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SPATIAL MAINTENANCE GOODS UNDER MONOPOLY

ABSTRACT

This study addresses the effects of monopolistic firms' location and guarantee time limit on pricing for goods requiring high maintenance expenditure, such as elevators, televisions and computers. A spatial maintenance model of two stages within a guaranteed time limit is outlined in this paper. Based on this model, location, maintenance commitment and pricing are all characterized under a monopoly situation. This paper outlines the optimal price and location for the monopolist. The effects of guarantee time limit on price are discussed.

Keywords: *Market structure, spatial maintenance commitment, guarantee time limit, game theory, monopoly*

JEL Classification: *C61, C72， D4, L1*

1. INTRODUCTION

Maintenance is legally required for many goods, such as elevators, televisions and computers. With a suitable guarantee time limit, a firm can improve profits and significantly encourage consumption. Maintenance has been extensively discussed in the management field and economics (Estelami, Grewal and Roggeveen, 2007; Indounas, 2008; Jin and Kato, 2006; Utaka, 2006). Most research has found a positive relationship between service guarantee and quality, while some research has questioned this relationship (Hays and Hill, 2006) and references therein. A recent paper discussed the price-matching guarantees (PMGs) of retailers and the potential negative effects on consumer perceptions (Estelami, Grewal and Roggeveen, 2007). Research in economics suggests that PMGs can support a mechanism of collusion among the retailers (Corts, 1997; Chen, 1995). The presence of PMGs by one retailer provides a disincentive to other retailers to lower their prices, because retailers' price will be matched by the PMG-offering retailer. Utaka (2006) established a multi-stage model to discuss warranties of durable goods in economics and made some interesting conclusions.

This paper focuses on the maintenance for durable goods with shipping costs. We hope to address the effects of transportation cost on the market under a monopoly and to develop a theory of warranties of durable goods with spatial competitions, since the shipping costs have significant effects on the price and location of firms (Ramcharan, 2009). Based on spatial competitions, optimal pricing strategies and location are addressed and captured. The effects of guarantee time limit on price and profits are characterized. Furthermore, our conclusions are useful for pricing durable goods in second hand markets. There is a vast literature about spatial competitions. Hotelling (1929) initially proposed a model which gives an excellent framework for research into spatial competitions. Vogel (2008) recently addressed product differentiations when spatial competitions were introduced. Loginova and Wang (2009) developed customization with spatial competition and derived some interesting results. Larralde, Stehlé and Jensen (2009) gave explicit solutions to a multi-dimensional Hotelling model with quadratic transportation costs. The Cournot equilibrium was recently addressed in a Hotelling model by Valverde and Escalona (2010). Heywood and Ye (2009) developed the theory of mixed oligopoly in spatial competitions and derived some interesting results about social welfare. Dixon (2010) discussed city planners and architects observing a variety of social phenomena. Chakravorty et al. (2008) investigated energy prices in a linear city. Nie (2010) developed a theory of technology spillover under spatial competitions.

Game theory approaches are employed in this work (Tirole, 1988; Nie, 2009; Anton and Das Varma, 2005; Dudine, Hendle, Lizzeri, 2006; Chen and Zhang, 2009). This paper is organized as follows: The model is outlined and discussed in Section 2. Analysis and main results are presented in Section 3. Optimal price and location are also outlined in Section 3. Some concluding remarks are provided in the final section.

2. THE MODEL

We assume that there is a unique producer in some industry and the monopoly firm faces demand for a corresponding good that requires high expenditure to maintain in the linear city $[0,1]$. We further assume that the guarantee time period is exactly T. Namely, it is free to maintain this product in time T after the consumer buys it. All consumers are uniformly distributed in the linear city. If the guarantee time limit expires, the consumer then has to pay maintenance costs. As an extreme case, $T=0$ is the case without a guarantee commitment.

The following notations are utilized in this paper:

p and q denote the price and quantity, respectively, of the good requiring high maintenance expenditures for the monopoly firm.

We assume that the probability to be maintained for each good observes an exponential distribution. Namely, the probability to repair at t is $\varphi(t,\lambda)=\lambda e^{-\lambda t}$ for $t \geq 0$, where λ is a constant dependent on the quality of the good and λ^{-1} is the average life expectancy of the corresponding good. The parameter λ depends on the technique of the monopoly firm. We further assume that $0 \leq T \leq \lambda^{-1}$ such that the guarantee time limit is shorter than the average life expectancy. Therefore, the probability to repair this good is from $t=0$ to $t=t_0$ is $\int_0^{t_0} \varphi(t,\lambda)dt = 1-e^{-\lambda t_0}$. The information about the product is known to both the producer and the consumers. The life cycle of many types of electronic ***products and other durable goods observes*** this kind of ***distribution.***

The firm locates at $z_1 \in [0,1]$ for sale and at $z_2 \in [0,1]$ for maintenance. The model is composed of two stages. In the first stage, the firm locates both for sale and for maintenance. In the second stage, the firm prices based on the following model.

The marginal cost incurred by production for each product is c_0, and c denotes the marginal cost to repair each time. We assume that the cost to repair the corresponding goods of quantity s each time is $c(s)=cs$. c_t is a constant and linear transportation cost incurred, which is different from that in a recent paper (Larralde, Stehlé and Jensen, 2009). Furthermore, the incurred transportation cost is wholly shouldered by consumers. We always assume that repairing the corresponding goods is cheaper than buying new goods. The quasi-linear consumer utility function, $u(q,T,x)$, in the consumption of good q with location at $x \in [0,1]$ is always employed. We further assume that u is continuously differentiable. The consumer model is presented as follows: Given any price p and guarantee time limit T, the consumer location at $x \in [0,1]$ chooses d to maximize utility, $d = d(p,T,x)$. The following utility maximization problem (UMP) is given:

$$\max_{d} \ u(d,T,x) = Ad - \frac{1}{2}d^2 - dp - cd\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t d\left|x - z_1\right| - c_t d\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt\left|x - z_2\right| \quad (1)$$

In (1), A is a positive constant, which is large enough such that the market size is full. $Ad - \frac{1}{2}d^2$ represents the utility of consumer lying at $x \in [0,1]$ to consumer goods d. dp is the price of goods of quantity d. $cd\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt$ stands for the expenditure to repair in the expected life cycle of goods d. $c_t d\left|x - z_1\right|$ represents the transportation cost to buy goods of quantity d. The term $c_t d\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt\left|x - z_2\right|$ indicates the transportation cost to repair goods of quantity d. $d(p,T,x)$ is the static demand function associated with (1). Denote the total demand $D(p,T) = \int_0^1 d(p,T,x)d\,x$. Given the price and guarantee time limit, the monopoly firm aims to maximize its objective function or corresponding profits:

$$\max_{p} \pi(p) = pq - cq\int_0^T \varphi(t,\lambda)dt - c_0 q. \quad (2)$$

In addition, for the purpose of tractability, the linear expenditure to maintain and the linear cost function is employed in the above model. The linear transportation cost is employed throughout this work. The results of this paper can be extended to situations without extreme curvature. The following notes are presented, which are satisfied in the above model. Noticeably, (1) indicates $d = A - p - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t\left|x - z_1\right| - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt\left|x - z_2\right|$, which is the first-optimal condition of (1).

Note 1 Because $\frac{\partial^2 u(d,T,x)}{\partial d^2} = -1$, $u(d,T,x)$ is concave and twice differentiable in d, it guarantees the existence of a unique solution for the consumer. Furthermore, $\frac{\partial u(d,T,x)}{\partial d} > 0$ and $d(p,T,x) > 0$ for all x because A is large enough.

Note 2 Because $d(p,T,x) > 0$, $D(p,T) = \int_0^1 d(p,T,x)d\,x$ yields $D(p,T) > 0$ for all p and T.

Note 3 From $d = A - p - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t\left|x - z_1\right| - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt\left|x - z_2\right|$, we obtain that $\pi(p)$ is concave and twice differentiable in p, which guarantees the existence of a unique solution for the above model.

Notes 1 and 3 demonstrate the existence of the unique solution to the above problem. Note 2 guarantees that the consumer consumes a positive quantity in the equilibrium state. If A is large enough, Notes 1 and 2 are met. The model is the extension of maintenance commitment of Nie (2010) to spatial competition. Furthermore, market clearing conditions, $q = D(p,T)$, always satisfy this characteristic because of monopolization.

3. MAIN RESULTS

We first focus on the demand based on (1). The equilibrium, including price and guarantee time limit, is then discussed.

3.1 DEMAND

Here, we capture the solution of (1) , $d(p,T,x)$ and $D(p,T)=\int_0^1 d(p,T,x)dx$ with comparative static analysis. We hope to grasp the features if p and T change. For the above model (1)-(2), the following conclusions about the demand functions are established.

Proposition 1 The demand function of the consumers satisfies the relationship $\frac{\partial D(p,T)}{\partial p}<0$ and $\frac{\partial D(p,T)}{\partial T}>0$.

Proof: See Appendix. ■

Remark: Conclusions in Proposition 1 illustrate that both lower price and longer guarantee time limits significantly increase demand, which is consistent with existing evidence (Nie, 2010). $\frac{\partial D(p,T)}{\partial p}<0$ is also the classic conclusion in economics.

Furthermore, according to (1), if the incurred transportation cost is entirely undertaken by consumers, we immediately have $\frac{\partial D}{\partial c_t}<0$. This indicates that lower marginal transportation cost leads to higher demand. Further, lower marginal transportation cost improves the consumers' utility.

The utility function of consumers is then addressed. When guarantee time limit changes, we hope to acknowledge the consumers' utility. Using the envelope theorem, consumer utility is discussed and the following conclusions are achieved.

Proposition 2 $\frac{\partial u}{\partial T}>0$ and $\frac{\partial u}{\partial c_t}<0$.

Proof: See in Appendix. ■

Remark: The above conclusions describe the relationship between the utility function and variables c_t and T. The conclusions in Propositions 1 and 2 simultaneously hold in the general hypothesis satisfying Notes 1 and 2. Longer guarantee time limit, higher utility of consumers and higher marginal transportation costs reduce consumers' utility.

Furthermore, we assert that the explicit function of demand meets

$$\frac{\partial u}{\partial q}=A-q-p-c\int_T^{\lambda^{-1}}\varphi(t,\lambda)dt-c_t\left|x-z_1\right|-c_t\int_0^{\lambda^{-1}}\varphi(t,\lambda)dt\left|x-z_2\right|=0, \quad (3)$$

$$d(p,T,x)=A-p-c\int_T^{\lambda^{-1}}\varphi(t,\lambda)dt-c_t\left|x-z_1\right|-c_t\int_0^{\lambda^{-1}}\varphi(t,\lambda)dt\left|x-z_2\right|. \quad (4)$$

By direct calculation, we obtain $\int_0^1 |x - z_1| dx = z_1^2 - z_1 + \frac{1}{2}$ and $\int_0^1 |x - z_2| dx = z_2^2 - z_2 + \frac{1}{2}$. The following formulation holds.

$$\begin{aligned} D(p,T) &= \int_0^1 d(p,T,x)d\,x \\ &= A - p - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t\int_0^1 |x - z_1| dx - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt \int_0^1 |x - z_2| dx \\ &= A - p - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t(z_1^2 - z_1 + \frac{1}{2}) - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(z_2^2 - z_2 + \frac{1}{2}) \end{aligned} \qquad (5)$$

We further have the relation $\frac{\partial D(p,T)}{\partial p} = -1$, $\frac{\partial D(p,T)}{\partial T} = c\varphi(T,\lambda)$, $\frac{\partial D(p,t)}{\partial c_t} = -(z_1^2 - z_1 + \frac{1}{2}) - \int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(z_2^2 - z_2 + \frac{1}{2}) < 0$, $\frac{\partial D(p,T)}{\partial z_1} = c_t(1 - 2z_1)$ and $\frac{\partial D(p,T)}{\partial z_2} = c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(1 - 2z_2)$. The following analyses are all based on these formulations.

3.2 EQUILIBRIUM

The equilibrium price and guarantee time limit are addressed. The model is analyzed by backward induction technology. The second stage is considered first. By virtue of market clearing conditions, $q = D(p,T)$, the profit function of the monopolist is

$$\max_p \pi(p) = pD(p,T) - cD(p,T)\int_0^T \varphi(t,\lambda)dt - c_0 D(p,T). \qquad (6)$$

According to (6), the profit function is concave and Note 3 is satisfied, which guarantees the existence and unique solution. The solution is determined by the first-order optimal condition of $\pi(p)$ as follows:

$$\frac{d\pi}{dp} = D(p,T) + p\frac{\partial D(p,T)}{\partial p} - [c\int_0^T \varphi(t,\lambda)dt + c_0]\frac{\partial D(p,T)}{\partial p} = 0. \qquad (7)$$

That is,

$$A - p - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t(z_1^2 - z_1 + \frac{1}{2}) - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(z_2^2 - z_2 + \frac{1}{2}) - p + [c\int_0^T \varphi(t,\lambda)dt + c_0] = 0.$$

We therefore achieve the optimal price of the monopolist.

$$p^* = \frac{A - c\int_T^{\lambda^{-1}} \varphi(t,\lambda)dt - c_t(z_1^2 - z_1 + \frac{1}{2}) - c_t\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(z_2^2 - z_2 + \frac{1}{2}) + [c\int_0^T \varphi(t,\lambda)dt + c_0]}{2}. \qquad (8)$$

The corresponding profits are

$$\pi(p^*) = p^* D(p^*,T) - cD(p^*,T)\int_0^T \varphi(t,\lambda)dt - c_0 D(p^*,T). \qquad (9)$$

For the optimal price and the corresponding profits, by virtue of comparative static analysis and the envelope theorem, the following conclusion holds.

Proposition 3 For the equilibrium price, we have $\frac{\partial p^*}{\partial c_t} < 0$, $\frac{\partial p^*}{\partial T} > 0$, $\frac{\partial p}{\partial z_1} = \frac{c_t}{2}(1-2z_1)$ and $\frac{\partial p}{\partial z_2} = \frac{c_t}{2}\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(1-2z_2)$. For profit function, using the envelope theorem, we achieve the relationship $\frac{\partial \pi}{\partial c_t} < 0$.

Proof: See in Appendix. ■

Remark: in the above demonstration, higher marginal transportation costs lower the price and profit of the firm. Longer guarantee time limit is associated with higher price, which is consistent with PMG. In the relationship $\frac{\partial p}{\partial z_1} = \frac{c_t}{2}(1-2z_1)\begin{cases} >0 & z_1 < 0.5 \\ <0 & z_1 > 0.5 \end{cases}$ and $\frac{\partial p}{\partial z_2} = \frac{c_t}{2}\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(1-2z_2)\begin{cases} >0 & z_1 < 0.5 \\ <0 & z_1 > 0.5 \end{cases}$, the monopolist's prices are highest when $z_1 = z_2 = 0.5$. Therefore, when location is closer to the middle point, the price increases under equilibrium price. When a firm's location and repairing location are both close to the middle point, demand improves and the monopolist benefits from this location.

We further discuss guarantee limit time T. According to the envelope theorem, the following relationship between a firm's profits and the guarantee limit time T is discussed. Because $\left.\frac{\partial \pi}{\partial p}\right|_{p^*} = 0$, we have

$$\begin{aligned}\frac{\partial \pi}{\partial T} &= \frac{\partial \pi}{\partial p}\frac{\partial p}{\partial T} + \frac{\partial \pi}{\partial D}\frac{\partial D(p,T)}{\partial T} - cD(p,T)\varphi(T,\lambda) \\ &= [p^* - c\int_0^T \varphi(t,\lambda)dt - c_0]\frac{\partial D(p,T)}{\partial T} - cD(p,T)\varphi(T,\lambda)\cdot \\ &= [p^* - c\int_0^T \varphi(t,\lambda)dt - c_0 - D(p^*,T)]c\varphi(T,\lambda)\end{aligned} \tag{10}$$

This analysis is summarized as follows:

Proposition 4 For profit function, if $p^* - c\int_0^T \varphi(t,\lambda)dt - c_0 - D(p^*,T) > 0$, we have $\frac{\partial \pi}{\partial T} > 0$ .Otherwise, we have $\frac{\partial \pi}{\partial T} \le 0$.

Remarks: The conclusion in the above proposition illustrates the relationship between the longer guarantee limit time and the firm's profits. If $p^* - c\int_0^T \varphi(t,\lambda)dt - c_0 - D(p^*,T) > 0$ or the demand is small, the firm is apt to longer guarantee a time limit. Otherwise, a longer guarantee time limit reduces the firm's profits.

The first stage is then discussed and the optimal location is focused on. The formulation indicates that $D(p,T)$ should obtain its maximization values at $z_1 = \frac{1}{2}$. $\frac{\partial p}{\partial z_1} = \frac{c_t}{2}(1-2z_1)$ also suggests that p should achieve its maximization at $z_1 = \frac{1}{2}$. We therefore make the conclusion that $\pi(p^*) = D(p^*,T)[p^* - c\int_0^T \varphi(t,\lambda)dt - c_0]$ reaches its maximization at $z_1 = \frac{1}{2}$. Similarly, $\pi(p^*) = D(p^*,T)[p^* - c\int_0^T \varphi(t,\lambda)dt - c_0]$ obtains its maximization at $z_2 = \frac{1}{2}$. The above analysis on the first stage is summarized as follows:

Proposition 5 Under uniform distribution of consumers, the optimal location of both sale and maintenance is the middle point of a linear city.

The two stage model is addressed and the optimal price and location for the monopolist are achieved. The above conclusions are useful for firms that are making location and price decisions.

4. CONCLUDING REMARKS

In this work, the theory of goods requiring high maintenance expenditure under spatial competition is developed for monopoly conditions, and the corresponding results are established. The optimal location and guarantee maintenance patterns are analyzed and compared. To our surprise, the optimal location is at the exact middle point of a linear city. To our knowledge, there exists no literature about spatial competitions on maintenance commitment; this study is the first to addresses this topic.

In this work, shipping costs are shouldered by consumers. If part of this cost is undertaken by the firm, similar techniques are adopted and the corresponding conclusions are achieved. The uniform distribution of consumers is discussed in this work and can be extended to general situations. This paper focuses on rational consumers, while, in many industries, shipping costs may be neglected according to experiments in behavioral economics (Hossain and Morgan, 2006). In summary, the consumers' nonstandard decision regarding shipping costs is a complex topic and will be the focus of our future research.

This paper assumes that complete information about the product is known to both producer and consumers. Incomplete information renders modeling more difficult and will be the subject of our future research.

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APPENDIX

Proof of Proposition 1

We first discuss $d(p,T,x)$, which is the solution to the first optimal conditions of (1). Then, $D(p,T)=\int_0^1 d(p,T,x)dx$ is addressed. $d(p,T,x)$ is the solution of (1) and the first order optimal conditions of (1) are outlined as follows:

$$f=\frac{\partial u(q,T,x)}{\partial q}=A-q-p-c\int_T^{\lambda^{-1}}\varphi(t,\lambda)dt-c_t\left|x-z_1\right|-c_t\int_0^{\lambda^{-1}}\varphi(t,\lambda)dt\left|x-z_2\right|=0\,.$$

From the above equation, we have $\frac{\partial f}{\partial q}=-1,\frac{\partial f}{\partial p}=-1$ and $\frac{\partial f}{\partial T}=c\varphi(T,\lambda)$. According to the implicit function theorem, there exists the unique explicit function $d(p,T,x)$, which is differentiable and the following relationships hold.

$$\frac{\partial d(p,T,x)}{\partial p}=-\frac{\frac{\partial f}{\partial q}}{\frac{\partial f}{\partial p}}=-1<0,$$

$$\frac{\partial d(p,T,x)}{\partial T}=-\frac{\frac{\partial f}{\partial q}}{\frac{\partial f}{\partial T}}>0\,.$$

Because $D(p,T)=\int_0^1 d(p,T,x)dx$ and $d(p,T,x)$ is continuously differentiable, we achieve the relationship $\frac{\partial D(p,T)}{\partial p}<0$ and $\frac{\partial D(p,T)}{\partial T}>0$.

Our conclusions are achieved and the proof is complete. We further point out that we can directly employ the comparative static analysis approach to obtain the above results. In general cases, the implicit function theorem seems to be more powerful that the comparative static analysis approach. ■

Proof of Proposition 2

We demonstrate the conclusions using the envelope theorem.

$$\frac{\partial u}{\partial T} = \frac{\partial u}{\partial d}\frac{\partial d}{\partial T} + \frac{\partial u}{\partial T} = cq\varphi(T,\lambda) > 0 \,,$$

$$\frac{\partial u}{\partial c_t} = \frac{\partial u}{\partial d}\frac{\partial d}{\partial c_t} + \frac{\partial u}{\partial c_t} = -q\left|x - z_1\right| - q\int_0^{\lambda^{-1}} \varphi(t,\lambda)d\,\left|xt - z_2\right| < 0 \,.$$

Our results are obtained and the proof is complete. ■

Proof of Proposition 3

We demonstrate this with comparative static analysis method. By virtue of (8), the comparative static analysis approach indicates the formulations $\frac{\partial p^*}{\partial T} = c\varphi(T,\lambda) > 0$,

$$\frac{\partial p^*}{\partial c_t} = \frac{-(z_1^2 - z_1 + \frac{1}{2}) - \int_0^{\lambda^{-1}} \varphi(t,\lambda)d\,(z_2^2 t - z_2 + \frac{1}{2})}{2} < 0 \,, \quad \frac{\partial p}{\partial z_1} = \frac{c_t}{2}(1 - 2z_1) \text{ and}$$

$$\frac{\partial p}{\partial z_1} = \frac{c_t}{2}\int_0^{\lambda^{-1}} \varphi(t,\lambda)dt(1 - 2z_2) \,.$$

According to (9), because $\left.\frac{\partial \pi}{\partial p}\right|_{p^*} = 0$, the envelope theorem suggests the following relationship:

$$\frac{\partial \pi}{\partial c_t} = \frac{\partial \pi}{\partial p}\frac{\partial p}{\partial c_t} + \frac{\partial \pi}{\partial D}\frac{\partial D}{\partial c_t} = \frac{\partial \pi}{\partial D}\frac{\partial D}{\partial c_t} = [\,p^* - c\int_0^T \varphi(t,\lambda)dt - c_0\,]\frac{\partial D}{\partial c_t} > 0 \,.$$

Our results are obtained and the proof is complete. ■

MONOPOL NAD ROBOM S PROSTORNIM ODRŽAVANJEM

SAŽETAK

Ovaj rad se bavi efektima koje lokacija i vremensko ograničenje garancije u monopolističkim tvrtkama imaju na formiranje cijena robe koja zahtijeva visoke troškove održavanja, kao što su liftovi, televizori i kompjuteri. U ovom radu iznosimo model prostornog održavanja u dva stupnja unutar garantiranog vremenskog ograničenja. Na osnovu ovog modela, lokacija, obveza održavanja i formiranje cijena se sve određuju u situaciji monopola. Rad donosi optimalne cijene i lokacije za monopolistu. Raspravlja se i o učincima vremenskog ograničenja garancije na cijenu.

Ključne riječi: *Struktura tržišta, obveza prostornog održavanja, vremensko ograničenje garancije, teorija igara, monopol*

JEL klasifikacija: *C61, C72， D4, L1*