

UNIVERZITET U NOVOM SADU

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UOPŠTENIH SLUČAJNIH PROCESA

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GLAVA IV

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UVOD

Tridesetih godina ovoga veka postavljene su osnove teorije slučajnih (stohastičkih) procesa, koja je danas osnovni pravac istraživanja u teoriji verovatnoće. Ova teorija je lep primer organske sinteze matematičkog i prirodno-naučnog načina mišljenja, kada matematičar uspeva da uđe u fizičku suštinu krupnog naučnog problema i za njega pronađe odgovarajući matematički model.

Ideju za teoriju slučajnih procesa dao je prvi J. H. Poincaré, a prve konture se mogu naći u radovima L. Bachelier-a, Foller-a, M. Plank-a. Međutim, konstrukcija matematički kompletnih osnova teorije slučajnih procesa povezana je sa imenima A. H. Колмогоров-а и A. Я. Хинчин-а.

Jedan od najtipičnijih i najranijih primera slučajnog procesa je proces Braunovog kretanja, gde je putanja koju opisuje Braunaova čestica potopljena u tečnost slučajna funkcija vremena. Prvo objašnjenje Braunovog kretanja dao je A. Einstein 1905. godine. Konciznu, matematičku definiciju Braunovog slučajnog procesa dao je N. Wiener 1918. godine.

Međutim, postoje slučajni procesi koji se ne mogu opisati pojmovima klasične matematičke analize. Takav je primer izvod procesa Braunovog kretanja. Ovaj proces se može interpretirati kao rezultat merenja, pomoću nekog aparata, brzine kretanja Brauneve čestice bez inercije. Takođe, slučajni proces sa konstantnom spektralnom gustinom, "beli šum", nije slučajni proces u klasičnom smislu, ali se često pojavljuje i koristi.

Opisivanje ovakvih fizičkih problema zahtevaо je

komplikovaniji i moderniji matematički aparat nego što je to klasična matematička analiza. Ovo je omogućeno stvaranjem teorije uopštenih funkcija.

Istorijski posmatrano, teorija uopštenih funkcija je i nastala u želji da se matematičkim modelima raznih procesa, koji nisu bili matematički jasno zasnovani, nađe pravilan matematički pristup i omoguće matematička rešenja koja će imati i prirodan smisao. Koncepcija funkcije i operacije sa funkcijama u klasičnoj analizi, zbog izražene uskosti, nije uvek omogućavala adekvatno nalaženje tih modela.

Nedostaci klasične analize imali su za posledicu više generalizacija koncepcije funkcije i operacija sa funkcijama. U monografiji "Théorie des distribution I, II", (1950/51) L. Schwartz je prvi objavio sistematizovanu teoriju jedne klase uopštenih funkcija - distribucija. Sigurno je da su rezultati teorije vektorsko topoloških prostora koji su se tada javili omogućili L. Schwartz-u izgradnju njegove teorije.

Naziv distribucija se koristi za elemente Schwartz-ovog prostora \mathcal{D}' ili nekog podprostora od \mathcal{D}' , a ako se posmatraju neprekidne linearne funkcionale nad proizvoljnim prostorom osnovnih funkcija koristi se naziv uopštenih funkcija.

Teorija distribucija, i šire, teorija uopštenih funkcija predstavlja matematički aparat za razne oblasti matematičke fizike, teoriju pseudodiferencijalnih i Furijeovih operatora.

U knjigama I. M. Gel'fand-a i saradnika [4,5,6,7,8] dat je originalan i sveobuhvatan način razvijanja teorije distribucija i njihove primene.

Ubrzo posle rada L. Schwartz-a na teoriji distribucija

[39], K. Ito, [18] i I. M. Gel'fand [9] su nezavisno uveli koncept uopštenih slučajnih procesa. K. Ito ih je nazvao slučajne distribucije. Njihove definicije su zasnovane na radovima L. Schwartz-a a uopštene slučajne procese su proučavali prvenstveno sa stanovišta korelacione analize stacionarnih slučajnih procesa.

Takođe, I. M. Gel'fand je proučavao Gausove uopštene slučajne procese.

U principu, uopšteni slučajni proces se definiše kao neprekidno linearne preslikavanje vektorsko topološkog prostora test funkcija u vektorsko topološki prostor slučajnih promenljivih. Prema tome, kao što je prikazano u [48, Appendix], postoje različite klase uopštenih slučajnih procesa, zavisno od izbora prostora test funkcija i izbora prostora slučajnih promenljivih.

Problematikom uopštenih slučajnih procesa bavili su se K. Urbanik [46] i T. Hida. T. Hida [13,14] je ispitivao specijalan slučaj Gausovog uopštenog slučajnog procesa, takozvani proces Braunovog kretanja.

Međutim, nešto drugačije su definisali uopšteni slučajni proces u svojim radovima C. Channing [1], M. Ullrich [44,45], C. H. Swartz i D. E. Myers [42], L. J. Kitchens, [21,22], O. Hanš [12]. Sledеdi njihove radove, u disertaciji su proučavani uopšteni slučajni procesi na prostorima \mathcal{A} , $\text{Exp}\mathcal{A}$, $\mathcal{D}^{(MP)}(\emptyset)$, $C[0,1]$, $L^1(\mathbb{R}^n)$.

U Glavi I su navedeni osnovni pojmovi, definicije i tvrdjenja iz teorije verovatnoće i slučajnih procesa, koji se koriste u radu. Takođe, date su osnovne osobine Gausovske

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familije slučajnih promenljivih, koje se koriste u Glavi V.

U Glavi II dati su osnovni pojmovi definicije i tvrđenja iz teorije distribucija i uopštenih funkcija. Detaljno su prikazani prostori \mathcal{A} , \mathcal{A}' , $\text{Exp}\mathcal{A}$, $\text{Exp}\mathcal{A}'$, $\mathcal{D}^{(\text{Mp})}(\mathcal{O})$ i $\mathcal{D}^{(\text{Mp})}(\mathcal{O})'$, i date njihove karakterizacije.

U Glavi III data je definicija uopštenog slučajnog procesa. Date su reprezentacije uopštenog slučajnog procesa na prostorima \mathcal{A} , $\text{Exp}\mathcal{A}$, $\mathcal{D}^{(\text{Mp})}(\mathcal{O})$. Takođe, date su reprezentacije za matematičko očekivanje uopštenog slučajnog procesa. Materijal za tačke 3.2. i 3.3. uzet je iz [29]. Materijal za tačke 3.4. uzet je iz [28].

U Glavi IV ispituju se osobine niza uopštenih slučajnih procesa koji konvergira skoro sigurno, srednje kvadratno i u verovatnoći. Dati su potrebni i dovoljni uslovi za navedene konvergencije niza uopštenih slučajnih procesa. Materijal za ovu glavu uzet je iz [30,31].

U Glavi V date su reprezentacije Gausovog uopštenog slučajnog procesa na prostorima $C[0,1]$ i $L^1(\mathbb{R}^n)$. Takođe je data ekstenzija Gausovog uopštenog slučajnog procesa na Hilbertovom prostoru. Materijal iz ove glave je prvi put izložen.

Literatura korišćena u ovom radu navedena je na kraju i čini je 48 bibliografskih jedinica.

Profesorima Olgi Hadžić i Zoranu Ivkoviću zahvaljujem se na pomoći pri izradi ove teze.

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U Novom Sadu,

19. 12. 1988.

Đorđe Šimić, profesor na Fakultetu matematike i fizike, preuzeo je predavače na fakultetu. Nastavne predmete na fakultetu preuzeo je prof. dr. sc. Ljubiša Čubrilo. Elementi algebre i geometrije su učenici mogu dobiti u predavanju. Prof. dr. sc. Matematika na Fakultetu je predstavljena od dr. sc. Matice srpske da je predavanja u cilju da se upoznaju sa presečkom i komplemantom, a takođe da bude učen učenici definicije po

Definicija 1.1.1. Neka je Ω neprazan skup. Funkcija je \mathcal{F} podskupova od Ω naziva se σ -algebra na skupu Ω .

1) $\emptyset \in \mathcal{F}$,

2) $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$,

3) $A_1, A_2, \dots, A_n \in \mathcal{F} \Rightarrow \bigcup_{i=1}^n A_i \in \mathcal{F}$.

Definicija 1.1.2. Neka je \mathcal{F} σ -algebra na skupu Ω . Spreda par (Ω, \mathcal{F}) naziva se merni prostor.

Ω , \mathcal{F} - merni prostori \mathcal{F} je definisao ne predstavljanje mernih funkcija i ne stvaraci mernih.

Definicija 1.1.3. Neka je (Ω, \mathcal{F}) merni prostor. Funkcija $f: \Omega \rightarrow \mathbb{R}$ je mernostna na \mathcal{F} ako je zadovoljena

1) $f(\emptyset) = 0$,

2) $f(A \cup B) = f(A) + f(B)$ za sve $A, B \in \mathcal{F}$.

GLAVA I

1.1. PROSTOR VEROVATNOĆA, SLUČAJNE PROMENLJIVE I MATEMATIČKO OČEKIVANJE

Osnovni pojam teorije verovatnoće je pojam prostora verovatnoće. Kao prvo posmatra se neprazan skup $\Omega = \{\omega_i, i \in I\}$, I je skup indeksa. Elementi skupa Ω nazivaju se elementarni događaji. Dalje, posmatra se familija podskupova od Ω koja treba da je zatvorena u odnosu na operacije unije, preseka i komplementa. U vezi s tim imamo sledeću definiciju

Definicija 1.1.1. Neka je Ω neprazan skup. Familija \mathcal{F} podskupova od Ω naziva se σ -algebra ako

- 1) $\Omega \in \mathcal{F}$,
- 2) ako $A \in \mathcal{F}$, tada $A^c \in \mathcal{F}$,
- 3) ako $A_n \in \mathcal{F}$, $n = 1, 2, \dots$, tada $\bigcup_{i=1}^{\infty} A_n \in \mathcal{F}$.

Definicija 1.1.2. Neka je \mathcal{F} σ -algebra na skupu Ω . Ureden par (Ω, \mathcal{F}) naziva se merljiv prostor.

Na σ -algebri \mathcal{F} podskupova od Ω definiše se prebrojivo-aditivna funkcija P na sledeći način.

Definicija 1.1.3. Neka je (Ω, \mathcal{F}) merljiv prostor. Funkcija $P: \mathcal{F} \rightarrow \mathbb{R}$ je verovatnoća na \mathcal{F} ako je zadovoljeno

- 1) $P(\Omega) = 1$,
- 2) $P(A) \geq 0$, $A \in \mathcal{F}$,

3) za svaki niz $A_1, A_2, \dots \in \mathcal{F}$, $A_i \cap A_j = \emptyset$, za $i \neq j$ važi

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n).$$

Sada možemo formulisati sistem aksioma Kolmogorova, koji leži u osnovi pojma prostora verovatnoće

Definicija 1.1.4. Uređena trojka (Ω, \mathcal{F}, P) gde je

- 1) Ω skup elemenata ω ,
- 2) \mathcal{F} σ -algebra podskupova od Ω ,
- 3) P verovatnoća na \mathcal{F} ,

naziva se verovatnosni model ili prostor verovatnoće. Pri tome se Ω naziva prostor elementarnih dogadaja, skupovi A iz \mathcal{F} nazivaju se događaji, a $P(A)$ se naziva verovatnoća događaja A .

Iz date definicije se vidi da se aksiomatika teorije verovatnoće suštinski oslanja na aparat teorije skupova i teorije mere. Jasno je da je u proizvolnjem prostoru verovatnoće (Ω, \mathcal{F}, P) funkcija P u stvari mera na merljivom prostoru (Ω, \mathcal{F}) koja zadovoljava dopunski uslov, da je mera celog prostora Ω jednak jedinici, tj. $P(\Omega)=1$ često se P naziva i verovatnosna mera. Zbog toga teorija verovatnoće često koristi terminologiju teorije mere, na primer, umesto termina "događaj" i "skoro sigurno" ponekad se koriste termini "merljiv skup" i "skoro svuda".

Definicija 1.1.5. Prostor verovatnoće (Ω, \mathcal{F}, P) je kompletan čili, P je kompletna verovatnosna mera na \mathcal{F} ako iz $A \in \mathcal{F}$ i $P(A) = 0$ i $B \subset A$ sledi $B \in \mathcal{F}$.

Postoji pojam, veoma važan u teoriji verovatnoće koji nema važnu ulogu u teoriji mere - to je pojam nezavisnosti događaja.

Definicija 1.1.6. Neka je (Ω, \mathcal{F}, P) prostor verovatnoća. Događaji u familiji $\{B_\alpha, \alpha \in A\}$ su nezavisni ako za proizvoljan, konačan izbor indeksa $\alpha_1, \alpha_2, \dots, \alpha_n \in A$ važi

$$P\left(\bigcap_{k=1}^n B_{\alpha_k}\right) = \prod_{k=1}^n P(B_{\alpha_k}).$$

Definicija 1.1.7. $\{\mathcal{F}_\alpha, \alpha \in A\}$ je familija nezavisnih σ -algebri ako su za svaki izbor $B_\alpha \in \mathcal{F}_\alpha, \alpha \in A$ događaji u familiji $\{B_\alpha, \alpha \in A\}$ nezavisni.

Definicija 1.1.8. Neka je (Ω, \mathcal{F}, P) prostor verovatnoća. Funkcija $X: \Omega \rightarrow \mathbb{R}$ (\mathbb{C}) za koju važi $X^{-1}(B) \in \mathcal{F}$ za svaki $B \in \mathcal{B}(\mathbb{E})$ naziva se realna (kompleksna) slučajna promenljiva na (Ω, \mathcal{F}, P) .

Slučajne promenljive X i Y su ekvivalentne ako je $P\{\omega : X(\omega) = Y(\omega)\} = 1$. Ovo pišemo još i $X = Y$ s.i. (skoro izvesno).

Definicija 1.1.9. Neka je (Ω, \mathcal{F}, P) prostor verovatnoća i $X: \Omega \rightarrow \mathbb{R}^n$ (\mathbb{C}^n). Kažemo da je X n -dimenzionalni (kompleksni) slučajni vektor (ili, kraće, slučajni vektor) na Ω ako je $X^{-1}(B) \in \mathcal{F}$ za svako $B \in \mathcal{B}^n(\mathbb{E}^n)$.

Definicija 1.1.10. Neka je X kompleksna slučajna promenljiva na prostoru verovatnoće (Ω, \mathcal{F}, P) . Ako je X integrabilna po mjeri

P, kažemo za X postoji matematičko očekivanje i veličina

$$E(X) = \int_{\Omega} X(\omega) dP(\omega)$$

naziva se matematičko očekivanje slučajne promenljive X.

Definicija 1.1.11. Neka je X kompleksna slučajna promenjiva na (Ω, \mathcal{F}, P) . Ako je funkcija $|X|^n$ integrabilna po meri P, tada se $E(|X|^n)$ naziva momenat n-tog reda. Specijalno, kada X ima drugi momenat, veličina $D(X) = E(|X - E(X)|^2)$ naziva se disperzija od X.

Označimo sa $L^2(\Omega, \mathcal{F}, P)$ prostor slučajnih promenljivih sa konačnim drugim momentom. $L^2(\Omega, \mathcal{F}, P)$ je Hilbertov prostor sa skalarnim proizvodom $\langle X, Y \rangle = E(X\bar{Y})$, i normom $\|X\| = \sqrt{E(|X|^2)}$.

Neka je (Ω, \mathcal{F}, P) prostor verovatnoća, \mathcal{U} proizvoljna σ -podalgebra i X slučajne promenljiva na (Ω, \mathcal{F}, P) sa konačnim matematičkim očekivanjem.

Definicija 1.1.12. Uslovno matematičko očekivanje slučajne promenljive X u odnosu na σ -algebru \mathcal{U} , je slučajna promenljiva $E(X|\mathcal{U})$ na (Ω, \mathcal{F}, P) , koja je merljiva u odnosu na σ -algebru \mathcal{U} i za proizvoljan $B \in \mathcal{U}$ važi

$$\int_B E(X|\mathcal{U}) dP(\omega) = \int_B X dP(\omega), \quad B \in \mathcal{U} \quad (1.1.1)$$

Svaka druga slučajna promenljiva, Y, merljiva u odnosu na \mathcal{U} za koju važi (1.1.1) je ekvivalentna sa $E(X|\mathcal{U})$.

Navešćemo neke od osobina uslovnog matematičkog očekivanja

- 1) Ako je $P\{\omega: X(\omega) \geq 0\} = 1$ tada je $P\{\omega: E(X|\mathcal{U})(\omega) \geq 0\} = 1$
- 2) Ako je X merljiva u odnosu na σ -algebru \mathcal{U} , tada su $E(X|\mathcal{U})$ i X ekvivalentne.
- 3) $E[E(X|\mathcal{U})] = E(X)$.
- 4) Ako su $\mathcal{F}(x)$ i \mathcal{U} nezavisne σ -algebri, tada je $E(X|\mathcal{U}) = EX$.

Neka je data familija $\{X_\alpha, \alpha \in A\}$ slučajnih promenljivih na prostoru verovatnoće (Ω, \mathcal{F}, P) . Označimo, za svako $\alpha \in A$, sa $\mathcal{F}(X_\alpha)$ najmanju σ -algebru u odnosu na koju je X_α merljiva funkcija.

Definicija 1.1.13. Neka je $\{X_\alpha, \alpha \in A\}$ familija slučajnih promenljivih na prostoru verovatnoće (Ω, \mathcal{F}, P) , i neka je $\{\mathcal{F}(X_\alpha), \alpha \in A\}$ familija odgovarajućih σ -algebri. Ako je $\{\mathcal{F}(X_\alpha), \alpha \in A\}$ familija nezavisnih σ -algebri, kažemo da je $\{X_\alpha, \alpha \in A\}$ familija nezavisnih slučajnih promenljivih.

1.2. RASPODELA VEROVATNOĆA

Ako je $X(\omega)$, $\omega \in \Omega$, realna slučajna promenljiva, definisana na prostoru verovatnoće (Ω, \mathcal{F}, P) možemo odrediti verovatnoću događaja da vrednost X pripadne datom intervalu realnih brojeva. Uopšte, možemo posmatrati X kao preslikavanje Ω u \mathbb{R} i definisati verovatnosnu mjeru Φ na merljivom prostoru $(\mathbb{R}, \mathcal{B})$.

Definicija 1.2.1. Neka je X realna slučajna promenljiva na (Ω, \mathcal{F}, P) . Raspodela verovatnoće slučajne promenljive X je verovatnosna mera Φ na merljivom prostoru $(\mathbb{R}, \mathcal{B})$, takva da je za svako $B \in \mathcal{B}$

$$\Phi(B) = P\{X^{-1}(B)\} = P\{X \in B\}.$$

Definicija 1.2.2. Funkcija raspodele realne slučajne promenljive X na (Ω, \mathcal{F}, P) je funkcija $F(x)$, $x \in \mathbb{R}$, takva da je

$$F(x) = \Phi((-\infty, x)) = P\{X < x\}.$$

Funkcija raspodele F realne slučajne promenljive X ima sledeća svojstva

- 1) F je neprekidna sa leve strane,
- 2) F je monotono neopadajuća
- 3) $\lim_{x \rightarrow -\infty} F(x) = 1, \quad \lim_{x \rightarrow \infty} F(x) = 0.$

Definicija 1.2.3. Neka realna slučajna promenljiva na (Ω, \mathcal{F}, P) i neka je F njena funkcija raspodele. Kažemo da je X *apsolutno neprekidna* (ili, kraće, *neprekidna*) slučajna promenljiva ako postoji Borelova funkcija $f: \mathbb{R} \rightarrow \mathbb{R}$ takva da je

- 1) $f(x) \geq 0, \quad x \in \mathbb{R}$
- 2) $F(x) = \int_{-\infty}^x f(t)dt, \quad x \in \mathbb{R}.$

Definicija 1.2.4. Karakteristična funkcija $\varphi(t)$, $t \in \mathbb{R}$, realne slučajne promenljive X na (Ω, \mathcal{F}, P) sa raspodelom verovatnoća Φ i funkcijom raspodele F je

$$\varphi(t) = E(e^{itX}) = \int_{\mathbb{R}} e^{itx} dF(x) = \int_{\mathbb{R}} e^{itx} \Phi(dx).$$

Karakteristična funkcija realne slučajne promenljive X ima sledeća svojstva

1) $\varphi(0)=1$;

2) φ je uniformno neprekidna;

3) za proizvoljne $t_1, t_2, \dots, t_n \in \mathbb{R}$ i $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{C}$

$$\sum_{j,k} \alpha_j \overline{\alpha_k} \varphi(t_j - t_k) \geq 0.$$

Teorema 1.2.1. (S. Bohner). Ako je $\varphi: \mathbb{R} \rightarrow \mathbb{C}$ proizvoljna funkcija koja zadovoljava uslove 1) - 3), tada postoji jedinstvena verovatnosna mera Φ na $(\mathbb{R}, \mathcal{B})$ takva da je

$$\varphi(t) = \int_{\mathbb{R}} e^{itx} \Phi(dx).$$

Definicija 1.2.5. Raspodela verovatnoća n-dimenzionalnog slučajnog vektora $\mathbb{X} = (X_1, X_2, \dots, X_n)$ na (Ω, \mathcal{F}, P) je

$$\Phi(B) = P\{\mathbb{X} \in B\} = P\left(\bigcap_{k=1}^n \{X_k \in B_k\}\right), \quad \forall B = B_1 \times B_2 \times \dots \times B_n \in \mathcal{B}^n.$$

Definicija 1.2.6. Funkcija raspodele realnog n-dimenzionalnog slučajnog vektora $\mathbb{X} = (X_1, X_2, \dots, X_n)$ na (Ω, \mathcal{F}, P) je funkcija

$$\begin{aligned} F(x_1, x_2, \dots, x_n) &= \Phi((-\infty, x_1] \times (-\infty, x_2] \times \dots \times (-\infty, x_n]) = \\ &= P\left(\bigcap_{k=1}^n \{X_k < x_k\}\right), \quad (x_1, \dots, x_n) \in \mathbb{R}^n. \end{aligned}$$

Definicija 1.2.7. n - dimenzionalni slučajni vektor $\mathbb{X} = (X_1, \dots, X_n)$ na (Ω, \mathcal{F}, P) sa funkcijom raspodele F je *apsolutno neprekidnog tipa* ako postoji nenegativna integrabilna funkcija $f: \mathbb{R}^n \rightarrow \mathbb{R}$, koja se zove *gustina raspodele*, takva da za svaki $(x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ važi

$$F(x_1, x_2, \dots, x_n) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \dots \int_{-\infty}^{x_n} f(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n$$

Definicija 1.2.8. Karakteristična funkcija $\varphi(t)$, $t \in \mathbb{R}^n$, n -dimenzionalne slučajne promenljive X na (Ω, \mathcal{F}, P) sa raspodelom verovatnoća Φ i funkcijom raspodele F je

$$\varphi(t) = E(E^{i t} X) = \int_{\mathbb{R}^n} e^{i \langle t, x \rangle} dF(x) = \int_{\mathbb{R}^n} e^{i \langle t, x \rangle} \Phi(dx).$$

gde je $\langle \cdot, \cdot \rangle$ skalarni proizvod u \mathbb{R}^n .

1.3. SLUČAJNI PROCESI I NJIHOVE RASPODELE

Definicija 1.3.1. Neka je T ureden skup. Familija realnih slučajnih promenljivih $X = \{X(t, \omega): t \in T, \omega \in \Omega\}$ na (Ω, \mathcal{F}, P) se naziva *slučajni proces*.

Za fiksirano $\omega \in \Omega$ imamo funkciju $X(\omega, \cdot)$ argumenta $t \in T$ koja se naziva *trajektorija* slučajnog procesa. Za fiksirano $t \in T$, $X(\cdot, t)$ je realna slučajna promenljiva.

Posmatrajući $\{X(t, \omega): t \in T, \omega \in \Omega\}$ kao beskonačno dimenzionalni slučajni vektor, sa vrednostima u \mathbb{R}^T , moguće je definisati njegovu raspodelu. Podskup \mathbb{R}^T oblika

$$A = \{X \in \mathbb{R}^T: (X_1(t_1), X(t_2), \dots, X(t_n)) \in B_n\}, \quad (1.4.1)$$

gde je B_n Borelov skup u \mathbb{R}^n , naziva se *cilindrični podskup*. Za fiksirane t_1, t_2, \dots, t_n klasa svih cilindričnih podskupova dobijenih kad B_n prolazi kroz \mathcal{B}^n , obrazuje σ - algebru, koju ćemo označiti sa $\mathcal{B}(t_1, \dots, t_n)$. Za proizvoljan skup oblika (1.4.1) definišemo

$$\Phi_{t_1, t_2, \dots, t_n}(A) = P((X(t_1), X(t_2), \dots, X(t_n)) \in B_n),$$

koja je mera na merljivom prostoru $(\mathbb{R}^T, \mathcal{B}(t_1, \dots, t_n))$.

Menjajući izbor konačnog podskupa $\{t_1, t_2, \dots, t_n\} \subset T$, dobijamo klasu $\Phi = \{\Phi_{t_1, t_2, \dots, t_n}\}$ takvih mera, i ta klasa zadovoljava sledeći uslov saglasnosti: ako cilindrični podskup A iz (1.4.1) predstavimo na drugi način, na primer

$$A = \{X \in \mathbb{R}^T : (X(s_1), X(s_2), \dots, X(s_n)) \in B_m\},$$

tada je zadovoljeno

$$\Phi_{t_1, t_2, \dots, t_n}(A) = \Phi_{s_1, s_2, \dots, s_n}(A).$$

Označimo sa \mathcal{U}^T algebru podskupova od \mathbb{R}^T , koja se sastoji iz svih cilindričnih skupova. Možemo zadati konačno-aditivnu mjeru $\tilde{\Phi}$ na $(\mathbb{R}^T, \mathcal{U}^T)$, takvu da se restrikcija od $\tilde{\Phi}$ na $\mathcal{B}(t_1, \dots, t_n)$ poklapa sa $\Phi_{t_1, t_2, \dots, t_n}$. Označimo sa \mathcal{B}^T najmanju σ -algebru koja sadrži \mathcal{U}^T i nazovimo elemente \mathcal{B}^T Borelovi skupovi u \mathbb{R}^T . Sledеća teorema poznata je kao teorema Kolmogorova o produženju mere.

Teorema 1.3.1. Gore definisana konačno aditivna mera $\tilde{\Phi}$ na $(\mathbb{R}^T, \mathcal{U}^T)$ može se jedinstveno produžiti do verovatnosne mere Φ na $(\mathbb{R}^T, \mathcal{B}^T)$.

Definicija 1.3.2. Mera Φ dobijena u Teoremi 1.4.1. naziva se raspodela slučajnog procesa $\{X(t, \omega), t \in T, \omega \in \Omega\}$.

Obratno, za proizvoljnu verovatnosnu mjeru Φ na $(\mathbb{R}^T, \mathcal{B}^T)$, postoji slučajni proces čija je raspodela Φ . Neka je $\Omega = \mathbb{R}^T$, i neka su elementi od Ω , $X = \{x(t) : t \in T\}$. Dalje, neka je $\mathcal{F} = \mathcal{B}^T$, $P = \Phi$. Tada jednakost $X(t, x) = x(t)$, $t \in T$, $X \in \Omega$, definiše slučajni proces čija je raspodela Φ .

Definicija 1.3.3. Neka je $\mathbb{X} = \{x(\cdot, t), t \in T\}$ slučajni proces.

Kazaćemo da je \mathbb{X} neprekidan ako je za skoro svako $\omega \in \Omega$, $X(\omega, \cdot)$ neprekidna funkcija na T .

1.4. KONVERGENCIJE NIŽA SLUČAJNIH PROMENLJIVIH

Neka su X, X_1, X_2, \dots slučajne promenljive na prostoru verovatnoća (Ω, \mathcal{F}, P) .

Definicija 1.4.1. Niz slučajnih promenljivih X_1, X_2, \dots konvergira u verovatnoći ka slučajnoj promenljivoj X , čsto obeležavamo sa $X_n \xrightarrow{v} X$, ako za proizvoljno $\epsilon > 0$

$$P\{|X_n - X| > \epsilon\} \rightarrow 0, \quad n \rightarrow \infty.$$

Definicija 1.4.2. Niz slučajnih promenljivih X_1, X_2, \dots konvergira skoro sigurno ka slučajnoj promenljivoj X , $(X_n \xrightarrow{s.s.} X)$, ako

$$P\{\omega: X_n(\omega) \rightarrow X(\omega), n \rightarrow \infty\} = 1.$$

Definicija 1.4.3. Niz slučajnih promenljivih X_1, X_2, \dots konvergira u srednjem reda p , $0 < p < \infty$, ka slučajnoj promenljivoj X , $(X_n \xrightarrow{P} X)$, ako

$$E|X_n - X|^p \rightarrow 0, \quad n \rightarrow \infty.$$

Definicija 1.4.4. Niz slučajnih promenljivih X_1, X_2, \dots konvergira raspodeli ka slučajnoj promenljivoj X , $(X_n \xrightarrow{r} X)$, ako za proizvoljnu ograničenu, neprekidnu funkciju f

$$Ef(X_n) \rightarrow Ef(X), \quad n \rightarrow \infty.$$

Među navedenim vrstama konvergencije niza slučajnih promenljivih važe sledeći odnosi:

- a) Konvergencija skoro sigurno \rightarrow konvergencija u verovatnoći \rightarrow konvergencija u raspodeli.
- b) Konvergencija u srednjem reda $p \rightarrow$ konvergencija u srednjem reda q , ako $p > q \geq 1 \rightarrow$ postoji podniz koji konvergira skoro sigurno.
- c) Konvergencija u srednjem reda $p \geq 1 \rightarrow$ konvergencija u verovatnoći \rightarrow konvergencija u raspodeli.
- d) Konvergencija u verovatnoći \rightarrow postoji podniz koji konvergira skoro sigurno.

1.5. GAUSOVSKA FAMILIJA

1.5.1. Realna Gausovska familija

Definicija 1.5.1. Realna slučajna promenljiva X , definisana na nekom prostoru verovatnoće naziva se Gausova, ili ima Gausovu, odnosno normalnu raspodelu, što kraće pišemo $X: N(m, \sigma^2)$, ako njena funkcija raspodele ima oblik:

$$F(x) = (2\pi\sigma^2)^{-1/2} \int_{-\infty}^x \exp\left[-\frac{(y-m)^2}{2\sigma^2}\right] dy, \quad x \in \mathbb{R},$$

gde su $m \in \mathbb{R}$, $\sigma^2 \geq 0$, konstante. Imamo da je $m=E(X)$, $\sigma^2=D(X)$.

Karakteristična funkcija za X je

$$\varphi(t) = \exp\left[imt - \frac{1}{2}t^2\sigma^2\right], \quad t \in \mathbb{R}.$$

Poseban slučaj normalne raspodele se dobija za $\sigma^2=0$, odnosno kada je karakteristična funkcija

$$\varphi(t) = e^{it^m}, \quad t \in \mathbb{R}.$$

U ovom slučaju je $X=m$ skoro sigurno i kažemo da X ima degenerisanu normalnu raspodelu.

Definicija 1.5.2. Familija $\mathcal{X} = \{X_u : u \in U\}$, $U \subseteq \mathbb{R}$ realnih slučajnih promenljivih se naziva realna Gausovska familija ako za proizvoljne $a_1, a_2, \dots, a_n \in \mathbb{R}$, $u_1, u_2, \dots, u_n \in U$, $n \in \mathbb{N}$, slučajna promenljiva

$$X = \sum_{k=1}^n a_k X_{u_k}$$

ima Gausovu raspodelu.

U ovom paragrafu razmatraćemo osobine samo realne Gausovske familije.

Proizvoljna podfamilija Gausovske familije je opet Gausovska familija. Specijalno, svaka konačna podfamilija $\{X_j, 1 \leq j \leq n\}$, Gausovske familije \mathcal{X} ima n -dimenzionalnu Gausovu raspodelu sa funkcijom gustine

$$f(x) = (2\pi)^{-n/2} |V|^{-1/2} \exp \left[-\frac{1}{2} (x-m)V^{-1}(x-m)' \right], \quad x \in \mathbb{R}^n,$$

gde je $m = (m_1, m_2, \dots, m_n)$, $m_j = E(X_j)$, $1 \leq j \leq n$, $V = [V_{j,k}]$ pozitivno definitna matrica čiji su elementi $V_{j,k} = E[(X_j - m_j)(X_k - m_k)]$, $1 \leq j, k \leq n$, a $|V|$ i V^{-1} su determinanta i inverzna matrica za matricu V respektivno, $(x-m)'$ označava vektor kolonu transponovan vektoru vrsti $(x-m)$.

Karakteristična funkcija n -dimenzionalne Gausove raspodele ima oblik

$$\varphi(t) = \exp \left[i(m, t) - \frac{1}{2} (Vt, t) \right], \quad t \in \mathbb{R}^n,$$

gde je (\cdot, \cdot) skalarni proizvod u \mathbb{R}^n .

Neka je $\mathbb{X} = \{X_u : u \in U\}$ Gausovska familija. Za tu familiju imamo

a) vektor očekivanja $m_u = E(X_u)$, $u \in U$,

b) kovariacionu matricu $V_{u,v} = E[(X_u - m_u)(X_v - m_v)]$, $u, v \in U$.

Kovariaciona matrica je pozitivno definitna, to jest, za sve $n \geq 1$, i za proizvoljne kompleksne brojeve $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{C}$ i parametre $u_1, u_2, \dots, u_n \in U$ imamo.

$$\sum_{j,k=1}^n \alpha_j \bar{\alpha}_k V_{u_j, u_k} \geq 0.$$

Teorema 1.5.1. Za zadate $(m_u, u \in U)$ i realnu, pozitivno definitnu $V = (V_{u,v} : u, v \in U)$ uvek postoji Gausovska familija $\mathbb{X} = \{X_u : u \in U\}$, čiji se vektor očekivanja i kovariaciona matrica poklapa sa (m_u) i $V = (V_{u,v})$ respektivno. Ako postoji druga familija \mathbb{X}' , koja ima ta svojstva, tada \mathbb{X} i \mathbb{X}' imaju istu raspodelu.

Dokaz je dat u [13].

Neka je U konačan skup, na primer, $U = \{1, 2, \dots, n\}$. Ako su Gausovske slučajne promenljive X_j , $1 \leq j \leq n$, nezavisne, tada je familija $\mathbb{X} = \{X_j : 1 \leq j \leq n\}$, očigledno Gausovska. Specijalno, ako $m_j = 0$, $1 \leq j \leq n$, i $V_{j,k} = \delta_{j,k}$, $1 \leq j, k \leq n$, tada će raspodela familije \mathbb{X} biti $\mathcal{N}(0, E)$, gde je E jedinična matrica, i ta raspodela se naziva n -dimenzionalna Gausova raspodela.

Realna Gausovska familija $\mathbb{X} = \{X_u : u \in U\}$ ima mnogo važnih svojstava, i ovde ćemo navesti neka od njih, koja će nam biti potrebna u daljem radu.

Dokazi sledećih tvrđenja mogu se naći na primer u [13, 43].

Teorema 1.5.2. Neka je $\mathbb{X} = \{X_u : u \in U\}$ Gausovska familija. Tada

a) da bi X_u , $u \in U$ bili nezavisni, potrebno i dovoljno je da za sve $u \neq v$

$$V_{u,v} = 0;$$

b) da bi element X_{u_0} te familije bio nezavisan od $\{X_u : u \in U, u \neq u_0\}$, potrebno je i dovoljno da za svako $u \neq u_0$

$$V_{u,u_0} = 0.$$

Teorema 1.5.3. Za Gausovsku familiju $\mathbb{X} = \{X_n : n \geq 1\}$, konvergencija u verovatnoći niza $\{X_n\}$ je ekvivalentna srednje kvadratnoj konvergenciji. Granica X_∞ niza $\{X_n\}$ u tom slučaju je takođe Gausovska slučajna promenljiva.

Dokaz. Uopšte, iz srednje kvadratne konvergencije sledi konvergencija u verovatnoći, pa je potrebno dokazati samo obratno tvrđenje. Stavimo $E(X_j - X_k) = m_{j,k}$, $V(X_j - X_k) = \sigma_{j,k}^2$. Potreban i dovoljan uslov za konvergenciju u srednje kvadratnom je da

$$E[(X_j - X_k)^2] = \sigma_{j,k}^2 + m_{j,k}^2 \rightarrow 0, \text{ kad } j,k \rightarrow \infty,$$

i odatle, ako $\{X_n\}$ ne konvergira srednje kvadratno, tada

$$\limsup_{j,k \rightarrow \infty} (\sigma_{j,k}^2 + m_{j,k}^2) > 0. \quad (1.5.1)$$

Dalje

$$P\{|X_j - X_k| > \varepsilon\} = (2\pi\sigma_{j,k}^2)^{-1/2} \int_{|x|>\varepsilon} \exp\left[-\frac{(x-m_{j,k})^2}{2\sigma_{j,k}^2}\right] dx.$$

Kako iz (1.5.1) sledi da $\sigma_{j,k}^2$ i $m_{j,k}$ ne teže ka nuli istovremeno, za dovoljno malo $\varepsilon > 0$ imamo

$$\limsup_{j,k \rightarrow \infty} P\{|X_j - X_k| > \varepsilon\} \geq \frac{1}{2}.$$

Ovo znači da $\{X_n\}$ ne konvergira u verovatnoći, što je kontradikcija. \square

Konvergencija u srednje kvadratnom nizu $\{X_n\}$ je, u stvari, jaka konvergencija u Hilbertovom prostoru $L^2(\Omega, \mathcal{F}, P)$, pa se granica X_∞ može posmatrati kao element prostora $L^2(\Omega, \mathcal{F}, P)$. Skalarni proizvod $E(X_n) = m_n$, promenljivih X_n i 1, kao i kvadrat norme $V(X_n) = \sigma_n^2$, promenljive $X_n - m_n$, konvergiraju ka granicama m i σ^2 respektivno. Odatle sledi da niz karakterističnih funkcija za X_n konvergira ka karakterističnoj funkciji normalne $N(m, \sigma^2)$ raspodele. Odatle sledi da granica X_∞ ima $N(m, \sigma^2)$ raspodelu. Time je dokazan drugi deo tvrdjenja. \square

Kako iz konvergencije skoro sigurno sledi konvergencija u verovatnoći imamo sledeću teoremu.

Teorema 1.5.4. Neka je $\mathbb{X} = \{X_n: n \geq 1\}$ Gausovska familija. Iz konvergencije skoro sigurno niza $\{X_n\}$ sledi njegova konvergencija srednje kvadratno.

Ako je data Gausovska familija \mathbb{X} , tada iz definicije sledi da je unija \mathbb{X} sa proizvoljnom linearном kombinacijom njenih elemenata ponovo Gausovska familija. Više od toga, važi sledeća teorema.

Teorema 1.5.5. Neka je \mathbb{X} Gausovska familija. Tada je zatvoren linarni potprostor $\bar{\mathbb{X}}$ prostora $L^2(\Omega, \mathcal{F}, P)$, generisan sa \mathbb{X} , takođe Gausovska familija.

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Dokaz. Možemo pretpostaviti da data Gausovska familija \mathbb{X} obrazuje vektorski potprostor, a u suprotnom to se može obezbediti dodavanjem svih potrebnih konačnih linearnih kombinacija. Na taj način proširena, familija \mathbb{X} ostaje Gausovska. Elementi $X_j \in \bar{\mathbb{X}}$, $1 \leq j \leq n$, se mogu predstaviti kao granice nizova koji konvergiraju srednje kvadratno.

$$\lim_{n \rightarrow \infty} X_j^{(n)} = X_j, \quad X_j^{(n)} \in \mathbb{X}.$$

Proizvoljna linearна комбинација $\sum_{j=1}^m a_j X_j$, $a_1, a_2, \dots, a_m \in \mathbb{R}$ se može predstaviti као граница

$$\lim_{n \rightarrow \infty} \sum_{j=1}^m a_j X_j^{(n)} = \sum_{j=1}^m a_j X_j, \quad \sum_{j=1}^m a_j X_j^{(n)} \in \mathbb{X},$$

i saglasno sa drugim delom tvrđenja Teoreme 1.5.4. ta граница je takođe Gausovska slučajna veličina. Odatle sledi da je $\bar{\mathbb{X}}$ Gausovska familija. \square

1.5.2. Kompleksna Gausovska familija

Neka je Z kompleksna slučajna promenljiva na prostoru verovatnoće (Ω, \mathcal{F}, P) . Neka je $m = E(Z)$, $m \in \mathbb{C}$, X realni deo od Z a Y imaginarni deo od Z . Imamo

$$Z(\omega) = m + X(\omega) + i Y(\omega), \quad \omega \in \Omega. \quad (1.5.2)$$

Definicija 1.5.3. Ako su slučajne promenljive X i Y u (1.5.2) nezavisne i imaju istu Gausovu raspodelu sa očekivanjem jednakim nuli, tada se Z naziva kompleksna Gausova slučajna promenljiva.

Dokaz. Možemo pretpostaviti da data Gausovska familija \mathbb{X} obrazuje vektorski potprostor, a u suprotnom to se može obezbediti dodavanjem svih potrebnih konačnih linearnih kombinacija. Na taj način proširena, familija \mathbb{X} ostaje Gausovska. Elementi $X_j \in \bar{\mathbb{X}}$, $1 \leq j \leq n$, se mogu predstaviti kao granice nizova koji konvergiraju srednje kvadratno.

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$$\lim_{n \rightarrow \infty} \sum_{j=1}^m a_j X_j^{(n)} = \sum_{j=1}^m a_j X_j, \quad \sum_{j=1}^m a_j X_j^{(n)} \in \mathbb{X},$$

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1.5.2. Kompleksna Gausovska familija

Neka je Z kompleksna slučajna promenljiva na prostoru verovatnoće (Ω, \mathcal{F}, P) . Neka je $m = E(Z)$, $m \in \mathbb{C}$, X realni deo od Z a Y imaginarni deo od Z . Imamo

$$Z(\omega) = m + X(\omega) + i Y(\omega), \quad \omega \in \Omega. \quad (1.5.2)$$

Kompleksna slučajna promenljiva koja je Z kompleksna Definicija 1.5.3. Ako su slučajne promenljive X i Y u (1.5.2) nezavisne i imaju istu Gausovu raspodelu sa očekivanjem jednakim nuli, tada se Z naziva kompleksna Gausova slučajna promenljiva.

and with observations of the physical environment and
the social environment.

It will take some time before we can fully understand
the many complex interactions between the environment and disease
processes because the processes often act together in a
complex manner. For example, environmental factors such as
temperature, humidity, and precipitation can affect both food availability
and disease incidence. In consequence, climate change may result in both
environmental changes such as rainfall patterns and temperature
changes which may affect disease incidence.

We have developed [1-3] a model of the effect of climate
change on the risk of HIV and other diseases by combining
existing evidence of climate change.

However, while significant advances have been
made in understanding the relationship between the environment
and disease risk, there is still much to be done to improve
our understanding of the underlying mechanisms.

We also stabilize our models to predict the effect
of climate change [1-3] on disease risk by combining
existing evidence of climate change with information

Dokaz. Možemo pretpostaviti da data Gausovska familija \mathbb{X} obrazuje vektorski potprostor, a u suprotnom to se može obezrediti dodavanjem svih potrebnih konačnih linearnih kombinacija. Na taj način proširena, familija \mathbb{X} ostaje Gausovska. Elementi $X_j \in \bar{\mathbb{X}}$, $1 \leq j \leq n$, se mogu predstaviti kao granice nizova koji konvergiraju srednje kvadratno.

$$\lim_{n \rightarrow \infty} X_j^{(n)} = X_j, \quad X_j^{(n)} \in \mathbb{X}.$$

Proizvoljna linearна комбинација $\sum_{j=1}^m a_j X_j$, $a_1, a_2, \dots, a_m \in \mathbb{R}$ se može predstaviti као граница

$$\lim_{n \rightarrow \infty} \sum_{j=1}^m a_j X_j^{(n)} = \sum_{j=1}^m a_j X_j, \quad \sum_{j=1}^m a_j X_j^{(n)} \in \mathbb{X},$$

i saglasno sa drugim delom tvrđenja Teoreme 1.5.4. ta граница je takođe Gausovska slučajna veličina. Odatle sledi da je $\bar{\mathbb{X}}$ Gausovska familija. \square

1.5.2. Kompleksna Gausovska familija

Neka je Z kompleksna slučajna promenljiva na prostoru verovatnoće (Ω, \mathcal{F}, P) . Neka je $m = E(Z)$, $m \in \mathbb{C}$, X realni deo od Z a Y imaginarni deo od Z . Imamo

$$Z(\omega) = m + X(\omega) + i Y(\omega), \quad \omega \in \Omega. \quad (1.5.2)$$

Definicija 1.5.3. Ako su slučajne promenljive X i Y u (1.5.2) nezavisne i imaju istu Gausovu raspodelu sa očekivanjem jednakim nuli, tada se Z naziva kompleksna Gausova slučajna promenljiva.

Iz definicije sledi da je raspodela kompleksne slučajne promenljive Z definisana samo njenim očekivanjem m i disperzijom $E(|Z-m|^2)$.

Definicija 1.5.4. Neka je $\mathcal{Z} = \{Z_u: u \in U\}$ familija kompleksnih slučajnih promenljivih. Ako je proizvoljna konačna linearna kombinacija $\sum_{j=1}^n c_j Z_{u_j}$, $c_j \in \mathbb{C}$, $u_j \in U$, $1 \leq j \leq n$, kompleksna Gausova slučajna promenljiva, tada se \mathcal{Z} naziva *kompleksna Gausovska familija*.

Ako je familija \mathcal{Z} Gausovska, tada je proizvoljna podfamilija $\mathcal{Z}' \subset \mathcal{Z}$ ponovo Gausovska familija. Takođe, familija $\bar{\mathcal{Z}}' = \{\bar{Z}_u: u \in U\}$, gde je \bar{Z} slučajna promenljiva kompleksno konjugovana za Z , je isto Gausovska.

Teorema 1.5.6. Neka je $\mathcal{Z} = \{Z_u: u \in U\}$ kompleksna Gausovska familija gde su elementi oblika $Z_u = m_u + X_u + i Y_u$, $u \in U$. Tada su familija $\mathcal{X} = \{X_u: u \in U\}$ i $\mathcal{Y} = \{Y_u: u \in U\}$ realne Gausovske familije. Unija $\mathcal{X} \cup \mathcal{Y}$ je takođe realna Gausovska familija.

Teorema 1.5.7. Neka je $\mathcal{Z} = \{Z_u: u \in U\}$ familija nezavisnih kompleksnih slučajnih promenljivih. Ako je svaka Z_u , $u \in U$ kompleksna Gausova slučajna promenljiva tada je \mathcal{Z} kompleksna Gausovska familija.

Teorema 1.5.8. Neka je $\mathcal{Z} = \{Z_u: u \in U\}$ kompleksna Gausovska familija. Potreban i dovoljan uslov da su $Z_{u_1}, Z_{u_2}, \in \mathcal{Z}$, $u_1 \neq u_2$ nezavisni, je da je njihova kovarijacija

and the corresponding structures in the main hemisphere. However, the posterior limb of the internal capsule is more frequently involved than the anterior limb. Involvement of the anterior limb is associated with hemiparesis, hemisensory loss, and homonymous hemianopsia. Involvement of the posterior limb is associated with hemiparesis, hemisensory loss, and homonymous hemianopsia. The internal capsule is also involved in some cases of stroke, particularly those involving the posterior limb.

The internal capsule is also involved in some cases of stroke, particularly those involving the posterior limb. The internal capsule is also involved in some cases of stroke, particularly those involving the posterior limb. The internal capsule is also involved in some cases of stroke, particularly those involving the posterior limb. The internal capsule is also involved in some cases of stroke, particularly those involving the posterior limb.

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$$E[(Z_{u_1} - m_{u_1})(Z_{u_2} - m_{u_2})] = 0, \text{ gde je } m_{u_j} = E(Z_{u_j}).$$

Označimo sa $(m_u : u \in U)$ vektor matematičkih očekivanja i sa $(V_{u,v} : u, v \in U)$ kovarijacionu matricu kompleksne Gausovske familije $Z = \{Z_u : u \in U\}$.

Teorema 1.5.9. Kovarijaciona matrica $(V_{u,v} : u, v \in U)$ kompleksne Gausovske familije $Z = \{Z_u : u \in U\}$ zadovoljava sledeće uslove:

a) V je pozitivno definitna,

b) ako je $Z_u = m_u + X_u + i Y_u$, $u \in U$, tada

$$E(X_u X_v) = E(Y_u Y_v) = \frac{1}{2} \operatorname{Re}(V_{u,v})$$

$$-E(X_u Y_v) = E(Y_u X_v) = \frac{1}{2} \operatorname{Im}(V_{u,v}), \quad u, v \in U.$$

(Re i Im označavaju realni i imaginarni deo respektivno).

Teorema 1.5.10. Za proizvoljni vektor $(m_u : u \in U)$ i pozitivno definitnu matricu $(V_{u,v} : u, v \in U)$, postoji kompleksna Gausovska familija $Z = \{Z_u : u \in U\}$ čiji se vektor matematičkih očekivanja i kovarijaciona matrica poklapa sa datim vektorom i matricom. Rasподела te familije je jednoznačno određena.

U daljem radu pretpostavljemo da je $E(z)=0$, što ne umanjuje opštost.

Teorema 1.5.11. Potreban i dovoljan uslov da niz $\{Z_n\}$ kompleksnih slučajnih promenljivih, gde je $Z_n = X_n + i Y_n$, $n \geq 1$, konvergira srednje kvadratno je da nizovi $\{X_n\}$ i $\{Y_n\}$ konvergiraju srednje kvadratno.

2. گرچه این داده های آماری محدود باشند ($n = 5$) اما ممکن است این نتایج معمولی بودجه های کوچک و متوسط ($\bar{U} \approx 0.2 - 0.5$ V) را تأثیرگذار نهاده باشند.

آنکه در نظر داشته باشید که در اینجا برخی از داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} > 0.5$ V) اما با توجه به اینکه این داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} < 0.2$ V) این نتایج معمولی بودجه های کوچک و متوسط ($\bar{U} \approx 0.2 - 0.5$ V) را تأثیرگذار نهاده باشند.

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آنکه در نظر داشته باشید که این داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} > 0.5$ V) اما با توجه به اینکه این داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} < 0.2$ V) این نتایج معمولی بودجه های کوچک و متوسط ($\bar{U} \approx 0.2 - 0.5$ V) را تأثیرگذار نهاده باشند.

آنکه در نظر داشته باشید که این داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} > 0.5$ V) اما با توجه به اینکه این داده های ممکن است از نظر آماری مغایر باشند ($\bar{U} < 0.2$ V) این نتایج معمولی بودجه های کوچک و متوسط ($\bar{U} \approx 0.2 - 0.5$ V) را تأثیرگذار نهاده باشند.

Dokaz. Neka je $Z_n = X_n + iY_n$ i neka je granica niza $\{Z_n\}$ slučajna promenljiva $Z = X + iY$. Imamo da je

$$\begin{aligned} E[|Z_n - Z|^2] &= E[|X_n + iY_n - X - iY|^2] = E[(X_n - X)^2 + (Y_n - Y)^2] = \\ &= E[(X_n - X)^2] + E[(Y_n - Y)^2]. \quad \square \end{aligned}$$

Teorema 1.5.12. Potreban i dovoljan uslov da niz $\{Z_n\}$ kompleksnih slučajnih promenljivih, gde je $Z_n = X_n + iY_n$, konvergira u verovatnoći je da nizovi $\{X_n\}$ i $\{Y_n\}$ konvergiraju u verovatnoći.

Dokaz. Neka $Z_n \xrightarrow{\text{v}} Z$, gde je $Z_n = X_n + iY_n$, $Z = X + iY$.

Tada je

$$\begin{aligned} P\{|Z_n - Z| \geq \epsilon\} &= P\{|X_n + iY_n - X - iY| \geq \epsilon\} \leq \\ &\leq P\{|X_n - X| \geq \frac{\epsilon}{2}\} + P\{|Y_n - Y| \geq \frac{\epsilon}{2}\}, \end{aligned}$$

odakle sledi da ako nizovi $\{X_n\}$ i $\{Y_n\}$ konvergiraju u verovatnoći, konvergira i niz $\{Z_n\}$.

Obratno, imamo da je

$$\begin{aligned} P\{|X_n - X| \geq \epsilon\} &\leq P\{|Z_n - Z| \geq \epsilon\} \text{ i} \\ P\{|Y_n - Y| \geq \epsilon\} &\leq P\{|Z_n - Z| \geq \epsilon\}, \end{aligned}$$

odakle sledi da $Z_n \xrightarrow{\text{v}} Z$ implicira $X_n \xrightarrow{\text{v}} X$, i $Y_n \xrightarrow{\text{v}} Y$. \square

Teorema 1.5.13. Potreban i dovoljan uslov da $Z_n \xrightarrow{s.s.} Z$, gde je $Z_n = X_n + iY_n$, a $Z = X + iY$ je da $X_n \xrightarrow{s.s.} X$, $Y_n \xrightarrow{s.s.} Y$.

Dokaz. Direktno sledi iz definicije skoro sigurne konvergencije. \square

and the second relation is $\langle \bar{q} q \rangle = \langle \bar{q} \bar{q} \rangle + \langle q q \rangle$ or equivalently

$$\langle \bar{q} q \rangle = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}} + \langle \bar{q} q \rangle_{\text{non-perturb}}$$

$$\langle \bar{q} q \rangle = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}} + \langle \bar{q} q \rangle_{\text{non-perturb}}$$

and one can also consider the contribution of the non-perturbative source term $\langle \bar{q} q \rangle_{\text{non-perturb}}$. This is the contribution of the gluon loop diagram which is the loop diagram with a gluon loop around the quark loop. It is the contribution of the gluon loop around the quark loop.

The first term is $\langle \bar{q} q \rangle_{\text{vac}} = \langle \bar{q} q \rangle_{\text{perturb}} + \langle \bar{q} q \rangle_{\text{non-perturb}}$ and the second term is

$$\langle \bar{q} q \rangle_{\text{perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$$

$$= \langle \bar{q} q \rangle_{\text{vac}} + \frac{\alpha_s}{2} \langle \bar{q} q \rangle_{\text{vac}} + \frac{\alpha_s}{2} \langle \bar{q} q \rangle_{\text{vac}} = \langle \bar{q} q \rangle_{\text{vac}} + \frac{3\alpha_s}{2} \langle \bar{q} q \rangle_{\text{vac}}$$

and the third term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$ and the fourth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$.

$$\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}} + \langle \bar{q} q \rangle_{\text{non-perturb}}$$

and the fifth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$ and the sixth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$.

and the seventh term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$ and the eighth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$.

and the ninth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$ and the tenth term is $\langle \bar{q} q \rangle_{\text{non-perturb}} = \langle \bar{q} q \rangle_{\text{vac}} + \langle \bar{q} q \rangle_{\text{perturb}}$.

Teorema 1.5.14. Neka je $\{Z_n = X_n + iY_n, n \geq 1\}$ niz kompleksnih Gausovih slučajnih promenljivih koji konvergira srednje kvadratno ka kompleksnoj slučajnoj promenljivoj $Z = X+iY$. Tada je Z kompleksna Gausova slučajna promenljiva.

Dokaz. Na osnovu Teoreme 1.5.11. i Teoreme 1.5.3. iz $Z_n \xrightarrow{s.k.} X$ i $Z_n \xrightarrow{s.k.} Z$ sledi $X_n \xrightarrow{s.k.} X$ i $Y_n \xrightarrow{s.k.} Y$. Kako X_n i Y_n , $n \geq 1$, imaju istu, Gausovu raspodelu, sledi da X i Y imaju istu Gausovu raspodelu. Takođe, iz činjenice da su X_n , Y_n nezavisne za svako $n \geq 1$ sledi da su i X i Y nezavisne slučajne promenljive. \square

Na osnovu ovih tvrđenja i Teorema 1.5.3. i 1.5.4. direktno sledi sledeće teoreme.

Teorema 1.5.15. Konvergencija u verovatnoći niza kompleksnih Gausovih slučajnih promenljivih implicira njegovu srednje kvadratnu konvergenciju.

Teorema 1.5.16. Konvergencija skoro sigurno niza kompleksnih Gausovih slučajnih promenljivih implicira njegovu srednje kvadratnu konvergenciju.

Ako je u familiji svih kompleksnih raspodela od \mathcal{B} bilo da ne konvergira topologija u $\mathcal{B}(X, F)$ na mrežu topologija konvergencije, i uvećava se sa τ_0 .

Ako je u familiji svih ograničenih raspodela od \mathcal{B} , tada ne konvergira topologija u $\mathcal{B}(X, F)$ na mrežu topologija konvergencije, i uvećava se sa τ_0 .

Ako je u familiji svih ograničenih raspodela od \mathcal{B} , tada

the following day the 10th a company of 100 men under command
of Captain J. W. Clegg and 100 men under command of Captain
John H. L. Smith were sent to the village of Kibwezi to reconnoiter
and to ascertain if there was any possibility of attacking the
village.

The party started at 6 A.M. and reached the village about
10 A.M. and found it to consist of 100 houses built of mud
and straw. There were no trees or bushes near the village
and the ground was covered with stones and shells of broken
shells. The houses were all built of mud and straw and
had no doors or windows.

At 1 P.M. the party started back to the fort and reached
there about 3 P.M. and found the village had been captured and
the inhabitants had fled.

On the 11th a company of 100 men under command of Captain J. W. Clegg and 100 men under command of Captain John H. L. Smith were sent to the village of Kibwezi to reconnoiter and to ascertain if there was any possibility of attacking the village.

The party started at 6 A.M. and reached the village about
10 A.M. and found it to consist of 100 houses built of mud
and straw. There were no trees or bushes near the village
and the ground was covered with stones and shells of broken

GLAVA II

U ovoj glavi daćemo neke osnovne pojmove teorije uopštenih funkcija i definisati prostore na kojima ćemo kasnije posmatrati uopštene slučajne procese. Za detaljnija objašnjenja videti [25,24,39,34,47].

2.1. OSNOVNI POJMOVI I PROSTOR DISTRIBUCIJA

1. Funkcije i prostori funkcija

Neka je (L, τ) vektorsko topološki prostor i $\sigma = \{B_\alpha : \alpha \in A\}$ familija ograničenih podskupova od L . Kažemo da je σ totalna familija ograničenih skupova ako je linearno proširenje od $\bigcup_{\alpha \in A} B_\alpha$ prostor L . Neka je (E, τ_1) vektorsko topološki prostor a (F, τ_2) lokalno konveksni prostor. Sa $\mathcal{L}(E, F)$ označimo skup neprekidnih linearnih preslikavanja iz (E, τ_1) u (F, τ_2) . Neka je σ totalna familija ograničenih skupova u E , S proizvoljan elemenat iz σ i V proizvoljna okolina nule u (F, τ_2) . Podskupovi od $\mathcal{L}(E, F)$ oblika $M(S, V) = \{f : f(S) \subset V\}$ čine bazu okolina nule u vektorskem prostoru $\mathcal{L}(E, F)$, u odnosu na koju je $\mathcal{L}(E, F)$ lokalno konveksni prostor.

Ako je σ familija svih konačnih podskupova od E tada se odgovarajuća topologija u $\mathcal{L}(E, F)$ naziva slaba topologija i obeležava se sa τ_s .

Ako je σ familija svih kompaktnih podskupova od E tada se odgovarajuća topologija u $\mathcal{L}(E, F)$ naziva topologija kompaktne konvergencije, i označava se sa τ_k .

Ako je σ familija svih ograničenih podskupova od E , tada

without retaining enough water weight from low water
depths so that they can't move up and down in elevation without
risking all available oxygen reserves. This causes us to need
more time to get back to the surface.

AN APPRAISE OF THEORETICAL PREDICTION

The first thing I wanted to do was to compare the predicted results with the observed. To do this I took the theoretical predictions and compared them with the observed data. The observed data was taken from the literature and the theoretical predictions were calculated using the same parameters. The results showed that the theoretical predictions were very close to the observed data. This indicates that the theoretical model is able to predict the behavior of the system accurately. The next step was to compare the theoretical predictions with the observed data at different elevations. The results showed that the theoretical predictions were very close to the observed data at all elevations. This indicates that the theoretical model is able to predict the behavior of the system accurately at all elevations. The final step was to compare the theoretical predictions with the observed data at different times. The results showed that the theoretical predictions were very close to the observed data at all times. This indicates that the theoretical model is able to predict the behavior of the system accurately at all times. The overall conclusion is that the theoretical model is able to predict the behavior of the system accurately.

se odgovarajuća topologija u $\mathcal{L}(E,F)$ naziva *jaka* topologija i označava se sa τ_b .

Između ovih topologija u $\mathcal{L}(E,F)$ postoji sledeći odnos:

$$\tau_s \leq \tau_k \leq \tau_b,$$

gde je simbolom $\tau_1 \leq \tau_2$ označeno da je topologija τ_2 finija od topologije τ_1 .

Neka je V vektorsko topološki prostor. Skup linearnih i neprekidnih funkcionala na V naziva se dualni prostor za V i obeležava sa V' .

$\text{supp } f$ - nosač funkcije f , odnosno zatvoreno zatvorenje skupa tačaka u kojima f nije nula

$\text{ess sup } |f(x)|$ - esencijalni supremum funkcije f

G - otvoren skup u \mathbb{R}^n

$C^m(G)$ - prostor m puta diferencijabilnih kompleksnih funkcija na G , $m \in \mathbb{N}_0$ ili $m=+\infty$

$C_c^\infty(G)$ - prostor C^∞ kompleksnih funkcija koje imaju kompaktan nosač. Elementi $C_c^\infty(G)$ se često nazivaju osnovne (test) funkcije

$C_c^\infty(K)$ - prostor C^∞ kompleksnih funkcija na \mathbb{R}^n koje nestaju identički van kompaktnog skupa K

$S \subset G$ - S je relativno kompaktan u G (zatvoreno od S u G je kompaktan skup u G)

Montelov prostor - lokalno konveksan, bačvast Hausdorfov prostor u kojem je svaki zatvoren i ograničen skup kompaktan. Strogi dual Montelovog prostora je takođe Montelov prostor.

পদ পরিবর্তন করে আবৃত করে দেওয়া হল। এইভাবে এক অন্য রীতে শব্দে শব্দ পরিবর্তন করা হল। এখন এই মুক্তি লক্ষণ পদটির একটি অন্য পুরোটা উপর আবৃত করে দেওয়া হল।

যদি এই অন্য পদটির অন্তর্ভুক্ত অবস্থায় একটি অন্য শব্দ আবৃত করে দেওয়া হল তবে এখন এই পদটি অন্য পদের অন্তর্ভুক্ত অবস্থায় একটি অন্য পুরোটা উপর আবৃত করে দেওয়া হল। এটা অন্য পদের অন্তর্ভুক্ত অবস্থায় একটি অন্য পুরোটা উপর আবৃত করে দেওয়া হল। এইভাবে একটি অন্য পদের অন্তর্ভুক্ত অবস্থায় একটি অন্য পুরোটা উপর আবৃত করে দেওয়া হল।

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Topologija u C^∞

Neka je K kompaktan podskup od G , $m \in \mathbb{N}_0$. Za $\varphi \in C^\infty(G)$ označimo

$$p_{m, K}(\varphi) = \max_{x \in K} \sum_{|\alpha| \leq m} |\partial^\alpha \varphi(x)|.$$

Tada, menjajući K i m , $p_{m, K}$ formira bazu neprekidnih seminormi u $C^\infty(G)$. U stvari, dovoljno je posmatrati samo slučaj kada K prolazi "iscrpljujući" niz kompaktnih podskupova od G , $\{K_\nu : \nu \in \mathbb{N}_0\}$ - što znači da je $K_\nu \subset K_{\nu+1}$ (K je unutrašnjost skupa K) i, da je svaki kompaktan podskup od G sadržan u nekom K_ν , $\nu \in \mathbb{N}_0$. Označimo $p_m = p_{m, K_m}$. Seminorme p_m definišu topologiju u $C^\infty(G)$. Baza okolina u $C^\infty(G)$ je

$$V_{m, \varepsilon}(\varphi_0) = \{\varphi \in C^\infty(G) : p_m(\varphi - \varphi_0) < \varepsilon\},$$

za odgovarajući izbor $m \in \mathbb{N}_0$, i $\varepsilon > 0$. Niz funkcija iz $C^\infty(G)$ $\{\varphi_n : n \in \mathbb{N}\}$ konvergira ka φ_0 iz $C^\infty(G)$ ako i samo ako, za svako $\alpha \in \mathbb{N}_0^n$, $\partial^\alpha \varphi_n$ konvergira ka $\partial^\alpha \varphi$ uniformno na svakom kompaktnom podskupu od G .

Topologija u $C^\infty(G)$ se može definisati pomoću metrike:

$$d(\varphi, \psi) = \sum_{m=0}^{\infty} 2^{-m} \inf(1, p_m(\varphi - \psi)).$$

Sve takve metrike su ekvivalentne, i sa takvom topologijom $C^\infty(G)$ je kompletan metrički prostor. Snabdeven sa prirodnom (t.j. C^∞) topologijom, $C^\infty(G)$ je Frechet-ov prostor, to jest lokalno konveksan vektorsko topološki prostor koji je metrizabilan i kompletan. U C^∞ topologiji svaki ograničen i zatvoren skup je kompaktan. To znači da je $C^\infty(G)$ sa navedenom topologijom Montelov prostor. Podskup u $C^\infty(G)$ je ograničen ako

of the kind, it is a good idea to consider what you can do

to increase your chances of getting a job.

There are many ways to increase your chances of getting a job. One way is to improve your skills. Another way is to network with other professionals in your field. You can also look for opportunities to work part-time or on a contract basis. It's important to keep your resume up-to-date and to apply for jobs as soon as possible. Good luck!

How to Get a Job in a Competitive Market

If you're looking for a job in a competitive market, it's important to stand out from the crowd. One way to do this is to focus on your strengths and weaknesses. Identify your strengths and weaknesses, and then highlight them in your resume and cover letter. This will help you stand out in a crowded market.

Another way to get a job in a competitive market is to network.

Networking is a great way to find job opportunities. You can join professional organizations, attend industry conferences, and participate in online forums. You can also ask for recommendations from your current employer or former employer. It's important to be persistent and patient when networking. It may take time to find the right opportunity, but it's worth it in the end.

je svaka seminorma $p_{m,K}$ ograničena na njemu.

Prirodna topologija prostora $C_c^\infty(G)$

Za bilo koji kompaktan podskup K od G , $C_c^\infty(K)$ je zatvoren, linearan, potprostor od $C_c^\infty(G)$ i snabdeven je indukovanim topologijom. U skupovnom smislu je

$$C_c^\infty(G) = \bigcup_{K \subset \subset G} C_c^\infty(K).$$

Važe sledeće osobine

- (1) Niz konvergira u $C_c^\infty(G)$ ako i samo ako je sadržan u $C_c^\infty(K)$ za neki kompaktan podskup K od G i konvergira u $C_c^\infty(K)$.
- (2) Podskup B od $C_c^\infty(G)$ je ograničen ako i samo ako je sadržan i ograničen u nekom $C_c^\infty(K)$.
- (3) Linearno preslikavanje $C_c^\infty(G)$ u proizvoljan lokalno konveksan prostor E je neprekidno ako i samo ako je njegova restrikcija na svaki podprostor $C_c^\infty(K)$, $K \subset \subset G$, neprekidna. (t.j. slika konvergentnog niza je konvergentan niz).
- (4) $C_c^\infty(G)$ je Montelov prostor.

2. Diferencijalni operatori na G

Neka je \mathcal{F} je definisana familija nombrova $\alpha_1, \alpha_2, \dots$

$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, $\alpha_1, \dots, \alpha_n \in \mathbb{N}_0$,

$$|\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n.$$

$$\partial_x^\alpha = \left(\frac{\partial}{\partial x_1} \right)^{\alpha_1} \cdots \left(\frac{\partial}{\partial x_n} \right)^{\alpha_n}, \quad x \in \mathbb{R}^n, \quad \alpha \in \mathbb{N}_0^n,$$

$$D_x^\alpha = \left(\frac{1}{i!} \frac{\partial}{\partial x_1} \right)^{\alpha_1} \cdots \left(\frac{1}{i!} \frac{\partial}{\partial x_n} \right)^{\alpha_n} = D^\alpha.$$

under der Bedingung gleich verhalten seien, so

ist die Verteilung auf die einzelnen Arbeit

gruppen in $\Omega^{(k)}$ so zu wählen, daß nicht alle Gruppenarbeiter in $\Omega^{(k)}$ die gleiche Anzahl von Arbeitstagen erledigen, und

$$\Omega^{(k)} \cap \{t = \theta\} \neq \emptyset \quad \text{und} \quad \text{dass keine Gruppe}$$

in $\Omega^{(k)}$ mehr als θ Tage arbeitet.

$\Omega^{(k)}$ ist zulässig, wenn es eine Gruppe in $\Omega^{(k)}$ mit $\leq 2\theta$ Tagen Arbeit hat und für jede Gruppe $t \in \Omega^{(k)}$ die Anzahl der Arbeitstage t nicht größer als θ ist. Eine solche Gruppe ist in $\Omega^{(k)}$ ein Kandidat für die Gruppe $\Omega^{(k)}$.

$\Omega^{(k)}$ ist zulässig, wenn es

eine Gruppe in $\Omega^{(k)}$ existiert, die höchstens θ Arbeitstage hat und für jede Gruppe $t \in \Omega^{(k)}$ die Anzahl der Arbeitstage t nicht größer als θ ist. Eine solche Gruppe ist in $\Omega^{(k)}$ ein Kandidat für die Gruppe $\Omega^{(k)}$.

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Linearni parcijalni diferencijalni operatori na G su polinomi od $D = (D_1, \dots, D_n)$ sa koeficijentima iz $C^\infty(\Omega)$, tako da je

$$P(x, D) = \sum_{|\alpha| \leq m} c_\alpha(x) D^\alpha.$$

Ako, za neko α , sa $|\alpha|=m$, c_α ne nestaje identički na G , m se naziva red od $P(x, D)$. Kada su koeficijenti konstante, pišemo $P(D)$.

Operator ${}^t P(x, D)$ transponovan za $P(x, D)$, definisan je sa

$${}^t P(x, D) u(x) = \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha [c_\alpha(x) u(x)]$$

Operator $P(x, D)^*$, adjungovani za $P(x, D)$ je definisan sa

$$P(x, D)^* = \overline{{}^t P(x, D)}.$$

Crta označava da su koeficijenti zamjenjeni sa svojim konjugovano kompleksnim vrednostima.

3. Topologija Schwartzovog prostora \mathcal{S}

Schwartzov prostor funkcija na \mathbb{R}^n brzoopadajućih u beskonačnosti je prostor

$$\mathcal{S} = \{\varphi \in C^\infty(\mathbb{R}^n) : q_m, M(\varphi) = \sup_{x \in \mathbb{R}^n} [(1+|x|^M) \sum_{|\alpha| \leq m} |\partial^\alpha \varphi(x)|] < \infty, m, M \in \mathbb{N}_0\}.$$

Topologija u \mathcal{S} je definisana familijom seminormi q_m, M , $m, M \in \mathbb{N}_0$. \mathcal{S} je Frechetov prostor. Svaki ograničen i zatvoren skup u \mathcal{S} je kompaktan. (Skup je ograničen u \mathcal{S} ako su seminorme ograničene na njemu).

Sledeća potapanja su sva neprekidna i slika im je gust skup

and so on. In this case, the first term in the expansion of $\langle \psi | \hat{H} | \psi \rangle$ is zero because the operator \hat{H} is Hermitian and the wavefunction ψ is an eigenstate of \hat{H} .

$$\langle \psi | \hat{H} | \psi \rangle = 0 = \langle \psi | \hat{H}^{\dagger} | \psi \rangle$$

so the next non-zero term in the expansion is the second term, which is proportional to $\langle \psi | \hat{V} | \psi \rangle$. This term is zero because the potential V is Hermitian and the wavefunction ψ is an eigenstate of V . Therefore, the expectation value of \hat{H} is zero.

$$\langle \psi | \hat{H} | \psi \rangle = \langle \psi | \hat{V} | \psi \rangle = \langle \psi | V | \psi \rangle$$

which is zero because the potential V is zero. Therefore, the expectation value of \hat{H} is zero. This is a general result: if a system is in an eigenstate of a Hermitian operator, the expectation value of that operator is zero.

For example, consider a particle in a one-dimensional box of length L . The wavefunction of the particle is given by $\psi(x) = A \sin(kx)$, where $k = \pi n / L$ and n is a positive integer. The expectation value of the position operator \hat{x} is given by

$$\langle \psi | \hat{x} | \psi \rangle = \int_{-\infty}^{\infty} \psi^*(x) x \psi(x) dx = \int_{-\infty}^{\infty} A \sin(kx)^* x A \sin(kx) dx = A^2 \int_{-\infty}^{\infty} \sin(kx)^* x \sin(kx) dx$$

which is zero because the function $x \sin(kx)$ is odd and the integral of an odd function over all real numbers is zero. This is a general result: if a system is in an eigenstate of a Hermitian operator, the expectation value of that operator is zero.

For example, consider a particle in a one-dimensional box of length L . The wavefunction of the particle is given by $\psi(x) = A \sin(kx)$, where $k = \pi n / L$ and n is a positive integer. The expectation value of the momentum operator \hat{p} is given by

$$(1) \quad C_c^\infty(G) \hookrightarrow L^p(G) \hookrightarrow L_{loc}^p(G), \quad (1 \leq p < +\infty)$$

$$(2) \quad C_c^\infty \hookrightarrow \mathcal{S} \hookrightarrow L^p, \quad (1 \leq p < +\infty).$$

4. Distribucije i prostori distribucija

$\mathcal{D}'(G)$ - prostor distribucija na G , odnosno prostor linearnih i neprekidnih preslikavanja iz $C_c^\infty(G)$ u \mathbb{C} , odnosno dual od $C_c^\infty(G)$, odnosno $\mathcal{L}(C_c^\infty(G), \mathbb{C})$

$\text{supp } T$ - nosač distribucije T , odnosno presek svih zatvorenih podskupova u čijem komplementu T identički nestaje

$\langle T, \varphi \rangle = T(\varphi) = (T, \bar{\varphi}) = \int T(x)\varphi(x)dx$ uparivanje između test funkcije φ i distribucije T . Tako, $T \in \mathcal{D}'(G), \varphi \in C_c^\infty(G)$ ili $T \in \mathcal{S}'(G), \varphi \in C^\infty(G)$

$\mathcal{S}'(G)$ - prostor distribucija sa kompaktnim nosačem u G , po definiciji dual od $C^\infty(G)$.

Konvergencija distribucija

Konvergencija distribucija je uniformna konvergencija na ograničenom podskupu od $C_c^\infty(G)$. Za nizove je ista kao i slaba konvergencija: $T_j \rightarrow T_0$, $j \rightarrow \infty$, ako i samo ako $\langle T_j, \varphi \rangle \rightarrow \langle T_0, \varphi \rangle$, $j \rightarrow \infty$, za svaku osnovnu funkciju φ .

Ograničeni skupovi distribucija

Skup B distribucija je ograničen ako i samo ako je za svako $\varphi \in C_c^\infty(G)$

1. $\text{H}_2\text{O} + \text{Na}_2\text{S} \rightarrow \text{NaHS} + \text{NaOH}$

2. $\text{NaHS} + \text{H}_2\text{O} \rightarrow \text{NaHSO}_3 + \text{H}_2$

3. $\text{NaHSO}_3 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 + \text{H}_2$

4. $\text{NaHSO}_4 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 \cdot \text{H}_2\text{O}$

5. $\text{NaHSO}_4 \cdot \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 + \text{H}_2\text{O}$

$\text{NaHSO}_4 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 \cdot \text{H}_2\text{O}$

6. $\text{NaHSO}_4 \cdot \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 + \text{H}_2\text{O}$

7. $\text{NaHSO}_4 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 \cdot \text{H}_2\text{O}$

8. $\text{NaHSO}_4 \cdot \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 + \text{H}_2\text{O}$

9. $\text{NaHSO}_4 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 \cdot \text{H}_2\text{O}$

10. $\text{NaHSO}_4 \cdot \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 + \text{H}_2\text{O}$

$\text{NaHSO}_4 + \text{H}_2\text{O} \rightarrow \text{NaHSO}_4 \cdot \text{H}_2\text{O}$

$$\sup_{T \in \mathcal{B}} |\langle T, \varphi \rangle| < +\infty.$$

Skupovi koji su ograničeni i zatvoreni u $\mathcal{D}'(G)$ (ili $\mathcal{E}'(G)$) su kompaktni.

Primena diferencijalnog operatora na distribuciju

Ako je $P(x, D)$ diferencijalni operator ne G , njegovo dejstvo na $T \in \mathcal{D}'(G)$ je definisano pomoću formule parcijalne integracije

$$\langle P(x, D), \varphi \rangle = \langle T, {}^t P(x, D) \varphi \rangle, \quad \varphi \in C_c^\infty(G).$$

Jasno je da $P(x, D)$ definiše linearno neprekidno preslikavanje od $\mathcal{D}'(G)$ (odnosno $\mathcal{E}'(G)$) u sebe. Specijalan slučaj je kada je red operatora nula, odnosno u tom slučaju imamo množenje sa C^∞ funkcijom ψ :

$$\langle \psi T, \varphi \rangle = \langle T, \varphi \psi \rangle,$$

koje definiše neprekidni endomorfizam od $\mathcal{D}'(G)$, ($\mathcal{E}'(G)$).

Primetimo da je $\text{supp } P(x, D)T \subset \text{supp } T$, odnosno diferencijalni operator smanjuje nosač.

Lokalna svojstva distribucije

Ako je data $T \in \mathcal{D}'(G)$ i $G' \subset G$, postoji konačan skup neprekidnih funkcija f_j i diferencijalnih operatora P_j na G , $j=1, 2, \dots, s$, tako da je na G'

$$T = \sum_{j=1}^s P_j f_j,$$

odnosno,

the CIO is still looking to the market to broaden its role beyond
the traditional labor relations function, but it has yet to do so.

It is important to note that the CIO's role in labor relations is not limited to collective bargaining.

The CIO also plays a key role in the development of labor standards. It has been instrumental in the development of national minimum wage laws, such as the Fair Labor Standards Act of 1938, which established a minimum wage of 25 cents per hour. It has also been involved in the development of other labor standards, such as the Occupational Safety and Health Act of 1970, which established standards for workplace safety and health. The CIO has also been involved in the development of labor standards for the construction industry, such as the Davis-Bacon Act of 1931, which establishes standards for wages and working conditions in the construction industry. The CIO has also been involved in the development of labor standards for agriculture, such as the Migrant Agricultural Worker Protection Act of 1970, which established standards for migrant agricultural workers.

The CIO also plays an important role in the development of labor standards for the service industry, such as the Minimum Wage Act of 1938, which established a minimum wage of 25 cents per hour. It has also been involved in the development of labor standards for the retail industry, such as the Retail Clerks International Union, which established standards for working conditions in retail stores.

The CIO has also been involved in the development of labor standards for the transportation industry, such as the Railway Labor Act of 1926, which established standards for working conditions in railroads and other transportation companies. It has also been involved in the development of labor standards for the energy industry, such as the National Energy Policy Development Group, which established standards for working conditions in the energy sector.

John G. T.
Editor

$$\langle T, \varphi \rangle = \sum_{j=1}^{\infty} \int f_j(x)^t P_j \varphi(x) dx, \quad \varphi \in C_c^\infty(G).$$

Ako T ima kompaktan nosač, može se uzeti da ova reprezentacija preko konačnog zbiru važi na celom G , a neprekidne funkcije f_j se mogu uzeti tako da se anuliraju van neke okoline od $\text{supp } T$.

Distribucije koje su funkcije

Kažemo da je distribucija T na Ω funkcija ako postoji funkcija f iz $L^1_{\text{loc}}(G)$ tako da je

$$\langle T, \varphi \rangle = \int f(x) \varphi(x) dx, \quad \varphi \in C_c^\infty(G).$$

Označimo takvu distribuciju T sa T_f . Tada je $f \rightarrow T_f$ linearna injekcija od $L^1_{\text{loc}}(G)$ u $\mathcal{D}'(G)$. Ovo preslikavanje definiše neprekidne injekcije u $\mathcal{D}'(G)$ prostora

$$C^m(G), (0 \leq m \leq +\infty); \quad C_c^\infty(G); \quad L^p_{\text{loc}}(G), (1 \leq p \leq +\infty).$$

Takođe, imamo neprekidnu injekciju u $\mathcal{D}'(G)$ prostora

$$C_c^m, (0 \leq m \leq +\infty); \quad L^p_c(G), (1 \leq p \leq +\infty).$$

Sve ove injekcije imaju guste slike.

Ako je Ω otvoren podskup od G , kažemo da je T (lokalno L^1) funkcija na Ω , ako je to tačno za restrikciju od T na Ω (odnosno na $C_c^\infty(\Omega)$). Na isti način može se kazati da je T funkcija neke druge vrste, na primer C^∞ funkcija.

Radonova mera je linearni neprekidni funkcional na prostoru C^0 neprekidnih funkcija na \mathbb{R}^n . C^0 je snabdeven topologijom uniformne konvergencije na kompaktnim podskupovima

od \mathbb{R}^n .

$\delta = \delta(x)$ - Dirakova mera u koordinatnom početku od \mathbb{R}^n . To je distribucija koja preslikava $\varphi \rightarrow \varphi(0)$, $\varphi \in C_c^\infty(\mathbb{R}^n)$.

$\delta_{x_0} = \delta(x-x_0)$ - Dirakova mera u tački x_0

$\delta^{(\alpha)} = \delta_x^\alpha \delta$ - α -ti izvod Dirakove mere δ

Ove distribucije nisu funkcije, za $\alpha \neq 0$, nisu čak ni Radonove mere.

2.2. PROSTORI \mathcal{A} i \mathcal{A}'

U ovom delu ćemo detaljno opisati prostore \mathcal{A} i \mathcal{A}' , na kojima ćemo u narednoj glavi definisati uopštene slučajne procese.

U [47] Zemanian je uveo prostore \mathcal{A} test funkcija, i njegov dual \mathcal{A}' . Sledеći njegove ideje konstruisaćemo skalu prostora \mathcal{A}_k , $k \in \mathbb{N}_0$, čiji se elementi mogu prikazati u obliku beskonačnog zbiru. Koristićemo notaciju iz [47].

Neka je I otvoren interval u skupu realnih brojeva i $L^2(I)$ prostor klasa ekvivalencije kvadratnih integrabilnih funkcija na I sa vrednostima u \mathbb{C} . Obeležimo normu u $L^2(I)$ sa

$$\|\varphi\|_0 = \left(\int_I |\varphi(t)|^2 dt \right)^{1/2}.$$

Neka je $C^\infty(I)$ skup beskonačno diferencijabilnih (glatkih) funkcija na I .

Neka je \mathcal{R} linearan diferencijalan samo-adjungovani operator oblika

$$\mathcal{R} = \Theta_0 D^{n_1} \Theta_1 \dots D^{n_\nu} \Theta_\nu,$$

takov da je

$$\mathcal{R} = \overline{\Theta}_\nu (-D)^{n_\nu} \dots (-D)^{n_2} \overline{\Theta}_1 (-D)^{n_1} \overline{\Theta}_0$$

$$\mathcal{R}^{k+1} = \mathcal{R}(\mathcal{R}^k), \quad k \in \mathbb{N},$$

$$\mathcal{R}^0 = J,$$

gde je J identički operator, $D = d/dx$, $n_k \in \mathbb{N}_0$, $k=1, \dots, \nu$, a Θ_k , $k=0, 1, \dots, \nu$, glatke funkcije bez nula na I . Pretpostavimo da postoje: niz realnih brojeva $\{\lambda_n: n \in \mathbb{N}_0\}$ i niz glatkih funkcija u $L^2(I)$, $\{\psi_n: n \in \mathbb{N}_0\}$ takvih da $|\lambda_n| \rightarrow \infty$, $n \rightarrow \infty$, i

$$\mathcal{R}\psi_n = \lambda_n \psi_n, \quad n \in \mathbb{N}_0.$$

Dalje, pretpostavimo da $\{\psi_n: n \in \mathbb{N}_0\}$ čini ortonormirani sistem u $L^2(I)$. Možemo prenumerisati λ_n i ψ_n tako da je $|\lambda_0| \leq |\lambda_1| \leq \dots \leq |\lambda_2| \leq \dots$.

Obeležimo

$$\tilde{\lambda}_n = \begin{cases} \lambda_n, & \lambda_n \neq 0 \\ 1, & \lambda_n = 0 \end{cases}, \quad n \in \mathbb{N}_0$$

$\{\tilde{\lambda}_n: n \in \mathbb{N}_0\}$ je niz takav da je $|\tilde{\lambda}_n| \rightarrow \infty$, $n \rightarrow \infty$.

Definišimo

$$\mathcal{A}_k = \{\varphi \in L^2(I): \varphi = \sum_{m=0}^{\infty} a_m \psi_m, \sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} < \infty\}, \quad k \in \mathbb{N}_0.$$

Prostor \mathcal{A}_k je Hilbertov prostor sa skalarnim proizvodom definisan na sledeći način

$$(\varphi, \psi)_k = \sum_{m=0}^{\infty} a_m \bar{b}_m \tilde{\lambda}_m^{2k}, \quad \varphi, \psi \in \mathcal{A}_k, \quad \varphi = \sum_{m=0}^{\infty} a_m \psi_m, \quad \psi = \sum_{m=0}^{\infty} b_m \psi_m.$$

Norma u \mathcal{A}_k je

$$\|\varphi\|_k = \left(\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} \right)^{1/2}, \quad \varphi \in \mathcal{A}_k.$$

Primetimo da je ortonormirani sistem u \mathcal{A}_k , $\tilde{\psi}_j = \psi_j \tilde{\lambda}_j^{-k}$,

$$\begin{aligned} & \text{if } \alpha \in \mathbb{R}^n \text{ and } \beta \in \mathbb{R}^{m \times n}, \text{ then } \alpha \beta = \beta \alpha \\ & \text{if } \alpha \in \mathbb{R}^n \text{ and } \beta \in \mathbb{R}^{m \times n}, \text{ then } \alpha \beta = \beta \alpha \end{aligned}$$

zur Verwendung mit den Ergebnissen der Gleichung (2.1) ist eine
Umformung von $\alpha \beta + \gamma \beta = (\alpha + \gamma)\beta$ zu zeigen und hierzu addieren wir $\alpha \beta$ zu
beidseitig $\alpha \beta$ und $\gamma \beta$. Wegen $\alpha \beta = \beta \alpha$ folgt aus der Addition die
 $\alpha \beta + \alpha \beta = \alpha \beta + \gamma \beta$ und weiter $\alpha \beta + \alpha \beta = \alpha \beta + (\gamma \beta + \alpha \beta)$ da $\alpha \beta$ einheitlich
 $\alpha \beta + \alpha \beta = \alpha \beta + \alpha \beta = \alpha \beta$

aus der Umformung $\alpha \beta + \gamma \beta = (\alpha + \gamma)\beta$ als umformungsweg zeigt sich
dass $\alpha \beta + \gamma \beta = (\alpha + \gamma)\beta$ ist. Hieraus folgt die Umformung $\alpha \beta + \gamma \beta = (\alpha + \gamma)\beta$ ist
gezeigt.

$$\alpha \beta + \gamma \beta = \left(\frac{\partial \alpha}{\partial x_1} x_1 + \dots + \frac{\partial \alpha}{\partial x_n} x_n \right) + \left(\frac{\partial \gamma}{\partial x_1} x_1 + \dots + \frac{\partial \gamma}{\partial x_n} x_n \right) = \alpha + \gamma$$

Wir zeigen nun $\alpha \beta = \beta \alpha$ für alle $\alpha \in \mathbb{R}^n$ und $\beta \in \mathbb{R}^{m \times n}$ ist zu zeigen dass $\alpha \beta = \beta \alpha$.

Wir schreiben $\alpha \beta = \alpha \beta$ und $\beta \alpha = \beta \alpha$ und zeigen dass $\alpha \beta = \beta \alpha$ ist.

$$\alpha \beta = \alpha \beta \Leftrightarrow \alpha \beta - \alpha \beta = 0 \Leftrightarrow \alpha \beta - \alpha \beta = \alpha(\beta - \beta) = \alpha \cdot 0 = 0$$

und $\beta \alpha = \beta \alpha \Leftrightarrow \beta \alpha - \beta \alpha = 0 \Leftrightarrow \beta \alpha - \beta \alpha = \beta(\alpha - \alpha) = \beta \cdot 0 = 0$

Wegen $\alpha \beta = \beta \alpha$ und $\beta \alpha = \alpha \beta$ ist $\alpha \beta = \beta \alpha$.

$$\alpha \beta = \beta \alpha \Leftrightarrow \alpha \beta - \beta \alpha = 0 \Leftrightarrow \alpha \beta - \beta \alpha = \alpha(\beta - \alpha) = \alpha \cdot 0 = 0$$

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$\alpha \beta = \beta \alpha$ und $\beta \alpha = \alpha \beta$ ist die Umformung $\alpha \beta = \beta \alpha$ ist gezeigt.

$j \in \mathbb{N}_0$. Očigledno je da je $\mathcal{A}_0 = L^2(I)$.

Neka su, za $k \in \mathbb{N}_0$

$$S = \{\varphi_s \in \mathcal{A}_k : \varphi_s = \sum_{m=0}^s a_m \psi_m, s \in \mathbb{N}_0, a_m \in \mathbb{C}, m \geq 0\}.$$

$$S_r = \{\varphi_s \in \mathcal{A}_k : \varphi_s = \sum_{m=0}^s (a_m + i b_m) \psi_m, s \in \mathbb{N}_0, a_m, b_m \in \mathbb{Q}\},$$

gde je \mathbb{Q} skup racionalnih brojeva. Skup S je gust, a skup S_r prebrojiv gust u \mathcal{A}_k , $k \in \mathbb{N}_0$. ($S_r \subset S$).

Operator $\tilde{\mathcal{R}}^n$, $n \in \mathbb{N}_0$ je definisan na skupu S . Iz činjenice da je preslikavanje $\mathcal{R}^n : S \rightarrow L^2(I)$, $n \in \mathbb{N}_0$, linearno i neprekidno, sledi da se $\tilde{\mathcal{R}}^n$, $n \leq k$, može linearno i neprekidno proširiti na prostor \mathcal{A}_k , $k \in \mathbb{N}_0$. Obeležimo to proširenje sa

$$\tilde{\mathcal{R}}^n, n \leq k. \text{ Neka je } \varphi_s = \sum_{m=0}^s a_m \psi_m \in S \text{ i } \varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}_k, k \in \mathbb{N}_0.$$

Imamo da $\varphi_s \rightarrow \varphi$, $s \rightarrow \infty$ u \mathcal{A}_k , pa je

$$\tilde{\mathcal{R}}^n \varphi = \tilde{\mathcal{R}}^n \left(\sum_{m=0}^s a_m \psi_m \right) = \lim_{s \rightarrow \infty} \left(\mathcal{R}^n \left(\sum_{m=0}^s a_m \psi_m \right) \right) = \sum_{m=0}^{\infty} a_m \tilde{\lambda}_m^n \psi_m.$$

Neka je $\varphi \in \mathcal{A}_k \cap C^\infty(I)$, $k \in \mathbb{N}_0$ i $(\tilde{\mathcal{R}}^n \varphi, \psi_m) = (\varphi, \tilde{\mathcal{R}}^n \psi_m)$, $n \leq k$, $m \in \mathbb{N}_0$. Tada je $\tilde{\mathcal{R}}^n \varphi = \mathcal{R}^n \varphi$, $n \leq k$. Dalje ćemo $\tilde{\mathcal{R}}$ obeležavati sa \mathcal{R} .

Definišimo prostore \mathcal{A}_{-k} , $k \in \mathbb{N}_0$ na sledeći formalan način

$$\mathcal{A}_{-k} = \{f : f = \sum_{m=0}^{\infty} b_m \psi_m, \sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} < \infty, b_m \in \mathbb{C}\}, k \in \mathbb{N}_0.$$

Prostor \mathcal{A}_{-k} , $k \in \mathbb{N}_0$ je vektorski prostor, sa operacijama definisanim na uobičajeni način. Na \mathcal{A}_{-k} , $k \in \mathbb{N}_0$ možemo definisati skalarni proizvod i normu na sledeći način.

Neka je $f = \sum_{m=0}^{\infty} b_m \psi_m$, $g = \sum_{m=0}^{\infty} c_m \psi_m \in \mathcal{A}_{-k}$, $k \in \mathbb{N}_0$, tada je

$$(f, g)_{-k} = \sum_{m=0}^{\infty} b_m \bar{c}_m \tilde{\lambda}_m^{-2k}, k \in \mathbb{N}_0$$

(O) $\mu_1 \approx 0.01$ mbar 100 $\mu\text{g} \approx 1$

$\mu\text{g} \approx 10^{-6}$ g/mole

$\mu\text{g} \approx 10^{-6}$ g/mole $\approx 10^{-6}$ mole/g

Quando se fizerem os cálculos para determinar quais são as espécies que possuem maior probabilidade de reagir com o gás a longo prazo, é importante levar em conta a constante de equilíbrio entre a reação e a sua inversa. A constante de equilíbrio é dada por $K = \frac{P_{\text{produtos}}}{P_{\text{reagentes}}}$, onde P é a pressão parcial da espécie em questão. Se a constante de equilíbrio é menor que 1,00, a reação é favorável, e vice-versa. Se a constante de equilíbrio é maior que 1,00, a reação é desfavorável.

Por exemplo, se $K = 0.001$, a reação é favorável, e se $K = 100$, a reação é desfavorável. Se a constante de equilíbrio é igual a 1,00, a reação é em equilíbrio. No entanto, é importante lembrar que a constante de equilíbrio só é válida para temperaturas específicas. Se a temperatura mudar, a constante de equilíbrio também mudará.

Portanto, é importante levar em conta a constante de equilíbrio quando se fizerem os cálculos para determinar quais espécies possuem maior probabilidade de reagir com o gás a longo prazo.

No entanto, é importante lembrar que a constante de equilíbrio só é válida para temperaturas específicas. Se a temperatura mudar, a constante de equilíbrio também mudará.

Portanto, é importante levar em conta a constante de equilíbrio quando se fizerem os cálculos para determinar quais espécies possuem maior probabilidade de reagir com o gás a longo prazo.

$\mu\text{g} \approx 10^{-6}$ g/mole $\approx 10^{-6}$ mole/g

$$\|f\|_{-k} = \left(\sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} \right)^{1/2}, \quad k \in \mathbb{N}_0.$$

Očigledno, \mathcal{A}_{-k} je Hilbertov prostor, $k \in \mathbb{N}_0$.

Neka je \mathcal{A}'_k , $k \in \mathbb{N}_0$, dualni prostor za \mathcal{A}_k . Važi sledeća teorema.

Teorema 2.2.1. Prostori \mathcal{A}'_k i \mathcal{A}_{-k} , $k \in \mathbb{N}_0$ su izometrični.

Dokaz. Neka je $f \in \mathcal{A}'_k$, $k \in \mathbb{N}_0$. Obeležimo $b_m = (f, \psi_m)$, $m \in \mathbb{N}_0$, i neka je $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}_k$, $k \in \mathbb{N}_0$. Kako je f linearno i neprekidno sledi da je

$$(f, \varphi) = \sum_{m=0}^{\infty} \bar{a}_m b_m.$$

Iz [25] sledi da je

$$\sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} < \infty.$$

Prema tome, postoji elemenat $g \in \mathcal{A}_{-k}$, $k \in \mathbb{N}_0$ tako da je

$$g = \sum_{m=0}^{\infty} b_m \psi_m.$$

Suprotno, ako je $g = \sum_{m=0}^{\infty} b_m \psi_m \in \mathcal{A}_{-k}$, $k \in \mathbb{N}_0$, tako da je za

$$\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}'_k,$$

$$\sum_{m=0}^{\infty} \bar{a}_m b_m \leq \infty,$$

tada preslikavanje $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \rightarrow \sum_{m=0}^{\infty} \bar{a}_m b_m$, $\varphi \in \mathcal{A}_k$, $k \in \mathbb{N}_0$

definiše elemenat iz \mathcal{A}'_k . Obeležimo taj elemenat sa f .

Očigledno je da $b_m = (f, \psi_m)$, $m \in \mathbb{N}_0$.

Prema tome, postoji obostrano jednoznačno preslikavanje iz \mathcal{A}'_k u \mathcal{A}_{-k} , $k \in \mathbb{N}_0$,

and the H_2O molecule is formed by the combination of two atoms of hydrogen and one atom of oxygen. This molecule is composed of three atoms, and it is the smallest molecule known. It is also the most abundant molecule in the universe. The oxygen atom has two electrons in its outer shell, and the hydrogen atoms each have one electron. When the two hydrogen atoms combine with the oxygen atom, they share their electrons, forming a covalent bond. This results in a molecule with a total of four electrons in its outer shell, which is a stable configuration. The resulting molecule is called water, and it is the most common substance on Earth.

$$f \rightarrow \sum_{m=0}^{\infty} b_m \varphi_m,$$

gde je $b_m = (f, \varphi_m)$, $m \in \mathbb{N}_0$. Očigledno je da je preslikavanje linearno. Pokazaćemo da je $|f|_k' = \|f\|_{-k}$ gde je $|f|_k'$ dualna norma u \mathcal{A}'_k .

Imamo da je

$$|(f, \varphi)| = \left| \sum_{m=0}^{\infty} b_m \bar{a}_m \right| \leq \left[\sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \left[\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} \right]^{1/2}$$

$$|(f, \varphi)| \leq \|f\|_{-k} \|\varphi\|_k$$

$$|f|_k' \leq \|f\|_{-k}.$$

Dalje, neka je $\varphi_s = \sum_{m=0}^s b_m \tilde{\lambda}_m^{-2k} \varphi_m \in \mathcal{A}_k$, $k \in \mathbb{N}_0$. Imamo da je

$$\|\varphi_s\|_k = \left[\sum_{m=0}^s |b_m|^2 |\tilde{\lambda}_m^{-2k}|^2 \tilde{\lambda}_m^{2k} \right]^{1/2} = \left[\sum_{m=0}^s |b_m|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2},$$

pa je

$$|f|_k' = \frac{(f, \varphi_s)}{\|\varphi_s\|_k} = \frac{\sum_{m=0}^s |b_m|^2 \tilde{\lambda}_m^{-2k}}{\left[\sum_{m=0}^s |b_m|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2}} = \\ \left[\sum_{m=0}^s |b_m|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \xrightarrow{s \rightarrow \infty} \left[\sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} = \|f\|_{-k}.$$

Sledi $|f|_k' = \|f\|_{-k}$, $k \in \mathbb{N}_0$. \square

Neka $X \hookrightarrow Y$ označava da se vektorsko topološki prostor X može linearno i neprekidno potopiti u vektorsko topološki prostor Y.

Lako se može pokazati da je

$$\dots \mathcal{A}_{k+1} \hookrightarrow \mathcal{A}_k \hookrightarrow \dots \mathcal{A}_1 \hookrightarrow \mathcal{A}_0 = L^2(I) \hookrightarrow \mathcal{A}_{-1} \hookrightarrow \dots \mathcal{A}_{-k} \hookrightarrow \mathcal{A}_{-(k+1)} \dots$$

Neka su

whereas the first two terms in (2.2) are $\sim (1,2,3) = \text{ad } \pi$ and the

third $\sim (1)$, so we get the required commutator relation

(2.1) in the form

as in section 1.

It is also clear that the commutator relation (2.1) holds for the Lie algebra \mathfrak{g} of the group G .

It remains to prove that the Lie bracket $[x,y]$ is given by the formula (2.1).

Let us consider the element $x = \sum_{i=1}^n x_i e_i$ of the Lie algebra \mathfrak{g} . Then

we have $[x,y] = \sum_{i=1}^n x_i [e_i, y] = \sum_{i=1}^n x_i e_i y - y e_i$ and

so we have to prove that $\sum_{i=1}^n x_i e_i y - y e_i = \sum_{i=1}^n x_i y e_i - y e_i e_i$.

It follows from the definition of the Lie bracket that

$[x_i, y] = \lim_{t \rightarrow 0} \frac{[e^{tx_i}, y] - y}{t} = \lim_{t \rightarrow 0} \frac{e^{tx_i} y - y e^{tx_i}}{t} = x_i y - y x_i$.

So we have $\sum_{i=1}^n x_i [e_i, y] = \sum_{i=1}^n x_i (x_i y - y x_i) = \sum_{i=1}^n x_i x_i y - \sum_{i=1}^n x_i y x_i$.

It follows from the definition of the Lie bracket that

$[x_i, x_j] = \lim_{t \rightarrow 0} \frac{[e^{tx_i}, e^{tx_j}] - e^{tx_j} e^{tx_i}}{t} = \lim_{t \rightarrow 0} \frac{e^{tx_i} e^{tx_j} - e^{tx_j} e^{tx_i}}{t} = x_i x_j - x_j x_i$.

So we have $\sum_{i=1}^n x_i x_i y = \sum_{i=1}^n x_i (x_i x_j - x_j x_i) y = \sum_{i=1}^n x_i x_i x_j y - \sum_{i=1}^n x_i x_j x_i y$.

It follows from the definition of the Lie bracket that

$[x_i, x_j] = \lim_{t \rightarrow 0} \frac{[e^{tx_i}, e^{tx_j}] - e^{tx_j} e^{tx_i}}{t} = \lim_{t \rightarrow 0} \frac{e^{tx_i} e^{tx_j} - e^{tx_j} e^{tx_i}}{t} = x_i x_j - x_j x_i$.

So we have $\sum_{i=1}^n x_i x_j x_i y = \sum_{i=1}^n x_i (x_i x_j - x_j x_i) y = \sum_{i=1}^n x_i x_i x_j y - \sum_{i=1}^n x_i x_j x_i y$.

$$\mathcal{A} = \bigcap_{k=0}^{\infty} \mathcal{A}_k = \{\varphi \in L^2(I) : \varphi = \sum_{m=0}^{\infty} a_m \psi_m, \forall k \in \mathbb{N}_0, \sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} < \infty\};$$

$$\mathcal{A}' = \bigcup_{k=0}^{\infty} \mathcal{A}_{-k} = \{f : f = \sum_{m=0}^{\infty} b_m \psi_m, \exists k \in \mathbb{N}_0, \sum_{m=0}^{\infty} |b_m|^2 \tilde{\lambda}_m^{-2k} < \infty\}.$$

Kako je $S \subseteq \mathcal{A}$, sledi da je \mathcal{A} gust u svakom \mathcal{A}_k , $k \in \mathbb{N}_0$.

Sledi da je \mathcal{A}_k , $k \in \mathbb{N}_0$, kompletiranje od \mathcal{A} u odnosu na normu $\|\cdot\|_k$. Tako iz Teoreme 2.2.1. sledi da je \mathcal{A}' dual od \mathcal{A} , videti takođe [25].

Iz Lemе 9.3.3. [47, p.316] sledi da su prostori \mathcal{A} i \mathcal{A}' identični sa prostorima Zemaniana \mathcal{A} i \mathcal{A}' , definisanim u [47; ch 9.3. i 9.6.].

Kako je $\mathcal{R}^n : \mathcal{A}_k \rightarrow L^2(I)$, $n \leq k$, $k \in \mathbb{N}_0$ možemo definisati $(\mathcal{R}^n)' : L^2(I) \rightarrow \mathcal{A}_{-k}$, $n \leq k$,

na sledeći način

$$((\mathcal{R}^n)')f, \varphi = (f, \mathcal{R}^n \varphi), \quad n \leq k, \quad \varphi \in \mathcal{A}_k, \quad f \in L^2(I).$$

Ako je $f \in L^2(I)$ oblika

$$f = \sum_{m=0}^{\infty} b_m \psi_m, \quad \sum_{m=0}^{\infty} |b_m|^2 < \infty,$$

i imamo da je

$$(\mathcal{R}^n)'f = \sum_{m=0}^{\infty} b_m \lambda_m^n \psi_m, \quad n \leq k, \quad f \in \mathcal{A}_{-k}, \quad k \in \mathbb{N}_0.$$

Očigledno je da se $(\mathcal{R}^n)'$ može definisati i na \mathcal{A}_{-p} , $p \in \mathbb{N}$, $n \leq p$, na sledeći način:

$$((\mathcal{R}^n)')f, \varphi = (f, \mathcal{R}^n \varphi), \quad \varphi \in \mathcal{A}_k, \quad f \in \mathcal{A}_{-p}, \quad n \leq \min\{k, p\}, \quad k, p \in \mathbb{N}_0,$$

i imamo da

$$(\mathcal{R}^n)' : \mathcal{A}_{-p} \rightarrow \mathcal{A}_{-p-n}.$$

Kako je formalno

$$(\mathcal{R}^n)'f = \sum_{m=0}^{\infty} b_m \lambda_m^n \psi_m, \quad f \in \mathcal{A}_{-k}, \quad n \leq k,$$

$\text{m} > \frac{\log \frac{1}{\delta}}{\log \frac{1}{\alpha}}$ \Rightarrow $m^2 > 10^6 \cdot \log \frac{1}{\delta} \Rightarrow m > 10^3 \cdot \log \frac{1}{\delta} = 10^3 \cdot \frac{1}{\delta}$

$\text{m} > \frac{\log \frac{1}{\delta}}{\log \frac{1}{\alpha}}$ \Rightarrow $m^2 > 10^6 \cdot \log \frac{1}{\delta} \Rightarrow m > 10^3 \cdot \frac{1}{\delta}$

so we can choose m large enough so that $m^2 \geq \frac{1}{\delta}$ and $m^2 \geq \frac{1}{\alpha}$.

Since α depends on n , we can choose m large enough so that $m^2 \geq \frac{1}{\alpha}$.

So we can choose m large enough so that $m^2 \geq \frac{1}{\delta}$ and $m^2 \geq \frac{1}{\alpha}$.

Since $m^2 \geq \frac{1}{\delta}$ and $m^2 \geq \frac{1}{\alpha}$, we have $m^2 \geq \frac{1}{\delta} \cdot \frac{1}{\alpha}$.

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obeležavaćemo $(\mathcal{R}^n)'$ sa \mathcal{R}^n .

Neka je $\Lambda = \{n \in \mathbb{N}_0 : \lambda_n = 0\}$, $\Lambda^c = \mathbb{N}_0 \setminus \Lambda$. Važi sledeća reprezentaciona teorema.

Teorema 2.2.2. Neka je $f \in \mathcal{A}_{-k}$, $k \in \mathbb{N}_0$, oblika $f = \sum_{m=0}^{\infty} b_m \psi_m$, i

neka je $F = \sum_{m \in \Lambda^c} b_m \lambda_m^{-k} \psi_m$. Tada je $F \in L^2(I)$ i

$$f = \mathcal{R}^k F + \sum_{m \in \Lambda} b_m \psi_m.$$

Dokaz je dat u [47, Teorema 9.6.2.].

U [47, Lemma 9.3.1.] pokazana je sledeća teorema

Teorema 2.2.3. Prostor \mathcal{A} je kompletan, Frešeov prostor.

Daćemo nekoliko primera operatora \mathcal{R} i odgovarajućih nizova $\{\psi_n : n \in \mathbb{N}_0\}$ i $\{\lambda_n : n \in \mathbb{N}_0\}$.

1. Furieovi polinomi

1.a. Prvi oblik

$$I = (-\pi, \pi)$$

$$\mathcal{R} = -iD = i^{-1/2} Di^{-1/2}$$

$$\psi_n(x) = \frac{e^{inx}}{\sqrt{2\pi}}, \quad n=0, \pm 1, \pm 2, \dots$$

$$\lambda_n = n.$$

1.b. Drugi oblik

$$I = (0, \pi),$$

$$\mathcal{R} = D^2,$$

$$\psi_0(x) = \pi^{-1/2}, \quad \psi_n(x) = \sqrt{\frac{2}{\pi}} \cos nx, \quad n=1, 2, 3, \dots$$

$$\lambda_n = -n^2, \quad n=0, 1, 2, \dots$$

1.c. Treći oblik

$$I = (0, \pi),$$

$$\mathcal{R} = D^2,$$

$$\psi_n(x) = \sqrt{\frac{2}{\pi}} \sin nx, \quad n=1, 2, \dots$$

$$\lambda_n = -n^2, \quad n=1, 2, 3, \dots$$

2. Lagerovi polinomi

$$I = (0, \infty),$$

$$\mathcal{R} = x^{-\alpha/2} e^{x/2} D x^{\alpha+1} e^{-x} D x^{-\alpha/2} e^{x/2} = x D^2 + D - \frac{x}{4} - \frac{\alpha}{4x} + \frac{\alpha+1}{2},$$

$$\alpha > -1,$$

$$\psi_n(x) = \left[\frac{\Gamma(n+1)}{\Gamma(\alpha+n+1)} \right]^{1/2} x^{\alpha/2} e^{-x/2} L_n^\alpha(x), \quad n=0, 1, 2, \dots$$

$$L_n^\alpha(x) = \sum_{m=0}^{\infty} \binom{n+\alpha}{n-m} \frac{(-x)^m}{m!}$$

$$\lambda_n = -n, \quad n=0, 1, 2, \dots$$

3. Ermitovi polinomi

$$I = (-\infty, \infty)$$

$$\mathcal{R} = e^{x^2/2} D e^{-x^2} D e^{x^2/2} = D^2 - x^2 + 1$$

$$\psi_n(x) = \frac{e^{-x^2/2} H_n(x)}{[2^n n! \sqrt{\pi}]^{1/2}}, \quad n=0, 1, 2, \dots$$

$$H_n(x) = n! \sum_{m=0}^{[n/2]} \frac{(-1)^m (2x)^{n-2m}}{m! (n-2m)!}$$

$$\lambda_n = -2n.$$

4. Ležandrovi polinomi

$$I = (-1, 1)$$

$$\mathcal{R} = D(x^2 - 1)D,$$

$$\psi(x) = \sqrt{n + \frac{1}{2}} P(x), \quad n=0, 1, 2, \dots$$

$$P_n(x) = 2^{-n} \sum_{m=0}^{[n/2]} (-1)^m \binom{n}{m} \binom{2n-2m}{n} x^{n-2m}$$

W. L. G. 1947 2,2

$$\langle R, S \rangle = 1$$

$$R = 2$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R}$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R}$$

$$\frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R} = \frac{1}{R} \sin \alpha \sin \frac{\pi}{R}$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\langle R, S \rangle = 1$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \frac{\partial \langle R, S \rangle}{\partial R} \frac{\partial \sin \alpha}{\partial R} = \langle R, S \rangle$$

$$\frac{\partial \langle R, S \rangle}{\partial R} \frac{\partial \sin \alpha}{\partial R} = \frac{\partial \langle R, S \rangle}{\partial R} \frac{\partial \sin \alpha}{\partial R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\langle R, S \rangle = 1$$

$$\langle R, S \rangle = 1$$

$$\dots \text{B.L.G.} \sin \alpha \sin \frac{\pi}{R} = \langle R, S \rangle$$

$$\dots \text{B.L.G.} \left(\frac{\partial \sin \alpha}{\partial R} \right) \left(\frac{\pi}{R} \right) \langle R, S \rangle = \frac{\partial \langle R, S \rangle}{\partial R} \frac{\partial \sin \alpha}{\partial R} = \langle R, S \rangle$$

$$\lambda_n = n(n+1).$$

5. Polinomi Čebiševa

$$I = (-1, 1)$$

$$R = (1-x^2)^{1/4} D(1-x)^{1/2} D(1-x)^{1/4},$$

$$\psi_0(x) = \pi^{-1/2} (1-x^2)^{-1/4},$$

$$\psi_n(x) = \sqrt{\frac{2}{\pi}} (1-x^2)^{-1/4} T_n(x), \quad n=1, 2, \dots$$

$$T_n(x) = \sum_{m=0}^{[n/2]} \frac{(-1)^m (n-m-1)!}{m! (n-2m)!} (2x)^{n-2m},$$

$$\lambda_n = -n^2, \quad n=0, 1, 2, \dots$$

Još primera se može naći u [47, 3].

2.3. PROSTORI $\mathcal{D}_K^{(M_p)}$ I $\mathcal{D}^{(M_p)}(\emptyset)$

Prostori $\mathcal{D}^{(M_p)}(\emptyset)$ dati su u [24].

Neka je \emptyset otvoren podskup od \mathbb{R}^n i K regularan kompaktan skup u \emptyset u smislu Whitney-a. Ovo znači da je K kompaktan skup u \mathbb{R}^n sa sledećom osobinom: postoji $c > 0$ tako da su bilo koje dve tačke x, y bilo koje povezane komponente $L \subset K$ povezane lukom u L čija je dužina manja ili jednaka od $c|x-y|$. Dalje ćemo sa K uvek obeležavati regularan kompaktan skup. Neka je $\{M_p : p \in \mathbb{N}\}$ niz pozitivnih brojeva tako da je

$$(M.1) \quad M_p^2 \leq M_{p-1} M_{p+1}, \quad p \in \mathbb{N};$$

$$(M.2) \quad M_p \leq AH^p M_{p-1}, \quad \text{za neko } A > 0, \quad H > 0, \quad p \in \mathbb{N}_0.$$

$$(M.3) \quad \sum_{p=1}^{\infty} M_{p-1}/M_p < \infty.$$

Neka je $C(K)$ skup neprekidnih funkcija na K snabdeven

general population 20

$$(1.2)^{-1} \approx 2$$

$$e^{(1.2)(1.2)} = e^{(1.2)^2} = e^{2.4} = 10.59$$

$$e^{(1.2)(1.2)} = 10.59$$

so the general population is the initial 20% increased by 10.59% giving us 20.1059 which is quite reasonable. So, we have the exponential function $y = e^{kt}$ which is the same as $y = e^{kt}$ where k is the growth rate. This is a simple exponential relationship as $y = e^{kt}$ is a simple exponential relationship that has either a positive or negative linear growth or a constant non-increasing growth rate. It can start at zero or it can start at some value other than zero. The exponential function is a very useful function in many applications.

$$y = e^{kt} \text{ where } k = 1.2 \text{ (0.12)}$$

$$y = e^{1.2t} \text{ where } t = \text{time in years} \text{ (0.12)}$$

$$y = e^{1.2t} \text{ (0.12)}$$

Exponential growth is an important consideration when trying to predict

normom

$$\|f\|_{C(K)} = \sup_{x \in K} |f(x)|.$$

Neka je $h > 0$. Obeležimo sa X_h prostor beskonačno diferencijabilnih funkcija φ sa $\text{supp } \varphi \subset K$ tako da je

$$\|\varphi\|_{X_h} = \sup_{\alpha \in \mathbb{N}_0^n} \left\{ \frac{h^{|\alpha|} \|D^\alpha \varphi\|_{C(K)}}{M^{|\alpha|}} \right\} < \infty,$$

i

$$\frac{h^{|\alpha|} \|D^\alpha \varphi\|_{C(K)}}{M^{|\alpha|}} \rightarrow 0, \quad |\alpha| \rightarrow \infty.$$

Prostor X_h je Banahov. Prostori $\mathcal{D}_K^{(MP)}$ i $\mathcal{D}^{(MP)}(\emptyset)$ se definišu na sledeći način [24, pp. 44, 77].

$$\mathcal{D}_K^{(MP)} = \varprojlim_{j \rightarrow \infty} X_j,$$

$$\mathcal{D}^{(MP)}(\emptyset) = \varinjlim_{K \subset \subset \emptyset} \mathcal{D}_K^{(MP)},$$

gde su \varinjlim i \varprojlim induktivna, odnosno projektivna topološka granica.

$K \subset \subset \emptyset$ znači da K pripada familiji regularnih podskupova od \emptyset čija je unija \emptyset . Ovi prostori imaju mnoge "lepe" osobine, videti [24]. Na primer, ovi prostori su separabilni i kompletни.

Strogi dual od $\mathcal{D}^{(MP)}(\emptyset)$, prostor $(\mathcal{D}^{(MP)}(\emptyset))'$ je prostor Beurling-ovih ultradistribucija.

Obeležimo sa Y prostor nizova $(\psi_\alpha) = \{\psi_\alpha : \alpha \in \mathbb{N}_0^n\}$ iz $C(K)$ za koje je

$$\|(\psi_\alpha)\|_Y = \sup_{\alpha \in \mathbb{N}_0^n} \|\psi_\alpha\|_{C(K)} < \infty$$

i

normom

$$\|f\|_{C(K)} = \sup_{x \in K} |f(x)|.$$

Neka je $h > 0$. Obeležimo sa X_h prostor beskonačno diferencijabilnih funkcija φ sa $\text{supp } \varphi \subset K$ tako da je

$$\|\varphi\|_{X_h} = \sup_{\alpha \in \mathbb{N}_0^n} \left\{ \frac{h^{|\alpha|} \|D^\alpha \varphi\|_{C(K)}}{M^{|\alpha|}} \right\} < \infty,$$

i

$$\frac{h^{|\alpha|} \|D^\alpha \varphi\|_{C(K)}}{M^{|\alpha|}} \rightarrow 0, \quad |\alpha| \rightarrow \infty.$$

Prostor X_h je Banahov. Prostori $\mathcal{D}_K^{(MP)}$ i $\mathcal{D}^{(MP)}(\emptyset)$ se definišu na sledeći način [24, pp. 44, 77].

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$$\|(\psi_\alpha)\|_Y = \sup_{\alpha \in \mathbb{N}_0^n} \|\psi_\alpha\|_{C(K)} < \infty$$

i

measured reaction of the reaction $\text{O}_2 + \text{CH}_3 \rightarrow \text{CH}_2 + \text{H}_2\text{O}$

$$\left(\frac{\text{d}[\text{CH}_2]}{\text{d}t} \right) = k_1 [\text{CH}_3] [\text{O}_2]$$

$$k_1 = \frac{1}{2} \cdot \frac{\text{d}[\text{CH}_2]}{\text{d}t} \cdot \frac{1}{[\text{CH}_3][\text{O}_2]}$$

estimated at (3) $10^{-10} \text{ cm}^3 \text{ mole}^{-1} \text{ sec}^{-1}$ measured at 20°C

$$k_1 = \frac{1}{2} \cdot \frac{\text{d}[\text{CH}_2]}{\text{d}t} \cdot \frac{1}{[\text{CH}_3][\text{O}_2]}$$

the second order reaction constant k_2 of equation (2) is given by

as the second order rate coefficient of the reaction $\text{CH}_2 + \text{O}_2 \rightarrow \text{CH}_3 + \text{HO}_2$ measured "and" equal to $1.0 \times 10^{-10} \text{ cm}^3 \text{ mole}^{-1} \text{ sec}^{-1}$ at 20°C

using (3) $10^{-10} \text{ cm}^3 \text{ mole}^{-1} \text{ sec}^{-1}$ as the second

order rate coefficient of the reaction $\text{CH}_3 + \text{O}_2 \rightarrow \text{CH}_2 + \text{HO}_2$ measured in dissociation

(2) $\text{CH}_3 + \text{O}_2 \rightarrow \text{CH}_2 + \text{HO}_2$ energy values γ are selected

as follows: $\gamma_{\text{CH}_3} = 10.0$, $\gamma_{\text{O}_2} = 10.0$, $\gamma_{\text{CH}_2} = 10.0$,

$$\lim_{|\alpha| \rightarrow \infty} \|\psi_\alpha\|_{C(K)} = 0.$$

Prostor Y je separabilan Banahov prostor. Neka je d_h , $h > 0$, preslikavanje iz X_h u Y definisano sa

$$d_h(\varphi) = \left(\frac{h^{|\alpha|(-1)} |\alpha| D^\alpha \varphi}{M_{|\alpha|}} \right), \quad \varphi \in X_h.$$

Obeležimo $\bar{X}_h = d_h(X_h) \subset Y$.

Propozicija 2.3.1. Važi da je

$$D_K^{(MP)} \xrightarrow{i_1} X_h \xrightarrow{d_h} \bar{X}_h \xrightarrow{i_2} Y,$$

gde su i_1 i i_2 neprekidna potapanja a d_h je izometrija.

Dokaz. Neka je $\varphi \in D_K^{(MP)}$, i $l > k$.

Tada je

$$\frac{h^{|\alpha|} \|D^\alpha \varphi\|_{C(K)}}{M_{|\alpha|}} \leq \left(\frac{h}{l}\right)^{|\alpha|} \|\varphi\|_{X_l} \rightarrow 0, \quad |\alpha| \rightarrow \infty,$$

što implicira da je i_1 neprekidno potapanje.

Druga dva tvrđenja lako sledi.

2.4. PROSTORI Exp^A i $\text{Exp}^{A'}$

Neka je $\{\alpha_n: n \in \mathbb{N}_0\}$ niz kompleksnih brojeva različitih od nule. Označimo sa $S(\alpha_n)$ i $S^X(\alpha_n)$ prostor nizova [25] definisanih sa:

$$\{a_m\} = \{a_m: m \in \mathbb{N}_0\} \in S(\alpha_n)$$

ako i samo ako je za svako $k \in \mathbb{N}_0$

$\hat{D} = \text{diag}_n(\hat{\pi})$ will
be used.

As it is right to admit, many varieties and mixtures of \hat{D} to obtain
no conclusion $\hat{\theta}$ of all observations

$$\hat{\theta}^{\text{MLE}} = \underset{\theta}{\operatorname{arg\,max}} \ell(\theta) = (\hat{\theta})_{\text{MLE}}$$

$\hat{\theta} = (\hat{\theta}_{\text{MLE}}, \hat{\theta}_{\text{MLR}})$ are called

the maximum likelihood estimator

and the maximum likelihood ratio estimator

respectively. In this paper we consider the maximum likelihood estimator

of θ .

Therefore, we have $\hat{\theta}^{\text{MLE}} = (\hat{\theta}_{\text{MLE}}, \hat{\theta}_{\text{MLR}})^T$

coinciding with formula (1) in the introduction with
the same number of observations and sample

size n and m respectively.

It is well known that the maximum likelihood estimator is the best estimator in the class of unbiased estimators. This is due to the fact that the maximum likelihood estimator is the minimum variance unbiased estimator (MLE). It follows from theorems about the maximum likelihood estimator and its properties.

$$Cov(\hat{\theta}) = (\hat{\theta}^{\text{MLE}} - \theta_{\text{MLE}})(\hat{\theta}^{\text{MLE}} - \theta_{\text{MLE}})^T$$

which is called the covariance matrix of the maximum likelihood estimator.

$$\sum_{m=0}^{\infty} |a_m|^2 |\alpha_m|^{2k} < \infty;$$

$$\{b_m\} = \{b_m : m \in \mathbb{N}_0\} \in S^x(\alpha_n)$$

ako i samo ako postoji $k \in \mathbb{N}_0$

$$\sum_{m=0}^{\infty} |b_m|^2 |\alpha_m|^{-2k} < \infty.$$

Neka su I , \mathcal{R} , $\{\psi_m : m \in \mathbb{N}_0\}$, $\{\lambda_m : m \in \mathbb{N}_0\}$ kao u delu 2.2.

ove glave, i neka je

$$\tilde{\lambda}_m = \begin{cases} |\lambda_m|, & \lambda_m \neq 0 \\ 1, & \lambda_m = 0 \end{cases}, \quad m \in \mathbb{N}_0$$

Označidemo sa $\exp_p \mathcal{A}$, $p \in \mathbb{N}$, potprostor od $L^2(I)$ definisan na sledeći način (videti [34]).

$$\exp_p \mathcal{A} = \left\{ \varphi \in L^2(I) : \varphi = \sum_{m=0}^{\infty} a_m \psi_m, \sum_{m=0}^{\infty} |a_m|^2 \exp_p \tilde{\lambda}_m^{2k} < \infty \right\}, \quad p \in \mathbb{N}$$

gde je

$$\exp_p \tilde{\lambda}_m = \underbrace{\exp (\exp \dots (\exp}_{p} \tilde{\lambda}_m)), \quad p \in \mathbb{N}.$$

Dual prstora $\exp_p \mathcal{A}$, označidemo sa $\exp_p \mathcal{A}'$. U [34] je dokazano da ako $f \in \exp_p \mathcal{A}'$, tada je

$$f = \sum_{m=0}^{\infty} b_m \psi_m, \quad (2.4.1)$$

gde red konvergira slabo u $\exp_p \mathcal{A}'$ i gde je

$$b_m = (f, \psi_m), \quad m=0,1,2,\dots$$

$$\{b_m\} \in S^x(\exp_p \tilde{\lambda}_n);$$

Obrnuto, ako $\{b_m\} \in S^x(\exp_p \tilde{\lambda}_n)$, tada je redom u (2.4.1) jedinstveno definisan elemenat iz $\exp_p \mathcal{A}'$.

$$m \geq \frac{1}{2} \left\| \omega^* \right\|^2 \left\| \omega^* \right\| \frac{1}{\lambda}$$

$$\text{Cost} \approx 1.16 \approx \text{cost} \approx 1.03$$

After 3 iterations the error is 0.03

After 4 iterations the error is 0.01

After 5 iterations the error is 0.005

After 6 iterations the error is 0.0015 which is the result

After 7 iterations the error is 0.0005

After 8 iterations the error is 0.0001

After 9 iterations the error is 0.00005

Implementation of the backtracking step is as follows:

Let $\alpha = \left(m - \frac{\lambda}{2} \left\| \omega^* \right\|^2 \right) \frac{1}{\lambda} \leq 1$. Then choose

~~the smallest $\alpha > 0$ such that~~

at least one of the two constraints in your iteration limit

is satisfied, i.e. either $\omega^T \omega \leq 1$ or $\omega^T b \leq c$.

Otherwise, let $\alpha = \frac{m}{\lambda}$ and repeat the iteration.

Also, when $\alpha = 0$, open a new iteration and then stop

the algorithm.

~~After 10 iterations the cost is 0.00005~~

Finally, it makes no sense to keep going in this case because

the gradient becomes zero for all

Označimo sa $\exp_{p,k} \mathcal{A}$, $k \in \mathbb{N}_0$, $p \in \mathbb{N}$, potprostor od $L^2(I)$ definisan sa,

$$\exp_{p,k} \mathcal{A} = \left\{ \varphi \in L^2(I) : \varphi = \sum_{m=0}^{\infty} a_m \psi_m, \sum_{m=0}^{\infty} |a_m|^2 \exp(2k(\underbrace{\exp \dots (\exp}_{p-1} \tilde{\lambda}_m)) < \infty \right\},$$

$$(\text{za } p=1, \sum_{m=0}^{\infty} |a_m|^2 \exp(2k\tilde{\lambda}_m) < \infty).$$

U skupovnom smislu je

$$\exp_p \mathcal{A} = \lim_{k \rightarrow \infty} \exp_{p,k} \mathcal{A}.$$

Propozicija 2.4.1.

(1) Prostori $\exp_{p,k} \mathcal{A}$, $k \in \mathbb{N}_0$ su Banahovi prostori.

(2) Potapanja

$$i_k : \exp_{p,k+1} \mathcal{A} \rightarrow \exp_{p,k} \mathcal{A}, \quad k \in \mathbb{N}_0$$

su kompaktne.

Dokaz. Prvi deo tvrđenja se može dokazati na standardan način a drugi sledi iz teoreme Kolmogorova koja daje potrebne i dovoljne uslove za kompaktnost u prostorima nizova. □

Skup E , gde je

$$E = \left\{ \varphi \in \exp_{p,k} \mathcal{A} : \varphi_s = \sum_{m=0}^s a_m \psi_m, s \in \mathbb{N}_0, a_m \in \mathbb{C}, m \in \mathbb{N}_0 \right\}.$$

je gust u svakom prostoru $\exp_{p,k} \mathcal{A}$, $k \in \mathbb{N}_0$.

Projektivni niz

$$\{\exp_{p,k} \mathcal{A} : k \in \mathbb{N}_0\}$$

je redukovani [24, p. 33.]. Tako, Propozicija 2.4.1. implicira da je

$$\exp_p \mathcal{A}' = (\lim_{k \rightarrow \infty} \exp_{p,k} \mathcal{A})' = \lim_{k \rightarrow \infty} \exp_{p,k} \mathcal{A}',$$

2001 No telecaching. 100% of the animals were captured.

100% successful

$$\left\{ \text{if } \left(\text{Catched} = \text{Successful} \right) \text{Then } \frac{\theta}{\text{Catched}} \text{ else } \frac{\theta}{\text{Not Caught}} \right\} \text{where } \theta = 100$$

$$\text{do } \left\{ \text{Catched} = \frac{\theta}{\text{Catched}} \right\} \text{ until } \text{Catched} = 100$$

100% capture guaranteed. UNTIL

100% capture guaranteed. UNTIL

100% capture guaranteed.

100% capture guaranteed. UNTIL

100% capture guaranteed.

100% capture guaranteed. UNTIL

100% capture guaranteed.

$$\left\{ \text{if } \left(\text{Catched} = \text{Successful} \right) \text{Then } \frac{\theta}{\text{Catched}} \text{ else } \frac{\theta}{\text{Not Caught}} \right\} = 100$$

100% capture guaranteed. UNTIL

100% capture guaranteed. UNTIL

100% capture guaranteed. UNTIL

100% capture guaranteed. UNTIL

100%

100% capture guaranteed. UNTIL

u smislu jake topologije, gde su

$$\exp_{p,k} \mathcal{A}' = \left\{ f = \sum_{m=0}^{\infty} b_m \psi_m : \sum_{m=0}^{\infty} |b_m|^2 \exp(-2k(\underbrace{\exp \dots (\exp}_{p-1} \tilde{\lambda}_m)) < \infty \right\}, k \in \mathbb{N}_0$$

Propozicija 2.4.2.

- (1) Niz $\{\exp_p \mathcal{A} : p \in \mathbb{N}\}$ je projektivan i redukovani u odnosu na potapanja

$$i_p : \exp_{p+1} \mathcal{A} \rightarrow \exp_p \mathcal{A}$$

koja su kompaktne;

- (2) $\text{Exp} \mathcal{A} = \lim_{\substack{\leftarrow \\ p \rightarrow \infty}} \exp_p \mathcal{A}$ je Freše-Švacov prostor;

- (3) $\text{Exp} \mathcal{A}' = (\lim_{\substack{\leftarrow \\ p \rightarrow \infty}} \exp_p \mathcal{A})' = \lim_{\substack{\rightarrow \\ p \rightarrow \infty}} \exp_p \mathcal{A}'$ u smislu jake topologije;

- (4) ako je $\{b_n : n \in \mathbb{N}_0\}$ niz kompleksnih brojeva tako da za neko $p \in \mathbb{N}$ i $k \in \mathbb{N}_0$

$$\sum_{m=0}^{\infty} |b_m|^2 \exp(-2k(\underbrace{\exp \dots (\exp}_{p-1} \tilde{\lambda}_m)) < \infty, \quad (2.4.2.)$$

tada $\sum_{m=0}^{\infty} b_m \psi_m$ konvergira slabo u $\text{Exp} \mathcal{A}'$ nekom elementu f .

Obratno, ako $f \in \text{Exp} \mathcal{A}'$, postoji $p \in \mathbb{N}$ i $k \in \mathbb{N}_0$ tako da za kompleksne brojeve $b_n = (f, \psi_m)$, $m \in \mathbb{N}_0$, (2.4.2) važi i

$$f = \sum_{m=0}^{\infty} b_m \psi_m$$

u smislu slabe konvergencije u $\text{Exp} \mathcal{A}'$.

Dokaz. (1) Prostor E je gust potprostor u $\exp_p \mathcal{A}$, $p \in \mathbb{N}$.

Slično kao u Propoziciji 2.4.1. može se pokazati da je i_p , $p \in \mathbb{N}$ kompaktno

(2) sledi iz [47, p.103, 1.8],

(3) sledi iz (1);

on wing configuration and volume of

$$\text{lift} = \left\{ C_L \cdot \frac{\rho}{2} \cdot V^2 \cdot S \cdot \left(1 - \frac{C_D}{C_L} \cdot \frac{V^2}{V_{ref}^2} \right) \right\} \cdot \eta \cdot \text{area}$$

lift is proportional

volume is proportional to chord length so lift is proportional to chord

proportional to

lift is proportional to chord

lift is proportional to chord

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared

lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared

lift is proportional to chord squared so lift is proportional to chord squared

lift is proportional to chord squared

(4) sledi iz (3). \square

Iz Propozicije 2.4.2. direktno sledi:

Propozicija 2.4.3.

(1) $\text{Exp}^A = \lim_{\substack{\leftarrow \\ p \rightarrow \infty}} \exp_{p,p}^A$ i potapanja

$i_p: \exp_{(p+1),(p+1)}^A \rightarrow \exp_{p,p}^A, p \in \mathbb{N},$

su kompaktna,

(2) $\text{Exp}^A' = \lim_{\substack{\rightarrow \\ p \rightarrow \infty}} \exp_{p,p}^A'$ u smislu jake topologije.

Iz Propozicije 2.4.3. sledi da je Exp^A Montelov prostor
sto opet implicira da je Exp^A' Montelov prostor. Tako,
Propozicija 2.4.3. direktno implicira :

Propozicija 2.4.4.

(1) Slaba i stroga sekvensijalna konvergencija u Exp^A' su
ekvivalentne;

(2) Exp^A i Exp^A' su refleksivni.

Uvek ćemo pretpostaviti da je topologija u Exp^A' jaka
dualna topologija.

Iz Propozicije 2.4.3. sledi

Propozicija 2.4.5. Niz $\{f_n\}$ u Exp^A' konvergira ka $f \in \text{Exp}^A'$
ako i samo ako za neko $p \in \mathbb{N}$ i $k \in \mathbb{N}_0$, $f_n \in \exp_{p,k}^A$, $n \in \mathbb{N}$,
 $f \in \exp_{p,k}^A$ i $f_n \rightarrow f$ u $\exp_{p,k}^A$.

Propozicija 2.4.5. Linearan operator $L: \text{Exp}^A' \rightarrow \text{Exp}^A'$ je
neprekidan ako i samo ako za svaki niz $\{f_n\}$ iz Exp^A' i
 $f \in \text{Exp}^A'$, takve da $f_n \rightarrow f$ u Exp^A' , važi da $Lf_n \rightarrow Lf$ u
 Exp^A' .

o (ED) et their CO

likely reflecting the CO's influence on the

CO's role in decision

making. In this particular case, the CO's CO

had a very strong influence on the CO's behavior, likely reflecting the CO's desire to maintain a positive relationship with the CO's CO.

Overall, the findings suggest that the CO's CO's influence on the CO's behavior is significant and may be more pronounced than previously thought.

However, it is important to note that the CO's CO's influence on the CO's behavior is likely to be influenced by the CO's CO's own characteristics, such as their level of experience and their ability to influence the CO's behavior.

Overall, the findings suggest that the CO's CO's influence on the CO's behavior is significant and may be more pronounced than previously thought.

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GLAVA III

3.1. UOPŠTENI SLUČAJNI PROCESI

U ovoj glavi daćemo definiciju uopštenog slučajnog procesa kao i razlike reprezentacije uopštenih slučajnih procesa na prostoru Zemaniana \mathcal{A} , prostoru $\mathcal{D}^{(MP)}(\mathcal{O})$ i $\text{Exp}\mathcal{A}$.

Neka je (Ω, \mathcal{F}, P) prostor verovatnoće i neka je P kompletna mera. U daljem radu smatraćemo da je (Ω, \mathcal{F}, P) fiksiran.

Neka je V vektorsko-topološki prostor test funkcija i V' njegov dualni prostor.

Definicija 3.1.1. Uopšteni slučajni proces na $\Omega \times V$ je preslikavanje $\xi: \Omega \times V \rightarrow \mathbb{C}$ tako da je

- (i) $\forall \varphi \in V$, $\xi(\cdot, \varphi)$ kompleksna slučajna promenljiva,
- (ii) $\forall \omega \in \Omega$, $\xi(\omega, \cdot)$ je elemenat iz V' .

Uopšteni slučajni proces ξ na $\Omega \times V$ ćemo kraće zapisati u.s.p. ξ na $\Omega \times V$. Dalje, posmatraćemo samo kompleksne slučajne promenljive, pa ćemo u daljem tekstu izostavljati reč "kompleksan" sem ako je to neophodno. "Klasičan" slučajni proces na $\Omega \times \mathbb{R}$ ili $\Omega \times \mathbb{R}^n$ ćemo nazivati slučajan proces.

Raspodela uopštenog slučajnog procesa je verovatnosna mera na prostoru uopštenih funkcija i strogo govoreći, taj prostor je potrebno izabrati specijalno za svaki pojedinačni proces.

Neka je I konačni ili beskonačni interval u skupu \mathbb{R} (\mathbb{R}^n) i neka je $H=L^2(I)$. Posmatrajmo trojku $V \subseteq H \subseteq V'$ gde su oba

III AWARD

DISCUSSION OF THE WINNING DESIGN

The committee commended the winning design because it
utilized the available space effectively and efficiently. It was suggested
that a 100' wide runway be considered extending the existing
runway 9' at each of the outermost corners (extending about
10' from the present 90' to the outermost corner) and that U turns
be eliminated from landing patterns to prevent the possibility
of aircraft crashing into the outermost corner of the field. The
committee also recommended that the outermost corner of the field be
utilized for parking aircraft and that the outermost corner of the
runway be used for landing aircraft. The committee further
recommended that the outermost corner of the runway be used
for parking aircraft and that the outermost corner of the runway be
utilized for landing aircraft.

The committee commended the V-shaped landing pattern and found
it to be the most effective landing pattern. The V-shaped landing pattern
was recommended because it is more efficient than the U-shaped landing
pattern. The committee also recommended that the outermost corner of
the runway be used for parking aircraft and that the outermost corner of the
runway be utilized for landing aircraft. The committee further
recommended that the outermost corner of the runway be used
for parking aircraft and that the outermost corner of the runway be
utilized for landing aircraft.

The committee commended the V-shaped landing pattern and
the outermost corner of the runway for its effectiveness in preventing
aircraft from crashing into the outermost corner of the runway.

potapanja neprekidna, gde je V prebrojivo normiran Hilbertov nuklearni prostor i V' njegov dual. Dualni prostor V' je osnovni prostor na kome će biti lokalizovana raspodela uopštenog slučajnog procesa.

Kanonička bilinearna forma koja povezuje V i V' se obeležava sa

$$\langle f, \varphi \rangle, \quad f \in V', \quad \varphi \in V.$$

Specijalno, ako je $f \in H$, tada se $\langle f, \varphi \rangle$ poklapa sa skalarnim proizvodom u H .

Neka su $\varphi_1, \varphi_2, \dots, \varphi_n \in V$. Zajednička raspodela slučajnog vektora $(\xi(\omega, \varphi_1), \xi(\omega, \varphi_2), \dots, \xi(\omega, \varphi_n))$ je jednoznačno određena karakterističnom funkcijom

$$\int_{\Omega} \exp \left[i \sum_{j=0}^n t_j \xi(\omega, \varphi_j) \right] dP(\omega), \quad z_1, \dots, z_n \in \mathbb{R}.$$

Kako je ξ linearno po φ , ova karakteristična funkcija je u stvari karakteristična funkcija slučajne promenljive $\xi(\cdot, \sum_{j=0}^n t_j \varphi_j)$. Takođe, imamo da $\sum_{j=0}^n t_j \varphi_j \in V$, pa odatle sledi da je raspodela posmatranog uopštenog slučajnog procesa ξ potpuno određena funkcionalom

$$C_{\xi}(\varphi) = \int_{\Omega} \exp [i \xi(\omega, \varphi)] dP(\omega).$$

Funkcional $C_{\xi}(\varphi)$, $\varphi \in V$, naziva se karakteristični funkcional uopštenog slučajnog procesa ξ i ima sledeće osobine:

- (1) $C_{\xi}(\varphi)$ je neprekidan po $\varphi \in V$;
- (2) $C_{\xi}(\varphi)$ je pozitivno definitan, t.j. za proizvoljno $n \in \mathbb{N}$, proizvoljne $a_1, a_2, \dots, a_n \in \mathbb{C}$ i $\varphi_1, \varphi_2, \dots, \varphi_n \in V$ je

vermedellit nördligen av Kyrönström vid enig annan tillräcklig utsträckning
av V. vid denna tidpunkt kunde vagnen V. i. rörelsen tillståndet
avseende det första året vid den tiden att vagnen kunde
köras med goda hastigheter och med goda
och vid denna tidpunkt vid denna
tidpunkt vid denna

V. 1. V. 2. V. 3. V. 4.

är beträffande en expedition d. 20. decr. 1860 vid vilken denna vagn
var medtagen. Denna expedition bestod i huvudsak
av en resa från Stockholm till Östersund. H. d. underhållande
berättelser om vagnens tillhörsallt. V. 1. var vagnen vid denna
resan i särskilt goda tillstånd. V. 2. var vagnen vid denna
resan i särskilt goda tillstånd. V. 3. var vagnen vid denna
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resan i särskilt goda tillstånd.

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är därför att denna resan är en resa från Stockholm till Östersund.

ispunjeno

$$\sum_{j,k} \alpha_j \bar{\alpha}_k C_\xi(\varphi_j - \varphi_k) \geq 0;$$

$$(3) \quad C_\xi(0) = 1.$$

Ovi uslovi su isti kao i u konačno dimenzionalnom slučaju (Definicija 1.2.4.), pa je, slično kao u Teoremi 1.2.1. potrebno naći verovatnosnu mjeru μ , tako da je

$$C_\xi(\varphi) = \int_V e^{i\langle f, \varphi \rangle} d\mu(f),$$

gde je C_ξ zadati funkcional koji zadovoljava (1)-(3).

Međutim, mere na beskonačno dimenzionalnim prostorima se razlikuju od mera na konačno dimenzionalnim prostorima. Teoremu Bohnera (Teorema 1.2.1.) koja daje uzajamno jednoznačnu korespondenciju između raspodela i karakterističnih funkcija potrebno je modifikovati. Ovde ćemo samo navesti teoremu, a detaljan dokaz dat je u [13].

Neka je B Borelov podskup u \mathbb{R}^n . Podskup od V' oblika

$$A_{\varphi_1, \dots, \varphi_n, B} = \{f \in V': \langle f, \varphi_1 \rangle, \dots, \langle f, \varphi_n \rangle \in B\} \quad (3.1.1)$$

naziva se cilindrični skup. Ako su svi φ_i , $i=1,2,\dots,n$ u (3.1.1) izabrani iz konačno dimenzionalnog potprostora F prostora V , kažemo da cilindrični skup ima bazu u F . Familija svih cilindričnih skupova koji imaju bazu u jednom, fiksiranom F obrazuje σ -algebru \mathcal{U}_F . Unija

$$\mathcal{U} = \bigcup_{F \subset V} \mathcal{U}_F$$

gde F prolazi po svim konačno dimenzionalnim podprostorima F je algebra podskupova u V' . Neka je \mathcal{V} najmanja σ -algebra koja sadrži \mathcal{U} .

the species. The last two species are described by the author in the present paper.

Cirrophilus ciliatus is a small beetle, 2.5 to 3.0 mm. long, with a broad, flat body, and a short, slightly convex, apical lobe on the elytra.

The head is broad, with a deep, longitudinal, median depression.

The antennae are inserted in the middle of the head, above the mouth, and consist of eleven segments, the first three being longer than the others.

The pronotum is broad, with a deep, longitudinal, median depression, and the sides are slightly raised, so that the sides of the pronotum are slightly convex (fig. 1).

The elytra are broad, with a deep, longitudinal, median depression, and the sides are slightly raised, so that the sides of the elytra are slightly convex (fig. 1).

The legs are short, and the tarsi are slightly curved, so that the tarsal claws are directed forward.

The wings are broad, with a deep, longitudinal, median depression, and the sides are slightly raised, so that the sides of the wings are slightly convex (fig. 1).

The body is black, with a few pale yellowish hairs on the head, pronotum, and elytra, and a few pale yellowish hairs on the antennae, legs, and tarsi.

The body is black, with a few pale yellowish hairs on the head, pronotum, and elytra, and a few pale yellowish hairs on the antennae, legs, and tarsi.

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The body is black, with a few pale yellowish hairs on the head, pronotum, and elytra, and a few pale yellowish hairs on the antennae, legs, and tarsi.

Teorema 3.1.1. Neka je $C(\varphi)$ karakteristični funkcional na V , t.j. funkcional koji zadovoljava uslove (1), (2), (3). Tada postoji jedinstvena verovatnosna mera μ na (V, \mathcal{F}) takva da je

$$C(\varphi) = \int_V e^{i\langle f, \varphi \rangle} d\mu(f).$$

U daljem radu ćemo često koristiti probabilističku Hahn-Banach -ovu teoremu, pa ćemo je ovde navesti. Ovu teoremu je dokazao Hanš u [12]. Nešto drugačiju formu, koju ćemo ovde navesti dao je Ullrich u [45].

Teorema 3.1.2. Neka je V separabilan Banahov prostor i neka je $W \subset V$ proizvoljna mnogostruktost u V . Neka je ξ u.s.p. na $\Omega \times W$, za koji postoji slučajna promenljiva S takva da za svako $\omega \in \Omega, v \in W$,

$$|\xi(\omega, v)| \leq S(\omega) \|v\|.$$

Tada postoji u.s.p. ξ^* definisan na $\Omega \times V$ tako da je za svako $\omega \in \Omega, v \in W$

$$\xi^*(\omega, v) = \xi(\omega, v),$$

a za svako $\omega \in \Omega, v \in V$,

$$|\xi^*(\omega, v)| \leq S(\omega) \|v\|.$$

3.2. REPREZENTACIJE UOPŠTENOG SLUČAJNOG PROCESA NA $\Omega \times \mathcal{A}$

Teorema 3.2.1. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Tada za svako $\varepsilon > 0$ postoji $k = k(\varepsilon) \in \mathbb{N}_0$, postoji skup $B \in \mathcal{F}$, takav da je $P(B) \geq 1 - \varepsilon$, i niz slučajnih promenljivih $\{c_m : m \in \mathbb{N}_0\}$ tako da je

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$$(1) \quad \xi(\omega, \varphi) = \sum_{m=0}^{\infty} c_m(\omega) (\varphi_m, \varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_n^{-2k} \right]^{1/2} < k, \quad \omega \in B.$$

Dokaz. Dokaz je sličan dokazu Leme 4. i Teoreme 1. u [45].

Videti takođe [1, 42, 29].

Za svako $\omega_0 \in \Omega$ imamo da je $\xi(\omega_0, \cdot) \in \mathcal{A}'$. Tada sledi da postoji $C(\omega_0)$ i $k(\omega_0)$ tako da je

$$|\xi(\omega_0, \varphi)| \leq C(\omega_0) \|\varphi\|_{k(\omega_0)}, \quad \varphi \in \mathcal{A}'.$$

Neka je

$$A_N(\varphi) = \{\omega \in \Omega : |\xi(\omega, \varphi)| \leq N \|\varphi\|_N, \quad N \in \mathbb{N}_0, \quad \varphi \in \mathcal{A}\},$$

$$A_N = \bigcap_{\varphi \in \mathcal{A}} A_N(\varphi), \quad N \in \mathbb{N}_0.$$

Tada je

$$A_N = \bigcap_{\varphi \in S_r} A_N(\varphi), \quad N \in \mathbb{N}_0$$

gde je S_r prebrojiv gust skup u \mathcal{A} . (Videti tačku 2.2. Glave II.) Kako je S_r prebrojiv, sledi da je A_N merljiv skup. Dalje, imamo da je

$$A_N \subset A_{N+1}, \quad i \quad \Omega = \bigcup_{N=0}^{\infty} A_N.$$

Dakle, za dato $\varepsilon > 0$, postoji $k = k(\varepsilon) \in \mathbb{N}_0$, tako da je $P(A_k) \geq 1 - \varepsilon$. Ako obeležimo $A_k = B$, dobijamo da je

$$|\xi(\omega, \varphi)| \leq k \|\varphi\|_k, \quad \varphi \in \mathcal{A}, \quad \omega \in B.$$

Za $\varphi \in \mathcal{A}$, definišimo

$$\xi_1(\omega, \varphi) = \begin{cases} \xi(\omega, \varphi), & \omega \in B \\ 0, & \omega \notin B. \end{cases} \quad (3.2.1)$$

Algebraic theory defined for $\mathbb{R} = \{0, \infty\}$, $\mathbb{C} =$

$$\mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \times \left[\begin{matrix} \mathbb{R} & \mathbb{R} \\ \mathbb{R} & \mathbb{R} \end{matrix} \right] \quad \text{etc.}$$

Let \mathcal{A} be a \mathbb{R} -algebra. A \mathbb{R} -valued function f on \mathcal{A} is called

(\mathcal{A}, \mathbb{R})-smooth if f

is smooth about $0 \in \mathbb{R}$ and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$.

A \mathbb{R} -valued function f on \mathcal{A} is called \mathbb{C} -smooth if it is smooth about $0 \in \mathbb{R}$ and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$.

Let \mathcal{A} be a \mathbb{R} -algebra. A \mathbb{R} -valued function f on \mathcal{A} is called

\mathbb{R} -analytic

if there exists a neighborhood U of $0 \in \mathbb{R}$ such that f is \mathbb{R} -smooth on U and $f(x) = \lim_{y \rightarrow x} f(y)$ for all $x \in U$.

A \mathbb{R} -valued function f on \mathcal{A} is called \mathbb{C} -analytic if it is \mathbb{R} -analytic about $0 \in \mathbb{R}$ and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$.

Let \mathcal{A} be a \mathbb{R} -algebra. A \mathbb{R} -valued function f on \mathcal{A} is called

\mathbb{R} -holomorphic if it is \mathbb{R} -analytic and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$ for all α .

A \mathbb{R} -valued function f on \mathcal{A} is called \mathbb{C} -holomorphic if it is \mathbb{R} -holomorphic about $0 \in \mathbb{R}$ and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$ for all α .

Let \mathcal{A} be a \mathbb{R} -algebra. A \mathbb{R} -valued function f on \mathcal{A} is called

\mathbb{R} -meromorphic if it is \mathbb{R} -analytic and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$ for all α .

A \mathbb{R} -valued function f on \mathcal{A} is called \mathbb{C} -meromorphic if it is \mathbb{R} -meromorphic about $0 \in \mathbb{R}$ and satisfies the condition $\partial^\alpha f / \partial x^\alpha(0) = 0$ for all α .

Neka je, za $\omega \in \Omega$

$$S(\omega) = \sup \{ |\xi_1(\omega, \varphi)|, \varphi \in \mathcal{A}, \|\varphi\|_k \leq 1 \} =$$

$$= \sup \{ |\xi_1(\omega, \varphi)|, \varphi \in S_r, \|\varphi\|_k \leq 1 \}.$$

Imamo da je $S(\cdot)$ merljiva funkcija, $S(\cdot) \leq k$ i

$$|\xi_1(\omega, \varphi)| \leq S(\omega) \|\varphi\|_k, \varphi \in \mathcal{A}, \omega \in B.$$

Prema probabilističkoj Hahn-Banach -ovojoj teoremi, ξ_1 se može proširiti na \mathcal{A}_k , tako da norma ostaje očuvana. Obeležimo ovo proširenje sa $\tilde{\xi}_1$. Imamo da je

$$|\tilde{\xi}_1(\omega, \varphi)| \leq S(\omega) \|\varphi\|_k, \varphi \in \mathcal{A}_k, \omega \in \Omega.$$

Neka je $\ell^2 = \{ \{a_m, m \in \mathbb{N}_0 \} : a_m \in \mathbb{C}, \sum_{m=0}^{\infty} |a_m|^2 < \infty \}$

Preslikavanje iz $\Omega \times \mathcal{A}_k$ u $\Omega \times \ell^2$ definisano na sledeći način,

$$\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}_k$$

$$i: (\omega, \varphi) \rightarrow (\omega, \{\tilde{\lambda}_m^k a_m\}),$$

je izometrija prostora $\Omega \times \mathcal{A}_k$ i $\Theta = i(\Omega \times \mathcal{A}_k) \subset \Omega \times \ell^2$.

Možemo definisati u.s.p. ξ_2 na Θ na sledeći način

$$\xi_2(\omega, \{\tilde{\lambda}_m^k a_m\}) = \tilde{\xi}_1(\omega, \varphi), \varphi \in \mathcal{A}_k, \omega \in \Omega,$$

gde je $(\omega, \{\tilde{\lambda}_m^k a_m\}) = i(\omega, \varphi)$. Tada je

$$|\xi_2(\omega, \{\tilde{\lambda}_m^k a_m\})| \leq S(\omega) \left[\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} \right]^{1/2}, \omega \in \Omega.$$

Prema probabilističkoj Hahn-Banach -ovojoj teoremi ξ_2 se može proširiti na $\Omega \times \ell^2$. Obeležimo ovo proširenje sa $\tilde{\xi}_2$. Imamo da je

$$\tilde{\xi}_2(\omega, \{\tilde{\lambda}_m^k a_m\}) = \xi_2(\omega, \{\tilde{\lambda}_m^k a_m\}), \quad (\omega, \{\tilde{\lambda}_m^k a_m\}) \in \Theta, \omega \in \Omega,$$

10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23.

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10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23.

i

$$|\tilde{\xi}_2(\omega, \{b_n\})| \leq S(\omega) \|\{b_m\}\|_{l^2}, \quad \{b_m\} \in l^2, \quad \omega \in \Omega.$$

Za svako $\omega \in \Omega$, $\tilde{\xi}_2(\omega, \cdot)$ je neprekidan i linearan funkcional na l^2 . Prema tome, postoji niz $\{\tilde{c}_m(\omega)\}$: $m \in \mathbb{N}_0$ tako

da je $\sum_{m=0}^{\infty} |\tilde{c}_m(\omega)|^2 < \infty$, $\omega \in \Omega$, i

$$\tilde{\xi}_2(\omega, \{b_m\}) = \sum_{m=0}^{\infty} \tilde{c}_m(\omega) \bar{b}_m, \quad \{b_m\} \in l^2, \quad \omega \in \Omega.$$

Na analogan način možemo definisati u.s.p. na $\Omega \times L^2(I)$,

koji ćemo obeležiti na isti način,

$$\tilde{\xi}_2: \Omega \times L^2(I) \rightarrow \mathbb{C},$$

$$\tilde{\xi}_2(\omega, \varphi) = \tilde{\xi}_2(\omega, \{b_m\}), \quad \varphi = \sum_{m=0}^{\infty} b_m \psi_m \in L^2(I), \quad \{b_m\} \in l^2$$

Kako je $\tilde{\xi}_2(\cdot, \varphi)$ slučajna promenljiva za svako $\varphi \in L^2(I)$, sledi, za $\varphi = \psi_m$, da su $\tilde{c}_m(\cdot) = \tilde{\xi}_2(\cdot, \psi_m)$, $m \in \mathbb{N}_0$ slučajne promenljive. Staviš, dualna norma

$$|\tilde{\xi}_2(\omega, \cdot)|_{L^2(I)} = \left[\sum_{m=0}^{\infty} |\tilde{c}_m(\omega)|^2 \right]^{1/2} = R(\omega), \quad \omega \in \Omega.$$

Imamo, za $\omega \in B$, $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in A$,

$$\xi(\omega, \varphi) = \xi_1(\omega, \varphi) = \tilde{\xi}_1(\omega, \varphi) = \xi_2(\omega, \{\tilde{a}_m\}) = \tilde{\xi}_2(\omega, \{\tilde{a}_m\}) =$$

$$= \sum_{m=0}^{\infty} \tilde{c}_m(\omega) \tilde{a}_m = \sum_{m=0}^{\infty} \tilde{c}_m(\omega) \tilde{\lambda}_m^k (\psi_m, \varphi) = \sum_{m=0}^{\infty} c_m(\omega) (\psi_m, \varphi),$$

gde je $c_m(\omega) = \tilde{c}_m(\omega) \tilde{\lambda}_m^k$, $m \in \mathbb{N}_0$. \square

$$G \times \mathbb{R}^n \ni (x, u) \mapsto (x, u) \in G \times \{0\} \subset \mathbb{R}^n$$

Lemma 1. π_1 is a closed subgroup of G , and π_1 is a closed subgroup of π_1 for every $n \in \mathbb{N}$.

Proof. Let $x \in G$ and $u \in \mathbb{R}^n$. Then $(x, u) \in G \times \mathbb{R}^n$.

Let $y \in G$ and $v \in \mathbb{R}^n$. Then $(y, v) \in G \times \mathbb{R}^n$.

$(x, u) \cdot (y, v) = (xy, u+v)$ and $(y, v) \cdot (x, u) = (yx, v+u)$.

Since $(x, u) \cdot (y, v) = (y, v) \cdot (x, u)$, π_1 is a closed subgroup of G .

Let $x \in G$ and $u \in \mathbb{R}^n$. Then $(x, u) \in G \times \mathbb{R}^n$.

Let $y \in G$ and $v \in \mathbb{R}^n$. Then $(y, v) \in G \times \mathbb{R}^n$.

$(x, u) \cdot (y, v) = (xy, u+v) = (y, v) \cdot (x, u)$ and $(y, v) \cdot (x, u) = (yx, v+u) = (x, u) \cdot (y, v)$.

π_1 is a closed subgroup of G .

Let $x \in G$ and $u \in \mathbb{R}^n$. Then $(x, u) \in G \times \mathbb{R}^n$.

Let $y \in G$ and $v \in \mathbb{R}^n$. Then $(y, v) \in G \times \mathbb{R}^n$.

$(x, u) \cdot (y, v) = (xy, u+v) = (y, v) \cdot (x, u)$ and $(y, v) \cdot (x, u) = (yx, v+u) = (x, u) \cdot (y, v)$.

π_1 is a closed subgroup of G .

π_1 is a closed subgroup of G .

Reprezentaciju u.s.p. ξ možemo proširiti na skup $\Omega \setminus A$, gde je $P(A) = 0$, ako pretpostavimo da važi još jedan dodatni uslov.

Teorema 3.2.2. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Pretpostavimo da postoji slučajna promenljiva R , skup $A \in \mathcal{F}$, $P(A) = 0$ i $k \in \mathbb{N}_0$ tako da je $|\xi(\omega, \varphi)| \leq R(\omega) \|\varphi\|_k$, za $\omega \in \Omega \setminus A$. Tada postoji niz slučajnih promenljivih $\{c_m : m \in \mathbb{N}_0\}$ tako da je

$$(1) \quad \xi(\omega, \varphi) := \sum_{m=0}^{\infty} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega \setminus A, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} < R(\omega), \quad \omega \in \Omega \setminus A.$$

Dokaz. Dokaz je isti kao dokaz Teoreme 3.2.1. ako stavimo da je u (3.2.1)

$$\xi_1(\omega, \varphi) = \begin{cases} \xi(\omega, \varphi), & \omega \in \Omega \setminus A \\ 0, & \omega \in \Omega \setminus A \end{cases} \quad \square$$

Neka je $\tilde{\mathcal{R}}$ operator definisan u paragrafu 2.2. druge glave.

Na skupu uopštenih slučajnih procesa može se definisati diferencijalni operator $\tilde{\mathcal{R}}^k$, $k \in \mathbb{N}_0$, na sledeći način

$$\tilde{\mathcal{R}}^k \xi(\omega, \varphi) = \xi(\omega, \mathcal{R}^k \varphi), \quad \omega \in \Omega, \quad \varphi \in \mathcal{A}.$$

$$\tilde{\mathcal{R}}^{k+1} = \tilde{\mathcal{R}}(\tilde{\mathcal{R}}^k), \quad k \in \mathbb{N},$$

$$\tilde{\mathcal{R}}^0 = J.$$

Dalje ćemo $\tilde{\mathcal{R}}$ obeležavati isto sa \mathcal{R} .

the first stage of the primary equation is given by the following equality
which holds true for all values of the corresponding angle θ in (A)4 at any
time t .

After some calculations we get that the right-hand side of (A)5 is equal to
 $(\partial \phi / \partial t) + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w$, where $u = \cos(\theta)$, $v = \sin(\theta) \cos(\varphi)$, $w = \sin(\theta) \sin(\varphi)$. The left-hand side of (A)5 is equal to
 $(\partial \phi / \partial t) + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w - (\partial \phi / \partial x) \cdot u^2 - (\partial \phi / \partial y) \cdot v^2 - (\partial \phi / \partial z) \cdot w^2$. Thus, we have

$$(\partial \phi / \partial t) + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w - (\partial \phi / \partial x) \cdot u^2 - (\partial \phi / \partial y) \cdot v^2 - (\partial \phi / \partial z) \cdot w^2 = 0.$$

From this it follows that the second term in the right-hand side of (A)5 is equal to zero. Then we have

$$\partial \phi / \partial t + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w = 0.$$

$$\partial \phi / \partial t + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w = 0.$$

Since $\partial \phi / \partial t$ is a function of x , y , z and t , we have

which implies that $\partial \phi / \partial t$ is a function of x , y , z and t . Then we have

$$(\partial \phi / \partial t) + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w = 0.$$

$$(\partial \phi / \partial t) + (\partial \phi / \partial x) \cdot u + (\partial \phi / \partial y) \cdot v + (\partial \phi / \partial z) \cdot w = 0.$$

$$L = 0.$$

Thus, the derivative L does not

Sledeća teorema je analogna Teoremi 2.2.2. odnosno Teoremi 9.6.2. [47, ch. 9.6]. Kao i ranije, neka je $\Lambda = \{n \in \mathbb{N}_0 : \lambda_n \neq 0\}$, $\Lambda^c = \mathbb{N}_0 \setminus \Lambda$.

Teorema 3.2.3. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, broj $k_0 = k_0(\varepsilon) \in \mathbb{N}_0$, i za svako $k \geq k_0$ u.s.p. ξ_k na $\Omega \times L^2(I)$ i slučajne promenljive c_m , $m \in \Lambda$, tako da je

$$\xi(\omega, \varphi) = R^k \xi_k(\omega, \varphi) + \sum_{n \in \Lambda} c_n(\omega) (\psi_n, \varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A}.$$

Dokaz. Iz Teoreme 3.2.1. sledi da postoji $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$ i $k_0 = k_0(\varepsilon)$, i niz slučajnih promenljivih $\{c_m : m \in \mathbb{N}_0\}$ tako da je

$$\xi(\omega, \varphi) = \sum_{m=0}^{\infty} c_m(\omega) (\psi_m, \varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A},$$

$$\left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} \leq k_0, \quad \omega \in B.$$

Neka je, za $k \geq k_0$,

$$b_n(\omega) = \begin{cases} c_m(\omega) \tilde{\lambda}_m^{-k}, & \omega \in B \\ 0, & \omega \notin B \end{cases}.$$

Imamo da je

$$\xi_k(\omega, \varphi) = \sum_{m=0}^{\infty} b_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega, \quad \varphi \in \mathcal{A} \text{ u.s.p. na } \Omega \times L^2(I).$$

Dalje je,

$$R^k \xi_k(\omega, \varphi) = \xi_k(\omega, R^k \varphi) = \sum_{m=0}^{\infty} b_m(\omega) \lambda_m^k (\psi_m, \varphi) = \sum_{n \in \Lambda} c_n(\omega) (\psi_n, \varphi),$$

whereas $\langle u, v \rangle_{\omega}$ is induced by the ω -symplectic structure above it.

$$\langle u, v \rangle_{\omega} = \int_{\Omega} \omega \wedge d\omega(u, v) \wedge d\omega(v, w) \wedge d\omega(w, u)$$

Since ω is exact and $d\omega = 0$ on \mathcal{M} , we have $\langle u, v \rangle_{\omega}$ is induced by $\omega \wedge d\omega$ and $\omega \wedge d\omega = 0$. So $\langle u, v \rangle_{\omega}$ is quite interesting. Furthermore, since $(1)^3 I = 0 = \omega^3$, we have $\langle u, v \rangle_{\omega} = 0$ always if ω is closed. In other words, $\langle u, v \rangle_{\omega} = 0$ if $\omega = d\alpha$.

$$\langle u, v \rangle_{\omega} = \int_{\Omega} \omega \wedge d\omega(u, v) = \langle u, v \rangle_{d\alpha} = \langle u, v \rangle_{\alpha}$$

As ω is a closed 2-form, ω is ω -symplectic and ω induces a ω -closed differential form $d\omega$. Therefore $\langle u, v \rangle_{\omega}$ is induced by $\langle u, v \rangle_{d\omega}$ and $\langle u, v \rangle_{d\omega} = 0$.

$$\langle u, v \rangle_{\omega} = \int_{\Omega} \omega \wedge d\omega(u, v) = \langle u, v \rangle_{d\omega} = \frac{\partial}{\partial \theta} \left[\phi(\theta) \frac{\partial}{\partial \theta} \right]_{\theta=0}$$

$$\langle u, v \rangle_{\omega} = \int_{\Omega} \omega \wedge d\omega(u, v) = \left[\phi(\theta) \frac{\partial}{\partial \theta} \right]_{\theta=0} = \phi'(0)$$

which is called the ω -action.

$$\begin{aligned} \langle u, v \rangle_{\omega} &= \int_{\Omega} \omega \wedge d\omega(u, v) \\ &= \left. \int_{\Omega} \omega \wedge d\omega(u, v) \right|_{\theta=0} = \phi'(0) \end{aligned}$$

which is called the ω -symplectic form.

$$(1)^3 I = 0 \text{ on } \mathcal{M} \text{ implies } \langle u, v \rangle_{\omega} = \langle u, v \rangle_{d\omega} = \langle u, v \rangle_{\alpha}$$

which is called

$$\langle u, v \rangle_{\omega} = \langle u, v \rangle_{d\omega} = \langle u, v \rangle_{\alpha} = \left. \int_{\Omega} \omega \wedge d\omega(u, v) \right|_{\theta=0} = \langle u, v \rangle_{\alpha}^{(d\omega)}$$

pa sledi

$$\xi(\omega, \varphi) = \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi). \quad \square$$

Napomenimo samo da se može pokazati, (vidi dokaz Teoreme 3.2.6.) da je $\xi_k = \int_{\Omega} X_k(\omega, t) R^k \varphi(t) dt$, gde je X_k funkcija definisana sa:

$$X_k(\omega, t) = \sum_{m=0}^{\infty} b_m(\omega) \psi_m(t), \quad \omega \in \Omega, \quad t \in I.$$

pa je gornja reprezentacija u stvari oblika:

$$\xi(\omega, \varphi) = \int_{\Omega} X_k(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega, \quad \varphi \in \mathcal{A}. \quad (3.2.2)$$

Slično kao u Teoremi 3.2.2., dodavanjem još jednog uslova možemo proširiti reprezentaciju (3.2.2) na skup mere jedan.

Teorema 3.2.4. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Pretpostavimo da postoji slučajna promenljiva R tako da je $E(R) < \infty$, skup $A \in \mathcal{F}$, sa $P(A) = 0$, broj $k_0 \in \mathbb{N}_0$, tako da je $|\xi(\omega, \varphi)| \leq R(\omega) \|\varphi\|_{k_0}$, $\omega \in \Omega \setminus A$, $\varphi \in \mathcal{A}$. Tada za svako $k \geq k_0$, postoji funkcija $X_k: \Omega \times I \rightarrow \mathbb{C}$, i za $m \in \Lambda$ slučajne promenljive c_m , nezavisne od k tako da je

$$\xi(\omega, \varphi) = \int_{\Omega} X_k(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega \setminus A, \quad \varphi \in \mathcal{A}.$$

U sledećim teoremmama daćemo reprezentaciju preko neprekidnog slučajnog procesa X_k .

Da bismo dobili reprezentaciju preko neprekidnog

that is

$$Q_2 \cdot (x, u) \cdot Q_1 \cdot u = (x, u)^2.$$

Because x is full, it has a unique value for an ideal multiplication

$$\text{ideal}(x) \cdot u = \text{ideal}(x)u^2 \cdot \text{ideal}(x)u^{-2} = u^2 \cdot u^{-2} = 1 \in \mathcal{O}_K^\times$$

(as multiplication

$$\text{ideal}(x) \cdot u = (Q_1 \cdot u) \cdot Q_2 \cdot u = (x, u)^2 \neq$$

and this is an ideal multiplication showing x is a unit.

$$x = u \cdot \text{ideal}(X) \cdot u^{-1} = \text{ideal}(X) \cdot u^2 = (x, u)^2$$

as $x, u \in \text{ideal}(X)$.
Because x is a unit, x^{-1} is also a unit.
Hence, since x^{-1} is a unit of multiplication, we can multiply

the multiplication by x^{-1} to get another ideal multiplication
 $\text{ideal}(x) \cdot u = \text{ideal}(x) \cdot x^{-1} \cdot \text{ideal}(X) \cdot u = \text{ideal}(X) \cdot u$
 $\text{ideal}(x) \cdot u = (x, u)^2$ as the value $x^{-1} \cdot x = 1$ and $x \in \mathcal{O}_K^\times$ as
a unit. Hence $x^{-1} \cdot \text{ideal}(X) \cdot u = \text{ideal}(X) \cdot u$, so $x^{-1} \cdot \text{ideal}(X) \cdot u$ is a
multiplication, and multiplication by x^{-1} is a unit \rightarrow x^{-1} is a unit
as required.

$$x = u \cdot \text{ideal}(X) \cdot u^{-1} = \text{ideal}(X) \cdot u^2 = (x, u)^2$$

because x is a unit, x^{-1} is also a unit.
Hence, since x^{-1} is a unit,
 $x^{-1} \cdot \text{ideal}(X) \cdot u = \text{ideal}(X) \cdot u$

slučajnog procesa, potrebno je staviti neke uslove na nizove $\{\lambda_m: m \in \mathbb{N}_0\}$ i $\{\psi_m: m \in \mathbb{N}_0\}$.

Pretpostavimo da nizovi $\{\lambda_m: m \in \mathbb{N}_0\}$ i $\{\psi_m: m \in \mathbb{N}_0\}$ zadovoljavaju sledeće uslove:

(*) postoji $s_0 \in \mathbb{N}_0$ i konstanta K tako da je za $s \geq s_0$

$$\sup |\psi_m(t) \tilde{\lambda}_m^{-s}| : m \in \mathbb{N}_0, t \in I \} < K;$$

(**) postoji $p_0 \in \mathbb{N}_0$ tako da je za $p \geq p_0$

$$\sum_{m=0}^{\infty} \tilde{\lambda}_m^{-2p} < \infty.$$

Uslovi (*) i (**) nisu suviše restriktivni. Na primer Ermitov, Lagerov, Furieov ortonormirani sistem zadovoljava ove uslove. U [47, ch.9.8] i [3, ch.10.18], mogu su naći i drugi ortonormirani sistemi koji zadovoljavaju ove uslove.

Reprezentacija u.s.p. preko neprekidnog slučajnog procesa može se dati, kao i ranije, na skupu proizvoljno velike verovatnoće, i na skupu verovatnoće jedan. Dokazi su slični dokazima Teorema 3.2.3. i 3.2.4., pa ćemo prvu teoremu navesti a dokaz dati za drugu.

Teorema 3.2.5. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1-\varepsilon$, broj $k_0 = k_0(\varepsilon) \in \mathbb{N}_0$, a za svako $k \geq k_0$ postoji neprekidan slučajni proces $X_k: \Omega \times I \rightarrow \mathbb{C}$, i slučajne promenljive c_m , $m \in \Lambda$, tako da

$$\xi(\omega, \varphi) = \int_X X_k(\omega, t) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi), \quad \omega \in B, \varphi \in \mathcal{A},$$

gde je $s \geq s_0$, s_0 iz (*), $p \geq p_0$, p_0 iz (**).

mentar na evolução entre 1990 e 2000, quando o percentual de pessoas

com 20 ou mais anos é de 14,9%

em 1990 e de 16,6% em 2000, resultando na consequente

redução substancial das taxas

de mortalidade infantil.

As taxas de mortalidade infantil são altas no Brasil, com

16,6% em 2000 (IBGE, 2001).

As taxas de mortalidade infantil no Brasil são superiores

à média das Américas (IBGE, 2001).

Além disso, a mortalidade infantil é maior entre os negros do que entre os brancos.

Portanto, é preciso que os governos e as organizações sociais promovam

programas que visem a redução da mortalidade infantil, tanto entre os negros quanto entre os brancos.

Além disso, é necessário que os governos e as organizações sociais

busquem formas de aumentar a expectativa de vida entre os negros e os brancos.

Portanto, é preciso que os governos e as organizações sociais

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busquem formas de aumentar a expectativa de vida entre os negros e os brancos.

Teorema 3.2.6. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Prepostavimo da postoji slučajna promenljiva R za koju je $E(R) < \infty$, skup $A \in \mathcal{F}$, sa $P(A) = 0$, broj $k_0 \in \mathbb{N}_0$, tako da je $|\xi(\omega, \varphi)| \leq R(\omega) \|\varphi\|_{k_0}$, $\omega \in \Omega$, $\varphi \in \mathcal{A}$. Tada, za svako $k \geq k_0$ postoji neprekidan slučajan proces $X_k: \Omega \times I \rightarrow \mathbb{C}$, i slučajne promenljive c_m , $m \in \Lambda$, nezavisne od k tako da je

$$\xi(\omega, \varphi) = \int_I X_k(\omega, t) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega \setminus A, \quad \varphi \in \mathcal{A},$$

gde je $s \geq s_0$, s_0 iz (*), a $p \geq p_0$, p_0 iz (**).

Dokaz. Iz Teoreme 3.2.2. sledi da postoji niz slučajnih promenljivih $\{c_m, m \in \mathbb{N}_0\}$ tako da je

$$\xi(\omega, \varphi) = \sum_{m=0}^{\infty} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega \setminus A, \quad \varphi \in \mathcal{A},$$

$$\left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} \leq R(\omega), \quad \omega \in \Omega \setminus A.$$

Neka je $k \geq k_0$ fiksirano. Definišimo, za $s \geq s_0$, $p \geq p_0$,

$$X_k(\omega, t) = \sum_{m=0}^{\infty} c_m(\omega) \tilde{\lambda}_m^{-(k+p+s)} \psi_m(t), \quad \omega \in \Omega, \quad t \in I.$$

Imamo, da je za $\omega \in \Omega \setminus A$

$$\sum_{m=0}^{\infty} |c_m(\omega) \tilde{\lambda}_m^{-(k+p+s)} \psi_m(t)| \leq K \sum_{m=0}^{\infty} |c_m(\omega) \tilde{\lambda}_m^{-k} \tilde{\lambda}_m^{-p}| \leq$$

$$\leq K \left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \left[\sum_{m=0}^{\infty} \tilde{\lambda}_m^{-2p} \right]^{1/2} < \infty.$$

Sledi da je za svako $\omega \in \Omega \setminus A$ $X_k(\omega, \cdot)$ neprekidna funkcija. Kako je $X_k(\cdot, t)$, $t \in I$ merljiva, sledi da je X_k merljiva kao

for the corresponding θ , ϕ in the set $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$.
 It is a direct consequence of the fact that θ and ϕ are continuous variables. However, $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is not closed under addition and $(0, 0)$ is an accumulation point. As it is a closed set, there must be an element in \mathbb{R}^2 which is not in $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ but which is an accumulation point. This contradicts the fact that \mathbb{R}^2 is closed under addition. As a result, $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is closed under addition.

Thus, the set $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is closed under addition.

Similarly, with respect to the second condition, we can show that \mathbb{R}^2 is closed under multiplication. Let $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ be a closed set. Then, $(0, 0)$ is an accumulation point. As it is a closed set, there must be an element in \mathbb{R}^2 which is not in $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ but which is an accumulation point. This contradicts the fact that \mathbb{R}^2 is closed under multiplication. As a result, $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is closed under multiplication.

Therefore, $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is closed under both addition and multiplication.

As a result, $\{(\theta, \phi) \in \mathbb{R}^2 : \theta \in [0, 2\pi], \phi \in [-\pi, \pi]\}$ is a closed set.

$$\approx \left[\exp^{j\theta} \left(\frac{\partial}{\partial \theta} \right) \left(\exp_{\theta, \phi} \right) \right] \left[\exp^{j\phi} \left(\frac{\partial}{\partial \phi} \right) \left(\exp_{\theta, \phi} \right) \right]$$

$$= \exp^{j\theta} \left[\left(\exp_{\theta, \phi} \right) \left(\frac{\partial}{\partial \theta} \right) \right] \exp^{j\phi} \left[\left(\exp_{\theta, \phi} \right) \left(\frac{\partial}{\partial \phi} \right) \right] \approx \exp_{\theta + j\phi, \phi}$$

which is the definition of $\exp_{\theta + j\phi, \phi}$. This is a closed set as it is the set of all points $\theta + j\phi$ where $\theta \in [0, 2\pi]$ and $\phi \in [-\pi, \pi]$.

funkcija na $\Omega \times I$. Za $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}$ imamo

$$\left[\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{-2(k+p+s)} \right]^{1/2} = C < \infty.$$

Takođe,

$$\left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2(k+p+s)} \right]^{1/2} \leq R(\omega), \quad \omega \in \Omega \setminus A.$$

Dakle,

$$\begin{aligned} & \int_{\Omega \setminus A} \int_I |X_k(\omega, t) \mathcal{R}^{k+p+s} \varphi(t)| dt dP(\omega) \leq \\ & \leq \int_{\Omega \setminus A} \left[\left(\int_I |X_k(\omega, t)|^2 dt \right)^{1/2} \left(\int_I |\mathcal{R}^{k+p+s} \varphi(t)|^2 dt \right)^{1/2} \right] dP(\omega) \leq \\ & \leq \int_{\Omega \setminus A} \left[\left(\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2(k+p+s)} \right)^{1/2} \left(\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{-2(k+p+s)} \right)^{1/2} \right] dP(\omega) \leq \\ & \leq C \int_{\Omega \setminus A} |R(\omega)| dP(\omega) < \infty. \end{aligned}$$

Prema Fubinijevoj teoremi [37], sledi da

$X_k(\cdot, \cdot) \mathcal{R}^{k+p+s} \varphi(\cdot) \in L^1(\Omega \setminus A \times I)$, i po istoj teoremi, da je

$$\xi_k(\cdot, \varphi) = \int_I X_k(\cdot, t) \mathcal{R}^{k+p+s} \varphi(t) dt.$$

slučajna promenljiva za svako $\varphi \in \mathcal{A}$. Sledi da je ξ_k u.s.p. na $\Omega \times L^2(I)$. Osigledno je da je

$$\xi_k(\omega, \varphi) = \sum_{m=0}^{\infty} c_m(\omega) \tilde{\lambda}_m^{-(k+p+s)} (\psi_m, \mathcal{R}^{k+p+s} \varphi) + \sum_{m \in \Lambda^c} c_m(\omega) (\psi_m, \varphi),$$

pa je

$$\xi(\omega, \varphi) = \xi_k(\omega, \varphi) + \sum_{m \in \Lambda} c_m(\omega) (\psi_m, \varphi), \quad \omega \in \Omega \setminus A, \varphi \in \mathcal{A}. \quad \square$$

debut $\mathbb{W} \otimes \mathbb{V}$ en \mathbb{R}^n et \mathbb{R}^m et \mathbb{R}^{n+m}

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \leq \frac{1}{1 - \frac{1}{1600}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$\text{max}(\text{min}(A, D)) = 0.999$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

$$A \times D \geq 0 \geq 1600 \geq \left[\frac{\text{max}(\text{min}(A, D))}{\text{min}(A, D)} \right]^{\frac{n}{m}}$$

3.3. USLOVNO MATEMATIČKO OČEKIVANJE UOPŠTENOG SLUČANOG PROCESA

3.3.1. MATEMATIČKO OČEKIVANJE UOPŠTENOG SLUČAJNOG PROCESA

NA $\Omega \times \mathcal{A}$

Neka je V vektorsko topološki prostor a V' njegov dualni prostor.

Definicija 3.3.1. Neka je ξ u.s.p. na $\Omega \times V$, ako je

(ci) $\forall \varphi \in V \quad E\xi(\cdot, \varphi) < \infty$,

(cii) preslikavanje $V \ni \varphi \rightarrow E\xi(\cdot, \varphi) \in \mathbb{C}$ je linearno i neprekidno tada se funkcija $E[\xi] \in V'$ definisana na sledeći način

$$E[\xi](\varphi) = \int_{\Omega} \xi(\omega, \varphi) dP(\omega), \quad \varphi \in V$$

naziva matematičko očekivanje uopštenog slučajnog procesa ξ .

Teorema 3.3.1. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$ i neka su zadovoljeni uslovi Teoreme 3.2.2.. Ako je $E(R) < \infty$, tada postoji $E(c_m)$, $m \in \mathbb{N}_0$, postoji $E[\xi]$ i dato je sa

$$E[\xi](\varphi) = \sum_{m=0}^{\infty} E(c_m)(\psi_m, \varphi), \quad \varphi \in \mathcal{A},$$

takođe postoji $p \in \mathbb{N}_0$ tako da je

$$\sum_{m=0}^{\infty} |E(c_m)|^2 \tilde{\lambda}_m^{-2p} < \infty.$$

Dokaz. Kako je u Teoremi 3.2.2.

$$\left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} = S(\omega) < \infty, \quad \omega \in \Omega \setminus A$$

sledi da je

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$$\left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \leq R(\omega), \quad \omega \in \Omega \setminus A,$$

pa je, za svako $m \in \mathbb{N}_0$, $|c_m(\omega)| \leq |\tilde{\lambda}_m|^k R(\omega)$, $\omega \in \Omega \setminus A$.

Sledi da je

$$|E(c_m)| = \left| \int_{\Omega} c_m(\omega) dP(\omega) \right| \leq |\tilde{\lambda}_m|^k \int_{\Omega} R(\omega) dP(\omega) < \infty.$$

Dalje, za $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}$, $\omega \in \Omega$,

$$|E\xi(\omega, \varphi)| = \left| \int_{\Omega} \left[\sum_{m=0}^{\infty} c_m(\omega)(\psi_m, \varphi) \right] dP(\omega) \right| \leq$$

$$\leq \int_{\Omega \setminus A} \left[\sum_{m=0}^{\infty} |c_m(\omega)| |\tilde{a}_m| \right] dP(\omega) \leq$$

$$\leq \int_{\Omega \setminus A} \left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \left[\sum_{m=0}^{\infty} |a_m|^2 \tilde{\lambda}_m^{2k} \right]^{1/2} dP(\omega) \leq$$

$$\leq C \int_{\Omega \setminus A} \left[\sum_{m=0}^{\infty} |c_m(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} dP(\omega) < \infty.$$

Prema Fubinijevoj teoremi je, za $\varphi \in \mathcal{A}$, $\omega \in \Omega$,

$$E[\xi](\varphi) = \int_{\Omega} \xi(\omega, \varphi) dP(\omega) =$$

$$= \int_{\Omega \setminus A} \left[\sum_{m=0}^{\infty} c_m(\omega)(\psi_m, \varphi) \right] dP(\omega) =$$

$$= \sum_{m=0}^{\infty} \left[\int_{\Omega \setminus A} c_m(\omega) dP(\omega) \right] (\psi_m, \varphi) =$$

$$= \sum_{m=0}^{\infty} E(c_m)(\psi_m, \varphi).$$

Iz činjenice da je $\sum_{m=0}^{\infty} E(c_m)(\psi_m, \varphi)$ konačno za svako $\varphi \in \mathcal{A}$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(\frac{1}{2}) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0}$$

$\phi^{-1}(x) = 0.001 \approx 0.001 \cdot 10^3 \approx 1000$ where $x = 0.001$, and
 $\phi'(0_0) = 1$, so this is

$$\phi^{-1}(0.001) \approx 1000 \cdot 10^3 \approx 10^7 = 10^7 \cdot 10^3$$

$\phi^{-1}(0.001) \approx 10^7$ where $x = 0.001$, and $\phi'(0_0) = 1$, and
 $\phi^{-1}(0.001) \approx 10^7$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$\phi^{-1}(0.001) \approx 10^7$ where $x = 0.001$, and $\phi'(0_0) = 1$, and
 $\phi^{-1}(0.001) \approx 10^7$

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 $\phi^{-1}(0.001) \approx 10^7$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

$$P = \phi^{-1}(0.001) \approx \left[\phi^{-1}(0_0) \left(\phi'(0_0) \right)^{-\frac{1}{2}} \right]_{0=0} \cdot \phi'(0_0) \approx 10^7 \cdot 10^3 = 10^{10}$$

i iz [25 ch. 30], sledi da postoji $p \in \mathbb{N}_0$ tako da je

$$\sum_{m=0}^{\infty} |E(c_m)|^2 \tilde{\lambda}_m^{-2p} < \infty. \quad \square$$

Primetimo da je za matematičko očekivanje u.s.p. na $\Omega \times \mathcal{A}$ dovoljno tražiti samo da je $E[\xi]$ funkcional na V . Očigledno, $E[\xi]$ je linearno, a neprekidnost se lako može pokazati.

Zaista, neka $\varphi_n \rightarrow \varphi$ u \mathcal{A} i neka je $\varphi_n = \sum_{m=0}^{\infty} a_m^n \psi_m$, $\varphi = \sum_{m=0}^{\infty} a_m \psi_m$.

Tada je

$$|E[\xi](\varphi_n) - E[\xi](\varphi)| \leq \sum_{m=0}^{\infty} |E(c_m)(\psi_m, \varphi_n - \varphi)| \leq$$

$$\leq \left[\sum_{m=0}^{\infty} |E(c_m)|^2 \tilde{\lambda}_m^{-2p} \right]^{1/2} \left[\sum_{m=0}^{\infty} |a_m^n - a_m| \tilde{\lambda}_m^{2p} \right]^{1/2} \rightarrow 0, \quad n \rightarrow \infty.$$

Teorema 3.3.1. Neka je $\tilde{\lambda}_m > 0, m \in \mathbb{N}_0$ i $\sum_{m=0}^{\infty} \tilde{\lambda}_m^{-2p} < \infty$ nepraktičan situacijni

proses. Neka postoji odgovarajuća posredna funkcija R na $\mathbb{R}(P)$ i u takoj

Teorema 3.3.2. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$. Neka su ispunjeni uslovi Teoreme 3.2.6.. Ako postoji $E[X_k(\cdot, t)]$ za svako $k \geq k_0$, $E[\xi]$ postoji i dato je sa

$$E[\xi](\varphi) = \int_{\Omega} E[X_k(\cdot, t)] R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} E(c_m)(\psi_m, \varphi), \quad \varphi \in \mathcal{A}.$$

Dokaz. Iz Teoreme 3.3.1. sledi da $E(c_m)$ postoji za svako

$m \in \mathbb{N}_0$, pa i za $m \in \Lambda$.

Primenjujući Fubinijevu teoremu, imamo za svako $\varphi \in \mathcal{A}$,

$$E[\xi](\varphi) = \int_{\Omega} \xi(\omega, \varphi) dP(\omega) = \int_{\Omega} \xi(\omega, \varphi) dP(\omega) =$$

$$\int_{\Omega \setminus A} \left[\int_I X_k(\omega, t) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega)(\psi_m, \varphi) \right] dP(\omega) =$$

we also point out in the following subsections how the results of section 3

$$= \omega \times 4 \pi \rho^2 |C_0| \Omega^2$$

can be used to approximate contributions of the other terms in the energy budget. We also demonstrate that by using these estimates one can obtain a reasonable value which can be considered as a contribution of the total energy density of the system. In addition, it is shown that the energy density of the scalar field is small.

$$\omega \times 4 \pi \rho^2 |C_0| \Omega^2 \ll \omega \times 4 \pi \rho^2 |C_0| \Omega^2$$

$$= 100 (1.1) \times 100 (1.1) \times 100 (1.1)$$

$$\text{and } \omega \times 4 \pi \rho^2 |C_0| \Omega^2 = \left[\frac{\omega}{\rho} \right]^{1/2} \left[\frac{\rho}{C_0} \right]^{1/2} \left[\frac{\Omega^2}{\omega} \right]^{1/2}$$

therefore we have $\omega \times 4 \pi \rho^2 |C_0| \Omega^2 \ll \omega \times 4 \pi \rho^2 |C_0| \Omega^2$ because $\omega \times 4 \pi \rho^2 |C_0| \Omega^2$ is given by $(\omega \times 4 \pi \rho^2 |C_0| \Omega^2)^2 \ll (\omega \times 4 \pi \rho^2 |C_0| \Omega^2)^2$ because $\omega \times 4 \pi \rho^2 |C_0| \Omega^2$ is given by $(\omega \times 4 \pi \rho^2 |C_0| \Omega^2)^2 \ll (\omega \times 4 \pi \rho^2 |C_0| \Omega^2)^2$.

$$= 100 (1.1) \times 100 (1.1) \times 100 (1.1) \times 100 (1.1)$$

where we find the total energy density of the system. The energy density of the system is given by

$$\omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2$$

$\approx \omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2$

$$= \omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2 + \omega \times 4 \pi \rho^2 |C_0| \Omega^2$$

$$= \omega \times 4 \pi \rho^2 [C_0 (1.1) \times 100 (1.1)] + \omega \times 4 \pi \rho^2 [C_0 (1.1) \times 100 (1.1)] + \omega \times 4 \pi \rho^2 [C_0 (1.1) \times 100 (1.1)]$$

$$= \omega \times 4 \pi \rho^2 [C_0 (1.1) \times 100 (1.1) \times 100 (1.1)]$$

$$\int_I \left[\int_{\Omega \setminus A} X_k(\omega, t) dP(\omega) \right] R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} \int_{\Omega \setminus A} c_m(\omega) dP(\omega) (\psi_m, \varphi) =$$

$$\int_I E[X_k(\cdot, t) R^{k+p+s} \varphi(t)] dt + \sum_{m \in \Lambda} E(c_m)(\psi_m, \varphi). \quad \square$$

3.3.2. USLOVNO MATEMATIČKO OČEKIVANJE

Daćemo reprezentacionu teoremu za uslovno matematičko očekivanje u.s.p na $\Omega \times \mathcal{A}$, u odnosu na σ -podalgebru \mathcal{U} od \mathcal{F} . Ova teorema je slična Teoremi 5. u [21]. Prvo ćemo dokazati sledeću teoremu.

Teorema 3.3.3. Neka je $X(\omega, t): \Omega \times I \rightarrow \mathbb{C}$ neprekidan slučajni proces. Neka postoji slučajna promenljiva R , sa $E(R) < \infty$, tako da je za svako $t \in I$, $|X(\cdot, t)| \leq c(t)R(\cdot)$, skoro sigurno na Ω , gde je $c(t)$ funkcija ograničena na I . Tada za proizvoljnu σ -podalgebru \mathcal{U} od \mathcal{F} , funkcija $g(\omega, t) = E(X(\omega, t)|\mathcal{U})$ je $\mathcal{U} \times \mathcal{B}(I)$ merljiva, gde je $\mathcal{B}(I)$ σ -algebra generisana Borelovim skupovima u I .

Dokaz. Za svako $t \in I$ imamo da je $g(\cdot, t)$ \mathcal{U} -merljiva funkcija. Pokazaćemo da je za skoro svako $\omega \in \Omega$, $g(\omega, \cdot)$ neprekidna na I . Neka je $\{t_n: n \in \mathbb{N}_0\}$ niz u I takav da $t_n \rightarrow t_0$, i za svako $\omega \in \Omega$ stavimo $Y_n(\omega) = X(\omega, t_n)$. Kako je $X(\omega, \cdot)$ neprekidno za skoro svako $\omega \in \Omega$ sledi da

$$Y_n(\cdot) \xrightarrow{s.s.} Y_0(\cdot) = X(\cdot, t_0), \quad n \rightarrow \infty.$$

Dalje, kako je za $t \in I$, $|X(\cdot, t)| \leq c(t)R(\cdot)$ skoro

$$= \left(\frac{1}{\lambda} \int_{\Omega} \phi(x) \partial_x \phi(x) dx + \frac{1}{\lambda} \int_{\Omega} \phi(x) \partial_y \phi(x) dy \right) \left[\frac{\partial_x \phi(x)}{\lambda} + \frac{\partial_y \phi(x)}{\lambda} \right] \frac{1}{\lambda}$$

$$= \frac{1}{\lambda^2} \int_{\Omega} \phi(x) \partial_x \phi(x) dx + \frac{1}{\lambda^2} \int_{\Omega} \phi(x) \partial_y \phi(x) dy \left[\frac{\partial_x \phi(x)}{\lambda} + \frac{\partial_y \phi(x)}{\lambda} \right]$$

Seja $\phi(x) = \phi_0(x) e^{-\lambda|x|}$, com $\phi_0(x)$ contínua e de suporte compacto. A equação acima se transforma em

$\lambda^2 \int_{\Omega} \phi_0(x) \partial_x \phi_0(x) dx + \lambda^2 \int_{\Omega} \phi_0(x) \partial_y \phi_0(x) dy = \lambda^2 \int_{\Omega} \phi_0(x) \partial_x \phi_0(x) dx + \lambda^2 \int_{\Omega} \phi_0(x) \partial_y \phi_0(x) dy$

Integrando termo a termo, obtemos que λ^2 é menor ou igual a λ^2 . Se $\lambda^2 < \lambda^2$, teríamos que λ^2 é menor que λ^2 , o que é contraditório. Assim, λ^2 é maior ou igual a λ^2 . Isto implica que λ é menor ou igual a λ . Portanto $\lambda = \lambda$.

Portanto $\lambda = \lambda$. Isto é, se temos $\lambda = \lambda$ para um certo campo ϕ , é porque ϕ é o campo que minimiza a função J entre todos os campos que satisfazem a condição $\int_{\Omega} \phi(x) \partial_x \phi(x) dx = 0$. Portanto λ é o menor valor de λ para o qual $\lambda = \lambda$.

Portanto $\lambda = \lambda$ se e só se λ é o menor valor de λ para o qual $\lambda = \lambda$.

sigurno na Ω , prema Fatou-Lebesgue teoremi o konvergenciji [26, ch. 25.1] i kako je R integrabilno, imamo da je

$$E(Y_n | \mathcal{U}) \xrightarrow{s.s.} E(Y_0 | \mathcal{U}).$$

Dakle, za skoro svako $\omega \in \Omega$, $g(\omega, t_n) = E(X(\omega, t_n) | \mathcal{U}) = E(Y_n | \mathcal{U})$ konvergira ka $g(\omega, t_0) = E(Y_0 | \mathcal{U})$, pa je $g(\omega, \cdot)$ neprekidno za skoro svako $\omega \in \Omega$. Sledi da je $g(\cdot, \cdot) : \mathcal{U} \times \mathcal{B}(I)$ merljiva funkcija. Takođe, kako je $X(\omega, \cdot)$ neprekidna, sledi da je $X : \mathcal{F} \times \mathcal{B}(I)$ merljiva funkcija. \square

Neka je $P_{\mathcal{U}}$ restrikcija od P na \mathcal{U} definisana sa

$$P_{\mathcal{U}}(B) = P(B), \quad B \in \mathcal{U}.$$

Definicija 3.3.2. Neka je ξ u.s.p. na $\Omega \times \mathcal{A}$ i neka je \mathcal{U} σ -podalgebra od \mathcal{F} . Uсловно математичко очекivanje (условно очекivanje) од ξ у односу на \mathcal{U} , обељавамо га са $E[\xi | \mathcal{U}](\varphi)$, $\varphi \in \mathcal{A}$, је за свако $\varphi \in \mathcal{A}$, \mathcal{U} - мерljива функција дефинисана до на $P_{\mathcal{U}}$ еквиваленцију на следећи начин

$$\int_B E[\xi | \mathcal{U}](\varphi) dP_{\mathcal{U}}(\omega) = \int_B \xi(\omega, \varphi) dP(\omega), \quad B \in \mathcal{U}, \varphi \in \mathcal{A}.$$

Teorema 3.3.4. Neka je ξ u.s.p. на $\Omega \times \mathcal{A}$. Pretpostavimo да постоји slučajна променљива R , таква да је $E(R) < \infty$, скуп $A \in \mathcal{F}$, такав да је $P(A)=0$, и број $k_0 \in \mathbb{N}_0$ тако да је $|\xi(\omega, \varphi)| \leq R(\omega) \|\varphi\|_{k_0}$, $\omega \in \Omega \setminus A$, $\varphi \in \mathcal{A}$. Тада за $k \geq k_0$, постоји непрекидан slučajни процес $X_k(\omega, t)$ на $\Omega \times I$ и slučajне променљиве c_m , $m \in \Lambda$, тако да за $\omega \in \Omega \setminus A$, и $\varphi \in \mathcal{A}$,

it, interpretation. A logical consequence of this is that if we can prove
that $\phi \rightarrow \psi$ is true, then ϕ must be true.

$$C_1 \phi \rightarrow \psi \vdash C_1 \psi$$

$C_1 \phi \rightarrow \psi = C_1(C_0 \wedge \phi) \rightarrow \psi = C_0 \wedge \phi \rightarrow \psi$ is a tautology since all of the
other components ($C_0 \wedge \phi$) are true. So $C_1 \phi \rightarrow \psi = C_0 \wedge \phi \rightarrow \psi$ is a tautology
and therefore $C_1 \phi \rightarrow \psi \vdash C_1 \psi$. This means that if ϕ is a tautology, then
it is also a tautology. This is a contradiction, so ϕ is not a tautology.
Therefore, $\phi \rightarrow \psi$ is not a tautology, so $\phi \rightarrow \psi$ is not true. This contradicts
our assumption that $\phi \rightarrow \psi$ is true, so $\phi \rightarrow \psi$ is false.

$$\neg(\phi \rightarrow \psi) \vdash \phi \wedge \neg\psi$$

Let's do another proof by contradiction. Suppose that $\phi \wedge \neg\psi$ is true. Then it follows
that ϕ is true and $\neg\psi$ is true. Since $\neg\psi$ is true, it follows that ψ is false. Since ψ is false,
then $\psi \rightarrow \phi$ is true. This contradicts our assumption that $\phi \wedge \neg\psi$ is true. Therefore,
 $\phi \wedge \neg\psi$ is not true, so $\phi \wedge \neg\psi$ is false.

$$\neg(\phi \wedge \neg\psi) \vdash \neg(\phi \wedge \neg\psi) = (\phi \rightarrow \psi) \vdash \psi$$

Let's do another proof by contradiction. Suppose that $\neg(\phi \wedge \neg\psi)$ is true. Then it follows
that $\phi \wedge \neg\psi$ is false. Since $\phi \wedge \neg\psi$ is false, it follows that either ϕ is false or $\neg\psi$ is true.
If ϕ is false, then $\phi \rightarrow \psi$ is true. This contradicts our assumption that $\neg(\phi \wedge \neg\psi)$ is true.
Therefore, $\phi \wedge \neg\psi$ is not true, so $\phi \wedge \neg\psi$ is false.

$$(1) \quad \xi(\omega, \varphi) = \int_{\mathbb{B}} X_k(\omega, t) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega)(\psi_m, \varphi),$$

gde je $s \geq s_0$ iz (*) a $p \geq p_0$ iz (**). Dalje,

$$(2) \quad E[\xi | \mathcal{U}](\varphi) = \int_{\mathbb{B}} E(X_k(\omega, t) | \mathcal{U}) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} E(c_m(\omega) | \mathcal{U})(\psi_m, \varphi).$$

Dokaz. Neka je $k \geq k_0$, fiksirano. Tada (1) sledi iz Teoreme 3.3.6.. Prema Teoremi 3.3.3. $E(X_k(\cdot, \cdot) | \mathcal{U})$, je merljiva funkcija na $\mathcal{U} \times \mathcal{B}(\mathbb{I})$, a X_k je $\mathcal{F} \times \mathcal{B}(\mathbb{I})$ merljiva. Prema Teoremi 3.3.1., $E(c_m) < \infty$, $m \in \Lambda$. Na isti način kao u Teoremi 3.2.6. može se pokazati da je

$$\left| \int_{\mathbb{B}} \left[\int_{\mathbb{I}} X_k(\omega, t) R^{k+p+s} \varphi(t) dt \right] dP(\omega) \right| \leq K \int_{\mathbb{B}} |R(\omega)| dP(\omega) < \infty.$$

Iz teoreme Fubinija sledi da je, za $\varphi \in \mathcal{A}$,

$$\begin{aligned} \int_{\mathbb{B}} E[\xi | \mathcal{U}](\varphi) dP_{\mathcal{U}}(\omega) &= \int_{\mathbb{B}} \xi(\omega, \varphi) dP(\omega) = \\ &= \int_{\mathbb{B}} \left[\int_{\mathbb{I}} X_k(\omega, t) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_m(\omega)(\psi_m, \varphi) \right] dP(\omega) = \\ &= \int_{\mathbb{I}} \left[\int_{\mathbb{B}} X_k(\omega, t) dP(\omega) R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} \left[\int_{\mathbb{B}} c_m(\omega) dP(\omega) \right] (\psi_m, \varphi) \right] (\psi_m, \varphi) = \end{aligned}$$

$$= \int_{\mathbb{I}} \left[\int_{\mathbb{B}} E(X_k(\omega, t) | \mathcal{U}) dP_{\mathcal{U}}(\omega) \right] R^{k+p+s} \varphi(t) dt +$$

$$+ \sum_{m \in \Lambda} \left[\int_{\mathbb{B}} E(c_m(\omega) | \mathcal{U}) dP_{\mathcal{U}}(\omega) \right] (\psi_m, \varphi) =$$

$$= \int_{\mathbb{I}} \left[\int_{\mathbb{B}} E(X_k(\omega, t) | \mathcal{U}) dP_{\mathcal{U}}(\omega) \right] R^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} E(c_m(\omega) | \mathcal{U})(\psi_m, \varphi) \Big] dP_{\mathcal{U}}(\omega). \square$$

$$\left[\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$

which shows that ϕ_1 is a solution of the Laplace equation.

$$\left[\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] \phi_2(x, y) = 0$$

and since ϕ_2 satisfies the Laplace equation it is a solution of the problem. Therefore we have shown that if ϕ_1 and ϕ_2 are two solutions of the Laplace equation in a domain D , then $\phi_1 - \phi_2$ is also a solution of the Laplace equation in D .

$$\left[\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] (\phi_1 - \phi_2)(x, y) = 0$$

Since $\phi_1 - \phi_2$ is a solution of the Laplace equation in D , it must be zero at the boundary of D . This shows that ϕ_1 and ϕ_2 are linearly independent in D .

$$= \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right]$$

$$= \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial x}$$

$$= \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial y}$$

$$= \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial x} + \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial y}$$

$$= \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial x} + \text{const} \left[\sin \alpha_1 \cos \alpha_2 \left(\frac{\partial}{\partial x} \right)^2 + \sin \alpha_2 \cos \alpha_1 \left(\frac{\partial}{\partial y} \right)^2 \right] \frac{\partial}{\partial y}$$

3.4. REPREZENTACIJA UOPŠTENOG SLUČAJNOG PROCESA

NA $\Omega \times \mathcal{D}^{(MP)}(\emptyset)$ I $\Omega \times \text{Exp}^A$

3.4.1. REPREZENTACIJA U.S.P. NA $\Omega \times \mathcal{D}^{(MP)}(\emptyset)$

U sledećoj teoremi koristićemo ideje dokaza Teoreme 8.1.

[24] i Teoreme 6.1. [1].

Teorema 3.4.1. Neka je ξ u.s.p. na $\Omega \times \mathcal{D}^{(MP)}(\emptyset)$. Za svaki regularan, relativno kompaktan otvoren podskup G od \emptyset i za svako $\varepsilon > 0$ postoji $B \in \mathcal{F}$ i konačna slučajna Radonova mera $\nu_\alpha(\omega, dx)$ na $C(G)$, $\alpha \in \mathbb{N}_0^n$, tako da

$$(1) \quad P(B) \geq 1 - \varepsilon;$$

$$(2) \quad \text{za svako } \omega \in B \text{ postoji } L > 0 \text{ i } c > 0 \text{ tako da}$$

$$\|\nu_\alpha(\omega, dx)\|_{C^1(G)} < cL^{|\alpha|}/M_{|\alpha|}, \alpha \in \mathbb{N}_0^n;$$

$$(3) \quad \text{za svako } \omega \in B \text{ i } \varphi \in \mathcal{D}^{(MP)}(G)$$

$$\xi(\omega, \varphi) = \sum_{\alpha \in \mathbb{N}_0^n} \langle D^\alpha \nu_\alpha(\omega, dx), \varphi(x) \rangle \quad (3.4.1)$$

Staviš, za svako $\omega \in B$ red u (3.4.1) konvergira ka $\xi(\omega, \cdot)$ u smislu jake topologije u $(\mathcal{D}^{(MP)}(\emptyset))'$.

Dokaz. Kako je G regularan, relativno kompaktan otvoren podskup od \emptyset , njegovo zatvorene, \bar{G} , je regularan kompaktan podskup od \emptyset . Obeležimo \bar{G} sa K . Neka je ξ_1 restrikcija od ξ na $\Omega \times \mathcal{D}_K^{(MP)}$. ξ_1 je u.s.p. na $\Omega \times \mathcal{D}_K^{(MP)}$.

Pokazaćemo da se ξ_1 može prikazati u obliku (3.4.1) u

3. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

4. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

5. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

6. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

7. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

8. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

9. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

10. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

11. $\delta = \frac{1}{2} \cdot 10^{-10}$, $\gamma = 0.1$, $\alpha = 10^4$, $\beta = 10^3$

jakoj topologiji prostora $(\mathcal{D}_K^{(MP)})'$. Kako je inkluzija $\mathcal{D}^{(MP)}(G) \rightarrow \mathcal{D}_K^{(MP)}$ neprekidna, pa preslikava ograničene skupove u ograničene, sledi da (3.4.1) važi i u jakoj topologiji prostora $(\mathcal{D}^{(MP)}(G))'$.

Iz definicije i topološke strukture prostora $\mathcal{D}_K^{(MP)}$ sledi da za svako $\omega \in \Omega$ postoji $c(\omega) > 0$ i $j(\omega) \in \mathbb{N}$ tako da

$$|\xi_1(\omega, \varphi)| \leq c(\omega) \|\varphi\|_{x_j(\omega)}, \quad \varphi \in \mathcal{D}_K^{(MP)}.$$

Neka je

$$A_N(\varphi) = \{\omega \in \Omega : |\xi_1(\omega, \varphi)| < N \|\varphi\|_{x_N}\}, \quad \varphi \in \mathcal{D}_K^{(MP)}, \quad N \in \mathbb{N}.$$

i

$$A_N = \bigcap_{\varphi \in \mathcal{D}_K^{(MP)}} A_N(\varphi), \quad N \in \mathbb{N}.$$

Kako je $\mathcal{D}_K^{(MP)}$ separabilan, imamo da

$$A_N = \bigcap_{\varphi \in R} A_N(\varphi) \in \mathcal{F},$$

gde je R gust, prebrojiv skup u $\mathcal{D}_K^{(MP)}$.

Tako iz

$$\Omega = \bigcup_{N=1}^{\infty} A_N, \quad A_N \subset A_{N+1}, \quad N \in \mathbb{N},$$

dobijamo da za dato $\varepsilon > 0$ postoji $r \in \mathbb{N}$ tako da je $P(A_r) \geq 1 - \varepsilon$.

Ako obeležimo $B = A_r$, imamo da je za $\omega \in B$ i $\varphi \in \mathcal{D}_K^{(MP)}$,

$$|\xi_1(\omega, \varphi)| < r \|\varphi\|_{x_r}.$$

Definišimo na $\Omega \times \mathcal{D}_K^{(MP)}$

$$\tilde{\xi}_1(\omega, \varphi) = \begin{cases} \xi_1(\omega, \varphi), & \omega \in B \\ 0, & \omega \notin B \end{cases}, \quad \varphi \in \mathcal{D}_K^{(MP)}.$$

selected at will by the professor. It is important to note
that the probability distribution of the parameter θ is (Θ) is
the same for each of the three distributions.

Let us now consider the following situation: we have a
sample of size $n = 10$, i.e. 10 observations x_1, x_2, \dots, x_{10} are ob-

tained from a population with density function $f(x; \theta) = (\theta x + 3)$. We want to estimate the parameter θ .

It is clear that the maximum likelihood estimate of θ is given by the formula $\hat{\theta} = \bar{x} - 3$, where $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ is the sample mean.

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Neka je

$$\begin{aligned} S(\omega) &= \sup \{ |\tilde{\xi}_1(\omega, \varphi)| : \varphi \in \mathcal{D}_K^{(MP)}, \|\varphi\|_{X_r} \leq 1 \} = \\ &= \sup \{ |\tilde{\xi}_1(\omega, \varphi)| : \varphi \in R, \|\varphi\|_{X_r} \leq 1 \}, \quad \omega \in \Omega. \end{aligned}$$

pa je $S(\cdot)$ merljiva funkcija, i $S(\cdot) \leq r$.

$\mathcal{D}_K^{(MP)}$ je potprostor od X_r . Po probabilističkoj Hahn-Banach-ovojoj teoremi $\tilde{\xi}_1$ se može proširiti na $\Omega \times X_r$. Obeležimo ovo proširenje sa ξ_2 . Tada je

$$|\xi_2(\omega, \varphi)| \leq S(\omega) \|\varphi\|_{X_r}, \quad \varphi \in X_r, \quad \omega \in \Omega.$$

Kako je d_r izometrija "na" prostora X_r i \tilde{X}_r (paragraf 2.3. Glava II), preslikavanje $F: \Omega \times \tilde{X}_r \rightarrow \mathbb{C}$ definisano sa

$$F(\omega, (\tilde{\varphi}_\alpha)) = \xi_2(\omega, \varphi), \quad \text{gde je } \varphi = d_r^{-1}((\tilde{\varphi}_\alpha)),$$

je u.s.p. na $\Omega \times \tilde{X}_r$.

Po probabilističkoj Hahn-Banach-ovojoj teoremi F se može proširiti na $\Omega \times Y$. Obeležimo ovo proširenje sa \tilde{F} . \tilde{F} je u.s.p. na $\Omega \times Y$ tako da je

$$|\tilde{F}(\omega, (\psi_\alpha))| \leq S(\omega) \|(\psi_\alpha)\|_Y, \quad \omega \in \Omega, \quad (\psi_\alpha) \in Y.$$

Za svako $\omega \in \Omega$, $\tilde{F}(\omega, \cdot)$ je linearna i neprekidna funkcionala na Y pa je \tilde{F} oblika

$$\tilde{F}(\omega, \cdot) = \sum_{\alpha \in \mathbb{N}_0^n} F_\alpha(\omega, \cdot),$$

i

$$\|\tilde{F}(\omega, \cdot)\|_Y = \sum_{\alpha \in \mathbb{N}_0^n} \|F_\alpha(\omega, \cdot)\|_{C(K)} = S(\omega),$$

gde su $F_\alpha(\omega, \cdot)$, $\alpha \in \mathbb{N}_0^n$, neprekidne i linearne funkcionele na potprostorima $Y_\alpha \subset Y$, $\alpha \in \mathbb{N}_0^n$, i gde je

$$Y_\alpha = \{(\psi_\beta, \beta \in \mathbb{N}_0^n) : \psi_\beta \in C(K), \psi_\beta \equiv 0 \text{ za } \beta \in \mathbb{N}_0^n \setminus \{\alpha\}\}.$$

Za svako $\alpha \in \mathbb{N}_0^n$ prostor Y_α je izometričan sa $C(K)$.

Obeležićemo $[\varphi]_\alpha$ element iz Y_α koji odgovara $\varphi \in C(K)$. Kako je $\tilde{F}(\omega, \cdot)|_{Y_\alpha} = F_\alpha(\omega, \cdot)$, $\omega \in \Omega$, sledi da su F_α u.s.p. na $\Omega \times Y_\alpha$, $\alpha \in \mathbb{N}_0^n$, odnosno na $\Omega \times C(K)$.

Dalje, Lemma 5.2. [1] implicira da za svako $\alpha \in \mathbb{N}_0^n$ postoji jedna i samo jedna slučajna Radonova mera $\nu_\alpha(\omega, dx)$ tako da je

$$F_\alpha(\omega, [\psi]_\alpha) = \int_K \psi(x) \nu_\alpha(\omega, dx), \quad \omega \in \Omega, \quad \psi \in C(K).$$

Tako, za $\omega \in B$ i $\varphi \in \mathcal{D}_K^{(MP)}$,

$$\xi_1(\omega, \varphi) = \tilde{\xi}_1(\omega, \varphi) = \xi_2(\omega, \varphi) =$$

$$= \tilde{F}(\omega, \left(\frac{r^{| \alpha |} (-1)^\alpha D^\alpha \varphi}{M^{| \alpha |}} \right))$$

$$= \sum_{\alpha \in \mathbb{N}_0^n} \int_K \frac{r^{| \alpha |} (-1)^\alpha}{M^{| \alpha |}} D^\alpha \varphi(x) \nu_\alpha(\omega, dx)$$

$$= \sum_{\alpha \in \mathbb{N}_0^n} \langle \frac{r^{| \alpha |}}{M^{| \alpha |}} D^\alpha \nu_\alpha(\omega, dx), \varphi(x) \rangle$$

$$= \sum_{\alpha \in \mathbb{N}_0^n} \langle D^\alpha \tilde{\nu}_\alpha(\omega, dx), \varphi(x) \rangle,$$

gde je

$$\tilde{\nu}_\alpha(\omega, dx) = \frac{r^{| \alpha |}}{M^{| \alpha |}} \nu_\alpha(\omega, dx),$$

i

$$\sum_{\alpha \in \mathbb{N}_0^n} \left\| \frac{M^{| \alpha |}}{r^{| \alpha |}} \tilde{\nu}_\alpha(\omega, dx) \right\|_{C'(K)} < \infty.$$

Što implicira (2). \square

QCD on noncommutative space-time is a theory of
strong interactions among fields at distance $\sim \alpha_s$ characterized
 $\sim \alpha_s$ by gluons and $\sim \alpha_s^{-1}$ by gluon-gluon vertices.

QCD is a renormalizable theory
which is free of infrared divergences.
Renormalization constants contain other terms in addition to the mass and coupling constant renormalizations.

$$\langle QCD \rangle_{\text{ren}} = \langle QCD \rangle_{\text{bare}} + \delta \langle QCD \rangle_{\text{mass}} + \delta \langle QCD \rangle_{\text{coupling}}$$

Renormalization of the coupling constant is given by the relation

$$\frac{\partial \langle QCD \rangle_{\text{ren}}}{\partial \lambda} = \frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{mass}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{coupling}}}{\partial \lambda} = 0$$

$$\frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial \lambda} = \frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{mass}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{coupling}}}{\partial \lambda} = 0$$

Renormalization of the mass is given by the relation

$$\frac{\partial \langle QCD \rangle_{\text{ren}}}{\partial m} = \frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial m} + \frac{\partial \delta \langle QCD \rangle_{\text{mass}}}{\partial m} + \frac{\partial \delta \langle QCD \rangle_{\text{coupling}}}{\partial m} = 0$$

$$\frac{\partial \langle QCD \rangle_{\text{ren}}}{\partial m} = \left[\frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial m} + \frac{\partial \delta \langle QCD \rangle_{\text{mass}}}{\partial m} + \frac{\partial \delta \langle QCD \rangle_{\text{coupling}}}{\partial m} \right] = 0$$

Renormalization of the coupling constant is given by the relation

$$\frac{\partial \langle QCD \rangle_{\text{ren}}}{\partial \lambda} = \left[\frac{\partial \langle QCD \rangle_{\text{bare}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{mass}}}{\partial \lambda} + \frac{\partial \delta \langle QCD \rangle_{\text{coupling}}}{\partial \lambda} \right] = 0$$

3.4.2. REPREZENTACIJA U.S.P. NA $\Omega \times \text{Exp}^A$

Na sličan način kao i u Teoremi 3.4.1. i Teoremi 3.2.1. možemo pokazati sledeću teoremu.

Teorema 3.4.2. Ako je ξ u.s.p. na $\Omega \times \text{Exp}^A$ tada za svako $\varepsilon > 0$ postoji $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, postoji $k = k(\varepsilon)$, i niz slučajnih promenljivih $\{b_m : m \in \mathbb{N}_0\}$ tako da za svako $\omega \in B$ i $\varphi \in \text{Exp}^A$

$$(1) \quad \xi(\omega, \varphi) = \sum_{m=0}^{\infty} b_m(\omega)(\psi_m, \varphi);$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |b_m(\omega)|^2 (\exp_k |\tilde{\lambda}_n|)^{-k} \right]^{1/2} < \infty.$$

Štaviše, za svako $\omega \in B$ niz u (1) konvergira ka $\xi(\omega, \cdot)$ u smislu jake topologije prostora $(\text{Exp}^A)'$.

Dalje ćemo navesti neke primene reprezentacije u.s.p. ξ na $\Omega \times \text{Exp}^A$ dobijene u Teoremi 3.4.2., na rešavanje stohastičkih evolucionih jednačina.

Neka je $\xi(\cdot, \cdot) = \sum_{m=0}^{\infty} b_m(\cdot)(\psi_m, \cdot)$ u.s.p. na $\Omega \times \text{Exp}^A$, i neka

je T polinom takav da važi sledeći uslov.

Ako je neko $\omega \in \Omega$ i $m \in \mathbb{N}_0$ $b_m(\omega) \neq 0$, tada je $T(\lambda_m) \neq 0$.

Tada stohastička diferencijalna jednačina

$$T(\mathcal{R})u(\omega, \varphi) = \xi(\omega, \varphi), \quad \omega \in \Omega, \quad \varphi \in \text{Exp}^A,$$

gde je $T(\mathcal{R})u(\omega, \varphi) = u(\omega, T(\mathcal{R})\varphi)$, $\omega \in \Omega$, $\varphi \in \text{Exp}^A$, ima rešenje

4.00 hours. The 4 hours is 1 and 1/2 an eighth of an hour.

Therefore, there are 16 hours remaining.

Now we can divide 1600 by 16 and get 100. This is the number of hours that have passed since the time when the alarm was set. Since the alarm was set at 10:00 AM, the time is now 10:00 AM + 100 hours = 10:00 AM + 4 hours = 1:00 PM.

$$\text{Time} = \frac{\text{Number of hours}}{\text{Hours per day}} = \frac{100}{24} = 4\frac{1}{3}$$

$$\text{Time} = \left| \frac{\text{Number of hours}}{\text{Hours per day}} \right| = \left| \frac{100}{24} \right| = 4\frac{1}{3}$$

In other words, the time has passed 100 hours or 4 full days plus 1/3 of a day.

Therefore, the alarm clock was set at 10:00 AM on the previous day. Since the alarm was set at 10:00 AM, it must have been set at 10:00 AM on the previous day.

Since 1 hour is 60 minutes, $\frac{1}{3}$ of an hour is 20 minutes.

Therefore, the alarm clock was set at 10:20 AM on the previous day.

Therefore, the answer is 10:20 AM on the previous day.

Therefore, the answer is 10:20 AM on the previous day.

$$\text{Number of hours} = \frac{\text{Number of minutes}}{\text{Minutes per hour}} = \frac{120}{60} = 2$$

Therefore, the answer is 120 minutes or 2 hours.

$$u(\omega, \varphi) = \left(\sum_{m=0}^{\infty} \frac{b_m(\omega)}{T(\lambda_m)} \psi_m, \varphi \right).$$

Ako je za neko $\omega \in \Omega$ $b_m(\omega)=0$ i $T(\lambda_m)=0$ stavljamo da je
 $b_m(\omega)/T(\lambda_m)=0$, $m \in \mathbb{N}_0$.

Rešenje u je u.s.p. za koji Teorema 3.4.2. implicira:

Za svako $\varepsilon > 0$ postoji $B \in \mathcal{F}$ i $k=k(\varepsilon)$

(i) tako da je $P(B) \geq 1-\varepsilon$, i $\sum_{m=0}^{\infty} |u_m(\omega)|^2 (\exp_k |\tilde{\lambda}_m|)^{-2k} < \infty$,

gde je $u_m(\omega) = b_m(\omega)/T(\lambda_m)$, $m \in \mathbb{N}_0$, $\omega \in \Omega$.

Obeležimo sa, E_1^k diferencijalni operator beskonačnog reda

oblika

$$E_1^k (e^R)^k = e^{kR} = \sum_{m=0}^{\infty} \frac{k^m R^m}{m!}$$

a sa E_p^k operator

$$E_p^k = \exp k(\underbrace{\exp \dots \exp R}_{p})$$

$$= \sum_{m_1=0}^{\infty} \frac{k^{m_1}}{m_1!} \sum_{m_2=0}^{\infty} \frac{m_1^{m_2}}{m_2!} \dots \sum_{m_p=0}^{\infty} \frac{m_p^{m_{p-1}}}{m_p!} R^{m_p}.$$

Na sličan način možemo posmatrati i rešiti stohastičku
diferencijalnu jednačinu beskonačnog reda

$$T_0(R)u + T_1(E_1)u + \dots + T_s(E_s)u = Hu = \xi.$$

Tačnije, ova jednačina je oblika

$$Hu(\omega, \varphi) \equiv u(\omega, H\varphi) = \xi(\omega, \varphi), \quad \omega \in \Omega, \quad \varphi \in \text{Exp} \mathcal{A},$$

gde je ξ dati u.s.p. na $\Omega \times \text{Exp} \mathcal{A}$, a T_0, T_1, \dots, T_s proizvoljni
polinomi [33].

$$\left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix} \right) = (0,0,0)$$

so, the matrix form of $\text{diag}(v_1, v_2, v_3)$ is $\text{diag}(v_1, v_2, v_3)$ where v_1, v_2, v_3 are vectors.

$$v_1 = \alpha \cdot \text{diag}(v_1, v_2, v_3)$$

similarly $v_2 = \beta \cdot \text{diag}(v_1, v_2, v_3)$ and $v_3 = \gamma \cdot \text{diag}(v_1, v_2, v_3)$

(which is the definition of eigenvectors)

$$\text{diag}(v_1, v_2, v_3) = \frac{1}{\alpha} v_1 + \frac{1}{\beta} v_2 + \frac{1}{\gamma} v_3$$

$$\text{diag}(v_1, v_2, v_3) = \frac{1}{\alpha} v_1 + \frac{1}{\beta} v_2 + \frac{1}{\gamma} v_3 = (\alpha, \beta, \gamma) \cdot \text{diag}(v_1, v_2, v_3)$$

Since $\text{diag}(v_1, v_2, v_3)$ is a linear combination of v_1, v_2, v_3 it is also an eigenvector.

$$\text{diag}(v_1, v_2, v_3) = \frac{1}{\alpha} v_1 + \frac{1}{\beta} v_2 + \frac{1}{\gamma} v_3 = \frac{1}{\alpha} \text{diag}(v_1, v_2, v_3) + \frac{1}{\beta} \text{diag}(v_1, v_2, v_3) + \frac{1}{\gamma} \text{diag}(v_1, v_2, v_3)$$

so, $\text{diag}(v_1, v_2, v_3)$ is an eigenvector of A with eigenvalue $\alpha + \beta + \gamma$.

Other eigenvalues = ?

Let's do the same for $\text{diag}(v_1, v_2, v_3)$

$$\text{diag}(v_1, v_2, v_3) = \frac{1}{\alpha} v_1 + \frac{1}{\beta} v_2 + \frac{1}{\gamma} v_3 = \frac{1}{\alpha} \text{diag}(v_1, v_2, v_3) + \frac{1}{\beta} \text{diag}(v_1, v_2, v_3) + \frac{1}{\gamma} \text{diag}(v_1, v_2, v_3)$$

which shows $\text{diag}(v_1, v_2, v_3)$ is an eigenvector of A with eigenvalue $\alpha + \beta + \gamma$.

$$\text{diag}(v_1, v_2, v_3) = \alpha \cdot \text{diag}(v_1, v_2, v_3) + \beta \cdot \text{diag}(v_1, v_2, v_3) + \gamma \cdot \text{diag}(v_1, v_2, v_3)$$

which is a linear combination of v_1, v_2, v_3

$$\text{diag}(v_1, v_2, v_3) = \alpha \cdot \text{diag}(v_1, v_2, v_3) + \beta \cdot \text{diag}(v_1, v_2, v_3) + \gamma \cdot \text{diag}(v_1, v_2, v_3)$$

so, $\text{diag}(v_1, v_2, v_3)$ is an eigenvector of A with eigenvalue $\alpha + \beta + \gamma$.

Dalje, posmatrajmo stohastičku diferencijalnu jednačinu

$$(ii) \frac{\partial u(\omega, t, \varphi)}{\partial t} = T(\mathcal{R}) u(\omega, t, \varphi), \quad \omega \in \Omega, \quad t \in [0, \infty), \quad \varphi \in \text{Exp} \mathcal{A},$$

$$u(\omega, \varphi) = \xi(\omega, \varphi), \quad \omega \in \Omega, \quad \varphi \in \text{Exp} \mathcal{A},$$

gde je ξ dati u.s.p. na $\Omega \times \text{Exp} \mathcal{A}$.

Pretpostavimo da je $u(\omega, \cdot, \varphi)$ glatka funkcija na $[0, \infty)$ za svako $\omega \in \Omega, \varphi \in \text{Exp} \mathcal{A}$, i za svako $\omega \in \Omega, t \in [0, \infty), \varphi \in \text{Exp} \mathcal{A}$:

$$T(\mathcal{R})u(\omega, t, \varphi) = u(\omega, t, T(\mathcal{R})\varphi).$$

Koristeći rezultate u [33] dobijamo da je

$$u(\omega, t, \varphi) = \left(\sum_{m=0}^{\infty} (\exp(T(\lambda_m)t)) b_m(\omega) \psi_m, \varphi \right),$$

$$b_m(\omega) = \xi(\omega, \psi_m), \quad m \in \mathbb{N}_0, \quad \omega \in \Omega,$$

rešenje jednačine u (ii). Za koeficijente

$u_m(\omega, t) = (\exp(T(\lambda_m)t)) b_m(\omega), \quad m \in \mathbb{N}_0$, važi (i) ako t pripada ograničenom skupu u $[0, \infty)$. U ovom slučaju B i k ne zavise od tog ograničenog skupa.

Na sličan način možemo rešiti stohastičku diferencijalnu jednačinu oblika

$$\frac{\partial u(\omega, t, \varphi)}{\partial t} = (T_0(\mathcal{R}) + T_1(E_1) + \dots + T_s(E_s)) u(\omega, t, \varphi),$$

$$\omega \in \Omega, \quad t \in [0, \infty), \quad \varphi \in \text{Exp} \mathcal{A},$$

$$u(\omega, t, \varphi) = \xi(\omega, \varphi), \quad \omega \in \Omega, \quad \varphi \in \text{Exp} \mathcal{A},$$

gde je ξ dati u.s.p. na $\Omega \times \text{Exp} \mathcal{A}$.

containing, which is consistent with our previous calculations, and

$$\text{Im}(\omega) = \eta_{\text{eff}}(0,0) = 2.10 \approx \alpha - \text{Im}(q,T,\omega)\nu(\text{RGT}) = \frac{\text{Im}(q,T,\omega)\nu}{\pi}, \quad (113)$$

$$\text{Im}(\omega) = \eta_{\text{eff}}(0,0) \approx \alpha - \text{Im}(q,\omega)\nu = \langle q,T,\omega \rangle \nu$$

where $\eta_{\text{eff}}(0,0)$ is the effective Josephson energy, α is the superconducting gap, and $\langle q,T,\omega \rangle \nu$ is the contribution from the quasiparticle current.

As we can see, the effective Josephson energy $\eta_{\text{eff}}(0,0)$ is equal to the difference between the superconducting gap and the quasiparticle current.

$$\text{Im}(q(T),T,\omega)\nu = \langle q,T,\omega \rangle \nu(\text{RGT})$$

and, according to (113), the constant term (RGT) is independent of the temperature.

$$\langle q_{\text{eff}},T,\omega \rangle \nu(\text{RGT}) = \frac{\text{Im}(q(T),T,\omega)\nu}{\pi \nu}$$

$$= \alpha - \omega_{\text{eff}} + \omega - \langle q,T,\omega \rangle \nu = (\omega_{\text{eff}} - \omega) + \langle q,T,\omega \rangle \nu$$

where $\omega_{\text{eff}} = \min(\omega_{\text{eff}}(0,0), \omega_{\text{eff}}(0,0))$ is the effective energy gap at zero temperature, and $\langle q,T,\omega \rangle \nu = \text{Im}(q(T),T,\omega)\nu = \langle q,T,\omega \rangle \nu(\text{RGT})$ is the average value of the quasiparticle current $\langle q,T,\omega \rangle \nu$ in the superconducting gap.

It follows from the above that the current $\eta_{\text{eff}}(0,0)$ is equal to the sum of the superconducting gap and the average value of the quasiparticle current.

$$\langle q_{\text{eff}},T,\omega \rangle \nu(\text{RGT}) = \langle q,T,\omega \rangle \nu + \langle q,T,\omega \rangle \nu = \frac{\text{Im}(q,T,\omega)\nu}{\pi}$$

$$\text{Im}(\omega) = \eta_{\text{eff}}(0,0) = \alpha - \langle q,T,\omega \rangle \nu$$

$$\text{Im}(\omega) = \eta_{\text{eff}}(0,0) = \alpha - \text{Im}(q,T,\omega)\nu = \langle q,T,\omega \rangle \nu$$

As we can see, the superconducting gap and the average value of the quasiparticle current are equal.

GLAVA IV

4.1. KONVERGENCIJE NIZA U.S.P. NA $\Omega \times \mathcal{A}$

U svom radu [22], L.J. Kitchens je dao definicije konvergencije u verovatnoći, srednje kvadratno i skoro sigurno, niza $\{\xi_n, n \geq 1\}$ uopštenih slučajnih procesa na $\Omega \times \mathcal{K}\{M_p\}$. U [22] su takođe dobijene karakterizacije niza u.s.p. na $\Omega \times \mathcal{K}\{M_p\}$ koji konvergira na neki od navedenih načina.

Sledeći njegova istraživanja, u ovoj glavi daćemo karakterizacije niza $\{\xi_n, n \geq 1\}$ u.s.p. na $\Omega \times \mathcal{A}$, koji konvergira u verovatnoći, skoro sigurno i srednje kvadratno. Niz u.s.p. ćemo kraće zapisivati $\{\xi_n\}$.

4.1.1. OSNOVNE DEFINICIJE

Definicija 4.1.1. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira ka u.s.p. ξ na $\Omega \times \mathcal{A}$ u verovatnoći (\mathcal{A}') ako za svako $\varepsilon > 0$ postoji $k \in \mathbb{N}_0$ tako da

$$\lim_{n \rightarrow \infty} P\{\omega \in \Omega : \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi) - \xi(\omega, \varphi)| \geq \varepsilon\} = 0.$$

Kraće pišemo $\xi_n \xrightarrow{\mathcal{A}'} \xi$.

Definicija 4.1.2. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira ka u.s.p. ξ na $\Omega \times \mathcal{A}$ srednjem (\mathcal{A}') ako postoji $k \in \mathbb{N}_0$ tako da

$$\lim_{n \rightarrow \infty} \int_{\Omega} \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi) - \xi(\omega, \varphi)| dP(\omega) = 0.$$

N = G OR N/G AND GLICKSEN'S METHOD

It is well known that in the case of a finite group G , the number N of irreducible characters of G is equal to the number of conjugacy classes of G . Let us consider the case of a finite abelian group G of order n . Then G has n conjugacy classes, which are the cyclic subgroups of G . Let $\chi_1, \chi_2, \dots, \chi_n$ be the characters of G . Then χ_i is the character of the subgroup $\langle g_i \rangle$ where g_i is a generator of $\langle g_i \rangle$. It is well known that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. Hence $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. This shows that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. This shows that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. This shows that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. This shows that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$.

GLICKSEN'S METHOD

Let us consider the case of a finite abelian group G of order n . Then G has n conjugacy classes, which are the cyclic subgroups of G . Let $\chi_1, \chi_2, \dots, \chi_n$ be the characters of G . Then $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. Hence $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. This shows that $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$.

Let us consider the case of a finite abelian group G of order n . Then G has n conjugacy classes, which are the cyclic subgroups of G . Let $\chi_1, \chi_2, \dots, \chi_n$ be the characters of G . Then $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$. Hence $\chi_i(g_j) = \chi_i(g_j)$ if $g_j \in \langle g_i \rangle$ and $\chi_i(g_j) = 0$ if $g_j \notin \langle g_i \rangle$.

Kraće pišemo $\xi_n \xrightarrow{1} \xi$, (\mathcal{A}').

Osigledno, konvergencija u verovatnoći (srednjem) (\mathcal{A}') implicira takozvanu slabu konvergenciju u verovatnoći (srednjem):

Definicija 4.1.3. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira slabo u verovatnoći (\mathcal{A}') ka u.s.p. ξ na $\Omega \times \mathcal{A}$ ako za svako $\varphi_0 \in \mathcal{A}$, $\xi_n(\cdot, \varphi_0) \xrightarrow{v} \xi(\cdot, \varphi_0)$.

Definicija 4.1.4. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira slabo u srednjem (\mathcal{A}') ka u.s.p. ξ na $\Omega \times \mathcal{A}$ ako za svako $\varphi_0 \in \mathcal{A}$, $\xi_n(\cdot, \varphi_0) \xrightarrow{1} \xi(\cdot, \varphi_0)$.

Definicija 4.1.5. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira ka u.s.p. ξ na $\Omega \times \mathcal{A}$ skoro sigurno (\mathcal{A}') ako postoji skup $A \in \mathcal{F}$ takav da je $P(A) = 0$ i da za $\omega \in \Omega \setminus A$, $\xi_n(\omega, \cdot) \rightarrow \xi(\omega, \cdot)$ slabo u \mathcal{A}' .

Kraće pišemo $\xi_n \xrightarrow{s.s.} \xi$, (\mathcal{A}').

Definicija 4.1.6. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira ka u.s.p. ξ na $\Omega \times \mathcal{A}$ ograničeno u verovatnoći (\mathcal{A}') ako

(i) $\xi_n \xrightarrow{v} \xi$, (\mathcal{A}'),

(ii) postoji skup $A \in \mathcal{F}$, takav da je $P(A) = 0$ i da je za $\omega \in \Omega \setminus A$ skup $\{\xi_n(\omega, \cdot)\}$ je ograničen u \mathcal{A}' .

Kraće pišemo $\xi_n \xrightarrow{v.o.} \xi$, (\mathcal{A}').

Definicija 4.1.7. Niz u.s.p. $\{\xi_n\}$ na $\Omega \times \mathcal{A}$ konvergira ka ξ na $\Omega \times \mathcal{A}$ ograničeno u srednjem (\mathcal{A}') ako

$\{n\} \rightarrow \{n\}$ mapping above

$\{n\}$ designated above such that $n \in \{n\}$ never holds for $n \in \mathbb{N}$

Mapping π is bijective and $\pi(n)$ is maximal among

all $m \in \mathbb{N}$ such that $n \in \{m\}$ holds for some $k \in \mathbb{N}$

and $\pi(n) = n$ if and only if $n \in \{n\}$ holds for all $k \in \mathbb{N}$

and $\pi(n) < n$ if and only if $n \in \{n\}$ holds for some $k \in \mathbb{N}$

and $\pi(n) > n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for some $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for all $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for some $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for all $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for some $k \in \mathbb{N}$

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and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

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and $\pi(n) \neq n$ if and only if $n \in \{n\}$ holds for no $k \in \mathbb{N}$

(i) $\xi_n \xrightarrow{1} \xi$, (\mathcal{A}'),

(ii) postoji skup $A \in \mathcal{F}$, takav da je $P(A) = 0$ i da je za $\omega \in \Omega \setminus A$ skup $\{\xi_n(\omega, \cdot)\}$ ograničen u \mathcal{A}' .

Kraće pišemo $\xi_n \xrightarrow{1 \circ} \xi$, (\mathcal{A}').

Očigledno da $\xi_n \xrightarrow{v \circ} \xi$ implicira $\xi_n \xrightarrow{v} \xi$ i $\xi_n \xrightarrow{1 \circ} \xi$ implicira $\xi_n \xrightarrow{1} \xi$.

Kako niz u.s.p. $\{\xi_n\}$ konvergira ka u.s.p. ξ ako i samo ako niz u.s.p. $\{\xi_n - \xi\}$ konvergira ka nuli, dalje ćemo posmatrati samo slučaj kada niz u.s.p. $\{\xi_n\}$ konvergira ka nuli.

4.2. KONVERGENCIJA SKORO SIGURNO (\mathcal{A}')

Teorema 4.2.1. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Sledeći uslovi su ekvivalentni.

A. Niz $\{\xi_n\}$ konvergira ka nuli skoro sigurno (\mathcal{A}').

B. (i) Za svako $\varphi \in \mathcal{A}$, $\{\xi_n(\cdot, \varphi)\}$ konvergira ka nuli skoro sigurno.

(ii) Postoji skup $A \in \mathcal{F}$ takav da je $P(A)=0$, i za svako $\omega \in \Omega \setminus A$ skup $\{\xi_n(\cdot, \varphi)$, $n \in \mathbb{N}\}$ je ograničen u \mathcal{A}' .

C. (i) Za svako $\varphi \in \mathcal{A}$, niz $\{\xi_n(\cdot, \varphi)\}$ konvergira ka nuli skoro sigurno.

(ii) Za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, i broj $k_0 \in \mathbb{N}_0$, oba nezavisni od n , takvi da je $P(B) \geq 1-\varepsilon$, i za svako $\omega \in B$, $\varphi \in \mathcal{A}$, je $|\xi_n(\omega, \varphi)| \leq k_0 \|\varphi\|_{k_0}$.

Dokaz. Dokaz je sličan dokazu Teoreme 2.2. u [22]. Pokazaćemo da $A \Rightarrow C \Rightarrow B \Rightarrow A$.

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and we see $\phi = \lambda\beta$ is a value of ϕ in A since β is in C_{112} .
The ϕ -function $\phi(\cdot, \omega_0)$ sends $\lambda \neq 0$ to
 $\phi(\lambda) = e^{\frac{1}{2}\lambda^2}$ making short

calculations for ϕ analogous to those in the preceding LEMMA.
Thus $\phi(\lambda)$ is a value of ϕ in A since λ is in C_{112} .
Since $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A since
 λ is in C_{112} , there must exist $\lambda \in C_{112}$ such that $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .
Thus $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .
Thus $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .

LEMMA 33. If $\phi(\lambda)$ is a value of ϕ in A then $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .
PROOF. Let $\phi(\lambda)$ be a value of ϕ in A . Then $\phi(\lambda)$ is a value of ϕ in A .
Since $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A ,
 $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .

LEMMA 34. If $\phi(\lambda)$ is a value of ϕ in A then $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .
PROOF. Let $\phi(\lambda)$ be a value of ϕ in A . Then $\phi(\lambda)$ is a value of ϕ in A .
Since $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A ,
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Thus $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .

LEMMA 35. If $\phi(\lambda)$ is a value of ϕ in A then $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .
PROOF. Let $\phi(\lambda)$ be a value of ϕ in A . Then $\phi(\lambda)$ is a value of ϕ in A .
 $\phi(\lambda)$ is a value of ϕ in A and $\phi(\lambda)$ is a value of ϕ in A .

A. \Rightarrow C. Pretpostavimo da $\xi_n \xrightarrow{s.s.} \xi$, (\mathcal{A}'). Tada C.(i) odmah sledi. Imamo da je za svako $\omega \in \Omega \setminus A$ niz $\{\xi_n(\omega, \cdot)\}$ ograničen, a kako je $\mathcal{A}' = \bigcup_{k=0}^{\infty} \mathcal{A}'_k$, sledi da postoji nenegativan prirodan broj $k = k(\omega)$, koji ne zavisi od n , takav da je $|\xi_n(\omega, \varphi)| \leq k(\omega) \|\varphi\|_{k(\omega)}$, $\varphi \in \mathcal{A}$. Dalje, kao u [28, Theorem 2.], videti takođe [1,12], označimo

$$A_N(\varphi) = \{\omega \in \Omega : |\xi_n(\omega, \varphi)| \leq N \|\varphi\|_N\}, \quad \varphi \in \mathcal{A}, \quad N \in \mathbb{N}.$$

$$A_N = \bigcap_{\varphi \in S_r} A_N(\varphi).$$

Imamo da je A_N merljiv skup, da je $\Omega \setminus A \subset \bigcup_{N=1}^{\infty} A_N$.

i $A_N \subset A_{N+1}$, $N \in \mathbb{N}$. Odatle dobijamo da za dato $\varepsilon > 0$ postoji $k \in \mathbb{N}_0$ takav da je $P(A_k) \geq 1 - \varepsilon$. Stavimo $B = A_k$ i C.(ii) sledi.

C. \Rightarrow B. Uslovi C.(i) i B.(i) su isti, a da bismo pokazali da C.(ii) \Rightarrow B.(ii), odaberimo $\varepsilon = 1/p$, $p \in \mathbb{N}$. Tada postoji B_p i k_p , nezavisni od n , takvi da je $P(B_p) \geq 1 - 1/p$, i za $\omega \in B_p$, $|\xi_n(\omega, \varphi)| \leq k_p \|\varphi\|_{k_p}$, $\varphi \in \mathcal{A}$. Neka je $A = \Omega \setminus \bigcup_{p=1}^{\infty} B_p$. Tada je $P(A) = 0$ i B.(ii) sledi.

B. \Rightarrow A. Iz B.(i) sledi da za svako $\varphi \in S_r$ postoji skup $A_\varphi \in \mathcal{F}$, takav da je $P(A_\varphi) = 0$ i za svako $\omega \in \Omega \setminus A_\varphi$, $\xi_n(\omega, \varphi) \rightarrow 0$. Neka je $A' = A \cup (\bigcup_{\varphi \in S_r} A_\varphi)$, (A je skup iz uslova B.(ii)). Imamo da je $P(A') = 0$, i za svako $\varphi \in S_r$ i $\omega \in \Omega \setminus A'$, $\xi_n(\omega, \varphi) \rightarrow 0$. Kako iz uslova B.(ii) sledi da je za svako $\omega \in \Omega \setminus A'$ niz $\{\xi_n(\omega, \cdot)\}$ ograničen u \mathcal{A}' , na osnovu Banach-Steinhausove teoreme sledi da za svako $\omega \in \Omega \setminus A'$, $\xi_n \xrightarrow{s.s.} 0$ (\mathcal{A}'). \square

函数 $C(D)$ 使 $\{0\} \times A \xrightarrow{\text{id}_A} A$ 为常数映射。如果 $A \times B \neq \emptyset$ 且 $\text{id}_{A \times B}$ 为常数映射， $A \times B = \emptyset$ 。如果 $A \times B \neq \emptyset$ 且 $\text{id}_{A \times B}$ 不为常数映射，则 $A \times B = \emptyset$ 。

如果 $\{0\} \times A \neq \emptyset$ 且 $\text{id}_{\{0\} \times A}$ 为常数映射，则 $\{0\} \times A = \emptyset$ 。如果 $\{0\} \times A \neq \emptyset$ 且 $\text{id}_{\{0\} \times A}$ 不为常数映射，则 $\{0\} \times A \neq \emptyset$ 。

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Teorema 4.2.2. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Tada $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}') ako i samo ako $\forall \epsilon > 0$ postoji skup $C \in \mathcal{F}$, takav da je $P(C) \geq 1-\epsilon$, postoji $k_0 \in \mathbb{N}_0$, takav da za $k > k_0$ i $\omega \in C$,

$$(1) \quad \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)| \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Prepostavimo da $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}'). Tada iz Teoreme 4.2.1. deo C.(ii) sledi da postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1-\epsilon$, broj $k_0 \in \mathbb{N}_0$ takvi da za $\omega \in B$, $\varphi \in \mathcal{A}$, $n \in \mathbb{N}$, $|\xi_n(\omega, \varphi)| \leq k_0 \|\varphi\|_k$. Dalje, iz Teoreme 3.2.1. sledi da za svako $n \in \mathbb{N}$ postoji niz slučajnih promenljivih $\{c_{m,n}, m \in \mathbb{N}_0\}$ takav da je za svako $\omega \in B$, $\varphi \in \mathcal{A}$,

$$\xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega)(\psi_m, \varphi), \quad (4.2.1)$$

i za svako $n \in \mathbb{N}$, i $k \geq k_0$, k fiksirano,

$$\left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} < k. \quad (4.2.2)$$

Kako $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}'), postoji $A \in \mathcal{F}$, takav da je $P(A) = 0$ i da za $\omega \in \Omega \setminus A$, $\xi_n(\omega, \varphi) \rightarrow 0$ za svako $\varphi \in \mathcal{A}$. Stavimo $C = B \setminus A$. Tada je $P(C) \geq 1-\epsilon$ jer je $P(\cdot)$ kompletna mera. Stavljujući $\varphi = \psi_m$, $m \in \mathbb{N}_0$ u (4.2.1.), dobijamo, za $\omega \in C$.

$$\xi_n(\omega, \psi_m) = c_{m,n}(\omega) \rightarrow 0, \quad n \rightarrow 0, \quad m \in \mathbb{N}_0. \quad (4.2.3)$$

Neka je $k > k_0$ fiksirano. Imamo, za $\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}$, $\omega \in C$,

$$|\xi_n(\omega, \varphi)| = \left| \sum_{m=0}^{\infty} c_{m,n}(\omega) (\psi_m, \varphi) \right| =$$

$$= \left| \sum_{m=0}^{\infty} c_{m,n}(\omega) \tilde{\lambda}_m^{-k} \tilde{\lambda}_m^k a_m \right| \leq$$

ESTA E UNA DIFERENCIA ENTRE EL COEFICIENTE ALTO DE INTERACCIÓN
DE LA ECUACIÓN LINEAL Y EL DE LA CURVA EN LA FIGURA 10. ESTA DIFERENCIA
SE PUEDE VER EN LA TABLA 10, DONDE SE HA COMPARADO EL COEFICIENTE ALTO

DE LA ECUACIÓN LINEAL CON EL COEFICIENTE ALTO DE LA CURVA EN LA TABLA 10.

$$\text{TABLA 10. } \frac{\partial \mu}{\partial x} = \left(\frac{\partial \mu}{\partial x} \right)_{\text{lineal}} \quad \text{que} \quad \left(\frac{\partial \mu}{\partial x} \right)_{\text{curva}}$$

ESTA TABLA MUESTRA QUE EL COEFICIENTE ALTO DE LA ECUACIÓN LINEAL ES MAYOR QUE EL COEFICIENTE ALTO DE LA CURVA EN LA FIGURA 10. ESTA DIFERENCIA SE PUEDE VER EN LA TABLA 10, DONDE SE HA COMPARADO EL COEFICIENTE ALTO DE LA ECUACIÓN LINEAL CON EL COEFICIENTE ALTO DE LA CURVA EN LA FIGURA 10.

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$$\left(\frac{\partial \mu}{\partial x} \right)_{\text{lineal}} = \left(\frac{\partial \mu}{\partial x} \right)_{\text{curva}}$$

$$\left(\frac{\partial \mu}{\partial x} \right)_{\text{lineal}} = \left(\frac{\partial \mu}{\partial x} \right)_{\text{curva}}$$

$$\left(\frac{\partial \mu}{\partial x} \right)_{\text{lineal}} = \left(\frac{\partial \mu}{\partial x} \right)_{\text{curva}}$$

$$\leq \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \|\varphi\|_k.$$

Dalje,

$$\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} =$$

$$\sum_{m=0}^{m_0} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} + \tilde{\lambda}_{m_0+1}^{-2} \sum_{m=m_0+1}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2(k-1)},$$

gde je m_0 izabrano tako da

$$\tilde{\lambda}_{m_0+1}^{-2} \leq \left(\frac{\varepsilon}{2k}\right)^2.$$

Iz (4.2.3.) sledi da postoji $n_0 = n_0(\varepsilon)$ takvo da je, za

$\omega \in C$

$$\sum_{m=0}^{m_0} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} < \left(\frac{\varepsilon}{2}\right)^2, \quad n \geq n_0.$$

Dakle, imamo da za svako $\varepsilon > 0$, $\omega \in C$, $\varphi \in A$, $k > k_0$,

$$|\xi_n(\omega, \varphi)| \leq \varepsilon \|\varphi\|_k,$$

pa sledi (1).

Obrnuto, pretpostavimo da je (1) ispunjeno. Tada C.(ii) odmah sledi.

Za svaki $p \in \mathbb{N}$ izaberimo $\varepsilon = 1/p$. Iz (1) sledi da za $p \in \mathbb{N}$ postoji C_p i k_p takvi da je $P(C_p) \geq 1 - 1/p$, i za $\omega \in C_p$ $\sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)| \rightarrow 0$. Neka je $A = \bigcup_{p=1}^{\infty} C_p$, tada je $P(A) = 0$. Za svako $\omega \in \Omega \setminus A$ postoji $p(\omega)$ takvo da $\omega \in C_{p(\omega)}$ i postoji $k_{p(\omega)}$, takvo da $\sup_{\substack{\|\varphi\|_k \leq 1 \\ p(\omega)}} |\xi_n(\omega, \varphi)| \rightarrow 0$. Dakle, za dato $\varphi \in A$, i za

$\omega \in \Omega \setminus A$

$$|\xi_n(\omega, \varphi)| \leq \sup_{\substack{\|\varphi\|_k \leq 1 \\ p(\omega)}} |\xi_n(\omega, \varphi)| \|\varphi\|_{k_{p(\omega)}} \rightarrow 0,$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\partial_x \partial_y^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \right) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right) \alpha$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \partial_x \partial_y^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \frac{\partial}{\partial x} \frac{\partial}{\partial z} \alpha$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial x} \partial_y^{-1} \partial_z^{-1} (\alpha)_{x,y,z} + \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \partial_y^{-1} \partial_z^{-1} (\alpha)_{x,y,z}$$

all summands are of size $\leq \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha| dx dy dz$

$\leq \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha| dx dy dz \leq \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\alpha| dx dy dz$

so that the desired value is obtained by lemma C.E.B.M.30 with $C = 1/\pi$, $\delta = 1/2$, $\epsilon = 1/2$, $M = 1$.

Since $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} = \partial_z^{-1} \partial_z^{-1} \alpha$ we have $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} = \partial_z^{-1} \partial_z^{-1} \alpha$

by applying theorem C.D.8 to α and $\partial_z^{-1} \partial_z^{-1} \alpha$ above we obtain (since

$$\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \geq - \frac{1}{2} \|\alpha\|_2$$

and since $\partial_z^{-1} \partial_z^{-1} \alpha = \alpha$ we have $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \geq - \frac{1}{2} \|\alpha\|_2$ also by lemma C.D.8.

It follows that $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} = \alpha$ and $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \geq - \frac{1}{2} \|\alpha\|_2$ also by lemma C.D.8.

Since $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} = \alpha$ we have $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} = \alpha$ and $\partial_z^{-1} \partial_z^{-1} (\alpha)_{x,y,z} \geq - \frac{1}{2} \|\alpha\|_2$ also by lemma C.D.8.

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pa C.(i) sledi. □

U Teoremi 3.2.3. data je reprezentacija u.s.p. ξ pomoću običnog slučajnog procesa. U sledećoj teoremi biće dat potreban u dovoljan uslov da u ovom slučaju niz $\{\xi_n\}$ konvergira skoro sigurno (\mathcal{A}').

Teorema 4.2.3. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Tada $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}') ako i samo ako za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$ sa $P(B) \geq 1 - \varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su i B i k_0 nezavisni od n ; za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n}(\omega) : n \in \mathbb{N}\}$, i za svako $k > k_0$ postoji niz funkcija $X_{k,n}$ na $\Omega \times I$, $n \in \mathbb{N}$, tako da, za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \int_I X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

postojeći tako da $\omega \in B$, $\varphi \in \mathcal{A}$,

$$(2) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} < k, \quad \omega \in \Omega,$$

$$(3) \quad \|X_{k,n}(\cdot, \cdot)\|_{L^2} \xrightarrow{s.s.} 0,$$

$$(4) \quad \text{za } \omega \in C, \quad \sum_{m \in \Lambda} c_{m,n}(\omega) \rightarrow 0, \quad n \rightarrow \infty,$$

gde je C skup iz Teoreme 4.2.2..

Dokaz. Pretpostavimo da $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}') i neka je $\varepsilon > 0$ dato. Iz C(ii) Teoreme 4.2.1. sledi da postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, postoji broj $k_0 \in \mathbb{N}_0$, oba nezavisna od n , takva da $|\xi_n(\omega, \varphi)| \leq \|\varphi\|_{k_0}$, $\omega \in B$, $\varphi \in \mathcal{A}$. Tada iz Teoreme 3.2.3. sledi da za svaki $n \in \mathbb{N}$ i za $k \geq k_0$ postoji funkcija $X_{k,n}$ na $\Omega \times I$, i slučajne promenljive $c_{m,n}$ na Ω , $m \in \Lambda$, tako da je (1) i (2) ispunjeno.

Während die in der vorliegenden Arbeit erarbeiteten Ergebnisse über die Verteilung von O_2 -Rückständen im Boden und deren Beeinflussung durch verschiedene Faktoren im Rahmen der Bodenökologie eine wichtige Basis für die Anwendung der Ergebnisse der hier vorgelegten Untersuchungen bilden, ist die hier vorgenommene Untersuchung der Verteilung von O_2 -Rückständen im Boden auf die Bodenökologie und auf die Bodenökophysiologie im weitesten Sinn nicht unbedeutend.

Die Ergebnisse der hier vorgenommenen Untersuchungen zeigen, dass die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von den Bodenfaktoren, die die Verteilung bestimmen, unterschiedlich ist. So ist z.B. die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von der Bodentiefe und dem Wassergehalt des Bodens, während sie in Abhängigkeit von der Bodentemperatur und dem Wassergehalt des Bodens nicht so stark variiert. Die Ergebnisse der hier vorgenommenen Untersuchungen können als Grundlage für die Anwendung der Ergebnisse der hier vorgenommenen Untersuchungen auf die Bodenökologie und auf die Bodenökophysiologie dienen.

Die Ergebnisse der hier vorgenommenen Untersuchungen zeigen, dass die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von den Bodenfaktoren, die die Verteilung bestimmen, unterschiedlich ist. So ist z.B. die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von der Bodentiefe und dem Wassergehalt des Bodens, während sie in Abhängigkeit von der Bodentemperatur und dem Wassergehalt des Bodens nicht so stark variiert. Die Ergebnisse der hier vorgenommenen Untersuchungen können als Grundlage für die Anwendung der Ergebnisse der hier vorgenommenen Untersuchungen auf die Bodenökologie und auf die Bodenökophysiologie dienen.

Die Ergebnisse der hier vorgenommenen Untersuchungen zeigen, dass die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von den Bodenfaktoren, die die Verteilung bestimmen, unterschiedlich ist. So ist z.B. die Verteilung von O_2 -Rückständen im Boden in Abhängigkeit von der Bodentiefe und dem Wassergehalt des Bodens, während sie in Abhängigkeit von der Bodentemperatur und dem Wassergehalt des Bodens nicht so stark variiert. Die Ergebnisse der hier vorgenommenen Untersuchungen können als Grundlage für die Anwendung der Ergebnisse der hier vorgenommenen Untersuchungen auf die Bodenökologie und auf die Bodenökophysiologie dienen.

Dalje, kako je

$$\|X_{k,n}(\omega, \cdot)\|_{L^2} = \begin{cases} \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)|, & \omega \in B \\ 0, & \omega \notin B \end{cases}, \quad (4.2.5)$$

imamo, prema (1), Teorema 4.2.2., da $\|X_{k,n}(\omega, \cdot)\|_{L^2} \rightarrow 0$ za $\omega \in \Omega \setminus (B \cap A)$, (A je skup iz Definicije 4.1.5.). Kako je $P(B \cap A) = 0$ sledi (3).

Kako i u Teoremi 4.2.2. možemo dobiti da za svako $m \in \Lambda$, $c_{m,n}(\omega) \rightarrow 0$, $\omega \in C = B \setminus A$, kako je Λ konačno, (4) sledi.

Obrnuto, neka su uslovi (1) - (4) ispunjeni. Uslovi (3) i (4) impliciraju (1) iz Teoreme 4.2.2. pa iz iste teoreme sledi da $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}'). □

U slučaju da je niz $\{\xi_n\}$ u.s.p. na $\Omega \times \mathcal{A}$ prikazan preko niza neprekidnih slučajnih procesa na $\Omega \times I$ imamo samo potreban uslov za konvergenciju skoro sigurno (\mathcal{A}').

Pretpostavimo, kao i ranije, da su uslovi (*) i (**) iz Glave III zadovoljeni.

Teorema 4.2.4. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}') tada za svako $\varepsilon < 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, postoji broj $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n}(\omega) : n \in \mathbb{N}\}$ na Ω , i za svako $k > k_0$ niz neprekidnih slučajnih procesa $X_{k,n}$ na $\Omega \times I$, tako da za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \int_{\mathbb{I}} X_{k,n}(\omega, t) \mathcal{R}^{k+p+s} \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$$\left\{ \begin{array}{l} \text{if } n = 0 \\ \text{if } n \neq 0 \end{array} \right. \quad \left. \begin{array}{l} \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \sum_{k=1}^{\infty} \frac{1}{k^2} \\ \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \end{array} \right\} = \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \cdot \frac{1}{d} \sum_{k=1}^{\infty} \frac{1}{k^2}$$

so $\sigma_0(n) = \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \sum_{k=1}^{\infty} \frac{1}{k^2}$. This shows that $\sigma_0(A \cap B) = A \cap B$ if and only if $A \cap B$ is relatively prime to n , i.e. $(A \cap B) \times D = n$.

It follows from the previous section that σ_0 is bounded and each σ_0 -invariant subgroup A of \mathbb{Z} either $A \times D = \mathbb{Z} \times n$, or $D = \langle m \rangle$, and σ_0 has no nontrivial subgroups. So σ_0 makes up about $\frac{1}{2}$ of all σ_0 -invariant subgroups of \mathbb{Z} . It follows that σ_0 is σ_0 -invariant by σ_0 -invariantness of σ_0 :

$$\sigma_0(\sigma_0(n)) = \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \sum_{k=1}^{\infty} \frac{1}{k^2}$$

which implies $\sigma_0(n) = n$ for all $n \in \mathbb{Z}$ since σ_0 is injective. Since σ_0 is σ_0 -invariant it follows that σ_0 is σ_0 -invariant. This completes the proof.

End of section 3.1.1. \square

It follows that σ_0 is σ_0 -invariant. Now let $n \in \mathbb{Z}$. Then $\sigma_0(n)$ is σ_0 -invariant. So $\sigma_0(n)$ is σ_0 -invariant. Since σ_0 is σ_0 -invariant, $\sigma_0(\sigma_0(n)) = \sigma_0(n)$. This implies $\sigma_0(n) = n$. This completes the proof.

$$\sigma_0(\sigma_0(n)) = \frac{1}{2} \sum_{d|n} \mu(d) \cdot \frac{1}{d} \sum_{k=1}^{\infty} \frac{1}{k^2} = (\sigma_0(n))^2 = n^2$$

Since σ_0 is σ_0 -invariant, $\sigma_0(n) = n$.

$\omega \in B$, $\varphi \in \mathcal{A}$, gde je $s \geq s_0$ iz (*), $p \geq p_0$ iz (**),

$$(2) \|x_{k,n}(\omega, \cdot)\|_{L^2} \leq k, \quad \omega \in \Omega,$$

(3) $\{x_{k,n}(\omega, \cdot), n \geq 1\}$ je jednako neprekidan na I za $p > p_0$, $\omega \in \Omega \setminus A$,

(4) za $\omega \in \Omega \setminus (B \cap A)$, $x_{k,n}(\omega, \cdot) \rightarrow 0$, na I , $n \rightarrow \infty$,

(5) za svako $t \in I$ $x_{k,n}(\cdot, t) \rightarrow 0$, na $\Omega \setminus A$, $n \rightarrow \infty$,

$$(6) \sum_{m \in \Lambda} c_{m,n}(\omega) \rightarrow \infty, \quad n \rightarrow \infty, \quad \omega \in B \setminus A$$

gde je A skup iz Definicije 4.1.5.

Dokaz. Pretpostavimo da $\xi_n \xrightarrow{s.s.} 0$, (\mathcal{A}'). Tada iz C(ii),

Teorema 4.2.1. i Teoreme 3.2.5. sledi (1) i (2), gde je za $n \in \mathbb{N}_0$ i $k \geq k_0$

$$x_{k,n}(\omega, t) = \begin{cases} \sum_{m=0}^{\infty} c_{m,n}(\omega) \tilde{\lambda}_m^{-(k+p+s)} \psi_m(t), & \omega \in B \\ 0 & \omega \notin B \end{cases}, \quad t \in I.$$

Neka je $t \in I$. Za $\omega \notin B$, $x_{k,n}(\omega, t) = 0$, a $\omega \in B$

$$|x_{k,n}(\omega, t)| = \left| \sum_{m=0}^{\infty} c_{m,n}(\omega) \tilde{\lambda}_m^{-(k+p+s)} \psi_m(t) \right| \leq$$

$$\leq K \sum_{m=0}^{\infty} |c_{m,n}(\omega) \tilde{\lambda}_m^{-(k+p)}| < \varepsilon, \quad \text{za } n \geq n_0(\varepsilon).$$

na isti način kao i u Teoremi 4.2.2., jer je $k+p > k_0$, pa (4) sledi. Uslov (6) sledi na isti način kao i uslov (4) u Teoremi 4.2.3.

Da bismo pokazali (3) uočimo da iz uslova (**) sledi, za $p > p_0$,

then ω is a η -periodic function, i.e. $\omega(x) = \omega(x + \eta)$.

$$B(\omega)_{\eta,\delta} = \{(\omega)_n\}_{n \in \mathbb{Z}} \quad (\text{C2})$$

and it has continuous extension to $\{1 \leq n < (\omega)_n\}_{n \in \mathbb{Z}}$. (C3)

$$\lim_{n \rightarrow -\infty} (\omega)_n = 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} (\omega)_n = \omega_0 < 0 \quad (\text{C4})$$

where $\omega_0 = \lim_{n \rightarrow -\infty} (\omega)_n$, $\omega_0 < 0$ since $(\omega)_n > 0$ for all $n \in \mathbb{Z}$. (C5)

Let ω be a η -periodic function, i.e. $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C6)

Then ω is a η -periodic function if and only if $\omega \in \{\omega\}_{\eta,\delta} \cap \mathbb{Z}$. (C7)

Thus, ω is a η -periodic function if and only if $\omega \in \{\omega\}_{\eta,\delta} \cap \mathbb{Z}$. (C8)

Since ω is a η -periodic function, we have $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C9)

Let ω be a η -periodic function, i.e. $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C10)

Then ω is a η -periodic function if and only if $\omega \in \{\omega\}_{\eta,\delta} \cap \mathbb{Z}$. (C11)

$$\begin{aligned} \omega &= \omega_n \quad \text{for all } n \in \mathbb{Z} \\ \Rightarrow \omega &= \left\{ \omega_n \right\}_{n \in \mathbb{Z}} \end{aligned} \quad \left. \begin{array}{l} \text{continuous function} \\ \text{extension to } \{1 \leq n < (\omega)_n\}_{n \in \mathbb{Z}} \end{array} \right\} \in \{\omega\}_{\eta,\delta} \cap \mathbb{Z}$$

Since ω is a η -periodic function, we have $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C12)

$$\Rightarrow \left[\{\omega_n\}_{n \in \mathbb{Z}} \right] = \left[\{\omega_{n+1}\}_{n \in \mathbb{Z}} \right] = \dots = \left[\{\omega_m\}_{m \in \mathbb{Z}} \right]$$

$$\left[\{\omega_n\}_{n \in \mathbb{Z}} \right] = \left[\{\omega_{n+1}\}_{n \in \mathbb{Z}} \right] = \dots = \left[\{\omega_m\}_{m \in \mathbb{Z}} \right]$$

Since ω is a η -periodic function, we have $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C13)

Since ω is a η -periodic function, we have $\omega(x) = \omega(x + \eta)$ for all $x \in \mathbb{R}$. (C14)

$$\sum_{m=0}^{\infty} \tilde{\lambda}_m^{-2p} = A < \infty.$$

Možemo odabrat i₀ tako da je

$$\left[\min_{m \geq i_0} \tilde{\lambda}_m^2 \right]^{-1} < \frac{\varepsilon^2}{4AK^2k^2} .$$

Konstanta K je iz uslova (*). Niz $\tilde{\lambda}_m$, $m \in \mathbb{N}_0$ je monotono neopadajući pa je $\tilde{\lambda}_{i_0}^2 = \min_{m \geq i_0} \tilde{\lambda}_m^2$.

Kako su $\psi_m(\cdot)$, $m \in \mathbb{N}_0$ neprekidne funkcije, za svako $t, t' \in I$ i svako $\varepsilon > 0$ postoji $\delta = \delta(\varepsilon, t)$ tako da je

$$\sum_{m=0}^{i_0-1} |\psi_m(t) - \psi_m(t')|^2 \tilde{\lambda}_m^{-2(s+p)} < \frac{\varepsilon^2}{2k^2} ,$$

ako je $|t-t'| < \delta(\varepsilon, t)$. Sada imamo za $t, t' \in I$, $|t-t'| < \delta(\varepsilon, t)$, $\omega \in B$

$$\begin{aligned} |x_{k,n}(\omega, t) - x_{k,n}(\omega, t')| &\leq \sum_{m=0}^{\infty} |c_{m,n}(\omega)| |\tilde{\lambda}_m|^{-(k+p+s)} |\psi_m(t) - \psi_m(t')| \leq \\ &\leq \left(\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right)^{1/2} \left(\sum_{m=0}^{\infty} |\psi_m(t) - \psi_m(t')|^2 \tilde{\lambda}_m^{-2(p+s)} \right)^{1/2} \leq \\ &\leq k \left(\sum_{m=0}^{i_0-1} |\psi_m(t) - \psi_m(t')| \tilde{\lambda}_m^{-2(p+s)} + 2K^2 \sum_{m=i_0}^{\infty} \tilde{\lambda}_m^{-2p} \right)^{1/2} \leq \\ &\leq k \left[\frac{\varepsilon^2}{2k^2} + \frac{2K^2}{\tilde{\lambda}_{i_0}^2} \sum_{m=i_0}^{\infty} \tilde{\lambda}_m^{-2(p+s)} \right]^{1/2} \leq \\ &\leq k \left[\frac{\varepsilon^2}{2k^2} + \frac{\varepsilon^2}{2k^2} \right]^{1/2} = \varepsilon. \end{aligned}$$

Kako je još $x_{k,n}(\omega, \cdot) = 0$ za $\omega \in B$, sledi (3), odnosno da je niz $\{x_{k,n}(\omega, \cdot)\}$ jednako neprekidan na I.

Dalje, za $t_0 \in I$, $x_{k,n}(\omega, t_0) = 0$ za $\omega \in B$, a za $\omega \in B \setminus A$,

and the initial value problem

$$\begin{cases} \frac{d}{dt} \tilde{\phi}_t(\omega) = \tilde{\phi}_t(\omega) \tilde{A} \\ \tilde{\phi}_0(\omega) = \tilde{\phi}_0(\omega) \end{cases}$$

is equivalent with the evolution of the solution

$$\tilde{\phi}_t(\omega) = \tilde{\phi}_0(\omega) e^{\int_0^t \tilde{A}(s) ds}$$

of the differential equation $\dot{\phi}_t(\omega) = \phi_t(\omega) A_t(\omega)$. The initial condition is given by $\tilde{\phi}_0(\omega) = \phi_0(\omega)$.

Let us now consider the evolution of the solution $\phi_t(\omega)$ of the differential equation $\dot{\phi}_t(\omega) = \phi_t(\omega) A_t(\omega)$ under the condition that $A_t(\omega)$ is bounded.

Since $A_t(\omega)$ is bounded, we have $\|A_t(\omega)\|_{\infty} \leq C$ for all $t \in [0, T]$, where $C > 0$ is a constant.

Then we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

Since $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$, we have $\|\phi_t(\omega)\|_{\infty} \leq \|A_t(\omega)\|_{\infty} \|\phi_0(\omega)\|_{\infty}$ for all $t \in [0, T]$.

$t \in I$,

$$|X_{k,n}(\omega, t_0)| = |X_{k,n}(\omega, t_0) - X_{k,n}(\omega, t) + X_{k,n}(\omega, t)| \leq \\ \leq |X_{k,n}(\omega, t_0) - X_{k,n}(\omega, t)| + |X_{k,n}(\omega, t)|.$$

Iz (3) imamo da je $|X_{k,n}(\omega, t_0) - X_{k,n}(\omega, t)| \leq \varepsilon/2$, kada je $|t-t_0| < \delta(\varepsilon, t_0)$, a iz (4) da je $|X_{k,n}(\omega, t)| \leq \varepsilon/2$, za $n \geq n_0(\varepsilon)$, pa (5) sledi. \square

4.3. KONVERGENCIJA U VEROVATNOĆI (\mathcal{A}')

Teorema 4.3.1. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{v.o.} 0, (\mathcal{A}')$ tada za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, takav da je $P(B) \geq 1-\varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , i za svako $n \in \mathbb{N}$ postoji niz slučajnih promenljivih $\{c_{m,n}: m \in \mathbb{N}_0\}$ takođe da je, za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega) (\psi_m, \varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} \leq k_0, \quad \omega \in B,$$

(3) za svako $\delta > 0$

$$P\left\{\omega \in B: \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} > \delta\right\} \rightarrow 0, \quad n \rightarrow \infty$$

$$(4) \quad P\left\{\omega \in B: |c_{m,n}(\omega)| > \delta\right\} \rightarrow 0, \quad n \rightarrow \infty, \quad m \in \mathbb{N}_0.$$

Dokaz. Pretpostavimo da $\xi_n \xrightarrow{v.o.} 0, (\mathcal{A}')$, i neka je $\varepsilon > 0$ dato. Uslov (ii) iz Definicije 4.1.6. je isti kao i uslov B.(ii) iz Teoreme 4.2.1. pa je prema tome ekvivalentan uslovu C.(ii) iste teoreme. Prema tome, postoji skup $B \in \mathcal{F}$, takav da

$$\begin{aligned} & \left(\partial_{\alpha} (\partial_{\beta} \omega)_{\alpha, \beta} \right)_{\alpha, \beta} K + (\partial^{\alpha} \omega)_{\alpha, \beta} K + (\partial_{\beta} \omega)_{\alpha, \beta} K = (\partial_{\alpha} \partial_{\beta} \omega)_{\alpha, \beta} K \\ & |\partial_{\alpha} (\partial_{\beta} \omega)_{\alpha, \beta}| + |\partial^{\alpha} \omega|_{\alpha, \beta} K + |\partial_{\beta} \omega|_{\alpha, \beta} K \leq \end{aligned}$$

Now we have $\|\partial_{\alpha} (\partial_{\beta} \omega)_{\alpha, \beta}\| \leq C_1 \|\omega\|_{\alpha, \beta} K$ because $(\partial_{\alpha} \partial_{\beta} \omega)_{\alpha, \beta}$ is bounded by $(\partial_{\beta} \omega)_{\alpha, \beta} K$ up to the term $\|\omega\|_{\alpha, \beta} K$ (see (2.2)). So, $\|\partial_{\alpha} (\partial_{\beta} \omega)_{\alpha, \beta}\| \leq C_1 \|\omega\|_{\alpha, \beta} K$. Now we have $\|\partial^{\alpha} \omega\|_{\alpha, \beta} \leq C_1 \|\omega\|_{\alpha, \beta} K$ and $\|\partial_{\beta} \omega\|_{\alpha, \beta} \leq C_1 \|\omega\|_{\alpha, \beta} K$.

Step 3. Let us prove that ω is a distributional solution of (2.2). We will prove that ω is a weak solution of (2.2).

Let $\phi \in C_c^{\infty}(\Omega)$ be a smooth function such that $\int_{\Omega} \phi dx = 1$. Then we can find a smooth function $\psi \in C_c^{\infty}(\Omega)$ such that $\int_{\Omega} \psi dx = 1$ and $\partial_{\beta} \psi = \delta_{\beta} \phi$. This follows from the fact that ω is a solution of (2.2).

Then we have $\int_{\Omega} \omega \cdot \partial_{\beta} \phi dx = \int_{\Omega} \omega \cdot \partial_{\beta} (\psi \phi) dx = \int_{\Omega} (\omega \cdot \partial_{\beta} \psi) \phi dx$.

$$\int_{\Omega} (\omega \cdot \partial_{\beta} \psi) \phi dx = \int_{\Omega} \omega \cdot \partial_{\beta} \psi dx = \int_{\Omega} \omega \cdot \partial_{\beta} \phi dx.$$

So, ω is a weak solution of (2.2).

$$\begin{aligned} & \text{Let } \omega = \left\{ \begin{array}{ll} \frac{\partial_{\alpha} \omega}{\alpha!} \left[\frac{\partial_{\beta} \omega}{\beta!} \left(\frac{\partial_{\gamma} \omega}{\gamma!} \left[\dots \left(\frac{\partial_{\theta} \omega}{\theta!} \right) \right] \right) \right] & \text{if } \omega \neq 0 \\ 0 & \text{if } \omega = 0 \end{array} \right\} \quad \text{and } \omega' = \left\{ \omega \in \{ \omega \in C_c^{\infty}(\Omega) \mid \omega \neq 0 \} \right\} \quad \text{if } \omega \neq 0 \end{aligned}$$

$\omega \in \omega$ if and only if $\omega \in \{ \omega \in C_c^{\infty}(\Omega) \mid \omega \neq 0 \}$ and $\omega' \in \omega'$. So, ω is a distributional solution of (2.2).

je $P(B) \geq 1-\varepsilon$, postoji $k_0 \in \mathbb{N}_0$, nezavisni od n , takvi da za $\omega \in B$ i $\varphi \in \mathcal{A}$ važi $|\xi_n(\omega, \varphi)| \leq k_0 \|\varphi\|_{k_0}$. Na osnovu toga (1) i (2) sledi direktno iz Teoreme 3.2.1.

Iz dokaza Teoreme 3.2.1. imamo da je

$$\sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| = \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2}, \quad \omega \in B.$$

Dakle, za $\delta > 0$ je

$$\begin{aligned} P\left\{\omega \in B: \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} \geq \delta \right\} &= \\ &= P\left\{\omega \in B: \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| \geq \delta \right\} \leq \\ &\leq P\left\{\omega \in \Omega: \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| \geq \delta \right\} \rightarrow 0, \quad n \rightarrow \infty, \end{aligned}$$

pa važi (3).

Za $\varphi = \psi_m$, iz (1) dobijamo $\xi_n(\cdot, \psi_m) = c_{m,n}(\cdot)$, $m \in \mathbb{N}_0$. Kako konvergencija niza $\{\xi_n\}$ ograničeno u verovatnoći implicira slabu konvergenciju u verovatnoći imamo da je

$$P\{\omega \in B: |c_{m,n}(\omega)| \geq \delta\} \leq P\{\omega \in \Omega: |\xi_n(\omega, \psi_m)| \geq \delta\} \rightarrow 0,$$

$n \rightarrow \infty$, pa (4) sledi. \square

Da bismo pokazali suprotno, potreban nam je dodatni uslov.

Teorema 4.3.2. Niz $\{\xi_n\}$ u.s.p. na $\Omega \times \mathcal{A}$ konvergira ka nuli ograničeno u verovatnoći (\mathcal{A}') ako za svako $n \in \mathbb{N}$ postoji niz slučajnih promenljivih $\{c_{m,n}: m \in \mathbb{N}_0\}$ tako da su ispunjeni sledeći uslovi:

Postoji $k_0 \in \mathbb{N}_0$ tako da za svako $p \in \mathbb{N}$ postoji skup $B_p \in \mathcal{F}$, sa $P(B_p) \geq 1 - 1/p$, i da je za $n \in \mathbb{N}$,

on an equal or the difference will be at least $n-1$ & $(n)^{n-1}$ &
the original number will contain $n-1$ digits. Now if $n-1$ is even
then the last digit of the number will be even & if $n-1$ is odd
then the last digit of the number will be odd.

Now $\left(\frac{n}{2} \right)^{n-1} \times \left(\frac{n}{2} + 1 \right) = \left(\frac{n+1}{2} \right)^n$ &
so we have $(n-1)^{n-1} < \left(\frac{n+1}{2} \right)^n$ which
implies $n^{n-1} < \left(\frac{n+1}{2} \right)^{n+1}$ i.e. $n < 6$. So required

number is $\left\{ \left(\frac{n}{2} - 1 \right)^{n-1} \times \left(\frac{n}{2} + 1 \right) \right\}_{n=3}^6$.
So the required numbers are $2^2 \times 3^2 = 36$, $3^2 \times 5^2 = 225$,
 $4^2 \times 7^2 = 784$, $5^2 \times 9^2 = 8100$ & $6^2 \times 11^2 = 14580$.
So the required numbers are 36, 225, 784, 8100 & 14580.

Ques. If n is a natural number such that $n^2 + 1$ is divisible by 3, then find the smallest value of n for which $n^2 + 1$ is divisible by 9.

Sol. If $n^2 + 1$ is divisible by 3, then $n^2 \equiv -1 \pmod{3}$ & if n is smallest such number then $n^2 \equiv -1 \pmod{9}$ because if $n^2 \equiv -1 \pmod{9}$ then $n^2 + 1 \equiv 0 \pmod{9}$ which contradicts the fact that $n^2 + 1$ is not divisible by 9. So the required value of n is 2.

$$(1) \quad \xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega) (\psi_m, \varphi), \quad \omega \in B_p, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} < k_0, \quad \omega \in B_p,$$

(3) za svako $\delta > 0$

$$P\left\{\omega \in B_p : \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right] > \delta\right\} \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Neka je $\varepsilon = 1/p$, $p \in \mathbb{N}$. Tada za svako $p \in \mathbb{N}$ postoji skup $B_p \in \mathcal{F}$, sa $P(B_p) \geq 1 - 1/p$ tako da važi (1), (2), (3). Neka

je $\Omega_1 = \bigcup_{p=1}^{\infty} B_p$. Imamo da je $P(\Omega) = 1$, i za $\omega \in \Omega$, $\varphi \in \mathcal{A}$,

$$|\xi_n(\omega, \varphi)| \leq \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} \|\varphi\|_{k_0} \leq k_0 \|\varphi\|_{k_0},$$

tako da je uslov (ii) Definicije 4.1.6. ispunjen, gde je $A = \Omega \setminus \Omega_1$.

Da bismo pokazali da je ispunjen i uslov (i) Definicije 4.1.6., pretpostavimo da je $\varepsilon > 0$ dato. Tada postoji $p \in \mathbb{N}$ tako da je $P(B_p) \geq 1 - \varepsilon/2$, odnosno $P(B_p^c) < \frac{\varepsilon}{2}$. Takođe iz (3) sledi da za svako $\delta > 0$ postoji $n_0 = n_0(\varepsilon, \delta)$, tako da za $n \geq n_0$

$$P\left\{\omega \in B_p : \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} > \delta\right\} < \frac{\varepsilon}{2}.$$

kako je

$$\sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| = \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2}, \quad \omega \in B_p,$$

imamo da je za svako $n \geq n_0$, $\delta > 0$,

$$P\left\{\omega \in \Omega_1 : \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| > \delta\right\} =$$

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \lim_{m \rightarrow \infty} \frac{1}{m^n} \sum_{n_1, \dots, n_m} \langle \phi(n_1, \dots, n_m), \psi(n_1, \dots, n_m) \rangle_n$$

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \lim_{m \rightarrow \infty} \left[\frac{1}{m^n} \sum_{n_1, \dots, n_m} \langle \phi(n_1, \dots, n_m), \psi(n_1, \dots, n_m) \rangle_n \right]$$

$\phi, \psi \in \mathcal{H}$ erlaubt

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \left\{ h \in \left[\frac{1}{n^{2n}} \sum_{n_1, \dots, n_n} \langle \phi(n_1, \dots, n_n), \psi(n_1, \dots, n_n) \rangle_n \right] \mid \text{definiert} \right\}$$

Dieser Wert ist ein Grenzwert, der abhängt von n , aber nicht von m . Es ist also ein eindeutiger Wert.

Um dies zu zeigen, sei $\phi, \psi \in \mathcal{H}$ und $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$ definiert.

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \left[\frac{1}{n^{2n}} \sum_{n_1, \dots, n_n} \langle \phi(n_1, \dots, n_n), \psi(n_1, \dots, n_n) \rangle_n \right] \in \{ \langle \phi, \psi \rangle_n \}$$

Wir zeigen, dass $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$ ein Grenzwert ist, der unabhängig von n ist.

Seien $\phi, \psi \in \mathcal{H}$ und $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$ definiert. Wir wollen zeigen, dass $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$ ein Grenzwert ist. Dazu wählen wir eine Folge von n -en, die gegen ∞ geht. Seien n_1, \dots, n_m die ersten m Glieder dieser Folge. Dann ist $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes m}}$ definiert. Da $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes m}}$ ein Grenzwert ist, existiert $\langle \phi, \psi \rangle_m$, der unabhängig von m ist. Da $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes m}}$ ein Grenzwert ist, existiert $\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$, der unabhängig von n ist.

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \left[\frac{1}{n^{2n}} \sum_{n_1, \dots, n_n} \langle \phi(n_1, \dots, n_n), \psi(n_1, \dots, n_n) \rangle_n \right] \in \{ \langle \phi, \psi \rangle_n \}$$

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \left[\frac{1}{n^{2n}} \sum_{n_1, \dots, n_n} \langle \phi(n_1, \dots, n_n), \psi(n_1, \dots, n_n) \rangle_n \right] \in \{ \langle \phi, \psi \rangle_n \} \subset \{ \langle \phi, \psi \rangle_n \}$$

$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}}$ ist also ein Grenzwert.

$$\langle \phi, \psi \rangle_{\mathcal{H}^{\otimes n}} = \left[\frac{1}{n^{2n}} \sum_{n_1, \dots, n_n} \langle \phi(n_1, \dots, n_n), \psi(n_1, \dots, n_n) \rangle_n \right] \in \{ \langle \phi, \psi \rangle_n \}$$

$$\begin{aligned}
&= P\left\{\omega \in B_p : \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 |\tilde{\lambda}_m|^{-2k_0} \right]^{1/2} > \delta \right\} + \\
&+ P\left\{\omega \in B_p : \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| > \delta \right\} < \\
&< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \quad \square
\end{aligned}$$

Teorema 4.3.3. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{v.o.} 0$, (\mathcal{A}'), tada za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n} : n \in \mathbb{N}\}$, i za svako $k \geq k_0$ postoji niz funkcija, $X_{k,n}$ na $\Omega \times I$, tako da je, za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \int_{\Omega} X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$$\omega \in B, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} < k, \quad \omega \in \Omega,$$

$$(3) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \xrightarrow{v} 0, \quad n \rightarrow \infty,$$

$$(4) \quad \text{za svako } \delta > 0$$

$$P\left\{\omega \in B : \left| \sum_{m \in \Lambda} |c_{m,n}(\omega)| \right| > \delta \right\} \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Kako $\xi_n \xrightarrow{v.o.} 0$, (\mathcal{A}') ispunjen je uslov (ii) iz Definicije 4.1.6.. Ovaj uslov je ekvivalentan uslovu C.(ii) Teoreme 4.2.1. pa na osnovu Teoreme 3.2.3. imamo da je (1) i (2) ispunjeno, gde je, za $n \in \mathbb{N}$, $k > k_0$

$$\begin{aligned} & \left\{ \theta \in \mathbb{R}^n \mid \theta^T (\theta)_{\text{diag}} \theta \leq 1, \theta \geq 0 \right\} \cap \\ & \left\{ \theta \in \mathbb{R}^n \mid \theta^T (\theta)_{\text{diag}} \theta \leq 1, \theta \geq 0, \theta_i = 0 \right\} \cap \\ & \left\{ \theta \in \mathbb{R}^n \mid \theta^T (\theta)_{\text{diag}} \theta \leq 1, \theta \geq 0, \theta_i = 0 \right\} \end{aligned}$$

with $\theta_i = 0$ for all $i \neq j$. This is a convex set, which is called the n -dimensional simplex. It is the feasible region of linear programming problems with $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ as the objective function and $\theta \geq 0$ as the equality constraint. The simplex is the convex hull of the vertices $(0,0,\dots,0)$, $(1,0,\dots,0)$, $(0,1,0,\dots,0)$, ..., $(0,0,\dots,0,1)$.

Let $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ be a linear programming problem with $\theta \geq 0$ as the equality constraint.

Then $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex function of θ and $\theta \geq 0$ is a convex set.

Therefore, the feasible region of $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex set.

Since $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex function of θ and $\theta \geq 0$ is a convex set,

the feasible region of $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex set.

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Since $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex function of θ and $\theta \geq 0$ is a convex set,

the feasible region of $\theta^T (\theta)_{\text{diag}} \theta \leq 1$ is a convex set.

$$X_{k,n}(\omega, t) = \begin{cases} \sum_{m=0}^{\infty} c_{m,n}(\omega) \tilde{\lambda}_m^{-k} \psi_m(t), & \omega \in B \\ 0 & , \quad \omega \notin B \end{cases}, \quad t \in I.$$

Tako imamo da je

$$\|X_{k,n}(\omega, \cdot)\|_{L^2}^2 = \begin{cases} \sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} & , \quad \omega \in B, \quad t \in I \\ 0 & , \quad \omega \in B, \quad t \in I \end{cases} < k,$$

a za $\omega \in B, \varphi \in \mathcal{A}$

$$\|X_{k,n}(\omega, \cdot)\|_{L^2} = \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)|.$$

Dakle, kako je za svako $\delta > 0$

$$P\{\omega \in B: \|X_{k,n}(\omega, \cdot)\|_{L^2} > \delta\} = 0.$$

imamo, za to isto $\delta > 0$,

$$P\{\omega \in \Omega: \|X_{k,n}(\omega, \cdot)\|_{L^2} > \delta\} = P\{\omega \in B: \|X_{k,n}(\omega, \cdot)\|_{L^2} > \delta\} =$$

$$= P\{\omega \in B: \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)| > \delta\} \leq P\{\omega \in \Omega: \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)| > \delta\} \rightarrow 0, \quad n \rightarrow \infty,$$

odnosno, važi (3).

(4) sledi na isti način kao i (4) u Teoremi 4.3.1., jer je Λ konačan skup.

Teorema 4.3.4. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Tada $\xi_n \xrightarrow{v.o.} 0$, (\mathcal{A}') ako postoji $k_0 \in \mathbb{N}_0$ tako da za svaki $p \in \mathbb{N}$ postoji $B_p \in \mathcal{F}$, sa $P(B_p) \geq 1 - 1/p$, i za $n \in \mathbb{N}$, $k \geq k_0$ postoji niz funkcija $X_{k,n} : \Omega \times I \rightarrow \mathbb{C}$ i za svako $n \in \Lambda$ niz slučajnih promenljivih $\{c_{m,n} : m \in \mathbb{N}_0\}$ tako da je

$$(1) \quad \xi_n(\omega, \varphi) = \int_I X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$$\omega \in B_p, \quad \varphi \in \mathcal{A},$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

$$\left\{ \begin{array}{l} \text{if } \alpha = 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha > 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \\ \text{if } \alpha < 0, \text{ then } \langle \alpha, \beta \rangle_{\alpha, \beta} = \langle \alpha, \beta \rangle_{\alpha, \beta} \end{array} \right\} = \langle \alpha, \beta \rangle_{\alpha, \beta}$$

QD 3: Linear Combinations

Linear combinations in QD 1 and entries listed in Table 6.3.

Suppose scalars λ and

$$\begin{aligned} &\text{and } \alpha, \beta \in G \text{ are elements such that } \alpha \neq 0 \text{ and } \beta \neq 0 \text{ are linearly independent.} \\ &\text{Then } \lambda \alpha + \beta \text{ is also } \neq 0 \text{ if and only if } (\lambda, 1) \neq (0, 1) \text{ or } \lambda \neq -1. \\ &\text{Moreover, if } \lambda \neq -1 \text{ then } \lambda \alpha + \beta = \lambda(\alpha + \beta) + (\lambda - 1)\beta \text{ and } \lambda \neq -1 \text{ if and only if } \lambda(\alpha + \beta) \neq 0. \\ &\text{Consequently, when } \lambda \neq -1 \text{ we have } \lambda \alpha + \beta = \lambda(\alpha + \beta) + (\lambda - 1)\beta \text{ which implies that } \\ &\text{the scalar } \lambda \text{ and the element } \beta \text{ are linearly independent.} \\ &\text{Consequently, if } \lambda \neq -1 \text{ then } \langle \lambda \alpha + \beta, \beta \rangle_{\lambda \alpha + \beta, \beta} = \langle \lambda \alpha + \beta, \beta \rangle_{\lambda \alpha + \beta, \beta} = 1. \end{aligned}$$

On the other hand,

if

$$\langle \lambda \alpha + \beta, \beta \rangle_{\lambda \alpha + \beta, \beta} = 1$$

then $\lambda \alpha + \beta = \beta$ and $\lambda \alpha = 0$.

$$(2) \|X_{k,n}(\omega, \cdot)\|_{L^2} < k, \quad \omega \in \Omega,$$

$$(3) \|X_{k,n}(\omega, \cdot)\|_{L^2} \xrightarrow{v} 0, \quad n \rightarrow \infty,$$

(4) za svako $\delta > 0$

$$P\left\{\omega \in B_p : \left| \sum_{m \in \Lambda} c_{m,n}(\omega) \right| > \delta\right\} \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Dokaz je isti kao dokaz Teoreme 4.3.2. jer je

$$\|X_{k,n}(\omega, \cdot)\|_{L^2} = \begin{cases} \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)|, & \omega \in B_p \\ 0, & \omega \notin B_p \end{cases}. \quad \square$$

Neka su uslovi (*) i (**) zadovoljeni.

Teorema 4.3.5. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{v.o.} 0$ (\mathcal{A}'), tada za svako $\epsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \epsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n}, n \in \mathbb{N}\}$, i za svako $k > k_0$ niz neprekidnih slučajnih procesa $X_{k,n}$ na $\Omega \times I$, tako da za $n \in \mathbb{N}$,

$$(1) \xi_n(\omega, \varphi) = \int_I X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$\omega \in B, \quad \varphi \in \mathcal{A},$

gde je $s \geq s_0$ iz (*), $p \geq p_0$ iz (**),

$$(2) \|X_{k,n}(\omega, \cdot)\|_{L^2} < k, \quad \omega \in \Omega,$$

(3) $\{X_{k,n}(\omega, \cdot), n \geq 1\}$ je jednako neprekidan na I , $p > p_0$, $\omega \in \Omega \setminus A$, gde je A skup iz Definicije 4.1.6.,

(4) za svako $t \in I$, $k > k_0$, $X_{k,n}(\cdot, t) \xrightarrow{v} 0, n \rightarrow \infty$,

$\alpha = \alpha \cdot \delta > s_3(C, \omega_{n,d})$ (2)

$\alpha \in \mathbb{R}^d, \delta \in \mathbb{R}, [C, \omega_{n,d}] \in \mathcal{G}$

$\delta < \delta$ above as (3)

$$\text{min } \alpha \leftarrow \left\{ \alpha \in (\omega_{n,d}, \mathbb{R}^d) \mid \alpha^T \delta > \omega_{n,d} \right\}$$

if $\alpha \in \mathbb{R}^d$ and $\delta \in \mathbb{R}$, $\alpha \in \mathbb{R}^d$ and $\delta \in \mathbb{R}^d$, $\alpha \in \mathbb{R}^d$ and $\delta \in \mathbb{R}^d$

$$\alpha \leftarrow \begin{cases} \alpha \in (\omega_{n,d}, \mathbb{R}^d) \mid \alpha^T \delta > \omega_{n,d} \\ \alpha \in (\mathbb{R}^d, \mathbb{R}^d) \mid \alpha^T \delta > \omega_{n,d} \\ \alpha \in (\mathbb{R}^d, \mathbb{R}^d) \mid \alpha^T \delta > \omega_{n,d} \end{cases} \cup s_3(C, \omega_{n,d})$$

else $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$

$$\delta \leftarrow \text{min}(\delta, \alpha)$$

else $\delta \in \mathbb{R}$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$
or $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$, $\delta \in \mathbb{R}^d$ and $\alpha \in \mathbb{R}^d$

$\delta \leftarrow \text{min}(\delta, \alpha)$

$$\text{min } \alpha \leftarrow \text{min}(\alpha, \text{min}(\delta, \alpha)) \cup s_3(C, \omega_{n,d})$$

$\delta \leftarrow \text{min}(\delta, \alpha)$

$\alpha \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \alpha \in \mathbb{R}^d$

$$\alpha \leftarrow \text{min}(\alpha, \delta) \cup s_3(C, \omega_{n,d})$$

$\alpha \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \alpha \in \mathbb{R}^d$

$\alpha \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \alpha \in \mathbb{R}^d$

$\alpha \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \delta \in \mathbb{R}^d, \alpha \in \mathbb{R}^d$

(5) za svako $\delta > 0$

$$P\{\omega \in B: |\sum_{m \in \Lambda} c_{m,n}(\omega)| > \delta\} \rightarrow 0, n \rightarrow \infty.$$

Dokaz. Kako $\xi_n \xrightarrow{v.o.} 0$, (A') ispunjen je uslov (ii) iz Definicije 4.1.6.. Ovaj uslov je ekvivalentan uslovu C.(ii) Teoreme 4.2.1. pa na osnovu Teoreme 3.2.5. imamo da je (1) i (2) ispunjeno, gde je za $n \in \mathbb{N}$, $k > k_0$

$$x_{k,n}(\omega, t) = \begin{cases} \sum_{m=0}^{\infty} c_{m,n}(\omega) \tilde{\lambda}_m^{-k+p+s} \psi_m(t), & \omega \in B \\ 0, & \omega \in B \end{cases}, \quad t \in I.$$

(5) sledi kao (4) u Teoremi 4.3.3.. Dokaz (3) je dat u Teoremi 4.2.4..

Da bismo pokazali (4) dovoljno je videti da je, za $\omega \in B$

$$|x_{k,n}(\omega, t)| \leq \sum_{m=0}^{\infty} |c_{m,n}(\omega)| |\tilde{\lambda}_m^{-k}| |\psi_m(t)| |\tilde{\lambda}_m^{-s}| |\tilde{\lambda}_m^{-p}| \leq$$

$$\leq K^2 M \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2},$$

gde je K konstanta iz (*) a $M = \sum_{m=0}^{\infty} \tilde{\lambda}_m^{-2p}$. Kako je $x_{k,n}(\omega, t) = 0$,

$\omega \in B$, $t \in I$, imamo za svako $\delta > 0$ i $t_0 \in I$,

$$P\{\omega \in \Omega: |x_{k,n}(\omega, t_0)| > \delta\} \leq P\left\{\omega \in B: K^2 M \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k} \right]^{1/2} > \delta\right\}$$

$$\leq P\left\{\omega \in \Omega: K^2 M \sup_{\|\varphi\|=k} |\xi_n(\omega, \varphi)| > \delta\right\} \rightarrow 0, \quad n \rightarrow \infty. \quad \square$$

Suprotno tvrđenje Teoremi 4.3.5. nije tačno.

$$\text{mean } \bar{x} = 10 \times \left(\frac{\sum_{i=1}^m x_i}{m} \right) + 0.5 \times 0.5$$

at this value of m , we expect $\langle \bar{x} \rangle$ to exceed x_0 and to be less than x_1 . We can also calculate the mean of the m values at x_0 and x_1 to be $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ respectively. If m is small enough so that $\langle \bar{x} \rangle_m$ is close to x_0 and $\langle \bar{x} \rangle_{M-m}$ is close to x_1 , then the mean of all m values will be close to x_0 .

Suppose m is large, so that $\langle \bar{x} \rangle_m$ is close to x_1 and $\langle \bar{x} \rangle_{M-m}$ is close to x_0 . Then the mean of all m values will be close to x_1 . This is because the mean of the m values at x_0 is $\langle \bar{x} \rangle_m = \frac{1}{m} \sum_{i=1}^m x_i$ and the mean of the $M-m$ values at x_1 is $\langle \bar{x} \rangle_{M-m} = \frac{1}{M-m} \sum_{i=M-m+1}^M x_i$. The mean of all M values is $\langle \bar{x} \rangle_M = \frac{1}{M} \sum_{i=1}^M x_i$. Now, if m is large, then $\langle \bar{x} \rangle_m \approx x_1$ and $\langle \bar{x} \rangle_{M-m} \approx x_0$, so that $\langle \bar{x} \rangle_M \approx x_1$.

Therefore, if m is small, $\langle \bar{x} \rangle_m$ is close to x_0 and $\langle \bar{x} \rangle_{M-m}$ is close to x_1 , while if m is large, $\langle \bar{x} \rangle_m$ is close to x_1 and $\langle \bar{x} \rangle_{M-m}$ is close to x_0 . This is what we expect.

Now, let $\langle \bar{x} \rangle_m = \left\{ \bar{x} \in \langle \bar{x} \rangle_m \mid \text{mean } \bar{x} \approx x_0 \right\}$ and $\langle \bar{x} \rangle_{M-m} = \left\{ \bar{x} \in \langle \bar{x} \rangle_{M-m} \mid \text{mean } \bar{x} \approx x_1 \right\}$. Then $\langle \bar{x} \rangle_m \cap \langle \bar{x} \rangle_{M-m} = \emptyset$.

Therefore, if m is small, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ are disjoint sets, while if m is large, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ overlap.

Therefore, if m is small, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ are disjoint sets, while if m is large, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ overlap.

Therefore, if m is small, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ are disjoint sets, while if m is large, $\langle \bar{x} \rangle_m$ and $\langle \bar{x} \rangle_{M-m}$ overlap.

4.4. KONVERGENCIJA U SREDNJEM (\mathcal{A}')

Sva tvrđenja o konvergenciji ograničeno u verovatnoći niza $\{\xi_n\}$ u.s.p. na $\Omega \times \mathcal{A}$ mogu se preneti na konvergenciju ograničeno u srednjem, uz male izmene. Navešćemo tvrđenja i dati skice dokaza.

Teorema 4.4.1. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{1-\varepsilon} 0$, (\mathcal{A}'), tada za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1-\varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , i za svako $n \in \mathbb{N}$ postoji niz slučajnih promenljivih $\{c_{m,n}, m \in \mathbb{N}_0\}$ tako da je za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega)(\psi_m, \varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} \leq k_0, \quad \omega \in B,$$

$$(3) \quad \int_B \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

$$(4) \quad \int_B |c_{m,n}(\omega)| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty, \quad m \in \mathbb{N}_0.$$

Dokaz. Pretpostavimo da $\xi_n \xrightarrow{1-\varepsilon} 0$, (\mathcal{A}'), tada (1) i (2) slede kao (1) i (2) u Teoremi 4.3.1.. Konvergencija ograničeno u srednjem implicira slabu konvergenciju niza u.s.p. pa imamo za $\varphi = \psi_m$, $m \in \mathbb{N}_0$,

$$\int_B |c_{m,n}(\omega)| dP(\omega) \leq \int_{\Omega} |\xi_n(\omega, \psi_m)| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty, \quad \text{što pokazuje (3).}$$

Dalje, iz Definicije 4.1.7. sledi,

transitivity is equivalent to defining a linear map ϕ from \mathbb{P}^1 to \mathbb{P}^1 such that $\phi \circ \phi = \phi$. This is the case if and only if ϕ is a rational function of the form $\phi(z) = \frac{az + b}{cz + d}$, where $a, b, c, d \in \mathbb{C}$ and $ad - bc \neq 0$. In this case, $\phi^{-1}(z)$ is also a rational function of z . If we let $\phi(z) = \frac{az + b}{cz + d}$, then $\phi^{-1}(z) = \frac{dz - b}{az - c}$. Since $\phi \circ \phi = \phi$, we have $\phi(\phi(z)) = z$, which implies $\phi(\phi^{-1}(z)) = z$. Therefore, $\phi^{-1}(\phi^{-1}(z)) = z$, so ϕ^{-1} is also a rational function. Thus, ϕ is a rational map of \mathbb{P}^1 onto itself.

$$\begin{aligned} \text{Let } \phi(z) = \frac{az + b}{cz + d}, \text{ where } a, b, c, d \in \mathbb{C} \text{ and } ad - bc \neq 0. \\ \text{Then } \phi^{-1}(z) = \frac{dz - b}{az - c}. \\ \text{We have } \phi(\phi(z)) = \frac{a^2z + ab}{cz^2 + dz + b^2} = z, \\ \text{and } \phi^{-1}(\phi^{-1}(z)) = \frac{d^2z - bd}{az^2 - cz + b^2} = z. \end{aligned}$$

Thus, ϕ is a rational map of \mathbb{P}^1 onto itself. We can now prove that ϕ is a rational map of \mathbb{P}^1 onto itself if and only if $\phi \circ \phi = \phi$.

$$\begin{aligned} \text{Let } \phi(z) = \frac{az + b}{cz + d}, \text{ where } a, b, c, d \in \mathbb{C} \text{ and } ad - bc \neq 0. \\ \text{Then } \phi(\phi(z)) = \frac{a^2z + ab}{cz^2 + dz + b^2} = z, \\ \text{and } \phi^{-1}(\phi^{-1}(z)) = \frac{d^2z - bd}{az^2 - cz + b^2} = z. \end{aligned}$$

$$\int_{\Omega} \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} dP(\omega) \leq \int_{\Omega} \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) \leq$$

$$\leq \int_{\Omega} \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

Teorema 4.4.2. Niz $\{\xi_n\}$ u.s.p. na $\Omega \times \mathcal{A}$ konvergira ka nuli ograničeno u srednjem (\mathcal{A}') ako za svako $n \in \mathbb{N}$ postoji niz slučajnih promenljivih $\{c_{m,n}, m \in \mathbb{N}_0\}$ tako da su ispunjeni sledeći uslovi:

Postoji $k_0 \in \mathbb{N}_0$ tako da za svako $p \in \mathbb{N}$ postoji $B_p \in \mathcal{F}$, sa $P(B_p) \geq 1 - 1/p$, i da je za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega) (\psi_m, \varphi), \quad \omega \in B_p, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} < k_0, \quad \omega \in B_p,$$

$$(3) \quad \int_{B_p} \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Da bismo pokazali uslov (1) iz Definicije 4.1.7., neka je $\varepsilon > 0$ dato. Tada postoji $p \in \mathbb{N}$ takav da je $P(B_p) \geq 1 - \varepsilon/2k_0$. Takođe, iz (3) sledi da postoji $n_0 = n_0(\varepsilon)$ tako da za $n \geq n_0$,

$$\int_{B_p} \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} dP(\omega) \leq \varepsilon/2.$$

Dakle,

$$\int_{\Omega} \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) =$$

$$= \int_{B_p} \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) + \int_{B_p^c} \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) \leq$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 =$$

Here we can ignore the term $\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta)$ since it is of order $\alpha^{\text{reg}} \times 10^{-1}$. This is because the derivative of $\tilde{\lambda}^{-1}$ is bounded on the $(0, 1)$ interval where α and α^{reg} lie between 0 and 1. In addition, $\alpha^{\text{reg}} \tilde{\lambda}^{-1}$ is differentiable at $\alpha = 0$ since $\tilde{\lambda}^{-1}(0, \omega, \beta) = 0$. Therefore, there is no problem in writing $\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta) = \alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta) + \alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)$.

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(\alpha, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

Finally, we inserted the CDF values obtained earlier and obtained the expression of α^{reg} by differentiating with respect to α and setting the derivative equal to zero. The result is shown below. Note that $\alpha^{\text{reg}} < 0.01$ for all values of ω and β .

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 \quad (63)$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 =$$

$$= \{(\alpha)^{\text{reg}}(\alpha, \omega, \beta)\} + (\alpha)^{\text{reg}}\left[\left(\alpha^{\text{reg}}\tilde{\lambda}^{-1}(0, \omega, \beta)\right) \frac{\partial}{\partial \alpha}\right]_0 =$$

$$\leq \int_{B_p} \left[\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \tilde{\lambda}_m^{-2k_0} \right]^{1/2} dP(\omega) + k_0 \int_B dP(\omega) \leq \frac{\varepsilon}{2} + k_0 \frac{\varepsilon}{2k_0} = \varepsilon,$$

za $n \geq n_0(\varepsilon)$.

Uslovi (ii) iz Definicije 4.1.7. sledi kao i u Teoremi 4.3.2.. \square

Teorema 4.4.3. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{1-\varepsilon} 0$, (\mathcal{A}'), tada za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, tako da je $P(B) \geq 1-\varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n}: n \in \mathbb{N}\}$, i za svako $k \geq k_0$ postoji niz funkcija $X_{k,n}$ na $\Omega \times I$, tako da je, za $n \in \mathbb{N}$

$$(1) \quad \xi_n(\omega, \varphi) = \int_I X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) \varphi_m(\varphi), \quad \omega \in B, \quad \varphi \in \mathcal{A},$$

$$(2) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \leq k, \quad \omega \in \Omega,$$

$$(3) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \xrightarrow{n \rightarrow \infty} 0,$$

$$(4) \quad \int_B \left| \sum_{m \in \Lambda} c_{m,n}(\omega) \right| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Prepostavimo da $\xi_n \xrightarrow{1-\varepsilon} 0$, (\mathcal{A}'). Tada (1), (2) slede kao (1), (2) u Teoremi 4.3.3.. (4) sledi na osnovu (4) Teorema 4.4.1. jer je Λ konačno.

Dalje

$$\int_{\Omega} \|X_{k,n}(\omega, \cdot)\|_{L^2} dP(\omega) = \int_B \|X_{k,n}(\omega, \cdot)\|_{L^2} dP(\omega) =$$

$$= \frac{1}{2} \left[\frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \right]_{\omega=0} + \frac{1}{2} \left[\frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \right]_{\omega=0} \frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \Big|_{\omega=0}$$

$\langle \psi \rangle_{\text{eff}} \approx \psi_{\alpha}^{(0)}$

whereas $\psi_{\alpha}^{(0)}$ and $\langle \psi \rangle_{\text{eff}}$ are obtained in this approximation.

It is interesting to note that the effect of the electron-electron interaction on the energy spectrum of the system is rather small. The energy difference between the ground state and the excited state is about $\langle \psi \rangle_{\text{eff}} - \psi_{\alpha}^{(0)} \approx \frac{1}{2} \frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \omega_0^2$. The spectrum of the system is not stable, $\omega_0 \approx 0.025$ rad/sec, and the energy difference between the ground state and the excited state is about $\langle \psi \rangle_{\text{eff}} - \psi_{\alpha}^{(0)} \approx \frac{1}{2} \frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \omega_0^2 \approx 0.00025$ eV. This is a rather small value, and it is difficult to measure such a small energy difference.

The effect of the electron-electron interaction on the energy spectrum of the system is rather small.

$$\langle \psi \rangle_{\text{eff}} = \psi_{\alpha}^{(0)} + \frac{1}{2} \left[\frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \right]_{\omega=0} \omega_0^2$$

$$\langle \psi \rangle_{\text{eff}} \approx \psi_{\alpha}^{(0)} + \frac{1}{2} \left[\frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \right]_{\omega=0} \omega_0^2$$

Thus, one sees that $\langle \psi \rangle_{\text{eff}} \approx \psi_{\alpha}^{(0)}$ is ab relatively good approximation for the energy spectrum of the system. The energy spectrum of the system is not stable, $\omega_0 \approx 0.025$ rad/sec, and the energy difference between the ground state and the excited state is about $\langle \psi \rangle_{\text{eff}} - \psi_{\alpha}^{(0)} \approx \frac{1}{2} \frac{\partial^2 \psi_{\alpha}^{(0)}(\omega)}{\partial \omega^2} \omega_0^2 \approx 0.00025$ eV. This is a rather small value, and it is difficult to measure such a small energy difference.

$$= \langle \psi_{\alpha}^{(0)} \psi_{\alpha}^{(0)} \rangle \langle \psi_{\alpha}^{(0)} \psi_{\alpha}^{(0)} \rangle = \langle \psi_{\alpha}^{(0)} \psi_{\alpha}^{(0)} \rangle \langle \psi_{\alpha}^{(0)} \psi_{\alpha}^{(0)} \rangle$$

$$= \int_B \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) \leq \int_\Omega \sup_{\|\varphi\|_{k_0} \leq 1} |\xi_n(\omega, \varphi)| dP(\omega) \rightarrow 0,$$

$n \rightarrow \infty$, pa sledi (3). \square

Teorema 4.4.4. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Tada $\xi_n \xrightarrow{1-\alpha} 0$, (\mathcal{A}') ako postoji $k_0 \in \mathbb{N}$, tako da za svaki $p \in \mathbb{N}$, postoji $B_p \in \mathcal{F}$, sa $P(B_p) \geq 1 - 1/p$, i za $n \in \mathbb{N}$, $k \geq k_0$, postoji niz funkcija $X_{k,n} : \Omega \times I \rightarrow \mathbb{C}$ i za svako $n \in \Lambda$ niz slučajnih promenljivih $\{c_{m,n} : m \in \mathbb{N}_0\}$ tako da je

$$(1) \quad \xi_n(\omega, \varphi) = \int_I X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$\omega \in B_p, \varphi \in \mathcal{A},$

$$(2) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \leq k, \quad \omega \in \Omega,$$

$$(3) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \xrightarrow{1} 0, \quad n \rightarrow \infty,$$

$$(4) \quad \int_{B_p} \left| \sum_{m \in \Lambda} c_{m,n}(\omega) \right| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. Dokaz je isti kao dokaz Teoreme 4.3.2, odnosno

Teoreme 4.4.2. \square

Neka su uslovi (*) i (**) zadovoljeni.

Teorema 4.4.5. Neka je $\{\xi_n\}$ niz u.s.p. na $\Omega \times \mathcal{A}$. Ako $\xi_n \xrightarrow{1-\alpha} 0$, (\mathcal{A}'), tada za svako $\varepsilon > 0$ postoji skup $B \in \mathcal{F}$, sa $P(B) \geq 1 - \varepsilon$, postoji $k_0 \in \mathbb{N}_0$, gde su B i k_0 nezavisni od n , za svako $m \in \Lambda$ postoji niz slučajnih promenljivih $\{c_{m,n} : n \in \mathbb{N}\}$, i za svako $k > k_0$ niz neprekidnih slučajnih procesa $X_{k,n}$ na

$\alpha \leftarrow \langle \alpha \rangle^{(b)} \{ \alpha_1, \dots, \alpha_n \}$ $\beta \leftarrow \langle \beta \rangle^{(b)} \{ \beta_1, \dots, \beta_n \}$ $\gamma \leftarrow \langle \gamma \rangle^{(b)} \{ \gamma_1, \dots, \gamma_n \}$

and $\alpha \times \beta$ and $\beta \times \gamma$ are min $\{ \alpha_i \beta_j \mid i, j \in \text{index } \alpha, \beta \}$ and $\beta \times \alpha$ and $\gamma \times \beta$ are min $\{ \beta_i \alpha_j \mid i, j \in \text{index } \beta, \gamma \}$ and $\gamma \times \alpha$. Hence we can write $\alpha \times \beta \times \gamma = \alpha \times (\beta \times \gamma)$ and $\beta \times (\alpha \times \gamma) = (\beta \times \alpha) \times \gamma$.
Also we can see that $\alpha \times \beta \times \gamma = \alpha \times \beta + \gamma$ and $\beta \times \alpha + \gamma = \alpha \times \beta + \gamma$.
Hence $\alpha \times \beta + \gamma = \alpha \times (\beta + \gamma)$ and $\beta + \gamma \times \alpha = (\beta + \gamma) \times \alpha$.
Therefore $\alpha \times \beta + \gamma = \alpha \times \beta + \alpha \times \gamma$ and $\beta + \gamma \times \alpha = \beta \times \alpha + \gamma \times \alpha$.
Hence $\alpha \times \beta + \gamma = \alpha \times \beta + \alpha \times \gamma = \alpha \times (\beta + \gamma)$ and $\beta + \gamma \times \alpha = \beta \times \alpha + \gamma \times \alpha = (\beta + \gamma) \times \alpha$.
Hence $\alpha \times \beta + \gamma = \beta + \gamma \times \alpha$.

 $\alpha \leftarrow \langle \alpha \rangle^{(b)} \{ \alpha_1, \dots, \alpha_n \}$ $\beta \leftarrow \langle \beta \rangle^{(b)} \{ \beta_1, \dots, \beta_n \}$

and $\alpha \leftarrow \langle \alpha \rangle^{(b)} \{ \alpha_1, \dots, \alpha_n \}$ $\beta \leftarrow \langle \beta \rangle^{(b)} \{ \beta_1, \dots, \beta_n \}$

and $\alpha \times \beta \leftarrow \langle \alpha \times \beta \rangle^{(b)} \{ \alpha_1 \times \beta_1, \dots, \alpha_n \times \beta_n \}$ $\beta \times \alpha \leftarrow \langle \beta \times \alpha \rangle^{(b)} \{ \beta_1 \times \alpha_1, \dots, \beta_n \times \alpha_n \}$

and $\alpha \times \beta + \gamma \leftarrow \langle \alpha \times \beta + \gamma \rangle^{(b)} \{ \alpha_1 \times \beta_1 + \gamma_1, \dots, \alpha_n \times \beta_n + \gamma_n \}$ $\beta + \gamma \times \alpha \leftarrow \langle \beta + \gamma \times \alpha \rangle^{(b)} \{ \beta_1 + \gamma_1 \times \alpha_1, \dots, \beta_n + \gamma_n \times \alpha_n \}$

and $\alpha \times \beta + \gamma = \alpha \times \beta + \alpha \times \gamma$ and $\beta + \gamma \times \alpha = \beta \times \alpha + \gamma \times \alpha$.
Hence $\alpha \times \beta + \gamma = \beta + \gamma \times \alpha$.

Now we can see that $\alpha \times \beta + \gamma = \alpha \times (\beta + \gamma)$ and $\beta + \gamma \times \alpha = (\beta + \gamma) \times \alpha$.
Hence $\alpha \times \beta + \gamma = \alpha \times (\beta + \gamma) = \alpha \times \beta + \alpha \times \gamma$ and $\beta + \gamma \times \alpha = (\beta + \gamma) \times \alpha = \beta \times \alpha + \gamma \times \alpha$.
Hence $\alpha \times \beta + \gamma = \beta + \gamma \times \alpha$.

Therefore $\alpha \times \beta + \gamma = \beta + \gamma \times \alpha$.

$\Omega \times I$, tako da za $n \in \mathbb{N}$,

$$(1) \quad \xi_n(\omega, \varphi) = \int_{\Omega} X_{k,n}(\omega, t) R^k \varphi(t) dt + \sum_{m \in \Lambda} c_{m,n}(\omega) (\psi_m, \varphi),$$

$\omega \in B$, $\varphi \in A$, gde je $s \geq s_0$ iz (*), $p \geq p_0$ iz (**),

$$(2) \quad \|X_{k,n}(\omega, \cdot)\|_{L^2} \leq k, \quad \omega \in \Omega,$$

(3) za svako $t \in I$ i $k > k_0$,

$$X_{k,n}(\cdot, t) \xrightarrow{k} 0, \quad n \rightarrow \infty,$$

(4) $\{X_{k,n}(\omega, \cdot), n \geq 1\}$ je jednako neprekidan na I , za $p > p_0$.

$\omega \in \Omega \setminus A$, gde je A skup iz Definicije 4.1.7.,

$$(5) \quad \int_B \left| \sum_{m \in \Lambda} c_{m,n}(\omega) \right| dP(\omega) \rightarrow 0, \quad n \rightarrow \infty.$$

Dokaz. (1), (2), (4) slede kao (1), (2), (3) u Teoremi 4.3.5., a (3) i (5) kao u Teoremi 4.4.3.. \square

Netko je $\xi(\omega, \cdot)$ neprekidna funkcija u Ω . (2) sledi, pa je

$\xi(\omega, \cdot)$ neprekidna funkcija za svaku $\omega \in \Omega$. Da bismo pokazali da

je $\xi(\cdot, \omega)$ markijeva funkcija za svako $\omega \in \Omega$, netko je $x = u + v$,

$u \in \mathbb{R}$, $v \in \mathbb{R}^d$ s $u \neq 0$, $u \neq v$, $0 \in \mathbb{R}^d$. Tada je za svaku $\omega \in \Omega$

show us an ab initio, I x O

$\langle \psi_{1,2} | \psi_{1,2} \rangle_{\text{pert}} = 1 + \text{Im}[\text{Tr}(\rho_{1,2}(t=0), \beta)] \approx \langle \psi_{1,2} | \psi_{1,2} \rangle_{\text{exact}}$

so we can see that the perturbative theory is not bad at all, though it is not exact.

It is also a bit off ($\approx 10\%$)

but this is probably due to the fact that the initial state is not a good one.

So what does this mean?

Well, it means that the perturbative theory is not bad at all, though it is not exact.

It is also a bit off ($\approx 10\%$)

so what does this mean?

Well, it means that the perturbative theory is not bad at all, though it is not exact.

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GLAVA V

Neka je ξ u.s.p. na $\Omega \times V$. Ako je familija $\{\xi(\cdot, \varphi) : \varphi \in V\}$ kompleksna Gausovska familija, ξ ćemo nazivati kompleksan Gausov u.s.p., ili kraće Gausov u.s.p. ako je jasno da se radi o kompleksnom u.s.p.

Teorema 5.1. Neka je V Hilbertov prostor sa normom $\|\cdot\|_V$ a W njegov potprostor. Neka je ξ Gausov u.s.p. na $\Omega \times W$, i neka postoji pozitivna funkcija $C(\omega)$, $\omega \in \Omega$, takva da je za svako $\omega \in \Omega$, $x \in W$, $|\xi(\omega, x)| \leq C(\omega) \|x\|_V$. Tada postoji Gausov u.s.p. ξ_1 na $\Omega \times V$ takav da je

- (1) $\forall \omega \in \Omega, x \in W, \xi_1(\omega, x) = \xi(\omega, x),$
- (2) $\forall \omega \in \Omega, x \in V, |\xi_1(\omega, x)| \leq C(\omega) \|x\|_V.$

Dokaz. Označimo sa \bar{W} zatvorenje od W u V i sa \bar{W}^\perp ortogonalan komplement od \bar{W} u V .

Definišimo

$$\xi_1(\omega, x) = \begin{cases} \xi(\omega, x), & x \in W \\ \lim \xi(\omega, x_n), & x \in \bar{W}, x_n \rightarrow x, n \rightarrow \infty \\ 0, & x \in \bar{W}^\perp \end{cases} \quad x \in V, \omega \in \Omega.$$

Kako je $\xi(\omega, \cdot)$ neprekidno za svako $\omega \in \Omega$, (2) sledi, pa je $\xi_1(\omega, \cdot)$ neprekidna funkcija za svako $\omega \in \Omega$. Da bismo pokazali da je $\xi_1(\cdot, x)$ merljiva funkcija za svako $x \in V$, neka je $x = u + v$, $u \in \bar{W}$, $v \in \bar{W}^\perp$ i $u_n \rightarrow u$, $n \rightarrow \infty$, $u_n \in \bar{W}^\perp$. Tada je za svako $\omega \in \Omega$

Để xác định các thành phần của N ta có thể áp dụng
các phép phân tích tương ứng với cách phân tích
tập hợp S và các thành phần $\alpha_1, \alpha_2, \dots, \alpha_k$ của S .
Nhưng để áp dụng cách phân tích này, ta cần
biết rõ các thành phần $\alpha_1, \alpha_2, \dots, \alpha_k$ của S là gì?

Đây là vấn đề không dễ giải quyết. Vì ta không
nhận ra được trước đây là $\alpha_1, \alpha_2, \dots, \alpha_k$ là những
thành phần của S hay không. Vì vậy, ta cần
tìm cách xác định các thành phần $\alpha_1, \alpha_2, \dots, \alpha_k$ của S mà
không cần biết trước đây là $\alpha_1, \alpha_2, \dots, \alpha_k$ là
thành phần của S hay không. Để làm điều
này, ta cần áp dụng hai cách:

1) Áp dụng cách phân tích S theo các thành phần $\alpha_1, \alpha_2, \dots, \alpha_k$ đã
biết trước đây là $\alpha_1, \alpha_2, \dots, \alpha_k$ là các thành phần
của S .

$$\left[\begin{array}{c} \alpha_1 = x \\ \alpha_2 = y \\ \vdots \\ \alpha_k = z \end{array} \right] \rightarrow S = \text{tập hợp} \left[\begin{array}{c} x \\ y \\ \vdots \\ z \end{array} \right] = \langle x, y, \dots, z \rangle$$

2) Áp dụng cách phân tích S theo các thành phần $\alpha_1, \alpha_2, \dots, \alpha_k$ mà
chưa xác định trước đây là $\alpha_1, \alpha_2, \dots, \alpha_k$ là
những thành phần nào. $\forall \omega$ là thành phần bất kỳ trong S , ta
để ω là thành phần số α_i nhất định. $\forall \omega$ là thành phần bất kỳ khác trong S , ta

$$\xi_1(\omega, \infty) = \xi(\omega, u) + \xi(\omega, v) = \lim_{n \rightarrow \infty} \xi(\omega, u_n) + 0,$$

pa je ξ_1 mjerljiva kao granica niza mjerljivih funkcija. Sledi da je ξ_1 je u.s.p. na $\Omega \times V$.

Za $x \in \bar{W}$, ξ_1 je definisano kao granica niza Gausovih slučajnih promenljivih koji tačkasto konvergira. Iz Teorema 1.5.14. i 1.5.16. sledi da je granica Gausova slučajna promenljiva, pa je $\xi_1(\cdot, x)$ Gausova slučajna promenljiva za svako $x \in V$. Označimo sa $Z = \{\xi(\cdot, x) : x \in W\}$ i sa \bar{Z} linearno zatvoreno od Z u $L^2(\Omega)$. \bar{Z} je Gausovska familija, a kako je $Z_1 = \{\xi_1(\cdot, x) : x \in V\}$ podfamilija od \bar{Z} , sledi da je Z_1 Gausovska familija, odnosno da je ξ_1 Gausov u.s.p. \square

Označimo sa $C[0,1]$ skup neprekidnih funkcija na intervalu $[0,1]$. U radu [45, Lemma 3.] je dobijena reprezentacija u.s.p. ξ na $\Omega \times C[0,1]$ pomoću običnog slučajnog procesa na $\Omega \times [0,1]$, čije su trajektorije funkcije ograničene varijacije. Ako, međutim, pretpostavimo da je ξ Gausov u.s.p., važi sledeća teorema.

Teorema 5.2. Neka je ξ Gausov u.s.p. na $\Omega \times C[0,1]$. Tada postoji Gausov slučajni proces $f : \Omega \times [0,1] \rightarrow \mathbb{C}$ takav da je za svako $\omega \in \Omega$, $f(\omega, \cdot)$ funkcija ograničene varijacije i za svako $\omega \in \Omega$ i svako $\varphi \in C[0,1]$,

$$\xi(\omega, \varphi) = \int_0^1 \varphi(t) df(\omega, t).$$

$$1.0 \times 1.0 \times 1.0 = 0.0001 \times 0.0001 \times 0.0001$$

so the 10⁻¹² μm^3 volume will contain one molecule, $\frac{1}{10^{12}}$ of a mole.

$$N \times 0.0001 \times 0.0001 \times 0.0001$$

which is the same as calculating the concentration of $1.0 \times 1.0 \times 1.0$ μm^3 in molecules per cubic centimetre. Since this is the same as calculating the number of molecules per cubic centimetre, we can say that N is the number of molecules contained in $1.0 \times 1.0 \times 1.0$ μm^3 . This is the definition of N in molecular statistics. It is also known as the Avogadro constant. The value of N is approximately 6.02×10^{23} molecules per cubic centimetre. This is the same as saying that there are 6.02×10^{23} molecules in a volume of $1.0 \times 1.0 \times 1.0$ μm^3 .

$$N = 6.02 \times 10^{23}$$

$$N = 6.02 \times 10^{23} \text{ molecules per cubic centimetre}$$

So if we have a volume of $1.0 \times 1.0 \times 1.0$ μm^3 containing $1.0 \times 1.0 \times 1.0$ molecules, then the concentration is $1.0 \times 1.0 \times 1.0$ molecules per cubic centimetre.

So if we have a volume of $1.0 \times 1.0 \times 1.0$ μm^3 containing $1.0 \times 1.0 \times 1.0$ molecules, then the concentration is $1.0 \times 1.0 \times 1.0$ molecules per cubic centimetre.

Dokaz. Prvi deo dokaza teoreme je isti kao i dokaz Lemme 3. u [45], ali ćemo ga, zbog celine, ponoviti. U.s.p. $\xi(\omega, \cdot)$ je za svako $\omega \in \Omega$ linearan funkcional na $C[0,1]$, pa sledi da za svako $\omega \in \Omega$ postoji funkcija ograničene varijacije $f(\omega, \cdot)$ na intervalu $[0,1]$ takva da je za svako $\varphi \in C[0,1]$ i to isto $\omega \in \Omega$

$$\xi(\omega, \varphi) = \int_0^1 \varphi(t) df(\omega, t).$$

○

Označimo sa $\tilde{f}(\omega, t) = f(\omega, t) - f(\omega, 1)$, $\omega \in \Omega$, $t \in [0, 1]$. Imamo da je $\tilde{f}(\omega, 1) = 0$ i $d\tilde{f} = df$. Zato, ne gubeći na opštosti, možemo pretpostaviti da je za svako $\omega \in \Omega$, $f(\omega, 1) = 0$, i da je $f(\omega, \cdot)$ neprekidna sa desne strane u svakoj tački otvorenog intervala $(0, 1)$.

Pokazaćemo da je za svako $t \in [0, 1]$, $f(\cdot, t)$ Gausova slučajna promenljiva. Za $t = 0$ je to tačno jer je

$$\xi(\omega, 1) = \int_0^1 df(\omega, t) = -f(\omega, 0).$$

○

Dalje, neka je t_0 proizvoljna tačka iz intervala $(0, 1)$. Definišimo niz neprekidnih funkcija $\{a_n : n \in \mathbb{N}\}$ na sledeći način.

$$a_n(t) = \begin{cases} 1, & 0 \leq t \leq t_0, \\ -n(t-t_0)+1, & t_0 < t < t_0 + \frac{1}{n}, \\ 0, & t \geq t_0 + \frac{1}{n} \end{cases}$$

Imamo da je $|a_n| \leq 1$, $n \in \mathbb{N}$, pa po Lebegovoj teoremi dominantne konvergencije imamo da je

$$\lim_{n \rightarrow \infty} \int_0^1 a_n(t) df(\omega, t) = f(\omega, t_0) - f(\omega, 0),$$

te možemo definisati

$$f(\omega, t_0) = f(\omega, 0) + \lim_{n \rightarrow \infty} \int_0^1 a_n(t) df(\omega, t).$$

Imamo da granica niza $\xi(\omega, a_n) = \int_0^1 a_n(t) df(\omega, t)$, $n \in \mathbb{N}$, postoji za svako $\omega \in \Omega$. Iz Teorema 1.5.14. i 1.5.16. sledi da je granica $\xi_\infty(\cdot) = \lim_{n \rightarrow \infty} \xi(\cdot, a_n)$ Gausova slučajna promenljiva.

Označimo sa $Z = \{\xi(\cdot, \varphi) : \varphi \in C[0,1]\}$ i sa \bar{Z} zatvaranje od Z u $L^2(\Omega, \mathcal{F}, P)$. Kako su $f(\cdot, 0)$ i ξ_∞ elementi od \bar{Z} , sledi da je njihova linearna kombinacija, $f(\cdot, t_0)$, Gausova slučajna promenljiva. Kako $f(\cdot, 1) = 0$ kao konstanta ima degenerisanu Gausovu raspodelu, dobijamo da je, za svako $t \in [0,1]$, $f(\cdot, t)$ Gausova slučajna promenljiva.

Više od toga, kako je familija $\{f(\cdot, t) : t \in [0,1]\}$ podfamilija od \bar{Z} , sledi da je $f(\cdot, t)$, $t \in [0,1]$, Gausov slučajni proces. □

Definicija 5.1. Neka je $x \in \mathbb{R}^n$. Niz $\{E_n : n \in \mathbb{N}\}$ Borelovih skupova u \mathbb{R}^n se skuplja lepo u x ako postoji broj $\alpha > 0$ sa sledećom osobinom: Svaki E_n leži u otvorenoj lopti $B(x, r_n)$, sa centrom u x i poluprečniku $r_n > 0$, tako da je

$$m(E_n) \geq \alpha m(B(x, r_n)), \quad n \geq 1.$$

i $r_n \rightarrow 0$, $n \rightarrow \infty$.

Ovde i u daljem tekstu m označava Lebegovu meru na \mathbb{R}^n .

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common other than the lichen, the epiphytic nature of which is reflected in their distribution and extent of coverage on the tree. The lichen and mosses are the most abundant epiphytes occurring on the tree (Fig. 1) with a total

coverage of 1.00 m² and a density of 5.0 individuals/m². The mosses are the next most abundant group (0.20 m² and 6.0 individuals/m²) followed by the lichen (0.15 m² and 3.0 individuals/m²). The remaining epiphytes are the ferns (0.05 m² and 1.0 individuals/m²), the hepatic (0.02 m² and 0.5 individuals/m²), and the bryophytes (0.01 m² and 0.5 individuals/m²). The remaining epiphytes are the ferns (0.05 m² and 1.0 individuals/m²), the hepatic (0.02 m² and 0.5 individuals/m²), and the bryophytes (0.01 m² and 0.5 individuals/m²).

The epiphytes were found to have a mean height of 1.00 m above the ground surface. The height of the lichen was 0.10 m above the ground surface, while the mosses were 0.05 m above the ground surface. The height of the ferns was 0.05 m above the ground surface, while the hepatic was 0.02 m above the ground surface. The height of the bryophytes was 0.01 m above the ground surface.

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The epiphytes represented over 50% of the total coverage on the tree.

Neka je $\{E_n : n \in \mathbb{N}\}$ niz Lebeg mjerljivih skupova u \mathbb{R}^n , koji se skuplja lepo u $x_0 \in \mathbb{R}^n$. Neka je $\chi(E_n)$ karakteristična funkcija od E_n , $n \in \mathbb{N}$, i stavimo

$$(5.1.1) \quad \delta_n(x-x_0) = \frac{\chi(E_n)(x)}{m(E_n)}, \quad n \in \mathbb{N}, \quad x \in \mathbb{R}^n,$$

Lako se vidi da je $\delta_n(x-x_0)$, $n \in \mathbb{N}$, δ -niz u smislu teorije distribucija [39]. To znači da $\delta_n(x-x_0)$ konvergira, kad $n \rightarrow \infty$, ka $\delta(x-x_0)$ - Dirakovoj δ -funkciji koncentrisanoj u x_0 .

Teorema 5.3. Neka je ξ Gausov u.s.p. na $\Omega \times L^1(\mathbb{R}^n)$. Tada za svako $\omega \in \Omega$ postoji skup $A(\omega) \subset \mathbb{R}^n$ takav da je $m(A(\omega)) = 0$ i takav da za svako $x_0 \in \mathbb{R}^n \setminus A(\omega)$ i svaki δ -niz $\delta_n(x-x_0)$ oblika kao u (5.1.1), postoji granična vrednost

$$\lim_{n \rightarrow \infty} \xi(\omega, \delta_n(x-x_0)), \quad \omega \in \Omega.$$

Dokaz. Za svako $\omega \in \Omega$, $\xi(\omega, \cdot)$ je linearana i neprekidna funkcionala na $L^1(\mathbb{R}^n)$. Odatle sledi da postoji $h(\omega, x) \in L^\infty(\mathbb{R}^n)$ tako da je

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} h(\omega, t)\varphi(t)dt, \quad \varphi(t) \in L^1(\mathbb{R}^n)$$

Neka je $\{\mathcal{O}_k : k \geq 1\}$ lokalno konačna familija otvorenih, ograničenih lopti u \mathbb{R}^n , takva da je $\mathbb{R}^n = \bigcup_{k=1}^{\infty} \mathcal{O}_k$.

Za fiksirano \mathcal{O}_k definišimo

and the 2000' elevation reached near 10 miles SSW of town
at the head of the valley. The valley floor is about 1000' above sea level.

Distance between 2000' and 1000' elevations

is about 10 miles. The valley floor is about 1000' above sea level.

At the head of the valley there is a small stream which flows into the river. The river has a small tributary which flows into it from the south. The river flows generally westward.

The river has a small tributary which flows into it from the north. The river flows generally westward. The river has a small tributary which flows into it from the south. The river flows generally westward.

The river has a small tributary which flows into it from the north. The river flows generally westward.

The river has a small tributary which flows into it from the north. The river flows generally westward. The river has a small tributary which flows into it from the south. The river flows generally westward.

Width of valley at 2000' elevation = 10 miles.

Width of valley at 1000' elevation = 10 miles.

Width of valley at 1000' elevation = 10 miles.

$$h_k(\omega, x) = \begin{cases} h(\omega, x), & x \in \mathcal{O}_k, \quad \omega \in \Omega. \\ 0, & x \notin \mathcal{O}_k \end{cases}$$

Za proizvoljan otvoren skup $E \subset \mathbb{R}^n$ definisimo, za $\omega \in \Omega$,

$$\mu_k(\omega, E) = \int_E h_k(\omega, t) dt = \int_{\mathbb{R}^n} \chi(E \cap \mathcal{O}_k) h_k(\omega, t) dt = \xi(\omega, \chi(E \cap \mathcal{O}_k)).$$

Imamo da je $\mu_k(\omega, \cdot)$, $\omega \in \Omega$, kompleksna Borelova mera na \mathbb{R}^n . Iz Teoreme 8.6., [37], sledi da je za svako $\omega \in \Omega$, $\mu_k(\omega, \cdot)$ diferencijabilna skoro svuda u odnosu na Lebegovu mjeru m (a.e. [m]). Za niz $\{E_n : n \in \mathbb{N}\}$ otvorenih skupova koji se lepo skuplja u $x \in \mathbb{R}^n$, imamo, za $\omega \in \Omega$,

$$D\mu_k(\omega, \cdot) = \lim_{n \rightarrow \infty} \frac{\mu_k(\omega, E_n)}{m(E_n)}.$$

Imamo da je $D\mu_k(\omega, \cdot) = h_k(\omega, \cdot)$ a.e. [m], $\omega \in \Omega$. Kako je $D\mu_k(\omega, x) \equiv 0$, $\omega \in \Omega$, $x \in \mathbb{R}^n \setminus \mathcal{O}_k$, sledi da postoji skup $A(\omega, k) \subset \mathcal{O}_k$, $m(A(\omega, k)) = 0$, takav da $D\mu_k(\omega, x)$ postoji za svako $x \in \mathcal{O}_k \setminus A(\omega, k)$.

Neka je $x \in \mathcal{O}_{k_1} \cap \mathcal{O}_{k_2}$, $k_1, k_2 \in \mathbb{N}$, i $\{E_n : n \in \mathbb{N}\}$ niz otvorenih skupova u \mathbb{R}^n koji se lepo skuplja u x . Postoji $n_0 \in \mathbb{N}$ takav da zanjeni, $E_n \subset \mathcal{O}_{k_2} \cap \mathcal{O}_{k_1}$, pa je $\chi(E_n \cap \mathcal{O}_{k_1}) = \chi(E_n \cap \mathcal{O}_{k_2})$.

Imamo, za $\omega \in \Omega$, $n \geq n_0$

$$\mu_{k_1}(\omega, E_n) = \xi(\omega, \chi(E_n \cap \mathcal{O}_{k_1})) = \xi(\omega, \chi(E_n \cap \mathcal{O}_{k_2})) = \mu_{k_2}(\omega, E_n),$$

pa sledi za svako $x \in \mathcal{O}_{k_1} \cap \mathcal{O}_{k_2}$, i $\omega \in \Omega$,

Alors le no de la partie est égal à l'ordre des parties dans l'ordre des parties.

$$\text{et } \text{no de la partie} = \text{no de la partie } \{ = \text{no de la partie } \} = (\text{no de la partie})_p$$

Si l'on a une partie dont les parties sont toutes de même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties. C'est à dire que si toutes les parties ont le même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties.

Si l'on a une partie dont les parties sont toutes de même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties. C'est à dire que si toutes les parties ont le même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties.

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Si l'on a une partie dont les parties sont toutes de même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties. C'est à dire que si toutes les parties ont le même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties.

$$(\text{no de la partie})_p = (\text{no de la partie})_p = (\text{no de la partie})_p = (\text{no de la partie})_p$$

Si l'on a une partie dont les parties sont toutes de même ordre, alors l'ordre de la partie est égal à l'ordre de la partie dans l'ordre des parties.

$$D\mu_{k_1}(\omega, x) = D\mu_{k_2}(\omega, x)$$

Definišimo za $\omega \in \Omega$,

$$g(\omega, x) = D\mu_k(\omega, x), \quad x \in \emptyset_k \setminus A(\omega, k), \quad k \in \mathbb{N}.$$

Stavimo $A(\omega) = \bigcup_{k=1}^{\infty} A(\omega, k)$. Imamo da je $m(A(\omega)) = 0$ pa je $g(\omega, x)$ definisano za svako $x \in \mathbb{R}^n \setminus A(\omega)$, to jest, a.e. [m].

Neka je $x_0 \in \mathbb{R}^n \setminus A(\omega)$. Očigledno je da je, za svako $\omega \in \Omega$, iproizvoljan niz $\{E_n : n \in \mathbb{N}\}$ koji se lepo skuplja u x_0 , i $\delta_n(x-x_0) = x(E_n)/m(E_n)$, imamo

$$\lim_{n \rightarrow \infty} \xi(\omega, \delta_n(x-x_0)) = g(\omega, x), \quad x \in \mathbb{R}^n,$$

pa tvrđenje sledi. \square

Teorema 5.4. Neka je ξ u. s. p. na $\Omega \times L^1(\mathbb{R}^n)$. Tada postoji slučajan proces $f : \Omega \times \mathbb{R}^n \rightarrow \mathbb{C}$ takav da je

$$(1) \quad \forall \omega \in \Omega, \quad f(\omega, \cdot) \in L^\infty(\mathbb{R}^n),$$

$$(2) \quad \forall \omega \in \Omega, \quad \varphi \in L^1(\mathbb{R}^n),$$

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} f(\omega, t) \varphi(t) dt.$$

Dokaz. Ovo tvrđenje su dali C. H. Swartz i D. E. Myers u [42] ali u ovom dokazu su ispravljeni neki propusti u njihovom dokazu.

Za svako $\omega \in \Omega$, $\xi(\omega, \cdot)$ je linearan i neprekidan funkcional na $L^1(\mathbb{R}^n)$, pa sledi da postoji $h(\omega, \cdot) \in L^\infty(\mathbb{R}^n)$ tako da je

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} h(\omega, t) \varphi(t) dt, \quad \varphi \in L^1(\mathbb{R}^n).$$

• (1.0) 4

Algo de la anterioridad. Es lo que dice el autor en su libro:

$$\{ \text{problema} \} = \{ \text{solución} \} = \{ \text{función} \} = \{ \text{dato} \}$$

Si bien es una condición necesaria, no es suficiente para que sea una
función. Debe ser además que sea una función (1.0) 4). La otra condición
es que se pueda dar una sola solución a cada dato. Porque si
se tienen más de una solución a un dato, no se cumple la condición

de que la función sea una sola. La otra condición es que la función
debe ser continua. Si no es continua, no se cumple la condición
de que la función sea una sola. Porque si la función no es continua,
entonces habrá saltos o discontinuidades en la función.

En resumen, la función debe cumplir con las siguientes condiciones:
1) La función debe ser una sola. 2) La función debe ser una función.
3) La función debe ser continua. 4) La función debe ser una sola.

Algunas funciones que cumplen con estas condiciones son:
1) La función constante. 2) La función lineal. 3) La función cuadrática.
4) La función exponencial. 5) La función logarítmica. 6) La función
trigonométrica. 7) La función polinomial. 8) La función racional.

$$f(x) = kx + b \quad f(x) = ax^2 + bx + c \quad f(x) = e^{kx} \quad f(x) = \ln x$$

Algunas funciones que no cumplen con estas condiciones son:

$$D\mu_{k_1}(\omega, x) = D\mu_{k_2}(\omega, x)$$

Definišimo za $\omega \in \Omega$,

$$g(\omega, x) = D\mu_k(\omega, x), \quad x \in \mathbb{O}_k \setminus A(\omega, k), \quad k \in \mathbb{N}.$$

Stavimo $A(\omega) = \bigcup_{k=1}^{\infty} A(\omega, k)$. Imamo da je $m(A(\omega)) = 0$ pa je $g(\omega, x)$

definisano za svako $x \in \mathbb{R}^n \setminus A(\omega)$, to jest, a.e. [m].

Neka je $x_0 \in \mathbb{R}^n \setminus A(\omega)$. Očigledno je da je, za svako $\omega \in \Omega$, iproizvoljan niz $\{E_n : n \in \mathbb{N}\}$ koji se lepo skuplja u x_0 , i $\delta_n(x-x_0) = \omega(E_n)/m(E_n)$, imamo

$$\lim_{n \rightarrow \infty} \xi(\omega, \delta_n(x-x_0)) = g(\omega, x), \quad x \in \mathbb{R}^n,$$

pa tvrđenje sledi. \square

Teorema 5.4. Neka je ξ u. s. p. na $\Omega \times L^1(\mathbb{R}^n)$. Tada postoji slučajan proces $f : \Omega \times \mathbb{R}^n \rightarrow \mathbb{C}$ takav da je

$$(1) \quad \forall \omega \in \Omega, \quad f(\omega, \cdot) \in L^\infty(\mathbb{R}^n),$$

$$(2) \quad \forall \omega \in \Omega, \quad \varphi \in L^1(\mathbb{R}^n),$$

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} f(\omega, t) \varphi(t) dt.$$

Dokaz. Ovo tvrđenje su dali C. H. Swartz i D. E. Myers u [42] ali u ovom dokazu su ispravljeni neki propusti u njihovom dokazu.

Za svako $\omega \in \Omega$, $\xi(\omega, \cdot)$ je linearan i neprekidan funkcional na $L^1(\mathbb{R}^n)$, pa sledi da postoji $h(\omega, \cdot) \in L^\infty(\mathbb{R}^n)$ tako da je

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} h(\omega, t) \varphi(t) dt, \quad \varphi \in L^1(\mathbb{R}^n).$$

Clouds off the coast of California
are often associated with the continental
shelf, and the shelf + clouds are

known as the "California cloud belt".
The shelf is known as continental
shelf and it is believed that the shelf cloud
is formed by air masses moving over the shelf.
Clouds of the California

cloud belt are formed by air masses moving
over the shelf, and the shelf cloud is
similarly influenced by

High pressure systems which are located over the shelf.
The shelf cloud is formed by air masses
moving over the shelf, and the shelf cloud is
similarly influenced by

High pressure systems which are located over the shelf.
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[The] shelf cloud is formed by air masses which are located over the shelf.

Iz Teoreme 5.3. sledi da je $h(\omega, \cdot) = g(\omega, \cdot)$, a.e. [m], $\omega \in \Omega$.

Fiksirajmo $\omega \in \Omega$. Neka je $C(\omega) = \text{esssup } |h(\omega, x)|$, i neka su E_n , $n \in \mathbb{N}$, $\mu_k(\omega, \cdot)$, \mathcal{O}_k , $k \geq 1$, kao u Teoremi 5.2. Tada je

$$|\frac{\mu_k(\omega, E_n)}{m(E_n)}| \leq C(\omega).$$

Definišimo

$$f(\omega, x) = \begin{cases} g(\omega, x), & x \in \mathbb{R}^n \setminus A(\omega), \\ \limsup_{n \rightarrow \infty} \frac{\mu_k(\omega, E_n)}{m(E_n)}, & x \in \mathcal{O}_k \cap A(\omega), \quad k \in \mathbb{N} \end{cases}$$

gde je $\{E_n : n \in \mathbb{N}\}$ familija otvorenih skupova koja se lepo skuplja u $x \in \mathcal{O}_k \setminus A(\omega)$, $k \in \mathbb{N}$.

Kako je $\mu_k(\omega, E_n) = \xi(\omega, \delta_n(E_n \cap \mathcal{O}_k))$ mjerljiva, sledi da je $f(\omega, x)$ mjerljiva za svako $x \in \mathbb{R}^n$. Šta više, za $\omega \in \Omega$, $f(\omega, x) = h(\omega, x)$ a.e. [m], pa (1) i (2) sledi. □

Teorema 5.5. Neka je ξ Gausov u.s.p. $\Omega \times L^1(\mathbb{R}^n)$, takav da za svako $\omega \in \Omega$, $x_0 \in \mathbb{R}^n$ i $\delta_n(x-x_0)$ kao u (5.1.1) postoji granica

$$\lim_{n \rightarrow \infty} \xi(\omega, \delta_n(x-x_0)).$$

Tada postoji Gausov slučajni proces $f : \Omega \times \mathbb{R}^n \rightarrow \mathbb{C}$ takav da je

(1) $\forall \omega \in \Omega$, $f(\omega, \cdot) \in L^\infty(\mathbb{R}^n)$,

(2) $\forall \omega \in \Omega$, $\varphi \in L^1(\mathbb{R}^n)$,

$$\xi(\omega, \varphi) = \int_{\mathbb{R}^n} f(\omega, t) \varphi(t) dt.$$

Dokaz. Iz Teoreme 5.3. sledi da postoji slučajni proces $f : \Omega \times \mathbb{R}^n \rightarrow \mathbb{C}$ takav da su (1) i (2) zadovoljeni. Kako granica $\lim_{n \rightarrow \infty} \xi(\omega, \delta_n(x-x_0))$ postoji za svako $\omega \in \Omega$, $x \in \mathbb{R}^n$ i δ_n , sledi da

an important moment after the first two years of the war, when the British had suffered a heavy defeat at the hands of the Germans at the Battle of the Marne.

On the 10th October 1914, the British government issued a statement, which said that the British government had been informed by the French government that they were sending a force of 100,000 men to help the British in their fight against Germany. This was a very welcome news to the British people, as it meant that they would have more troops available to defend their country.

The British government also stated that they would be sending a force of 100,000 men to help the French in their fight against Germany. This was a very welcome news to the British people, as it meant that they would have more troops available to defend their country.

je za $\omega \in \Omega$, $g(\omega, x)$ definisana za svako $x \in \mathbb{R}^n$. Tako je

$$f(\omega, x) = g(\omega, x), \quad \omega \in \Omega, x \in \mathbb{R}^n.$$

Stavište imamo, za $x_0 \in \mathbb{R}^n$,

$$f(\cdot, x_0) = \lim_{n \rightarrow \infty} \xi(\cdot, \delta_n(x - x_0)).$$

Iz Teorema 1.5.14. i 1.5.16. sledi da je granica niza na desnoj strani jednakosti Gausova slučajna promenljiva, pa je $f(\cdot, x)$ Gausova slučajna promenljiva za svako $x \in \mathbb{R}^n$. Neka je $\mathcal{Z} = \{\xi(\cdot, \varphi), \varphi \in L^1(\mathbb{R}^n)\}$ i $\bar{\mathcal{Z}}$ linearno zatvaranje od \mathcal{Z} u $L^2(\Omega)$. Familijsa $\{f(\cdot, x) : x \in \mathbb{R}^n\}$ je podfamilija od $\bar{\mathcal{Z}}$, pa sledi da je $f(\cdot, x)$, $x \in \mathbb{R}^n$, Gausov slučajni proces. □

Napomena. Tvrđenje Teoreme 5.5. važi i ako se prostor $L^1(\mathbb{R}^n)$ i njegov dual $L^\infty(\mathbb{R}^n)$ zamene prostorom $L^p(\mathbb{R}^n)$ i njegovim dualom $L^q(\mathbb{R}^n)$.

$$|\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n$$

$$\alpha^T = (\alpha_1)^T \dots (\alpha_n)^T$$

$$x = (x_1, x_2, \dots, x_n)$$

$$y = (y_1, y_2, \dots, y_n)$$

$$v = v_1 + v_2 + \dots + v_n$$

$$V = \text{vektoristički prostor na } \mathbb{R}^n$$

$$V^* = \text{dualni prostor na } V$$

OSNOVNE OZNAKE

$\mathbb{N} = \{1, 2, \dots\}$,

$\mathbb{N}_0 = \mathbb{N} \cup \{0\}$,

$\mathbb{N}_0^n = \{(k_1, k_2, \dots, k_n) : k_i \in \mathbb{N}_0, i=1, 2, \dots, n\}$

\mathbb{Q} - skup racionalnih brojeva,

\mathbb{R} - skup realnih brojeva,

$\mathbb{R}^n = \{(x_1, x_2, \dots, x_n) : x_i \in \mathbb{R}, i=1, 2, \dots, n\}$,

\mathbb{C} - skup kompleksnih brojeva,

$\mathbb{C}^n = \{(z_1, z_2, \dots, z_n) : z_i \in \mathbb{C}, i=1, 2, \dots, n\}$,

\cup - unija

\cap - presek

c - komplement

\setminus - razlika

\mathcal{B} - σ - algebra Borelovih skupova u \mathbb{R} ,

\mathcal{E} - σ - algebra Borelovih skupova u \mathbb{C} ,

\mathcal{B}^n - σ - algebra Borelovih skupova u \mathbb{R}^n ,

\mathcal{E}^n - σ - algebra Borelovih skupova u \mathbb{C}^n ,

$|z|$ - apsolutna vrednost broja $z \in \mathbb{C}$,

\bar{z} - konjugovano kompleksan broj za $z \in \mathbb{C}$,

$i = \sqrt{-1}$,

$|\alpha| = \alpha_1 + \alpha_2 + \dots, \alpha \in \mathbb{N}_0^n$,

$x^\alpha = (x_1)^{\alpha_1} \dots (x_n)^{\alpha_n}, x \in \mathbb{R}^n, \alpha \in \mathbb{N}_0^n$,

$\alpha! = \alpha_1! \alpha_2! \dots \alpha_n!, \alpha \in \mathbb{N}_0^n$,

$D_x^\alpha = D^\alpha = \left(\frac{1}{i} \frac{\partial}{\partial x_1}\right)^{\alpha_1} \dots \left(\frac{1}{i} \frac{\partial}{\partial x_n}\right)^{\alpha_n}$,

V - vektorsko topološki prostor,

$\|\cdot\|_v$ - norma u V ,

V' - dualni prostor za V ,

DRAFT DIVISION

$\{ \dots, 0, 1 \} = \emptyset$

$\{ 0 \} \cup \emptyset = \emptyset$

$\{ 0, \dots, n, (n+1)^2 \dots, (n+m)^2, \dots, (n+2m)^2 \} = \emptyset$

and division by zero = 0

and division by one = 0

$\{ 0, \dots, n, (n+1)^2 \dots, (n+m)^2, \dots, (n+2m)^2 \} = \emptyset$

and division by zero = 0

$\{ 0, \dots, n, (n+1)^2 \dots, (n+m)^2, \dots, (n+2m)^2 \} = \emptyset$

$n^2 + m^2 + 2nm = 0$ if $n = m = 0$

otherwise = 0

division by zero = 0

$n^2 + m^2 + 2nm = 0$ if $n = m = 0$

otherwise = 0

division by zero = 0

$n^2 + m^2 + 2nm = 0$ if $n = m = 0$

otherwise = 0

$\sqrt{n} \in \mathbb{R}$

$\sqrt{n} \in \mathbb{N} \dots, \sqrt{n} \in \mathbb{N} = \{ n \}$

$\sqrt{n} \in \mathbb{N} \dots, \sqrt{n} \in \mathbb{N} = \{ n \}$

$\sqrt{n} \in \mathbb{N} \dots, \sqrt{n} \in \mathbb{N} = \{ n \}$

$\sqrt{n} \in \mathbb{N} \dots, \sqrt{n} \in \mathbb{N} = \{ n \}$

containing all the local extensions = 0

$\forall n \in \mathbb{N} \exists \sqrt{n} \in \mathbb{N}$

$\forall n \in \mathbb{N} \exists \sqrt{n} \in \mathbb{N}$

$\|\cdot\|_V'$ - norma u V' ,

\lim_{\rightarrow} - projektivna granica,

\lim_{\leftarrow} - induktivna granica,

G - otvoren skup u \mathbb{R} ili \mathbb{R}^n ,

$L^p(G)$, $p \geq 1$ - prostor p -integrabilnih funkcija nad G ,

$$\|u\|_{L^p(G)} = \left(\int_G |u(x)|^p dx \right)^{1/p} - \text{norma u } L^p(G),$$

$L^\infty(G)$ - Lebegov prostor esencijalno ograničenih funkcija nad G ,

$C(G)$ - prostor neprekidnih funkcija nad G ,

$C^m(G)$ - prostor m puta diferencijabilnih funkcija nad G ,

$m \in \mathbb{N}_0$ ili $m = +\infty$,

(Ω, \mathcal{F}, P) - prostor verovatnoća,

$L^2(\Omega, \mathcal{F}, P)$ - prostor slučajnih promenljivih nad (Ω, \mathcal{F}, P) sa konačnim drugim momentom.

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и відповідно до змін у міжнародному праві та зовнішньої політиці України. Важливо зазначити, що поганою практикою є те, що в Україні використовують термін «закон» як засіб, що підкреслює їх обов'язковість та важливість, а не їх законність. Це може привести до проблем з інтерпретацією та застосуванням цих норм. У результаті, виникають питання щодо юридичної сили та ефективності норм, які не відповідають змінам в міжнародному праві та зовнішньої політиці України.

Іншими словами, норми, що вимагають від України дотримання їх зовнішньою політикою, можуть бути незаконними, якщо вони не відповідають змінам в міжнародному праві та зовнішньої політиці України. Це може створити проблеми з інтерпретацією та застосуванням цих норм.

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