Consensus Reaching in Multiple Attribute Group Decision Making: A Multi-Stage Optimization Feedback Mechanism with Individual Bounded Confidences

Quanbo Zha, Yucheng Dong, Francisco Chiclana, Enrique Herrera-Viedma

***Abstract*—Existing consensus models focus on improving the group consensus level, but ignore whether a higher group consensus level means higher mutual acceptance of decision makers. In the field of opinion dynamics, the bounded confidence model asserts that the decision makers will accept the preferences of others within a neighborhood of theirs with width a certain confidence level. Inspired by this research methodology, this paper develops a consensus model to address the acceptance issue based on individual bounded confidences. Specifically, a bounded confidence-based consensus measure is designed to measure the level of group mutual acceptance, and a multi-stage optimization feedback mechanism based on individual bounded confidences is proposed to maximize the group mutual acceptance and minimize the amount of preference adjustment. A numerical example and a simulation analysis are included to illustrate the use of the model and to justify its effectiveness, respectively.**

***Index Terms*—Group decision making; Consensus; Bounded confidence; Multi-Stage Optimization**

I. INTRODUCTION

TO obtain a solution with agreement in group decision making (GDM), it is necessary to include a consensus process within the resolution procedure [1]-[5], with tools to support the enhancement of consensus level among the group via an iterative process of preference adjustment. Consensus measurement and feedback mechanism are two key phases normally included in consensus process. The first phase is to measure and quantify the consensus level among the group, while the second phase is often embodied in the form of consensus rules that generate preference adjustment recommendations to increase the group consensus level.

Currently, measurement of consensus is mainly based on the use of a distance function in two main methods: (1) Measurement based on the distances between the individual decision makers' preferences and the collective preference. Spillman et al.'s research on consensus within the fuzzy sets framework [6] being one of the earliest approaches to develop a distance based consensus measure; other notable examples of developing distance based consensus measures are Herrera et al.'s linguistic preferences consensus measures [7] and Ben-Arieh and Chen's order based consensus measure and mean based consensus measure [8]. (2) Measurement based on pairwise distances between decision makers' preferences. Kacprzyk and Fedrizzi et al. [9] developed the consensus measurement to capture the similarity between decision makers' preferences from the perspective of "soft"; Herrera et al. [10] proposed consensus measurement for linguistic preferences based on the concept of coincidence of linguistic values; while Chen et al. [11] investigated consensus measurement based on deviation and overlap degrees in GDM with uncertain linguistic terms.

Feedback mechanism is mainly expressed in the form of consensus rules and includes two types: (1) The first type of rule is known as the identification rule and direction rule (IR-DR) [12]-[14], where IR identifies the decision makers in unacceptable states of consensus levels who are advised by DR to adjust their preferences in the appropriate direction. Herrera-Viedma et al. [13] studied the feedback mechanism in the form of IR-DR for pursuing higher consensus level in a multigranular linguistic preference relation framework; Zhang et al. [15] used IR-DR within the uncertain 2-tuple linguistic preference relations framework; while Dong et al. [16] and Wu et al. [17] proposed trust relationships consensus models with IR-DR. (2) The second type of rule is known as the optimization-based consensus rule [18]-[20], because it aims to

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minimize the adjustments/costs. Zhang et al. [53] stuided minimum cost consensus models and their economic significance; while Ben-Arieh and Easton [22] investigated the minimum cost issue with multiple attributes; Zhang et al. [21] developed the minimum adjustments in a 2-rank context; while Wu et al. [23] studied this issue in GDM with trust relationships.

In the existing consensus models, the willingness to accept the feedback recommendations has been studied in the form of bounded confidence in recent years. In opinion dynamics, the bounded confidence model defines this psychological behavior, that is, decision maker will only accept the preferences within their bounded confidence [24]-[26]. Zhang et al. [27] developed a two-stage consensus model with bounded confidence; Liang et al. [28] studied this issue in minimum adjustments consensus model with time constraints; Zha et al. [29] proposed a bounded confidence learning mechanism in GDM; Zhang et al. [56] studied GDM with bounded confidence within linguistic preference context; Zhang et al. [57] considered leadership and bounded confidence in social network GDM; while Dong et al. [54] proposed a hybrid GDM framework using bounded confidence to obtain stable opinions. However, there are still some limitations in the existing studies:

(1) In the existing consensus models, the measurement of consensus are based on a distance function as described above, which may not ensure that decision makers will accept the final decision result. Indeed, even if the preferences of two decision makers are similar, there is still a certain distance between their preferences that may be larger than their respective bounded confidence levels leading to their disagreement. On the other hand, even if the preferences of two decision makers are not similar, the two may agree with each other because of their larger psychological bottom line.

(2) In a consensus process, the feedback mechanism improves the similarities of the preferences of decision makers without considering the improvement of decision makers' mutual acceptance. However, the mutual acceptance among decision makers is one of the characteristics describing consensus, since it affects decision makers' satisfaction on the final decision results. In other words, the existing feedback mechanisms can effectively enhance the similarity of group preferences but not the mutual acceptance among decision makers.

To overcome the above limitations, this paper proposes a bounded confidence based consensus model with multi-stage optimization feedback mechanism (MOFMCM) for multiple attribute GDM (MAGDM) problems, aiming at helping promote the level of group mutual acceptance improvement. The specific contributions of this paper are:

(1) A consensus measurement methodology to quantify the level of group mutual acceptance based on the bounded confidence model in opinion dynamics;

(2) A multi-stage optimization feedback mechanism based on bounded confidence: (i) maximizing the level of mutual acceptance, (ii) minimizing the preference adjustments after maximizing acceptance, (iii) maximizing the similarity of the group when mutual acceptance cannot be improved, and (iv) minimizing preference adjustments after maximizing similarity;

(3) Simulation and comparison analysis methodology to justify the effectiveness of MOFMCM in improving group mutual acceptance.

The rest of this article is arranged as follows. In Section II, the general consensus process framework, the minimum adjustment consensus model, and the bounded confidence model are presented. The resolution process for the MAGDM problem with bounded confidence is described in Section III. Section IV illustrates the use of the MOFMCM and analyzes its effectiveness in increasing mutual acceptance. Finally, conclusions are drawn in Section V.

II. PRELIMINARIES

This section reviews briefly the main architecture of a general consensus process in MAGDM (Section II-A), the minimum adjustment consensus model (Section II-B), and the bounded confidence model (Section II-C).

A. A General Consensus Process in MAGDM

The main objective of an MAGDM problem is to arrive at a consensus-based solution from multiple alternatives on multiple attributes. The typical elements of an MAGDM problem are:

(1) A group of decision makers $D=\{d_1,d_2,\cdots,d_r\}(r\ge 2)$ with associated weights $\omega=\{\omega_1,\omega_2,\cdots,\omega_r\}$ subject to constraints: $\omega_k\ge 0(k=1,2,\cdots,r)$; $\sum_{k=1}^{r}\omega_k=1$.

(2) A finite set of alternatives $X=\{x_1,x_2,\cdots,x_m\}(m\ge 2)$, which are the potential solutions of the MAGDM problem;

(3) A set of attributes $A=\{a_1,a_2,\cdots,a_n\}(n\ge 2)$ with associated weights $w=\{w_1,w_2,\cdots,w_n\}(n\ge 2)$ subject to the constraints: $w_j\ge 0(j=1,2,\cdots,n)$; $\sum_{j=1}^{n}w_j=1$.

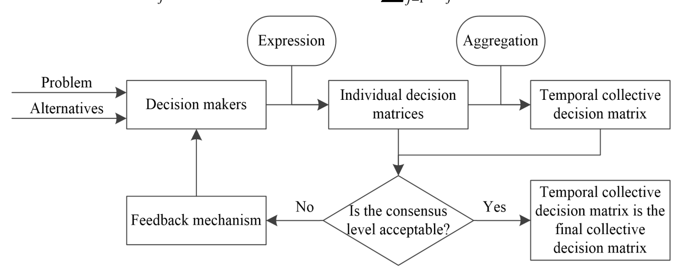

Fig.1. A general consensus process framework

A consensus process is an iterative process of preference adjustment with the purpose of improving the group consensus level. Fig.1 shows its most general basic process, which consists of two fundamental phases:

(1) *Consensus measurement*: This phase quantifies the group consensus level by the first method as described before [14], [31], [39], [48]. Let $V^k=(v_{ij}^k)_{m\times n}$ be the multiple attribute decision matrix expressed by decision maker $d_k\in D$, where $v_{ij}^k\in[0,1]$ denotes the evaluation value for the alternative $x_i\in X$ with respect to the attribute $a_j\in A$. Without loss of generality, this paper uses the weighted average (WA) operator [32] (different aggregation operators are applicable) to compute the collective decision matrix $V^c=(v_{ij}^c)_{m\times n}$:

$$v_{ij}^c=f_\omega(v_{ij}^1,v_{ij}^2,\cdots,v_{ij}^r)=\sum_{k=1}^{r}\omega_k v_{ij}^k \tag{1}$$

Applying the Manhattan distance, the individual consensus level of $d_k \in D$ is:

$$CL(d_k) = 1 - \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} | v_{ij}^k - v_{ij}^c |}{mn} \quad (2)$$

The weighted average of the individual consensus levels is the group consensus level:

$$CL = \sum_{k=1}^{r} \omega_k cl(d_k) \quad (3)$$

The larger $CL \in [0,1]$, the higher the group consensus level. Let $\mu \in [0,1]$ be 'the group consensus threshold' (closer to 1). If $CL < \mu$, then the next consensus fundamental phase is activated because of the unacceptable or unsatisfactory current group consensus level. Otherwise, the current collective decision matrix is considered as the final one.

(2) *Feedback mechanism*: This phase provides feedback recommendations for increasing the group consensus level. IR and DR are two classical consensus rules employed in this phase [12], [13]. IR is employed for identifying the decision maker(s) with unsatisfactory consensus level(s). DR provides feedback recommendations of preference adjustment, $\overline{V}^k = (\overline{v}_{ij}^k)_{m\times n}$, to the identified decision makers $d_k \in D$ with the goal of increasing their consensus levels:

$$\overline{v}_{ij}^k \in [\min(v_{ij}^k, v_{ij}^c), \max(v_{ij}^k, v_{ij}^c)] \quad (4)$$

When an satisfactory group consensus level is achieved, via the iterative application of the above two consensus phases, a selection process is activated to derive a final ranking of the alternatives (x_i) based on their corresponding dominance values (Q_i) [33], [34]:

$$Q_i = WA_w(v_{i1}^c, v_{i2}^c, \cdots, v_{in}^c) = \sum_{j=1}^{n} w_j v_{ij}^c \quad (5)$$

B. The Minimum Adjustment Consensus Model

As described in section II-A, the general consensus model uses the IR-DR to promote the group to achieve a consensus. However, IR and DR based feedback mechanism may cause a large amount of loss of the original preference information. To improve the efficiency of group consensus, Dong et al. [18] proposed an original minimum adjustment consensus model. To simplify the illustration here, this model is illustrated in the form of a multiple attribute decision matrix as the preference of decision maker.

This model retains the original preferences of decision makers as much as possible. If $V^k = (v_{ij}^k)_{m\times n}$ and $\overline{V}^k = (\overline{v}_{ij}^k)_{m\times n}$ are the original and adjusted multiple attribute decision matrix mentioned in Section II-A, then the objective function to optimize is the distance between V^k and $\overline{V}^k$, $D(V^k, \overline{V}^k)$, i.e.

$$\min \sum_{k=1}^{r} D(V^k, \overline{V}^k) \quad (6)$$

Meanwhile, the individual consensus level should be at an acceptable level, i.e.

$$cl'(d_k) \le \alpha \quad (7)$$

where $cl'(d_k) = \sum_{i=1}^{m}\sum_{j=1}^{n} | \overline{v}_{ij}^k - \overline{v}_{ij}^c | / mn$; $\overline{V}^c$ is the collective decision matrix obtained from the adjusted multiple attribute decision matrices via the aggregation function $f_\pi(\overline{V}_1, \cdots, \overline{V}_r)$; and $\alpha \in [0,1]$ (closer to 0) is the consensus threshold. Thus, the optimal adjusted preferences, $\{\overline{V}_1, \cdots, \overline{V}_r\}$, which are the feedback recommendations, are obtained by solving the below optimization model:

$$\min \sum_{k=1}^{r} D(V^k, \overline{V}^k)$$
$$\begin{cases} s.t. \quad \sum_{i=1}^{m}\sum_{j=1}^{n} | \overline{v}_{ij}^k - \overline{v}_{ij}^c | / mn \le \alpha \quad k = 1, \cdots, r \\ \overline{V}^c = f_\pi(\overline{V}_1, \cdots, \overline{V}_r) \end{cases} \quad (8)$$

So far, the minimum adjustment consensus model has been widely studied in the following scenarios [46]: (1) using linguistic preferences (Dong et al. [18]; Wu et al. [40]); (2) using preference relations (Zhang et al. [41]; Wu et al. [42]; Zhang et al. [43]); (3) using heterogeneous preference representation structures (Chen et al. [44]; Zhang et al. [45]); (4) in MAGDM (Zha et al. [30]; Yu et al. [47]); (5) using a multi-stage optimization strategy (Zhang et al. [19]; Wu et al. [50]); (6) in classification-based GDM (Zhang et al. [21]; Chen et al. [49]); (7) in social network GDM (Wu et al. [23]; Cheng et al. [51]); (8) in large-scale GDM (Zha et al. [30]; Xiao et al. [52]); and (9) in opinion dynamic GDM (Liang et al. [28]; Chen et al. [39]; Dong et al. [53]).

C. Bounded Confidence Model

In opinion dynamics, Hegselmann-Krause (HK) model [35] and Deffuant-Weisbuch (DW) model [36], [37], are two widespread bounded confidence models. Both models study individual willingness of accepting opinions, and argue that decision makers will only be influenced by the recommendations similar to their own opinions. Specifically, the bounded confidence of a decision maker is the critical value to judge whether a recommendation is acceptable to the decision maker.

Let $V^R = (v_{ij}^R)_{m\times n}$ be the feedback recommendation for adjusting decision matrix $V^k = (v_{ij}^k)_{m\times n}$ of decision maker d_k in a consensus process. Let $B = \{b_1^k, b_2^k, \cdots, b_n^k\}$ be the bounded confidence set of decision maker d_k associated with the attributes $A = \{a_1, a_2, \cdots, a_n\} (n \ge 2)$. According to the bounded confidence model, decision maker d_k will accept the feedback recommendation v_{ij}^R when the distance between v_{ij}^R and v_{ij}^k, $D(v_{ij}^R, v_{ij}^k)$, is less than or equal to b_j^k, i.e.

$$| v_{ij}^R - v_{ij}^k | \le b_j^k \quad (9)$$

In both HK and DW models, the bounded confidence influences the convergence time and the distribution of final opinions [35]-[37]. The larger the bounded confidence value, the smaller the number of opinion clusters and the larger the opinion cluster size. When the bounded confidence takes a sufficiently large value, an opinion cluster is formed between the decision makers, that is, consensus is reached. In GDM problems, consensus model has begun to pay attention to the bounded confidence model [27], [29], [30], [54]. Zha et al. [29] provided a learning algorithm to find out the unknown bounded confidence. However, in most existing studies, the research paradigm of GDM with bounded confidence assumes that the bounded confidences are given or known. Therefore, in this paper, we follow the research line of the GDM with bounded confidence and assume that the bounded confidences are known.

III. Consensus Model with Multi-stage Optimization Feedback Mechanism

This section proposes a consensus model with individual bounded confidences based on a multi-stage optimization feedback mechanism.

A. Framework of the MOFMCM Model

In a consensus process under bounded confidence context, decision makers will accept or reject feedback recommendations according to their own bounded confidences [29], [30]. However, the existing bounded confidence based consensus models ignore the mutual acceptance among decision makers. Even if the distance-based consensus is at a high level, decision makers may be unsatisfied with the final solution since other decision makers' preferences may be unacceptable for them based on their own bounded confidences. Therefore, it is meaningful to measure the consensus level taking into account the mutual acceptance of decision makers. Thus, this section develops a consensus-based solution for MAGDM problem with individual bounded confidences, where the consensus level and the multi-stage optimization feedback mechanism are both designed based on the level of mutual acceptance among decision makers.

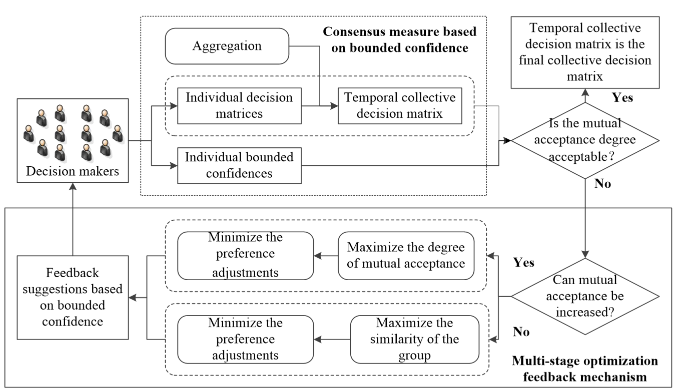

Fig. 2. Framework of the MOFMCM model

The MOFMCM model framework shown in Fig. 2 includes two key phases: (1) Consensus measure based on bounded confidence, and (2) multi-stage optimization feedback mechanism. In the first phase group consensus level is measured based on the mutual acceptance level among the group. The second phase is divided into two parts: (1) A two-stage optimization based on mutual acceptance to improve the mutual acceptance between decision makers, and (2) a two-stage optimization based on similarity which is activated when the first part cannot be improved, to improve the similarity between decision makers.

B. Consensus Measure based on Bounded Confidence

As mentioned in the introduction, two main methods for measuring group consensus level aim to narrow the preference distance between decision makers. However, they ignored whether decision makers agreed to the current level of consensus at each round. In other words, the decision makers' preferences may be relatively similar but with a context of mutual rejection among them; on the contrary, the decision makers' preferences may not be similar but within a context of mutual acceptance of them. This is illustrated with the following example.

Example 1: Let $B^1=\{0.1,0.05,0.6\}$ and $B^2=\{0.14,0.04,0.56\}$ be the bounded confidences of decision makers d_1 and d_2, respectively. And their decision matrices are as follows:

$$V^1=\begin{pmatrix}0.25 & 0.63 & 0.90\\ 0.70 & 0.53 & 0.35\\ 0.81 & 0.39 & 0.15\end{pmatrix},\quad V^2=\begin{pmatrix}0.67 & 0.56 & 0.58\\ 0.24 & 0.46 & 0.80\\ 0.41 & 0.45 & 0.63\end{pmatrix}.$$

Based on Eq. (6), we draw the following observations:

(1) Decision makers d_1 and d_2 are far away from each other with respect to their evaluations with respect to attribute a_1, and each one of them consider the other's opinions are unacceptable as suggestions.

(2) Decision makers d_1 and d_2 have close evaluations with respect to attribute a_2, but each one of them consider the other's opinions unacceptable as suggestions.

(3) Decision makers d_1 and d_2 are still far apart in their evaluations with respect to attribute a_3, but they are mutually acceptable.

In what follows, the group consensus level is derived from the measurements of the levels of mutual acceptance among decision makers based on individual bounded confidences.

Let $B=\{b_1^k,b_2^k,\cdots,b_n^k\}$ be the bounded confidence set of decision maker d_k associated with the set of attributes $A=\{a_1,a_2,\cdots,a_n\}$ $(n\geq 2)$. Let $AD^{lk}=(ad_{ij}^{lk})_{m\times n}$ $(k,l=1,2,\cdots,r;k\neq l)$ be a 0-1 matrix to represent the decision maker d_l acceptance of decision maker d_k: $ad_{ij}^{lk}=1$ denotes decision maker d_l accepts decision maker d_k evaluation value for alternative $x_i\in X$ with respect to the attribute $a_j\in A$; otherwise, $ad_{ij}^{lk}=0$. According to the bounded confidence model, we have

$$ad_{ij}^{lk}=\begin{cases}1 & \text{if } |v_{ij}^k-v_{ij}^l|\leq b_j^l\\ 0 & \text{if } |v_{ij}^k-v_{ij}^l|> b_j^l\end{cases}\tag{10}$$

The following acceptance levels are defined:

(1) The acceptance level from decision maker d_l to decision maker d_k:

$$AL^{lk}=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}ad_{ij}^{lk}}{mn}\tag{11}$$

(2) The group's acceptance level of decision maker d_k:

$$AL(d_k) = \frac{\sum_{l=1,l\neq k}^{r} AL^{lk}}{r-1} \tag{12}$$

(3) The group mutual acceptance level:

$$AL = \frac{\sum_{k=1}^{r} AL(d_k)}{r} \tag{13}$$

Obviously, $AL \in [0,1]$. And larger AL values indicate higher mutual acceptance levels. It can be seen from Eqs. (10)-(13) that under the same preference, the greater the boundary trust, the greater the degree of mutual acceptance. In this paper, we use the group mutual acceptance level to measure consensus instead of the traditional method described by Eq. (3). For notation simplicity, we still use $\mu \in [0,1]$ to denote the mutual acceptance threshold. If $AL < \mu$, then group mutual acceptance is unsatisfactory, and the feedback phase is activated. Otherwise, Eq. (5) will be used to obtain the dominance values of alternatives and the final solution to the MAGDM problem.

C. *Multi-stage Optimization Feedback Mechanism*

Existing feedback mechanisms aim to increase the distance based group consensus level by improving the similarity between decision makers without considering the decision makers' recognition of the consensus level improvement. This section proposes a multi-stage optimization feedback mechanism based on bounded confidence, which includes two parts: (1) a two-stage optimization based on mutual acceptance, and (2) a two-stage optimization based on similarity.

As shown in Example 1, the mutual acceptance of attribute a_1 between the two decision makers cannot be improved. This situation may happen for the entire group. When the *two-stage optimization based on mutual acceptance* cannot improve the group mutual acceptance, the MOFMCM model will activate the *two-stage optimization based on similarity* to enhance the similarity of the decision makers' decision matrices and lay the foundation for increasing the mutual acceptance among decision makers in next round.

(1) *A two-stage optimization based on mutual acceptance.* This consists of two consecutive stages: Stage (i) to maximize the mutual acceptance, and Stage (ii) to minimize the preference adjustments based on Stage (i).

Stage (i) aims to maximize the group mutual acceptance, i.e., the feedback decision matrices $\overline{V}^k$ and $\overline{V}^l$ should maximize the number of 1 elements in $AD^{lk} = (ad_{ij}^{lk})_{m\times n}$ $(k,l = 1,2,\cdots,r; k \neq l)$:

$$\max AL \quad (\text{or } \max \frac{1}{mnr(r-1)} \sum_{l=1,l\neq k}^{r} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} ad_{ij}^{lk}) \tag{14}$$

$$ad_{ij}^{lk} = \begin{cases} 1 & \text{if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| \le b_j^l \\ 0 & \text{if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| > b_j^l \end{cases} \tag{15}$$

In order to make the feedback decision matrix $\overline{V}^k$ within the acceptable range of the decision maker d_k , the feedback decision matrix $\overline{V}^k$ and the decision matrix V^k need to meet the following constraint:

$$|\overline{v}_{ij}^k - v_{ij}^k| \le b_j^k \tag{16}$$

Based on Eqs. (14)-(16), the mutual acceptance maximization model becomes:

$$\min -\frac{1}{mnr(r-1)} \sum_{l=1,l\neq k}^{r} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} ad_{ij}^{lk}$$

$$\begin{cases} s.t.\ \ |\overline{v}_{ij}^k - v_{ij}^k| \le b_j^k & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & k=1,2,\cdots,r \\ ad_{ij}^{lk} = \begin{cases} 1 \text{ if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| \le b_j^l \\ 0 \text{ if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| > b_j^l \end{cases} & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & l,k=1,2,\cdots,r; l\neq k \\ 0 \le \overline{v}_{ij}^k \le 1 & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & k=1,2,\cdots,r \end{cases} \tag{17}$$

The optimal solution for AL denoted by AL^* , is the solution of model (17), which stands the highest level of mutual acceptance that the group can reach. Nevertheless, multiple solutions for $\overline{V}^k$ $(k = 1,2,\cdots,r)$ may exist in model (17). Therefore, we further optimize the optimal solution of the feedback decision matrices through *Stage* (ii).

Stage (ii) aims to minimize the group preference adjustments on the basis of the optimal solution AL^* , i.e., the distances between the feedback decision matrix $\overline{V}^k$ and the decision matrix V^k needs to be minimized:

$$\min \frac{1}{rmn} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} |\overline{v}_{ij}^k - v_{ij}^k| \tag{18}$$

where ad_{ij}^{lk} is computed by Eq. (15), while constraint (16) being still valid at Stage (ii). To achieve the highest level of mutual acceptance, the following constraint is to be met:

$$\frac{1}{mnr(r-1)} \sum_{l=1,l\neq k}^{r} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} ad_{ij}^{lk} = AL^* \tag{19}$$

According to Eqs. (15)-(19), the preference adjustment minimization model becomes:

$$\min \frac{1}{rmn} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} |\overline{v}_{ij}^k - v_{ij}^k|$$

$$\begin{cases} s.t.\ \ |\overline{v}_{ij}^k - v_{ij}^k| \le b_j^k & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & k=1,2,\cdots,r \\ ad_{ij}^{lk} = \begin{cases} 1 \text{ if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| \le b_j^l \\ 0 \text{ if } |\overline{v}_{ij}^k - \overline{v}_{ij}^l| > b_j^l \end{cases} & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & l,k=1,2,\cdots,r; l\neq k \\ \frac{1}{mnr(r-1)} \sum_{l=1,l\neq k}^{r} \sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} ad_{ij}^{lk} = AL^* & \\ 0 \le \overline{v}_{ij}^k \le 1 & i=1,2,\cdots,m; j=1,2,\cdots,n; \\ & k=1,2,\cdots,r \end{cases} \tag{20}$$

The optimal value for $\overline{V}^k$ $(k = 1,2,\cdots,r)$ denoted $\overline{V}^{k*}$, is the solution of model (20). Model (20) has a closed bounded non-empty feasible region. Therefore, it can be known from the Weierstrass theorem that the optimal solution(s) of model (20) exists. However, model (20) may still have multiple optimal solutions. For this kind of issues, it can be further optimized based on these optimal solutions. For example, Chandran et al. [58] developed a two-stage approach to pursue a unique optimal solution; Dong and Herrera-Viedma [59] also agreed with this solution approach. Specifically, let $\overline{V}^*$ be the optimal solution set of model (20), and then the unique optimal solution can be obtain based on model (20'):

$$\min_{\bar{V}^k* \in \bar{V}^*} \max_k D(\bar{V}^k*, V^k) \tag{20'}$$

Model (20') further minimizes the maximal distances between $\bar{V}^k*$ and V^k, and can be similarly solved. This paper does not focus on the uniqueness issue, so we do not discuss this issue in detail here.

To improve the group mutual acceptance, it is recommended that the adjusted decision matrix value $v_{ij}^{k\prime}$ of decision maker d_k follows the rule below:

$$v_{ij}^{k\prime} \in [\min(v_{ij}^k, \bar{v}_{ij}^k*), \max(v_{ij}^k, \bar{v}_{ij}^k*)] \tag{21}$$

(2) *A two-stage optimization based on similarity.* This part also includes two stages of optimization: Stage (i) to maximize the similarity of the group, and stage (ii) to minimize preference adjustments.

Stage (i) aims to increase the proximity of the decision makers' decision matrices by minimizing the distance between the individual feedback decision matrices $\bar{V}^k (k = 1, 2, \cdots, r)$ and their corresponding collective decision matrix $\bar{V}^c$, $D(\bar{V}^k, \bar{V}^c) = \sum_{i=1}^m \sum_{j=1}^n | \bar{v}_{ij}^k - \bar{v}_{ij}^c | \big/ m \cdot n$, i.e.,

$$\min \sum_{k=1}^r D(\bar{V}^k, \bar{V}^c) \tag{22}$$

Constraint (16) is used here to ensure that the adjusted decision matrix is within the bounded confidence range of the decision maker. Based on Eq. (1), we have $\bar{v}_{ij}^c = \sum_{k=1}^r \omega_k \bar{v}_{ij}^k$. Then, the model similarity maximization model becomes:

$$\min \sum_{k=1}^r D(\bar{V}^k, \bar{V}^c)$$
$$\begin{cases} s.t. \ \ | \bar{v}_{ij}^k - v_{ij}^k | \le b_j^k & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n \\ \bar{v}_{ij}^c = \sum_{k=1}^r \omega_k \bar{v}_{ij}^k & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n \\ 0 \le \bar{v}_{ij}^k \le 1 & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n \end{cases} \tag{23}$$

The optimal values for $\bar{V}^k$ and $\bar{V}^c$, denoted as $\bar{V}^k**$ and $\bar{V}^c**$, are the solution for model (23). Based on Eqs. (2) and (3), the highest level of group similarity (i.e., group consensus level) $CL*$ is:

$$CL* = 1 - \frac{\sum_{k=1}^r \sum_{i=1}^m \sum_{j=1}^n | \bar{v}_{ij}^k ** - \bar{v}_{ij}^c ** |}{rmn} \tag{24}$$

However, model (23) may multiple solutions for $\bar{V}^k$ $(k = 1, 2, \cdots, r)$. Therefore, *Stage* (ii) further minimizes the preference adjustments based on the highest group similarity to optimize the optimal solution for $\bar{V}^k$ $(k = 1, 2, \cdots, r)$. To achieve this, Eq. (19) is employed with the group similarity subject to the following constraint:

$$1 - \frac{1}{rmn} \sum_{k=1}^r \sum_{i=1}^m \sum_{j=1}^n | \bar{v}_{ij}^k - \bar{v}_{ij}^c | = CL* \tag{25}$$

According to Eqs. (16) and (25), the preference adjustment minimization model based on the highest group similarity become:

$$\min \frac{1}{rmn} \sum_{k=1}^r \sum_{i=1}^m \sum_{j=1}^n | \bar{v}_{ij}^k - v_{ij}^k |$$
$$\begin{cases} s.t. \ \ | \bar{v}_{ij}^k - v_{ij}^k | \le b_j^k & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ k = 1, 2, \cdots, r \\ \bar{v}_{ij}^c = \sum_{k=1}^r \omega_k \bar{v}_{ij}^k & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ k = 1, 2, \cdots, r \\ 1 - \frac{1}{rmn} \sum_{k=1}^r \sum_{i=1}^m \sum_{j=1}^n | \bar{v}_{ij}^k - \bar{v}_{ij}^c | = CL* & \\ 0 \le \bar{v}_{ij}^k \le 1 & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ k = 1, 2, \cdots, r \end{cases} \tag{26}$$

The optimal value for $\bar{V}^k$ $(k = 1, 2, \cdots, r)$, namely $\bar{V}^k**'$, is the solution of model (23). To improve the group similarity, it is recommended that the adjusted decision matrix value $v_{ij}^{k\prime}$ of decision maker d_k follows the rule below:

$$v_{ij}^{k\prime} \in [\min(v_{ij}^k, \bar{v}_{ij}^k**'), \max(v_{ij}^k, \bar{v}_{ij}^k**')] \tag{27}$$

D. Mixed 0-1 Linear Programming Associated with the MOFMCM Model

To facilitate solving model (17), this is transformed into a mixed 0-1 linear programming model. Lemma 1 is the theoretical basis for equivalent transformation.

Lemma 1: Let M be a large enough number. When $\bar{v}_{ij}^k$ and $\bar{v}_{ij}^l$ satisfy constraint (28), we have

$$ad_{ij}^{lk} = \begin{cases} 1 & \text{if } | \bar{v}_{ij}^k - \bar{v}_{ij}^l | \le b_j^l \\ 0 & \text{if } | \bar{v}_{ij}^k - \bar{v}_{ij}^l | > b_j^l \end{cases}.$$

$$\begin{cases} \bar{v}_{ij}^k - \bar{v}_{ij}^l \ge (y_{ij}^{lk} - 1)M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ \bar{v}_{ij}^k - \bar{v}_{ij}^l < y_{ij}^{lk} M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ \bar{v}_{ij}^k - \bar{v}_{ij}^l \le b_j^l + (2 - y_{ij}^{lk} - ad_{ij}^{lk})M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ \bar{v}_{ij}^k - \bar{v}_{ij}^l > b_j^l - (1 - y_{ij}^{lk} + ad_{ij}^{lk})M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ -\bar{v}_{ij}^k + \bar{v}_{ij}^l \le b_j^l + (y_{ij}^{lk} + 1 - ad_{ij}^{lk})M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ -\bar{v}_{ij}^k + \bar{v}_{ij}^l > b_j^l - (y_{ij}^{lk} + ad_{ij}^{lk})M & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \\ y_{ij}^{lk}, rd_{ij}^{lk} \in \{0, 1\} & i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \ l, k = 1, 2, \cdots, r; l \ne k \end{cases} \tag{28}$$

Proof: From $\bar{v}_{ij}^k - \bar{v}_{ij}^l \ge (1 - y_{ji}^{lk})M$ and $\bar{v}_{ij}^k - \bar{v}_{ij}^l < y_{ji}^{lk} M$, we obtain that: (1) $y_{ij}^{lk} = 1$ implies $\bar{v}_{ij}^k - \bar{v}_{ij}^l \ge 0$; (2) while $y_{ij}^{lk} = 0$ implies $\bar{v}_{ij}^k - \bar{v}_{ij}^l < 0$.

Furthermore, from $\bar{v}_{ij}^k - \bar{v}_{ij}^l \le b_j^l + (2 - y_{ji}^{lk} - ad_{ij}^{lk})M$ and $\bar{v}_{ij}^k - \bar{v}_{ij}^l > b_j^l - (1 - y_{ji}^{lk} + ad_{ij}^{lk})M$, we have: (3) When $y_{ij}^{lk} = 1$, from (1) we have $\bar{v}_{ij}^k - \bar{v}_{ij}^l \ge 0$. Then, $ad_{ij}^{lk} = 1$ can obtain $0 \le \bar{v}_{ij}^k - \bar{v}_{ij}^l \le b_j^l$ and $\bar{v}_{ij}^k - \bar{v}_{ij}^l \ge 0 > b_j^l - M$; while $ad_{ij}^{lk} = 0$ implies $0 \le \bar{v}_{ij}^k - \bar{v}_{ij}^l \le M$ and $\bar{v}_{ij}^k - \bar{v}_{ij}^l > b_j^l \ge 0$; (4) When

$y_{ij}^{lk}=0$, $\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}<0$ can be obtained from (2). Then, $ad_{ij}^{lk}=1$ implies $\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}<0\le b_j^l+M$ and $0>\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}>b_j^l-2M$; while $ad_{ij}^{lk}=0$ can obtain $\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}<0\le b_j^l+2M$ and $0>\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}>b_j^l-M$.

Then, it is $ad_{ij}^{lk}=\begin{cases}1 & \text{if } 0\le \bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\le b_j^l\\ 0 & \text{if } \bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}>b_j^l\ge 0\end{cases}$.

Subsequently, from $-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l+(1+y_{ji}^{lk}-ad_{ij}^{lk})M$ and $-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}>b_j^l-(y_{ji}^{lk}+ad_{ij}^{lk})M$, we have that: (5) When $y_{ij}^{lk}=1$, based on (1), $\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\ge 0$. Furthermore, $ad_{ij}^{lk}=1$ implies $-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le 0\le b_j^l+M$ and $b_j^l-2M<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le 0$; by $ad_{ij}^{lk}=0$, it can be guaranteed that $-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le 0\le b_j^l+2M$ and $b_j^l-M<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le 0$; (6) When $y_{ij}^{lk}=0$, based on (2), $-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}>0$. Then, $ad_{ij}^{lk}=1$ guarantees $0<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l$ and $b_j^l-M<0<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}$; while $ad_{ij}^{lk}=0$ guarantees $0<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l+M$ and $0<b_j^l<-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}$.

Therefore, we have $ad_{ij}^{lk}=\begin{cases}1 & \text{if } 0\le -\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l\\ 0 & \text{if } -\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}>b_j^l>0\end{cases}$.

Then, constraint (25) implies $ad_{ij}^{lk}=\begin{cases}1 & \text{if } |\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}|\le b_j^l\\ 0 & \text{if } |\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}|>b_j^l\end{cases}$.

This completes the Proof of Lemma 1.

Proposition 1: Let M be a large enough number. The model (17) can be transformed into the below 0-1 mixed linear programming model (29).

$$\min -\frac{1}{mnr(r-1)}\sum_{l=1,l\ne k}^{r}\sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} rd_{ij}^{lk}$$
$$\begin{cases}
s.t.\quad \bar{v}_{ij}^{k}-v_{ij}^{k}\le b_j^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
\bar{v}_{ij}^{k}-v_{ij}^{k}\ge -b_j^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\ge (y_{ij}^{lk}-1)M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}< y_{ij}^{lk}M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\le b_j^l+(2-y_{ij}^{lk}-ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}> b_j^l-(1-y_{ij}^{lk}+ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l+(y_{ij}^{lk}+1-ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}> b_j^l-(y_{ij}^{lk}+ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
0\le \bar{v}_{ij}^{k}\le 1 & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
y_{ij}^{lk}, ad_{ij}^{lk}\in\{0,1\} & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k
\end{cases} \quad (29)$$

Proposition 2: Let $f_{ij}^k=\bar{v}_{ij}^{k}-v_{ij}^{k}$, $u_{ij}^k=|f_{ij}^k|$, and M be a large enough number. Then model (20) can be transformed into the below 0-1 mixed linear programming model (30).

$$\min \frac{1}{rmn}\sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} u_{ij}^{k}$$
$$\begin{cases}
s.t.\quad \bar{v}_{ij}^{k}-v_{ij}^{k}\le u_{ij}^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
-\bar{v}_{ij}^{k}+v_{ij}^{k}\le u_{ij}^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
\bar{v}_{ij}^{k}-v_{ij}^{k}\le b_j^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
\bar{v}_{ij}^{k}-v_{ij}^{k}\ge -b_j^k & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\ge (y_{ij}^{lk}-1)M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}< y_{ij}^{lk}M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}\le b_j^l+(2-y_{ij}^{lk}-ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\bar{v}_{ij}^{k}-\bar{v}_{ij}^{l}> b_j^l-(1-y_{ij}^{lk}+ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}\le b_j^l+(y_{ij}^{lk}+1-ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
-\bar{v}_{ij}^{k}+\bar{v}_{ij}^{l}> b_j^l-(y_{ij}^{lk}+ad_{ij}^{lk})M & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k\\
\frac{1}{mnr(r-1)}\sum_{l=1,l\ne k}^{r}\sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n} ad_{ij}^{lk}=AL* & \\
0\le \bar{v}_{ij}^{k}\le 1 & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ k=1,2,\cdots,r\\
y_{ij}^{lk}, ad_{ij}^{lk}\in\{0,1\} & i=1,2,\cdots,m;\ j=1,2,\cdots,n;\ l,k=1,2,\cdots,r;l\ne k
\end{cases} \quad (30)$$

Proof: The main part of the proof of Propositions 1 and 2 can be obtained from the proof of Lemma 1, so the detailed description is omitted.

E. Algorithm for the MOFMCM Model

Based on the above descriptions, the framework of MOFMCM model is detailed in Algorithm I. Due to the time limitation in the actual consensus process, the maximum number of iterations of Algorithm I usually does not exceed 5 rounds [27], [30]. Therefore, the key to solving the computational complexity of Algorithm I depends on its linear programming models. Dantzig [55] pointed out that the average time complexity of the simplex algorithm to solve linear programming model is $O(n)$, which shows that the average time complexity of our linear programming models using Dantzig's method is also $O(n)$.

Algorithm 1: MOFMCM model

Input: Initial decision matrices $\{V^1,V^2,\cdots,V^r\}$, individual bounded confidences $\{B^1,B^2,\cdots,B^r\}$, mutual acceptance threshold μ, decision makers' weights $\{\omega_1,\omega_2,\cdots,\omega_m\}$, attributes' weights $\{w_1,w_2,\cdots,w_n\}$.

Output: The ranking of alternatives.

Step 1: Let $t=0$ and $V^{k,t}=(v_{ij}^{k,t})_{m\times n}=(v_{ij}^{k})_{m\times n}$ $(k=1,2,\cdots,r)$.

Step 2: Using Eqs.(10)-(12), obtain the group acceptance level of decision make d_k $(k=1,2,\cdots,r)$ at round t $AL^t(d_k)=\sum_{l=1,l\ne k}^{r}AL^{lk\,t}\big/(r-1)$, where $AL^{lk,t}=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}ad_{ij}^{lk,t}}{mn}$ and $ad_{ij}^{lk,t}=\begin{cases}1 & \text{if } |v_{ij}^{k,t}-v_{ij}^{l,t}|\le b_j^l\\ 0 & \text{if } |v_{ij}^{k,t}-v_{ij}^{l,t}|>b_j^l\end{cases}$.

Step 3: Apply Eq. (10) to compute the group mutual acceptance at round t:

$AL^t = \sum_{k=1}^{r} AL^t(d_k) \big/ r$.

Step 4: If $AL^t < \mu$, i.e., AL^t at round t is unacceptable, then go to Step 5; otherwise, go to Step 9.

Step 5: Solve model (17) and obtain AL^t* . If $AL^t*=AL^t$, then go to Step 7; otherwise, continue Step 6.

Step 6: Solve model (20) and obtain $\bar{V}^{k,t}*$ $(k=1,2,\cdots,r)$. The recommended adjusted decision matrix $V^{k,t+1}=(v_{ij}^{k,t+1})_{m\times n}$ follow the rule: $v_{ij}^{k,t+1} \in [\min(v_{ij}^{k,t}, \bar{v}_{ij}^{k,t}*), \max(v_{ij}^{k,t}, \bar{v}_{ij}^{k,t}*)]$. Go to Step 8.

Step 7: Solve model (23) to obtain $\bar{V}^{k,t}**$ and $\bar{V}^{c,t}**$. Compute CL^t* based on Eq. (24). Solve model (26) to obtain $\bar{V}^{k,t}**'$ $(k=1,2,\cdots,r)$. The recommended adjusted decision matrices $V^{k,t+1}=(v_{ij}^{k,t+1})_{m\times n}$ follow the rule: $v_{ij}^{k,t+1} \in [\min(v_{ij}^{k,t}, \bar{v}_{ij}^{k,t}**'), \max(v_{ij}^{k,t}, \bar{v}_{ij}^{k,t}**')]$.

Step 8: Let $t=t+1$; and go back to Step 2.

Step 9: Apply Eq. (1) to aggregate $\{V^{1,t}, V^{2,t}, \cdots, V^{r,t}\}$ into $V^{c,t}=(v_{ij}^{c,t})_{m\times n}$, where $v_{ij}^{c,t}=\sum_{k=1}^{r}\omega_k v_{ij}^{k,t}$. Then, output the ranking of alternatives derived from the dominance value $Q_i=\sum_{j=1}^{n} w_j v_{ij}^{c,t}$.

IV. Experimental Analysis

A numerical analysis is included first in this section to illustrate the usage of the MOFMCM model. Then, simulation analysis I is proposed to study the influence of the bounded confidence on the MOFMCM model, while simulation analysis II is designed to compare the consensus effectiveness of the MOFMCM model and the general consensus reaching model (abbreviated as the GCR).

A. Numerical Analysis

A gearbox manufacturing enterprise needs EPM software to be supplied by one of four software suppliers $X=\{x_1,x_2,x_3,x_4\}$. A manager and five experts from different departments (information; project management; financial; planning; collaborators) use four qualitative attributes to evaluate and compare the four suppliers: after-sales service and training (a_1), core function (a_2), technical support level (a_3) and software cost (a_4). The weights associated with the experts and attributes are $\omega=\{1/4,1/4,\cdots,1/4\}$ and $w=\{0.15,0.25,0.2,0.4\}$, respectively. In this MAGDM problem, the mutual acceptance threshold is set as $\mu=0.8$, while the individual bounded confidences and initial decision matrices are listed in Tables 1 and 2, respectively. Using Eq. (1), the collective decision matrix of initial decision matrices, $V^{c,0}$, is obtained as shown in Table 3. Applying Eq. (5), we can obtain the dominance values of alternatives: $Q_1=0.513$, $Q_2=0.464$, $Q_3=0.512$, $Q_4=0.475$. Then, the corresponding initial alternative ordering is: $x_1 \succ x_3 \succ x_4 \succ x_2$.

Table 1. individual bounded confidences $B^k (k=1,2,\cdots,4)$

	a_1	a_2	a_3	a_4
B^1	0.36	0.35	0.30	0.32
B^2	0.35	0.40	0.43	0.45
B^3	0.30	0.34	0.28	0.27
B^4	0.05	0.20	0.15	0.13

Table 2. initial decision matrices $V^k (k=1,2,\cdots,4)$

	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	d_1				d_2			
x_1	0.45	0.90	0.70	0.75	0.65	0.20	0.20	0.60
x_2	0.65	0.50	0.45	0.80	0.10	0.30	0.50	0.55
x_3	0.80	0.25	0.20	0.80	0.20	0.60	0.25	0.20
x_4	0.90	0.15	0.90	0.65	0.30	0.50	0.85	0.45
	d_3				d_4			
x_1	0.50	0.55	0.40	0.45	0.30	0.50	0.65	0.30
x_2	0.30	0.15	0.30	0.55	0.50	0.70	0.10	0.45
x_3	0.70	0.65	0.55	0.75	0.35	0.70	0.75	0.35
x_4	0.65	0.40	0.55	0.45	0.95	0.15	0.30	0.10

Table 3. The collective decision matrix $V^{c,0}$

	a_1	a_2	a_3	a_4
x_1	0.48	0.54	0.49	0.53
x_2	0.39	0.41	0.34	0.59
x_3	0.51	0.55	0.44	0.53
x_4	0.70	0.30	0.65	0.41

(1) First round: Using Eqs. (10)-(13), the group mutual acceptance is obtained: $AL^1=0.57$. The experts are advised to adjust decision matrices since $AL^1<\mu$. Solving model (17), we have $AL^1*=0.98>AL^1$. Then, the feedback decision matrices $\bar{V}^{k,1}*$ listed in Table 4 are obtained by solving model (20).

Table 4. The feedback decision matrices $\bar{V}^{k,1}*$ at first round

	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	d_1				d_2			
x_1	0.45	0.65	0.65	0.60	0.55	0.35	0.45	0.60
x_2	0.50	0.50	0.30	0.75	0.25	0.40	0.30	0.55
x_3	0.60	0.50	0.35	0.65	0.35	0.60	0.55	0.33
x_4	0.90	0.15	0.70	0.33	0.65	0.15	0.73	0.45
	d_3				d_4			
x_1	0.50	0.55	0.45	0.45	0.30	0.50	0.55	0.30
x_2	0.30	0.40	0.30	0.55	0.50	0.55	0.20	0.45
x_3	0.60	0.65	0.55	0.65	0.35	0.65	0.65	0.35
x_4	0.65	0.30	0.50	0.45	0.90	0.15	0.40	0.33

(2) Second round: Based on the feedback recommendation, the decision matrices are adjusted to the ones of Table 5. The mutual acceptance at this round is still below the threshold: $AL^2=0.75<\mu$. Maximum mutual acceptance $AL^2*=1>AL^2$ is obtained by solving model (17). Solving model (23), we have the optimal solution $\bar{V}^{k,2}*$ of Table 6.

(3) Third round: At this round, the adjusted decision matrices $V^{k,3}$ are obtained and listed in Table 7 leading to a satisfactory mutual acceptance: $AL^3=0.802>\mu=0.8$. Applying Eq. (1), the collective decision matrix $V^{c,3}$ is obtained (Table 8), from which the dominance values of alternatives are obtained applying Eq. (5): $Q_1=0.497$, $Q_2=0.460$, $Q_3=0.543$,

$Q_4 = 0.412$. Then, we have the alternative ordering $x_3 \succ x_1 \succ x_2 \succ x_4$, and the gearbox manufacturing enterprise ultimately choose x_3 as its software supplier.

Table 5. The adjusted decision matrices $V^{k,2}$

	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	d_1				d_2			
x_1	0.45	0.70	0.66	0.63	0.57	0.32	0.40	0.60
x_2	0.53	0.50	0.33	0.76	0.22	0.38	0.34	0.55
x_3	0.64	0.45	0.32	0.68	0.32	0.60	0.49	0.30
x_4	0.90	0.15	0.74	0.39	0.58	0.22	0.75	0.45
	d_3				d_4			
x_1	0.50	0.55	0.43	0.45	0.30	0.50	0.63	0.30
x_2	0.30	0.31	0.30	0.55	0.50	0.67	0.12	0.45
x_3	0.64	0.65	0.55	0.69	0.35	0.69	0.73	0.35
x_4	0.65	0.34	0.52	0.45	0.94	0.15	0.32	0.15

Table 6. The feedback decision matrices $\overline{V}^{k,2}*$ at second round

	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	d_1				d_2			
x_1	0.45	0.65	0.63	0.60	0.55	0.35	0.43	0.60
x_2	0.50	0.50	0.32	0.75	0.25	0.38	0.32	0.55
x_3	0.63	0.45	0.53	0.68	0.38	0.60	0.53	0.36
x_4	0.90	0.15	0.52	0.39	0.65	0.22	0.52	0.45
	d_3				d_4			
x_1	0.50	0.55	0.43	0.45	0.30	0.50	0.53	0.30
x_2	0.30	0.38	0.30	0.55	0.50	0.53	0.22	0.45
x_3	0.64	0.65	0.55	0.68	0.38	0.60	0.63	0.38
x_4	0.65	0.30	0.52	0.45	0.90	0.15	0.42	0.15

Table 7. The adjusted decision matrices $V^{k,3}$

	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	d_1				d_2			
x_1	0.45	0.66	0.64	0.61	0.55	0.34	0.43	0.60
x_2	0.51	0.50	0.32	0.75	0.24	0.38	0.32	0.55
x_3	0.64	0.45	0.49	0.68	0.37	0.60	0.52	0.35
x_4	0.90	0.15	0.56	0.39	0.64	0.22	0.57	0.45
	d_3				d_4			
x_1	0.50	0.55	0.43	0.45	0.30	0.50	0.61	0.30
x_2	0.30	0.36	0.30	0.55	0.50	0.64	0.14	0.45
x_3	0.64	0.65	0.55	0.68	0.36	0.67	0.71	0.36
x_4	0.65	0.31	0.52	0.45	0.93	0.15	0.34	0.15

Table 8. The collective decision matrix $V^{c,3}$

	a_1	a_2	a_3	a_4
x_1	0.45	0.51	0.53	0.49
x_2	0.39	0.47	0.27	0.58
x_3	0.50	0.59	0.57	0.52
x_4	0.78	0.21	0.50	0.36

B. Simulation Analysis I

Simulation analysis I is designed to study the influence of the bounded confidence on the consensus effectiveness of the MOFMCM model. In this simulation, the decision matrices are randomly generated with initial values. Specifically, the settings and issues involved in Simulation analysis I are as follows:

(1) The bounded confidences are set within $[\delta_{\min}, \delta_{\max}]$, namely $b_j^k \in [\delta_{\min}, \delta_{\max}]$, with the aim of studying the influence of different levels of bounded confidence on consensus reaching.

(2) To automatically obtain the adjusted decision matrices without changing the essence of the MOFMCM model, Eqs. (21) and (27) are replaced with Eqs. (31) and (32), respectively.

$$v_{ij}^{k\prime} = \alpha_k v_{ij}^k + (1-\alpha_k)\overline{v}_{ij}^k * \; (i = 1, \cdots, m; j = 1, \cdots, n) \tag{31}$$

$$v_{ij}^{k\prime} = \alpha_k v_{ij}^k + (1-\alpha_k)\overline{v}_{ij}^k **' \; (i = 1, \cdots, m; j = 1, \cdots, n) \tag{32}$$

where $\alpha_k \in [0,1]$ is a randomly generated parameter to derive feedback decision matrices.

Let $r = 5$, and $T = 5$. We run Simulation analysis I 1000 times to obtain the mutual acceptance average value, AL^t , under different levels of $[\delta_{\min}, \delta_{\max}]$. The obtained results are shown in Fig. 3, from where it is observed that the values of AL^t increase when the values of interval $[\delta_{\min}, \delta_{\max}]$ increase (The same is true at $t = 0$). This indicates that the group will achieve a higher mutual acceptance level when the decision makers are more acceptable on the attributes.

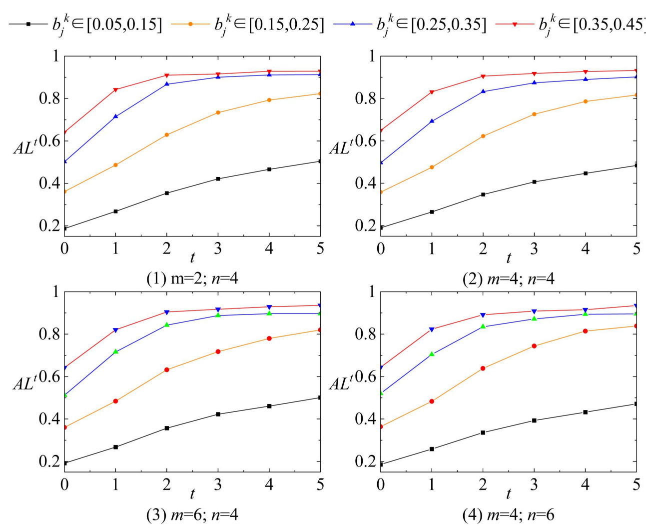

Fig. 3. The mutual acceptance AL^t under different levels of interval $[\delta_{\min}, \delta_{\max}]$

Simulation analysis I: The influence of bounded confidence on the MOFMCM model

Input: the number of decision makers r , the number of alternative m , the number of attributes n , the maximum number of rounds T , the bounded confidence interval $[\delta_{\min}, \delta_{\max}]$.

Output: the mutual acceptance in each round $AL^t \, (t = 0, 1, 2 \cdots)$.

Steps 2, 3, 5 and 8 are the same as those of Algorithm 1, and the other steps are described below.

Step 1: Uniformly and randomly generate v_{ij}^k of $V^k=(v_{ij}^k)_{m\times n}$ $(k=1,2,\cdots,r)$ from $[0,1]$ and $b_j^k(j=1,2,\cdots,n)$ from $[\delta_{\min},\delta_{\max}]$. Let $t=0$ and $V^{k,t}=(v_{ij}^{k,t})_{m\times n}=(v_{ij}^k)_{m\times n}$.

Step 4: If $t<T$, run Step 5; otherwise, run Step 9.

Step 6: Solve model (17) to derive optimal solution $\overline{V}^{k,t}*$. Based on Eq. (31), generate the adjusted decision matrices $V^{k,t+1}=(v_{ij}^{k,t+1})_{m\times n}$, where $v_{ij}^{k,t+1}=\alpha_k v_{ij}^{k,t}+(1-\alpha_k)\overline{v}_{ij}^{k,t}*$. Go to Step 8.

Step 7: Solve model (23) to obtain $\overline{V}^{k,t}**$ and $\overline{V}^{c,t}**$. Obtain CL^t* using Eq. (24). Solve model (26) to obtain $\overline{V}^{k,t}**'$. Use Eq. (31) to generate the adjusted decision matrix $V^{k,t+1}=(v_{ij}^{k,t+1})_{m\times n}$, where $v_{ij}^{k,t+1}=\alpha_k v_{ij}^{k,t}+(1-\alpha_k)\overline{v}_{ij}^{k,t}**'$.

Step 9: Output the mutual acceptance $AL^t(t=0,1,2\cdots)$.

C. Simulation Analysis II

Simulation analysis II is used to compare the consensus effectiveness of the MOFMCM model and the GCR model, in which the initial decision matrices are still randomly generated. And the effectiveness is represented by the following two indicators:

(1) The mutual acceptance level AL^t at each round;

(2) The preference adjustment PA^t at each round, as per the following expression: $PA^t=\sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}|v_{ij}^{k,t}-v_{ij}^{k,t-1}|$.

Simulation analysis II is a modified version of Simulation analysis I to obtain the preference adjustment PA^t, with modifications being:

(1) Add computation of the preference adjustment PA^t at Steps 6 and 7.

(2) Step 9 is modified to: Output the mutual acceptance AL^t and the preference adjustment PA^t.

Simulation experiment II' is used to analyze the effectiveness of the GCR model, which follows the framework of the general consensus process shown of Fig. 1. To align the GCR model with the MOFMCM model, IR of the GCR model identifies all decision makers. Simulation experiment II' is also a modified version of Simulation analysis I, by replacing Steps 4-9 with Steps 4'-6' below.

Steps 4': If $t<T$, continue Step 5'; otherwise, go to Step 6'.

Steps 5': Using Eq. (1) compute $V^{c,t}$, where $v_{ij}^{c,t}=\sum_{k=1}^{r}\omega_k v_{ij}^{k,t}$. Obtain the adjusted decision matrices $V^{k,t+1}=(v_{ij}^{k,t+1})_{m\times n}$, where

$$\begin{cases} v_{ij}^{k,t+1}=\alpha_k v_{ij}^{k,t}+(1-\alpha_k)v_{ij}^{c,t} & \text{if } b_j^k\geq|v_{ij}^{k,t}-v_{ij}^{c,t}| \\ v_{ij}^{k,t+1}=v_{ij}^{k,t} & \text{if } b_j^k<|v_{ij}^{k,t}-v_{ij}^{c,t}| \end{cases}.$$

Compute $PA^{t+1}=\sum_{k=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}|v_{ij}^{k,t+1}-v_{ij}^{k,t}|$. Let $t=t+1$; and go to Step 2.

Steps 6': Output the mutual acceptance AL^t and the preference adjustment PA^t.

Let $r=5$, $m=5$, $n=4$, and $T=5$. We run simulation analyses II and II' 1000 times to obtain the average values of the mutual acceptance AL^t and the preference adjustment PA^t under different bounded confidence intervals $[\delta_{\min},\delta_{\max}]$. The simulation results are shown in Figs. 4 and 5, from which the following observations are drawn:

(1) From Figs. 4(1), 4(2), 4(4), 5(1), 5(2), and 5(4), the average values for AL^t and PA^t of the GCR model are lower than those of the MOFMCM model. This is because that the feedback recommendations generated by the GCR model are unacceptable for decision makers with low bounded confidence levels. And this resulted in them not changing their preferences, and the level of group mutual acceptance hardly increased. On the contrary, the feedback recommendations of the MOFMCM model are accepted by the decision makers, thereby adjusting their preferences and increasing the level of group mutual acceptance.

(2) For a large bounded confidence interval $[\delta_{\min},\delta_{\max}]$ (see Fig. 4(3) and Fig. 5(3)), the average values for AL^t of the GCR model are lower than those of the MOFMCM model, while the average values for PA^t of the GCR model are greater than those of the MOFMCM model. Therefore, compared with the GCR, the MOFMCM can reach a higher level of mutual acceptance more efficiently.

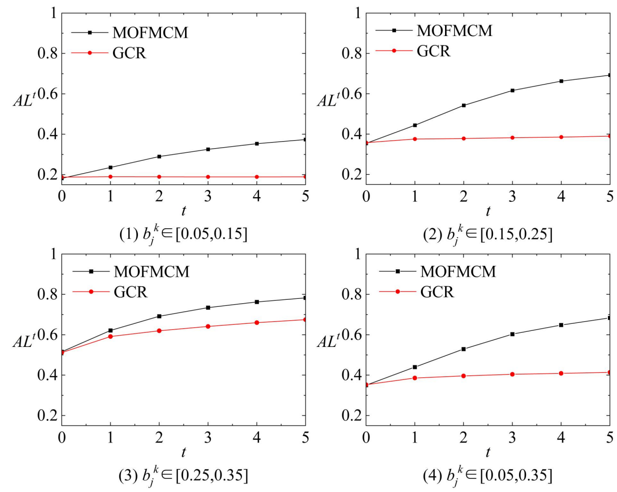

Fig. 4. Average AL^t values in MOFMCM and GCR models under different bounded confidence intervals $[\delta_{\min},\delta_{\max}]$

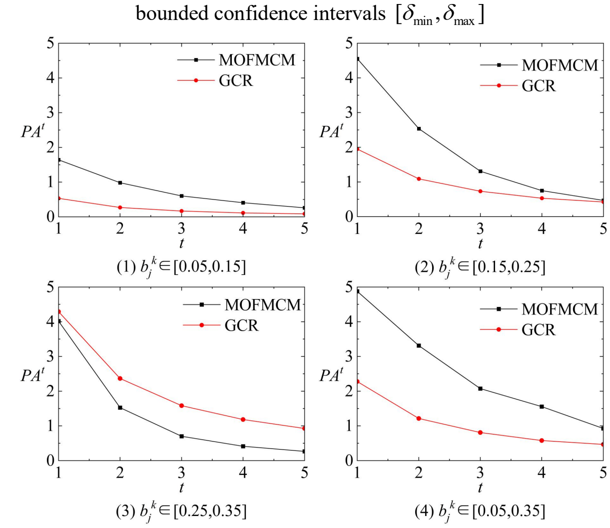

Fig. 5. Average PA^t values in MOFMCM and GCR models under different bounded confidence intervals $[\delta_{\min},\delta_{\max}]$

T is the number of iterations in simulation analysis, usually no more than 5 rounds in practical consensus processes. Based on the description of computational complexity before, we have that the average time complexity of our linear programming models in Simulation analyses I and II is $O(n)$ by the Dantzig's method [55]. Furthermore, computational complexity is generally considered in the worst case, in which, the GCR model can not converge to consensus, while the MOFMCM model can achieve. The reason is that the GCP model may always produces unacceptable feedback recommendations as shown in Fig .4(1), and the MOFMCM model produces acceptable feedback recommendations based on bounded confidence. In summary, the feedback recommendations of the MOFMCM model are easier to be accepted to promote the group mutual acceptance improvement.

V. Conclusion

This paper proposed a multi-stage optimization feedback mechanism based consensus model in MAGDM that aims to help decision makers improve the level of group mutual acceptance based on their individual bounded confidences. The MOFMCM model is based on the group consensus level being related to the level of mutual acceptance, and measures the consensus level from the perspective of the mutual acceptance based on individual bounded confidences. The multi-stage optimization feedback mechanism of MOFMCM considers the willingness of decision makers to accept recommendations from the perspective of bounded confidence. In comparison with the similarity based feedback mechanisms, the priority here is given to a two-stage optimization from the perspective of mutual acceptance. Furthermore, the effectiveness of the MOFMCM model is verified through simulation and comparison analyses.

The social network has been studied widely in GDM in recent years [1], [16], [17], [23], [25], [26], [39]. However, these studies rarely combine social networks with individual bounded confidences for research. Our future work will focus on investigating the evolution of the preferences in GDM, based not only on the bounded confidences of the decision makers, but also on the existence of social relationships between decision makers.

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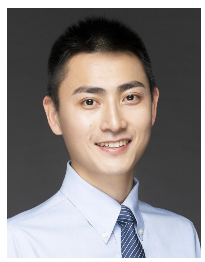

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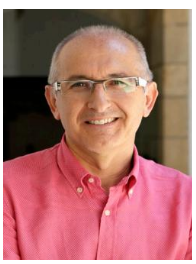

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