## MIRNA GRŽANIĆ MARKO DELIMAR TOMISLAV CAPUDER

mirna.grzanic@fer.hr marko.delimar@fer.hr tomislav.capuder@fer.hr
Faculty of Electrical Engineering and Computing, University of Zagreb

## FINANCIAL TRANSMISSION AND STORAGE RIGHTS

## SUMMARY

The paper presents concepts of Financial Transmission Rights (FTRs) and Financial Storage Rights (FSRs) as key market concepts for alleviating congestion issues in transmission networks. These instruments are in place in markets where prices differ depending on the location/node due to congestions. They serve as a tool for transmission system operators TSO (or independent system operators; ISOs) for eliminating congestions by remunerating entities who make it possible. The paper further discusses different aspects of FTRs, which are traditional financial instruments used to hedge the risk of high cost occurrence associated with transmission congestion. By owning and trading with FTRs, through auction or via bilateral contracts, market participants can gain additional profit. More variable and uncertain power system environment, characterized by high penetration of renewable energy sources (RES), creates potential for storage units to assist TSO/ISO in maximizing social welfare through FSR. As storage has the capability to move energy in time, it can alleviate transmission lines congestion and create profit through intertemporal arbitrage (by load shifting and peak shaving) improving return rate of its investment. These concepts are additionally explained by intuitive examples showing how, when congestion occurs and TSO/ISO awards market participants who own transmission and storage rights, price volatility is reduced.

Key words: auction markets, congestion management, financial storage rights, financial transmission rights, strategic bid

## 1. INTRODUCTION

Congestion management and financial transmission rights (FTRs) are key financial elements ensuring reliable and secure operation of power systems in deregulated electricity market [1]. However, the presumption of their existence is that markets incorporate Locational Marginal Pricing mechanisms in which prices differ depending on the both the location in power system and time of electricity production (examples of such systems are New England, North and South America and Nordic countries). These price differences are a result of transmission line congestion and FTRs are financial instruments that allow market participants to hedge against the extra cost caused by congestions. FTRs entitle their owners to the revenue collected during the congestion time, meaning market participants can buy or sell this right for any line in the transmission grid in order to avoid congestion and at the same time maximize their own profit. The FTR owners receive payments based on price differences between two nodes for which the right is bought. As the prices can vary, FTRs protect their owners from price volatility and provide them trading at the price agreed in contracts.

In power systems with larger integration of renewable energy sources, new challenges arise when it comes to power system operation and participant behavior [2]. This is where energy storage can find its business case, by enhancing power system flexibility, efficiency and reliability [3]. From the market perspective, transmission lines move power spatially while energy storage has the additional capability to move power forward also in time; this is called energy arbitrage [4]. Storage units have high initial investment costs and low marginal operating costs. Using storage only for one service highly underestimates their value and does not justify investments in such units. Financial storage rights (FSRs) are seen as a potential service to increase storage profitability; the same way market participants hedge against the congestion and price volatility in transmission lines with FTRs, they can also hedge against congestion by utilizing storage units and maximize revenues through provision of additional services.

The paper is organized as follows; Section 2 gives a general overview of different market principles and transmission rights pricing, explaining the main concepts and differences between market set-ups. Section 3 explains the concept of Financial Transmission Rights and mathematical modelling background supported with examples. Section 4 explains Energy Storage Rights with auction examples. Section 5 concludes the article.

## 2. LOCATIONAL MARGINAL PRICING

### 2.1. Pricing transmission congestion

There are three different methods for pricing in transmission system: locational or nodal pricing, zonal pricing and uniform pricing. In locational pricing, the price of electricity at each bus reflects the marginal cost of providing electricity at that location. It is used in the USA, South America, New England and Nordic countries. It maximizes social welfare considering transmission and generators constraints, as well as losses in the power system and is performed by the system operator. If the system is not congested all prices are equal, however if the transmission line is congested or there is a loss of a transmission line, nodal prices will differ by location. These locational prices are based on the principles of economic dispatch. Transmission congestion prevents energy from low-cost generators from meeting all load requirements and, consequently, from clearing the market. This means that low-cost generators dispatch will be constrained to lower power output values (or even shut down) and higher-cost generators will be dispatched to serve load and will thereby raise the market price.

Zonal pricing is similar to locational pricing. Several buses are grouped into zones and there are price differences between the zones. Prices are calculated from simplified models. Zonal pricing implies higher operating cost [5]. When uniform pricing is used and location is neglected social welfare is often reduced even if the transmission is not congested, which can give incorrect investment signals in the long term [6].

### 2.2. Locational Marginal Prices

Prices are determined by generator bids for producing energy. If there is no congestion, the charge for using transmission lines is zero and marginal prices at all nodes are the same (this does not include charges for using transmission grid for transferring electricity). However, in case of congestions the transmission network usage charges are defined as the incremental cost of redispatch that satisfies transmission lines limit. This means that if transmission lines are congested, some low-cost generators will have to be replaced by more expensive generators resulting in higher marginal prices. Locational Marginal Prices (LMPs) values are calculated for different time periods and are a result of the optimization process which finds least cost dispatch while maintaining production consumption equilibrium and satisfies all system technical constraints [7]. There are many factors that affect LMPs: the electrical characteristics of the system, generators' bid prices, generators' limit, the transmission system elements that are experiencing congestion (line thermal limit) and the losses in the system (if they are taken in the account) [8]. The calculation process combines shadow prices of transmission system constraints with the impact on transmission constraints due to supplying additional load at a
bus to determine the LMPs values. In short, LMPs can be defined as "the price of supplying an additional MW of load at each location (bus) in the system. [9]" This results in generators selling energy at one price (the price calculated for that location) and suppliers (or consumers) buying energy at a different price because of congestion and differences in LMPs. As a way of protecting buyers from price volatility efficient mechanisms such as FTRs are put in place to deal with congestions in transmission system.

## 3. FINANCIAL TRANSMISSION RIGHTS

FTRs are used to mitigate market participants' exposure to price risks when congestion occurs in transmission network, they are a financial entitlement used for hedging against congestion charges. The economic value of an FTR is determined by the difference in the hourly locational marginal prices between its source and sink and by the MW reservation level of the financial transmission right. The FTR is independent of actual energy delivery since its value is determined by the reservation MW [10]. The marginal cost of energy (energy component of LMP) has the same value throughout the system. On the other hand, values of loss and congestion components may vary from one location to another. The congestion component of the LMP is calculated as a congestion difference between sink and source location (bus), while the difference between loss components of sink and source LMPs determines the loss component of the LMP differences. However, currently FTRs are determined only according to the congestion components of LMP differences. Energy component of the LMP differential is always zero. This is because the energy component of the LMP at a certain location is always the same as that at any other location [11].

When losses are neglected, nodal price difference reflects the existence of transmission constraints or transmission congestion (1):

$$
\Delta L M P=W_{i}-W_{j} \#(1)
$$

where $W_{i}-W_{j}$ is price difference in congestion cost between bus $i$ and bus $j$.
FTRs can be acquired through four market mechanisms: Long-term FTR Auctions, Annual FTR Auction, Monthly FTR Auction or FTR Secondary market.

### 3.1. Obligation and Option

There are two different types of Financial Transmission Rights: obligation and option. The hourly economic value of FTR Obligation is based on the MW reservation and the price difference between sink (point of delivery) and the source point (point of receipt). The hourly economic value of an FTR Obligation is positive (a benefit) when the path designated in the FTR is in the same direction as the
congested flow. This occurs when the price at the sink point is higher than the price at the source point. An FTR Obligation is negative (a liability) if the designated path is in the direction opposite of the congested flow (price at the source point is higher than the price at the sink source), as shown in Figure 1.a) and 1.b). Since FTR payoff is related to LMP differences, potential FTR owners will try to forecast LMP in order to choose FTR injection and withdrawal locations with only positive payoff.

(a)

(b)

Figure 1 a) FTR as a benefit b) FTR as a liability [12]
The hourly economic value of an FTR Option is based on the FTR MW reservation and the price difference between sink point (point of delivery) and the source point (point of receipt). The hourly economic value of an FTR Option can only be positive. This occurs when path designated in the FTR is in the same direction as the congested flow. When the designated path is in the direction opposite to the congested flow the economic value of an FTR Option is zero.

FTR is a financial instrument. The payment ISO collects from congestion charges will be allocated to the FTR owners regardless if they actually use the transmission lines or not. Payment is independent of real physical power transfer since it is based on reservation of MW for specified lines.

### 3.2. Mathematical background for calculating FTRs

DC optimal power flow method is used to calculate minimal cost of generation and financial transmission rights. The goal is to minimize generators cost functions for meeting the load (2):

$$
\operatorname{minimize} \sum_{i \in G} f_{i}\left(p_{g_{i}}\right) \#(2)
$$

where $G$ is the set of all generators in the system, $f_{i}\left(p_{g_{i}}\right)$ is generator cost function for generator at node $i$ and $p_{g_{i}}$ is the amount of power produced by the generator at the node $i$.
$\lambda_{i}$ is nodal price for node $i$ (dual variable). The power injected in the node or withdrawn from the node $p_{i}$ is equal to sum of all power flows on the transmission lines $p_{i j}$ that start (end) at that node (3):

$$
\lambda_{i}: p_{i}=\sum_{j} p_{i j} \#(3)
$$

Generators have lower $p_{\min }$ and upper $p_{\max }$ power limit (4):

$$
p_{\min } \leq p_{g_{i}} \leq p_{\max } \#(4)
$$

$\mu_{i j}$ is the dual variable for line shadow price. Power flow on each transmission line is constrained by thermal capacity limit ${ }_{i j}$ (5):

$$
\mu_{i j}: p_{i j} \leq \text { limit }_{i j} \#(5)
$$

The complementarity conditions enforce that the inner product of an inequality constraint and the primal or dual variable is zero, and the nonnegativity of both the inequality constraint and primal or dual variable. This means that either the inequality constraint holds as an equality, i.e. is binding, or the primal or dual variable is zero. If $p_{i j}<$ limit $_{i j}$, then the dual variable for shadow price $\mu_{i j}$ is equal to zero. If $p_{i j}=$ limit $_{i j}$, then the value of $\mu_{i j}$ is greater than zero and that is the value for financial transmission right [13].

### 3.3. 3-bus example

There are two generators in the system, one located at the bus 1 with marginal cost $10 € / \mathrm{MWh}$ and the other at the bus 2 with marginal cost $20 € / \mathrm{MWh}$. Load is located at the bus 3. Transmission lines capacity between two nodes are 40 MW and the impedances are equal for all three lines. Nodal price can be a combination of two marginal costs of generators and it can be lower than the cheapest generator's marginal cost and higher than the most expensive marginal cost. The system is shown in Figure 2:

Generator $G_{1}$ and $G_{2}$ each have the capacity of 100 MW . In the first case example, load at the bus 3 is equal to 55 MW . The power flow and nodal prices are calculated in MATLAB. The best solution for this case is to dispatch generator $G_{1}$ for serving the load at the bus 3. Transmission line constraints are not violated. Nodal price is $10 € / \mathrm{MWh}$ at each bus.


Figure 2 3-bus system
In the second case load is increased by 5 MW . Generator $G_{1}$ produces 60 MW and nodal prices are not unique in the entire system. Nodal price at bus 1 is 10 $€ / \mathrm{MWh}$, at bus 2 is $20 € / \mathrm{MWh}$ and at bus 3 is $30 € / \mathrm{MWh}$. Increasing the load for 1 MW at bus 1 can be served by Generator $G_{1}$. If load at bus 3 is increased for 1 MW , generator $G_{2}$ must increase its power production for 2 MW and generator $G_{1}$ must decrease the power generation by 1 MW . That is the reason why the nodal price at bus 3 is $30 € / \mathrm{MWh}$. Generator's profit at the bus 1 is $10 € / \mathrm{MWh} * 60 \mathrm{MW} * 1 \mathrm{~h}=600 €$. The load has to pay $30 € / \mathrm{MWh} * 60 \mathrm{MW} * 1 \mathrm{~h}=1800 €$. The difference between generators' profit and consumers' cost is collected by the system operator. The FTR price for transmission line between bus 1 and bus 3 is $30 € / \mathrm{MWh}$. If that price is multiplied by amount of power transferred over that line ( 40 MW ), $1200 €$ revenue is collected for consumer at bus 3 . If the consumer at bus 3 buys the FTR from bus 1 to bus 3 , he can hedge against the price volatility.

### 3.4. IEEE 9-bus example without congestion

9-bus example is shown in Figure 3:


Figure 3 9-bus example

The generators' cost function, transmission line limits and reactance are given in Table I and Table II:

Table I Generators' cost function

| Generator | Cost function ( $€$ ) |
| :---: | :--- |
| $\mathrm{G}_{1}$ | $0.11 \mathrm{p}^{2}+5 \mathrm{p}+150$ |
| $\mathrm{G}_{2}$ | $0.085 \mathrm{p}^{2}+1.2 \mathrm{p}+600$ |
| $\mathrm{G}_{3}$ | $0.1225 \mathrm{p}^{2}+1 \mathrm{p}+335$ |

where $p$ is the amount of produced power in MW.
Table II Transmission line characteristics

| Line | From | To | Reactance <br> (p.u.) | Limit <br> (MW) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 4 | 0.0576 | 250 |
| 2 | 2 | 8 | 0.0625 | 250 |
| 3 | 3 | 6 | 0.0586 | 300 |
| 4 | 4 | 5 | 0.092 | 250 |
| 5 | 4 | 9 | 0.085 | 250 |
| 6 | 5 | 6 | 0.17 | 150 |
| 7 | 6 | 7 | 0.1008 | 60 |
| 8 | 7 | 8 | 0.072 | 250 |
| 9 | 8 | 9 | 0.161 | 250 |

Load is located at buses 5, 7 and 9, as shown in Figure 3, and the generators are located at buses 1,2 and 3 . The objective is to minimize the generation cost considering transmission line limits. In the first example, 90 MW load is located at bus 5, 100 MW load at bus 7 and 125 MW load at bus 9, as shown in Table III. After running the optimization algorithm, the minimum cost of supplying the load in the system is $5216.03 €$ and generators are dispatched as shown in Table III:

## Table III Load and generators' production

| Load bus | Load (MW) | Gen bus | Gen (MW) |
| :---: | :---: | :---: | :---: |
| 5 | 90 | 1 | 86.56 |
| 7 | 100 | 2 | 134.38 |
| 9 | 125 | 3 | 94.06 |

The system is operating within technical limits and constraints and prices are equal at every bus ( $24.04 €$ ).

Power flow on each line and comparison with thermal line limits is shown in Table IV:

## Table IV Power flow compared to line limit

| Line | From | To | Limit (MW) | Power flow (MW) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 4 | 250 | 86.5614 |
| 2 | 2 | 8 | 250 | 134.3633 |
| 3 | 3 | 6 | 300 | 94.0753 |
| 4 | 4 | 5 | 250 | 33.7322 |
| 5 | 4 | 9 | 250 | 52.8292 |
| 6 | 5 | 6 | 150 | -56.2678 |
| 7 | 6 | 7 | 60 | 37.8075 |
| 8 | 7 | 8 | 250 | -62.1925 |
| 9 | 8 | 9 | 250 | 72.1708 |

### 3.5. IEEE 9-bus example with congestion

Load is increased in all buses of the system. Loads at the buses 5,7 and 9 are $155 \mathrm{MW}, 190 \mathrm{MW}$ and 200 MW , respectively. The production cost is $12619.1 €$ and the generators dispatch is shown in Table V:

Table V Load and generators dispatch for constrained case

| Load bus | Load (MW) | Gen bus | Gen (MW) |
| :---: | :---: | :---: | :---: |
| 5 | 155 | 1 | 157.14 |
| 7 | 190 | 2 | 243.96 |
| 9 | 200 | 3 | 143.90 |

Figure 4 shows absolute value of power flow in constrained case. Thermal limit on line 7 is enforced:


Figure 4 Power flows in constrained case

This results in different nodal prices as shown in Table VI:
Table VI Nodal prices

| Bus | Price (€/MWh) |
| :---: | :---: |
| 1 | 39.56 |
| 2 | 42.67 |
| 3 | 36.26 |
| 4 | 39.56 |
| 5 | 38.41 |
| 6 | 36.26 |
| 7 | 43.58 |
| 8 | 42.67 |
| 9 | 40.64 |

After the optimization process, shadow price for the constrained line is 8.6095 $€ / M W$. Every participant who requested the FTR for that line will be paid $8.6095 € / \mathrm{MW}$ times reserved amount of MW.

Market participants can hedge against the price differences or make profit by owning FTRs. Next section demonstrates maximization of revenue from owning the FTR through auctions.

### 3.6. Auction model

The main goal of auction process is maximizing ISO profit. Bidders make offers of lower and upper amount of MW they are willing to buy and the price they are willing to pay. There are 3 types of FTRs that can be bought in an auction: flowbased financial transmission option or flowgate rights (in this case presented as bidder $a_{i}$ ), point-to-point financial transmission obligation (presented as $b_{i}$ ) and point-to-point financial transmission option (presented as $c_{i}$ ). Market participants develop a bidding strategy by calculating the flowgate capacity required by their transaction and often change the bid price or quantity required to adjust their portfolios. Willing to obtain more rights or trying to get a better price for desired right, bidders attempt to eliminate other markets participants, as explained later in 3.7. The auction model is taken from [14].

The objective is presented as (6):

$$
\max \sum_{i} a_{i} \cdot \operatorname{price}\left(a_{i}\right)+\sum_{i} b_{i} \cdot \operatorname{price}\left(b_{i}\right)+\sum_{i} c_{i} \cdot \operatorname{price}\left(c_{i}\right) \#(6)
$$

where $a_{i}, b_{i}$ and $c_{i}$ are required amount of MW in auction for flowgate right, point-to-point obligation and point-to-point option, respectively and price $\left(a_{i}\right)$, $\operatorname{price}\left(b_{i}\right)$ and price $\left(c_{i}\right)$ are prices that bidders are willing to pay for a certain right.

Constraints are lower and upper bounds of required MW (7):

$$
\text { lower }_{\text {oound }_{i}} \leq x_{i} \leq \text { upper }_{\text {bound }_{i}}, \forall \mathrm{i}, x \in \mathrm{a}, \mathrm{~b}, \mathrm{c} \#(7)
$$

There is also constraint connected with flowgate limit on every line in each direction (the FTR flow on the same line does not have to be equal for both directions). This can be described by equation (8) and described as:

$$
\sum_{i} a_{i} \cdot \alpha_{i}+\sum_{i} b_{i} \cdot \beta_{i}+\sum_{i} c_{i} \cdot \gamma_{i} \leq \text { line }_{\text {limit }} \#(8)
$$

where $a_{i}, b_{i}$ and $c_{i}$ are required quantities of MW in auction for flowgate right, point-to-point obligation and point-to-point option, respectively. $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are coefficients that determine power flow for flowgate right, point-to-point obligation and point-to-point option.

3 -bus system is shown in Figure 5 with equal transmission line reactance of $X=1 \mathrm{p} . \mathrm{u}$. for all lines and thermal line limits of 100 MW :


Figure 5 3-bus system and thermal limits
When bidder $a_{i}$ is willing to buy a flowgate right for certain line and direction, his coefficient for that flowgate will be 1 and for the other lines 0 . Flowgate or flow based option is an option for a line in specified direction. As one can see in Table VII bidder $a_{1}$ is willing to buy the right for flowgate 1->3, bidder $a_{2}$ is willing to buy the right for flowgate $3->2$ and bidder $a_{3}$ is willing to buy the right for flowgate $3->1$. Bidder $b_{1}$ is buying point-to-point obligation from node 1 to node 3. As it is a point-to-point obligation, there is negative Power Transfer Distribution Factor (PTDF) on opposite direction. This means if required FTR is in the same direction as congested flow, he will receive payment, and if it is in opposite direction he has to pay the ISO. If he injected 1 MW of power in node 1 and extract it from node 3 , that will increase a power flow on line $1->3$ by $2 / 3 \mathrm{MW}$ (in opposite direction $-2 / 3 \mathrm{MW}$ ), on line $1->2$ and $2->3$ by $1 / 3 \mathrm{MW}$ (with negative sign in opposite directions). Bidder $c_{1}$ is willing to buy a point-to-pont option from node 1 to node 2. As he is buying an option, there is just a positive sign because if required FTR is in opposite direction as the congested flow, he does not have to pay the ISO anything. Injecting 1 MW at node 1 and extracting from node 2 will cause a power flow of $2 / 3$ MW on the line $1->2$ and $1 / 3 \mathrm{MW}$ on the lines $3->2$ and $1->3$.

Table VII Flowgate and PTDFs factors for each bid

| Flowgate | Bidder $\mathrm{a}_{1}$ | Bidder $\mathrm{a}_{2}$ | Bidder $\mathrm{a}_{3}$ | Bidder $\mathrm{b}_{1}$ | Bidder $\mathrm{c}_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1->2 | 0 | 0 | 0 | 1/3 | 2/3 |
| $2->1$ | 0 | 0 | 0 | -1/3 | 0 |
| $2->3$ | 0 | 0 | 0 | 1/3 | 0 |
| $3->2$ | 0 | 1 | 0 | -1/3 | 1/3 |
| 1->3 | 1 | 0 | 0 | 2/3 | 1/3 |
| $3->1$ | 0 | 0 | 1 | -2/3 | 0 |

Lower bounds are FTRs that bidders already own and do not want to trade with. Bidding prices, lower and upper bound of required FTRs are given in Table VIII:

## Table VIII Bidding prices, lower and upper bounds of FTRs

| Bidder | $\mathbf{a}_{1}$ | $\mathbf{a}_{2}$ | $\mathbf{a}_{3}$ | $\mathbf{b}_{1}$ | $\mathbf{c}_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bid (€/MW) | 4 | 7 | 8 | 10 | 5 |
| Lower bound (MW) | 0 | 0 | 50 | 20 | 0 |
| Upper bound (MW) | 100 | 100 | 100 | 80 | 50 |

After the auction, comparing the line limits with power flow in Table IX, one can notice that flowgate $1->3$ is congested and the shadow price is $4 € / \mathrm{MW}$ (given later in the section).

Table IX Line limit, actual power flow and shadow prices

| Line | Line limit (MW) | Power flow (MW) | Shadow price (€/MW) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 - > 2}$ | 100 | 60 | 0 |
| $\mathbf{2 - > 1}$ | 100 | -26.6667 | 0 |
| $\mathbf{2 - > 3}$ | 100 | 26.6667 | 0 |
| $\mathbf{3 - > 2}$ | 100 | 90 | 0 |
| $\mathbf{1 - > 3}$ | 100 | 100 | 4 |
| $\mathbf{3 - > 1}$ | 100 | 46.6667 | 0 |

Awarded FTRs, upper bound of required FTRs and ISO revenue from auction are given in Table X :

> Table X Awarded FTRs and ISO receipt

| Bidder | $\mathbf{a}_{\mathbf{1}}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{3}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{c}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Upper bound (MW) | 100 | 100 | 100 | 80 | 50 |
| Awarded MW | 30 | 100 | 100 | 80 | 50 |
| ISO revenue (€) | 120 | 0 | 0 | 213.33 | 66.67 |

As one can notice from Table X, ISO revenue depends on bidding prices and MW awarded to each bidder. The main goal of the auction is to maximize the profit taking into account transmission line limit. As shown in Table IX, thermal limit of line $1->3$ is violated and shadow price for that line is not 0 . Shadow prices depend on the latest bid which could not be awarded. For example, we can notice that bidder $a_{1}$ got 30 MW (and he was willing to buy 100 MW ). His bidding price was 4 $€ / \mathrm{MW}$ for flowgate $1->3$. After the auction, the shadow price for flowgate $1->3$ is 4 $€ / \mathrm{MW}$ because this is the last bid that could not be awarded (he could not get the required amount because of congestion, his price reflected the shadow price for that flowgate).

Bidders have to be careful in auction process. For example, bidder $a_{1}$ wanted to buy 100 MW for flow gate $1->3$. As that flowgate is very valuable, he pays dearly for his 30 MW because this flowgate is also very valuable to other bidders. In next hour auction bidder $a_{1}$ can change his strategy and bid only for 30 MW and if others do not change their strategy, he would get 30 MW of that right. However, now the shadow price will be 0 and he would not have to pay anything for holding that right because there are no more rights that have to be awarded for that flowgate.

As it can be seen in Table X, bidders $a_{2}$ and $a_{3}$ do not have to pay the ISO for awarded FTRs because there are more rights available. This can also be subject to changes in next hour auction. Flowgates $3->2$ and $3->1$ can become more valuable and price can change.

Bidders $b_{1}$ and $c_{1}$ have to pay for holding the congested flowgate 1->3 depending on the PTDFs factor. Bidder $b_{1}$ has to pay $80 \mathrm{MW} * 2 / 3 * 4 € / \mathrm{MW}=213.33 €$ and bidder $c_{1} 50 \mathrm{MW}^{*} 1 / 3^{*} 4 € / \mathrm{MW}=66.67 €$.

Bidder $a_{1}$ could not get the full 100 MW for his right since his bidding price was too low. As ISO objective is to maximize the profit from auction, and bidders $b_{1}$ and $c_{1}$ had higher prices, they are the first to be awarded their FTRs.

If bidder $b_{1}$ changes his bid price (e.g. from $10 € / \mathrm{MW}$ to $2 € / \mathrm{MW}$ wishing to pay less for FTRs) situation will change as shown in Table XI and XII:

Table XI Changes in awarded FTRs and ISO receipt

| Bidder | $\mathbf{a}_{1}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{3}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{c}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Awarded MW | 50 | 100 | 100 | 50 | 50 |
| ISO revenue (€) | 200 | 200 | 0 | 100 | 100 |
| Previous awarded MW | 30 | 100 | 100 | 80 | 50 |
| Previous ISO revenue (€) | 120 | 0 | 0 | 213.33 | 66.67 |

Table XII Changes in power flow and shadow prices

| Line | Line limit (MW) | Power flow (MW) | Shadow price (€/MW) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 - > 2}$ | 100 | 50 | 0 |
| $\mathbf{2 - > 1}$ | 100 | -16.6667 | 0 |
| $\mathbf{2 - > 3}$ | 100 | 16.6667 | 0 |
| $\mathbf{3 - > 2}$ | 100 | 100 | 2 |
| $\mathbf{1 - > 3}$ | 100 | 100 | 4 |
| $\mathbf{3 - > 1}$ | 100 | 66.6667 | 0 |

ISO would award more MW to bidder $a_{1}$ since this would increase its profit. As one can notice, shadow prices and power flow changed as well.

### 3.7. Bidder $a_{1}$ strategy for awarding more MW

Bidder $a_{1}$ would like to gain more than 50 MW of FTR on line $1->3$. To achieve this, he can chooses from different strategies. His first option would be rising his bidding price from $4 € / \mathrm{MW}$ to $7 € / \mathrm{MW}$. The choice of $7 € / \mathrm{MW}$ follows from the logic of $6 € / \mathrm{MW}$ being sufficient only for awarding him 50 MW , as shown in the previous case. The shadow price for flowgate $1->3$ is $6 € / \mathrm{MW}$ and ISO receives $300 €$ from bidder $a_{1}, 200 €$ from bidder $b_{1}$ and $100 €$ from bidder $c_{1}$. Rising his price to $7 € / \mathrm{MW}$, he gets 70 MW for flowgate $1->3$. At the same time ISO reduces FTRs to the bidder $b_{1}$ from 50 MW to 20 MW . The shadow price for flowgate $1->3$ is now $7 € / \mathrm{MW}$ and ISO receives $490 €$ from bidder $a_{1}, 93.33 €$ from bidder $b_{1}$ and $116.67 €$ from bidder $c_{1}$.

Second choice for bidder $a_{1}$ is to buy the FTR for different flowgate and try to eliminate other market players. He wants to buy 70 MW for flowgate 1->2 (this bid is presented as $a_{4}$ ). Choosing the right bidding price is the most important if he chooses this strategy. He bids $4 € / \mathrm{MW}$ for flowgate $1->3$ (as he did in previous auction) and he bids $3 € / \mathrm{MW}$ for flowgate $1->2$. This bid is too low to eliminate other market participants and he cannot get more than 50 MW for the flowgate 1->3 (it is still more profitable for ISO to award more MW to bidders $b_{1}$ and $c_{1}$ ). This means he has to give a higher bid of $4 € / \mathrm{MW}$. Results of the auction and payments to the ISO compared to the previous case are given in Table XIII:

Table XIII Compared awarded MW and payment to ISO

| Bidder | $\mathbf{a}_{\mathbf{1}}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{3}$ | $\mathbf{a}_{4}$ | $\mathbf{b}_{\boldsymbol{1}}$ | $\mathbf{c}_{\boldsymbol{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Awarded MW (case 1) | 70 | 90 | 100 | - | 20 | 50 |
| Payment to ISO (€) | 490 | 490 | 0 | - | 46.67 | 233.33 |
| Awarded MW (case 2) | 70 | 100 | 100 | 70 | 30 | 30 |
| Payment to ISO (€) | 280 | 500 | 0 | 210 | 80 | 150 |

As the results show, better strategy for a bidder is to participate in auctions by bidding for different rights instead of increasing the bid for the same right. As opposed to the previous example, now bidder $a_{1}$ is awarded 70 MW for the flowgate $1->3$ and he needs to pay $280 €$ for holding that FTR, instead of $490 €$. He could additionally make more profit by selling the right for flowgate $1->2$ through bilateral contract if he does not need that right. By choosing the right bidding price for the flowgate 1->2, he was awarded more MW for the flowgate 1->3 and paid less for it. The shadow prices for the flowgates $1->2,3->2$ and $1->4$ are $3 € / \mathrm{MW}, 5 € / \mathrm{MW}$ and $4 € / \mathrm{MW}$, respectively.

### 3.8. Rising shadow prices

Assume that the second scenario explained in section 3.7. occurs. Only flowgates $3->2$ and $1->3$ are congested and shadow prices are $2 € / \mathrm{MW}$ and $4 € / \mathrm{MW}$. Shadow price for flowgate $2->3$ is 0 which means there are more rights on that flowgate that can be awarded. Only bidder $b_{1}$ is holding point-to-point obligation for that flowgate. The flowgate is 16.67 MW and there is additional 83.33 MW that can be awarded. If new bidder wants to buy flow-based option for flowgate $2->3$, regardless the price he offers, he can get a maximum of 93.33 MW. This comes from the fact that bidder's $b_{1}$ lower bound is $20 \mathrm{MW}\left(20^{*} 1 / 3=6.667 \mathrm{MW}\right)$ and does not want to trade with that amount. Depending on the price and the upper bound of required amount of MW offered by new bidder for flowgate 2->3, flowgates, awarded FTRs and shadow prices change. Table XIV presents shadow prices and power flow on flowgates when bidding price for flow-based option $2->3$ is $4 € / \mathrm{MW}$ and required upper bound is 80 MW . Compared to the previous case, shadow prices are still the same for flowgate $3->2$ and $1->3(2 € / \mathrm{MW}$ and $4 € / \mathrm{MW})$. Power flow changed only on the flow gate $2->3$ and is increased by 80 MW .

Table XIV Shadow prices and power flow for bidding price $4 € / M W$ and upper bound $80 M W$

| Flow gate | Shadow price (€/MW) | Power flow (MW) |
| :---: | :---: | :---: |
| $\mathbf{1 - > 2}$ | 0 | 50 |
| $\mathbf{2 - > 1}$ | 0 | -16.667 |
| $\mathbf{2 - > 3}$ | 0 | 96.667 |
| $3->\mathbf{2}$ | 2 | 100 |
| $\mathbf{1 - > 3}$ | 4 | 100 |
| $\mathbf{3 - > \mathbf { 1 }}$ | 0 | 66.667 |

If the new bidder increases his required upper bound up to 100 MW and the price remains the same ( $4 € / \mathrm{MW}$ ), he will be rewarded 83.33 MW (others get the same amount as in previous case), but his upper bound reflects the shadow prices, as shown in Table XV:

Table XV Shadow prices and power flow for bidding price $4 € / M W$ and upper bound 1000 MW

| Flow gate | Shadow price (€/MW) | Power flow (MW) |
| :---: | :---: | :---: |
| $\mathbf{1 - > 2}$ | 0 | 50 |
| $\mathbf{2 - > 1}$ | 0 | -16.667 |
| $\mathbf{2 - > 3}$ | 4 | 100 |
| $3->\mathbf{2}$ | 6 | 100 |
| $\mathbf{1 - > 3}$ | 4 | 100 |
| $3->\mathbf{1}$ | 0 | 66.667 |

If the new bidder wants more MW, he should rise his price. Shadow prices and power flow, when the price is $5 € / \mathrm{MW}$ and upper bound 100 MW , is shown in Table XVI:

Table XV Shadow prices and power flow for bidding price $5 € / M W$ and upper bound 1000 MW

| Flow gate | Shadow price (€/MW) | Power flow (MW) |
| :---: | :---: | :---: |
| $\mathbf{1 - > 2}$ | 0 | 40 |
| $\mathbf{2 - > 1}$ | 0 | -6.667 |
| $\mathbf{2 - > 3}$ | 4 | 100 |
| $3->\mathbf{2}$ | 7 | 100 |
| $\mathbf{1 - > 3}$ | 5 | 100 |
| $\mathbf{3 - > 1}$ | 0 | 86.667 |

This also changes the awarded MW for bidders as shown in Table XVII (second and third column presents the situation when bidding price is $4 € / \mathrm{MW}$ and $5 € / \mathrm{MW}$, and the upper bound is 100 MW ):

## Table XVII Differences in awarded MW depending on different bidding price

| Bidder | Awarded MW | Awarded MW |
| :---: | :---: | :---: |
| $\mathbf{a}_{\mathbf{1}}$ | 50 | 70 |
| $\mathbf{a}_{\mathbf{2}}$ | 100 | 90 |
| $\mathbf{a}_{3}$ | 100 | 100 |
| $\mathbf{a}_{4}$ | 83.333 | 93.333 |
| $\mathbf{b}_{\mathbf{1}}$ | 50 | 20 |
| $\mathbf{c}_{\boldsymbol{1}}$ | 50 | 50 |

The new bidder will receive 93.333 MW , which is maximum since bidder $b_{1}$ already owns 6.667 MW for the specified flowgate. His bidding price changed awarded FTRs to bidders $a_{1}, a_{2}$ and $b_{1}$. As one can notice, this can also be a good strategy for bidder $a_{1}$ if he wants to get more MW for flowgate $1->3$. If bidder $a_{1}$ request 100 MW for flowgate $2->3$ at the price $5 € / \mathrm{MW}$ or higher, he will get 70 MW on the flowgate $1->3$. As the shadow price is $5 € / \mathrm{MW}$, he needs to pay $350 €$ for 70 MW and $373.33 €$ for 93.333 MW on the flowgate $2->3$. This is a suboptimal case is compared to the one described in section 3.8. On the other hand, if he would choose to sell the right for flowgate $2->3$ via bilateral contract, there is an opportunity to increase the profit and still get 70 MW for flowgate 1->3.

### 3.9. FTR auction in 6 bus system

Line parameters are obtained from [15] and shown in Table XVIII.

Table XVIII Line parameters 6 bus system

| Line | From bus | To bus | X (p.u.) | Flow limit (MW) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 0.20 | 100 |
| 2 | 1 | 4 | 0.20 | 100 |
| 3 | 1 | 5 | 0.30 | 100 |
| 4 | 2 | 3 | 0.25 | 60 |
| 5 | 2 | 4 | 0.10 | 60 |
| 6 | 2 | 5 | 0.30 | 60 |
| 7 | 2 | 6 | 0.20 | 60 |
| 8 | 3 | 5 | 0.26 | 60 |
| 9 | 3 | 6 | 0.10 | 60 |
| 10 | 4 | 5 | 0.40 | 60 |
| 11 | 5 | 6 | 0.30 | 60 |

6-bus system is given in Figure 6:


Figure 6 6-bus system [15]
Generators' cost function is a quadratic function given in Table XIX, as well as lower and upper bound:

## Table XVIII Generators' parameters

| Generator | $\mathbf{P}_{\min }$ (MW) | $\mathbf{P}_{\max }$ (MW) | Cost function |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 50 | 200 | $213.1+11.669 \mathrm{p}_{1}+0.00533 \mathrm{p}_{1}{ }^{2}$ |
| $\mathbf{2}$ | 37.5 | 150 | $200.0+10.333 \mathrm{p}_{2}+0.00889 \mathrm{p}_{2}{ }^{2}$ |
| $\mathbf{3}$ | 45 | 180 | $240.0+10.833 \mathrm{p}_{3}+0.00741 \mathrm{p}_{3}{ }^{2}$ |

where $p_{1}, p_{2}, p_{3}$ are power produced by generator at bus 1,2 and 3 , respectively. Load at the bus 4,5 and 6 is $80 \mathrm{MW}, 90 \mathrm{MW}$ and 70 MW . Generator at the bus 1 produces 50 MW , generator at bus 2 produces 101.696 MW and generator
at the bus 3 produces 88.304 MW. Thermal line limits are not violated and nodal prices are the same all over the network $12.14 € / \mathrm{MWh}$.

If the demand increases, congestion will occur and the nodal price will be different depending on nodes. Assume there is $120 \mathrm{MW}, 120 \mathrm{MW}$ and 100 MW load at buses 4,5 and 6 . Lines 5 (between bus 2 and 4) and 9 (between bus 3 and 6 ) are congested and nodal prices are different. Shadow prices for congested lines are 3.86 $€ / \mathrm{MW}$ and $1.59 € / \mathrm{MW}$. ISO collects $327.10 €$ for transmission congestion.

If participants want to hedge against price volatility, they can buy flow based option, firm point-to-point option or obligation. As lines 5 and 9 are congested, market participants compete to buy rights for those lines. Bidder $a_{1}$ wants to buy flow based option for flow gate $2->4$; his bidding price is $3.5 € / \mathrm{MW}$ and maximum bound of FTR he is willing to buy is 10 MW . Bidder $a_{2}$ competes for flowgate3->6 with the bidding price $1.5 € / \mathrm{MW}$ and upper bound 15 MW . They chose their prices according to the shadow prices in congested system. If they make bids with higher prices they can get more rights, but also they cawilln pay more than the right is worth. Bidders $b_{1}$ and $b_{2}$ are buying point-to-point obligation ( $b_{1}$ from bus 2 to 4 and bidder $b_{2}$ from bus 3 to 6 ). Upper bounds for both are 60 MW and bidding prices are $2 € / \mathrm{MW}$ and $3 € / \mathrm{MW}$. Flow based options and calculated PTDFs are shown in Table XX:

Table XIX Flow based options and PTDFs for auction

| Flowgate | Bidder $\mathrm{a}_{1}$ | Bidder a2 | Bidder $\mathrm{b}_{1}$ | Bidder $\mathrm{b}_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $1->2$ | 0 | 0 | -0.1555 | +0.004 |
| $2->1$ | 0 | 0 | +0.1555 | -0.004 |
| $1->4$ | 0 | 0 | +0.1895 | +0.001 |
| $4->1$ | 0 | 0 | -0.1895 | -0.001 |
| $1->5$ | 0 | 0 | -0.0337 | -0.005 |
| $5->1$ | 0 | 0 | +0.0337 | +0.005 |
| $2->3$ | 0 | 0 | +0.0384 | -0.1508 |
| $3->2$ | 0 | 0 | -0.0384 | +0.1508 |
| $2->4$ | 1 | 0 | +0.69 | -0.006 |
| $4->2$ | 0 | 0 | -0.69 | +0.006 |
| $2->5$ | 0 | 0 | +0.07 | -0.0077 |
| $5->2$ | 0 | 0 | -0.07 | +0.0077 |
| $2->6$ | 0 | 0 | +0.045 | +0.1675 |
| $6->2$ | 0 | 0 | -0.045 | -0.1675 |
| $3->5$ | 0 | 0 | +0.0438 | +0.1362 |
| $5->3$ | 0 | 0 | -0.0438 | -0.1362 |
| $3->6$ | 0 | 1 | -0.006 | +0.7120 |
| $6->3$ | 0 | 0 | +0.006 | -0.7120 |
| 4->5 | 0 | 0 | -0.12 | -0.0043 |
| $5->4$ | 0 | 0 | +0.12 | +0.0043 |
| $5->6$ | 0 | 0 | -0.04 | +0.1193 |
| $6->5$ | 0 | 0 | +0.04 | -0.1193 |

After the auction, ISO awards maximum amount to every bidder. Shadow prices are 0 because there are more rights that can be awarded and none of them has to pay for awarded FTRs. Bidder $a_{1}$ gets 10 MW and receives $38.6 €$ for holding the FTR for flow gate $2->4$. Bidder $a_{2}$ gets 15 MW and receives $23.85 €$. Bidder $b_{1}$ gets for flow gate 2->4 41.4 MW and $159.80 €$. Bidder $b_{2}$ gets 42.7 MW for flow gate $3->6$ and $67.92 €$.

If in the next auction bidders $a_{1}$ and $a_{2}$ will try to get more FTRs, meaning they will rise their upper bound to e.g. 30 MW . ISO awards them 30 MW and 17.564 MW. Bidder $b_{1}$ gets 44 MW and bidder $b_{2}$ gets 60 MW . Shadow prices for flow gates $2->4$ and $3->6$ are $2.91 € / \mathrm{MW}$ and $1.5 € / \mathrm{MW}$. Bidder $a_{1}$ pays $87.35 €(30 \mathrm{MW}$ at the price $2.91 € / \mathrm{MW}$ ) and receives $115.80 €$ for holding the FTR ( 30 MW multiplied by $3.86 € / \mathrm{MW})$. Bidder $a_{2}$ pays $26.32 €$ and receives $27.90 €$. Bidder $b_{1}$ pays $88.35 €$ and gets $117.19 €$. Bidder $b_{2}$ pays $64.08 €$ and gets $67.92 €$.

## 4. ENERGY STORAGE

Utilizing electricity storage flexibility can increase the reliability of supply for the consumers, especially with the increased integration of renewable energy sources characterized by limited controllability and predictability. Flexibility is the ability to provide upward and downward power adjustments to deal with short-term imbalances between generation and consumption of electric energy. This flexibility can be provided by flexible generation and consumption and electricity storage, but can also be activated in neighboring regions through interconnection capacity and by further integration of adjacent markets [16]. Electricity storage has the ability to compensate temporary power surpluses and shortages by decoupling the generation of electric energy from its consumption over time. The extent of this ability is limited by storage capacity. Although there is an increasing need for flexibility, market participants are incentivized to integrate new flexible resources only if the investment is profitable.

### 4.1. Financial storage rights

Model for calculating Financial Storage Rights is obtained from [6]. System is modeled with multiperiod DC optimal power flow. The objective is to minimize the intertemporal production cost. Constraints and dual variables are given in (9):

$$
\lambda_{i, t}: p_{i, t}=\sum_{j \varepsilon S_{i}}\left(\text { charge }_{j, t}+\text { discharge }_{j, t}\right)+\sum_{j} p_{i j, t} \#(9)
$$

Where $\lambda_{i, t}$ is nodal price for bus $i$ in the time $t$. Power $p_{i, t}$ injected in the bus or withdrawn from bus $i$ is equal to sum of all power flow on the transmission lines
that start (end) at that bus plus power charged into storage chargej,t or discharged from storage discharge $j, t$ (of course only if storage is installed at that bus).

All generators have lower $p_{\min }$ and upper $p_{\max }$ power limit, modelled by (10):

$$
p_{\min } \leq p_{g_{i, t}} \leq p_{\max } \#(10)
$$

$\mu_{i j, t}$ is dual variable for line shadow price of line between buses $i$ and $j$. Power flow on each transmission line is constrained by thermal capacity, modelled by (11):

$$
\mu_{i j, t}: p_{i j, t} \leq \text { limit }_{i j} \#(11)
$$

State of charge of storage $i\left(\mathrm{SoCi}_{\mathrm{i}, \mathrm{t}+1}\right)$ in the time $t+1$ is equal to state of charge $S o C_{i, t}$ in previous time step $t$ and reduced by leakage coefficient $\alpha_{i}$, plus charging (charge $i_{i, t}$ ) reduced by charge loss coefficient $\eta_{i}^{+}$and discharging (discharge ${ }_{i, t}$ ) reduced by discharge loss coefficient $\eta_{i}^{-}$(12):

$$
\operatorname{SoC}_{i, t+1}=\alpha_{i} \cdot \text { SoC }_{i, t}+\eta_{i}^{+} \cdot \text { charge }_{i, t}+\eta_{i}^{-} \cdot \text { discharge }_{i, t} \#(12)
$$

Storage state-of-charge at the beginning and at the end of the day is the same, as modelled by (13):

$$
S o C_{i, 0}=S_{S O}^{i, 24} 1=0 \#(13)
$$

Charging charge $i_{i, t}$ and discharging discharge $i_{i, t}$ are constrained by charge charge_max i,t and discharge rate limits dicharge_max ind $^{(14)}$ and (15):

$$
\begin{gathered}
\vartheta_{i}^{+}: \text {charge }_{i, t} \leq \text { charge_max }_{i, t} \#(14) \\
\vartheta_{i}^{-}: \text {discharge }_{i, t} \geq \text { dicharge_max }_{i, t} \#(15)
\end{gathered}
$$

$\vartheta_{i}^{+}$and $\vartheta_{i}^{-}$are dual variables for charging and discharging. If storage is charged or discharged at the maximum rate, charge ${ }_{i, t}=$ charge_max $_{i, t}$ or discharge $i_{i, t}=$ discharge_max $i_{i, t}$, dual variables $\vartheta_{i}^{+}$and $\vartheta_{i}^{-}$and are grater then 0 . If congestion occurs, the owner of a power capacity right (PCR) collects $\vartheta_{i}^{+} \cdot \overline{\operatorname{charge}}_{i, t}-$ $\vartheta_{i}^{-} \cdot \overline{\text { discharge }}_{i, t}$.
$\overline{\operatorname{charge}}_{i, t}$ and $\overline{\text { dıscharge }}_{i, t}$ are quantities of power that power capacity right owner has.

Charging is always positive, meaning charge_max $i_{i, t}$ has positive values as modelled by (16). Discharging is negative and dicharge_max $i_{i, t}$ is a negative values (17).

$$
\begin{gathered}
\text { charge }_{i, t} \geq 0 \#(16) \\
\text { discharge }_{i, t} \leq 0 \#(17)
\end{gathered}
$$

State of charge $S_{i, t}$ is constrained by storage capacity SoC_max in $_{i, t}$ (18):

$$
\delta_{i, t}: \text { SoC }_{i, t} \leq \text { SoC_max }_{i, t} \#(18)
$$

State of charge is always grater or equal to zero (19):

$$
S o C_{i, t} \geq 0 \#(19)
$$

$\delta_{i, t}$ is dual variable for energy capacity right. If storage is congested $\operatorname{SoC} C_{i, t}=$ SoC_max ${ }_{i, t}$ and $\delta_{i, t}$ is greater than 0 and owner of energy capacity right collects $\delta_{i, t} \cdot \overline{S o C}_{i, t}$.
$\overline{S O C}_{i, t}$ is the quantity of energy that owner of the energy capacity rights owns and it has to be less than SoC_max it .

Together, power capacity right and energy capacity right are financial storage rights. If the owner of storage rights has right for power and energy and if congestion occurs in storage, he will collect $\vartheta_{i}^{+} \cdot \overline{\operatorname{charge}}_{i, t}-\vartheta_{i}^{-} \cdot \overline{\operatorname{dscharge}}_{i, t}+\delta_{i, t}$. $\overline{S o C}_{i, t}$.

### 4.2. 3-bus system with one energy storage

A 3-bus system, as shown in Figure 2, is analyzed. All transmission lines have the same reactance $X=1 p . u$. and thermal limit of 60 MW . Generators are located at the bus 1 and 2. Generator costs are $10 € / \mathrm{MWh}$ and $20 € / \mathrm{MWh}$. Load and storage are located at bus 3. Energy storage is characterized as follow: capacity of storage is 18 MWh, charge and discharge rate limits are 10 MW and 9 MW , charge loss coefficient is 0.95 and discharge loss coefficient is $1 / 0.85$. Energy leakage coefficient is 0.9 . We consider the case with 6 -time periods. Results are shown in Table XXI, Table XXII and Table XXIII:

Table XXI Generation, load, charging, discharging, state of charge

| Time (h) | $\mathbf{G}_{\mathbf{1}}(\mathbf{M W )}$ | $\mathbf{G}_{\mathbf{2}}$ (MW) | Load(MW) | Charge(MW) | Discharge (MW) | SoC (MWh) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 30 | 0 | 20 | 10 | 0 | 9.5 |
| 2 | 77.2675 | 25.4650 | 110 | 0 | -7.2675 | 0 |
| 3 | 59.9415 | 0 | 50 | 9.9415 | 0 | 9.4444 |
| 4 | 80 | 0 | 70 | 10 | 0 | 18 |
| 5 | 69 | 42 | 120 | 0 | -9 | 5.6118 |
| 6 | 74.2930 | 31.4140 | 110 | 0 | -4.2930 | 0 |

Table XXII Nodal prices

| Time (h) | Bus 1 (€/MWh) | Bus 2 (€/MWh) | Bus 3 (€/MWh) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 10 | 10 | 10 |
| $\mathbf{2}$ | 10 | 20 | 30 |
| $\mathbf{3}$ | 10 | 10 | 10 |
| $\mathbf{4}$ | 10 | 10 | 10 |
| $\mathbf{5}$ | 10 | 20 | 30 |
| $\mathbf{6}$ | 10 | 20 | 30 |

Table XXII Power flow

| Time (h) | Line 12 (MW) | Line 23 (MW) | Line 13 (MW) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 10 | 10 | 20 |
| $\mathbf{2}$ | 17.2675 | 42.7325 | 60 |
| $\mathbf{3}$ | 19.9805 | 19.9805 | 39.9610 |
| $\mathbf{4}$ | 26.6667 | 26.6667 | 53.333 |
| $\mathbf{5}$ | 9 | 51 | 60 |
| $\mathbf{6}$ | 14.2930 | 45.707 | 60 |

Total cost is $5882.6 €$. As one can notice, congestion and different nodal prices occur in second, fifth and sixth hour. Line shadow prices for the congested line, in all three cases, are $30 € / \mathrm{MW}$. When congestion occurs, owner of the Financial Transmission Right for the line 13 will be paid $30 € / \mathrm{MW}$ times the amount of reserved MW on the transmission line 13.

There is also congestion of energy storage. As one can notice, in first and fourth hour, energy storage is charged at the maximum rate 10 MW and shadow prices for power capacity right are $11.80 € / \mathrm{MW}$ and $1.11 € / \mathrm{MW}$. In fifth hour energy storage is discharged at maximum rate of -9 MW and shadow price is $3 € / \mathrm{MW}$. In fifth hour energy storage is full and the shadow price for energy capacity right is 8.96 €/MWh.

Because of congestion and price differences in system, payment collected from load exceeds payment to generators. ISO has $5717.4 €$ surplus. Amount of $5400 €$ is reserved for FTRs owners and $317.4 €$ for Storage Rights owners ( $161.26 €$ for power capacity right and $156.14 €$ for energy capacity right).

If someone holds the transmission right for line 13 in second, fifth or sixth hour, he will be paid $30 €$ for each MW he owns. If storage right owner holds the right for power and energy capacity in fifth hour, he will be paid $3 € / \mathrm{MW}$ for reserved charging MW and 8.96€/MWh for reserved MWh.

### 4.3. 30-bus system

30-bus system is shown in Figure 7:


Figure 7 30-bus system [17]
There are 6 generators in the system located at buses 1, 2, 13, 22, 23 and 27. Generators' cost functions are given in Table XXIV:

Table XXIIIV Generators' cost function

| Generator | Cost function (€) |
| :---: | :---: |
| $\mathbf{G}_{\mathbf{1}}$ | $0.02 \mathrm{p}_{1}{ }^{2}+10 \mathrm{p}_{1}$ |
| $\mathbf{G}_{\mathbf{2}}$ | $0.0175 \mathrm{p}_{2}{ }^{2}+15 \mathrm{p}_{2}$ |
| $\mathbf{G}_{\mathbf{1 3}}$ | $0.0625 \mathrm{p}_{13}{ }^{2}+15 \mathrm{p}_{13}$ |
| $\mathbf{G}_{\mathbf{2}}$ | $0.00834 \mathrm{p}_{22^{2}}{ }^{2} 17 \mathrm{p}_{22}$ |
| $\mathbf{G}_{\mathbf{2 3}}$ | $0.025 \mathrm{p}_{23}{ }^{2}+20 \mathrm{p}_{23}$ |
| $\mathbf{G}_{\mathbf{2 7}}$ | $0.025 \mathrm{p}_{27}{ }^{2}+16 \mathrm{p}_{27}$ |

Energy storages are located at buses 3 and 30. Storage parameters (charge and discharge rate limit, energy capacity, charge and discharge loss coefficient and energy leakage coefficient) are given in Table XXV:

Table XXIV Storage parameters

| Storage | Charge <br> limit <br> (MW) | Discharge <br> limit <br> (MW) | Energy <br> capacity <br> (MWh) | Charge <br> loss <br> coefficient | Discharge <br> loss <br> coefficient | Energy <br> leakage <br> coefficient |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S}_{3}$ | 20 | -15 | 100 | 0.98 | 1 | 0.98 |
| $\mathrm{~S}_{30}$ | 15 | -20 | 100 | 0.98 | 1 | 0.98 |

Load is shown in Table XXVI:

Table XXVI Load characteristics

| Hour | Load <br> (MW) | Hour | Load <br> (MW) | Hour | Load <br> (MW) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 523.60 | $\mathbf{9}$ | 652.70 | $\mathbf{1 7}$ | 751.50 |
| $\mathbf{2}$ | 458.40 | $\mathbf{1 0}$ | 723.80 | $\mathbf{1 8}$ | 773.40 |
| $\mathbf{3}$ | 428.10 | $\mathbf{1 1}$ | 749.10 | $\mathbf{1 9}$ | 789.40 |
| $\mathbf{4}$ | 404.90 | $\mathbf{1 2}$ | 800.00 | $\mathbf{2 0}$ | 855.50 |
| $\mathbf{5}$ | 404.20 | $\mathbf{1 3}$ | 809.30 | $\mathbf{2 1}$ | 910.80 |
| $\mathbf{6}$ | 421.40 | $\mathbf{1 4}$ | 779.60 | $\mathbf{2 2}$ | 897.80 |
| $\mathbf{7}$ | 433.50 | $\mathbf{1 5}$ | 758.30 | $\mathbf{2 3}$ | 852.70 |
| $\mathbf{8}$ | 498.90 | $\mathbf{1 6}$ | 740.60 | $\mathbf{2 4}$ | 756.50 |

Line parameters (reactance and thermal limit) are shown in Table XXVII:
Table XXVII Line parameters

| Line | From <br> bus | To <br> bus | Reactance <br> p.u. | Limit <br> MW | Line | From <br> bus | To <br> bus | Reactance <br> p.u. | Limit <br> MW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 1 | 2 | 0.26 | 87 | $\mathbf{2 2}$ | 12 | 13 | 0.22 | 80 |
| $\mathbf{2}$ | 1 | 3 | 0.19 | 112 | $\mathbf{2 3}$ | 12 | 14 | 0.13 | 60 |
| $\mathbf{3}$ | 2 | 4 | 0.17 | 120 | $\mathbf{2 4}$ | 12 | 15 | 0.27 | 60 |
| $\mathbf{4}$ | 2 | 5 | 0.24 | 90 | $\mathbf{2 5}$ | 12 | 16 | 0.21 | 50 |
| $\mathbf{5}$ | 2 | 6 | 0.20 | 130 | $\mathbf{2 6}$ | 14 | 15 | 0.28 | 50 |
| $\mathbf{6}$ | 3 | 4 | 0.18 | 95 | $\mathbf{2 7}$ | 15 | 18 | 0.27 | 60 |
| $\mathbf{7}$ | 4 | 6 | 0.24 | 95 | $\mathbf{2 8}$ | 15 | 23 | 0.15 | 65 |
| $\mathbf{8}$ | 4 | 12 | 0.22 | 90 | $\mathbf{2 9}$ | 16 | 17 | 0.22 | 50 |
| $\mathbf{9}$ | 5 | 7 | 0.28 | 70 | $\mathbf{3 0}$ | 18 | 19 | 0.20 | 50 |
| $\mathbf{1 0}$ | 6 | 7 | 0.24 | 75 | $\mathbf{3 1}$ | 19 | 20 | 0.18 | 50 |
| $\mathbf{1 1}$ | 6 | 8 | 0.21 | 70 | $\mathbf{3 2}$ | 21 | 22 | 0.27 | 80 |
| $\mathbf{1 2}$ | 6 | 9 | 0.56 | 95 | $\mathbf{3 3}$ | 22 | 24 | 0.33 | 50 |
| $\mathbf{1 3}$ | 6 | 10 | 0.21 | 70 | $\mathbf{3 4}$ | 23 | 24 | 0.38 | 50 |
| $\mathbf{1 4}$ | 6 | 28 | 0.11 | 70 | $\mathbf{3 5}$ | 24 | 25 | 0.21 | 55 |
| $\mathbf{1 5}$ | 8 | 28 | 0.26 | 70 | 36 | 25 | 26 | 0.40 | 55 |
| $\mathbf{1 6}$ | 9 | 11 | 0.24 | 75 | $\mathbf{3 7}$ | 25 | 27 | 0.42 | 90 |
| $\mathbf{1 7}$ | 9 | 10 | 0.26 | 75 | $\mathbf{3 8}$ | 27 | 28 | 0.60 | 55 |
| $\mathbf{1 8}$ | 10 | 20 | 0.13 | 60 | $\mathbf{3 9}$ | 27 | 29 | 0.45 | 50 |
| $\mathbf{1 9}$ | 10 | 17 | 0.20 | 60 | $\mathbf{4 0}$ | 27 | 30 | 0.20 | 50 |
| $\mathbf{2 0}$ | 10 | 21 | 0.20 | 60 | $\mathbf{4 1}$ | 29 | 30 | 0.30 | 55 |
| $\mathbf{2 1}$ | 10 | 22 | 0.19 | 65 | - | - | - | - | - |

The results are shown in Figure 8:


Figure 8 Load and generation curve
As one can notice that in the first, second and third hour generation is equal to the demand. Because system contains energy storages, from fourth to eighth hours generators produce more energy (demand is low and cheap generators can produce more energy to save money in the future when demand will be higher and system will need to use more expensive generators). Saved energy is discharged from storages during tenth, eleventh, twelfth and thirteenth hour. During four hours before daily peak, generators also produce more energy and then during twenty-first, twenty-second and twenty-third hour storages are discharged again.

Line 1 (between bus 1 and 2) is congested during the whole day and line 2 (between bus 1 and 3) is congested from thirteenth to twenty-fourth hour. When congestion occurs, there are differences between nodal prices and generators are redispatched from optimal production to satisfy line thermal limit. Line shadow price for line 1 and line 2 are shown in Table XXVIII and Table XXIX:

Table XXVIII Shadow prices for line 1

| Hour | €/MW | Hour | €/MW | Hour | €/MW |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 3.62 | $\mathbf{9}$ | 5.15 | $\mathbf{1 7}$ | 4.94 |
| $\mathbf{2}$ | 2.91 | $\mathbf{1 0}$ | 5.67 | $\mathbf{1 8}$ | 4.89 |
| $\mathbf{3}$ | 2.66 | $\mathbf{1 1}$ | 5.92 | $\mathbf{1 9}$ | 4.84 |
| $\mathbf{4}$ | 2.62 | $\mathbf{1 2}$ | 6.46 | $\mathbf{2 0}$ | 4.95 |
| $\mathbf{5}$ | 2.51 | $\mathbf{1 3}$ | 6.32 | $\mathbf{2 1}$ | 4.61 |
| $\mathbf{6}$ | 2.61 | $\mathbf{1 4}$ | 5.68 | $\mathbf{2 2}$ | 4.71 |
| $\mathbf{7}$ | 2.70 | $\mathbf{1 5}$ | 5.89 | $\mathbf{2 3}$ | 4.28 |
| $\mathbf{8}$ | 3.50 | $\mathbf{1 6}$ | 5.19 | $\mathbf{2 4}$ | 5.94 |

Table XXVIII Shadow prices for line 2

| Hour | €/MW | Hour | €/MW |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 3}$ | 1.31 | $\mathbf{1 9}$ | 5.79 |
| $\mathbf{1 4}$ | 2.52 | $\mathbf{2 0}$ | 7.64 |
| $\mathbf{1 5}$ | 1.02 | $\mathbf{2 1}$ | 10.52 |


| $\mathbf{1 6}$ | 2.64 | $\mathbf{2 2}$ | 9.94 |
| :--- | :--- | :--- | :--- |
| $\mathbf{1 7}$ | 3.82 | $\mathbf{2 3}$ | 9.64 |
| $\mathbf{1 8}$ | 4.80 | $\mathbf{2 4}$ | 0.84 |

Charging and discharging at bus 3 is shown in Figure 9 and state of charge during the day in Figure 10:


Figure 9 Charging and discharging - Storage 3


Figure 10 State of charge - Storage 3
Energy capacity is 100 MWh and, as we can see in Figure 10, storage is never full. Storage is charging in fifth, sixth, seventh and eight hour at maximum rate and is discharging in ninth, tenth, twelfth, thirteenth, twenty-first and twentysecond hour at maximum rate. Owner of the power capacity right will collect for each hour price shown in Table XXX times reserved MW of charging/discharging capacity.

Table XXIX Price for power capacity right - Storage 3

| Hour | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{2 1}$ | $\mathbf{2 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PCR <br> (€/MW) | 0.0089 | 0.2627 | 0.5593 | 0.2393 | 0.0494 | 0.1271 | 0.0098 | 0.1756 | 0.7442 | 0.0053 |

Charging and discharging at the bus 30 is shown in Figure 11 and state of charge during the day in Figure 12:


Figure 11 Charging and discharging - Storage 30


Figure 12 State of charge - Storage 30
Storage is charging at maximum rate during the fourth, fifth, sixth, seventh and eight hour and is discharging at the maximum rate during tenth and twelfth hour. Owner of a power capacity right will collect revenue as shown in Table XXXI times reserved amount of MW:

Table XXXI Price for power capacity right - Storage 30

| Hour | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{1 0}$ | $\mathbf{1 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PCR (€/MW) | 0.1597 | 0.4099 | 0.6638 | 0.9625 | 0.5375 | 0.1063 | 0.0734 |

### 4.4. Auction for Power Capacity Rights

The objective of the auction is to maximize profit. Bidders submit their offers for charging and discharging capacity they want to reserve and the price they are willing to pay. If charging and discharging for each hour are at the lower level than rate limits, bidders will get their maximum amount of preferred right and do not need to pay for those rights because there are still more rights that can be awarded. Maximum desirable capacity for charging has a positive sign and for discharging a negative sign.

Energy storage is empty in the beginning and it is characterized by energy capacity, charge and discharge rate limits and energy leakage coefficient. There is no charge and discharge loss coefficients. As charging and discharging have opposite sign and they are simultaneously feasible, more rights can be awarded. Dual variables are related to charge and discharge rate limits. For the purpose of this model, it is assumed that charging and discharging is the same variable. If the variable is positive, the storage is charging, and if it is negative, storage is discharging.

### 4.5. Auction example

Storage energy capacity is 60 MWh , energy leakage coefficient is 0.9 , charge and discharge rate limits are 15 MW and -10 MW. Storage is operating for four hours.

There are three bidders ( $a_{1}, a_{2}$, and $a_{3}$ ) competing for charging capacity and two bidders for discharging capacity ( $b_{1}$ and $b_{2}$ ). Their prices and desirable power capacity are shown in Table XXXII and Table XXXIII:

## Table XXXII Bidding prices ( $€ / M W$ )

| Bidder | $\mathbf{a}_{\mathbf{1}}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{\mathbf{3}}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{b}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hour 1 | 0 | 6 | 7 | 5 | 8 |
| Hour2 | 8 | 5 | 3 | 4 | 2 |
| Hour 3 | 0 | 0 | 5 | 5 | 15 |
| Hour 4 | 4 | 6 | 2 | 10 | 5 |

Table XXXIII Desirable charging and discharging capacity (MW)

| Bidder | $\mathbf{a}_{\mathbf{1}}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{\mathbf{3}}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{b}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hour 1 | 0 | 14 | 12 | -7 | -8 |
| Hour2 | 12 | 10 | 11 | -8 | -5 |
| Hour 3 | 0 | 0 | 8 | -10 | -10 |
| Hour 4 | 10 | 12 | 10 | -5 | -10 |

Awarded charging and discharging capacity is shown in Table XXXIV:
Table XXXIIIV Awarded charging and discharging capacity (MW)

| Bidder | $\mathbf{a}_{\mathbf{1}}$ | $\mathbf{a}_{\mathbf{2}}$ | $\mathbf{a}_{\mathbf{3}}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{b}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hour 1 | 0 | 14 | 12 | -7 | -8 |
| Hour2 | 12 | 10 | 6 | -8 | -5 |
| Hour 3 | 0 | 0 | 8 | -8 | -10 |
| Hour 4 | 10 | 12 | 8 | -5 | -10 |

As one can see, bidder $a_{3}$ did not get desirable amount of charging capacity in second and fourth hour because storage is charging at the rate limit 15 MW (red color in the Table XXXIV shows the differences between desirable and awarded amount of MW). Shadow price for second and fourth hour is $3 € / M W$ and $2 € / M W$. In the third hour storage is discharging at rate limit -10 MW and discharging shadow price for third hour is $5 € / \mathrm{MW}$. In the first hour, storage is not congested and every bidder gets desirable capacity without paying for that right (there are still more rights that can be awarded). Table XXXV shows how much each bidder has to pay for awarded capacity rights:

Table XXXIV Payment ( $€$ )

| Bidder | $\mathbf{a}_{1}$ | $\mathbf{a}_{2}$ | $\mathbf{a}_{3}$ | $\mathbf{b}_{\mathbf{1}}$ | $\mathbf{b}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hour $\mathbf{1}$ | 0 | 0 | 0 | 0 | 0 |
| Hour2 | 36 | 30 | 18 | 24 | 15 |
| Hour 3 | 0 | 0 | 40 | 40 | 50 |
| Hour 4 | 20 | 24 | 16 | 10 | 20 |

## 5. CONCLUSION

When transmission lines are congested, differences in LMPs occur. Low-cost generators power is reduced (or are shut down) and high-cost generators are redispatched. Generators sell energy at one price and load buys at the different one. Payments which ISO collects from loads exceed payment to generators and the surplus ISO collects during congestion can be awarded to the market participants to hedge against congestion charges and price volatility. FTRs are financial instrument that enables market participants to avoid risk in price differences. Market players can require FTRs in the auction process (annual, monthly or daily auction) or via bilateral contracts. Once FTRs are awarded in auction, they can easily be traded in different auctions or bilateral contracts. Market players can require FTRs to avoid paying extra cost due to congestion or make profit by reserving some amount of MW (because FTRs are based on reserved amount of MW, not actual MW delivered). This means that everyone can participate in FTRs auction or bilateral contracts to increase their profit. For have more FTRs awarded,
market participants need to act strategically. This means they will change their bidding prices or even submit offers for other flowgates or point-to-point rights.

Energy storage are sources of flexibility and reliability, they enable lower operating cost in the power system and decrease generation during daily's peaks. Storages usually profit through intertemporal arbitrage (load shifting and peak shaving). Storages can be compared to transmission lines. Transmission lines move power spatially and storages move power forward in time. When storages are congested, owners of power and energy capacity rights (together known as financial storage rights) collect money from ISO. Passive storages profit through sales of rights.

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