Damir Tokic, Ph. D.
Assistant Professor of Finance
University of Houston – Downtown
Houston, USA
tokicd@uhd.edu

VALUATION OF GROWTH FIRMS: THEORETICAL MODELING

UDK/UDC: 65.016
JEL klasifikacija/JEL classification: D92
Prethodno priopćenje/ Preliminary Communication
Primljeno/Received: 21. rujna 2005./ September 21, 2005
Prihvaćeno za tisak/Accepted for publishing: 22. studenog 2005./ November 22, 2005

Summary

This study models the value of growth firms using the modified investment opportunities approach to valuation. The proposed model suggests that the value of a growth firm is function of: 1) profit margins, 2) investments in growth, and 3) the level of growth opportunity. Theoretical predictions suggest that the value is maximized when: 1) the growth opportunity exists and profitability multipliers are significant, 2) profit margins are high, and 3) the investment in growth is optimal.

Key words: growth firms, profit margins, growth opportunity, profitability multipliers.

1. INTRODUCTION

During the late 1990's, prices of technology-based stocks have been soaring to what seem to be irrational levels. These stocks had historically extremely high valuation multiples, such as Price/Sale and Price/Earnings. Many analysts have attempted to reason such valuations. However, the problem was that there was a lack of consensus model to explain values of these firms. Thereby, investors rushed in these shares based on pure speculation. As a result, the tech bubble inflated beyond the logic, peaked in March of 2000 and then collapsed, erasing over 75% of investor wealth. In the aftermath of the bubble, academia and financial community have debated on proper valuation metrics behind the dot.com bubble and questioned what went wrong. The debate has not produced any new meaningful model. It appears that the "new" economy has been once again replaced with the "old" economy and traditional valuation modeling. However, a deeper analysis of traditional literature opens the possibility that there could be a twist or a modification to traditional modeling that would provide us with better understanding of valuation of glamorous "growth" firms.

This study models the value of growth firms using the model based on Miller and Modigliani (1961) investment opportunity approach. According to the proposed model, the value of growth firms is maximized when the investment in growth is optimal, profit margins are high, and profitability multipliers are significant. These fundamentals theoretically support high valuation multiples.

Next, the paper explains the theoretical derivation of the valuation model. Then, it discusses major theoretical predictions of the model. Finally, the simulation analysis is performed to support the theoretical predictions.

2. THEORETICAL MODEL DERIVATION

The investment opportunities approach (Miller and Modigliani, 1961; Solomon, 1963) proposes that the value of a growth firm, V_j, is equal to the present value of cash flows from assets in place, V_1, and the present value of "growth" opportunity, V_2, as illustrated in equation (1):

$$V_j = V_1 + V_2 \tag{1}$$

Solomon (1963) presents in equation (2) a simple growth situation assuming permanent growth and a fixed amount of investment per annum.

$$V = \frac{E}{k} + (\frac{r-k}{k})(\frac{bE}{k}), \tag{2}$$

The first right hand side (RHS) term represents the present value of earnings from assets in place (E, standing for earnings and, k, for the normal rate of return). The second RHS term represents the profitability index $(\frac{r-k}{k})$, where r stands for the rate of return higher than normal, multiplied by the present value of total investment that amounts to the "b" percentage of earnings retained, bE.

2.1. Zero dividend policy assumption

Growth firms historically do not pay out any dividends due to their desperate need for additional capital to invest in growth. Therefore, it is safe to assume that these firms retain and reinvest a hundred percent of earnings, which means "b" in Solomon's model is equal to one.

With the assumption that b=1, the original investment opportunities approach to the valuation of growth shares in equation (2) transforms to the modified version in equation (3).

$$V = \frac{E}{k}(\frac{r}{k}) \tag{3}$$

To further simplify the model, the profitability multiplier $\frac{r}{k}$ in equation (3) is identical to the profitability multiplier m in equation (4).

$$V = \frac{E}{k}m \tag{4}$$

The version of the investment opportunities approach in equation (4) suggests that the value of a growth firm is a function of expected earnings, the risk-adjusted discount rate, and the profitability multiplier as long as the opportunity for abnormal returns exists. The presence of the profitability multiplier suggests that investors should pay a premium for earnings of growth firms, relative to ordinary firms. In other words, P/E ratios of growth firms should be higher than P/E ratios of firms without a growth opportunity, a finding of Malkiel (1963) and Miller and Modigilani (1961).

2.2. R&D and Advertising as Investments in Intangible Assets Assumption

There is a large body of literature that suggests that R&D and advertising expenses should be capitalized due to their intangible capital feature (Weiss, 1969; Hirschey, 1982; Connolly and Hirschey, 1984; Chauvin and Hirschey 1993). In addition, these studies suggest that R&D and advertising represent an actual investment in intangible assets. Thereby, the total investment, I, for growth firms equals to the retained earnings, RE, plus investments in intangible assets, such as R&D, (RD), and advertising, (A), as illustrated in equation (5). Since retained earnings are limited to zero in case of negative or zero profitability, the total investment cannot be negative in case those losses exceed the investments in R&D and advertising.

$$I = RE + RD + A, \text{ and } RE = 0 \text{ when } E \leq 0 \tag{5}$$

Equations (6), (7) and (8) adjust the earnings with R&D and advertising:

$$V = \frac{E}{k} + \left(\frac{r-k}{k}\right)\left(\frac{E+RD+A}{k}\right) \tag{6}$$

$$V = \frac{E}{k} + \left(\frac{r-k}{k}\right)\left(\frac{E}{k}\right) + \left(\frac{r-k}{k}\right)\frac{RD}{k} + \left(\frac{r-k}{k}\right)\frac{A}{k} \tag{7}$$

$$V = \frac{E}{k} + \left(\frac{r}{k}-1\right)\left(\frac{E}{k}\right) + \left(\frac{r}{k}-1\right)\frac{RD}{k} + \left(\frac{r}{k}-1\right)\frac{A}{k} \tag{8}$$

Equations (9), (10) and (11) simplify the model by substituting the profitability multiplier “m” for the $\frac{r}{k}$ ratio.

$$V = \frac{E}{k} + (m-1)\left(\frac{E}{k}\right) + (m-1)\frac{RD}{k} + (m-1)\frac{A}{k} \tag{9}$$

$$V = \frac{E}{k}(1+m-1) + (m-1)\frac{RD+A}{k} \tag{10}$$

$$V = \frac{E}{k}m + \frac{RD+A}{k}(m-1) \tag{11}$$

Finally, equation (11) presents the proposed modified investment opportunities approach. The value of a growth firm is still a function of earnings, risk-adjusted discount rate, and the profitability multiplier (abnormal return opportunity). However, in addition to these variables, the model suggests that the present value of R&D and advertising investments directly adds to the value of a firm. Even though the present value of these intangible investments is reduced by what Mao (1966) calls the index of profitability, or m-1, it still adds the overall value of a firm. Note that, as soon as the opportunity for abnormal returns disappears ($r = k$, $m = 1$), the value of a firm is calculated using the traditional present value approach, excluding the value of growth.

2.3. Constant Growth Assumption

Mao (1966) and Taylor (1974) modify the simple constant growth model of Miller and Modigliani (1961) and Solomon (1963), and propose a model with temporary growth. Their models have three stages of growth: exponential, constant, and declining. During the exponential growth stage, firms invest heavily in growth. As growth slow down to a constant rate, the investment in growth slows down accordingly. During the declining growth stage, investments in growth become insignificant. After the declining growth stage, a firm becomes mature and invests only for replacement purposes. This is the case when $r = k$ and $m= 1$.

However, there is strong consensus in the literature that the reduction in investments such as R&D and advertising leads to a decrease in market value (Chan, Lakonishok, and Sougiannis, 2001; Ikenberry, Lakonishok, and Vermaelen, 1995; Loughran and Ritter, 1995; and Lakonishok and Lee 2001). Therefore, the question is what happens to the firm's value once the growth rate diminishes? Mueller's (1972) life-cycle theory suggests that firms created with single products go bankrupt once the growth opportunity for their products matures. In response, single-product growth firms must continuously innovate to extend/expend their product mix. In other words, these firms must keep active R&D programs and advertising campaigns to develop new products and reach new customers. Consequently, the opportunity for growth must be on average constant and permanent. Otherwise, the value of these firms would quickly approach zero.

Strategically, growth firms should maintain a portfolio of R&D projects. Some of those projects will face declining growth, while others may face constant or even exponential growth potential. Projects with declining growth potential should be aggressively advertised to produce profits as cash cows, since markets for those products are already mature. Any further R&D investments in projects with declining growth potential may be inefficient. On the other hand, projects with an exponential growth potential should be heavily supported with R&D investments. These are innovative products not available on the market yet, but with enormous potential once developed. Growth firms should not advertise these projects because there could be potential classified information embedded in the initial stage of development that could be leaked to their competitors. Projects with constant growth potential should be supported with additional R&D funds for upgrades and improvements. Also, these projects require certain level of advertising because the product is already in the marketplace.

Risk-wise, projects with an exponential growth rate will be the riskiest because there is high uncertainty regarding the market success and application of these projects. Similarly, projects with a declining growth rate will have the lowest risk rate because there is a current market for these products, which proved to be marketable and profitable. Figure 1 describes the proposed R&D and advertising budget allocation and associated risk.

Figure 1.

R&D and advertising budget allocation and risk for Internet firms

	R&D	Advertising	Risk
Exponential Growth	$RD_{\exp}$	$Adv_{\exp} = 0$	$m_{\exp}$ - Highest
Constant Growth	RD_{con}	Adv_{con}	m_{con}
Declining Growth	$RD_{dec} = 0$	Adv_{dec}	m_{dec} - Lowest

With the assumption that investment in each growth stage carries a different risk rate, ($m_{\exp} > m_{con} > m_{dec}$), the modified investment opportunities approach in equation (11) takes the form of the model in equation (12):

$$V = \frac{E}{k}\left(\left(\frac{RD_{\exp}m_{\exp}}{(RD+A)_{total}}\right)+\left(\frac{(RD_{con}+A_{con})m_{con}}{(RD+A)_{total}}\right)+\left(\frac{A_{dec}m_{dec}}{(RD+A)_{total}}\right)\right)+ \frac{RD_{\exp}}{k}(m_{\exp}-1)+\frac{(RD_{con}+A_{con})}{k}(m_{con}-1)+\frac{A_{dec}}{k}(m_{dec}-1) \quad (12)$$

The term ($\frac{RD_{\exp}m_{\exp}}{(RD+A)_{total}}$) represents the risk-adjusted investment in R&D for projects with exponential growth, as a percentage of total R&D and advertising investments. Following the same interpretation of other terms, the profitability multiplier, m, in equation (11), becomes the weighted average profitability multiplier, m_{wa} , in equation (13).

$$V = \frac{E}{k}m_{wa} + \frac{RD_{\exp}}{k}(m_{\exp}-1)+\frac{(RD_{con}+A_{con})}{k}(m_{con}-1)+\frac{A_{dec}}{k}(m_{dec}-1) \quad (13)$$

The final version of the modified investment opportunities approach assumes a zero dividend policy, investment in intangible assets feature of R&D and advertising, and a permanently constant growth opportunity.

3. THEORETICAL PREDICTIONS OF THE MODEL

The modified investment opportunities approach in equation (13) suggests that it is theoretically possible to achieve relatively high valuation multiples (such as price/sales ratio) given the following fundamentals:

1. High profitability multipliers

Profitability multipliers are theoretically estimated as a proxy for the growth opportunity. For example, Internet stocks had an enormous growth opportunity in mid nineties due to expected increase of the Internet adoption in the United States. In 1995, less than 10% of U.S. population was on-line. At the time the expectations were that the on-line population would annually more than double until 2000, when it would reach over 50% of total U.S. population. Therefore high profitability multipliers for Internet firms could have been justified during late 1990's.

2. High net profit margins

Firms that have high profit margins have low business risk and low financial risk. Therefore these firms are able to convert a high percentage of sales into earnings. Once these sales increase (due to a growth opportunity) future earnings will increase as well. Consequently, a firm with higher net profit margins should have comparable higher P/E and P/S ratios. Growth firms with low profit margins may not be able to convert a growth opportunity into earnings, and could potentially become overvalued in the short run.

3. Optimal investment in growth

Overinvestment in growth causes negative profitability, and consequently decreases the stock price. Underinvestment in growth maximizes short-term profits, at the expense of future growth and profits, causing the stock price to decrease as well. The optimal level of investment in growth maximizes short-term profitability and the value of growth simultaneously. For example, an R&D portfolio should include projects in different growth stages, some which create current cash flow and generate profits, and some that are expected to generate profits in the future.

4. SIMULATION ANALYSIS

The theoretical model in equation (13) is translated into a simulation model in equation (14). Each variable is normalized with sales. Figure 2 illustrates model variables with their measurements and explanations. The data used for simulation is obtained from Microsoft income statements from fiscal 2000 to fiscal 2003, for total of 4 years. This period (2000-2003) includes the peak of the dot.com bubble in 1999 (fiscal 2000), the dot.com meltdown (fiscal 2001 and 2002) and the price recovery (fiscal 2003).

$$PS = \frac{ES}{k}m + \frac{wRDSe \times RDS}{k}(me-1) + \frac{(wRDSc \times RDS) + (wASc \times AS)}{k}(mc-1) + \frac{wASd \times AS}{k}(md-1)$$

where;

$$m = \frac{wRDSe \times RDS \times me}{RDS + AS} + \frac{(wRDc \times RDS + wASc \times AS) \times mc}{RDS + AS} + \frac{wASd \times AS \times md}{RDS + AS}$$

The purpose of the simulation analysis is to support the theoretical prediction of the model in equation (13). Step one of the simulation simply plugs in the data into a model using the spreadsheets and compares the estimated P/S ratio with the actual P/S ratio. The major objective is to estimate the size of profitability multipliers in order to explain the actual P/S ratio. The sensitivity analysis does not attempt to accurately estimate the value of Microsoft stock.

Figure 2.

Variables used in the simulation analysis.

Variable	Explanation
PS actual	Actual Price/Sales ratio for Microsoft in each fiscal year from 2000 to 2003.
PS estimated	Estimated Price/Sales ratio for Microsoft using the model developed in this study.
ES	Earnings/Sales ratio or profit margin
M	Weighted average profitability multiplier
K	Arbitrary chosen equity premium for holding MSFT stock versus the risk free bond.
RDS	R&D investments as percentage of sales.
AS	Advertising (marketing and sales) as percentage of sales
wRDSe	Percentage of total R&D invested in projects that grow at an exponential rate. Assumption is 50%. (w = .5)
wRDSc	Percentage of total R&D invested in projects that grow at a constant rate. Assumption is 50%. (w = .5)
wASc	Percentage of total Advertising invested in projects that grow at a constant rate. Assumption is 50%. (w = .5)
wASd	Percentage of total Advertising invested in projects that grow at a declining rate. Assumption is 50%. (w = .5)
Me	Profitability multiplier for investments that grow at an exponential rate.
Mc	Profitability multiplier for investments that grow at a constant rate.
Md	Profitability multiplier for investments that grow at a declining rate.

Table 1 illustrates the simulation where it is assumed that the value of Microsoft stock is simply the present value of earnings in perpetuity, with arbitrary assigned 10% discount rate. This rate is kept constant throughout the simulation. The profitability multiplier for each level of a growth opportunity is set to 1. The estimated P/S ratio undershoots the actual P/S ratio in each year. However, it does show the bubble in 2000, meltdown in 2001/2002 and recovery in 2003.

The simulation presented in Table 2 attempts to estimate the actual P/S ratio in each year by assigning profitability multipliers to each level of growth. The assumption is that 50% of R&D investments is invested in projects with exponential growth rate and 50% in projects with the constant growth rate. Similarly, 50% of advertising budget is invested in projects with the constant growth rate and 50% in the projects with the declining growth rate. (This assumption is held constant in each simulation.) The results show that at the peak of the bubble in 2000, the weighted average profitability multiplier (wapm) had to be 4.36 to justify the actual P/S ratio. The wapm decreased to 3.36 and 3.06 in 2001 and 2002, respectively. In 2003, the wapm had to be 2.02 to justify the 2003 actual P/S ratio.

Table 1.
Simulation with assumption that the value of a Microsoft stock is simply the present value of earnings in perpetuity,

	MSFT 2000	MSFT 2001	MSFT 2002	MSFT 2003
PS actual	27.11	16.89	13.99	9.12
PS estimated	**4.098**	**2.904**	**2.76**	**3.105**
ES	.4098	.2904	.276	.3105
M	1	1	1	1
K	.10	.10	.10	.10
RDS	.1643	.1731	.1518	.1447
AS	.1797	.1931	.1906	.2026
wRDSe	.08215	.08655	.0759	.07235
wRDSc	.08215	.08655	.0759	.07235
wASc	.08985	.09655	.0953	.1013
wASd	.08985	.09655	.0953	.1013
me	**1**	**1**	**1**	**1**
mc	**1**	**1**	**1**	**1**
md	**1**	**1**	**1**	**1**

Table 2.
Simulation estimation of the actual P/S ratio in each year by assigning profitability multipliers to each level of growth.

	MSFT 2000	MSFT 2001	MSFT 2002	MSFT 2003
PS actual	27.11	16.89	13.99	9.12
PS estimated	**27.12**	**16.90**	**14.00**	**9.13**
ES	.4098	.2904	.276	.3105
m	4.36	3.36	3.06	2.02
k	.10	.10	.10	.10
RDS	.1643	.1731	.1518	.1447
AS	.1797	.1931	.1906	.2026
wRDSe	.08215	.08655	.0759	.07235
wRDSc	.08215	.08655	.0759	.07235
wASc	.08985	.09655	.0953	.1013
wASd	.08985	.09655	.0953	.1013
me	**5.2**	**4.2**	**3.6**	**2.5**
mc	**4.1**	**3.1**	**2.9**	**1.9**
md	**3.1**	**2.6**	**2.0**	**1.6**

Table 3 analyzes the optimal level of investments in R&D and advertising and its effect on Microsoft P/S ratio in 2003. Column one shows that, by ignoring the value of growth whereby the wamp is 1, the estimated P/S ratio is almost three times lower than the actual P/S ratio. Considering the value of growth and assigning the profitability multiplies for each level of growth, resulting in wamp of 2.02, the estimated P/S ratio equals the actual P/S ratio (column two). The special case scenario where Microsoft decides to increase in investments in R&D and advertising to the point where earnings would become negative reduces the estimated P/S ratio by almost three times (column three). During the Internet bubble, it was common practice among dot.coms to overspend on R&D and advertising to gain the market share, which resulted in negative profitability for majority of firms. Investors initially ignored these losses focusing on the value of growth. Column four shows that by ignoring the losses from column three, the value of growth produces the P/S ratio of 8.31. However, this ratio is still smaller than 9.13, the estimated P/S from column two, suggesting that P/S ratio is maximized when the investment in growth is optimal. The optimal investment in growth is an efficient one. It maximizes the value of growth, while producing steady earnings.

Table 3.

Analysis of the optimal level of investments in R&D and advertising and its effect on Microsoft P/S ratio in 2003

	MSFT 2003	MSFT 2003	WHAT IF: CASE A	CASE A: VGO
PS actual	9.12	9.12	9.12	9.12
PS estimated	**3.105**	**9.13**	**3.11**	**8.31**
ES	.3105	.3105	**-.2107**	**ignore**
m	1	2.02	2.02	2.05
k	.10	.10	.10	.10
RDS	.1447	.1447	**.4873**	**.4873**
AS	.2026	.2026	**.5033**	**.5033**
wRDSe	.07235	.07235	.24365	.24365
wRDSc	.07235	.07235	.24365	.24365
wASc	.1013	.1013	.25665	.25665
wASd	.1013	.1013	.25665	.25665
me	**1**	**2.5**	**2.5**	**2.5**
mc	**1**	**1.9**	**1.9**	**1.9**
md	**1**	**1.6**	**1.6**	**1.6**

5. CONCLUSION

Modified version of investment opportunities approach assumes no dividends, treatment of R&D and advertising as investments in intangible assets, and permanent (constant) growth opportunity. Predictions of modified investment opportunities approach are able to theoretically explain the pricing of growth stocks. These predictions suggest that the value is maximized when: 1) the growth opportunity exists and profitability multipliers are high, 2) high profit margins exist, and 3) an optimal level in growth is achieved. The simulation analysis supports the theoretical predictions.

REFERENCES

Chan, Louis K.C., Joseph Lakonishok, and Theodore Sougiannis, 2001, The stock market valuation of research and development expenditures, *Journal of Finance*, 56, 2431 - 2456.

Chauvin, Keith W., and Mark Hirschey, 1993, Advertising, R&D expenditures and the market value of the firm, *Financial Management*, Winter, 128-140.

Connolly, Robert A., and Mark Hirschey, 1984, R&D, market structure, and profits: a value-based approach, *The Review of Economics and Statistics*, 66(4), 682-687.

Hirschey, Mark, 1982, Intangible capital aspects of advertising and R&D expenditures, *Journal of Industrial Economics*, 30(4), 375-390.

Ikenberry, David, Joseph Lakonishok, and Theo Vermaelen, 1995, Market underreaction to open market repurchases, *Journal of Financial Economics* 39, 405-435.

Lakonishok, Joseph, and Inmo Lee, 2001, Are insiders; trades informative?, *Review of Financial Studies* 14, 79-111.

Loughran, Tim, and Jay R.Ritter, 1995, The new issues puzzle, *Journal of Finance* 50, 23-51.

Malkiel, Burton G., 1963, Equity yields, growth, and the structure of share prices, *The American Economic Review* 53 (5), 1004-1031.

Mao, James C.T., 1966, The valuation of growth stocks: the investment opportunities approach, *Journal of Finance* 21, 95-102.

Miller, Merton H, and Franco Modigliani, 1961, Dividend policy, growth, and the valuation of shares, *Journal of Business* 34 (4), 411-433.

Mueller, Dennis C., 1972, A life cycle of the firm, *Journal of Industrial Economics* 20 (3), 1999-219.

Solomon, Ezra, 1963, *The theory of financial management*, New York: Columbia University Press.

Taylor, Walter, 1974, A note on Mao's growth stock-investment opportunities approach, *Journal of Finance* 29, 1573-1576.

Tokic, Damir, 2002, What went wrong with the dot.coms, *Journal of Investing*, 11(2), 52-56.

Weiss, Leonard W., 1969, Advertising, profits, and corporate taxes, *Review of Economics and Statistics*, 51, 421-430.

Damir Tokic
Docent na predmetu Financije
Sveučilište u Houstonu – Downtown
E-mail: tokicd@uhd.edu

VREDNOVANJE TVRTKI U RAZVOJU: TEORETSKO MODELIRANJE

Sažetak

U radu se daje model vrednovanja tvrtki u razvoju rabeći pristup modificiranih investicijskih prigoda. Predloženi model sugerira da je vrijednost tvrtke u razvoju funkcija: 1) profitnih marži, 2) investicija u porastu, i 3) razine prilika za rast. Teoretska predviđanja sugeriraju da se vrijednost maksimizira: 1) kada postoje prilike za rast i značajni multiplikatori profitabilnosti, 2) kada su visoke profitne marže, i 3) kada je investiranje u rast optimalno.

Ključne riječi: tvrtke u razvoju, profitne margine, mogućnosti razvoja, multiplikatori profita.

JEL classification: D92