How would you like it: cheaper or shorter

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Least cost scheduling techniques were accompanying the history of modern project management, however they have never gained much importance in the practice. Even in our days only a few computer application provide this kind of feature to the users. In this paper a generalized PDM least cost scheduling problem will be introduced, and a case study will be presented to demonstrate the effectiveness of the model. The case study is based on a highway construction project, where the least cost scheduling technique developed by the authors was used in applied in order to calculate the minimum direct cost solution to a given project duration. The authors came to a conclusion that least cost scheduling can be a useful tool in the cost planning of the projects, however further research are necessary (e.g. handle of the activity calendars) to make the model suitable for everyday use.

Keywords
project management, scheduling, cost

INTRODUCTION

The original CPM problem, developed by Kelley and Walker (Kelley at al. 1959) was a least cost scheduling problem. Some unusual characteristics and their implications in project management are discussed in (Weist, J.D. 1981). The basic hypothesis concerning the activities in the original CPM problem is, that it can be determined a normal duration and a normal cost related to the normal duration for each and every activity, and a crash duration and crash cost related to the crash duration. The crash cost always greater or equal to the normal cost (see Figure 1.), and the change of the cost is described by a linear function, within the interval of the normal and crash duration. The cost slope shows the increment of the cost for one day shortening of an activity duration, from the normal to the crash duration.
The goal in the CPM least cost scheduling model is to define the minimum direct cost to a given project duration, that is to define the curve of the minimum direct cost solution within the interval of the maximum and minimum project duration. The CPM technique has lost his importance from the mid sixties, early seventies, due to its rigid structure, problems of the graphical displaying of the network, and the poor modelling possibilities in describing logical relationships between activities. Instead a new technique, the so called Precedence Diagramming Method (PDM) started to disperse all over the world. One of the pioneers of the PDM technique was Roy (Roy, 1959).
In our days the majority of the available project management tools use the PDM technique as their basic model, due to the extended modelling function of the technique (compared to CPM technique), and the greater flexibility in modelling..

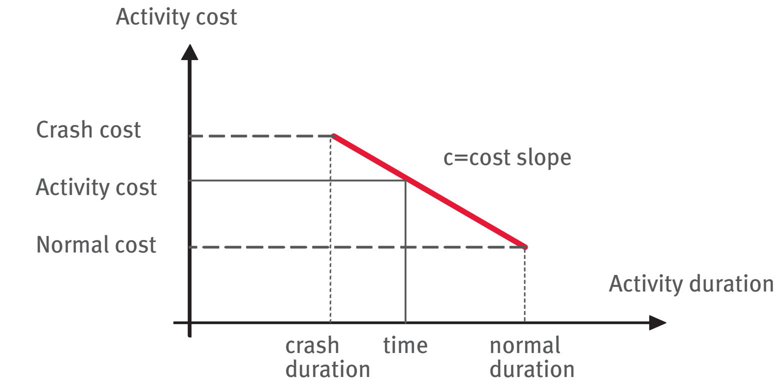

Figure 1. **Activity duration versus activity cost assumptions of the model**

The PDM technique gives more flexibility in modelling by introducing the minimal and maximal precedence relationships. The precedence relationships describe the minimum necessary, or maximum allowable time span between the start or finish times of the two activities, connected by the precedence relationships.

Minimal precedence relationships are well known by most of the planners, however maximal precedence relationships are considered relatively new in project management practice, only a small minority of the planners understand and apply them during their work. Further details on maximal precedence relationships can be obtained from the book of Hajdu. (Hajdu, 1996a)

The PDM Least Cost Scheduling Model

In a PDM network techniques for construction scheduling, arrows represent activities and logical relationships between activities and nodes present events. Let *a* and *b* assign to each activity as crash duration time and normal duration time. Assign to each activity a cost K_b (normal cost) to complete the activity at the normal duration and a cost K_a (crash cost)– greater than normal cost to complete at the minimum duration. Let us supposed that cost function of each activity is linear and its slope is,

$$\frac{K_b - K_a}{b - a} = tg\alpha = -c$$

where $c \geq 0$. For *a* given activity time „τ" the cost is $K_b+(b-\tau)c$. If *a* and *b* are nonpositve numbers let *c* be zero.

Let us see the model from the general contractor's point of view.

According to the contract between general contractor and client, accomplishment of event *i* will be occurred at time $e_i \geq 0$. General contractor also has contract with subcontractors. Let us assume that general contractor gets and/or pays D_i amount of money if event *i* has occurred. If $D_i > 0$ then client pays D_i amount of money to the general contractor. If $D_i < 0$ then general contractor pays D_i amount of money to the subcontractors. Based on the scheduling event i will be occurred at a time μ_i. Let interest rate per a day is q and $d_i = D_i q$. It means that d_i is a daily benefit or outcome depending of the sign of D_i and the sign of difference between μ_i and e_i. In an alternative reading let d_i *be a daily penalty for delayed delivery. From the general contractor point of view if* d_i *positive and* ($\mu_i > e_i$), *general contractor should pay to the client since event i delayed. If* d_i *positive and* ($\mu_i < e_i$), client pays to the general contractor daily d_i *amount of money because event i completed earlier. If* d_i *negative and* ($\mu_i > e_i$)*, it means that subcontractor pays to the general contractor because event i is delayed. s a daily penalty for delayed delivery. If* d_i *negative and* ($\mu_i < e_i$) *general contractor should pay to the subcontractor because event i completed earlier.*

Mathematically the sum of

$$K_{bij} + (b_{ij} - \tau_{ij})c_{ij} + (\mu_i - e_i)d_i$$

for each activity should be minimized.

Mathematical model

Denote [N,A] a directed graph (network) where N is a set of nodes and A is a set of arcs whose elements are ordered pairs of distinct nodes. Let it n be the number of nodes and m be the number of arcs. There is only one starting node s and one end node t in [N,A] directed graph. Directed graph contains no parallel arcs (i.e., two or more arcs with the same tail and head nodes. This assumption imposes no loss of generality. There is a path in a network from node s to every other node in the network.

Denote a_{ij}, τ_{ij}, b_{ij} integer values for all $ij \in A$ associated with network's arcs, where $a_{ij} \leq \tau_{ij} \leq b_{ij}$ for all $ij \in A$ and sign $a_{ij} = sign\ b_{ij}$ for all $ij \in A$, moreover given $c_{ij} \geq 0$ integer value for all $ij \in A$ associated with each network's arc that represents the cost of acceleration of activity *ij* if activity time is reduced by one unit time. If b_{ij} is negative then let $c_{ij} = 0$. For the sake of simplicity **a, b, c,** τ represent the corresponding vectors.

In engineering term τ_{ij} represents activity time of an activity $ij \in A$ with a_{ij}, b_{ij} lower and upper bound respectively normal and rush time. Denote k_{ij} the cost of activity *ij* for all $ij \in A$. Denote *p* the project duration time. Find to each node a μ_i, $\forall i \in N$ value. An obviously natural condition that $\tau_{ij} \leq \mu_j - \mu_i$ $\forall ij \in A$. The duration time of the network is *p*, where $p \leq \mu_t - \mu_s$. Let $\mu_s = 0$. Let [N,A] directed graph be supplemented with an arrow *(t,s)* for which $a_{ts} := -p$, $b_{ts} := 0$, $c_{ts} := 0$.

Remark. For a non-splitting $ij \in A$ activities $t_{ij} = \mu_j - \mu_i$ is a condition.

If $t_{ij} \leq \mu_j - \mu_i$, $a_{ij} \leq \tau_{ij} \leq b_{ij}$, and $t_{ji} \leq \mu_i - \mu_j$, $-b_i = a_{ij} \leq \tau_{ij} \leq b_{ij} = -a_{ji}$ then; $a_{ij} \leq \mu_j - \mu_i \leq b_{ij}$. Moreover if $c_{ij} > o$ then objective function attains its maximum value if $\tau_{ij} = \min(b_{ij}, \mu_j - \mu_i)$ for all $ij \in$ A, that is for all non-splitting activities $\tau_{ij} = \mu_j - \mu_i$ is satisfied. If $c_{ji} = o$, then τ_{ji} is arbitrary so let us choose τ_{ji}, that $\tau_{ji} = \mu_i - \mu_j$. For splitting activities $\tau_{ji} \leq \mu_j - \mu_i$ is the only condition.

We seek for all possible p values such a τ és μ systems that

$$\sum_{ij \in A} K_{bij} + \left(b_{ij} - \tau_{ij}\right) c_{ij} + \sum_{i \in N} \left(\mu_i - e_i\right) d_i$$

be minimal.

Least cost scheduling problem leads to the following mathematical model.

Mathematical model.

Given a directed network [N,A] with a_{ij}, b_{ij}, c_{ij} integer values where $c_{ij} \geq o$ for all $ij \in$ A and d_i, $\forall i \in N$ integer values where $\sum_{i \in N} d_i = 0$. Find μ_i for all $i \in N$; and τ_{ij}, for all $ij \in A$, for a given p (where $a_{ts} := -p$) project duration time that

$$\tau_{ij} \leq \mu_j - \mu_i \forall ij \in A$$
$$\tau_{ij} \leq b_{ij} \forall ij \in A$$
$$\tau_{ij} \geq a_{ij} \forall ij \in A$$
$$\mu_s = 0$$
$$-p \leq \mu_s - \mu_t$$

$$\sum_{ij \in A} K_{bij} + (b_{ij} - \tau_{ij}) c_{ij} + \sum_{i \in N} (\mu_i - e_i) d_i$$

should be minimized that is

$$\left\{ \sum_{ij \in A} c_{ij} \tau_{ij} - \sum_{i \in N} d_i \mu_i \right\}$$

should be maximized.

It is a dual of a special minimum cost flow problem that can be solve by wide range variety of algorithms. In our case study a PDM least cost scheduling technique was used to find the optimal cost solution in a given project duration interval. The algorithm originally was developed by Hajdu (Hajdu,1993) later some generalizations were developed by Hajdu and Malyusz (see below). The algorithm was used for this case study provides solutions for the following specific cases:

- Original CPM cost model (Kelley et al, 1959)
- CPM least cost scheduling, with penalty payment for late delivery, bonus payment for earlier delivery.
- PDM least cost scheduling: only minimal precedence relationships allowed, activities are carried out continuously, that is activity splitting is not allowed (EF-ES=t, LF-LS=t) (ES: Early Start; EF: Early Finish; LS: Late Start; LF: Late Finish; t: activity time (Hajdu,1993)
- PDM least cost scheduling: only minimal precedence relationships are allowed, splitting of activities are allowed, that is in case of some activities EF-ES ≥ t; and LF-LS≥ t. (Hajdu, 1996b)
- PDM least cost scheduling: both minimal and maximal type of precedence relationships are allowed, activity splitting is not allowed. (Malyusz, 2003)
- PDM least cost scheduling: both minimal and maximal type of precedence relationships are allowed, splitting of activities are allowed, that is, in case of some activities EF-ES≥t; and LF-LS≥t (Hajdu and Malyusz 2008b)
- PDM least cost scheduling allowing the application of time constraint in the network. (Hajdu and Malyusz, 2008a) The following constraints are handled in the model:
  - Must start on...
  - Must finish on..
  - Start earlier then...
  - Start later then...
  - Finish earlier then ....
  - Finish later then ...
  - Any combination of the aforementioned constraints

  Constraints above can be transformed to an inequality (1), (2) or (3) in mathematical model.
- PDM least cost scheduling accomplishing points 5,6 and 7 with penalty payment for late delivery and bonus payment for earlier delivery. (Hajdu and Malyusz 2008b)

Case Study

The development of an algorithm that solves the above mentioned generalizations, and the software application was finished in the summer of 2007, and after some sample testings we have decided to test the application on several real and possibly large-size project.

The following prerequisites were required towards the project to be chose:

- must be a construction project
- must be an ongoing project
- possible large scale project with long project duration (above 2 years)
- must be an original well defined and accurate baseplan (schedule), with more than 1000 activities

The project chosen for testing is a construction of a motorway in 6 km length. This project is a relatively small section of a huge construction programme, that aims to make a motorway ring in more than 150km-es around Budapest – capital of Hungary - in order to relieve the capital from the load of the - mainly- international transit traffic. The realization of the whole program started in the late 70's and will finish around 2025. The dense built area, the vast number of land owners (more than 30.000 owners on more than 20.000 properties) that makes the acquisition very slow, non-governmental-environmental organizations, make the whole programme very slow and expensive.

The start of this project was October, 2005 the planned finish was June, 2007. Due to some legal and technical problems the project has been rescheduled in January, 2007 at the stage of project completion below 10%. The new deadline is October, 2008. In the time of the submitting of this paper (July, 2008), it seems that de deadline will be met. At this time another 3 section, and a bridge over the river Danube will be finished, and the total length of the ring will be more than 90 km.

The length of this section is 6,7 km, contains 8 bridges, two junctions, replacement of more than 30 public utilities, crossing the planned road, and involves around 2million m3 earthwork. The contracted fee € 45million. The client is the National Infrastructure Development Ltd (NID), the contractors' consortia is formed by Porr, Teeraq-Asdag and Viadom Zrt. The leading firm is Porr Hungary, the Hungarian affiliate of Porr Ag. NID is responsible for managing all the government financed infrastructural projects (road and railway). Its current contracted portfolio is above € 10billion.

NID has a very strict requirements regarding project planning and monitoring, and requires from all the contracted partners to fulfil their regulations in these fields. The reason for this is that NID manages not only the project but the whole portfolio, therefore projects have to be managed and handled in the project management system in a uniformed way. This includes:

- the methodology of developing the schedule of quantities (A general schedule of quantities comprises around 3000 items, but in some cases can go above 15 000 items.)
- the methodology of establishing the bill of quantities (priced schedule of quantities).
- the methodology of preparing the baseline plan (A baseline plan generally consist of 1000 activities but in some cases can go over 5000 activities.)
- the methodology of monitoring, which is carried out monthly
- the methodology of handling claims and paid and unpaid extra works,
- the methodology of monthly actualization of schedule

These regulations are in use from the year of 2000, a non-adequate or late accomplishment costs a lot of money to the contractors, therefore they produce much better schedules comparing the average construction industry level.

The baseline plan that was the basis of our work was made in January 2007, and comprises 576 activities and 746 logical relationships. The schedule of quantities consisted of 1311 items. The WBS (Work Breakdown Structure) has been developed automatically from the schedule of quantities according to the standardized rules of NID. This project is considered as a relatively small one in NID's practice.

In the baseline plan there was one critical path with the length of 532 days. The plan has been developed in a scheduling tools used by NID and the contractors, which can handle maximal type of precedence relationships. Four different calendars were used in the network.

Preparations for least cost scheduling

Least cost scheduling requires the existence of normal time with the related normal cost, and crash time with the related crash cost for each activity, so the most important task during the preparation for least cost scheduling was the definition of these data. The second important part of the preparation was to standardization of the calendars, because the model has been developed within the frame of this research project do not handle different activity calendars. During the preparation of activity durations and activity costs we followed the principle, that the activity durations and costs in the baseline plan will serve as the normal duration and normal cost of our model, so our task was to define the crash duration and crash cost for each activity. For this two methods were applied:

- detail investigation of an activity
- estimation, based on experts opinion

Detail investigation has been carried out for only 20 activities. At the and of the investigation, crash durations were reduced to 70-90 percent of the normal durations, with the average 30-10 percent cost increment. It is important to notice that detailed investigation and estimations have been led to almost the same results.

During the process of the estimation several meetings with the chief engineer responsible for the construction were held in order to develop crash durations and crash costs. The result of the estimation was that activity durations could be reduced to 60-80 percent comparing to the original durations, that is normal duration, which resulted in an average 25 percent (10-40 percent) increment in cost.

As the least cost algorithm developed by the authors can not handle different calendars in the schedule, standardization of the calendars was the most time consuming activity during the preparation. It has involved the modification of the activity durations, modification of precedence relationships' lag time, and sometimes adding new relationships to the schedule. Our aim was to get the same start and finish time after the modification as they were before, that is to keep the results of the original schedule.

Least Cost Scheduling

The problem was solved with the least cost scheduling module of ProjectDirector 4.0. There were 34 breakpoints in the cost curve. The minimum project duration decreased to 530 from 467 days. The increment of the project direct cost in this interval was € 4 755 537, that is more than 63 days shortening in the project duration is possible and this costs less than 10% of the contracted fee. The results of the calculations are shown in Figure 5, and Table 1.

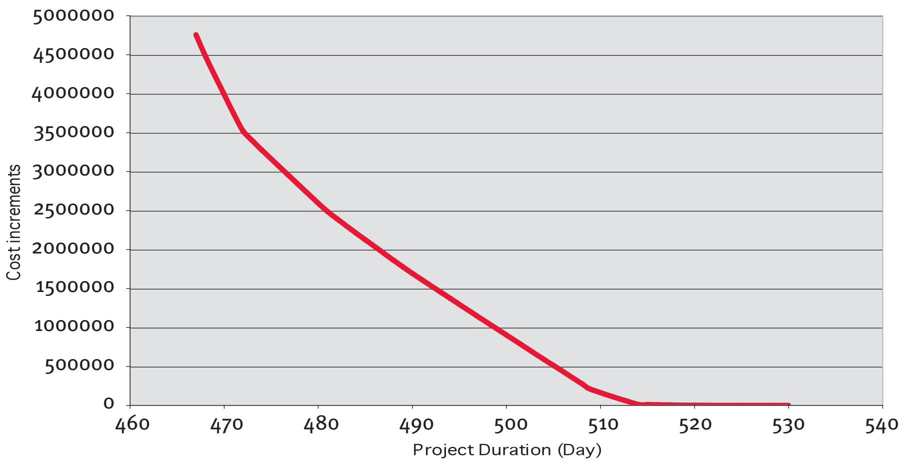

Figure 2: Project duration versus increments of project direct costs

Project dur. (day)	530	528	525	520	516	515	514	509	508	505	504	500
Cost increment (Euro)	0	0	0	1335	8491	10406	12 399	206 519	281 142	512763	589 987	902 575

499	495	494	493	491	490	489	488	485	484	482
981 326	1 296 358	1 375 184	1 454 473	1 613 789	1 693 539	1 777 483	1 862 311	2 125 045	2 212 995	2 392 723

481	480	478	474	473	472	471	470	469	468	467
2 485 808	2 594 560	2 821 060	3 277 044	3 396 876	3 522 697	3 748 535	3 988 563	4 232 704	4 484 993	4 755 537

Table 1. Result of PDM Least Cost Scheduling calculations. Project duration versus cost increment

CONCLUSIONS

The results of the calculations were promising. It can be stated that serious savings can be achieved by using a least cost scheduling model, if project speed up is necessary. The preparations especially the elimination of the different calendars were very time consuming. This took more than 70% of the total preparation time, therefore the work is much convenient if only one calendar is used in the schedule. During the calculations we came to the conclusions that in some cases precedence lag times depend on the duration of the preceding or succeeding activity. So far this was left out of consideration.

REFERENCES

Hajdu., M., 1993 "An algorithm for Solving the PDM Time -Cost Trade Off Problem", Periodica Politechnica Vol. 37.,No3., pp. 231-247, Budapest, Hungary

Hajdu, M, 1996a, Network Shceduling Techniques for Construction Project Management, Kluwer Academic Publisher, ISBN 0-7923-4309-3.

Hajdu, M 1996b „PDM Time Cost Trade Off: Activities Are Splittable or Non-Spittable", Optimization, Vol. 38. pp 155-171 Spinger

Hajdu M, Malyusz L 2008a "Constraints and Least Cost Scheduling" Approved for publication. To appear in Ybl Scientific Papers, Budapest, Hungary 2008,

Hajdu M, Malyusz L 2008b. "Some new generalization of the PDM Least Cost Scheduling Problem: Activity splitting together with maximal precedence relationships are allowed, penalty for late completion is introduced" Research Report carried out under the frame of the contract YDCM/01 between the Ybl Miklos School of Architecture and Civil Engineering, Budapest, Hungary

Kelley.J.,E., Walker.,M.,R., 1959. "Critical Path Planning and Scheduling", Proc. the Eastern Joint Computer Conference, Boston

Malyusz L., 2003 "An efficient network flow algorithm for MPM/PDM least cost scheduling problem" Proceedings of the 6th International Conference on Organisation, Technology and Management in Construction, Moscenicka Draga Croatia, 17-20 September 2003.

Roy, B 1959. "Theorie des graphes: Contribution de la theorie desígraphes a l'etudede certains problemes linearies" Comptes rendus des Seances de l'Acedemiedes Sciences, seence du Avril 1959, p2437-2449

Weist, J.D. 1981, "Precedence Diagramming Method: Some Unusual Characteristics and Their Implications for Project Management" Journal of Operations Management Vol.1. No3, Feb,1981, pp121-136.