

# MODELING, CONTROL AND NAVIGATION OF AN AUTONOMOUS QUAD-ROTOR HELICOPTER

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## ABSTRACT

Autonomous outdoor quad-rotor helicopters increasingly attract the attention of potential researchers. Several structures and configurations have been developed to allow 3D movements. The quadrotor helicopter is made of a rigid cross frame equipped with four rotors. The autonomous quad-rotor architecture has been chosen for this research for its low dimension, good manoeuvrability, simple mechanics and payload capability. This article presents the modelling, control and navigation of an autonomous outdoor quad-rotor helicopter.

## KEY WORDS

autonomous, quad-rotor helicopter, modelling control and navigation, 3D movements, sensor system

## CLASSIFICATION

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## **INTRODUCTION**

Autonomous outdoor quad-rotor helicopters increasingly attract the attention of potential researchers [1]. In fact, several industries require robots to replace men in dangerous, boring or onerous situations. A wide area of this research is dedicated to aerial platforms. Several structures and configurations have been developed to allow 3D movements, there are blimps, fixed-wing planes, single rotor helicopters, bird-like prototypes, quad-rotors, etc. The autonomous quadrotor helicopter is a small agile vehicle controlled by four rotors. The quadrotor architecture has low dimensions, good manoeuvrability, simple mechanics and payload capability [2, 3].

The quadrotor UAV is controlled by angular speeds of four motors, each motor produces a thrust and a torque, whose combination generates the main trust, the yaw torque, the pitch torque, and the roll torque acting on the quadrotor. Motors produce a force proportional to the square of the angular speed and the angular acceleration. Each motor produces a thrust and a torque, whose combination generates the main trust, the yaw torque, the pitch torque, and he roll torque acting on the quadrotor. Motors produce a force proportional to the square of the angular speed and the angular acceleration [4].

The quad-rotor helicopter configuration is well known and has been studied since the beginning of 1900s.

The study of kinematics and dynamics helps to understand the physics of the quadrotor and its behaviour [5-10]. Together with modelling, the determination of the control algorithm structure is very important.

The electrically powered four-rotor quad-rotor helicopter architecture has been chosen for this research for its low dimension, good maneuverability, simple mechanics and payload capability (Figure 1).

This structure can be attractive in several applications, in particular for surveillance, for imaging dangerous environments and for outdoor navigation and mapping [11, 12].



**Figure 1.** Quad-rotor helicopter.

The article is organized as follows:

Section 1: Introduction.

Section 2: The modeling of the autonomous quad-rotor helicopter is presented.

Section 3: The control strategy are presented.

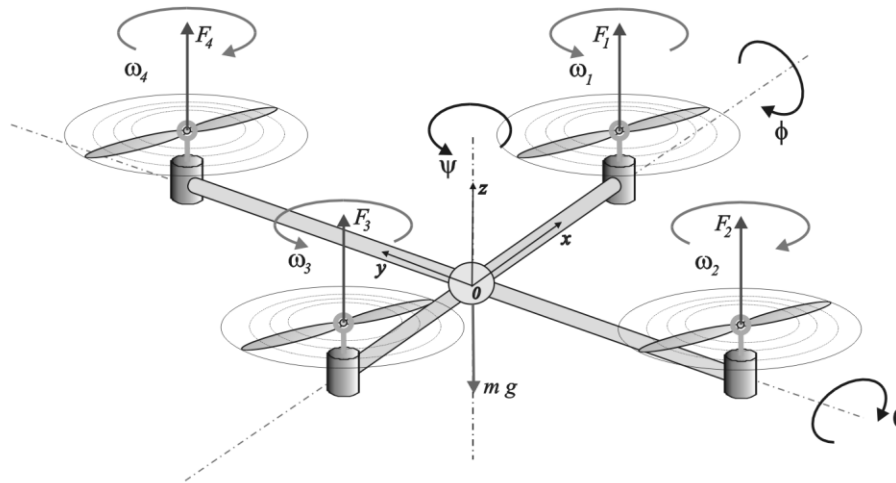
Section 4: The GPS navigation of the autonompus quad-rotor helicopter is illustrated.

Conclusions are given in Section 5.

## MODELING OF THE AUTONOMOUS QUAD-ROTOR HELICOPTER

The model of the quad-rotor helicopter and the rotational directions of the propellers can be seen in Figure 2. The rotor pair 2 and 4 rotates clockwise direction and the rotor pair 1 and 3, anticlockwise direction.

An autonomous quad-rotor helicopter has fixed pitch angle rotors and the rotor speeds are controlled in order to produce the desired lift forces.



**Figure 2.** The model of the autonomous quad-rotor helicopter.

The quadrotor helicopter has four actuators-brushless DC motors which exert lift forces  $F_1$ ,  $F_2$ ,  $F_3$ ,  $F_4$  proportional to the square of the angular velocities of the rotors. Actually, four motor driver boards are needed to amplify the power delivered to the motors. Their rotation is transmitted to the propellers which move the entire structure.

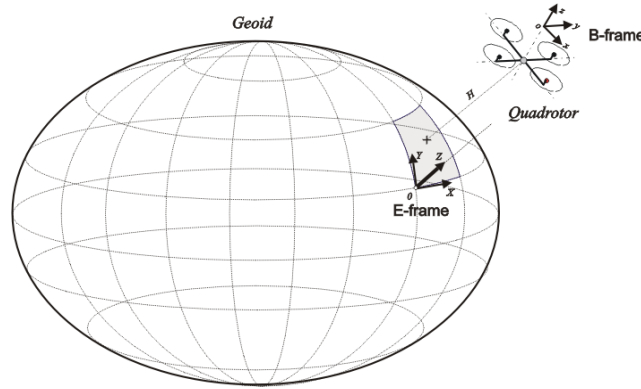
Two types of sensors are used for measuring the robot attitude and for measuring its height from the ground. For the first, an Inertial Measurement Unit (IMU) was adopted, while the distance was estimated with a Sound Navigation And Ranging (SONAR) and an InfraRed (IR) modules. There are: accelerometers and angular velocity sensors on the board of the quad-rotor helicopter.

The concept of the vision system is originated from motion-stereo approach. The camera is attached to the quadrotor helicopter. The data processing and the control algorithm are handled in the Micro Control Unit (MCU) which provides the signals to the motors [13-16].

## QUAD-ROTOR HELICOPTER COORDINATE SYSTEMS

To describe the motion of a 6 DOF rigid body it is usual to define two reference frames [17]:

- the earth inertial frame (E-frame), and
- the body-fixed frame (B-frame), Figure 3.



**Figure 3.** Earth- and Body-frame used for modeling of the quad-rotor system.

The equations of motion are more conveniently formulated in the B-frame because of the following reasons:

- the inertia matrix is time-invariant,
- advantage of body symmetry can be taken to simplify the equations,
- measurements taken on-board are easily converted to body-fixed frame, and
- control forces are almost always given in body-fixed frame.

The E-frame (OXYZ) is chosen as the inertial right-hand reference. Y points toward the North, X points toward the East, Z points upwards with respect to the Earth, and O is the axis origin. This frame is used to define the linear position (in meters) and the angular position (in radians) of the quad-rotor.

The B-frame (oxyz) is attached to the body, x points toward the center of gravity of the quad-rotor front, y points toward the quad-rotor left, z points upwards and o is the axis origin. The origin o is chosen to coincide with the center of the quad-rotor cross structure. This reference is righthand too. The linear velocity  $\mathbf{v}$  (m/s), the angular velocity  $\boldsymbol{\Omega}$  (rad/s), the forces  $\mathbf{F}$  (N) and the torques (Nm) are defined in this frame. The linear position of the helicopter (X, Y, Z) is determined by the coordinates of the vector between the origin of the B-frame and the origin of the E-frame according to the equation.

The angular position (or attitude) of the helicopter ( $\Phi$ ,  $\theta$ ,  $\psi$ ) is defined by the orientation of the B-frame with respect to the E-frame. This is given by three consecutive rotations about the main axes which take the E-frame into the B-frame. In this paper, the “roll-pitch-yaw” set of Euler angles were used. The vector that describes the quad-rotor position and orientation with respect to the E-frame can be written in the form:

$$\mathbf{s} = [X Y Z \Phi \theta \psi]^T. \quad (1)$$

The rotation matrix between the E- and B-frames has the following form:

$$\mathbf{R} = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\theta + c_\psi s_\theta s_\phi & s_\psi c_\theta + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\theta + s_\psi s_\theta s_\phi & -c_\psi c_\theta + s_\psi s_\theta c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}. \quad (2)$$

The corresponding transfer matrix has the form:

$$\mathbf{T} = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix}. \quad (3)$$

Where  $c_\theta$  and  $s_\theta$  represent  $\cos(\theta)$  and  $\sin(\theta)$  respectively.

## QUAD-ROTOR HELICOPTER KINEMATICS

The system Jacobian matrix, taking (2) and (3), can be written in the form:

$$\mathbf{J} = \begin{bmatrix} \mathbf{R} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{T} \end{bmatrix}, \quad (4)$$

where  $\mathbf{0}_{3 \times 3}$  is a zero-matrix. The generalized quad-rotor velocity in the B-frame has a form [18]:

$$\mathbf{v} = [\dot{x} \ \dot{y} \ \dot{z} \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T. \quad (5)$$

Finally, the kinematical model of the quad-rotor helicopter can be defined in the following way:

$$\dot{\mathbf{s}} = \mathbf{J} \cdot \mathbf{v}. \quad (6)$$

## QUAD-ROTOR HELICOPTER DYNAMICS

Dynamic modelling of the quadrotor helicopter is a well elaborated field of aeronautics. The dynamics of a generic 6 DOF rigid-body system takes into account the mass of the body  $m$  and its inertia matrix  $\mathbf{I}$ .

Two assumptions have been done in this approach:

- the first one states that the origin of the body-fixed frame is coincident with the center of mass (COM) of the body. Otherwise, another point (COM) should be taken into account, which could make the body equations considerably more complicated without significantly improving model accuracy,
- The second one specifies that the axes of the B-frame coincide with the body principal axes of inertia. In this case the inertia matrix  $\mathbf{I}$  is diagonal and, once again, the body equations become simpler.

Each rotor produces moments as well as vertical forces. These moments were observed experimentally to be linearly dependent on the forces at low speeds. There are four input forces and six output states ( $x, y, z, \psi, \theta, \phi$ ) and, therefore the quad-rotor is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counter clockwise, in order to balance the moments and to produce yaw motions as needed.

The equations of motion can be written using the force and moment balance, yielding:

$$\ddot{x} = \frac{\left( \sum_{i=1}^4 F_i \right) (c_\phi s_\theta c_\psi + s_\phi s_\psi) - K_x \dot{x}}{m}, \quad (7)$$

$$\ddot{y} = \frac{\left( \sum_{i=1}^4 F_i \right) (s_\phi s_\theta c_\psi + c_\phi s_\psi) - K_y \dot{y}}{m}, \quad (8)$$

$$\ddot{z} = \frac{\left( \sum_{i=1}^4 F_i \right) (c_\phi c_\psi) - K_z \dot{z} - G}{m}, \quad (9)$$

$$\ddot{\psi} = l \cdot \frac{(-F_1 + F_2 + F_3 - F_4 - K_\psi \dot{\psi})}{J_x}, \quad (10)$$

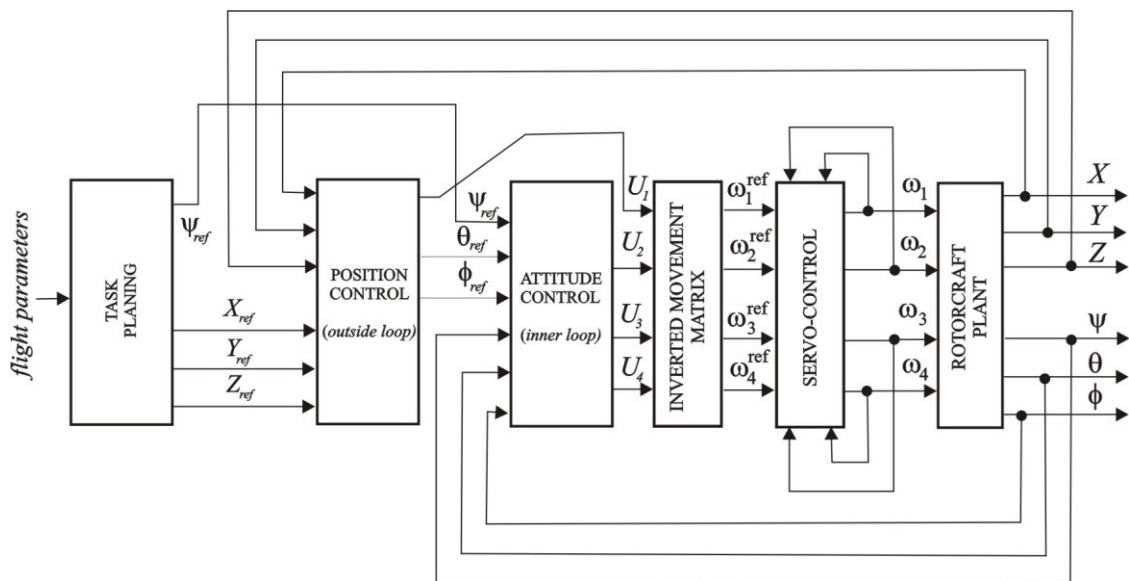
$$\ddot{\theta} = l \cdot \frac{(-F_1 - F_2 + F_3 + F_4 - K_\theta \dot{\theta})}{J_y}, \quad (11)$$

$$\ddot{\phi} = \frac{(-M_1 + M_2 + M_3 - M_4 - K_\phi \dot{\phi})}{J_z}. \quad (12)$$

The factors  $K_j$  in (7)-(12) given above are the air resistance coefficients to be determined experimentally.  $J_x$ ,  $J_y$ ,  $J_z$  are the moments of inertia with respect to the particular axes.

## MODELING OF THE CONTROL STRATEGY

Together with modeling, the determination of the control algorithm structure is very important for improving stabilization. Controlling an autonomous quad-rotor helicopter is basically dealing with highly unstable dynamics and strong axes coupling. In addition to this, any additional on-board sensor increases the autonomous quad-rotor helicopter total weight and therefore decreases its operation time. The control system of the autonomous quad-rotor helicopter requires accurate position and orientation information [14-18]. In this section we present a control strategy to stabilize of the quad-rotor. Figure 4 shows the block diagram of the quad-rotor control system.



**Figure 4.** The block diagram of the quad-rotor helicopter control system.

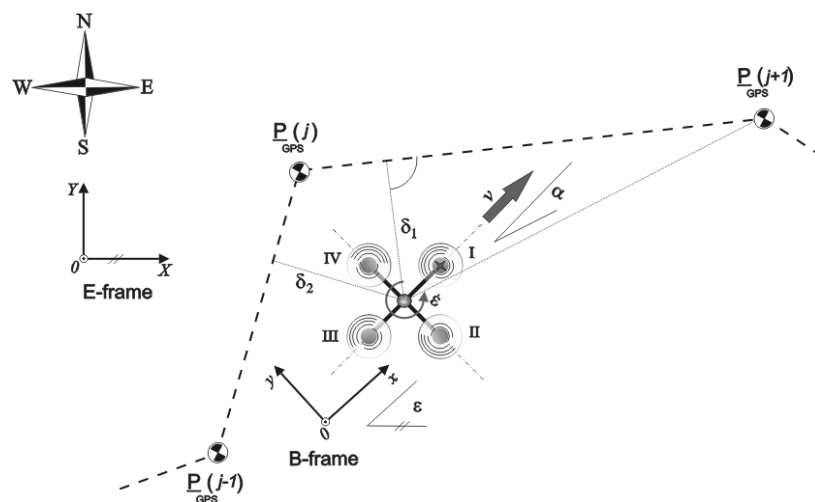
The task planning block is in debt to determine desired, i.e. referent 3D rotorcraft trajectory of flight as well as to propose the referent flight speed along the trajectory. The task planning block generates referent path based on flight parameters and quad-rotor task imposed. Position control block has to ensure accurate 3D trajectory tracking. It represents so called outside control loop. Based on sensory information (GPS, IR, SONAR) about the referent positions (speeds) and corresponding actual ones defined in the inertial coordinate system (E-frame), the position controller calculates referent attitude position of quad-rotor body (pitch and roll angle) that have to enable desired motion. Inner control block represents the core of the control scheme. It is responsible for the attitude control of quad-rotor system. Appropriate attitude control ensures in an indirect way required flight performances in the particular directions of motion such as longitudinal, lateral as well as vertical. Inner control block processes the task and sensor data and provides a signal for basic movements which

balances the position error. The essence of building control scheme presented in Figure 4 is that by controlling a body attitude (within an inner loop) it is enabled controlling of the rotorcraft movements in the coordinate directions co-linear with the axes of the inertial system. Inverted Movements Matrix block is used to compute the propeller's squared speed from the four basic movement signals.

Variety of control algorithms can be implemented within the flight controller presented in Figure 4. These are: (i) conventional PID regulator, (ii) backstepping method [19, 20] and (iii) knowledge-based Fuzzy Inference System (FIS) [4, 11]. The future of autonomous quad-rotor helicopter control lies in intelligent flight controllers that are programmed to specific flight missions [21-26].

## GPS NAVIGATION OF THE AUTONOMOUS QUAD-ROTOR HELICOPTER

The trajectory of the autonomous quad-rotor can be introduced by GPS coordinates (e. g.  $P_{GPS}(j)$ ) as shown in Figure 5. The autonomous quad-rotor helicopter is requested to track the imposed trajectory between the particular points ( $j=1, \dots, n$ ) with satisfactory precision, keeping the desired attitude and height of flight. The autonomous quad-rotor helicopter checks for the current position: X and Y by use of a GPS sensor and/or electronic compass. Also, the altitude is measured by a barometric sensor. An on-board microcontroller calculates the actual position deviation from the imposed trajectory given by successive GPS positions  $P_{GPS}(j)$ . It localizes itself with respect to the nearest trajectory segment, by calculation of the distances:  $\delta_1$  or  $\delta_2$ .



**Figure 5.** Quad-rotor localization and navigation with respect to the imposed GPS coordinates.

Gyroscopes provide angular velocity measurements with respect to inertial space. With recent developments in gyroscope technology, their usage in various fields is observably increasing. In combination with accelerometers, gyroscopes are used in position, velocity, and attitude computation in a variety of navigation and motion tracking applications for aircraft and robots. By providing angular velocity measurements, gyroscopes can also be used in angular orientation estimation. Using the gyroscope, the autonomous quad-rotor helicopter determines desired azimuth of flight  $\alpha$  (Figure 5) and keeps the desired direction of flight. The height of flight is also controlled to enable the performance of the imposed mission. The height of flight is also controlled to enable the performance of the imposed mission (task).

## CONCLUSIONS

In this article the modeling and navigation of an autonomous quad-rotor helicopter in a outdoor scenario is shown. The main aspects of modeling of rotorcraft kinematics and rigid body dynamics, spatial system localization and navigation of autonomous quad-rotor helicopter in outdoor scenario are considered in the paper. The control strategy is presented. The GPS navigation of the autonomous quad-rotor helicopter is illustrated.

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