

Simplified Harmonic Model for Full Wave Diode Rectifier Circuits

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Original scientific paper

This paper proposes a simplified equivalent circuit model that can be used in the presentation of characteristic harmonic components generated by full wave diode rectifiers. In the model, the load current is assumed to be constant as in many industrial applications. In order to obtain an accurate and exact analysis, the proposed model involves both current harmonics on AC side and voltage harmonics on DC side of the full wave rectifiers. The model depends on Fourier series expansion for the load voltage and the source current waveforms.

Key words: Harmonic Model, Equivalent Circuit, Full Wave Rectifier

Pojednostavljeni harmonički model za punovalni ispravljač s diodama. U radu je predložen pojednostavljeni ekvivalentni model električnog kruga koji se može koristiti za prikaz karakterističnih harmoničkih komponenata koje generira punovalni ispravljač. Pretpostavka modela je konstantan iznos struje tereta, što je česti slučaj u industrijskim primjenama. Kako bi se postigla vjerodostojnost modela, predloženi model uključuje harmonike struje na AC strani kao i harmonike napone na DC strani punovalnog ispravljača. Model ovisi o razvoju Fourierovog reda za valne oblike napona tereta i struje izvora

Ključne riječi: harmonički model, ekvivalentni električni krug, punovalni ispravljač

1 INTRODUCTION

Converters or rectifiers using semiconductor switching devices generate harmonics caused by the behavior of switching. The purpose of harmonic studies is to quantify the distortion in voltage and/or current waveforms at various locations in a power system. The need for a harmonic study may be indicated by excessive measured distortion in existing systems or by installation of harmonic producing equipment. One important step in harmonic studies is to characterize and to model harmonic-generating sources. Among the modern nonlinear loads, power electronic devices have a significant contribution in generating harmonics during their switching processes.

In general, rectifier circuits can be simply represented by a harmonic current source or a model that takes into account the interaction between the ac source network and the converter dc system. Some techniques for modeling the converter can be categorized as the frequency domain and the time-domain based models. The frequency domain models are further divided into current source model, transfer function model. Time domain models include representing the converter by a set of differential equations and the state-space model. Some of these models can be

integrated in harmonic power flow analysis while the others are not.

The most common technique for harmonic simulation is to treat the converter as known sources of harmonic currents or voltages with or without including phase angle information. The frequency-domain nodal equations for each harmonic are used to compute the network harmonic voltages via the system harmonic admittance or impedance matrix. The drawback of this method is that typical harmonic spectra are often used to represent the harmonic currents generated by the converter that ignores the interaction between the network and the converter. This prevents an adequate assessment of cases involving non-typical operating modes.

Ref. [1] presents a transfer function model based on the modulation theory that uses two transfer functions to relate the dc and ac sides of the converter. In [2]-[4], the authors propose efficient techniques by linearizing the interaction between the converter dc system and the ac network at different converter operating points of interests. The entire system is then solved via the harmonic coupling matrix equation to account for the interaction. In the iterative harmonic analysis, the converter is usually represented

by a fixed harmonic current source at each iteration. Ref. [5] proposes a harmonic Norton-equivalent circuit model which is developed to represent the converters of HVDC links under both unbalanced and distorted input conditions.

In the time domain, a converter can be expressed as a set of differential equations or state equations that govern its performance. After solving these equations, the harmonic spectra of converter input currents are generally found by the use of Fourier method such as fast Fourier transform [6]-[8]. This device model usually can be integrated with frequency-domain network model for harmonic power flow analysis or used in a time-domain simulation including the power system [9]. Ref. [10] presents a state-space model that includes the system frequency-dependent impedance with the converter which can be used in iterative harmonic analysis to improve convergence. The time domain method is accurate but the derivation and solution of the differential equations is a very time consuming and tedious process. The difficulty of applying this method becomes severe when a power system containing many converters is analyzed.

Many harmonic models have been proposed for representing rectifier circuits [11-16]. The most common model is in the form of a harmonic current source, which is specified by its magnitude and phase spectrum. Three basic approaches used to build detailed models include developing analytical formulae for the Fourier series as a function of terminal voltage and operating parameters for the harmonic source, developing analytical models for harmonic source operation and solving for its current waveform by a suitable iterative method, and solving for harmonic source steady-state current waveform with time-domain simulation [17-19].

In this paper, first, the conventional equivalent circuits of the rectifiers are pointed out, then a new equivalent circuit with respect to harmonic currents on the AC side and output voltage harmonics on DC side of the rectifiers is proposed.

2 CONVENTIONAL MODEL OF FULL-WAVE RECTIFIERS

The circuit configuration of a full wave diode rectifier, of which the equivalent circuit will be described in detail, is shown in Fig.1.

In a full wave diode rectifier, the output voltage, $u_d(t)$, is not dependent on the load. The source current, $i_s(t)$ has a bidirectional form of the load current, $i_d(t)$. The rectifier circuits are generally loaded by large valued inductors as in DC motors in industrial applications. In this situation, the amplitude of harmonic currents generated by a diode rectifier with a smoothing reactor on the DC side is almost constant even if a source impedance varies, because the

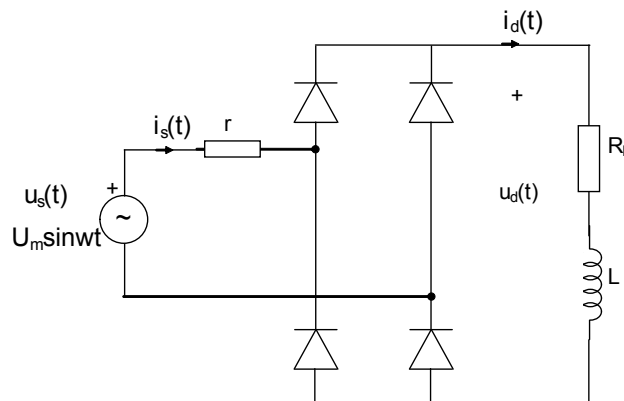


Fig. 1. Configuration of a full wave diode rectifier

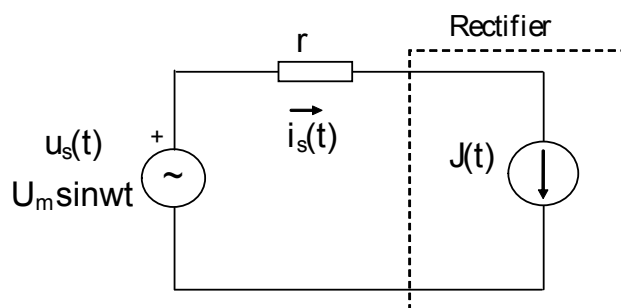


Fig. 2. Conventional equivalent circuit to $i_s(t)$ on AC side

impedance on the DC side is much larger than the source impedance on the AC side. Also, the time constant of the load is much larger than the source period. Due to the constant load current, I_d , in conventional equivalent circuits, the rectifier has been widely considered as an ideal current source for harmonics as follows.

With regard to this situation, the load voltage/current and the source voltage/current of Fig. 1 are available in many textbooks [20] and also given in Fig. 3.

The average values of the load voltage and load current in Fig. 3 are given as follows.

$$U_d = \frac{2U_m}{\pi}, \quad (1)$$

$$I_d = \frac{U_d}{R_L}. \quad (2)$$

3 PROPOSED MODEL

In conventional equivalent circuits of rectifiers, only the current harmonics on AC side are considered. Whereas, in rectifier circuits, there are both current harmonics on AC side and voltage harmonics on DC side. For

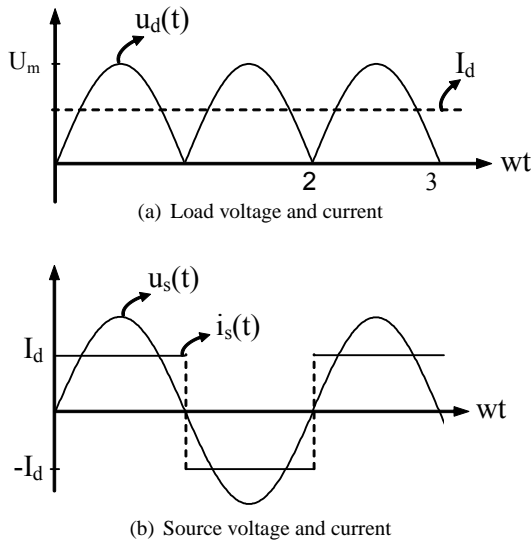


Fig. 3. Waveforms of a diode rectifier

an exact analysis, it is necessary to deal with both harmonics. For this purpose, a new equivalent circuit model containing both current harmonics and voltage harmonics is proposed. It depends on Fourier series expansion for the load voltage and the source current waveforms.

First, Let's consider the output voltage harmonics in DC side of the rectifier circuit in Fig. 1. In the rectifier circuits, an AC source is processed through a set of switches to create a well-defined waveform. We can represent the combined action of an actual source (AC source) and a set of switches by an equivalent source. The equivalent sources provide a very strong advantage: The new circuits are linear, and avoid the nonlinearity and complication of switches. We can use superposition, Laplace transforms, or other techniques from linear network analysis to analyze rectifiers. Based on superposition, a term-by-term for the Fourier series of the current and voltage in the rectifier circuits can be solved. Equivalent voltage source, $u_d(t)$, applied to the load in Fig. 1, is presented in Fig. 4.

The equivalent voltage source, $u_d(t)$, contains both the fundamental component and harmonic components of the voltage. The equivalent source waveform, accordingly the output voltage of the rectifier circuit, is given in Fig. 3a. The Fourier series expression relating to the voltage $u_d(t)$ is given by

$$u_D(t) = \frac{2U_m}{\pi} - \sum_{n=2,4,6\dots}^{\infty} \frac{2U_m}{\pi} \left(\frac{2}{n^2 - 1} \right) \cos(n\omega t). \tag{3}$$

Every component of Fourier series corresponds to a voltage source. They are symbolized as U_d , $u_{d2}(t)$,

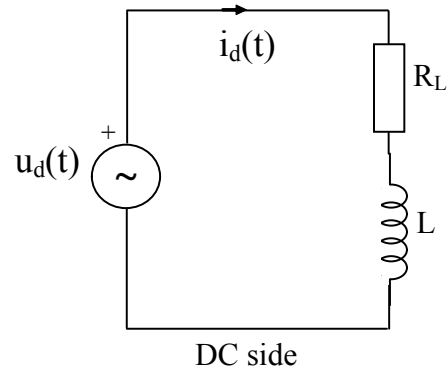


Fig. 4. Equivalent voltage source, $u_d(t)$, applied to the load

$u_{d4}(t), \dots$ in Fig.5, where U_d is average load voltage as DC source, $u_{d2}(t)$ is a AC source having ω_2 frequency, $u_{d4}(t)$ is a AC source having ω_4 frequency,...

Second, Let's deal with the current harmonics in AC side of the rectifier circuit in Fig.1. The source current on AC side, $i_s(t)$, is dependent on the load current, I_d . Although the current on DC side of the converter is constant, the current on AC side is bidirectional as shown in Fig. 3b. In other words, the source current has a bidirectional form of load current. The source current, $i_s(t)$, is expressed by Fourier series as follows.

$$i_s(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4}{n\pi} I_d \sin(n\omega t) i_s(t) = i_{sf}(t) + i_{sh}(t) \tag{4}$$

i_{sf} and i_{sh} represents the fundamental component, the harmonic component of the source current, respectively.

Since the source current on AC side is dependent on the load current, I_d , this current is modeled by a current controlled current source, which is controlled with load current. The dependent harmonic current source, $J_s(t)$, equal to $i_s(t)$, is shown in Fig.6. Although the load current is almost constant, the harmonic current source has a square wave form as in the source current of Fig. 3b.

Now, to derive the new equivalent circuit containing harmonic currents on the AC side and harmonic voltages on the DC side of full wave diode rectifier, we discuss an equivalent circuit shown in Fig. 7. This model, primarily, is a combination of Fig. 4 and Fig. 6.

Since the equivalent circuit shown in Fig. 7 contains both AC side and DC side of the rectifier circuit, it meets the need for exact analysis of the rectifier circuit. The switching circuit is converted into a linear circuit with regards to harmonics. Therefore, any circuit analysis technique can be used to analyze the rectifier circuit.

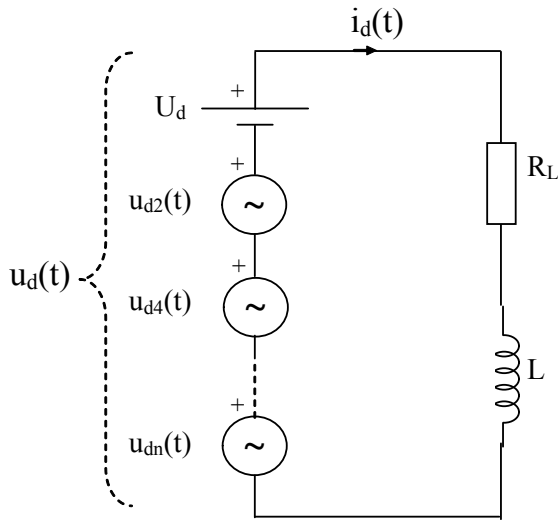


Fig. 5. Equivalent voltage sources corresponding to Fourier series

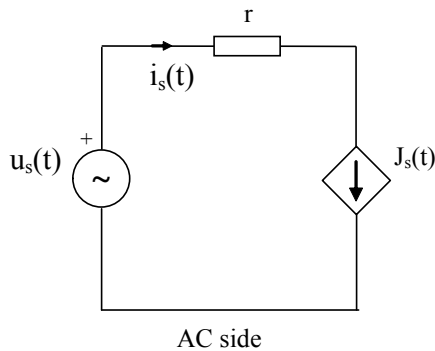


Fig. 6. Dependent harmonic current source

From the point of harmonic studies, the equivalent circuit can be divided into two circuits: one is an equivalent circuit to the fundamental component and the other is an equivalent circuit to harmonics.

First, the equivalent circuit to the fundamental components on both the AC side and the DC side of the rectifier is discussed. In this case, the first harmonic components of dependent current source, $J_s(t)$, and voltage source $u_d(t)$ in the model are considered: $J_{s1}(t)$, $u_{d1}(t)$. The source voltage, $u_s(t)$ remains unchanged. According to (3), Since $u_d(t)$ has even harmonics, $u_{d1}(t)$ is equal to 0. The equivalent circuit to the fundamental components is obtained as in Fig.8.

Next, the equivalent circuit to harmonic components on both the AC side and the DC side of the rectifier is discussed. This equivalent circuit can be obtained under the condition that the source voltage $u_s(t) = 0$, the first component of harmonic current source $J_{s1}(t) = 0$ in Fig.7. The equivalent circuit to harmonics is shown in Fig. 9.

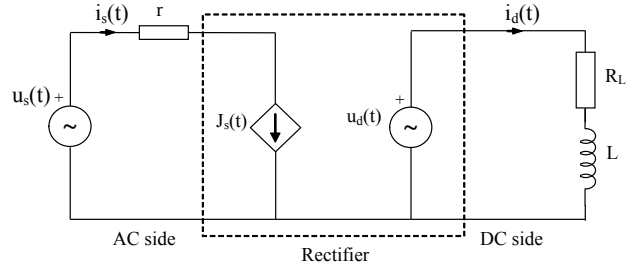


Fig. 7. Exact harmonic model of diode rectifier circuit

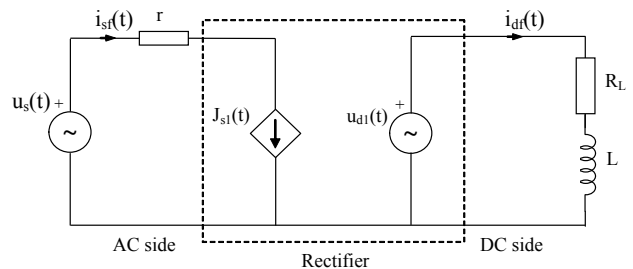


Fig. 8. Equivalent circuit to fundamental components on the DC and AC side

$J_{ah}(t)$ and $u_{dh}(t)$ represents the sources relating to harmonic components.

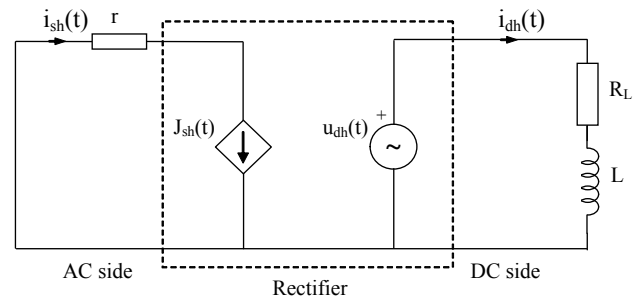


Fig. 9. Equivalent circuit to harmonic components on the DC and AC side

4 SIMULATION RESULTS

In Fig. 1, let the internal resistance r on the AC side of the rectifier be 1Ω and the resistance R_L on the DC side be 10Ω , the inductance L on the DC side be 1000mH , the max. value of the source voltage U_m be 100V and the frequency be 50Hz . The diodes are assumed to be ideal.

The load voltage, $u_d(t)$, and source current, $i_s(t)$, waveforms relating to the exact equivalent circuit in Fig. 7 are given in Fig. 10. It can be seen that the waveforms obtained from the proposed equivalent circuit model are the same as the waveforms of Fig. 3.

The fundamental component, $i_{sf}(t)$, of source current obtained from Fig. 8, is given in Fig. 11.

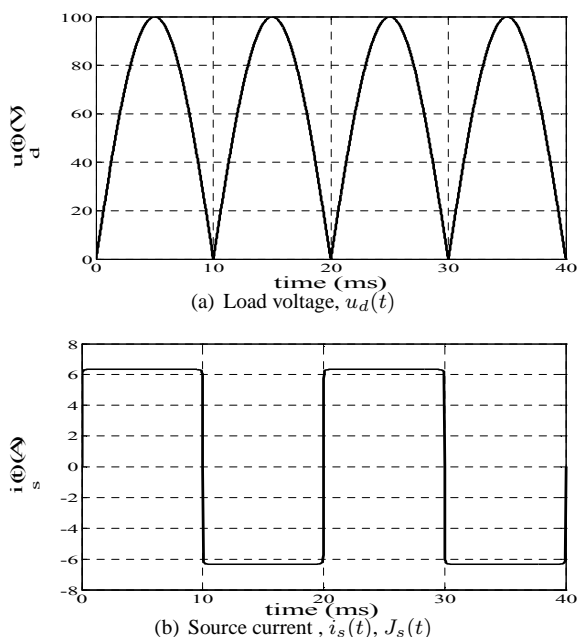


Fig. 10. Simulation results relating to the equivalent circuit

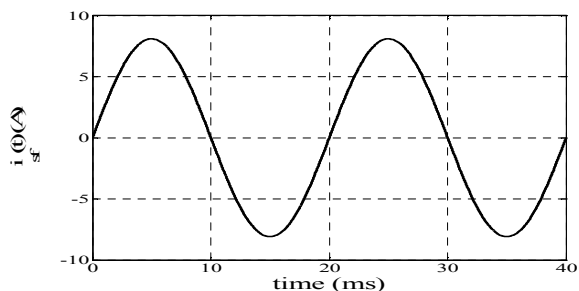


Fig. 11. Simulation result relating to fundamental component, $i_{sf}(t)$

The harmonic component, $i_{sh}(t)$, of source current obtained from Fig. 9, is given in Fig. 12.

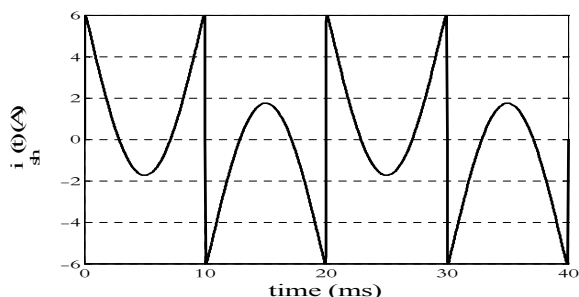


Fig. 12. Simulation result relating to harmonic component, $i_{sh}(t)$

5 CONCLUSION

The most important process in harmonic analysis is to obtain correct models relating to harmonic-generating sources. The paper describes the equivalent circuit containing current harmonics on the AC side and voltage harmonics on DC side of full wave diode rectifiers. The switching circuit is converted into a linear circuit with regards to harmonics. The desired harmonic components are easily obtained by the model. Simulation results show that the proposed equivalent circuit models properly current and voltage harmonics of the rectifier circuit.

REFERENCES

- [1] E. V. Larsen, U. H. Baker and J.C. McIver, "Low-order harmonic interactions on AC/DC system", *IEEE Trans. on Power Delivery*, vol. 4, no. 1, pp. 493-501, Jan. 1989.
- [2] A. R. Wood and J. Arrillaga, "The frequency dependent impedance of an HVDC converter," *IEEE Trans. on Power Delivery*, vol. 10, no. 3, pp.1635-1641, July 1995.
- [3] S. G. Jalali and R. H. Lasseter, "A study of nonlinear harmonic interaction between a single phase line-commutated converter and a power system", *IEEE Trans. on Power Delivery*, vol. 9, no. 3, pp. 1616-1624, July 1994.
- [4] J. Rittiger and B. Kulicke, "Calculation of HVDC-converter harmonics in frequency domain with regard to asymmetric and comparison with time domain simulations", *IEEE Trans. on Power Delivery*, vol. 10, no. 4, pp.1944-1949, Oct. 1995.
- [5] W. Xu, J. E. Drakos, Y. Mansour and A. Chang, "A three-phase converter model for harmonic analysis of HVDC system", *IEEE Trans. on Power Delivery*, vol. 9, no. 3, pp. 1724-1731, July 1994.
- [6] Y. Baghzouz, "An accurate solution to line harmonic distortion produced by ac/dc converters with overlap and dc ripple", *IEEE Trans. on Industrial Applications*, vol. 29, no.3, pp. 536-540, May/June 1993.
- [7] B. K. Perkins and M. R. Iravani, "Novel calculation of HVDC converter harmonics by linearization in the time-domain", *IEEE Trans. on Power Delivery*, vol. 12, no. 2, pp. 867-873, April 1997.
- [8] M. Grotzbach and C. Ried, "Investigation of AC/DC converter harmonics by an analytical based time-discrete approach", *IEEE Trans. on Power Delivery*, vol. 12, no.2, pp. 874-880, April 1997.
- [9] V. Sharma, R. J. Fleming and L. Niekamp, "An iterative approach for analysis of harmonic penetration in power transmission networks", *IEEE Trans. on Power Delivery*, vol. 6, no. 4, pp. 1698-1706, Oct. 1991.
- [10] G. Carpinelli, F. Gagliardi, M. Russo and D. Villacci, "Generalised convener models for iterative harmonic analysis in power system", *IEE Proc.-Gener. Transm.. Distrib.*, vol. 141, no. 5, pp. 445-451, Sept. 1994.

- [11] G. W. Chang and Y. Liu “Modeling of nonlinear components for harmonic studies”, *IEEE Power Engineering Society General Meeting*, Vol. 2, Taiwan, 2003.
- [12] L. Hu and R. E. Morrison, “AC side equivalent circuit-based method for harmonic analysis of a converter system,” *IEE Proc.—Electr. Power Appl.*, vol. 146, no. 1, pp. 103–110, Jan. 1999.
- [13] T. Tanaka, and H. Akagi, “Proposal for a New Equivalent Circuit of Diode Rectifiers with respect to Harmonics”, *Power Conversion Conference, IEEE*, pp. 508 - 513 , 1993, Yokohama.
- [14] R. Ghandehari, S. Mohamadian and A. Shoulaie, “A new approach to AC/DC converters modelling in time domain for harmonic analysis”, *Power Quality Conference, IEEE*, pp.1-5, 2010, Iran.
- [15] A. F. Gawad, “Studying the impact of different lighting loads on both harmonics and power factor” *Universities Power Engineering Conference, 2007. UPEC 2007*.
- [16] Z. M. Lian; X. B. Wu, X. I. Yan, “A novel smart power management integrated circuit used for fluorescent lamp”, *Power Elo. Motion Control Conf. China*, pp. 1568 – 1572, Vol.3, 2004
- [17] J. A. Fuentes, A. Gabaldon, F.J. Canovas, A. Molina, “Harmonic Model of Electronically Controlled Loads”, *Power Engineering Society Summer Meeting, IEEE*, vol. 3 pp 1805 – 1810, 2000.
- [18] N. K. Medora, and A. Kusko, “Computer-Aided Design and Analysis of Power-Harmonic Filters” *IEEE Trans. on industry Applications*, vol. 36. no. 2, pp. 604-613. Mar/Apr 2000.
- [19] IEEE PES Harmonic Working Group, “Characteristics and Modeling of Harmonic Sources – Power Electronic Devices”, *IEEE Trans. on Power Delivery*, vol. 4. no. 1, pp 791-800. Oct. 2001.
- [20] N. Mohan, T. M. Undeland and W. P. Robbins, “Power Electronics: Converters, Applications, and Design”, *John Wiley&Sons Inc.* 3rd Ed., 2003.



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