretical and practical reasoning, seeing logic as purely in the former arena. He also denies that there is such a thing as deductive reasoning. This would make him a nihilist on the ontic level, though he grants that deductive arguments exist as formal objects. Interestingly he goes further to question the appropriateness of logical systems for modelling (practical) reasoning (Harman 1986, p. 6). In his response to Field's decision theoretic response to his view he makes it clear that even the decision theoretic approach begs the question, as it "*assumes* that an ordinary person might 'employ a logic' – the very point at issue" (Harman 2009, p. 334). This clearly shows that he is a nihilist at the formal level. Harman is an ontic nihilist for theoretical reasoning, and a formal nihilist for practical reasoning.

7.4.3.3 Logical pluralism

Pinning down the conceptual claims at the heart of Beall and Restall's logical pluralism is not a straightforward task. They do claim that logic must be used in the analysis of the relationships expressed in arguments *we actually employ* (Beall and Restall 2006, p. 8). They appeal to the *pre-theoretic* notion of logical consequence (Beall and Restall 2006, p. 28). By this they mean that there is a notion of logical consequence that exists before we formalize, and it is this that we are trying to capture with our formal languages. Because of the diversity of *cases*, implied by the **GTT**, multiple languages will come out correct; correct for the multiple cases that are possible. When cases are taken to be total possible worlds, classical logic is the correct logic, similarly for mathematical constructions and intuitionistic logic and situations and relevant logic. This dovetails nicely with our notion of the conceptual and ontic as theoretic, and not technical. They have a fixed notion of the concept of logic, wrapped up in their notion of a pre-theoretic logical consequence. Thus there is a positive ontic claim as well. The formal level is where things differ from the standard monist story. Here the claim is that no one formal system of the type logic can fully capture the notion of logical consequence that exists. Rather, they claim, that formal systems of this type can capture slices of the pre-theoretic logical consequence based on the appropriate case at hand (Beall and Restall 2006, p. 97). So the type is correct but limited. Given that they split the field at the formal level, it is unsurprising that they make a multi-pronged claim of correctness at the epistemic level. Beall and Restall remain logical pluralists under the COFE framework.

7.4.3.4 Logic-as-modelling

Logic-as-modelling (LaM) is a recent theory of logics as mathematical models of a natural language or reasoning within a mathematical context. This leads to logics necessarily involving idealisations and abstractions. The intial application of this view was in dealing with vagueness in natural languages (Shapiro 2006). In other works they shift their focus to other modes of reasoning, specifically that of mathematical reasoning and other rigoroursly deductive enterprises (Shapiro 2014), (Kouri Kissel and Shapiro 2017).

Proponents of the LaM theory of logic dub it as a sort of logical pluralism. This is because just as there can be more than one successful model of a phenomenon, so too can there be more than one successful logic for a domain; logics are just a type of model. It is unlikely that one can speak of the one correct model, as there is almost always a gap between a model and what it is modelling (Shapiro 2006, p. 50). So for the LaMers the notion of correctness is relative to different theoretical goals, or different ways of idealising and abstracting the phenomena at hand (Cook 2010, p. 500). This seems to be straightforwardly a claim of goodness, rather than correctness.

In their recent article, “Logical pluralism and normativity”, Shapiro and Kouri add clarity on their notion of correctness via a discussion of the normativity of logic under their pluralist theory. The focus is clearly on mathematical reasoning, with natural language no longer in the scope of discussion. They claim that the correct logic is dependent on the domain of discourse, that is it is dependent on the mathematical theory in question, rejecting the generality and topic neutrality of logic as a result.

They take on Carnap’s idea that once we choose a linguistic framework we are voluntarily bound by the rules of that framework for the purposes that we are using that framework. The norms of this logic are constitutive for thought within the domain. They regulate how, and the way, we reason within the linguistic framework, so long as we are using it for the purpose we selected it for. They borrow Florian Steinberger’s term, the *Relativized Constitutivity Thesis*. “We cannot think clearly and rigorously in any given context without some formal rules in place" (Kouri Kissel and Shapiro 2017, p. 9). There are no universal logical norms that are constitutive of thought, but there are logical norms which are constitutive of thought within certain domains, namely different mathematical fields.

They propose that logics are rational reconstructions of the norms which are implicit in the various branches of mathematics, and perhaps science; taking the middle ground between descriptivism and prescriptivism, while leaning more towards the descriptive end. These reconstructions have the usual idealisations that go with any theorising about evaluative practices. As the modelling aspect of their pluralism demands. However, they also make the claim that “[e]ach of the logics is truth-preserving in its domain, in the sense that if the premises of valid argument (in the indicated logic) are true in its domain, then so is its conclusion" (Kouri Kissel and Shapiro 2017, p. 20). It is by way of this conclusion that we get that logics are constitutive for thought in certain domains, but there is no logic that is constitutive of thought generally. This is a clear rejection of a monist claim, but the final claim does indeed seem to be one of correctness and not goodness.

We still have the problem of the gap between the model and the modelled to deal with under this description of different logics being constitutive of thought within the various domains they represent. The notion of being ‘constitutive of thought’ is at the core of their claim of correctness. As the logics are rational reconstructive models they will contain idealisations, abstractions and perhaps errors. There is no guarantee that one’s model ‘gets it right’ in the relevant sense. If the logic (model) cannot do this then we cannot be sure that it is necessarily truth-preserving within the domain.

There seems to be a conflation at play here. By invoking the relatavised constitutivity thesis, we get a notion of being bound to the laws of the linguistic framework which we have chosen, and hence a sort of normativity and correctness. But logics are rational reconstructions of the informal reasoning that takes place within the branches of mathematics, and so they are not the framework that the mathematical reasoning is taking place in. The logics are post facto and idealised. When using the logics themselves, by the relatavised constituivity thesis, we will be bound by its rules in our reasoning, but this is separate from actual mathematical practice. The framework which would bind the mathematical reasoning is that which is being modelled, not the model itself. With this cleared, up the claim is deflated. The mathematical branches have their own sets of reasoning which are constitutive of thought within them; there are correct ways of doing maths within each area of mathematics. The logics may not be a proper reflection of reasoning in the mathematical branches they model, and thus may not be truth-preserving in the relevant sense. The claim of correctness has been deflated into one of goodness. Thus LaMers are in fact logical nihilists and not pluralists.

7.5 LaF and COFE

With the COFE framework of logical theories in hand, we can return to the topic of the previous chapter and ask what kind of logical theory is LaF. Given its close relation to LaM, it will be unsurprising that a claim of logical nihilism will be made, with a proviso.

In this section we will discuss how the LaF view dovetails nicely with the COFE framework due to their conceptual openness. We will then give evidence that the current logical nihilist views would fit nicely within the wider framework and under the LaF view as well. Finally, we will discuss the ontic backdoor argument available to those who are amenable to LaF but are nevertheless uncomfortable with logical nihilism that seems to follow.

The LaF theory of logic is an explication of the LaM view discussed above. It is not only an explication on formal grounds but also on its aims. LaF is a process-oriented view. Formalization often suffers from process product ambiguity, i.e. the process and the product are often conflated. This is why so much ink has been shed on the formal properties of systems, versus the process we undergo from our initial target to said systems. Logics are formalizations, that is they are structured models which represent some target phenomenon. This targeting is informed by the operative concept of logic at hand, a notion of ‘what follows from what’ (our loose notion of inference). With that fixed, we see logics as a specific class of model aimed at formalizing that concept, with the necessary idealisations and abstractions that come along with modelling of any sort. This conceptual openness of the LaF theory of logic matches well with the open-scoped nature of the COFE framework. LaF is an instance of using COFE against the area of logic. The COFE framework was not designed to require one to adopt the LaF theory of logic. It was designed to be able to appropriately categorise theories of logic based on the claims of correctness involved, after disambiguating correctness and goodness. LaF has a similar aim of conceptual openness in order to make sense of the various views of logic which exist historically and in the discourse, but it is a logically nihilistic view due to its lack of strong ontic commitment, with respect to logics.

7.5.1 Other Nihilists

We will now revisit the other logical nihilists and give some evidence that they would be open to the notion of logical nihilism that comes out of the COFE framework, as well as the LaF theory of logic. We will follow the same order as the previous section, starting with Russell, then Franks, and finally Cotnoir.

7.5.1.1 Russell

Russell's argument was against a logical nihilism based on the global principle of generality, which she thinks should be given up. The denial of the global principle of generality easily falls out of both COFE and LaF; it is a soft target. Under both views we need to fix the concept of logic at hand first, before discourse can fruitfully occur.

Russell offers Lakatosian lemma incorporation as a solution to clashes of logical laws, leading to a domain restriction of logical laws. The laws become truth-preserving with respect to the laws. So we end up with the principle of generality within those domains, which is trivial and unsurprising. Although she doesn't deal with the theoretical layer, the choice of domain roughly maps to the idea of fixing the concept of logic. Once we understand the concept of logic, we can pick out some domain. This is akin to how the concept of logic informs the purpose of logic; the lemma incorporation of Russell seems quite amenable to a LaF interpretation of logic.

> If our best laws turn out to have restricted domains, then this is better than no laws at all. We should abandon the generality of logic before we abandon logic. (Russell 2018, p. 321)

7.5.1.2 Franks

The logical nihilism of Franks is bound up in his view of logics as interesting structures to study, both themselves and the relationships between them. Though it seems primarily a view about logics as constructions, rather than formalizations, he does argue against the preconceived suppositions of there being a correct logic out there. This indicates a negative ontic claim, though paired with an unclear conceptual claim. He does refer to the logic 'out-there', pointing at perhaps a denial of logical realism as his nihilist claim. Overall, he continually points to the historical fact that logic, and our conception of it, has evolved and changed over time, lining up with our foundational arguments from chapter one that have led us to adopting the LaF framework.

Franks notes, and recommends following, the trend of seeing logic as the study of the relationship among various systems and their properties, indicating that that is how logicians themselves use the word. The approach of ignoring correctness stems from "an appreciation, fostered by the study of logic, that no one such system can have all the properties that might be useful and interesting" (Franks 2015, p. 163). This view lines up nicely with that of LaF, that different logics will have different properties that might prove interesting and fruitful. The gap between the two is that LaF is clear that we apply logics to a purpose and that comes with the baggage of modelling: idealisation and abstraction. Franks is not clear on what he takes the purpose of logic to be, if any. His open view towards logics as constructions and removing any limitations that might exist on these, coupled with his comments towards the ever-evolving concept of logic mirrors the openness of the concept of logic that underlies the LaF framework, but ultimately the focus of LaF on formalization, rather than construction, may not suit Franks' conception overall view on logic.

> Traditional debates about the scope and nature of logic do not do justice to the details of its maturation... [W]e ignore how modern logic has been shaped by developments in extra-logical culture. Similarly, questions about whether logic principally traces the structure of discursive thought or the structure of an impersonal world presuppose a logical subject matter unaffected by shifts in human interest and knowledge (Franks 2015, p. 163).

7.5.1.3 Cotnoir

The conception of logical nihilism within the COFE framework started with a generalisation of Cotnoir's **LN1**. Logical nihilism, under the COFE framework, starts with the necessary fixing of the concept of logic, and then proceeds to either a negative ontic or formal claim. If we have a positive ontic claim, then we are left with a type of **LN1**. Under this view, as Cotnoir fixes the concept of logical nihilism to natural language inference, it unsurprisingly fits into the COFE framework.

In his further discussion, Cotnoir notes the appealing nature of the logic as modelling approach. Applying formal techniques has yielded success in all areas of philosophy, but formal languages are representational, and so their applications have limits (Cotnoir 2018, p. 319). These limits on formal languages lead him to conclude that there are no correct, nor completely general, formal theories, but that doesn't preclude there being many useful and explanatory ones. This is an appeal to goodness, while rejecting correctness. This is precisely the role which LaF plays in the COFE framework. As the LaF theory is an explication of LaM and Cotnoir is amenable to the latter, he will no doubt be in favour of both LaF and COFE, though he may only be interested in logic with respect to natural language inference. The only barriers to such an acceptance would be if he denies the open-textured and polysemous nature of the concept of logic, choosing to fight the battle of correctness on the conceptual level, or if he disputes the definition of formalization.

> Formal logic has its best application in areas where we can regiment our language and revise our practice. But we shouldn't forget that not all of our inquiry is like this: sometimes regimentation leaves us with an expressive loss, and sometimes revising our practice is not possible or even desirable. Logical nihilism reminds us to respect the differences between model and reality.

7.5.2 The Ontic Backdoor

With the revised concept of correctness in hand, we have cleared up the logical pluralism landscape making clear on what grounds the debate with the monists and pluralists takes place. Some pluralists have been shifted to nihilists as a result. We have classed Carnap, LaM and LaF all as logical nihilists, rather than logical pluralists as they are commonly understood to be. In the previous chapters, we have made the case for the LaF view as a compelling and all-encompassing theory of logic. In both COFE and LaF we must settle the concept of logic we are using before we can coherently move to question of correctness, or formalize a logic at all.

There are advantages to the LaF theory of logic that might be appealing to some, though they might be squeamish about the idea of embracing logical nihilism as a result. Here is the one proviso we foreshadowed at the start. Both LaM and LaF are concerned with goodness rather than correctness. As such we removed them from the pluralist stage and onto the nihilist. Goodness and correctness are distinct notions; one could have different views with respect to each of them. This is the ontic backdoor.

Let us return to our first example, the logical realists. We have fixed the concept of logic, and we have a positive ontic claim: there are rules of logic out there in the fabric of the universe. The theoretical layer is clearly established. Moving to the technical layer, we make a negative formal sub-claim. There are no mathematical structures which totally capture the logic that is out there in the fabric of the universe.[16] From these combined claims we reach a form of LaF. Logics are formalizations that aim to model the logic that is out there in the universe, though they can never fully achieve this goal, due to the limitations of formalization. This is a form of instrumentalism about logics. There still cannot be a correct logical system, so there is a flavour of nihilism that goes along with this view. Yet, we still have a strong ontic notion of logic existing, and that non-formalized thing being correct for the universe. That is, we would be a formal nihilist and an ontic realist. So we see that we can be LaFers without being logical nihilists in the ontic sense. While the ontic backdoor is available, it is not an attractive choice for this LaFer. LaF is a theory about the goodness of logics, not their correctness. The ontic baggage of walking through the backdoor is needless given our concept of logics as idealised and abstracted representations.

[16]This can be watered down to ‘there are no mathematical structures that we know of’ for the same effect.

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