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NEW APPROACH TO ESTABLISHING POISSON'S RATIOS OF SOFT-WOOD COMPONENTS

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Summary

In literature, one can usually find experimental investigations into global mechanical behaviour of soft-wood specimens, but properties of their individual components, (namely the properties of the late-wood and the early-wood parts) are rarely presented. Therefore, information on the moisture content and on the density is presented here. In fact, the ratio of the early- and late-wood parts in the cross-sectional area represents a significant influential factor, especially in the case of soft-wood components. Based on previous experimental data bases, the authors offer an original and simple analytical method for establishing Poisson's ratios for new, untested soft-wood specimens.

Key words: *analytical calculus, Poisson's ratio, Video Image Correlation, experimental investigation*

1. Introduction

At the present moment, experimental investigations are focused mainly on the global mechanical behaviour [1; 4]; only a small number of experimental investigations have been focused on the elastic properties of individual components. The authors' innovative proposal consists of keeping the wood specimen intact and of evaluating individual elastic properties of the early and late rings [2] under these conditions. In [3] one can find how Young's modulus of the early- and late-wood parts can be established for the soft-wood species.

Softwood specimens (pine-wood) with dimensions of 20 mm x 20 mm x 50 mm ($R{*}T{*}L$) were tested under compression. These specimens contain several annual rings. Elastic properties of the specimen are determined by the elastic properties of different rings.

In a softwood specimen having a tangential-radial (*T-R*) cross-section, the early and late annual rings (the corresponding early- and late-wood segment/section/region) are parallel-connected (Figure 1). Figure 2 presents one of these characteristic elements. When the cross-section of a specimen is a longitudinal-tangential (*L-T*) one, adequate regions are serial-connected. Starting from this fact, it became useful to determine their individual elastic properties for more accurate and easier finite element modelling.

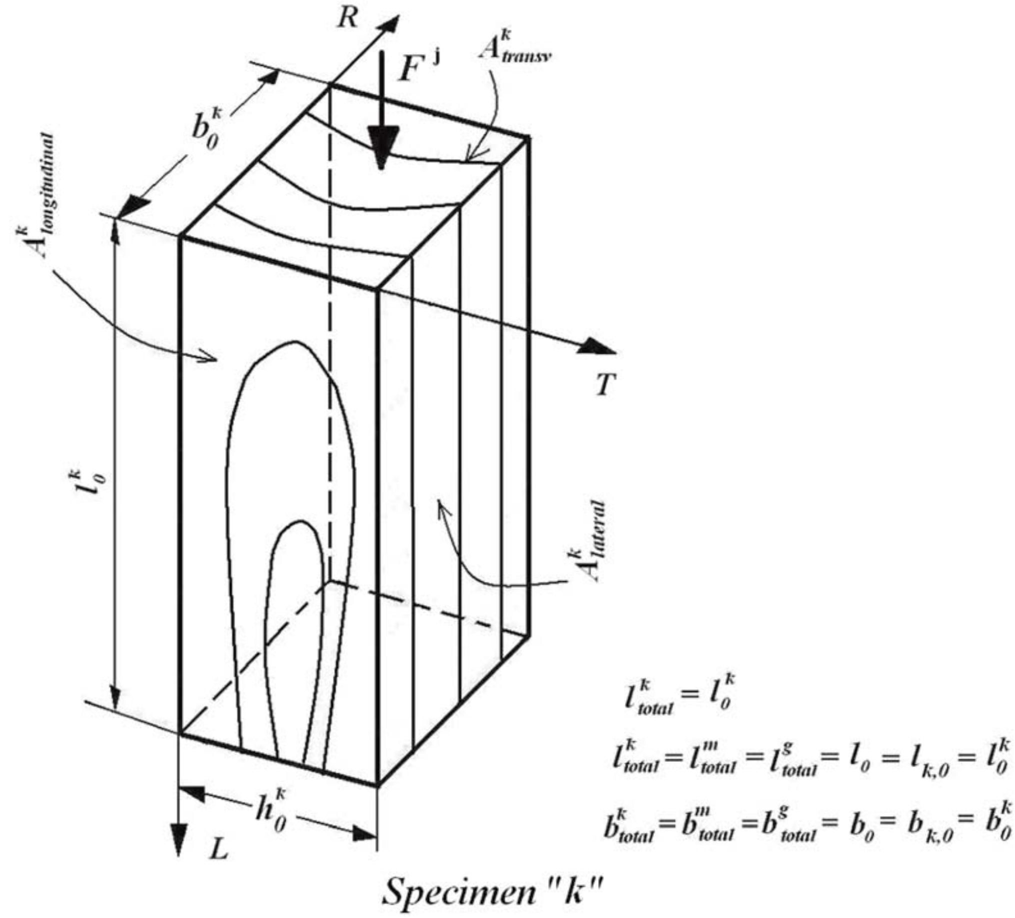

Fig. 1 Geometrical parameters of a soft-wood specimen with a *T-R* cross-section

2. The proposed analytical approach for tangential-radial (*T-R*) cross-section

Starting from the definition of Poisson's ratio

$$\nu_t = -\frac{\varepsilon_t}{\varepsilon_\ell}[-], \tag{1}$$

where $\varepsilon_t[-]$ represents the strain along the transversal direction, and

$\varepsilon_\ell = \frac{\Delta\ell}{\ell_0}[-]$ is the strain along the longitudinal direction,

one can obtain the following particular forms corresponding to transversal directions *b* and *h* (see Figure 1):

$$\nu_b = -\frac{\varepsilon_b}{\varepsilon_\ell}[-], \qquad \text{and} \qquad \nu_h = -\frac{\varepsilon_h}{\varepsilon_\ell}[-]. \tag{2}$$

In the case presented in Figure 1, i.e. in the case of wood materials, we have the following correspondence between the used notations:

$$\varepsilon_\ell \equiv \varepsilon_L \ ; \quad \varepsilon_b \equiv \varepsilon_R \ ; \quad \varepsilon_h \equiv \varepsilon_T \ ; \quad \nu_b \equiv \nu_{LR} ; \quad \nu_h \equiv \nu_{LT} . \tag{3}$$

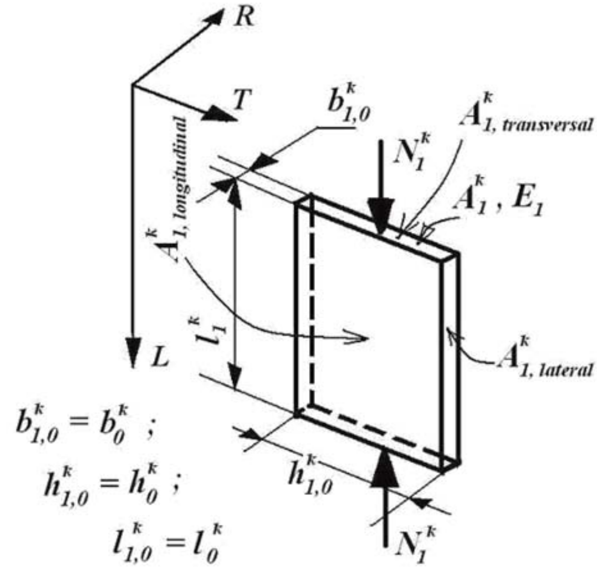

Fig. 2 A representative element (here: ***1*** - early-wood)

The authors highly recommend the use of specimen sets consisting of 3 specimens worked identically and made of the same wood item. The number of sets has to be large enough to enable further statistical evaluation. In each set, two specimens (denoted by ***k*** and ***m***) were used to determine the adequate mechanical parameters (here: Poisson's ratios), and the third one (noted with ***g***) served for validation of the obtained results. The specimens were compressed in a longitudinal direction. Based on the measured results (force-displacement $F - \Delta\ell$ curves), the authors developed and then applied a simple analytical procedure in order to obtain the individual properties of the early and the late segments/regions. The softwood specimen ***k*** (see Figure 1) can be considered as a system of two parallel-connected elements: ***1*** - early-wood, ***2*** - late-wood, each having a different cross-sectional area. They will work together and, for a given cross-section, they will offer nominal, global mechanical behaviour. If we change only their cross section ratio, we will obtain, of course, other global mechanical behaviours.

2.1. Calculation of Poisson's ratios along the *R* main direction

In the case of investigations along the *R* direction, one can observe that the two characteristic elements ***1*** and ***2*** are *serial-connected* because, in this direction, their transversal elongations and the corresponding strains have to be added together in order to obtain the total effect. Thus, their global effect (analysing the specimen marked ***k***) will be the algebraic sum of partial effects.

We have the following relationships:

$$\nu^k_{LR,global} = -\frac{\varepsilon^k_{R,global}}{\varepsilon^k_{L,global}}[-]; \qquad \varepsilon^k_{R,global} = \frac{\Delta b^k_{global}}{b^k_0}[-], \tag{4}$$

where, taking into consideration the Bernoulli hypothesis along the longitudinal main direction *L*, we have:

$$\varepsilon^k_{L,global} = \varepsilon^k_{L,1} = \varepsilon^k_{L,2} = \frac{\Delta\ell^k}{\ell^k_0}[-]. \tag{5}$$

To describe the serial connection of these elements (***1*** and ***2***) we follow the well-known steps presented below:

$$\Delta b_{global}^{k} = \Delta b_{1}^{k} + \Delta b_{2}^{k} , \tag{6}$$

and

$$\varepsilon_{R,global}^{k} \cdot b_{0}^{k} = \varepsilon_{R,1}^{k} \cdot b_{1,0}^{k} + \varepsilon_{R,2}^{k} \cdot b_{2,0}^{k} ; \tag{7}$$

$$\nu_{LR,global}^{k} \cdot \varepsilon_{L,global}^{k} \cdot b_{0}^{k} = \nu_{LR,1} \cdot \varepsilon_{L,1}^{k} \cdot b_{1,0}^{k} + \nu_{LR,2} \cdot \varepsilon_{L,2}^{k} \cdot b_{2,0}^{k} . \tag{8}$$

Finally (after the adequate reduction, taking into consideration the above-mentioned Bernoulli hypothesis) we obtain:

$$\nu_{LR,global}^{k} \cdot b_{0}^{k} = \nu_{LR,1} \cdot b_{1,0}^{k} + \nu_{LR,2} \cdot b_{2,0}^{k} . \tag{9,a}$$

By multiplying this relation by h_0^k, and based on the effective areas ($A_{1,0}^k$, $A_{2,0}^k$) of the components, which, in fact, offer together the total cross-sectional area $A_0^k = A_{1,0}^k + A_{2,0}^k$, we obtain the following expression:

$$\nu_{LR,global}^{k} \cdot A_{0}^{k} = \nu_{LR,1} \cdot A_{1,0}^{k} + \nu_{LR,2} \cdot A_{2,0}^{k} . \tag{9}$$

Similarly, for the specimen marked ***m*** we will obtain:

$$\nu_{LR,global}^{m} \cdot b_{0}^{m} = \nu_{LR,1} \cdot b_{1,0}^{m} + \nu_{LR,2} \cdot b_{2,0}^{m} , \tag{10,a}$$

and its final version will be:

$$\nu_{LR,global}^{m} \cdot A_{0}^{m} = \nu_{LR,1} \cdot A_{1,0}^{m} + \nu_{LR,2} \cdot A_{2,0}^{m} . \tag{10}$$

The system of equations (9)-(10) will offer the requested $\nu_{LR,1}$, $\nu_{LR,2}$ Poisson's ratios of the early- and the late-wood parts.

In order to predict the global ν_{LR} Poisson's ratio of an untested specimen ***n***, based on a previously collected/established data base of the $\nu_{LR,1}$, $\nu_{LR,2}$ values for the same wood species, we can apply the following *weighted value*:

$$\nu_{LR,global}^{n} = \frac{\nu_{LR,1} \cdot A_{1,0}^{n} + \nu_{LR,2} \cdot A_{2,0}^{n}}{A_{0}^{n}} , \tag{10,b}$$

where, of course, one has to establish the corresponding values of the component areas $\left(A_{1,0}^n ,\ A_{2,0}^n\right)$.

2.2. Calculation of Poisson's ratios along the *T* main direction

In this case, the characteristic elements ***1*** and ***2*** are parallel-connected. Consequently, for the specimen marked ***k***, we have the double equality

$$\Delta h_{global}^{k} = \Delta h_{1}^{k} = \Delta h_{2}^{k} , \tag{11}$$

where the following relations by definition can be applied:

$$\Delta h_{global}^k = \varepsilon_{T,global}^k \cdot h_0^k \; ; \quad \Delta h_1^k = \varepsilon_{T,1}^k \cdot h_{1,0}^k \; ; \quad \Delta h_2^k = \varepsilon_{T,2}^k \cdot h_{2,0}^k \; ;$$
$$\varepsilon_{T,1}^k = \nu_{LT,1} \cdot \varepsilon_{L,1}^k ; \qquad \varepsilon_{T,2}^k = \nu_{LT,2} \cdot \varepsilon_{L,2}^k . \tag{12}$$

So, relation (11) offers the following system for the requested Poisson's ratios:

$$\begin{cases} \nu_{LT,1} = \nu_{LT,global}^k \cdot \dfrac{h_0^k}{h_{1,0}^k} \; ; \\ \nu_{LT,2} = \nu_{LT,global}^k \cdot \dfrac{h_0^k}{h_{2,0}^k} \quad . \end{cases} \tag{13}$$

One can note that the analytical calculation described above is based on the experimentally measured values and their further analytical evaluation:

$$\varepsilon_{T,global}^k = \frac{\Delta h_{global}^k}{h_0^k} \; , \quad \text{and} \quad \nu_{LT,global}^k = \frac{\varepsilon_{T,global}^k}{\varepsilon_{L,global}^k} \; . \tag{14}$$

This calculation will be repeated for the specimen marked ***m***, and their average value will represent the requested values of Poisson's ratios.

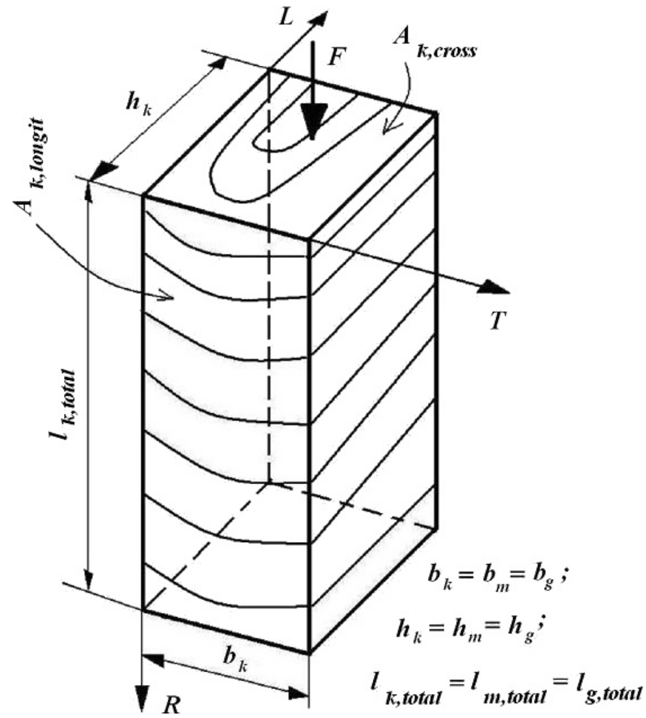

Fig. 3 Geometrical parameters of a soft-wood specimen with the *T-L* cross-section

In order to make an easier calculation and to predict the global $\nu_{LT,global}^n$ Poisson's ratio for some untested specimens, one has to multiply the lines of relations (13) applied for specimen ***n*** by $b_{1,0}^n$, and by $b_{2,0}^n$, and then to add them together term by term. Finally, after a simple calculation, one obtains the so-called *weighted value* of this Poisson's ratio:

$$\nu_{LT,global}^n = \frac{\nu_{LT,1} \cdot A_{1,0}^n + \nu_{LT,2} \cdot A_{2,0}^n}{A_0^n} \; . \tag{15}$$

3. The proposed analytical approach for tangential-radial (*T-L*) cross-section

In the calculation of Poisson's ratios along the *T* direction (in order to establish ε_T and ν_{RT}), or along the *L* direction (in order to establish ε_L and ν_{RL}), one can observe that the characteristic elements ***1*** and ***2*** are *parallel-connected*, and based on the presented strategies, one can perform a very simple calculation of these Poisson's ratios (see Figure 3).

4. Experimental results for the method validation

The authors performed some preliminary experimental investigations on sets of pine-wood specimens, each consisted of 3 specimens worked identically and made of the same wood-item.

The specimens, having dimensions of 20 *mm* x 20 *mm* x 50 *mm* ($R*T*L$), were subjected to uni-axial compression along their longitudinal direction; the number of sets was large enough for an adequate statistical evaluation.

In order to establish the displacement fields with an adequate accuracy and to determine the corresponding strains, in these experimental investigations the authors used a VIC-3D system (**V**ideo **I**mage **C**orrelation Spatial**, 3D** version) from ISI-Sys GmbH Kassel, Germany. Relative displacements of pairs of points (10-11), (11-12), (10-12), (7-8), (8-9). (7-9), (13-14). (14-15), and (13-15) were selected for the calculation of Poisson's ratios. These selected (marked) points on the specimen are shown in Figure 4. The rest of the marked points served both to establish Young's modulus and to perform calculation to verify the obtained results.

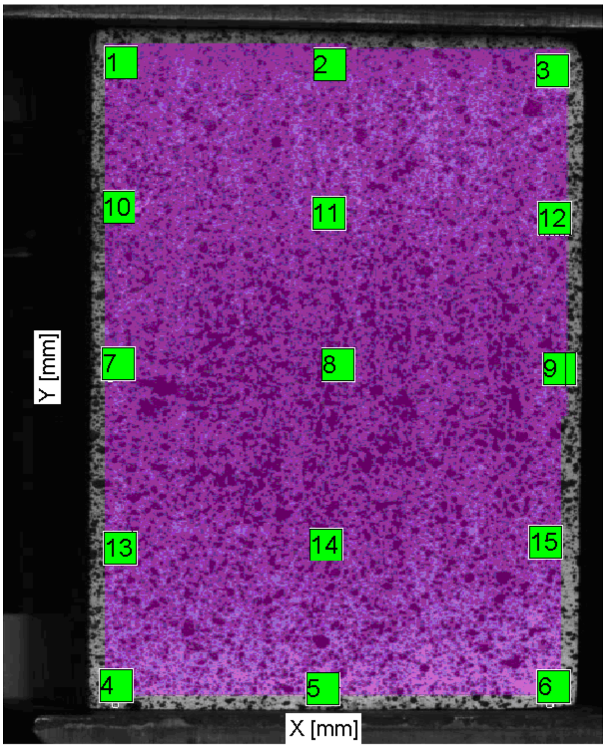

Fig. 4 The selected points for calculation (using the VIC-3D)

Thus, the presented strategy for ν_{LR} will be illustrated, where the mean values of the preliminary statistical calculation are included in Table 1.

Table 1 Initial data of the two pre-selected specimens

Specimen	$A_{global}\left[mm^2\right]$	$A_{1,0}\left[mm^2\right]$	$A_{2,0}\left[mm^2\right]$	$\nu_{LR,global}\left[-\right]$
Specimen ***k***	851	657	194	0.3947
Specimen ***m***	854	619	235	0.3999

Substituting these values in system (9) - (10) finally results in the requested values of $\nu_{LR,1} = 0.42513\left[-\right]$ and $\nu_{LR,2} = 0.31486\left[-\right]$.

Table 2 Initial data of the other two tested specimens

Specimen	$A_{global}\left[mm^2\right]$	$A_{1,0}\left[mm^2\right]$	$A_{2,0}\left[mm^2\right]$	$\nu^{exp}_{LR,global}\left[-\right]$	$\nu^{calc}_{LR,global}\left[-\right]$
Specimen ***l***	883	692	191	0.4239	0.401176 (5.3 %)
Specimen ***r***	865	681	184	0.4178	0.401571 (3.88 %)

In order to illustrate the efficiency of the described method, the mean values (based on the preliminary statistical calculation) of two other specimens, that is ***l*** and ***r*,** are included in Table 2. These specimens were tested in the similar manner, and finally the so-called global Poisson's ratios $\nu^{exp}_{LR,global}$ were obtained. Using the above-determined values of $\nu_{LR,1}$, and $\nu_{LR,2}$ the weighted global values $\nu^{calc}_{LR,global}$ were calculated, obtaining good agreement with the experimentally determined global values $\nu^{exp}_{LR,global}$.

5. Final remarks

In the presented strategy the authors analyzed and illustrated only the most often used fibre orientation (when the wood member is worked along its longitudinal direction and its cross-section has the radial and tangential natural main directions *R* and *T*).

Similar strategies were elaborated regarding the other two variants of working the wood specimens.

Due to the authors' experience in using VIC-3D system, it became possible to substitute the very expensive video extensometer with two simple supplementary unloaded reference plates with the same efficiency. The accuracy of measurements is about 1 *micron*. For smaller displacements, especially along the transversal directions, we can also use efficiently the ESPI/Shearography system. Using some adequate data bases it became possible to predict mechanical properties of some untested wood specimens [2, 3].

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