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Topology Control in Heterogeneous Wireless Networks: Problems and Solutions

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***Abstract*— Previous work on topology control usually assumes homogeneous wireless nodes with uniform transmission ranges. In this paper, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, each node selects a set of neighbors based on the locally collected information. We prove that (1) the topologies derived under DRNG and DLSS preserve the network connectivity; (2) the out degree of any node in the resulting topology by DLSS is bounded, while the out degree cannot be bounded in DRNG; and (3) the topologies generated by DRNG and DLSS preserve the network bi-directionality.**

I. Introduction

Energy efficiency [1] and network capacity are perhaps two of the most important issues in wireless ad hoc networks and sensor networks. Topology control algorithms have been proposed to maintain network connectivity while reducing energy consumption and improving network capacity. The key idea to topology control is that, instead of transmitting using the maximal power, nodes in a wireless multi-hop network collaboratively determine their transmission power and define the network topology by forming the proper neighbor relation under certain criteria.

By enabling wireless nodes to use adequate transmission power (which is usually much smaller than the maximal transmission power), topology control can not only save energy and prolong network lifetime, but also improve spatial reuse (and hence the network capacity) [2] and mitigate the MAC-level medium contention [3]. Several topology control algorithms [3]–[10] have been proposed to create power-efficient network topology in wireless multi-hop networks with limited mobility (a summary is given in Section III). However, most of them assume homogeneous wireless nodes with uniform transmission ranges (except [4]).

The assumption of homogeneous nodes does not always hold in practice, since even devices of the same type may have slightly different maximal transmission power. There also exist heterogeneous wireless networks in which devices have dramatically different capabilities, for instance, the communication network in the *Future Combat System* which involves wireless devices on soldiers, vehicles and UAVs. As will be exemplified in Section III, most existing algorithms cannot be directly applied to heterogeneous wireless multi-hop networks in which the transmission range of each node may be different.

To the best of our knowledge, this paper is the first effort to address the connectivity and bi-directionality issue in the heterogeneous wireless networks.

In this paper, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, the topology is constructed by having each node build its neighbor set and adjust its transmission power based on the locally collected information.

We are able to prove that (1) the topology derived under both DRNG and DLSS preserves network connectivity, i.e., if the original topology generated by having every node use its maximal transmission power is strongly connected, then the topologies generated by both DRNG and DLSS are also strongly connected; (2) the out degree of any node in the topology by DLSS is bounded, while the out degree of nodes in the topology by DRNG may be unbounded; and (3) the topology generated by DRNG and DLSS preserves network bi-directionality, i.e., if the original topology by having every node use its maximal transmission power is bi-directional, then the topology generated by either DRNG or DLSS is also bi-directional after some simple operations.

Simulation results indicate that, compared with the other known topology control algorithms that can be applied to heterogeneous networks, DRNG and DLSS have smaller average node degree (both logical and physical) and smaller average link length. The former reduces the MAC-level contention, while the latter implies a small transmission power needed to maintain connectivity.

The rest of the paper is organized as follows. In Section II, we give the network model. In Section III, we summarize previous work on topology control, and give examples to show why existing algorithms cannot be directly applied to heterogeneous networks. Following that, we present both the DRNG and DLSS algorithms in Section IV, and prove several of their useful properties in Section V. Finally, we evaluate the performance of the proposed algorithms in Section VI, and conclude the paper in Section VII.

II. Network Model

Consider a set of nodes(vertices), $V = \{v_1, v_2, \ldots, v_n\}$, which are randomly distributed in the 2-D plane. Assume the area that a transmission can cover is a disk. We define the range of a node v_i as the radius of the disk that v_i can cover using its maximal transmission power, denoted r_{v_i}. In a heterogeneous network, the transmission ranges of all nodes may not be the same. Let $r_{min} = \min_{v \in V}\{r_v\}$ and $r_{max} = \max_{v \in V}\{r_v\}$.

We denote the network topology generated by having each node use its own maximal transmission power as a simple directed graph $G = (V(G), E(G))$, where $E(G) = \{(u, v) : d(u, v) \leq r_u, u, v \in V(G)\}$ is the edge(link) set of G, and $d(u, v)$ is the Euclidean distance between node u and node v. Note that (u, v) is an ordered pair representing an edge from node u to node v, i.e., (u, v) and (v, u) are two different edges. A unique *id* (such as an IP/MAC address) is assigned to each node. Here we let $id(v_i) = i$ for simplicity.

We assume that the wireless channel is symmetric and obstacle-free, and each node is equipped with the capability to gather its location information via, for example, GPS for outdoor applications and pseudolite [11] for indoor applications, and many other lightweight localization techniques for wireless networks (see [12] for a summary).

Before delving into the technical discussion and algorithm description, we give the definition of several terms that will be used throughout the paper.

Definition 1 (Reachable Neighborhood): The reachable neighborhood N_u^R is the set of nodes that node u can reach using its maximal transmission power, i.e., $N_u^R = \{v \in V(G) : d(u, v) \leq r_u\}$. For each node $u \in V(G)$, let $G_u^R = (V(G_u^R), E(G_u^R))$ be an induced subgraph of G such that $V(G_u^R) = N_u^R$.

Definition 2 (Weight Function): Given two edges $(u_1, v_1), (u_2, v_2) \in E$ and the Euclidean distance function $d(\cdot, \cdot)$, weight function $w : E \mapsto R$ satisfies:

$$
\begin{aligned}
& w(u_1, v_1) > w(u_2, v_2) \\
\Leftrightarrow \quad & d(u_1, v_1) > d(u_2, v_2) \\
\text{or} \quad & (d(u_1, v_1) = d(u_2, v_2) \\
& \quad \&\& \ \max\{id(u_1), id(v_1)\} > \max\{id(u_2), id(v_2)\}) \\
\text{or} \quad & (d(u_1, v_1) = d(u_2, v_2) \\
& \quad \&\& \ \max\{id(u_1), id(v_1)\} = \max\{id(u_2), id(v_2)\} \\
& \quad \&\& \ \min\{id(u_1), id(v_1)\} > \min\{id(u_2), id(v_2)\}).
\end{aligned}
$$

This weight function ensures that two edges with different end-vertices have different weights. Note, however, that $w(u, v) = w(v, u)$.

Definition 3 (Neighbor Set): Node v is a *neighbor* of node u under an algorithm A, denoted $u \xrightarrow{A} v$, if and only if there exists an edge (u, v) in the topology generated by the algorithm. In particular, we use $u \rightarrow v$ to denote the neighbor relation in G. $u \xleftrightarrow{A} v$ if and only if $u \xrightarrow{A} v$ and $v \xrightarrow{A} u$. The *Neighbor Set* of node u is $N_A(u) = \{v \in V(G) : u \xrightarrow{A} v\}$.

Definition 4 (Topology): The topology generated by an algorithm A is a directed graph $G_A = (E(G_A), V(G_A))$, where $V(G_A) = V(G)$, $E(G_A) = \{(u, v) \in E(G) : u \xrightarrow{A}\}$.

Definition 5 (Radius): The radius, R_u, of node u is defined as the distance between node u and its farthest neighbor (in terms of Euclidean distance), i.e, $R_u = \max_{v \in N_A(u)}\{d(u, v)\}$.

Definition 6 (Connectivity): For any topology generated by an algorithm A, node u is said to be *connected to* node v (denoted $u \Rightarrow v$) if there exists a path $(p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)$ such that $p_i \xrightarrow{A} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_A), k = 0, 1, \ldots, m$. It follows that $u \Rightarrow v$ if $u \Rightarrow p$ and $p \Rightarrow v$ for some $p \in V(G_A)$.

Definition 7 (Bi-Directionality): A topology generated by an algorithm A is *bi-directional*, if for any two nodes $u, v \in V(G_A)$, $u \in N_A(v)$ implies $v \in N_A(u)$. In other words, the topology generated by A is bi-directional if all edges in the topology are bi-directional.

Definition 8 (Bi-Directional Connectivity): For any topology generated by an algorithm A, node u is said to be *bi-directionally connected to* node v (denoted $u \Leftrightarrow v$) if there exists a path $(p_0 = u, p_1, \ldots, p_{m-1}, p_m = v)$ such that $p_i \xleftrightarrow{A} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_A), k = 0, 1, \ldots, m$. It follows that $u \Leftrightarrow v$ if $u \Leftrightarrow p$ and $p \Leftrightarrow v$ for some $p \in V(G_A)$.

Deriving network topology consisting of only bi-directional links facilitates link level acknowledgment, which is a critical operation for packet transmissions and retransmissions over unreliable wireless media. Bi-directionality is also important in floor acquisition mechanisms such as the RTS/CTS mechanism in IEEE 802.11.

Definition 9 (Addition and Removal): The operation *Addition* is to add an extra edge (v, u) into G_A if $(u, v) \in E(G_A)$, $(v, u) \notin E(G_A)$, and $d(u, v) \leq r_v$. The operation *Removal* is to delete any edge $(u, v) \in E(G_A)$ if $(v, u) \notin E(G_A)$. Let G_A^+ and G_A^- denote the resulting topologies after applying *Addition* and *Removal* to G_A, respectively.

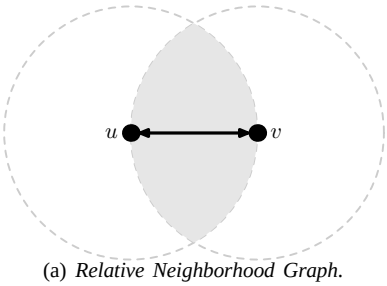
Both the *Addition* and *Removal* operations attempt to create a bi-directional topology by removing uni-directional edges or converting uni-directional edges into bi-directional. The resulting topology after *Removal* is alway bi-directional, although it may not be strongly connected. The resulting topology after *Addition* is not necessarily bi-directional, as it essentially tries to increases the transmission power of a node v to a level that may be beyond its capability.

III. Related Work and Why They Cannot be Directly Applied to Heterogeneous Networks

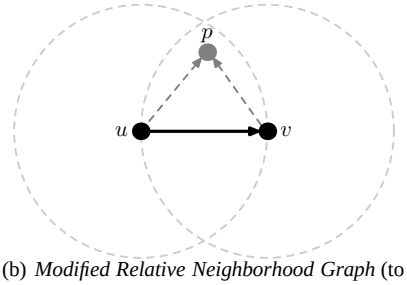
Several topology control algorithms [3]–[10] have been proposed. In this section, we first summarize these algorithm and then give examples on why they cannot be directly applied to heterogeneous networks.

A. Related Work

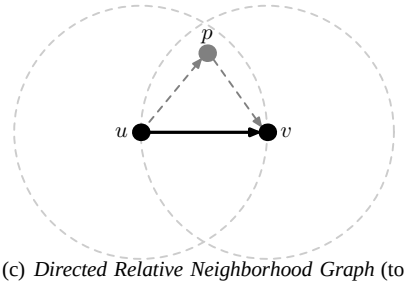
Rodoplu *et al.* [4] (denoted R&M) introduced the notion of *relay region* and *enclosure* for the purpose of power control. Instead of transmitting directly, a node chooses to



(a) *Relative Neighborhood Graph.*



(b) *Modified Relative Neighborhood Graph* (to be defined in Section III-B).



(c) *Directed Relative Neighborhood Graph* (to be defined in Section IV).

Fig. 1. The definition of the *Directed Relative Neighborhood Graph*.

relay through other nodes if less power will be consumed. It is shown in the paper that the network is strongly connected if every node maintains links with the nodes in its enclosure and the resulting topology is a minimum power topology. The major drawback is that it requires an explicit propagation channel model to compute the relay region (in the simulation study presented in Section VI, we assume that the free-space model is used), hence the resulting topology is sensitive to the model used in the computation. Also, it assumes there is only one data sink (destination) in the network.

Ramanathan *et al.* [5] presented two centralized algorithms to minimize the maximal power used per node while maintaining the (bi)connectivity of the network. They introduced two distributed heuristics for mobile networks. Both centralized algorithms require global information, and thus cannot be directly deployed in the case of mobility. On the other hand, the proposed heuristics cannot guarantee the preservation of the network connectivity.

COMPOW [3] and *CLUSTERPOW* [7] are approaches implemented in the network layer. Both hinge on the idea that if each node uses the smallest common power required to maintain network connectivity, the traffic carrying capacity of the entire network is maximized, the battery life is extended, and the MAC-level contention is mitigated. The major drawback is its significant message overhead, since each node has to run multiple daemons, each of which has to exchange link state information with their counterparts at other nodes.

CBTC(α) [6] is a two-phase algorithm in which each node finds the minimum power p such that some node can be reached in every cone of degree α. The algorithm has been proved to preserve network connectivity if $\alpha < 5\pi/6$. Several optimization methods (that are applied after the topology is derived under the base algorithm) are also discussed to further reduce the transmitting power.

To facilitate the following discussion, the definition of the *Relative Neighborhood Graph* (RNG) is given below.

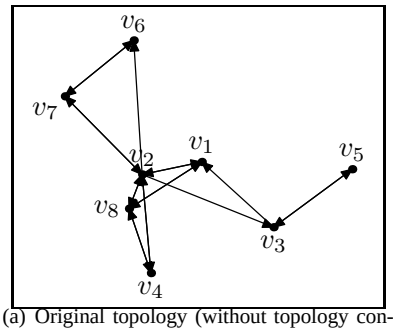
Definition 10 (Neighbor Relation in RNG): For RNG [13], [14], $u \overset{RNG}{\longleftrightarrow} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Or equivalently, there is no node inside the shaded area in *Fig. 1(a)*.

Borbash and Jennings [8] proposed to use RNG for the topology initialization of wireless networks. Based on the local knowledge, each node makes decisions to derive the network topology based on RNG. The network topology thus derived has been reported to exhibit good overall performance in terms of power usage, low interference, and reliability.

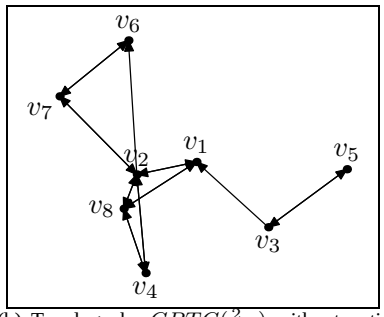
Li *et al.* [9] presented the Localized Delaunay Triangulationa, a localized protocol that constructs a planar spanner of the *Unit Disk Graph* (UDG). The topology contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. It is proved that the shortest path in this topology between any two nodes u and v is at most a constant factor of the shortest path connecting u and v in UDG. However, the notion of UDG and Delaunay triangulation cannot be directly extended to heterogeneous networks.

In [10], we proposed LMST (Local Minimum Spanning Tree) for topology control in homogeneous wireless multihop networks. In this algorithm, each node builds its local minimum spanning tree independently and only keeps on-tree nodes that are one-hop away as its neighbors in the final topology. It is proved that (1) the topology derived under LMST preserves the network connectivity; (2) the node degree of any node in the resulting topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links. Simulation results show that LMST can increase the network capacity as well as reduce the energy consumption.

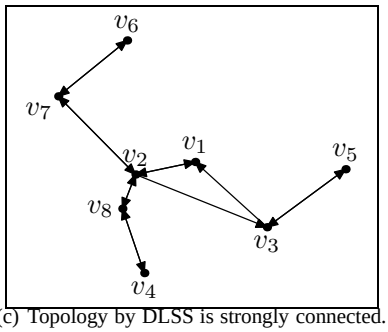
Instead of adjusting the transmission power of individual devices, there also exist other approaches to generate power-efficient topology. By following a probabilistic approach, Santi *et al.* derived the suitable common transmission range which preserves network connectivity, and established the lower and upper bounds on the probability of connectedness [15]. In [16], a "backbone protocol" is proposed to manage large wireless ad hoc networks, in which a small subset of nodes is selected to construct the backbone. In [17], a method of calculating the power-aware connected dominating sets was proposed to



(a) Original topology (without topology control) is strongly connected.

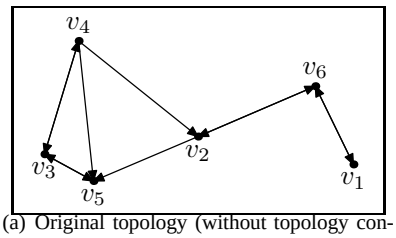


(b) Topology by $CBTC(\frac{2}{3}\pi)$ without optimization is not strongly connected: there is no path from v_1 to v_3.

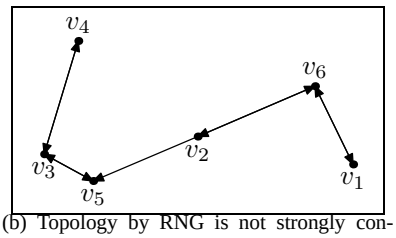


(c) Topology by DLSS is strongly connected.

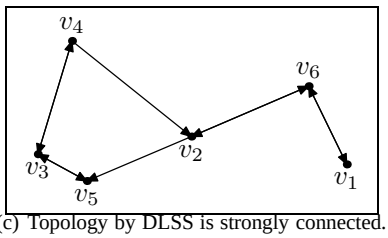
Fig. 2. An example that shows $CBTC(\frac{2}{3}\pi)$ may render disconnectivity in heterogeneous networks. There is no path from v_1 to v_3 due to the loss of edge (v_2,v_3), which is discarded by v_2 since v_1 and v_4 have already provided the necessary coverage.



(a) Original topology (without topology control) is strongly connected.

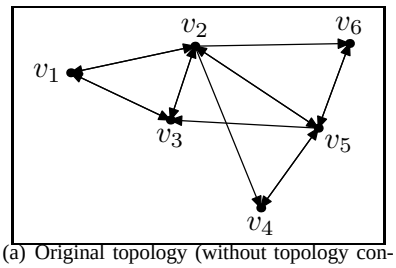


(b) Topology by RNG is not strongly connected: there is no path from v_5 to v_2.

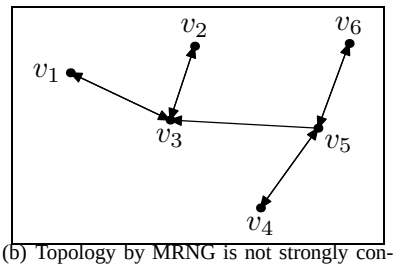


(c) Topology by DLSS is strongly connected.

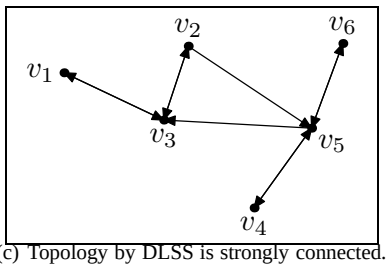
Fig. 3. An example that shows RNG may render disconnectivity in heterogeneous networks. There is no path from v_5 to v_2 due to the loss of edge (v_4,v_2), which is discarded since $|(v_4, v_5)| < |(v_4, v_2)|$, and $|(v_2, v_5)| < |(v_4, v_2)|$.



(a) Original topology (without topology control) is strongly connected.

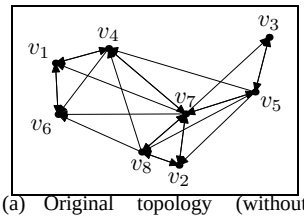


(b) Topology by MRNG is not strongly connected: there is no path from v_3 to v_5.

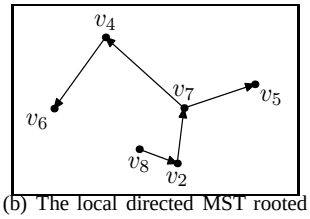


(c) Topology by DLSS is strongly connected.

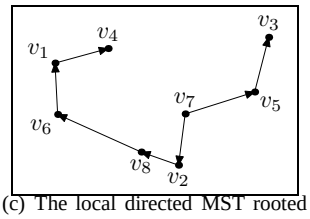
Fig. 4. An example that shows MRNG may render disconnectivity in heterogeneous networks. There is no path from v_3 to v_5 due to the loss of edge (v_2,v_5), which is discarded since $|(v_2, v_3)| < |(v_2, v_5)|$, and $|(v_5, v_3)| < |(v_2, v_5)|$.



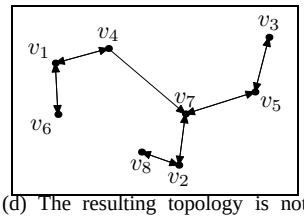
(a) Original topology (without topology control) is strongly connected.



(b) The local directed MST rooted at v_8.



(c) The local directed MST rooted at v_7.



(d) The resulting topology is not strongly connected: there is no path from v_7 to v_4.

Fig. 5. An example that shows the algorithm in which each node builds a local directed minimum spanning tree and only keeps the one-hop neighbors may result in disconnectivity.

establish an underlying topology for the network.

B. Why Existing Algorithms Cannot be Directly Applied to Heterogeneous Networks

Most existing topology control algorithms (except [4]) assume homogeneous wireless nodes with uniform transmission ranges. When directly applied to heterogeneous networks, these algorithms may render disconnectivity. In this subsection, we give several examples to motivate the need for new topology control algorithms for heterogeneous networks.

As shown in *Fig. 2* (a)-(b) (note that in Figs. 2–5 we use arrows to indicate the direction of the links to represent a link from u to v), the network topology derived under $CBTC(\frac{2}{3}\pi)$ (without optimization) may not preserver the connectivity, when the algorithm is directly applied to a heterogeneous network. $CBTC(\frac{5}{6}\pi)$ also has the same problem.

Similarly we show in *Fig. 3* (a)-(b) that the network topology derived under RNG may be disconnected when the algorithm is directly applied to a heterogeneous network. As RNG is defined for undirected graphs, one may tailor the definition of RNG for directed graphs.

Definition 11 (Neighbor Relation in MRNG): For *Modified Relative Neighborhood Graph* (MRNG), $u \xrightarrow{MRNG} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v), d(u,p) \leq r_u$ and $w(p,v) < w(u,v), d(v,p) \leq r_v$ (Fig. 1(b)).

As shown in Fig. 4 (a)-(b), the topology derived under MRNG may still be disconnected (we will give another variation of RNG for directed graphs in the next section).

One possible extension of LMST [10] is for each node to build a local *directed* minimum spanning tree [18]–[20] and keep only neighbors within one hop. Unfortunately, the resulting topology does not preserve the strong connectivity, as shown in *Fig. 5*. In the next section, we will improve on this approach to preserve the connectivity.

IV. DRNG AND DLSS

In this section, we propose two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS). In both algorithms, the topology is derived by having each node build its neighbor set and adjust its transmission power based on locally collected information. Several nice properties of both algorithms will be discussed in Section V.

Both algorithms are composed of three phases:

1) *Information Collection*: each node collects the local information of neighbors such as position and *id*, and identifies the *Reachable Neighborhood* N^R.
2) *Topology Construction*: each node defines (in compliance with the algorithm) the proper list of neighbors for the final topology using the information in N^R.
3) *Construction of Topology with Only Bi-Directional Links* (Optional): each node adjusts its list of neighbors to make sure that all the edges are bi-directional.

A. Information collection

The information needed by each node u for topology control is the information of its reachable neighborhood N^R. This can be obtained locally, in the case of homogeneous networks, by having each node broadcast periodically a Hello message using its maximal transmission power. The information contained in a Hello message should at least include the node *id* and the position of the node. These periodic messages can be sent either in the data channel or in a separate control channel. In heterogeneous networks, having each node broadcast a Hello message using its maximal transmission power may be insufficient. For example, as shown in Fig. 6, v_1 is unable to know the position of v_4 since v_4 cannot reach v_1. We will treat this issue rigorously in Section V-D. For the time being, we assume that by the end of the first phase every node u obtains its N_u^R.

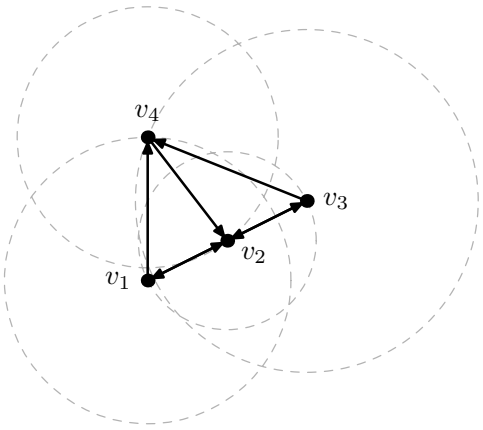


Fig. 6. An example that shows having each node broadcast a Hello message using its maximal transmission power may be insufficient for some nodes (e.g., node v_1) to know their reachable neighborhood. This figure also serves to show that given an arbitrary direct graph, it may be impossible to derive a bi-directional topology.

B. Topology construction

First we define the neighbor relation used in both algorithms.

Definition 12 (Neighbor Relation in DRNG): For *Directed Relative Neighborhood Graph* (DRNG), $u \xrightarrow{DRNG} v$ if and only if $d(u,v) \leq r_u$ and there does not exist a third node p such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v), d(p,v) \leq r_p$ (see Fig. 1(c)).

Definition 13 (Neighbor Relation in DLSS): For *Directed Local Spanning SubGraph* (DLSS), $u \xrightarrow{DLSS} v$ if and only if $(u,v) \in E(T_u)$, where T_u is obtained by applying Algorithm 1 to G_u^R. T_u is a directed local spanning subgraph that spans N_u^R. Hence node v is a neighbor of node u if and only if node v is on node u's directed local MST T_u, and is one-hop away from node u.

DLSS is a natural extension of LMST [10] for heterogeneous networks. Instead of computing a directed local MST (which minimizes the total cost of the all edges in the subgraph, and is shown to be wrong in Section III-B),

Algorithm 1 DLSS(u)

INPUT: G_u^R, the induced subgraph of G that spans the reachable neighborhood of u;

OUTPUT: $T_u = (V_{T_u}, E_{T_u})$, a local spanning subgraph of G_u^R;

```
1: V_{T_u} := V, E_{T_u} := ∅;
2: sort all edges in E(G_u^R) in the ascending order of weight (as defined in Definition 2);
3: for each edge (u, v) in the order do
4:    if u is not connected to v in T_u then
5:       E_{T_u} := E_{T_u} ∪ {(u, v)};
6:    end if
7:    if u is connected to all nodes in N_u^R then
8:       exit;
9:    end if
10: end for
```

each node u computes a directed local subgraph according to Algorithm 1 (which minimizes the maximum cost of all edges in the subgraph) and takes on-tree nodes that are one-hop away as its neighbors.

Each node can broadcast its own maximal transmission power in the Hello message. By measuring the receiving power of Hello messages, each node u can determine the specific power level required to reach each of its neighbors [10]. Node u then uses the power level that can reach its farthest neighbor as its transmission power. This approach can be applied without knowing the actual propagation model.

C. Construction of topology with only bi-directional edges

As illustrated in the previous section, some links in G_{DLSS} may be uni-directional. There also exist uni-directional links in G_{DRNG}. We can apply either *Addition* or *Removal* to G_{DLSS} and G_{DRNG} to obtain bi-directional topologies. We will discuss some properties of these solutions in Section V-B.

V. Properties of DRNG and DLSS

In this section, we discuss the connectivity, bi-directionality and degree bound of DLSS and DRNG. We always assume G is strongly connected, i.e., $u \Rightarrow v$ in G for any $u, v \in V(G)$.

A. Connectivity

Lemma 1: For any edge $(u, v) \in E(G)$, we have $u \Rightarrow v$ in G_{DLSS}.

Proof: Let all the edges $(u, v) \in E(G)$ be sorted in the increasing order of weight, i.e., $w(u_1, v_1) < w(u_2, v_2) < \ldots < w(u_l, v_l)$, where l is the total number. We prove by induction.

1) *Basis*: The first edge (u_1, v_1) satisfies $w(u_1, v_1) = \min_{(u,v)\in E(G)}\{w(u, v)\}$. According to Algorithm 1, (u_1, v_1) and (v_1, u_1) will be inserted into G_{DLSS}, i.e., $u_1 \xleftrightarrow{DLSS} v_1$.
2) *Induction*: Assume the hypothesis holds for all edges $(u_i, v_i), 1 \le i < k$, we prove $u_k \Rightarrow v_k$ in G_{DLSS}. If $u_k \xrightarrow{DLSS} v_k$, then $u_k \Rightarrow v_k$. Otherwise in the local topology construction of u, before edge (u_k, v_k) was inserted into T_{u_k}, there must already exist a path $p = (w_0 = u_k, w_1, w_2, \cdots, w_{m-1}, w_m = v_k)$ from u_k to v_k, where $(w_i, w_{i+1}) \in E(T_{u_k}), i = 0, 1, \cdots, m-1$. Since edges are inserted in a ascending order of weight, we have $w(w_i, w_{i+1}) < w(u_k, v_k)$. Applying the induction hypothesis to each pair $[w_i, w_{i+1}], i = 0, 1, \cdots, m-1$, we have $w_i \Rightarrow w_{i+1}$, thus $u_k \Rightarrow v_k$.

■

Theorem 1: G_{DLSS} preserves the connectivity of G, i.e., G_{DLSS} is strongly connected if G is strongly connected.

Proof: Suppose G is strongly connected. For any two nodes $u, v \in V(G)$, there exists at least one path $p = (w_0 = u, w_1, w_2, \cdots, w_{m-1}, w_m = v)$ from u to v, where $(w_i, w_{i+1}) \in E(G), i = 0, 1, \cdots, m-1$. Since $w_i \Rightarrow w_{i+1}$ by Lemma 1, we have $u \Rightarrow v$.

■

Lemma 2: Given three nodes $u, v, w \in V(G_{DLSS})$ satisfying $w(u, v) > w(u, w)$ and $w(u, v) > w(w, v)$, $d(w, v) \le r_w$, then $u \nrightarrow v$ in G_{DLSS}.

Proof: We only need to consider the case where $d(u, v) \le r_u$ since $d(u, v) > r_u$ would imply $u \nrightarrow v$. Consider the local topology construction of u. Before we insert (u, v) into T_u, the two edges (u, p) and (p, v) have already been processed since $w(u, p) < w(u, v)$ and $w(p, v) < w(u, v)$. Thus $u \Rightarrow p$ and $p \Rightarrow v$, which means $u \Rightarrow v$. Therefore, (u, v) should not be inserted into T_u according to Algorithm 1, i.e., $u \nrightarrow v$ in G_{DLSS}.

■

Theorem 2: The edge set of G_{DLSS} is a subset of the edge set of G_{DRNG}, i.e., $E(G_{DLSS}) \subseteq E(G_{DRNG})$.

Proof: We prove by contradiction. Given any edge $(u, v) \in E(G_{DLSS})$, assume $(u, v) \notin E(G_{DRNG})$. According to the definition of $DRNG$, there must exist a third node p such that $w(u, p) < w(u, v), d(u, p) \le r_u$ and $w(p, v) < w(u, v), d(p, v) \le r_p$. By Lemma 2, $u \nrightarrow v$ in G_{DLSS}, i.e., $(u, v) \notin E(G_{DLSS})$.

■

Theorem 3 (Connectivity of DRNG): If G is strongly connected, then G_{DRNG} is also strongly connected.

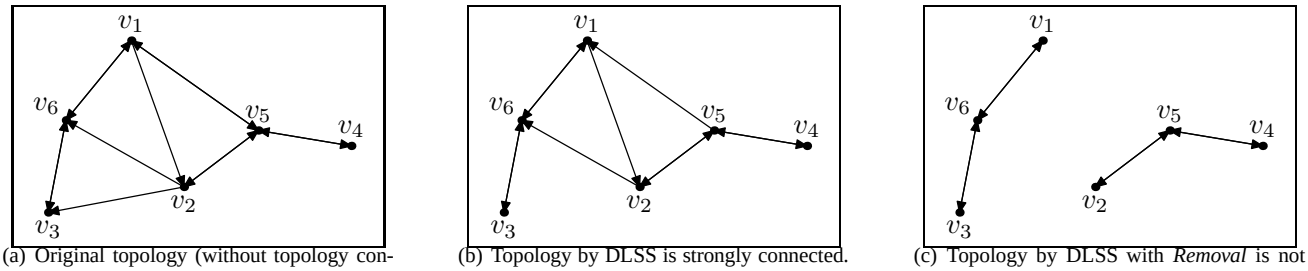
Proof: This is a direct result of *Theorem 1* and *Theorem 2*.

■

B. Bi-directionality

Now we discuss the bi-directionality property of DLSS and DRNG. Since *Addition* may not always result in bi-directional topologies, we first apply *Removal* to topologies by DLSS and DRNG. It turns out the simple *Removal* operation may lead to disconnectivity. Examples are given in Figs. 7–8 to show, respectively, that DLSS and DRNG with *Removal* may result in disconnectivity.

In general, G may not be bi-directional if the transmission ranges are non-uniform. Since the maximal transmission range can not be increased, it may be impossible to find a bi-directional connected subgraph of G for some cases. An

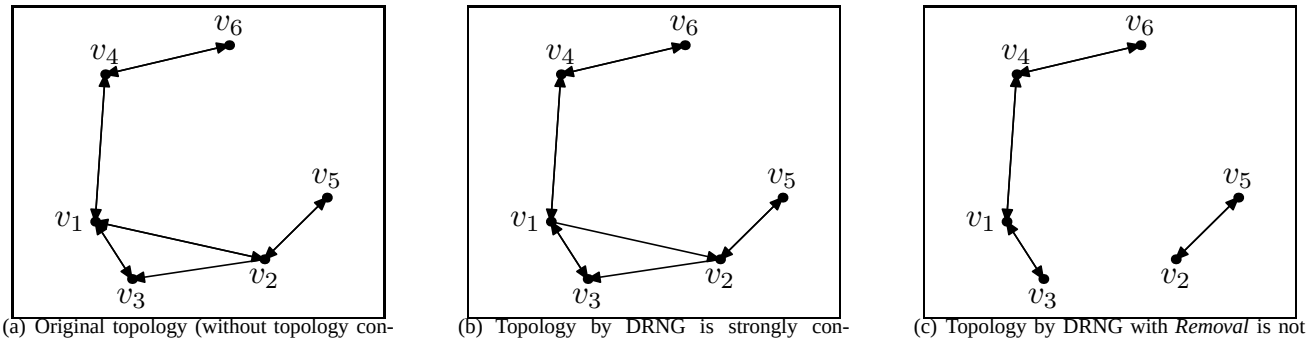


(a) Original topology (without topology control) is strongly connected.

(b) Topology by DLSS is strongly connected.

(c) Topology by DLSS with *Removal* is not strongly connected: there are 2 components.

Fig. 7. An example that shows DLSS with *Removal* may result in disconnectivity.



(a) Original topology (without topology control) is strongly connected.

(b) Topology by DRNG is strongly connected.

(c) Topology by DRNG with *Removal* is not strongly connected: there are 2 components.

Fig. 8. An example that shows DRNG with *Removal* may result in disconnectivity.

example is given in *Fig. 6*: v_1 can reach v_2 and v_4, v_2 can reach v_1 and v_3, v_3 can reach v_2 and v_4, and v_4 can reach v_2 only. *Addition* does not lead to bi-directionality since all edges entering or leaving v_4 are uni-directional with all nodes already transmitting with their maximal power. On the other hand, *Removal* will partition the network. In this example, although the graph G is strongly connected, its subgraph with the same vertex set cannot be both connected and bi-directional.

Now we show that bi-directionality can be ensured if the original topology is both strongly connected and bi-directional.

Theorem 4: If the original topology G is strongly connected and bi-directional, then G_{DLSS} and G_{DRNG} are also strongly connected and bi-directional after *Addition* or *Removal*.

Proof: Since $E(G_{DLSS}) \subseteq E(G_{DRNG})$, we have $E(G^-_{DLSS}) \subseteq E(G^+_{DLSS})$ and $E(G^-_{DLSS}) \subseteq E(G^-_{DRNG}) \subseteq E_(G^+_{DRNG})$. Therefore, we only need to prove that G^-_{DLSS} preserves the strong connectivity.

In the *Induction* step in Lemma 1, the only reason we cannot prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$ is that edge (v_k, u_k) may not exist. Given that G is bi-directional, we are able to prove that $u_k \overset{DLSS}{\longleftrightarrow} v_k$. Hence for any edge $(u, v) \in E(G)$, we have $u \Leftrightarrow v$ in G_{DLSS}. The removal of asymmetric edges in G_{DLSS} does not affect this property. Therefore, G^-_{DLSS} is still strongly connected.

■

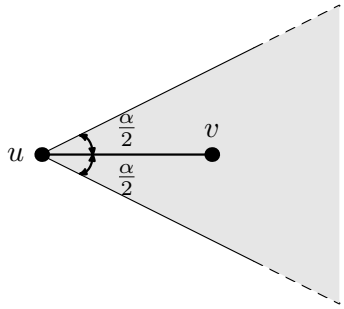


Fig. 9. The definition of $Cone(u, \alpha, v)$.

C. Degree Bound

It has been observed that any minimum spanning tree of a simple undirected graph in the plane has a maximum node degree of 6 [21]. However, this bound does not hold for directed graphs. An example is shown in Fig. 10, where node u has 18 neighbors. In this section, we will discuss the node degree in the topology by DLSS and DRNG.

Definition 14 (Disk): $Disk(u, r)$ is the disk centered at node u with a radius of r.

Definition 15 (Cone): $Cone(u, \alpha, v)$ is the unbounded shaded region shown in Fig. 9.

The following corollary is a direct result of Lemma 1.

Corollary 1: If v is a neighbor of u's in G_{DLSS}, and $d(u, v) \geq r_{min}$, then u can not have any other neighbor inside $Disk(v, r_{min})$.

Theorem 5: For any node $u \in V(G_{DLSS})$, the number of

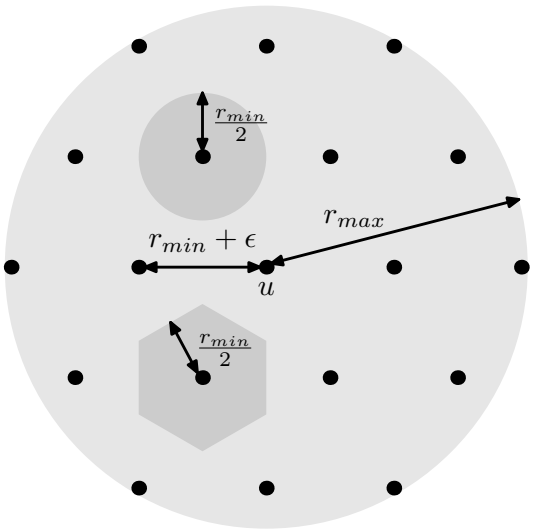


Fig. 10. An example that shows the out degree in a heterogenous network can be very large. The transmission range of u is r_{max} and the transmission range for all other nodes is r_{min}, where $r_{max} = 2(r_{min} + \epsilon)$, $\epsilon > 0$. All nodes are so arranged that the distance between any node and its closest neighbor is $r_{min} + \epsilon$. Therefore, the only links in the network are those from u to all the other nodes. Since relaying packets is impossible, u has to use its maximal transmission power and keeps all 18 neighbors.

neighbors in G_{DLSS} that are inside $Disk(u, r_{min})$ is at most 6.

Proof: Let $N(u)$ be the set of neighbors of u in G_{DLSS} that are inside $Disk(u, r_{min})$. Let the nodes in $N(u)$ be ordered such that for the ith node w_i and the jth node w_j ($j > i$), $w(u, w_j) > w(u, w_i)$. By Lemma 2, we have $w(u, w_j) \leq w(w_i, w_j)$ (otherwise $u \nrightarrow w_j$). Thus $\angle w_i u w_j \geq \pi/3$, i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have neighbors other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N(u)$, the maximal number of neighbors that u can have is at most 6.

■

Theorem 6: The out degree of node in G_{DLSS} is bounded by a constant that depends only on r_{max} and r_{min}.

Proof: For any node u in G_{DLSS}, there are at most 6 neighbors inside $Disk(u, r_{min})$ from Theorem 5. Also from Corollary 1, the set of disks $\{Disk(v, \frac{r_{min}}{2}) : v \in N_{DLSS}(u), v \notin Disk(u, r_{min})\}$ are disjoint. Therefore, the total number of neighbors of u is bounded by:

$$c_1 = 6 + \left\lceil \frac{\pi[(r_{max} + \frac{r_{min}}{2})^2 - (\frac{r_{min}}{2})^2]}{\pi(\frac{r_{min}}{2})^2} \right\rceil = 4\lceil \beta(\beta+1) \rceil + 6,$$

where $\beta = \frac{r_{max}}{r_{min}}$. Actually we can observe that Fig. 10 shows the scenario where the maximum out degree of u is achieved if $\epsilon \to 0$. Therefore, we can further tighten the bound. Since the hexagonal area (as shown in Fig. 10) centered at every neighbor of u is disjoint with each other, the total number of neighbors of u is bounded by:

$$c_2 = \left\lceil \frac{\pi(r_{max} + \frac{r_{min}}{\sqrt{3}})^2}{\frac{\sqrt{3}}{2} r_{min}^2} \right\rceil - 1 = \left\lceil \frac{2\pi}{\sqrt{3}} (\beta + \frac{1}{\sqrt{3}})^2 \right\rceil - 1.$$

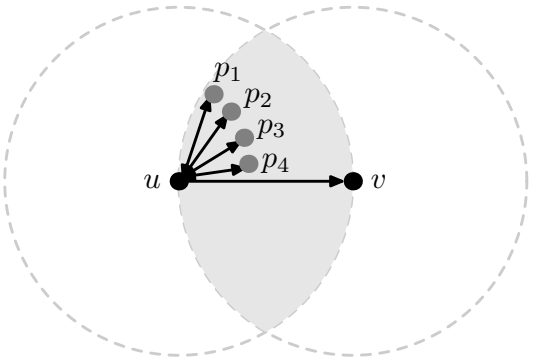


Fig. 11. The out degree may be unbounded in G_{DRNG}.

■

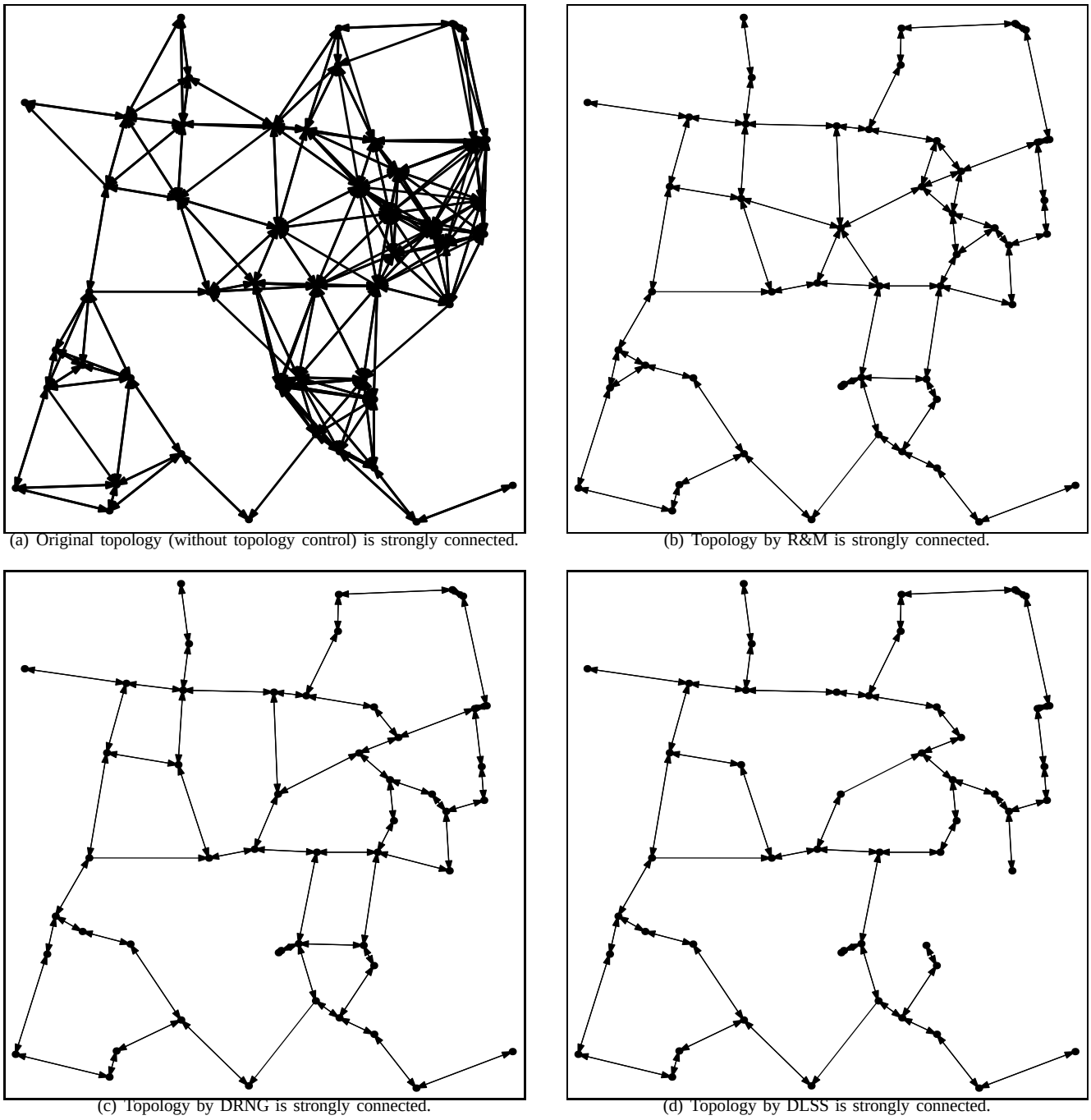
The bound given in Theorem 5 is actually quite large. We will show in Section VI that the average maximum degree is much smaller for networks with random distributed nodes. Also note that what has been discussed so far is actually the *logical* node degree, i.e., the number of logical neighbors. In practice, it is more important to consider the *physical* node degree, i.e., the number of nodes within the transmission radius. If omni-directional antennas are used, the physical degree cannot be bounded for an arbitrary topology. However, with the help of directional antennas, we will be able to bound the physical degree given that the logical degree is bounded under DLSS (except in some extreme cases, e.g., a large number of nodes are of the same distance from one node). The idea is that, when transmitting to a specific neighbor, node u should adjust the direction and limit the transmission power so that no other nodes will be affected.

Notice that the out degree is not bounded in G_{DRNG}. An example is given in Fig. 11. For all p_i that lies inside the shaded area, as long as $r_{p_i} < d(p_i, v)$, the edge (u, v) in G_{DRNG} will not exclude edges $(u, p_i), i = 1, 2, \ldots$. As a result, the out degree of u is unbounded.

D. Localized Algorithms

As mentioned in Section IV, in the case that nodes may have different maximal transmission powers, the operation of having each node u broadcast its own position information to all the other nodes within r_u is not sufficient to ensure each node u obtains the information of reachable neighborhood N_u^R (Fig. (6)). Fortunately with the desirable properties of DRNG and DLSS proved in Sections V-A and V-B, we show that it is sufficient for node u to collect neighborhood information only from nodes whose maximal transmission range covers node u. That is, the original information exchange algorithm that requires only "one-hop" information suffices.

Consider a directed simple graph with less edges: $G' = (V(G'), E(G'))$, where $E(G') = \{(u, v) : d(u, v) \leq \min(r_u, r_v), u, v \in V(G)\}$. For any edge $(u, v) \in E(G')$, since $d(u, v) \leq \min(r_u, r_v)$, we have $(v, u) \in E(G')$, which means G' is bi-directional. Define $N_u^{R'} = \{v \in V(G) : d(u, v) \leq \min(r_u, r_v)\}$, $r_u{}' = \max_{v \in N_u^{R'}}\{d(u, v)\}$, where



(a) Original topology (without topology control) is strongly connected.
(b) Topology by R&M is strongly connected.
(c) Topology by DRNG is strongly connected.
(d) Topology by DLSS is strongly connected.

Fig. 12. Topologies derived by R&M, DRNG, and DLSS.

$r_u{}' \leq r_u$ since for any $v \in N_u^{R'}$, $d(u,v) \leq r_u$. Let $r_{min}{}' = \min_{v \in V}\{r_v{}'\}$ and $r_{max}{}' = \max_{v \in V}\{r_v{}'\}$. By requiring each node u to broadcast its position and id to all other nodes within r_u, we are able to determine $N_u^{R'}$ and $r_u{}'$. We can then apply DRNG and DLSS on top of G' and prove that Theorems 1-5 still hold even if the original topology is G'.

Theorem 7: Theorems 1–6 still holds if the original topology is G'.

Proof: We replace G, r_u, N_u^R, r_{min}, and r_{max} with G', $r_u{}'$, $N_u^{R'}$, $r_{min}{}'$ and $r_{max}{}'$ in the proof of Lemma 1–2 and Theorem 1–6. Then following the same line of arguments, we can prove that they still hold if the original topology is G'. ■

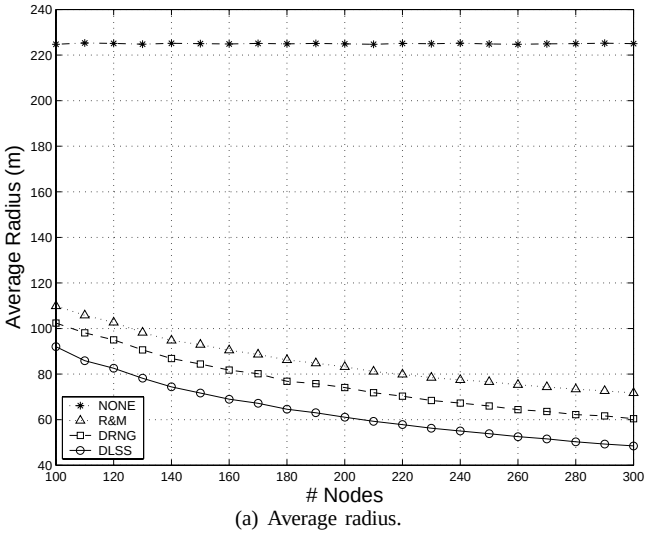
Theorem 8: If the original topology is G' (which is a subgraph of G), G_{DLSS} and G_{DRNG} are bi-directional after *Addition* or *Removal*.

Proof: We apply Theorem 4 to G', for G' is bi-directional. ■

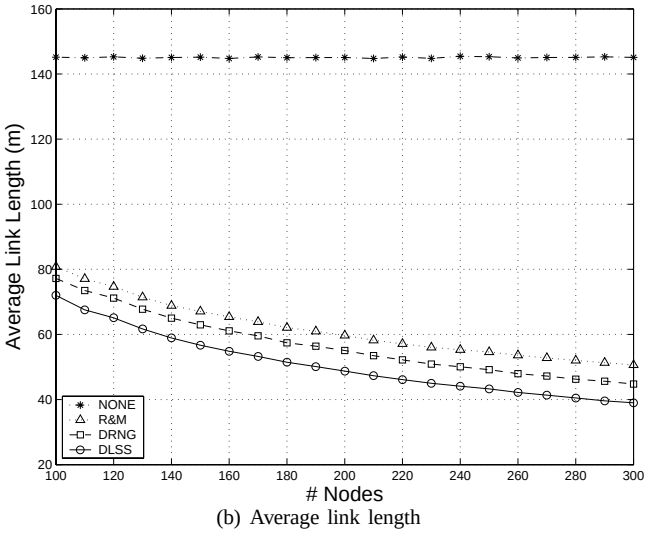
VI. Simulation Study

In this section, we evaluate the performance of R&M, DRNG, and DLSS by simulations. All three algorithms are known to preserve network connectivity in heterogeneous networks.

In the first simulation, 50 nodes are uniformly distributed in a $1000m \times 1000m$ region. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$. Fig. 12 gives the topologies derived using the maximal transmission power (labeled as NONE), R&M (under the two-ray ground model), DRNG, and DLSS for one simulation instance. As shown in Fig. 12, R&M, DRNG and LMST all significantly reduce the average node degree, while maintaining network connectivity. Moreover, both DRNG and DLSS outperforms R&M in the sense that fewer edges are formed in the topology.



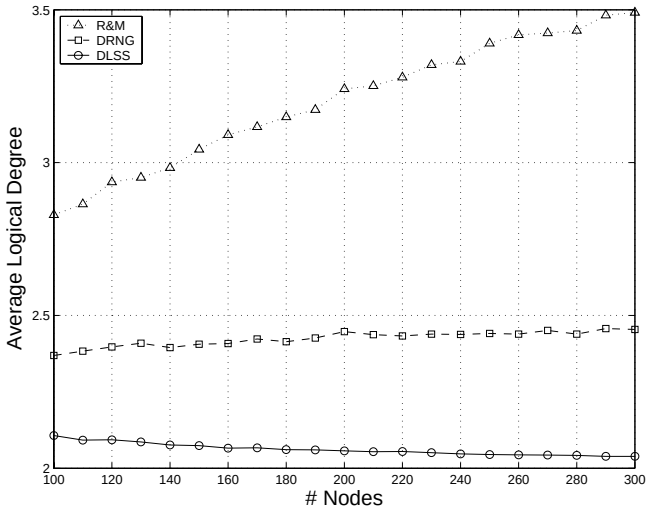
(a) Average radius.



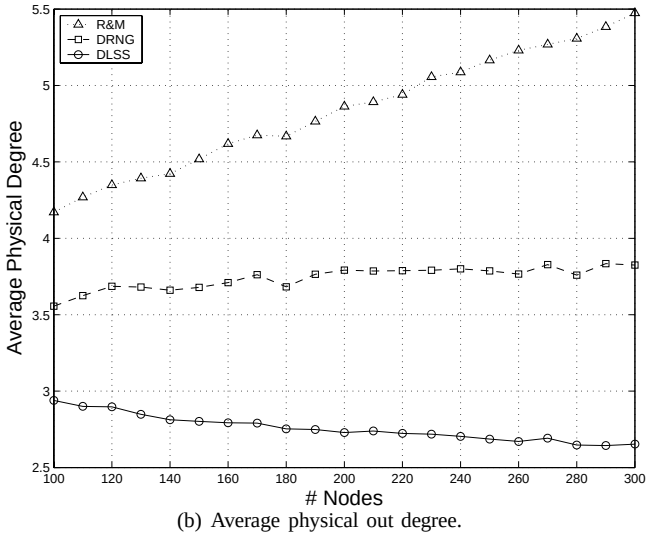
(b) Average link length

Fig. 13. Comparison of DLSS, DRNG and R&M with respect to average radius and average edge length.

In the second simulation, we vary the number of nodes in the region from 100 to 300, and each data point is an average of 50 simulation runs. The transmission ranges of nodes are uniformly distributed in $[200m, 250m]$. Fig. 13 shows the average radius and the average link length for the topologies derived under NONE(no topology control), R&M, DRNG, and DLSS. DLSS outperforms the others, which implies that DLSS can provide a better spatial reuse and use less energy to communicate.



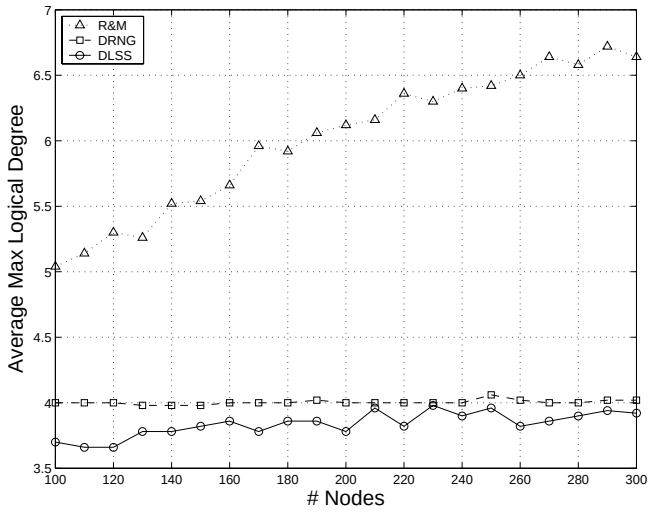
(a) Average logical out degree.



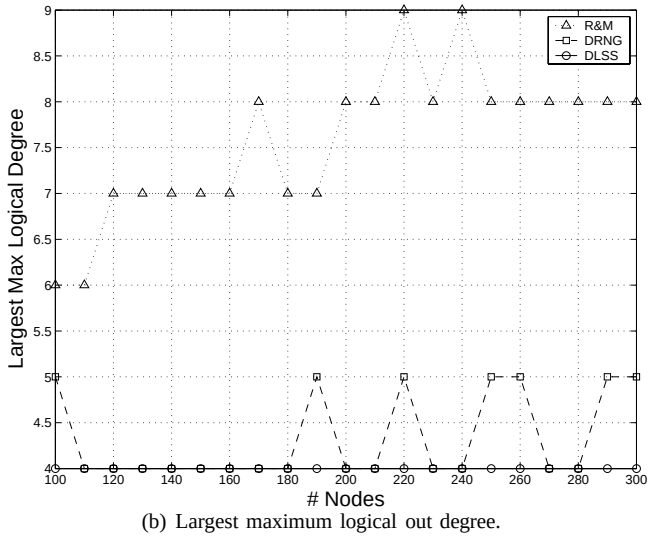
(b) Average physical out degree.

Fig. 14. Comparison of R&M, DRNG and DLSS with respect to average out degree.

We also compare the out degree of the topologies by different algorithms. The result of NONE is not shown because its out degrees increase almost linearly with the number of nodes and are significantly larger than those under R&M, DRNG, and DLSS. Fig. 14 shows the average logical/physical out degree for the topologies derived by R&M, DRNG, and DLSS. The average out degrees under R&M and DRNG increase with the increase in the number of nodes, while those under DLSS actually decrease. Fig .15 shows the average maximum logical degree and the largest maximum logical

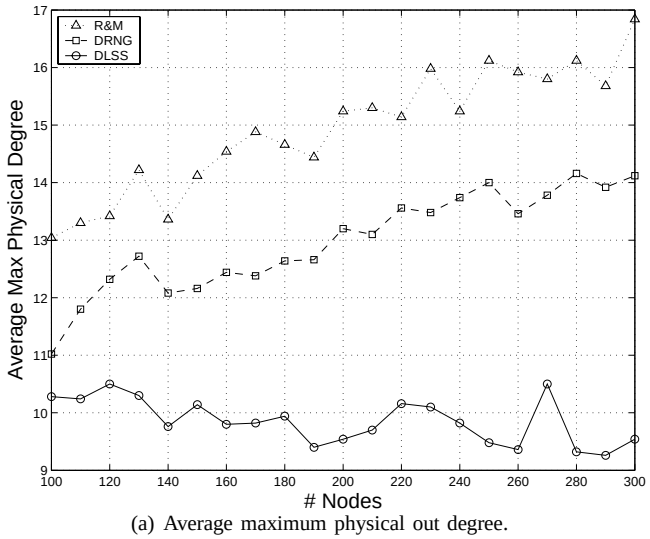


(a) Average maximum logical out degree.

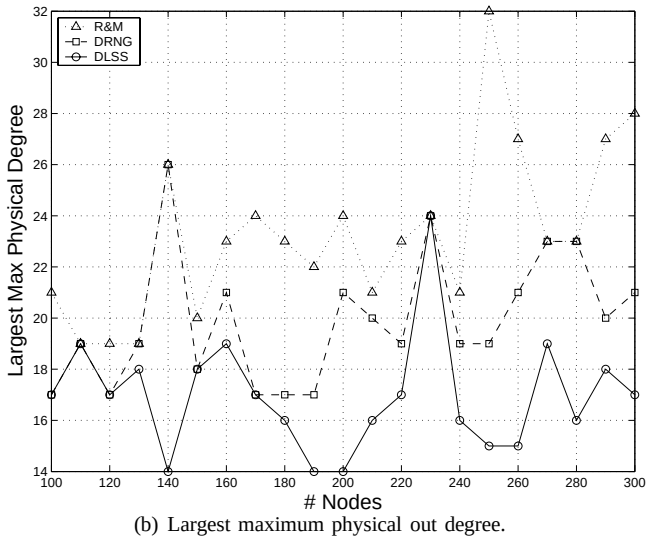


(b) Largest maximum logical out degree.

Fig. 15. Comparison of R&M, DRNG and DLSS with respect to the maximum logical degree.



(a) Average maximum physical out degree.



(b) Largest maximum physical out degree.

Fig. 16. Comparison of R&M, DRNG and DLSS with respect to the maximum physical degree.

out degree for each number of nodes. The largest maximum logical degree under DLSS is at most 4, and is well below the theoretical upper bound obtained in Theorem 6. Also DLSS has much smaller degrees than the other topologies. Similar results can be observed in Fig. 16 for physical degrees. The only difference is that the physical degrees are in general larger than the logical degrees for the same network.

VII. Conclusions

In this paper, we have proposed two local topology control algorithms, Directed Relative Neighborhood Graph (DRNG) and Directed Local Spanning Subgraph (DLSS), for heterogeneous wireless multi-hop networks in which each node may have different maximal transmission ranges. We show that as most existing topology control algorithms (except R&M [4]) do not consider the fact that nodes may have different maximal transmission ranges, they render disconnected network topology when directly applied to heterogeneous networks. Then we devise DRNG and DLSS and prove that (i) both DRNG and DLSS preserve network connectivity; (ii) both DRNG and DLSS preserve network bi-directionality if *Addition* and *Remove* operations are applied to the topologies derived under these algorithms; and (iii) the out degree of any node is bounded in the topology derived under DLSS, while that may be unbounded under DRNG. The simulation study validates the superiority of DRNG and DLSS over R&M.

As part of our future research, we will pursue the following open problems: (1) given a topology in which each node transmits with different maximal transmission power, what is the probability that the topology is bi-directional with respect to the distribution and the density of nodes, and the distribution of the transmission ranges? and (2) How will MAC-level interference affect network connectivity and bi-directionality?

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