PLACEMENT STUDY OF COMPOSITE VIBRATION ISOLATION SYSTEM FOR MACHINERY EQUIPMENT

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Abstract:

In traditional vibration isolation and placement studies, vibration participation of foundation is often ignored, and optimizations are focused on the improvements of isolated object and isolation system. In this paper, clamped thin plate was considered as the foundation, and "machinery equipment vibration isolator foundation" was proposed as a composite vibration isolation system. Based on the theory of mechanical four-pole connection, transmitted force from equipment to plate was derived, and the intermediate variable was the displacement admittance of contact points (isolators) on the plate. To peruse the optimal placement, a novel procedure has been programmed, and two cases of arrangement direction have been involved. This study could give a lot of inspiration for vibration isolation and placement strategies of engineering equipment so that extended researches based on this could be performed.

1 Introduction

Machinery equipment for engineering application includes rotating, reciprocating, impacting and other typical machines commonly used in manufacturing. Advances in science and technology are gradually prompting the fast equipment move towards a high-tech era. Vibration is a key problem in the process of utilizing machinery equipment, and isolating and attenuating the harmful vibration from equipment to surrounding environment has played a significant role.

Isolation methods are commonly utilized for vibration of machinery equipment, and the performance is closely related to an appropriate design. Optimal placement of vibration isolation system on the affiliated foundation is critical, which can greatly promote the isolation performance. Hui et al. [1] proposed a new design concept for arrangement of isolators on the floor, and the isolation performance was improved. In Ref. [2], dynamic sensitivity analysis method was used to derive optimization criteria for the placement of the piezoelectric stack actuator. In Ref. [3], multi degree of freedom suspension vibration isolation

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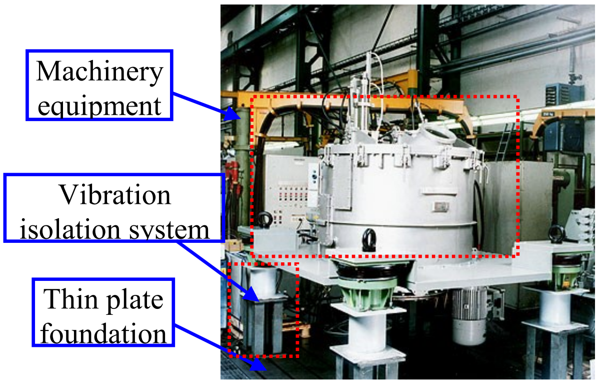
system was taken as the object and the vibration isolation system was built by the substructure method and the power flow transmissibility was used as the evaluation index; finally, optimal arrangement scheme of isolators was gained. In Ref. [4], the optimal isolator placement/position was determined and found slightly dependent upon pump power, and improvements in gain and noise figure were measured for the optimal isolator location. Chinedum et al. [5] used the 2D dynamics of a passively isolated system to show that coupling the vibration modes of the isolated system by altering the location of the isolators could reduce residual vibrations.

Seen from the aforementioned studies, participation of foundation vibration is not often taken into account when the placement study of an isolation system is performed, therefore the focus is the acquired optimal location on the foundation guided by some optimization strategies. However, when the external stimulus is high-frequency, and the foundation stiffness is low, then the isolated vibration cannot reach an ideal effect if traditional strategies are still adopted; they even make the dynamic performance deteriorate. Therefore, foundation vibration of placement optimization in engineering practice is critical. A thin plate is a common foundation form for machinery equipment, which is simply supported in view of computational simplicity in general. However, the clamped form cannot be ignored for its wide range of applications, such as the floors of industrial buildings. Amabili et al. [6] performed a study of large-amplitude forced vibration of a clamped stainless steel plate excited by centralized harmonic force. A comparative study of active and passive vibration isolation of the clamped thin plate, which was aimed at examining piezoelectric materials in active vibration control, was conducted in Ref. [7]. A free vibration problem of clamped thin plates using quasi Green function was solved in Ref. [8]. Arenas [9] derived the general expression of clamped thin plate vibration by means of virtual work principle, and it was not necessary to solve simultaneous equations, therefore, these solutions were easy to be calculated and programmed; meantime, complicated symmetric characteristic of Rayleigh-Ritz the method could be overcome here and it could create favourable conditions for equipment, isolators and thin plate foundation as a composite system.

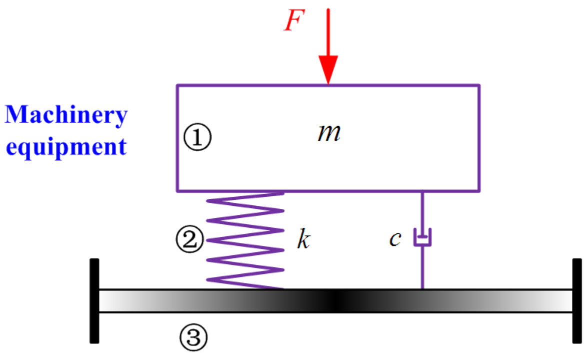
In this paper, a novel composite vibration isolation system for machinery equipment is proposed; and force transmissibility is derived by using / from admittance based on the mechanical four-pole connection theory, which effectively combines the equipment, isolators and thin plate foundation as a whole. Finally, the placement study is performed with a detailed procedure.

2 Composite vibration isolation system

Vibration isolation system of typical machinery equipment is shown in Fig. 1.



(a) Typical machinery equipment



(b) Simplified vibration isolation model

Figure 1. Composite vibration isolation system.

In Fig.1 (b), ① denotes the machinery equipment, ② denotes the isolators, including the stiffness and damping components, and ③ denotes the clamped rectangular thin plate foundation. M is the mass of machinery equipment which is supported by four isolators mounted on the thin plate, and the stiffness and damping can be denoted as k , c . The geometry of the plate is $a \times b \times h$, and the equipment can be simplified as a cuboid whose plane size is $e \times f$. The centralized harmonic excitation force generated by the equipment is $Fe^{j\omega t}$, F is the amplitude, and ω is the circular frequency. ①②③ considered as a composite isolation system which can be expressed in Cartesian coordinates are shown in Fig. 2; O is the coordinate origin, A, B, C and D are the four corners of equipment, respectively.

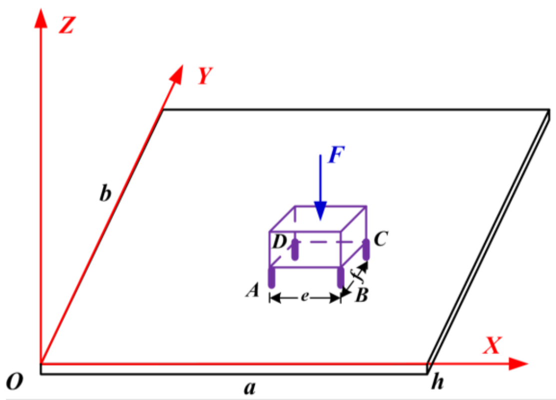


Figure 2. Schematic diagram of the vibration isolation system expressed in Cartesian coordinate system.

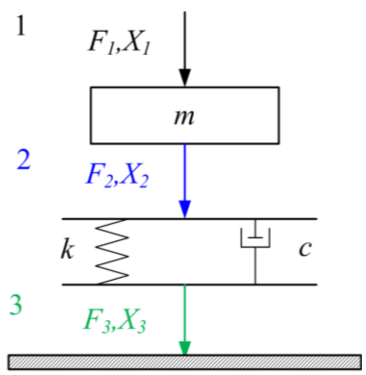


Figure 3. Schematic diagram of four-pole connection of the composite vibration isolation system.

According to the connection properties of mechanical four-pole parameters [10], the following can be derived,

$$\begin{bmatrix} F_1 \\ X_1 \end{bmatrix} = [m_{ij}][k_{ij}^*]\begin{bmatrix} F_3 \\ X_3 \end{bmatrix} \quad (1)$$

where $[m_{ij}]$ is the four-pole expression of mass, $[k_{ij}^*]$ is the four-pole expression of stiffness and damping in parallel, and $[m_{ij}] = \begin{bmatrix} 1 & -m\omega^2 \\ 0 & 1 \end{bmatrix}$, $[k_{ij}^*] = \begin{bmatrix} 1 & 0 \\ \frac{1}{k+ic\omega} & 1 \end{bmatrix}$. Thus Eq. (1) can be rewritten as,

$$\begin{aligned} \begin{bmatrix} F_1 \\ X_1 \end{bmatrix} &= \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}\begin{bmatrix} F_3 \\ X_3 \end{bmatrix} \\ &= \begin{bmatrix} \left(1 - \frac{m\omega^2}{k+ic\omega}\right) & -m\omega^2 \\ \frac{1}{k+ic\omega} & 1 \end{bmatrix}\begin{bmatrix} F_3 \\ X_3 \end{bmatrix} \end{aligned} \quad (2)$$

The following formula can be obtained easily from Eq. (2),

$$F_1 = \Omega_{11}F_3 + \Omega_{12}X_3 \quad (3)$$

Suppose $F_3 / X_3 = 1/M_A$, M_A denotes the displacement admittance of A point on the thin plate, thus Eq. (3) can be rewritten by dividing F_3,

$$F_3 = \frac{1}{\Omega_{11} + \Omega_{12}\frac{X_3}{F_3}}F_1 = \frac{1}{\Omega_{11} + \Omega_{12}M_A}F_1 \quad (4)$$

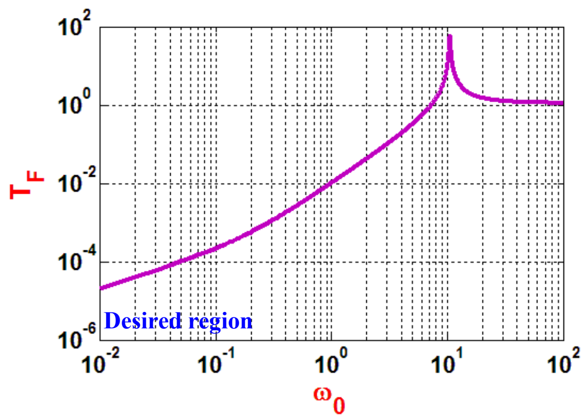
Thus the transmissibility is:

$$T_F = \left|\frac{F_3}{F_1}\right| = \frac{\sqrt{k^2 + c^2\omega^2}}{\sqrt{[(1-m\omega^2 M_A)k - m\omega^2]^2 + c^2\omega^2(1-m\omega^2 M_A)^2}} \quad (5)$$

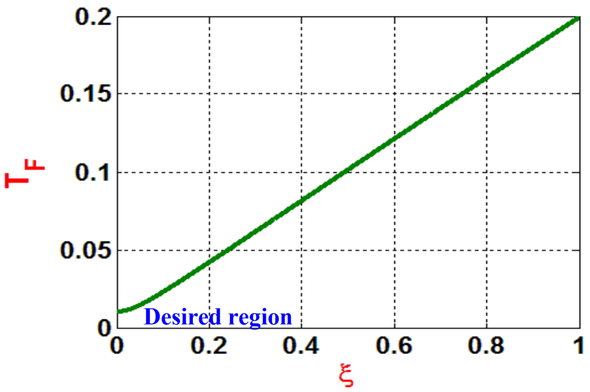
Suppose $\omega_0 = \sqrt{k/m}$, $\xi = c/(2m\omega_0)$, thus Eq. (5) can be rewritten as,

$$T_F = \frac{\omega_0\sqrt{\omega_0^2 + 4\xi^2\omega^2}}{\sqrt{[(1-m\omega^2 M_A)\omega_0^2 - \omega^2]^2 + 4\xi^2\omega^2\omega_0^2(1-m\omega^2 M_A)^2}} \quad (6)$$

Seen from Eq. (6), T_F is a complicated function with variables $\omega_0, \omega, m, \xi, M_A$, for certain working condition, ω, m will be regarded as prerequisites, then T_F can be denoted as $\Gamma(\omega_0, \xi, M_A)$.



(a) $T_F - \omega_0$



(b) $T_F - \xi$

Figure 4. Variation curves of T_F .

Seen from Fig. 4, selecting ω_0 and ξ from the desired region is intuitive for ideal transmissibility T_F . (Admittance M_A of connected points of isolators is a complicated intermediate variable, which will be investigated as follows).

3 Numerical simulation of clamped thin plate vibration

Arenas, J. P. derived a general displacement form of clamped thin plate vibration forced/caused by harmonic excitation,

$$X(x,y) = F_0 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\Psi_{mn}(x,y)\Psi(x',y')}{B(I_1 I_2 + 2I_3 I_4 + I_5 I_6) - \rho_s \omega^2 I_2 I_6} \quad (7)$$

where F_0 is the force amplitude; $B = Eh^3/[12(1-\upsilon^2)]$ is the bending stiffness of thin plate; E is the Young's modulus; υ is the Poisson's ratio; $\rho_s = \rho h$ is the surface density of thin plate, and ρ is the volume density; (x, y) is a random discrete point on the thin plate; (x', y') is the acting point of force on the thin plate. This analytical solution of a clamped form of a thin plate is aimed at simulating the foundation of industrial equipment installed upstairs.

Vibration mode function $\Psi_{mn}(x,y)$ can be expressed as,

$$\Psi_{mn}(x,y) = \theta_m(x)\xi_n(y)$$

$$\begin{aligned} \theta_m(x) &= J\left(\frac{\beta_m x}{a}\right) - \frac{J(\beta_m)}{H(\beta_m)} H\left(\frac{\beta_m x}{a}\right), \\ \zeta_n(y) &= J\left(\frac{\beta_n y}{b}\right) - \frac{J(\beta_n)}{H(\beta_n)} H\left(\frac{\beta_n y}{b}\right); \end{aligned} \quad (8)$$

$$J(\bullet) = \cosh(\bullet) - \cos(\bullet), H(\bullet) = \sinh(\bullet) - \sin(\bullet),$$

and β_i is the root of $\cosh(\beta)\cos(\beta) = 1$;

$$I_2 I_6 = \frac{ab}{\beta_m \beta_n} L_m L_n; I_3 I_4 = \frac{\beta_m \beta_n}{ab} R_m R_n;$$

$$I_1 = I_6\left(\frac{\beta_m}{a}\right)^4; I_5 = I_2\left(\frac{\beta_n}{b}\right)^4;$$

$$\begin{aligned} L_i = {} & \frac{(1+D_i^{\,2})\sinh(2\beta_i)}{4} + \sinh(\beta_i)[2D_i \sin(\beta_i) - (1-D_i^{\,2})\cos(\beta_i)] \\ & - (1+D_i^{\,2})\sin(\beta_i)\cosh(\beta_i) + (1-D_i^{\,2})\sin(\beta_i)\cos(\beta_i) + \beta_i \\ & - \frac{D_i[1+\cosh(2\beta_i)]}{2} + D_i\cos^2(\beta_i); \end{aligned}$$

$$\begin{aligned} R_i = {} & \frac{(1+D_i)^2\sinh(2\beta_i)}{4} - \frac{D_i\cosh(2\beta_i)}{2} - \frac{(1-D_i^{\,2})\sin(\beta_i)\cos(\beta_i)}{2} \\ & - D_i\cos^2(\beta_i) - D_i^{\,2}\beta_i + \frac{3D_i}{2}; D_i = \frac{J(\beta_i)}{H(\beta_i)}. \end{aligned}$$

Suppose that the generated force from machinery equipment to thin plate is excited at the center of the equipment, and according to the superposition principle, displacement of the point A excited by A, B, C and D respectively, equals to the algebraic sum of each force acting alone, thus the displacement of A can be written as,

$$X_A = X_{A\to A} + X_{B\to A} + X_{C\to A} + X_{D\to A} \quad (9)$$

In Eq. (9), subscript $A \to A$ denotes the displacement of A caused by the A point force, and the other meanings are similar. Then Eq. (9) can be further rewritten as,

$$X_{A\to A}=F_A\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_a,y_a)}{\Delta}$$

$$X_{B\to A}=F_B\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_b,y_b)}{\Delta}$$

(10)

$$X_{C\to A}=F_C\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_c,y_c)}{\Delta}$$

$$X_{D\to A}=F_D\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_d,y_d)}{\Delta}$$

where $\Delta=B(I_1I_2+2I_3I_4+I_5I_6)-\rho_s\omega^2I_2I_6$, (x_a,y_a) , (x_b,y_b), (x_c,y_c), (x_d,y_d) are the coordinates of A, B, C and D.

In practice, the infinite series expansion of Eq. (10) are often truncated into a finite term, and in this paper, $m,n=1,2,\cdots,6$, thus the admittance of point A can be derived as,

$$M_A=\frac{X_A}{F_A}=$$

(11)

$$\frac{1}{F_A}\left[\begin{array}{l}F_A\sum_{m=1}^{6}\sum_{n=1}^{6}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_a,y_a)}{\Delta}+F_B\sum_{m=1}^{6}\sum_{n=1}^{6}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_b,y_b)}{\Delta}\\+F_C\sum_{m=1}^{6}\sum_{n=1}^{6}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_c,y_c)}{\Delta}+F_D\sum_{m=1}^{6}\sum_{n=1}^{6}\frac{\Psi_{mn}(x_a,y_a)\Psi_{mn}(x_d,y_d)}{\Delta}\end{array}\right]$$

Similarly, M_B, M_C and M_D can be obtained. The computational chart of admittance is shown in Fig. 5.

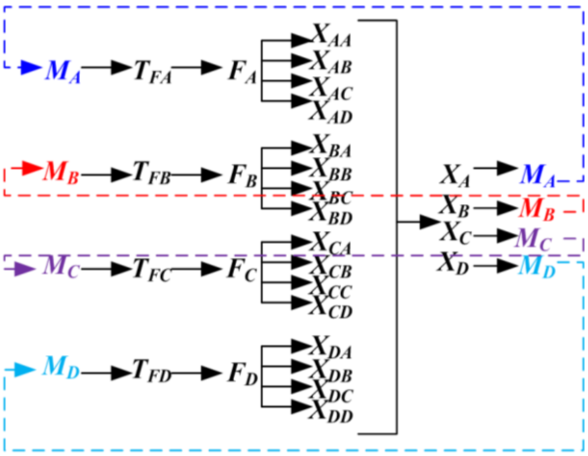


Figure 5. Computational chart of admittance.

Seen from Fig. 5, admittance of any point on the plate is an intermediate variable, which must be computed by an iteration process (such as *while/end* sentence in MATLAB tool). In fact, it is not necessary for this, and a phenomenon can support this standpoint, which can be shown in Fig. 6.

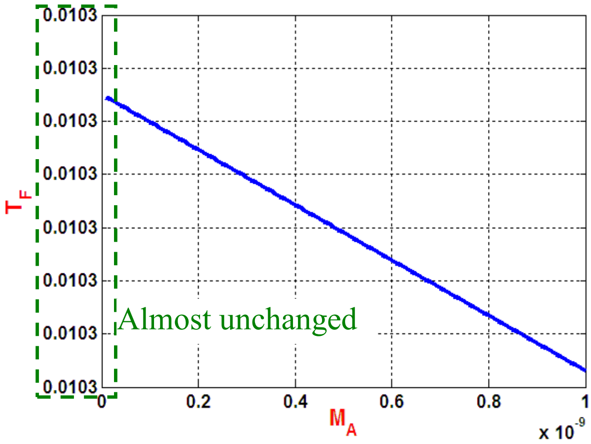


Figure 6. Variation of T_F with M_A.

For admittance of any point on the thin plate, its order of magnitude is approximately 10^{-9} or even smaller; seen from Fig. 6, the change of M_A will apparently not affect T_F ; meanwhile, in Eq.(6), M_A can be completely ignored because the order of magnitude is far different in practice, i.e. $(1-m\omega^2M_A\approx1)$.

4 Placement search of a proposed composite system

Step 1: Configure parameters of isolators according to Fig. 4, in which the desired region is shown; ω_0 and ξ are selected as 0.1 and 0.2 respectively.

In addition, equipment mass is supposed to be as $m=200$ kg , and plane size of the thin plate is $e\times f=0.90\times1.38$ m ; circular frequency of the generated force is $\omega=2\pi$ (rad/s) , and the force amplitude is 500 N . The basic parameters of the plate are set as

$$a=3.0\text{ m}, b=6.0\text{ m}, h=0.2\text{ m}, \rho=7800\text{ kg/m}^3,$$
$$\upsilon=0.33,$$
$$E=2.1\times10^{11}\text{Pa}.$$

Step 2: Discretize the thin plate, and the accuracy criteria of each edge can be defined as $\Delta x, \Delta y$ respectively, and then coordinates of any points on the plate can be obtained. The discrete operations are that,

1) if $a < b$, then $\Delta x = linspace(0, a, 68)$,
$\Delta y = linspace(0, b, round(b / a \times 44))$;

2) if $a \ge b$, then $\Delta y = linspace(0, b, 68)$,
$\Delta x = linspace(0, a, round(a / b \times 44))$;

($linspace(\cdot), round(\cdot)$) are the tools based on MATLAB).

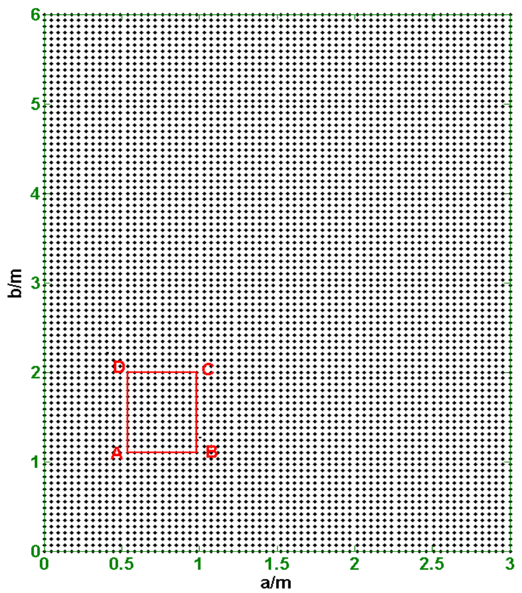


Figure 7. Discrete grid of the thin plate.

Step 3: Search the global and optimal placement of machinery equipment;

Iterative calculation:
for direction of edge *a*;
  for direction of edge *b*;
1) Generate supposed placement on the plate, x_A, x_B, x_C, x_D;
2) According to Sec.3, transmitted force from machinery equipment to A, B, C and D points on the plate respectively can be computed as F_A, F_B, F_C, F_D;
3) According to Eq. (7), the plate vibration forced by transmitted force of A, B, C and D respectively , can be computed;
4) Superpose vibration caused by the four points, and then the final vibration can be obtained;
5) Locate the peak, and record the placement, and a **3-dimensional array** is generated, [*x* coordinate, *y* coordinate, peak displacement];
  end
end
6) Search ends, and the minimum peak will be found out by comparing the generated array simultaneously, the optimal placement is also locked;
7) Utilize the obtained optimal placement, and compute the vibration of plate according to this.

5 Numerical study

The presented equipment is non-equilateral for/with the purpose of simulating a general case, and the following two placement conditions should be involved in:

Case1, the short size e of equipment is in parallel with the short size a of the thin plate;

Case2, the short size e of equipment is in parallel with the long size b of the thin plate.

Optimal placements of the proposed cases can be found according to Sec. 4, and the results are that:

Case1,

$$x_A = 2.0597 \text{ m}, y_A = 0.0000 \text{ m};$$
$$x_B = 2.9597 \text{ m}, y_B = 0.0000 \text{ m};$$
$$x_C = 2.9597 \text{ m}, y_C = 1.3800 \text{ m};$$
$$x_D = 2.0597 \text{ m}, y_D = 1.3800 \text{ m}.$$

The peak displacement of plate vibration is:

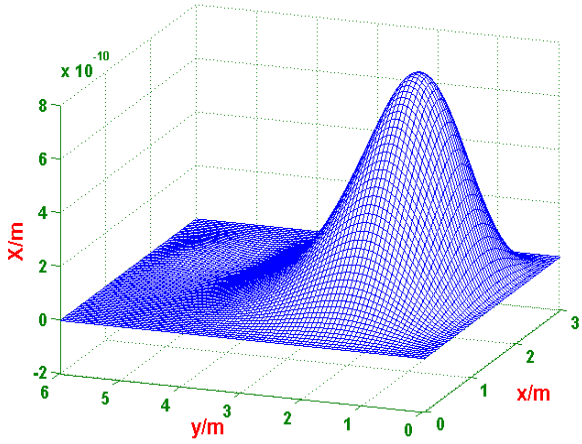
$$8.1744 \times 10^{-10} \text{ m}.$$

Case2,

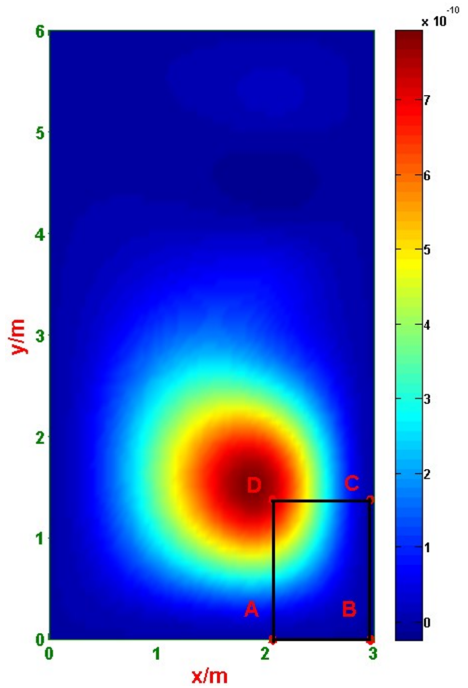
$$x_A = 0.8060 \text{ m}, y_A = 0.0000 \text{ m};$$
$$x_B = 2.1860 \text{ m}, y_B = 0.0000 \text{ m};$$
$$x_C = 2.1860 \text{ m}, y_C = 0.9000 \text{ m};$$
$$x_D = 0.8060 \text{ m}, y_D = 0.9000 \text{ m}.$$

The peak displacement of plate vibration is:

$$7.3758 \times 10^{-10} \text{ m}.$$

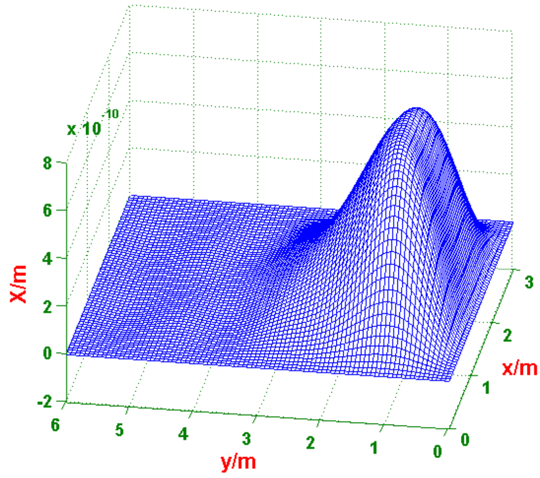


(a) 3D vision of plate vibration

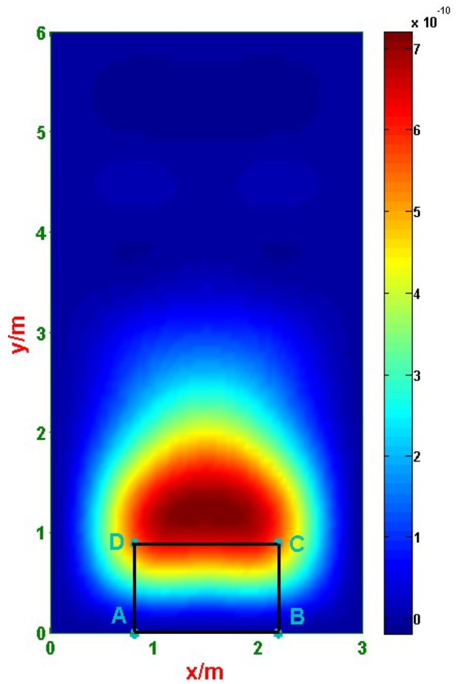


(b) 2D vision of plate vibration

Figure 8. Visualization of thin plate vibration (Case1).



(a) 3D vision of plate vibration



(b) 2D vision of plate vibration

Figure 9. Visualization of thin plate vibration (Case2).

Seen from the two cases of placement, the maximum of peak displacement of Case1 is lower than Case2 (the reduction ratio is about 10%), which can show that the arrangement direction is important for placement, but it is often ignored in practice.

In theory, more fine mesh of thin plate can lead to more reasonable location of machinery equipment; however, a moderate accuracy of discrete grid is adopted in this paper in view of computation efficiency. Meanwhile, arbitrary placement style can be adopted if the plate is of unlimited fine mesh, namely, the equipment is not required to be in parallel with the plate foundation; however, it is quite uneasy to perform this method with general mesh accuracy, because making the searched placement of thin plate match the size of equipment well is very difficult and the generated error may be large, therefore, this ideal strategy is not adopted here.

It is also worth mentioning that, the isolated equipment is considered as a four-point supporting here, but the optimization idea will be completely the same for other supporting methods (such as six-point, eight-point, etc).

6 Conclusions

In this paper, a novel composite vibration isolation system is proposed to perform a placement optimization of machinery equipment. The difference in traditional practices is that, the participation of foundation vibration is involved, and the whole idea is based on the mechanical four-pole connection. Two cases of arrangement direction are considered respectively, and the results can validate the importance. This study gives a pure numerical presentation of a novel idea, and this research opens up a new way that can inspire other ideas based on this topic, for example, an intelligent algorithm (such as colony algorithm [11]) can be adopted in optimizing the placement. Moreover, practical experiments can be conducted when all of the theoretical studies are completed.
In this paper, the interdisciplinary study of mechanical and civil engineering is presented as a unique perspective. Meantime, this novel study is also aimed at preparing the revising work of national code of China—"Code for designing the vibration isolation".

Acknowledgments

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