ISSN 0005-1144
ATKAFF 51(4), 353–360(2010)

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Modified Nodal Analysis-Based Determination of Transfer Functions for Multi-Inputs Multi-Outputs Linear Circuits

UDK 681.511.2
IFAC 2.3.1

Original scientific paper

A generalized method for determination of transfer functions of circuits with multi-inputs multi-outputs is introduced. The paper proposes a systematic and efficient formulation for generating the transfer matrix containing transfer functions, necessary to model these kinds of circuits. The modified nodal analysis, whose application is simpler than the state-space analysis, is used in obtaining the system equations. The method is a matrix-based approach. It is suitable for both symbolic manipulation and numeric processes. Furthermore, the frequency domain analysis is realized using the system model. Application examples are included to illustrate the method.

Key words: Transfer function, Transfer matrix, Modified nodal approach, Frequency analysis

Modificirana metoda čvorova u svrhu određivanja prijenosne funkcije linearnih sustava s više izlaza i više ulaza. Predstavljena je općenita metoda za određivanje prijenose funkcije sustava s više ulaza i više izlaza. U ovome članku predložena je sistematična i efikasna formulacija za računanje prijenosne matrice koja sadrži prijenosne funkcije, nužne za modeliranje sustava ovakvog tipa. Modificirana metoda čvorova, čija je primjena jednostavnija nego analiza u prostoru stanja, koristi se za određivanje jednadžbi sustava. Predložena metoda temelji se na matričnom računu. Prikladna je za simboličko i numeričko računanje. Nadalje, koristeći model sustava napravljena je i frekvencijska analiza. Također, priloženi su i primjeri radi ilustracije primjene metode.

Ključne riječi: prijenosna funkcija, prijenosna matrica, izmijenjena metoda čvorova, frekvencijska analiza

1 INTRODUCTION

The transfer functions are defined as the ratio of the output response to the input (source) in s-domain. The ability to use the transfer functions to calculate the steady-state responses of a circuit is important. If transfer functions are known, we can find the responses of circuits to all kinds of excitation sources. The transfer functions are also very useful tools in problems concerning the frequency responses of circuits. Many circuit characteristics such as voltage/current gains, poles/zeros of the circuits can be computed from transfer functions.

Several approaches to obtain the transfer functions are given in symbolic or numerical format. A graph-based approach is presented for the generation of exact symbolic network functions in the form of rational polynomials of the complex variable (s) in [1]. A symbolic method for generating a compact sequence of expressions for network functions of large-scale circuits is described in [2]. A unified approach to the approximate symbolic analysis of analog integrated circuits is given in [3]. The network functions and their sensitivities with respect to the elements are computed with a matrix-based method in [4]. Applications about the realization of several transfer functions are given in [5-6].

In this paper, the algebraic method for obtaining the transfer functions of linear or linearized time-invariant circuits with multi-inputs multi-outputs is proposed. For setting up the circuit equations, the modified nodal approach (MNA), the one of the most popular methods of circuit analysis, is used. The state variables method, the other popular method and based on the graph theoretical approach, was developed before the modified nodal analysis. It involves intensive mathematical process and has major limitations in the formulation of circuit equations. Some of these limitations arise because the state variables are capacitor voltages and inductor currents. Every circuit element cannot be easily included into the state equations. Because of the drawbacks of state variables analysis, the modified nodal analysis was first introduced by Ho et al. [7] and has been developed more by including many circuit elements (transformer, semiconductor devices, short circuit, etc.) into the system equations so far [8-11]. In this method, the system equations can be also obtained by inspection. It allows circuit equations to be easily and systematically obtained without any limitation. This method is used for circuit synthesis of passive descriptor systems in [12] and for computing the smallest, the largest and a given subset of the largest eigenvalues associated with linear time-invariant circuits in [13].

In this paper, it is shown how to use the advantages of modified nodal approach in obtaining the transfer functions and frequency-domain analysis of linear circuits with multi-inputs multi-outputs. The main contribution of the paper is that it gives a systematic formulation method in terms of variables of MNA. The transfer functions can be obtained as both symbolic and numeric with the proposed method.

In the circuits with multi-inputs multi-outputs, the use of the transfer matrix, containing all transfer functions, is required. The transfer functions, components of transfer matrix, can be found for only one input and one output. According to Superposition principle, the transfer matrix is expressed by taking into account all transfer functions together.

The paper is organized as follows: In Section 2, the structure of modified nodal approach, system equations in s-domain, the expressions relating to the transfer functions and frequency domain analysis are given. In Section 3, two application examples of the approach are given. Section 4 is the conclusion.

2 DESCRIPTION OF THE METHOD

The modified nodal equations and the output equations of a circuit with multi-inputs multi-outputs (Fig. 1) are given in s-domain, (1) and (2). The nodal and output equations together are called the system model. The circuit in Fig. 1 has p inputs and q outputs:

$$\begin{aligned} GX(s) + sCX(s) &= BU_i(s) \\ [G + sC]\,X(s) &= BU_i(s) \end{aligned} \tag{1}$$

$$Y(s) = TX(s), \tag{2}$$

where G, C, B, T are coefficient matrices. All conductances and frequency-independent values arising in the MNA formulation are stored in matrix G, capacitor and inductor values which are frequency-dependent in matrix C. $U_i(s)$ represents the inputs (voltage or current sources), $Y(s)$ represents the output variables (voltage/current). $X(s)$ is the unknown vector.

The transfer functions are defined as the ratio of the output responses to the inputs. The transfer matrix, H(s), containing all transfer functions can be expressed in terms of the matrices of MNA system, as follows. From (1):

$$X(s) = [G + sC]^{-1}\,BU_i(s), \tag{3}$$

the output equation is:

$$Y(s) = TX(s) = T\,[G + sC]^{-1}\,BU_i(s), \tag{4}$$

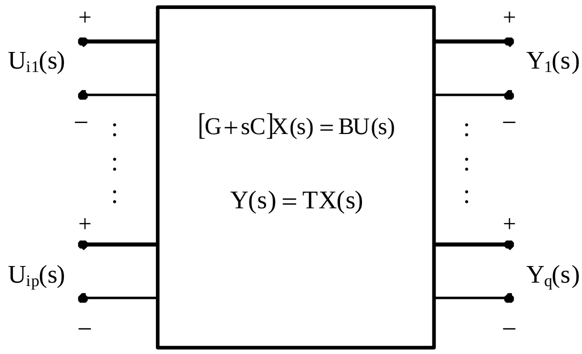

Fig. 1. Circuit with p inputs and q outputs

and transfer matrix:

$$H(s) = \frac{Y(s)}{U_i(s)} = T\,[G + sC]^{-1}\,B. \tag{5}$$

The unknown vector X(s) contains both voltage and current variables. MNA can handle all types of active and passive elements. It is a very important property of MNA.

Taking into account the types of variables, the unknown vector is partitioned as follows:

$$X(s) = \begin{bmatrix} X_1(s) \\ \cdots\cdots \\ X_2(s) \end{bmatrix}. \tag{6}$$

Here, $X_1(s)$ represents nodal voltage variables, $X_2(s)$ represents current variables relating to independent and controlled voltage sources, inductors, short circuit elements, etc, (7). If there are n nodes and m current variables in a circuit, $X_1(s)$ vector contains $n-1$ nodal voltage variables except reference node (ground) and X2(s) vector contains m current variables. Thus, the unknown vector X(s) contains $k = n - 1 + m$ variables, as in (8).

$$X_1(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_{n-1}(s) \end{bmatrix}, X_2(s) = \begin{bmatrix} I_1(s) \\ I_2(s) \\ \vdots \\ I_m(s) \end{bmatrix}, \tag{7}$$

$$X(s) = \begin{bmatrix} X_1(s) \\ \cdots\cdots \\ X_2(s) \end{bmatrix} = \begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_{n-1}(s) \\ \cdots\cdots\cdots \\ I_1(s) \\ I_2(s) \\ \vdots \\ I_m(s) \end{bmatrix}. \tag{8}$$

From (3), $X(s)$ vector is expressed as follows:

$$X(s) = \underbrace{[G + sC]^{-1} B}_{W(s)} U_i(s) = W(s)U_i(s). \quad (9)$$

The $W(s)$ matrix is of order $(n-1+m) \times p$, where $k = n-1+m$. It is created from coefficient matrices of system equations.

Let us consider (8) and (9) together:

$$X(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_{n-1}(s) \\ \cdots\cdots\cdots \\ I_1(s) \\ I_2(s) \\ \vdots \\ I_m(s) \end{bmatrix} = W(s)U_i(s)$$

$$= \begin{bmatrix} W_{11}(s) & ..... & W_{1p}(s) \\ W_{21}(s) & ..... & W_{2p}(s) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ W_{k1}(s) & ..... & W_{kp}(s) \end{bmatrix} \begin{bmatrix} U_{i1}(s) \\ U_{i2}(s) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ U_{ip}(s) \end{bmatrix}. \quad (10)$$

According to Superposition principle, since any output in Fig. 1 is a linear combination, the contribution of each input source is independent of all other inputs. This means that any output or any circuit variable can be found by finding the contribution from each source acting alone and then adding the individual responses to obtain the total response. Then, every component, W_{ij}, of matrix $W(s)$ represents the contributions of inputs.

Equation (10) is also expressed separately as follows:

$$\begin{aligned} U_1(s) &= W_{11}(s)U_{i1}(s) + W_{12}(s)U_{i2}(s) + \ldots \\ &\quad + W_{1p}(s)U_{ip}(s) \\ U_2(s) &= W_{21}(s)U_{i1}(s) + W_{22}(s)U_{i2}(s) + \ldots \\ &\quad + W_{2p}(s)U_{ip}(s) \\ &\vdots \\ U_{n-1}(s) &= W_{(n-1)1}(s)U_{i1}(s) + W_{(n-1)2}(s)U_{i2}(s) + \ldots \\ &\quad + W_{(n-1)p}(s)U_{ip}(s) \\ I_1(s) &= W_{n1}(s)U_{i1}(s) + W_{n2}(s)U_{i2}(s) + \ldots \\ &\quad + W_{np}(s)U_{ip}(s) \\ &\vdots \\ I_m(s) &= W_{k1}(s)U_{i1}(s) + W_{k2}(s)U_{i2}(s) + \ldots \\ &\quad + W_{kp}(s)U_{ip}(s). \end{aligned} \quad (11)$$

The elements of $X(s)$ vector in (10) or (11) are circuit variables. They are expressed in terms of the elements of $W(s)$ matrix and the inputs.

The elements of transfer matrix, $H(s)$, in (5) are determined in terms of $W(s)$ matrix as follows:

$$H(s) = \frac{Y(s)}{U_i(s)} = T\underbrace{[G + sC]^{-1} B}_{W(s)} = TW(s). \quad (12)$$

The output vector is

$$Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \\ \vdots \\ \vdots \\ Y_q(s) \end{bmatrix} = H(s)U_i(s)$$

$$= \begin{bmatrix} H_{11}(s) & ...... & H_{1p}(s) \\ H_{21}(s) & ...... & H_{2p}(s) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ H_{q1}(s) & ...... & H_{qp}(s) \end{bmatrix} \begin{bmatrix} U_{i1}(s) \\ U_{i2}(s) \\ \vdots \\ \vdots \\ U_{ip}(s) \end{bmatrix}. \quad (13)$$

The transfer matrix, $H(s)$, consists of the sum of transfer functions, $H_{ij}(s)$. This means that $H(s)$ can be found by finding the transfer function relating to every source and every output alone and then adding the individual responses. The individual transfer function is obtained by:

$$H_{ij}(s) = \left.\frac{Y_i(s)}{U_{ij}(s)}\right|_{\substack{U_{ij} \neq 0 \\ U_{i1} = U_{i2} = \cdots = U_{ip} = 0}} . \quad (14)$$

The transfer functions ($H_{ij}(s)$) relate inputs and outputs at different ports of a circuit. Fig. 2 shows the possible input-output configurations for a circuit with multi-inputs multi-outputs. Inputs are voltage sources (E_i) and/or current sources (J_i). Outputs are open circuit voltages (U_{oi}) and/or short circuit currents (I_{oi}) at desired ports.

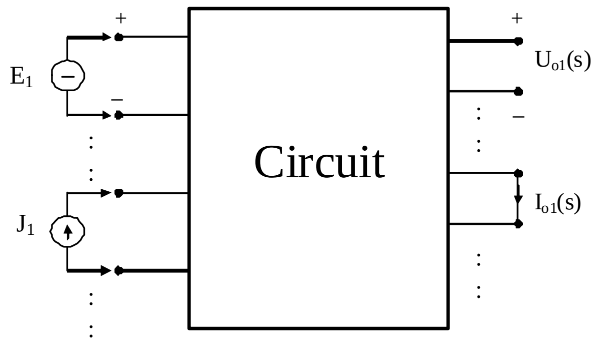

Fig. 2. Input-output configurations of a circuit with multi-inputs multi-outputs

There are four kinds of transfer functions according to input sources and output variables:

$$\text{Voltage transfer function: } H_V(s) = \frac{U_{oi}(s)}{E_i(s)} \tag{15a}$$

$$\text{Current transfer function: } H_I(s) = \frac{I_{oi}(s)}{J_i(s)} \tag{15b}$$

$$\text{Transfer impedance function: } H_Z(s) = \frac{U_{oi}(s)}{J_i(s)} \tag{15c}$$

$$\text{Transfer admittance function: } H_Y(s) = \frac{I_{oi}(s)}{E_i(s)}. \tag{15d}$$

The transfer functions are not inversions of each other. For generating the transfer functions, the voltage and/or current variables relating to output ports are obtained in terms of the elements of $W(s)$ matrix created and the sources, according to Fig. 2 and (11).

2.1 Frequency-Domain Response

For frequency response of system, we replace s by $j\omega$ in (3), (4), (15), respectively:

$$X(j\omega) = [G + j\omega C]^{-1} BU_i(j\omega) = W(j\omega)U_i(j\omega) \tag{16a}$$

$$Y(j\omega) = T\,[G + j\omega C]^{-1} BU_i(j\omega) = TW(j\omega)U_i(j\omega) \tag{16b}$$

$$H_V(j\omega) = \frac{U_{oi}(j\omega)}{E_i(j\omega)} \tag{16c}$$

$$H_I(j\omega) = \frac{I_{oi}(j\omega)}{J_i(j\omega)} \tag{16d}$$

$$H_Z(j\omega) = \frac{U_{oi}(j\omega)}{J_i(j\omega)} \tag{16e}$$

$$H_Y(j\omega) = \frac{I_{oi}(j\omega)}{E_i(j\omega)}. \tag{16f}$$

In this paper, transfer functions, elements of transfer matrix, and frequency domain responses relating to a circuit with multi-inputs multi-outputs are expressed systematically in terms of the elements of $W(s)$ matrix.

3 APPLICATION EXAMPLES

In this section, we give two examples in order to obtain transfer matrix containing transfer functions by the proposed method.

Example 1: Consider a linear RLC circuit having two inputs and two outputs in Fig. 3. The system equations, the transfer matrix containing four transfer functions ($U_o/E, U_o/J, I_o/E, I_o/J$) and the frequency response relating to the voltage transfer function will be obtained. Element values are $R_1 = R_2 = 5\ \Omega$, $C = 1$ F, $L = 2$ H.

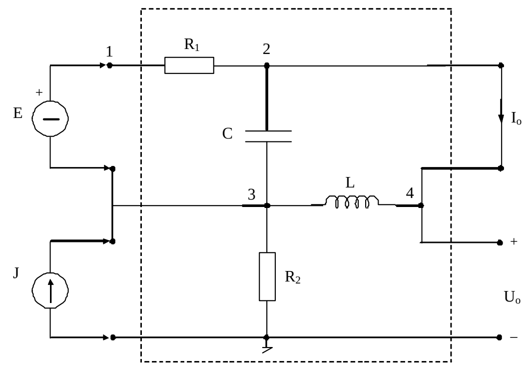

Fig. 3. Circuit for Example 1

The inputs of circuit are a voltage source, E, and a current source, J. The outputs of circuit are a open circuit voltage, voltage of node 4, and a short circuit current, I_o. The circuit has $n-1 = 4$ nonreference nodes. In the MNA system, $X_1(s)$ vector contains 4 nodal voltage variables. The current variables in $X_2(s)$ vector are I_L, I_o, I_E. Thus, in the circuit, $k = n - 1 + m = 7$.

Nodal (main) equations in s-domain:

$$1 \to G_1(U_1 - U_2) + I_E = 0$$
$$2 \to -G_1(U_1 - U_2) + sC(U_2 - U_3) + I_o = 0$$
$$3 \to G_2(U_3) - sC(U_2 - U_3) + I_L - I_E - J = 0$$
$$4 \to -I_L - I_o = 0.$$

Additional equations:

$$U_3 - U_4 = sLI_L, \cdots U_2 - U_4 = 0, \cdots U_1 - U_3 = E.$$

The overall equations constitute the MNA system (17). The output equations of system are given in (18). The system model containing both MNA equations and output equations can be given in matrix form, as in Fig. 1.

$$[G+sC]\,X(s)=BU_i(s)$$

$$\begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 0 & 1 \\ -G_1 & G_1+sC & -sC & 0 & 0 & 1 & 0 \\ 0 & -sC & G_2+sC & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & -sL & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ I_L \\ I_o \\ I_E \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} E \\ J \end{bmatrix} \tag{17}$$

The output equations:

$$Y(s)=TX(s)=\begin{bmatrix} U_o \\ I_o \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} U_1 & U_2 & U_3 & U_4 & I_L & I_o & I_E \end{bmatrix}^T. \tag{18}$$

The system model, (17) and (18), can be systematically obtained by inspection because of the advantages of MNA. By using this system model, the matrix $W(s)$ is created. Thus, the desired transfer functions in terms of the components of $W(s)$ are calculated systematically.

$$X(s)=\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ I_L \\ I_o \\ I_E \end{bmatrix}=\underbrace{[G+sC]^{-1}B}_{W(s)}U_i(s)=W(s)U_i(s)=\begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \\ W_{31}(s) & W_{32}(s) \\ W_{41}(s) & W_{42}(s) \\ W_{51}(s) & W_{52}(s) \\ W_{61}(s) & W_{62}(s) \\ W_{71}(s) & W_{72}(s) \end{bmatrix}\begin{bmatrix} E \\ J \end{bmatrix}, \tag{19}$$

where,

$$\begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \\ W_{31}(s) & W_{32}(s) \\ W_{41}(s) & W_{42}(s) \\ W_{51}(s) & W_{52}(s) \\ W_{61}(s) & W_{62}(s) \\ W_{71}(s) & W_{72}(s) \end{bmatrix}=\begin{bmatrix} 1 & R_2 \\ sL/\left[s^2LCR_1+sL+R_1\right] & R_2 \\ 0 & R_2 \\ sL/\left[s^2LCR_1+sL+R_1\right] & R_2 \\ -1/\left[s^2LCR_1+sL+R_1\right] & 0 \\ 1/\left[s^2LCR_1+sL+R_1\right] & 0 \\ -(s^2LC+1)/\left[s^2LCR_1+sL+R_1\right] & 0 \end{bmatrix}. \tag{20}$$

The elements of transfer matrix, $H(s)$, in (12) are determined in terms of $W(s)$ matrix as follows:

$$H(s)=\frac{Y(s)}{U_i(s)}=TW(s)=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \\ W_{31}(s) & W_{32}(s) \\ W_{41}(s) & W_{42}(s) \\ W_{51}(s) & W_{52}(s) \\ W_{61}(s) & W_{62}(s) \\ W_{71}(s) & W_{72}(s) \end{bmatrix}=\begin{bmatrix} W_{41}(s) & W_{42}(s) \\ W_{61}(s) & W_{62}(s) \end{bmatrix}. \tag{21}$$

The desired transfer functions are obtained as follows:

$$H_V(s) = \left.\frac{U_o}{E}\right|_{J=0} = W_{41}(s) = \frac{sL}{s^2LCR_1 + sL + R_1} \quad \rightarrow \quad H_V(s) = \frac{2s}{10s^2 + 2s + 5} \tag{22a}$$

$$H_Z(s) = \left.\frac{U_o}{J}\right|_{E=0} = W_{42}(s) = R_2 \quad \rightarrow \quad H_Z(s) = 5 \tag{22b}$$

$$H_Y(s) = \left.\frac{I_o}{E}\right|_{J=0} = W_{61}(s) = \frac{1}{s^2LCR_1 + sL + R_1} \quad \rightarrow \quad H_Y(s) = \frac{1}{10s^2 + 2s + 5} \tag{22c}$$

$$H_I(s) = \left.\frac{I_o}{J}\right|_{E=0} = W_{62}(s) = 0 \quad \rightarrow \quad H_I(s) = 0. \tag{22d}$$

The desired transfer matrix and output vector are given as follows:

$$Y(s) = H(s)U_i(s)$$

$$\begin{bmatrix} U_o \\ I_o \end{bmatrix} = \begin{bmatrix} \dfrac{2s}{10s^2+2s+5} & 5 \\ \dfrac{1}{10s^2+2s+5} & 0 \end{bmatrix} \begin{bmatrix} E \\ J \end{bmatrix}. \tag{23}$$

Substituting $s = j\omega$ in (22.a), the frequency response relating to the voltage transfer function is obtained:

$$H_V(jw) = \frac{2j\omega}{-10\omega^2 + 2j\omega + 5}. \tag{24}$$

For the voltage transfer function in (24), Bode plots of the frequency response are given in Fig. 4.

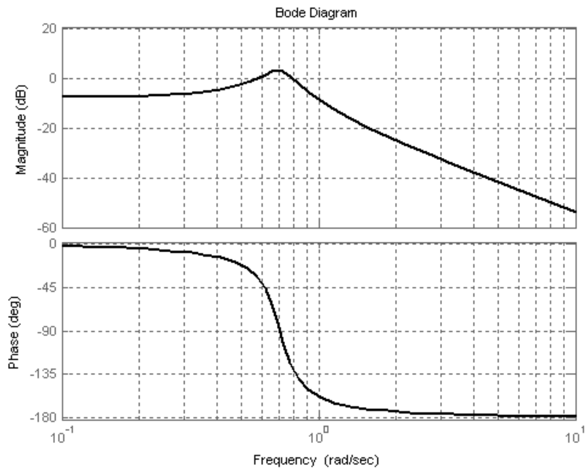

Fig. 4. Bode plots of the frequency response

Example 2: Consider a OP-AMP circuit having two inputs and one output in Fig. 5. The system equations and the transfer matrix will be obtained.

The circuit has $n-1 = 5$ nonreference nodes, including input-output terminals of Op Amp. Thus, in the MNA system, $X_1(s)$ vector contains 5 nodal voltage variables. The voltage and current constraints of ideal Op−Amp are $I_p = 0, I_n = 0, U_p - U_n = 0$. The current variables in $X_2(s)$ vector are I_{E1}, I_{E2}. In the circuit, $k = n - 1 + m = 7$.

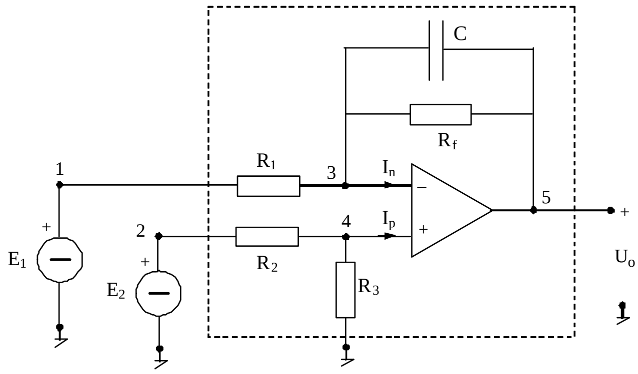

Fig. 5. Circuit for Example 2

Nodal (main) equations in s-domain:

$$\begin{aligned}
&1 \rightarrow G_1(U_1 - U_3) + I_{E1} = 0 \\
&2 \rightarrow G_2(U_2 - U_4) + I_{E2} = 0 \\
&3 \rightarrow G_f(U_3 - U_5) - G_1(U_1 - U_3) \\
&\qquad + sC(U_3 - U_5) + I_n = 0 \\
&4 \rightarrow G_3U_4 - G_2(U_2 - U_4) + I_p = 0.
\end{aligned}$$

Additional equations:

$$U_3 - U_4 = 0 \rightarrow \text{Op Amp constraint } I_p = 0, I_n = 0$$
$$U_1 = E_1, U_2 = E_2.$$

The overall equations constitute the MNA system (25). The output equation of the system is given in (26).

$$[G+sC]\,X(s)=BU_i(s)$$

$$\begin{bmatrix} \mathrm{G}_1 & 0 & -\mathrm{G}_1 & 0 & 0 & 1 & 0\\ 0 & \mathrm{G}_2 & 0 & -\mathrm{G}_2 & 0 & 0 & 1\\ -\mathrm{G}_1 & 0 & \mathrm{G}_1+\mathrm{G}_\mathrm{f}+sC & 0 & -\mathrm{G}_\mathrm{f}-sC & 0 & 0\\ 0 & -\mathrm{G}_2 & 0 & \mathrm{G}_2+\mathrm{G}_3 & 0 & 0 & 0\\ 0 & 0 & 1 & -1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\mathrm{U}_1\\ \mathrm{U}_2\\ \mathrm{U}_3\\ \mathrm{U}_4\\ \mathrm{U}_5\\ \mathrm{I}_{\mathrm{E}1}\\ \mathrm{I}_{\mathrm{E}2}\end{bmatrix}=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}\mathrm{E}_1\\ \mathrm{E}_2\end{bmatrix}. \tag{25}$$

The output equation $Y(s)=TX(s)$ is given as:

$$Y(s)=TX(s)=[U_o]=\begin{bmatrix}0&0&0&0&1&0&0\end{bmatrix}\begin{bmatrix}U_1&U_2&U_3&U_4&U_5&I_{E1}&I_{E2}\end{bmatrix}^T. \tag{26}$$

By using this system model, the matrix $W(s)$ is created. Thus, the transfer matrix in terms of the components of $W(s)$ is calculated systematically.

$$X(s)=\begin{bmatrix}\mathrm{U}_1\\ \mathrm{U}_2\\ \mathrm{U}_3\\ \mathrm{U}_4\\ \mathrm{U}_5\\ \mathrm{I}_{\mathrm{E}1}\\ \mathrm{I}_{\mathrm{E}2}\end{bmatrix}=\underbrace{[G+sC]^{-1}B}_{W(s)}U_i(s)=W(s)U_i(s)=\begin{bmatrix}W_{11}(s)&W_{12}(s)\\W_{21}(s)&W_{22}(s)\\W_{31}(s)&W_{32}(s)\\W_{41}(s)&W_{42}(s)\\W_{51}(s)&W_{52}(s)\\W_{61}(s)&W_{62}(s)\\W_{71}(s)&W_{72}(s)\end{bmatrix}\begin{bmatrix}E_1\\E_2\end{bmatrix}, \tag{27}$$

where,

$$\begin{bmatrix}W_{11}(s)&W_{12}(s)\\W_{21}(s)&W_{22}(s)\\W_{31}(s)&W_{32}(s)\\W_{41}(s)&W_{42}(s)\\W_{51}(s)&W_{52}(s)\\W_{61}(s)&W_{62}(s)\\W_{71}(s)&W_{72}(s)\end{bmatrix}=\begin{bmatrix}1&0\\0&1\\0&R_3/\left[R_2+R_3\right]\\0&R_3/\left[R_2+R_3\right]\\R_f/\left[R_1(1+sCR_f)\right]&\left[(R_1+R_f+sCR_1R_f)R_3\right]/\left[R_1(R_2+R_3)(1+sCR_f)\right]\\-1/R_1&R_3/\left[R_1(R_2+R_3)\right]\\0&-1/\left[R_2+R_3\right]\end{bmatrix}. \tag{28}$$

The elements of transfer matrix, $H(s)$, are determined in terms of $W(s)$ matrix as follows:

$$H(s)=\frac{Y(s)}{U_i(s)}=TW(s)=\begin{bmatrix}0&0&0&0&1&0&0\end{bmatrix}\begin{bmatrix}W_{11}(s)&W_{12}(s)\\W_{21}(s)&W_{22}(s)\\W_{31}(s)&W_{32}(s)\\W_{41}(s)&W_{42}(s)\\W_{51}(s)&W_{52}(s)\\W_{61}(s)&W_{62}(s)\\W_{71}(s)&W_{72}(s)\end{bmatrix}=\begin{bmatrix}W_{51}(s)&W_{52}(s)\end{bmatrix}. \tag{29}$$

The desired transfer matrix and the output vector are given as follows:

$$Y(s) = H(s)U_i(s)$$

$$U_o = \begin{bmatrix} \dfrac{R_f}{R_1(1+sCR_f)} & \dfrac{(R_1+R_f+sCR_1R_f)R_3}{R_1(R_2+R_3)(1+sCR_f)} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{30}$$

4 CONCLUSION

The main difficulty in determining the transfer functions in circuit analysis arises from obtaining the system equations. In general, the system equations are determined from state variable analysis having some structure restrictions. In this paper, an efficient and systematic approach for determining the transfer matrix of circuits with multi-inputs multi-outputs has been presented. The proposed method uses the modified nodal approach suitable for computer-aided analysis. It is based on the use of components of the matrix created, W(s), from the system equations. Determining transfer matrix containing transfer functions relating to the examples of passive and active circuits shows the efficiency of the given approach. For future work, a computer program about the transfer functions and frequency domain analysis of circuits with multi-inputs multi-outputs can be written by using the presented method. Moreover, the noise analysis, one of interesting applications of network analysis, can be also realized by this method.

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Received: 2010-03-16
Accepted: 2011-02-01