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MATHEMATICAL MODELING OF MACHINING BY DECOMPOSITION OF LATHE ON MODULES

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The machining is the most common way of final processing of the metallurgical semi-finished products. The article deals with mathematical modeling of virtual machining. A specific lathe with specific parameters was selected as the machining tool. Lathe, respectively its computational model was decomposed into modules that represent a natural group of module design where each module is sliding towards other, rotating or standing with a given dimension characteristics. Computational model of the machine tool should be as simple as possible, but must be designed so that features all the factors affecting the accuracy of the working face. Generally, mathematical model of inaccuracies of machining is possible to use for analytical detection of inaccuracies in machining parts for a particular model machine tools.

Key words: computational model, modeling of accuracy, virtual lathe, virtual machining

Matematko modeliranje strojne obrade pomoću raščlanjivanja na module. Strojna obrada najuobičajeniji je način konačne obrade metalurških poluproizvoda. Članak se bavi matematičkim modeliranjem virtualne strojne obrade. Posebna tokarilica sa specifičnim parametrima odabrana je kao alat za strojnu obradu. Tokarilica, odnosno njen računski model, bila je raščlanjena na module koji predstavljaju neku prirodnu skupinu modularne izvedbe gdje svaki modul klizi prema drugome, rotirajući ili stojeći uz dane dimenzijske karakteristike. Računski model tog strojnog alata trebao bi biti što jednostavniji, no mora biti izrađen tako da prikazuje sve faktore što utječu na preciznost radne površine. Općenito, matematički model nepreciznosti u strojnoj obradi moguće je koristiti za analitičku detekciju nepreciznosti u dijelovima strojne obrade za određeni model strojnih alata.

Ključne riječi: računski model, model preciznosti, virtualna tokarilica, virtualna strojna obrada

INTRODUCTION

Optimization of production machines is still hot topic in the economy of production, technology, logistics, and ultimately quality [1, 2]. The production quality is closely linked with precision of manufactured parts. Working accuracy of the machine qualifies the class of the production machine. Pre-design stage of the production machine is certified by experimental methods. [3-5].

During the examination of working accuracy of machine we need to determine the machine tool trajectory equation on mathematical model of the machine in workpiece coordinate system to determine the influence of inaccuracies linked to the individual modules of the machine for final machining inaccuracy [6-11]. The results must be transformed to the point of contact of the workpiece and tool and there they must be overlaid.

PROCEDURE FOR DATA COMPILATION OF MACHINE MODEL

- Define a sequence of stationary and moving modules of the machine - model bodies in the direction of the workpiece to the tool.
- Define coordinate systems of individual model bodies.
- Mathematically define mutual starting position of individual bodies of computational model by matrices of location $K_{i+1,i}$.
- Mathematically define the movements of individual modeled bodies.
- Mathematically define the transformation matrix of linear motion $T_{i+1,i}$ (t) and transformation matrix of rotational movements $R_{i+1,i}$ (t) of individual bodies of computational model.
- Define and mathematically express the deformation and inaccuracies linked to the individual elements of computational model of the machine and their time changes during the machining of examined area.

SIMPLIFIED COMPUTING MODEL OF MACHINE

A simplified computing model of lathe “EMCO PC TURN 50” consists of six model pieces (Figure 1). The machine has three controlled axes (X, Z, C). Individual model elements are gradually assembled away from the workpiece to the tool. Where:

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T_0 – workpiece (not depicted)
T_1 – spindle
T_2 – headstock
T_3 – lathe-bed
T_4 – longitudinal slide
T_5 – cross slide
T_6 – turret

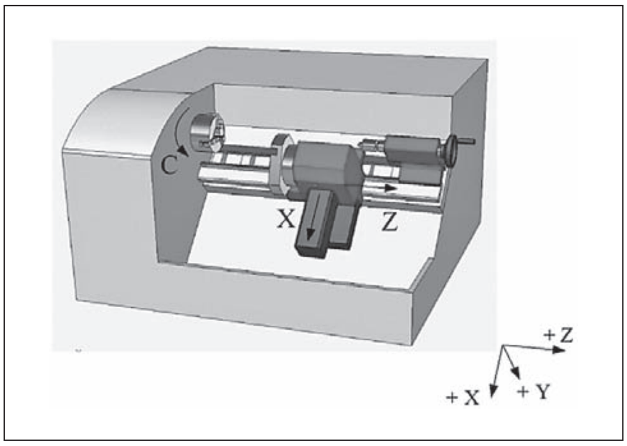

Figure 1 A simplified computing model of lathe – modules

Define the transformation matrix of rotating movements $R_{i+1,i}(t)$, transformation matrix of linear motion $T_{i+1,i}(t)$ and matrix of initial positions $K_{i+1,i}(t)$.

$$R_{65}(t) = E$$

$$T_{65}(t) = 0$$

$$K_{65}(t) = [0, \text{-}\,\mathrm{b}, 0]^T$$

$$R_{54}(t) = E$$

$$T_{54}(t) = [\pm s_5(t), 0, 0]^T$$

$$K_{54}(t) = [\mathrm{x}_{05}, 0, 0]^T$$

$$R_{43}(t) = E$$

$$T_{43}(t) = [0, 0, \pm s_4(t)]^T$$

$$K_{43}(t) = [\mathrm{d}, \mathrm{e}, \mathrm{z}_{04} + f]^T$$

$$R_{32}(t) = E$$

$$T_{32}(t) = 0$$

$$K_{32}(t) = [\text{-}\,\mathrm{k}, \mathrm{m}, 0]^T$$

$$R_{21}(t) = \begin{bmatrix} \cos\gamma_2(t) & -\sin\gamma_2(t) & 0 \\ \sin\gamma_2(t) & \cos\gamma_2(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{21}(t) = 0$$

$$K_{21}(t) = [0, 0, \text{-n}]^T$$

$$R_{10}(t) = E$$

$$T_{10}(t) = 0$$

$$K_{10}(t) = 0$$

While dimensions a, b, c, d, e, f, k, m, n resulting from the construction of machine, $s_4(t)$ is the immediate path of the model body T_4 in the direction of axe Z at the time t and $s_5(t)$ is the immediate path of the model body T_5 in the direction of axe X in the time t. Immediate values of angle $\gamma_2(t)$ are filled in the equation with a sign by the following: if the spindle rotates in the negative sense (necessary for turning), then the headstock makes relative rotational movement towards spindle in positive sense and therefore the angle $\gamma_2(t)>0$, has a positive sign (+), otherwise the minus sign (-).

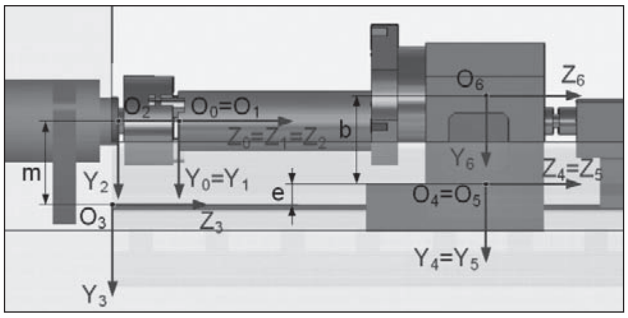

Figure 2 Model of lathe - delineation

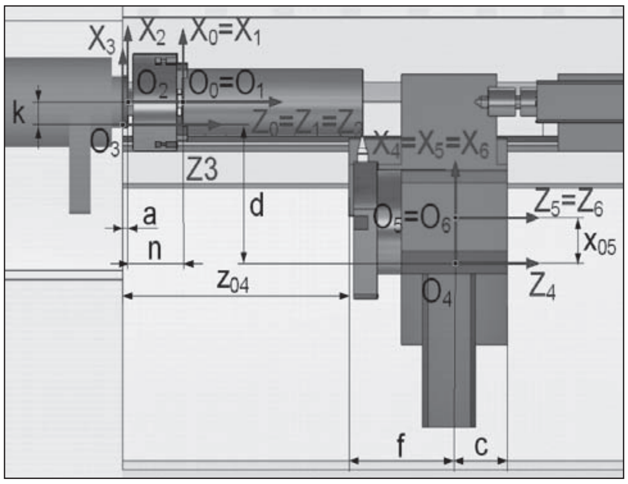

Figure 3 Model of lathe - footprint

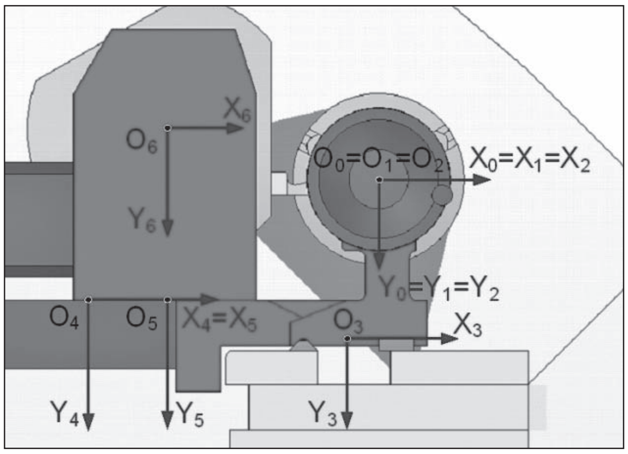

Figure 4 Model of lathe - sheer

All relationships have their general application and their specification will depend on the proposed conditions of machining workpiece model and its dimensions.

Arrangement of coordinate systems and the basic geometric characteristics are shown in Figures 2, 3, 4.

Point position vector of functional tool in the coordinate system of the model body T_6 is the model example

$\mathbf{r}_n = \mathbf{r}_6 = [63, 25, -88]^T$.

Substituting the numerical values of the dimensions of the machine construction and the dimensions and position of the model workpiece at the beginning of machining we get the expression matrix in numerical form (values in mm)

$K_{10} = [0, 0, 0]^T$,
$K_{21} = [0, 0, -50]^T$,
$K_{32} = [-5, 70, 0]^T$,
$K_{43} = [120, 17, 270]^T$,
$K_{54} = [12{,}5; 0; 0]^T$,
$K_{65} = [0, -75, 0]^T$.

As mutual rotational movements of modeling bodies were not considered all the transformation matrix of rotating movements are unitarily, thus

$R_{65}(t) = R_{54}(t) = R_{43}(t) = R_{32}(t) =$
$= R_{21}(t) = R_{10}(t) = E$.

Transformation matrix of linear motion of modeling bodies in a general form are defined as following

$$T_{65}(t) = T_{32}(t) = T_{21}(t) = T_{10}(t) = 0$$

$$T_{54}(t) = [\pm s_5(t), 0, 0]^T$$

$$T_{43}(t) = [0, 0, \pm s_4(t)]^T$$

Transformation matrix $T_{43}(t)$ can be specified by expression using the known displacement minute s_{zmin} = 300 mm.min^{-1}. Pathway of the model body T_4, traveled in time t [s] will be

$s_4(t) = s_{zmin} \,.\, t / 60 = 300t / 60$,

thus, the matrix $T_{43}(t)$ can be expressed as a function of time

$T_{43}(t) = [0, 0, 300\,t / 60]^T$.

In numerical modeling of trajectory of functional point of tool in the coordinate system of workpiece we will use the general expression of a position vector of the point in the coordinate systems of individual modeling bodies. Appling here

$$r_i(t) = \left(\prod_{j=i}^{n-1} R_{j+1,j}(t)\right) . r_n + T_{i+1,i}(t) + K_{i+1,i} +$$
$$+ \sum_{j=i}^{n-2}\left[\left(\prod_{k=i}^{j} R_{k+1,k}(t)\right) . \left(T_{j+2,j+1}(t) + K_{j+2,j+1}\right)\right]$$

and the coordinate system of workpiece is valid relationship

$$r_o(t) = \left(\prod_{i=1}^{n} R_{i,i-1}(t)\right) . r_n + T_{10}(t) + K_{10} +$$
$$+ \sum_{i=1}^{n-1}\left[\left(\prod_{j=1}^{i} R_{j,j-1}(t)\right) . \left(T_{i+1,i}(t) + K_{i+1,i}\right)\right]$$

As the number of modeling bodies equals n = 6, we substitute the relation for deriving $r_i(t)$ gradually i = 6, 5, 4, 3, 2, 1. Then for the correspondent position vector is valid

$\mathbf{r}_5(t) = \mathbf{r}_6 + K_{65}$,
$\mathbf{r}_4(t) = \mathbf{r}_6 + K_{65} + K_{54}$,
$\mathbf{r}_3(t) = \mathbf{r}_6 + K_{65} + K_{54} + K_{43} + T_{43}(t)$,
$\mathbf{r}_2(t) = \mathbf{r}_6 + K_{65} + K_{54} + K_{43} + K_{32} + T_{43}(t)$,
$\mathbf{r}_1(t) = \mathbf{r}_6 + K_{65} + K_{54} + K_{43} + K_{32} + K_{21} + T_{43}(t)$,
$\mathbf{r}_0(t) = \mathbf{r}_6 + K_{65} + K_{54} + K_{43} + K_{32} + K_{21} + T_{43}(t)$.

Substituting numerical values into the previous relations we get the individual position vectors in numerical form (values are expressed in mm)

$\mathbf{r}_6 = [63, 25, -88]^T$
$\mathbf{r}_5(t) = [60, 25, -88]^T$,
$\mathbf{r}_4(t) = [20, 25, -88]^T$,
$\mathbf{r}_3(t) = [140, 42, -88 - 300t / 60]^T$,
$\mathbf{r}_2(t) = [150, -28, -83 - 300t / 60]^T$,
$\mathbf{r}_1(t) = [150, -28, -33 - 300t / 60]^T$,
$\mathbf{r}_0(t) = [150, -28, -33 - 300t / 60]^T$.

Position vectors $\mathbf{r}_4$, $\mathbf{r}_5$, $\mathbf{r}_6$ are not dependent on time and they are constant. Time-dependent position vectors are only $\mathbf{r}_0$, $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, the only changeable component in the direction of Z axis.

Vector $\mathbf{r}_0(t)$ is the mathematical model of an ideal a functional points of tool path to the model workpiece machining. Components of vector $\mathbf{r}_0(t)$ in the directions of axes X_0 and Y_0 are not dependent of time, functional point of tool. Thus moves along a line parallel to the axis Z_0, which is the intersection of planes $x_0 = 150$ and $y_0 = -28$. The relevant coordinate z_0 is a function of time

$z_0(t) = -33 - 300t / 60$.

The line, along which the functional point of tool moves, is imagined physically as a set of points in space whose coordinates are: $z_0(t)$, $x_0 = 150$ a $y_0 = -28$. Tool -turning knife will make the helix, which may be in the optimal case taken as cylindrical plane. If we substitute the relation for the time t = 0, we get

$\mathbf{r}_0(0) = [150, -28, -33]^T$,

which is actually the tool position in coordinate system of workpiece at the beginning of machining. The same result would be reached if this starting mutual position would have an initial relative position of tool and workpiece expressed directly from the geometric interpretation.

Relevant mathematical expression can be written as relationship

$$r_o(0) = r_n + \sum_{i=1}^{n} K_{i,i-1}$$

The above mentioned considerations are also check ups of the correctness of the proposed computational model of the machine in terms of machining kinematics.

CONCLUSION

After obtaining the ideal tool path function it can be accessed towards the modal analysis of each module manufacturing machine. For modal analysis it is necessary to choose an appropriate tool, such as COSMOS program. After obtaining the particular values of deflection, deviation, respective inaccuracies of the model in virtual environment where the machine is tested under the virtual load it is appropriate to compare the machine with real experiment using appropriate measurement techniques. Previous comparisons of modal analysis and experimental measurements carried out in past, show real deviations in values of up to 10%.

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Note: The responsible for English language is Peter Kovkol, State Language School, Košice, Slovakia.