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DEVELOPING AN INTEGRATED AHP AND INTUITIONISTIC FUZZYTOPSIS METHODOLOGY

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Original scientific paper
This research gives an overview of the Analytic Hierarchy Process (AHP) and Intuitionistic FuzzyTOPSIS (IFT) methods. This study deals with an evaluation methodology based on the AHP-IFT where the uncertainties are handled with linguistic values. First, the supplier selection problem is formulated using AHP and, then, it is used to determine the weights of the criteria. Later, IFT is used to obtain full- ranking among alternatives based on opinion of the Decision Makers (DMs). The present model provides an accurate and easy classification in supplier attributes by those that have been prioritized in the hybrid model. A numerical example is given to clarify the main developed result in this paper.

Keywords: *Analytic Hierarchy Process (AHP), Intuitionistic FuzzyTOPSIS (IFT), Supply Chain Management (SCM), supplier selection*

Razvoj integrirane AHP i intuicijske fuzzyTOPSIS metodologije

Izvorni znanstveni članak
U ovom se istraživanju daje pregled analitičkog hijerarhijskog postupka (AHP) i intuicijskih FuzzyTOPSIS (IFT) metoda. Rad se bavi procjenom metodologije zasnovane na AHP-IFT gdje se nesigurnosti opisuju lingvističkim vrijednostima. Najprije se problem izbora dobavljača formulira primjenom AHP, a zatim se koristi za određivanje težina kriterija. Kasnije se IFT koristi za postizanje rangiranja među alternativama temeljenim na mišljenju donositelja odluka (DMs). Ovaj model omogućuje točnu i laku klasifikaciju svojstava dobavljača prema tome kako su rangirani u hibridnom modelu. Daje se numerički primjer kako bi se objasnio glavni dobiveni rezultat u radu.

Ključne riječi: *analitčki hijerarhijski postupak (AHP), intuicijski FuzzyTOPSIS (IFT), izbor dobavljača, upravljanje lancem nabave (SCM)*

1 Introduction

Multi - Criteria Decision - Making (MCDM) is a modelling and methodological tool for dealing with complex engineering problems [1]. Many mathematical programming models have been developed to address MCDM problems. However, in recent years, MCDM methods have gained considerable acceptance for judging different proposals. Intuitionistic fuzzy set (IFS) theory introduced by Atanassov [2] is an extension of the classical Fuzzy Set (FS), which is a suitable tool to deal with the vagueness and uncertainty decision information. Recently, some researchers have shown interest in the IFS theory and performed it in the field of MCDM [3 ÷ 11]. However, IFS has been applied to many areas such as medical diagnosis [12 ÷ 14], decision-making problems [15 ÷ 38], pattern recognition [39 ÷ 44], supplier selection [45, 46], enterprise partners selection [47], personnel selection [48], evaluation of renewable energy [49], facility location selection [50], web service selection [51], printed circuit board assembly [52], management information system [53] and project selection [54].

The AHP proposed by Saaty [55], is one of the most popular methods in those based on the preference relation in the decision-making process. The AHP is a well-known method for solving decision-making problems. In this method, the decision-maker (DM) performs pair-wise comparisons and, then, the pair-wise comparison matrix and the eigenvector are derived to specify the weights of each parameter in the problem. The weights guide the DM in choosing the superior alternative.

We shall study the AHP-IFT methodology where all the values are expressed in Intuitionistic fuzzy numbers collected. To do that, we first present the concept of AHP and determine the weight of criteria based on opinion of decision makers. Then, we introduce the concept of IFT and develop the model based on opinion of the decision makers. The rest of the paper is organized as follows: Section 2 provides materials and methods; mainly AHP, Fuzzy Set Theory (FST) and Intuitionistic Fuzzy Set (IFS). The AHP-IFT methodology is introduced in Section 3. How the proposed model is used in a numerical example is explained in Section 4. Finally, the conclusions are provided in the final section.

2 Preliminaries

2.1 Basic concept of AHP

The AHP is a general theory of measurement. It is used to derive relative priorities on absolute scale from both discrete and continuous paired comparisons in multilevel hierarchic structures. These comparisons may be taken from a fundamental scale that reflects the relative strength of preferences. The AHP has a special concern with deviation from consistency and the measurement of this deviation, and with dependence within and between the groups of elements of its structure. It has found its widest applications in MCDM. Generally, the AHP is a nonlinear framework for carrying out of both deductive and inductive thinking without use of the syllogism [56].

The AHP proposed by Saaty (1980) is a flexible, method for selecting among alternatives based on their relative performance with respect to criteria [57, 58].The AHP resolves complex decisions by structuring the alternatives into a hierarchical framework. The hierarchy is constructed through pair-wise comparisons of individual judgments rather than attempting to prioritize the entire list of decisions and criteria. This process has been given as follows [59]:

- Describe the unstructured problem,
- Detailed criteria and alternatives,
- Recruit pair wise comparisons among decision elements,

- Use the eigenvalue method to predict the relative weights of the decision elements,
- Compute the consistency properties of the matrix, and
- Collect the weighted decision elements.

The AHP techniques form a framework of the decisions that uses a one-way hierarchical relation with respect to decision layers. The hierarchy is constructed in the middle level(s), with decision alternatives at the bottom. The AHP method provides a structured framework for setting priorities on each level of the hierarchy using pair-wise comparisons that are quantified using a 1 ÷ 9 scale as demonstrated in Tab. 1.

Table 1 The 1 ÷ 9 Fundamental scale

Importance intensity	Definition
1	Equal importance
3	Moderate importance of one over another
5	Strong importance of one over another
7	Very strong importance of one over another
9	Extreme importance of one over another
2, 4, 6, 8	Intermediate values

2.2 FST

Zadeh (1965) introduced the FST to deal with the uncertainty and vagueness. A major contribution of FST is the capability of representing uncertain data. FST also allows mathematical operators and programming to be performed to the fuzzy domain. A FS is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging "between" zero and one [60 ÷ 61].

A tilde '~' will be placed above a symbol if the symbol shows a FST. A Triangular Fuzzy Number (TFN) $\tilde{M}$ is shown in Fig. 1. A TFN is denoted simply as (*a, b, c*). The parameters *a, b* and *c* ($a \le b \le c$), respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. The membership function of TFN is as follows.

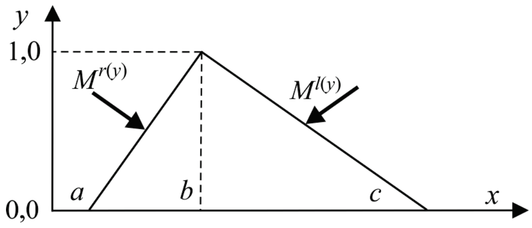

Figure 1 A TFN $\tilde{M}$

Each TFN has linear representations on its left and right side, such that its membership function can be defined as

$$\mu\left(\frac{x}{\tilde{M}}\right)=\begin{cases}0, & x<a,\\ \dfrac{x-a}{b-a}, & a\le x\le b,\\ \dfrac{c-x}{c-b}, & b\le x\le c,\\ 0, & x>c.\end{cases} \tag{1}$$

Left and right representation of each degree of membership as in the following:

$$\tilde{M}=M^{l(y)},M^{r(y)}=\left(a+(b-a)y,c+(b-c)y\right),\ y\in[0,1], \tag{2}$$

where $l(y)$ and $r(y)$ denote the left side representation and the right side representation of a fuzzy number (FN), respectively. Many ranking methods for FNs have been developed in the literature. These methods may provide different ranking results [62].

While there are various operations on TFNs, only the important operations used in this study are illustrated. Two positive TFNs (a_1, b_1, c_1) and (a_2, b_2, c_2) have been given as follows:

$$\begin{aligned}&(a_1,b_1,c_1)+(a_2,b_2,c_2)=(a_1+a_2,b_1+b_2,c_1+c_2),\\&(a_1,b_1,c_1)-(a_2,b_2,c_2)=(a_1-a_2,b_1-b_2,c_1-c_2),\\&(a_1,b_1,c_1)\cdot(a_2,b_2,c_2)=(a_1\cdot a_2,b_1\cdot b_2,c_1\cdot c_2),\\&(a_1,b_1,c_1)/(a_2,b_2,c_2)=(a_1/a_2,b_1/b_2,c_1/c_2).\end{aligned} \tag{3}$$

2.3 Basic concept of IFS

The following briefly introduces some necessary introductory basic concepts of IFS. IFS *A* in a finite set *R* can be written as:

$$\begin{aligned}&A=\{\langle r,\mu_A(r),\nu_A(r)\rangle\ |r\in R\},\\&\text{where}\\&\mu_A(r):\mu_A(r)\in[0,1],R\to[0,1],\\&\nu_A(r):\nu_A(r)\in[0,1],R\to[0,1],\end{aligned} \tag{4}$$

are membership function and non-membership function, respectively, such that

$$0\le\mu_A(r)\oplus\nu_A(r)\le1\qquad\forall r\in R\quad R\to[0,1]. \tag{5}$$

A third parameter of IFS is $\pi_A(r)$, known as the intuitionistic fuzzy index or hesitation degree of whether *r* belongs to *A* or not

$$\pi_A(r)=1-\mu_A(r)-\nu_A(r), \tag{6}$$

$\pi_A(r)$ is called the degree of indeterminacy of *r* to *A*.

It is obviously seen that for every $r\in R$:

$$0\le\pi_A(r)\le1. \tag{7}$$

If the $\pi_A(r)$ is small, knowledge about *r* is more certain. If $\pi_A(r)$ is great, knowledge about *r* is more uncertain. Obviously, when

$$\mu_A(r)=1-\nu_A(r), \tag{8}$$

for all elements of the universe, the ordinary FST concept is recovered [52].

Let *A* and *B* are IFSs of the set *R*, then multiplication operator is defined as follows (2).

$$A\oplus B=\left\{\mu_A(r)\cdot\mu_B(r),\nu_A(r)+\nu_B(r)-\nu_A(r)\cdot\nu_B(r)\,\middle|\,r\in R\right\}. \quad (9)$$

3 AHP- IFT hybrid method

To rank a set of alternatives, the AHP-IFT methodology as outranking relation theory was used to analyze the data of a decision matrix. We assume m alternatives and n decision criteria. Each alternative is evaluated with respect to the n criteria. All the values assigned to the alternatives with respect to each criterion form a decision matrix.

In this study, our model integrates two, well – known models, AHP and IFT methods. The evaluation of the study based on this hybrid methodology is given in Fig. 2. The procedure for AHP- IFT methodology ranking model has been given as follows.

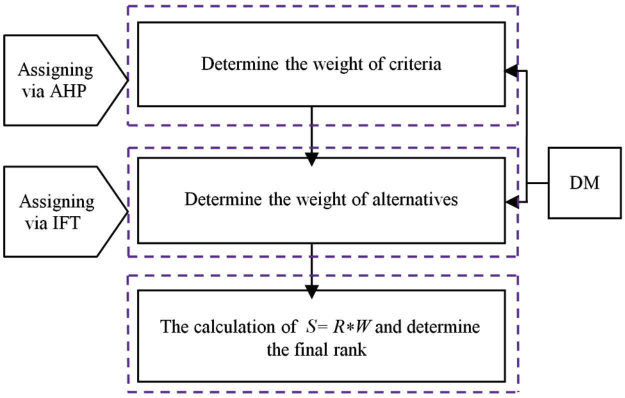

Figure 2 Schematic diagram of the AHP–IFT

Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be a set of criteria, it should be mentioned here that the presented approach mainly utilizes the IFT method presented in [45, 48 ÷ 50, 54]. The procedure for AHP-IFT methodlogy has been conducted in seven steps presented as follows:

Step 1: Determine the weight of criteria based on the opinion of decision makers (W).

In the first step, we assume that decision group contains $l = \{l_1, l_2, \dots, l_l\}$ DMs. The decision group or decision makers are given the task of forming individual pair-wise comparisons by using standard scale of nine levels given in Tab. 2.

Table 2 The 1 ÷ 9 Fundamental scale of absolute numbers

Importance intensity	Definition	Definition
1	Very bad (VB)	Equal mportance
3	Bad (B)	Moderate importance of one over another
5	Medium best (MB)	Strong importance of one over another
7	Good (G)	Very strong importance of one over another
9	Very good (VG)	Extreme importance of one over another

Step 2: Determine the weights of importance of DMs.

In the second step, we assume that decision group contains $l = \{l_1, l_2, \dots, l_l\}$ DMs. The importances of the DMs are considered as linguistic terms. These linguistic terms were assigned to IFN. Let $D_l = [\mu_l, \nu_l, \pi_l]$ be an intuitionistic fuzzy number for rating of k^{th} DM. Then the weight of l^{th} DM can be calculated as:

$$\lambda_l=\frac{\mu_l+\pi_l\left(\dfrac{\mu_l}{\mu_l+\nu_l}\right)}{\sum_{l=1}^{k}\left[\mu_l+\pi_l\left(\dfrac{\mu_l}{\mu_l+\nu_l}\right)\right]}, \quad (10)$$

where $\lambda_l\in[0,1]$ and $\sum_{l=1}^{k}\lambda_l=1$.

Step 3: Determine Intuitionistic Fuzzy Decision Matrix (IFDM).

Based on the weight of DMs, the aggregated intuitionistic fuzzy decision matrix (AIFDM) was calculated by applying intuitionistic fuzzy weighted averaging (IFWA) operator Xu [63]. In group decision-making process, all the individual decision opinions need to be fused into a group opinion to construct AIFDM.

Let $R^{(l)}=(r_{ij}^{(l)})_{m\times n}$ is an IFDM of each DM. $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k\}$ is the weight of DM.

$R=(r_{\text{ij}})_{m\times n}$, where

$$\begin{aligned} r_{ij}&=IFWA_\lambda\left(r_{ij}^{(1)},r_{ij}^{(2)},\dots,r_{ij}^{(l)}\right)=\\ &=\lambda_1 r_{ij}^{(1)}\oplus\lambda_2 r_{ij}^{(2)}\oplus\lambda_3 r_{ij}^{(3)}\oplus\dots\oplus\lambda_k r_{ij}^{(k)}= \quad (11)\\ &=\left[1-\prod_{l=1}^{k}(1-\mu_{ij}^{(l)})^{\lambda_l},\prod_{l=1}^{k}(\nu_{ij}^{(l)})^{\lambda_l},\prod_{l=1}^{k}(1-\mu_{ij}^{(l)})^{\lambda_l}-\prod_{l=1}^{k}(\nu_{ij}^{(l)})^{\lambda_l}\right].\end{aligned}$$

Step 4: The calculation of $S= R*W$.

In the step 4, the weights of criteria (W) with respect to IFDM (R) are defined as follows:

$$S=R*W. \quad (12)$$

Step 5: Determine intuitionistic fuzzy positive and negative ideal solution.

In this step, the intuitionistic fuzzy positive ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFNIS) have to be determined. Let J_1 and J_2 be benefit criteria and cost criteria, respectively. A^* is IFPIS and A^- is IFNIS. Then A^* and A^- are equal to:

$$A^*=(r'^*_1,r'^*_2,\dots,r'^*_n),\ r'^*_j=(\mu'^*_j,\nu'^*_j,\pi'^*_j),\ j=1,2,\dots,n, \quad (13)$$

and

$$A^-=(r'^-_1,r'^-_2,\dots,r'^-_n),\ r'^-_j=(\mu'^-_j,\nu'^-_j,\pi'^-_j),\ j=1,2,\dots,n, \quad (14)$$

where

$$\mu'^*_j=\left\{(\max_i\{\mu'_{ij}\} j\in J_1),(\min_i\{\mu'_{ij}\} j\in J_2)\right\}, \quad (15)$$

$$\nu'^*_j=\left\{(\min_i\{\nu'_{ij}\} j\in J_1),(\max_i\{\nu'_{ij}\} j\in J_2)\right\}, \quad (16)$$

$$\pi'^*_j=\left\{(1-\max_i\{\mu'_{ij}\}-\min_i\{\nu'_{ij}\} j\in J_1),(1-\min_i\{\mu'_{ij}\}-\max_i\{\nu'_{ij}\} j\in J_2)\right\}, \quad (17)$$

$$\mu'^-_j=\left\{(\min_i\{\mu'_{ij}\} j\in J_1),(\max_i\{\mu'_{ij}\} j\in J_2)\right\}, \quad (18)$$

$$\nu'^{-}_{j} = \left\{ \left(\max_{i} \{\nu'_{ij}\} \, j \in J_1 \right), \left(\min_{i} \{\nu'_{ij}\} \, j \in J_2 \right) \right\}, \tag{19}$$

$$\pi'^{-}_{j} = \left\{ \left(1 - \min_{i} \{\mu'_{ij}\} - \max_{i} \{\nu'_{ij}\} j \in J_1 \right), \left(1 - \max_{i} \{\mu'_{ij}\} - \min_{i} \{\nu'_{ij}\} j \in J_2 \right), \right\}. \tag{20}$$

Step 6: Determine the separation measures between the alternative.

Separation between alternatives on IFS, distance measures proposed by Atanassov [64], Szmidt and Kacprzyk [65], and, Grzegorzewski [66] including the generalizations of Hamming distance, Euclidean distance and their normalized distance measures can be used. After selecting the distance measure, the separation measures, S_i^* and S_i^-, of each alternative from IFPIS and IFNIS, are calculated.

$$S_i^* = \frac{1}{2} \sum_{j=1}^{n} \left[\left| \mu'_{ij} - \mu'^{*}_{j} \right| + \left| \nu'_{ij} - \nu'^{*}_{j} \right| + \left| \pi'_{ij} - \pi'^{*}_{j} \right| \right], \tag{21}$$

$$S_i^- = \frac{1}{2} \sum_{j=1}^{n} \left[\left| \mu'_{ij} - \mu'^{-}_{j} \right| + \left| \nu'_{ij} - \nu'^{-}_{j} \right| + \left| \pi'_{ij} - \pi'^{-}_{j} \right| \right]. \tag{22}$$

Step 7: Determine the final ranking.

In the final step, the relative closeness coefficient of an alternative A_i with respect to the IFPIS A^* is defined as follows:

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \text{ where } 0 \geq C_i^* \leq 1. \tag{23}$$

The alternatives were ranked according to descending order of C_i^*'s score.

4 Numerical examples

In this section, we will describe how an AHP- IFT methodlogy was applied via an example. Criteria to be considered in the selection of projects are determined by the expert team from a decision group. In our study, we employ six evaluation criteria. The attributes which are considered here in assessment of A_i (i = 1, 2,…, 6) are: (1) C_1 is benefit; (2) C_2,..., C_6 are cost . The committee evaluates the performance of alternatives A_i (i=1,2,…,4) according to the attributes C_j (j = 1, 2,…, 6) respectively. Therefore, one cost criterion, C_1, and five benefit criteria, C_2,...,C_6 are considered. After preliminary screening, four alternatives A_1, A_2, A_3, and A_4, remain for further evaluation. A team of four DMs such as DM_1, DM_2, DM_3, and DM_4 has been formed to select the most suitable alternative.

Now utilize the proposed AHP- IFT methodology to prioritize alternatives, the following steps were taken:

Table 3 The importance weight of the criteria

Criteria	DM_1	DM_2	DM_3	DM_4
C_1	*G*	*VG*	*VG*	*MB*
C_2	*VG*	*VG*	*VG*	*VG*
C_3	*MB*	*G*	*VG*	*MB*
C_4	*G*	*G*	*VG*	*G*
C_5	*G*	*VG*	*MB*	*G*
C_6	*MB*	*G*	*MB*	*VG*

The opinions of DMs on criteria were aggregated to determine the weight of each criterion.

$$W_{\{R_1, R_2, R_3, R_4, R_5, R_6\}} = \begin{bmatrix} 0{,}170 \\ 0{,}205 \\ 0{,}148 \\ 0{,}170 \\ 0{,}159 \\ 0{,}148 \end{bmatrix}^{\mathrm{T}}.$$

Degree of the DMs on group decision, shown in Tab. 4, and linguistic terms used for the ratings of the DMs, as in Tab. 5, respectively.

Table 4 Linguistic term for rating DMs

Linguistic terms	IFNs
Very important	(0.80, 0.10)
Important	(0.50, 0.20)
Medium	(0.50, 0.50)
Bad	(0.30, 0.50)
Very bad	(0.20, 0.70)

Table 5 The importance of DMs and their weights

	DM_1	DM_2	DM_3	DM_4
Linguistic terms	Very important	Medium	Important	Important
Weight	0,342	0,274	0,192	0,192

Construct the aggregated IFDM based on the opinions of DMs, the linguistic terms are shown in Tab. 6.

Table 6 Linguistic terms for rating the alternatives

Linguistic terms	IFNs
Extremely good (EG)	[1.00; 0.00; 0.00]
Very good (VG)	[0.85; 0,05; 0.10]
Good (G)	[0.70; 0.20; 0.10]
Medium bad (MB)	[0.50; 0.50; 0.00]
Bad (B)	[0.40; 0.50; 0.10]
Very bad (VB)	[0.25; 0.60; 0.15]
Extremely bad (EB)	[0.00; 0.90; 0.10]

The ratings given by the DMs to six alternatives are shown in Tab. 7.

Table 7 The ratings of the alternatives

Alternative	Criteria	DM_1	DM_2	DM_3	DM_4
A_1	C_1	*VG*	*VG*	*G*	*G*
	C_2	*G*	*VG*	*MB*	*MB*
	C_3	*VG*	*G*	*B*	*VG*
	C_4	*VG*	*VG*	*G*	*G*
	C_5	*VG*	*VG*	*MB*	*G*
	C_6	*G*	*VG*	*MB*	*MB*
A_2	C_1	*G*	*VG*	*MB*	*B*
	C_2	*VG*	*VG*	*G*	*MB*
	C_3	*VG*	*VG*	*B*	*B*
	C_4	*VG*	*VG*	*MB*	*G*
	C_5	*G*	*G*	*G*	*G*
	C_6	*VG*	*VG*	*MB*	*B*
A_3	C_1	*VG*	*VG*	*G*	*VG*
	C_2	*VG*	*G*	*G*	*VG*
	C_3	*VG*	*G*	*VG*	*G*
	C_4	*VG*	*VG*	*VG*	*VG*
	C_5	*VG*	*VG*	*VG*	*VG*
	C_6	*VG*	*VG*	*VG*	*VG*
A_4	C_1	*MB*	*G*	*MB*	*VG*
	C_2	*G*	*VG*	*G*	*G*
	C_3	*MB*	*VG*	*G*	*G*
	C_4	*VG*	*G*	*VG*	*VG*
	C_5	*VG*	*VG*	*G*	*VG*
	C_6	*VG*	*VG*	*VG*	*VG*

The aggregated IFDM based on aggregation of DMs' opinions was constructed as follows:

$$
R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad & \quad C_5 \quad & \quad C_6 \quad \end{array} \\
\begin{bmatrix}
(0.80,0.08,0.12) & (0.69,0.20,0.11) & (0.76,0.12,0.12) & (0.80,0.09,0.11) & (0.78,0.11,0.11) & (0.69,0.20,0.11) \\
(0.68,0.20,0.12) & (0.78,0.11,0.11) & (0.74,0.13,0.13) & (0.78,0.11,0.11) & (0.69,0.21,0.10) & (0.75,0.13,0.12) \\
(0.82,0.07,0.11) & (0.79,0.10,0.11) & (0.79,0.10,0.11) & (0.84,0.05,0.11) & (0.84,0.05,0.11) & (0.84,0.05,0.11) \\
(0.83,0.16,0.1) & (0.75,0.14,0.11) & (0.70,0.19,0.11) & (0.81,0.08,0.11) & (0.82,0.07,0.11) & (0.85,0.05,0.10) \\
(0.55,0.38,0.07) & (0.42,0.52,0.06) & (0.64,0.40,0.06) & (0.55,0.33,0.12) & (0.54,0.33,0.13) & (0.40,0.54,0.06) \\
(0.75,0.13,0.12) & (0.69,0.19,0.12) & (0.75,0.13,0.12) & (0.75,0.13,0.12) & (0.85,0.05,0.10) & (0.78,0.11,0.11)
\end{bmatrix}
\end{array}
$$

After the weights of the criteria and the rating of the projects were determined, the aggregated weighted IFDM was constructed as follows:

$$
R' = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad & \quad C_5 \quad & \quad C_6 \quad \end{array} \\
\begin{bmatrix}
(0.136,0.0136,0.020) & (0.141,0.041,0.023) & (0.112,0.018,0.018) & (0.136,0.015,0.019) & (0.124,0.017,0.017) & (0.102,0.030,0.016) \\
(0.116,0.034,0.020) & (0.160,0.023,0.023) & (0.110,0.019,0.019) & (0.133,0.019,0.019) & (0.110,0.033,0.016) & (0.111,0.019,0.018) \\
(0.139,0.012,0.019) & (0.162,0.021,0.023) & (0.117,0.015,0.016) & (0.143,0.009,0.019) & (0.134,0.008,0.017) & (0.124,0.007,0.016) \\
(0.141,0.027,0.017) & (0.154,0.029,0.023) & (0.104,0.028,0.016) & (0.138,0.014,0.019) & (0.130,0.011,0.017) & (0.126,0.007,0.015) \\
(0.094,0.065,0.012) & (0.086,0.107,0.012) & (0.095,0.059,0.009) & (0.094,0.056,0.020) & (0.086,0.052,0.021) & (0.059,0.080,0.009) \\
(0.128,0.022,0.020) & (0.141,0.039,0.025) & (0.111,0.019,0.018) & (0.128,0.022,0.020) & (0.135,0.008,0.016) & (0.115,0.016,0.016)
\end{bmatrix}
\end{array}
$$

Then IFPIS and IFNIS were provided as follows:

$$A^{*} = \{(0.141,0.012,0.847),(0.162,0.021,0.817),(0.117,0.015,0.868),(0.143,0.009,0.848),(0.135,0.008,0.857),(0.126,0.007,0.867)\}$$

$$A^{-} = \{(0.094,0.065,0.841),(0.086,0.107,0.807),(0.095,0.059,0.846),(0.094,0.056,0.850),(0.086,0.052,0.862),(0.059,0.080,0.861)\}$$

Negative and positive separation measures based on normalized Euclidean distance for each alternative and the relative closeness coefficient were calculated as shown in Tab. 8.

Table 8 Separation measures and the relative closeness coefficient of each alternative.

Alternatives	S^{*}	S^{-}	C_i^{*}
A_1	2,563	2,737	0,516
A_2	2,570	2,725	0,515
A_3	2,500	2,798	0,528
A_4	2,530	2,773	0,523

5 Conclusion

The AHP-IFT methodology has been emphasized in this paper. The purpose of the study was to use a MCDM Method which combines AHP and IFT methods to evaluate a set of alternatives in order to reach a suitable and best qualified alternative. In the evaluation process, the ratings of each alternatives, given with intuitionistic fuzzy information, were represented as IFNs. In this methodology, AHP is used to assign weights to the criteria, while IFT is employed to calculate the full-ranking of the altenatives. The AHP-IFT methodology was used to aggregate the rating of DMs. Multiple DMs are often preferred rather than a single DM to avoid to minimize the partiality in the decision process. Therefore, group decision making process for alternative selection is very useful. However, it combines the idea of different DMs by a scientific MCDM method. An actual life example information and performance are usually uncertain. Therefore, the DMs are unable to express their judgment on the alternative and criteria with crisp value and the evaluation is very often expressed in linguistic terms. AHP and IFT are suitable ways to deal with MCDM because it contains a vague perception of DMs` opinions. A numerical example was illustrated and finally the result as follow: Among 6 alternatives with respect to 6 criteria, after using this methodology, the best one is alternative 3 and alternative 4, alternative 6, alternative 1, alternative 2, alternative 5 will follow it respectively. The presented approach not only validates the methods, but also considers a more extensive list of benefit and cost oriented criteria, suitable for most suitable alternative selection. The AHP-IFT mothodology has capability to deal with similar types of the same situations with uncertainty in MCDM problems.

6 Reference

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