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FRACTAL DIMENSION ANALYSIS OF SOLAR GRANULATION- BOXCOUNTING DIMENSION

A. HANSLMEIER[1], B. LEMMER[1], D. UTZ[1], R. MULLER[2] and H. MUTHSAM[3]

[1] *Institut für Physik, IGAM*
Univ. Platz 5, 8010 Graz, Austria
[2] *Observatoire Pic du Midi,*
Av. d Azereix, 78000 Tarbes, France
[3] *Institut für Mathematik, Univ. Wien*
Oskar-Morgenstern Platz 1, A-1090 Wien, Austria

Abstract. The fractal dimension of high resolution Hinode solar granulation observations and numerical simulations is studied and the results are compared. These observations are not influenced by atmospheric seeing conditions and therefore allow a more realistic estimate of the fractal dimension than in previous works. Though arriving at similar results for observations and simulation data, non integer fractal dimension < 2, some differences in the numerical values occur, and these are discussed.

Key words: High resolution solar granulation - fractal analysis

1. Introduction

The solar granulation is the visible manifestation of convective overshooting motions in the solar photosphere. Due to the small spatial scale of these elements of about 1000 km, observations of solar granulation from ground based observatories are often strongly influenced by moderate seeing conditions. Therefore, a study of these elements can lead to wrong conclusions since a turbulent behavior found in the data can also be attributed to terrestrial atmospheric influence and not to the turbulent solar convective motions.

Studies of the fractal dimension of solar granulation were made for the first time by Roudier and Muller (1986). Using white light photographic data from the Pic du Midi observatory, they found the granules appear to have a critical scale of 1.37 arcsec, at which drastic changes in the properties of granules occur; in particular the fractal dimension changes at this critical scale. The granules smaller than this scale could be of turbulent origin.

Nesis *et al.* (1995) studied data from the VTT at Izana. By applying a rescaled range analysis to the granular velocity field in combination with a fractal analysis, they found a less turbulent nature inside granules than in intergranular lanes. Abramenko (2005) performed a multifractal analysis of solar magnetograms using SOHO/MDI high resolution magnetograms.

Paniveni *et al.* (2010) investigated the complexity of supergranular cells using the intensity patterns obtained at the Kodaikanal Solar Observatory during the solar maximum. They visually identified supergranular cells, from which a fractal dimension D for supergranulation is obtained according to the relation $P \sim A^{D/2}$, where A is the area and P the perimeter of the supergranular cells. A fractal dimension of about 1.12 for active region cells and about 1.25 for quiet region cells was obtained, a difference that could be attributed to the inhibiting effect of the magnetic field.

An excellent introduction into the field of fractals is given in the book by Mandelbrot (1982).

The motivation for performing such an analysis again was that now high resolution data without any terrestrial atmospheric disturbance are available and that numerical simulations show granulation at extremely high resolution.

In this paper we compare spatially highly resolved granulation observations from the Hinode SOT telescope with a highly resolved numerical HD-study of solar convection (ANTARES code, Muthsam *et al*, 2007).

2. The Fractal Dimension

A simple definition of a fractal can be given as:

A set is called a fractal if it displays self similarity. It can be split into parts and each part is approximately a reduced copy of the whole.

There exist different but similar definitions of a fractal. The box counting dimension is defined in the following way: Consider N number of boxes with size R. Suppose you start with a given number of boxes, say N_0 with a certain size R. The the size is modified and the number of boxes to cover a fractal is:

$$N = N_0 R^{-D_F} \tag{1}$$

D_F denotes the fractal dimension deduced from box-counting, D is the dimension of the physical space and in general $D_F < D$.

A more rigorous definition is the Minkowski-Bouligand dimension, also known as Minkowski dimension or box-counting dimension. Consider a set S in a Euclidean space R^n, or more generally in a metric space (X, d) then the dimension is given by

$$\dim_{\text{box}}(S) = \lim \epsilon \to 0 \frac{\log N(\epsilon)}{\log(1/\epsilon)} \tag{2}$$

where N is the number of boxes and ϵ the box size.

A different but similar approach to calculate the fractal dimension is the Hausdorff dimension. In this paper the box counting and the Hausdorff dimension are used.

3. Data Reduction

We used two sets of data:

- Hinode-SOT data of solar granulation observations; the observations where made in the blue continuum. Individual images from the Solar Optical Telescope at Hinode (diameter 50 cm) were selected, the filter in the blue continuum was centered at 450.45 nm with a bandwidth of 0.4 nm. The images contained 4094×2048 pixels corresponding to 218 × 109 arcsec.

- HD simulation ANTARES data: Antares is the acronym for A Numerical tool fro Astrophysical Research. The data are restricted to pure HD and not MHD but allow an extremely high spatial resolution of 1 pixel = 35 km, the total field being 512 × 512 pixels.

In order to perform a fractal analysis, the data were segmented. For the detection of granules, an algorithm according to Bovelet and Wiehr (2001) which was strongly optimized by Lemmerer *et al.* (2014) was used.

An example of a segmented ANTARES image is given in Fig. 1.

An example of a segmented Hinode image is given in 2

4. Results

The results of the fractal dimension analysis are given in Table I. Note that the ANTARES data consist of a time series of 10 images whereas the

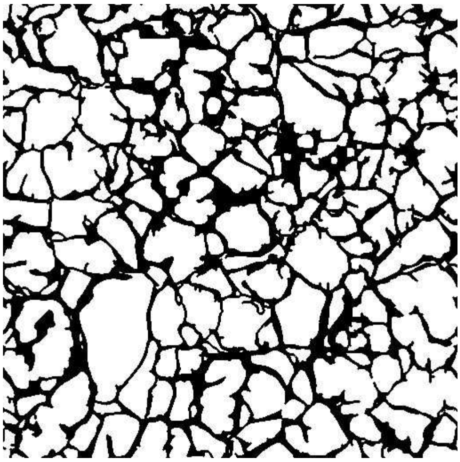

Figure 1: Sample of segmentation of an ANTARES image.

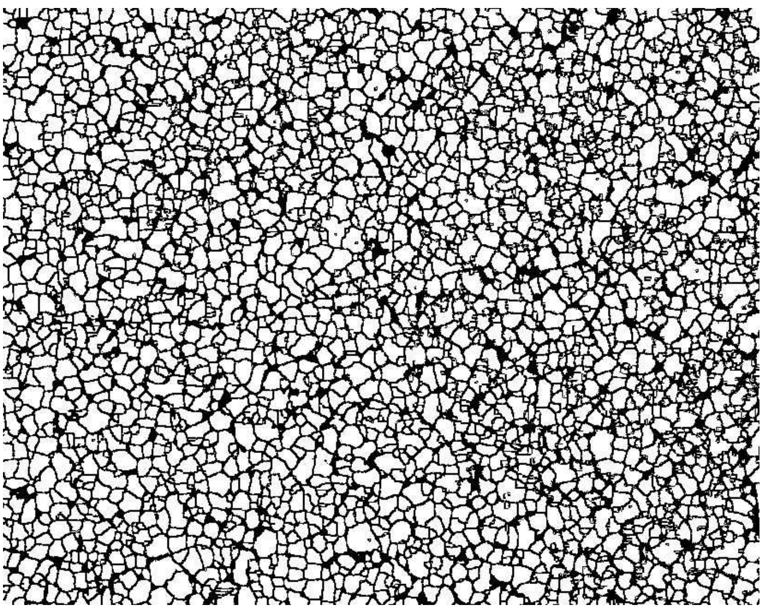

Figure 2: Sample of segmentation of an Hinode image.

Table I: Results of fractal analysis

Antares data (time series)	Hinode data
1.8085	1.8595
1.9406	1.8484
1.9413	1.8553
1.9408	1.8539
1.9421	1.8596
1.9426	1.8609
1.9407	1.8666
1.9390	1.8658
1.9411	1.8485
1.9387	1.8485

Hinode data were arbitrarily taken images at completely different times. The values for the simulation data and the observed data are quite similar. The fluctuations are smaller for the simulated time series data than for the randomly selected observed data.

In Fig. 3 the results of the Hausdorff dimension applied to the Hinode images for different selected sizes are given. This was done in order to understand the influence of the selected image size on the results. As it can be seen, the smaller the selected subimage, the smaller the values of the fractal dimension. A smaller selected field means that the importance of larger elements becomes less dominant.

5. Discussion and Conclusion

In this paper we compared fractal dimensions of different solar granulation sets. The values are quite similar for the observations and simulation. The Hinode observations contain randomly selected granulation images in order to show the dispersion of the results. The HD simulation data consist of a time series. Here the dispersion of the values appears to be smaller. An important result is further that the obtained values are dependent on the size of the subimages selected as it is shown by Fig. 3. The smaller the sub images the smaller the fractal values. In the smaller subimages the turbulent

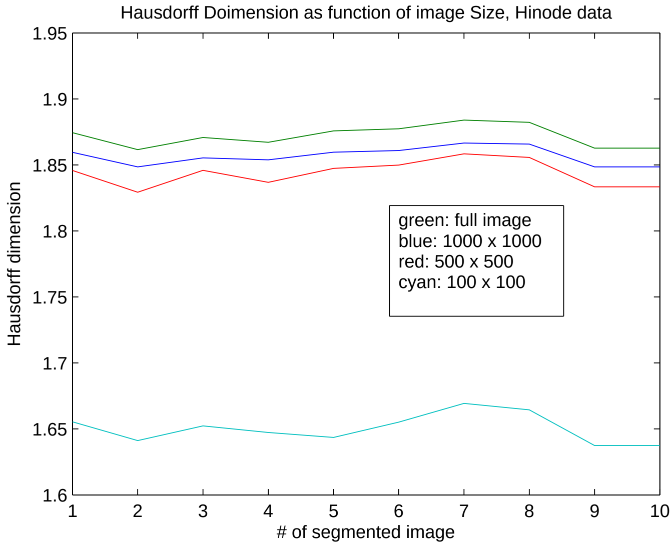

Figure 3: Fractal analysis: influence of different data size on fractal dimension.

nature seems to become dominant. The size of the HD simulation data is comparable to the smaller subimages however the fractal dimension is very similar to the large Hinode image field. Therefore turbulence in Hinode data seems to play a more significant role than in simulated data.

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