UDK 528.425.4:528.247:528.241:528.271(497.11)
Izvorni znanstveni članak

Optimum Resolution and Size of DTM during Modelling Topographic Effect

Jelena GUČEVIĆ, Vukan OGRIZOVIĆ, Siniša DELČEV, Violeta VASILIĆ – Belgrade[1]

ABSTRACT. Modelling the effects of topographic masses which bear local changes of the gravity field is exceptionally important in the contemporary scientific and practical implementations. The topography effect in this paper is analysed by the digital terrain model of the Republic of Serbia, developed on the basis of the topographic map at the scale of 1:25 000 and by the hypothesis on the equal density of surface layers of the Earth crust. The shortwave characteristic as an impact of the local topography was analysed in parts with different topography for the region of Belgrade and Paraćin. The gravity field parameters were determined in points of the reference GPS network of the Republic of Serbia and points of the Paraćin basis, with the purpose of comparing gravimetric vertical deflections with astro-geodetic vertical deflections.

Keywords: topographic effect, terrain correction, gravity reduction, gravity anomaly, vertical deflection.

1. Introduction

For surveyors, the prediction of Earth gravity field encompasses determination of unknown characteristics, above all: gravity anomaly, geoid height/anomaly height, and the components of vertical deflection. To know the geoid height in satellite-tracked points enables transformation of ellipsoid heights into the heights determined by mean sea level. To know vertical deflection provides possibility for orientation of reference surfaces, which is important in the process of reduction of geodetic observation to reference surface (Heiskanen and Moritz 1967).

Local topographic masses immensely influence vertical deflection, from 10 to 20 arc-seconds, while their influence on the geoid height is several centimetres.

[1] Assistant Professor dr. Jelena Gučević, Assistant Professor dr. Vukan Ogrizović, Assistant Professor dr. Siniša Delčev, Teaching Assistant Violeta Vasilić, dipl. ing. geod., Faculty of Civil Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, RS-11000 Belgrade, Serbia, e-mail: vasovic@grf.bg.ac.rs, vukan@grf.bg.ac.rs, delcev@grf.bg.ac.rs, tatic@grf.bg.ac.rs.

Naturally, the Earth is not a solid body and the gravity and shape of the Earth change in time. According to the contemporary knowledge, the Earth crust is comprised of plates of average density of 2.67 g/cm^3, which float in the matter of density of 3.27 g/cm^3. It is very difficult to determine the margin between the Earth's upper mantle (asthenosphere) and the Earth crust (lithosphere) (Vanicek and Krakiwisky 1980). Geological research shows that the thickness of the crust ranges from 10 to 80 km, and all the Earth deformations have impact on changes of the gravity field.

Elimination of the topography impact within the theory of Earth gravity field potential basically includes (Fig. 1):

- topographic effect, (Bouguer plate and terrain correction),
- effect of isostatic compensation (according to the Airy-Heiskanen and Pratt-Hayford model) and
- effect of residual topography of pre-determined mean elevation surface, (RTM effect introduced by Forsberg and Tscherning 1981).

The integral methods of Stokes and Vening-Meinesz basically use one type of data and predict other data on the basis of reduced gravity values. The reduction implies the obtainment of boundary value of gravity on the reference surface (Forsberg and Tscherning 1997). The value of normal acceleration in the point on the ellipsoid γ is subtracted from the acceleration on the reference surface g_P, and thus gravity anomalies are formed:

$$\Delta g_{topo} = g_P - \gamma.$$

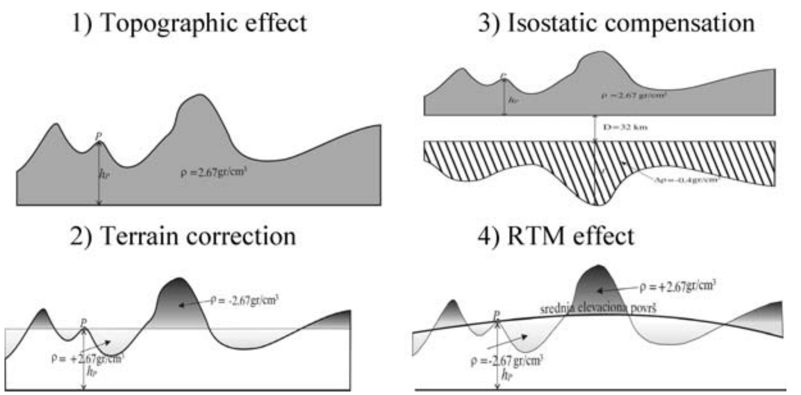


Fig. 1. *Modes of interpretation of topographical effects.*

Depending on the applied reductions, they are called as follows (Forsberg 1985):

1) Free air anomalies: $\Delta g_{AA} = g + F - \gamma$
2) Faye's anomalies: $\Delta g_{FA} = \Delta g_{AA} + c$
3) Bouguer anomalies: $\Delta g_{B} = \Delta g_{AA} - B + c$
4) Isostatic anomalies: $\Delta g_{I} = \Delta g_{AA} - A_T + A_C$
5) RTM anomalies: $\Delta g_{RTM} = \Delta g_{AA} - A_{RTM}$,

where: F is free air effect, c terrain correction, B Bouguer plate, A_T topography effect, A_C compensation impact, A_{RTM} impact of topographic masses with regard to the reference or mean elevation surface.

Prediction of the gravity field parameters includes determination of the anomaly potential and its functionals. For the determination, three sets of data are globally used: a) global geopotential model; b) terrestric, astronomic and satellite determinations and c) digital terrain model.

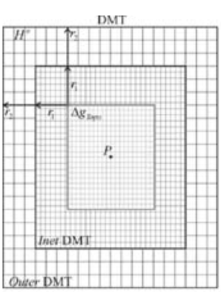
Out of each set of data, one tidal component of the gravity field is determined:

- Global component is determined with the use of global geopotential model (GGM) and is the result of incompliance of the Earth body with the body of ellipsoid at the global level. [Large number of models can be downloaded from the website of the International Geoid Service IGS (URL 1), or from the website of the late Professor H. G. Wenzel (URL 2)],
- Regional component, which is determined from the results of observation of gravity acceleration, as a consequence of regional incompliance of the Earth body with the ellipsoid body, and
- Local component, the result of impact of local topographic masses.

In literature, the presented components are often called long-wave, middle-wave and short-wave components. The technique for the prediction of funcionals of anomaly potential is known as *remove-restore technique* or *remove-restore method*.

With the use of the GGM coefficient, the impact of topography at the global level is eliminated at the first step of the *remove-restore method*. Determination of the local component encompasses: modelling topographic masses, determining their gravity influence, as well as forming residual observation results. The gravity influence of masses is calculated: in the vicinity of the point, up to 20 km (influence of local masses), and in the circle of several hundreds of kilometres (influence of remote masses).

Practical determination of the local component comprises of division of topographic masses to bodies of regular geometric shapes and iterative calculation of its gravity influence of inner and outer zones. The gravity influence can be calculated with the mass prism model (Fig. 2: a, b, c, d), line (Fig. 2. case e). The main characteristics of the mass prism topographic model are mid-height of the prism and mean density (Tsoulis 1999).



Itertion	distance [km] Iner r_1	Outer r_2
1.	10	20
2.	15	50
3.	50	100
4.	80	200
5.	80	250

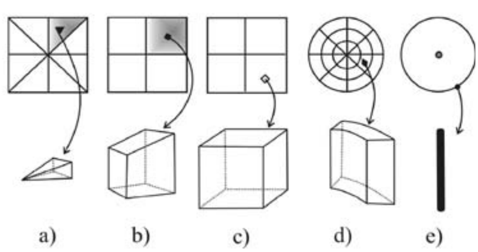


Fig. 2. *Calculating of topographic effect.*

Upon determining the global and local component, residual anomalies are formed:

$$\Delta g_R = \Delta g_{(i)} - \Delta g_{EGM\,96},\ i = \text{reduction scheme (AA, FA, B, I, RTM)}.$$

Residual anomalies are input values of the Stokes integral, and with some of the calculation techniques (integral formulas, FFT, collocation) the values of the regional components are calculated (Forsberg and Sideris 1993), (Haagmans et al. 1993) (Sideris and Tziavos 1988).

2. Data for Numerical Calculations

2.1. Gravimetrical Survey

Gravimetrical measuring in the territory of the former Yugoslavia started in 1951. In the period 1951-1953, the network of gravimetric points of 1^{st} order was set and measured by the Military Geographic Institute and Principal Geodetic Administration of Yugoslavia. Its absolute level was determined by connecting with the points of the French network and small and big base for calibration of gravimeter were set. (Bilibajkić et al. 1979).

The entire network was based on the Potsdam system of the absolute gravity value of one point only, determined in 1906 with the reversible pendulum. At the XV General Session in Moscow in 1971, the International Union of Geodesy and Geophysics adopted the network of basic gravimetric points. The formed network was called IGSN71[2], and it replaced the old Potsdam system (Bašić and Markovinović 2002). The IGSN71 network includes the points of Belgrade and Zagreb.

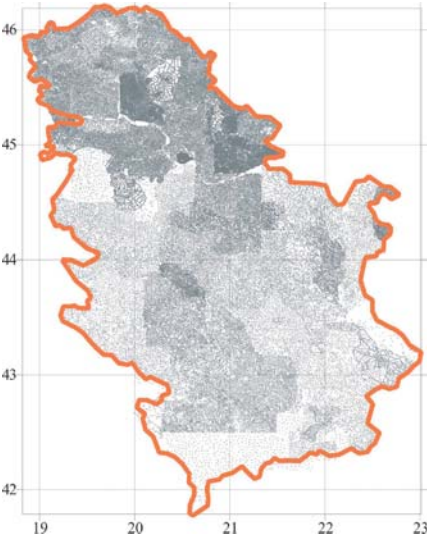


Fig. 3. *Arrange points gravimetric points determined in Serbia.*

[2] International Gravity Standardization Network 1971

They enabled determination of proportion of our gravimetric values and transformation into the ISGN71 system. In the territory of the Republic of Serbia, there are around 85 000 gravimetric points determined (Fig. 3), enabling research work in the field of geosciences. The positions of points of gravimetric survey were determined with the use of state position network with the reference surface of the Bessel ellipsoid. The heights of the points were determined by connecting to the height network of levelling in use (NVTI).

2.2. Digital Terrain Model

Works on the creation of the national digital terrain model for the territory of the Republic of Serbia started by the end of 2002, within the project of the Military Geographical Institute in Belgrade, first by scanning and vectorisation of topographic maps, at a scale of 1:25 000 (TK25), which cover the territory of entire state and serve as a basis for small-scale topographic survey.

In compliance with the methodology of the creation of digital terrain model, (Odalović et al. 2004), in the first phase some 864 maps were scanned, covering the territory of the Republic of Serbia. In the second phase, vectorisation of isohyps lines and hydrology was carried out, with the semi-automatic and automatic procedures, depending on the nature of the terrain and the quality of scanned surfaces. The obtained data were saved in the DXF format. All the vectorised surfaces were georeferenced within the state coordinate system, with the Digiscan software, with four joint points on the topographic map. The isohyps height, in meters, was attributed to the vectorised lines that were exported to the ASCII database of scattered points, with numbers, Y and X coordinates in the state coordination system and height.

Quality verification of vectorised cartographic surfaces in the third phase encompassed the implementation of exact and visual methods. The verification was first carried out in 125 control points within each topographic map TK25, on the total of 108 000 points for vectorised surfaces. In case there were height differences over 5 m registered in the control points, separation and re-verification was carried out in the part of the territory within which the deflection was registered.

Upon forming the final set of data of the scattered points, the creation and verification of the hybrid digital model was initiated for the territory of the Republic of Serbia. The very process of forming the digital model encompasses: selection of resolution, determination of altitude points and verification of the model quality.

The selection of resolution is in the function of density of the scattered points and the use of the digital model in the process of defining impact of topographic masses on functionals of anomaly potential. Density of points range from 30 to 3000 per square kilometre, which is in the function of pronounced changes of topography within topographic maps. The point altitude of the digital model is determined according to the principle of arithmetic mean of five closest points with known heights. The quality verification within partially created digital models was carried out for the resolution 1" × 1", with bilinear interpolation of the points of the gravimetric survey, according to the expression:

$$\Delta H^0_{GP} = H^0 - H^0_{DTM}$$

where H^0 is the height determined by measuring and H^0_{DTM} the value determined from digital terrain model. The conducted analyses show that 94.06% of the data that entered the analysis of accuracy of hybridly created digital model have differences less than 10 m according to absolute value.

In order to define the influence of remote masses, the additional information related to topography shape outside the Republic of Serbia was obtained by the model GTOPO30 (URL 3). It was carried out in accordance with experiences of other authors dealing with this issue.

2.3. Astronomic Determination

Astro-geodetic works in the territory of the former Yugoslav republics were initiated by Stevan Bošković. Historically speaking, the astro-geodetic works can fall into two categories: works of military services (S. Bošković and the Military Geographic Institute) and works of civil services – (Federal Geodetic Administration, Institute for Geodesy of the Faculty of Civil Engineering in Belgrade and Astronomic Observatory in Belgrade).

The works after the World War II are important for the research within this paper, those conducted within the Institute of Geodesy, on the points:

- Paraćin bases (145, 146) (Milovanović and Vasiljev 1984) and
- Reference GPS networks of the Republic of Serbia (R029, R050, R051 and R060), (Ogrizović 2003).

3. Data Preparation and Selection of Regions

Besides their collection and archiving, the preparation of the data necessary for defining tidal components in the first phase encompasses data transformation as well, including:

- Position data transformation, and
- Altitude transformation.

Position data transformation of the collected data implies the transformation of coordinates of the Gauss-Krüger projection on the Bessel non-geocentric ellipsoid into the geocentric coordinates of the WGS84 ellipsoid (Hofmann-Wellenhof 1994).

With the altitude transformation, heights of the points related to the NVTI system are transformed into the orthometric ones, with the reference surface of the geoid approximate the mean level of the Adriatic Sea (Vasović 2001). The mean level of the Adriatic Sea is the reference equipotential surface of the network of the second Levelling of high accuracy (NVTII).

The selection of characteristic regions for verification of short-wave component was carried out with regard to the shape of topographic masses and available data of astronomic determination. Consequently, the region of Belgrade and the region of Paraćin basis were selected, where astro-geodetic components of vertical deflections were determined (Fig. 4).

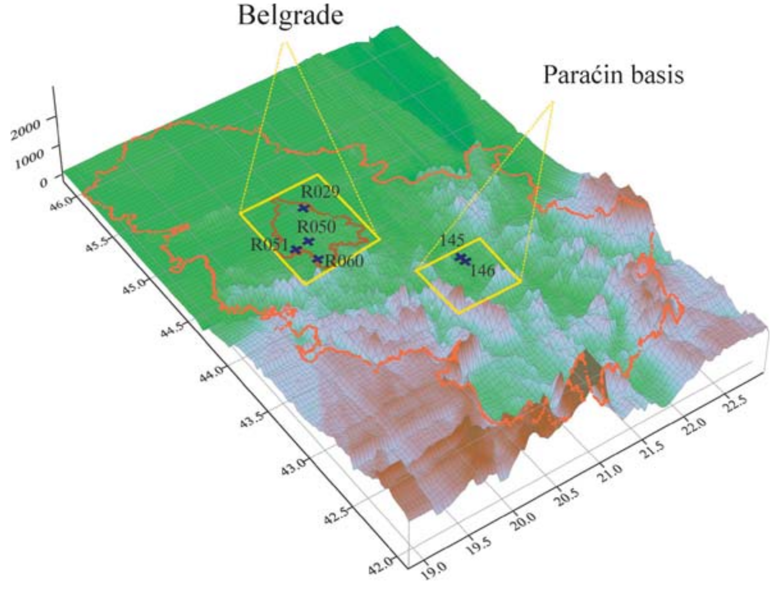


Fig. 4. *Selected areas for calculation.*

All numerical calculations of functionals of anomaly potentials were conducted with the programme packet GRAVSOFT (Tscherning 1994).

3.1. Forming Residual Anomalies

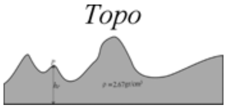
With the reduction in the "remove" process, the results of acceleration observation release the topography effect at the global and local level, and thus the residual results of observations are formed.

Prior to calculating topographic masses, the verification of the optimum resolution of the digital model of the selected regions of Belgrade and Paraćin was carried out. The conducted analyses show that with the calculation of the impact of inner DTM in Belgrade area the model of 3 second resolution can be used, and in the area of Paraćin basis, 2 second resolution can be used. For outer DTM, the model of resolution is 10 seconds (Gučević 2006). The calculation of the topographic masses impact is conducted iteratively. From these digital models, the size of the impact of topographic masses to functionals is clearly defined as: Δg_{topo} gravity change, N_{topo} geoid/anomaly height and ξ_{topo}, η_{topo} components of vertical deflection.

The impact of topographic masses is analysed as a *direct topographic effect* and the conducted analyses show the following (Table 1):

- For the Belgrade region, the stability in terms of the value of functionals is evident and the calculation of effects makes sense only up to 100 km. The changes of the functionals at longer distances are within the accuracy of short-wave component. The main statistics is shown at the Table 1.

Table 1. *Basic statistical data of topographic effect influence (ρ=2.67 g/cm^3).*

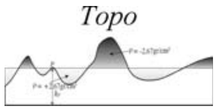
areas **Radius [km]**		Δg_{Topo} **[mGal]**	ξ_{Topo} **["]**	η_{Topo} **["]**	N_{Topo} **[m]**	**statistical**
Belgrade r_1 = 10 r_2 = 20		2.82	–2.68	–3.03	0.13	Min
		60.30	4.77	4.24	0.59	Max
		10.93	0.18	0.04	0.22	Average
		7.83	0.73	0.68	0.10	St. dev.
Belgrade r_1 = 50 r_2 = 100		5.56	–1.24	–2.48	0.70	Min
		66.72	6.09	6.41	1.97	Max
		13.71	1.34	0.80	1.26	Average
		7.86	1.00	1.29	0.30	St. dev.
Paraćin r_1 = 10 r_2 = 20		13.25	–6.51	–8.79	0.44	Min
		87.47	6.26	7.45	1.30	Max
		27.73	–0.12	–0.19	0.69	Average
		14.70	1.82	2.98	0.17	St. dev.
Paraćin r_1 = 80 r_2 = 250		11.62	–5.03	–4.45	2.88	Min
		88.11	9.17	9.72	3.84	Max
		29.27	2.36	3.34	3.08	Average
		14.85	2.02	2.25	0.27	St. dev.

- For the region of the Paraćin basis, changes in functionals are still present. It is necessary for the digital terrain model, created for up to 250 km, to be widened in order to determine the influence limit of outer zones. One of the reasons for instability in values can be the fact that in the part over 100 km to the east, only the data of the GTOPO30 models are used. The question is whether the observed instability is the consequence of insufficient size of the territory used in the process of integration or of the inadequately modelled shape of the topographic masses in the part related to the data only from the GTOPO30 model. The answer can be expected after the official exchange of information with the surrounding countries is carried out.

The terrain correlation calculated as an influence of discrepancy of topography and the Bouguer plate shows the following characteristics (Table 2):

- In the Belgrade region, the dominant values are in the part of close vicinity, up to 20 km. The calculation makes sense up to 100 km and the increased calculation distance, from 200 to 250 km does not bring any considerable changes.
- In the Paraćin basis region, dominant values are in the vicinity of 20 km, the expected stability is within the range of 100 to 200 km, and further changes of functionals are out of accuracy limit of the method of gravimetric determination.

Table 2. *Basic statistical data of terrain correlation influence (ρ=2.67 g/cm^3).*

areas **Radius [km]**		***c*** **[mGal]**	ξ_{Topo} ["]	η_{Topo} ["]	**statistical**
Belgrade r_1 = 10 r_2 = 20		0.00	–4.78	–3.25	Min
		1.61	2.68	3.04	Max
		0.15	–0.18	–0.03	Average
		0.32	0.73	0.68	St. dev.
Belgrade r_1 = 50 r_2 = 100		0.02	–7.09	–4.78	Min
		2.46	1.15	3.24	Max
		0.25	–1.48	–0.25	Average
		0.30	1.32	0.95	St. dev.
Paraćin r_1 = 10 r_2 = 20		0.21	–6.80	–8.19	Min
		7.00	6.71	9.66	Max
		1.12	0.10	0.33	Average
		1.13	1.97	3.36	St. dev.
Paraćin r_1 = 80 r_2 = 200		0.50	–10.90	–11.70	Min
		9.43	5.19	6.37	Max
		1.36	–1.89	–2.49	Average
		1.03	2.21	2.86	St. dev.

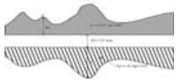
Isostatic compensation according to the Airy model shows the following characteristics (Table 3):

- In the Belgrade region, the biggest impact is up to 50 km, and reference values are reached at 100 km.
- In the Paraćin basis region, even values are reached at the distance of 200 km.

As it is already known, in theory all reductions can be considered equivalent and have to lead to the same reference surface, if the indirect effect is strictly implemented and calculated. However, there are certain restrictions that considerably reduce the member of practically usable reductions. Main requirements are: as a result, reduction must have acceleration anomalies, with small values and smooth surface, so that they can be easily interpolated; reduction has to correspond to the real model that can be geophysically and geologically interpreted; indirect effect should not be big. In order to analyse the presented requirements in accordance with possibilities, the following are formed:

- Free air anomalies,
- Faye's (Helmert) anomalies,
- Bouguer anomalies,
- Isostatic anomalies according to the Airy-Heiskanen model, and
- RTM anomalies,

Table 3. *Basic statistical data of isostatic compensation according to the Airy model.*

areas / **Radius [km]** Topo 	Δg_{Topo} **[mGal]**	ξ_{Topo} **["]**	η_{Topo} **["]**	N_{Topo} **[m]**	**statistical**
Belgrade $r_1=15$ $r_2=50$	–1.93	–1.93	–2.99	0.22	Min
	55.08	5.93	5.01	0.80	Max
	7.41	0.59	0.26	0.34	Average
	6.40	1.03	0.95	0.14	St. dev.
Belgrade $r_1=50$ $r_2=100$	–3.67	–1.82	–2.69	0.25	Min
	52.92	5.90	5.31	0.84	Max
	4.87	0.74	0.37	0.39	Average
	6.51	1.00	0.98	0.14	St. dev.
Paraćin $r_1=15$ $r_2=50$	–6.03	–6.67	–7.76	0.57	Min
	76.70	8.41	8.77	1.50	Max
	12.97	0.31	0.48	0.84	Average
	15.58	2.04	2.84	0.12	St. dev.
Paraćin $r_1=80$ $r_2=200$	–10.30	–5.90	–6.11	0.65	Min
	71.70	8.74	8.63	1.62	Max
	8.56	0.72	1.50	0.94	Average
	15.49	2.00	2.47	0.14	St. dev.

as well as their relevant residual anomalies. For the selected regions, the main statistical indicators are shown in Tables 4 and 5.

Free air anomalies generally exist in the process of calculating the reference surfaces and are an integral part of a presumption on the absence of masses above the reference surface. With the calculations of other authors and the presented statistics, the following can be concluded:

- Standard deviation of residual anomalies related to original ones has smaller values by 10-15%, which corresponds to the presumption,
- Helmert anomalies are easy to calculate since they are similar to free air anomalies by their values, differing only in the value of field correction,
- Bouguer anomalies have good interpolation characteristics (they are smooth and have big values) and geophysical importance, but it is well known that they have big indirect effect and thus are not used for geodetic purposes,
- Isostatic anomalies according to the Airy model are small and smooth, which can be concluded from their mean values and can be used for geodetic calculations,
- RTM anomalies provide the least standard deviation within forming of residual anomalies.

Table 4. *The main statistical indicators of region of Belgrade.*

scheme of reduction	anomalies	statistical		N = 8236	
		Min	Max	Average	St. dev.
Free air	$\Delta g_{AA} = g + F - \gamma$	–1.40	57.43	13.40	7.86
	$\Delta g_{AA} - \Delta g_{EGM\,96}$	–7.35	48.81	1.31	6.63
Faye's (Helmert)	$\Delta g_{FA} = \Delta g_{AA} + c$	–1.03	65.93	13.66	7.96
	$\Delta g_{FA} - \Delta g_{EGM\,96}$	–16.53	57.31	1.57	6.73
Bouguer	$\Delta g_{B} = \Delta g_{AA} - B + c$	7.76	87.35	14.46	8.36
	$\Delta g_{B} - \Delta g_{EGM\,96}$	–9.91	76.76	2.37	7.76
Airy-Heiskanen	$\Delta g_{I} = \Delta g_{AA} - A_T + A_C$	–13.85	43.86	4.31	7.58
	$\Delta g_{I} - \Delta g_{EGM\,96}$	–33.20	35.24	1.76	6.32
RTM	$\Delta g_{RTM} = \Delta g_{AA} - A_{RTM}$	–0.86	37.56	13.46	7.01
	$\Delta g_{RTM} - \Delta g_{EGM\,96}$	–16.85	27.10	1.38	5.93

Table 5. *The main statistical indicators of region of Paraćin basis.*

scheme of reduction	anomalies	statistical		N =1932	
		Min	Max	Average	St. dev.
Free air	$\Delta g_{AA} = g + F - \gamma$	–48.29	63.11	–6.87	20.15
	$\Delta g_{AA} - \Delta g_{EGM\,96}$	–10.01	89.89	20.79	18.84
Faye's (Helmert)	$\Delta g_{FA} = \Delta g_{AA} + c$	–46.99	72.55	–5.51	20.63
	$\Delta g_{FA} - \Delta g_{EGM\,96}$	–8.71	99.32	22.15	19.42
Bouguer	$\Delta g_{B} = \Delta g_{AA} - B + c$	–18.85	82.55	4.87	16.77
	$\Delta g_{B} - \Delta g_{EGM\,96}$	13.24	109.32	32.54	16.43
Airy-Heiskanen	$\Delta g_{I} = \Delta g_{AA} - A_T + A_C$	–75.58	47.25	–27.70	21.16
	$\Delta g_{I} - \Delta g_{EGM\,96}$	–37.30	74.02	–0.04	19.60
RTM	$\Delta g_{RTM} = \Delta g_{AA} - A_{RTM}$	–9.36	82.08	20.78	19.16
	$\Delta g_{RTM} - \Delta g_{EGM\,96}$	–47.64	52.45	–6.89	17.93

With the use of some of the calculating techniques (integral formulas, FFT, collocation…) prediction of the regional component is carried out from the formed residual observations.

Defining regional component and solving integral formulas of Stokes and Vening-Meinesz is carried out with the integration within the formed grid of residual anomalies. The residual anomalies necessary for the prediction of the regional component are formed at the points of gravimetric survey.

The implementation of the Helmert condensational reduction and isostatic reduction according to the Airy-Heiskanen model, define the components of vertical deflection on the geoid surface, i.e. Pizzetti deflection. Results of the functionals prediction are shown at the Tables 6 and 7.

Table 6. *Results of the functionals prediction by the Helmert condensational reduction.*

Nº	Global (EGM96)			Local			Regional			Total		
	ξ ["]	η ["]	N [m]	ξ_H^{IF} ["]	η_H^{IF} ["]	N_H^{IF} [m]	ξ_{VM} ["]	η_{VM} ["]	N_S [m]	ξ_H ["]	η_H ["]	N_H [m]
R029	0.92	3.13	43.85	1.43	–0.21	–0.001	2.39	–1.02	0.04	4.74	1.90	43.89
R050	1.65	2.95	44.32	–1.35	0.39	–0.001	–1.65	–2.15	0.05	–1.35	1.19	44.37
R051	1.75	2.36	44.47	2.25	0.27	–0.001	–0.69	0.36	–0.01	3.31	2.98	44.46
R060	2.88	3.21	44.57	3.77	0.76	–0.010	1.81	–1.32	0.05	8.45	1.89	44.61
145	1.28	1.26	44.39	–1.24	–0.36	0.03	–1.29	1.70	–0.27	–1.25	2.60	44.15
146	1.22	1.84	44.41	–1.10	–0.56	0.05	–1.40	1.77	–0.30	–1.28	3.05	44.16

Table 7. *Results of the functionals prediction by isostatic reduction according to the Airy-Heiskanen model.*

Nº	Global (EGM96)			Local			Regional			Total		
	ξ ["]	η ["]	N [m]	ξ_I^{IF} ["]	η_I^{IF} ["]	N_I^{IF} [m]	ξ_{VM} ["]	η_{VM} ["]	N_S [m]	ξ_I ["]	η_I ["]	N_I [m]
R029	0.92	3.13	43.85	1.76	0.16	0.295	1.54	–1.47	–0.36	4.22	1.11	43.79
R050	1.65	2.95	44.32	–0.58	1.40	0.480	–1.78	–1.96	–0.47	–0.72	2.39	44.33
R051	1.75	2.36	44.47	2.10	0.12	0.417	–1.04	1.22	–0.52	2.81	3.70	44.37
R060	2.88	3.21	44.57	2.54	–0.99	0.559	2.21	–1.25	–0.48	7.64	0.98	44.65
145	1.28	1.26	44.39	–0.91	–1.61	–1.99	–1.38	2.56	–0.11	–1.02	2.20	42.31
146	1.22	1.84	44.41	–1.10	–1.76	–1.98	–1.43	2.42	–0.13	–1.31	2.51	42.29

Differences in the functionals of anomaly potential, calculated with various methods of reduction are presented at the Table 8, and all within the accuracy limit of the gravimetric method of determination.

Due to different areas of integration, uneven density of gravimetric survey and use of global geopotential model that does not include the data from our territories, there are some differences in the functionals of anomaly potential. Deflections in the values of geoid height can be a result of inadequate calculation of the regional component, in terms of size of the region with the known anomaly values.

Table 8. *Differences in the functionals of anomaly potential.*

N°	$\Delta\xi_{H-I}$ **["]**	$\Delta\eta_{H-I}$ **["]**	ΔN_{H-I} **[m]**
R029	0.52	0.80	0.10
R050	–0.63	–1.20	0.04
R051	0.50	–0.72	0.09
R060	0.81	0.91	–0.04
145	–0.24	0.40	1.86
146	0.03	0.54	1.86

Table 9 shows accuracy of certain methods of determination of vertical deflection component, within which:

$\oiint_\sigma$ – marks numerical integration, Δg – gravity anomalies (Vening-Meinesz formulas) and DTM – impact of topography.

Table 9. *Accuracy of certain methods of determination of vertical deflection component.*

The methods of determination (*index*)	$\sigma_\xi^{(index)}$ ["]			$\sigma_\eta^{(index)}$ ["]		
	$\oiint_\sigma$	$\oiint_\sigma + \Delta g$	$\oiint_\sigma + \Delta g$ +DTM	$\oiint_\sigma$	$\oiint_\sigma + \Delta g$	$\oiint_\sigma + \Delta g$ +DTM
Astro-geodetic methods (A)	≤ 0.3	/	/	≤ 0.4	/	/
Gravimetric methods (G)	1.1	/	1.0	2.2	/	1.2
Geopotential model (GM)	1.1	1.1	0.9	2.2	2.2	1.1
Astro-gravimetric methods (AG)	0.6	/	0.6	0.7	/	0.6

The maximum allowed values of deflection differences calculated by gravimetric methods with differently calculated influence of topographic masses and components determined by astro-geodetic methods are calculated with the expression (Delčev 2001):

$$R_{\max} = 2\sigma_R,$$

where:

$$\sigma_R^2 = (\sigma_{(\xi,\eta)}^G)^2 + (\sigma_{(\xi,\eta)}^{AG})^2$$

In accordance with the values in Table 5, the maximum allowed deflection per component in the meridian direction is $R_{\max}^{\xi} = 2.15''$ and in the first vertical direction $R_{\max}^{\eta} = 2.53''$.

Starting from the accuracy of certain methods of determination of vertical deflection components (Table 9) mutual compatibility was analysed, first within the same method (gravimetric), and then compared with other determination method (astro-geodetic).

Differences of vertical deflection components determined by gravimetric methods (H-Helmert condensational reduction) and those determined by astro-geodetic methods (AG-astro-geodetic methods) presented in Table 10, point to excellent compatibility related to the component ξ, with all types of reduction of topographic masses, while related to η there are deviations larger than the maximum allowed values. The values with deviations beyond the allowed limits are marked with the slash line in Table 10. As a possible explanation, the method of time registration was emphasised, i.e. the moment of the star passing through the given almucantar.

Table 10. *Differences of vertical deflection components determined by gravimetric methods (H) and those determined by astro-geodetic methods (AG).*

	$\Delta\xi_{H-AG}$ ["]	$\Delta\eta_{H-AG}$ ["]	$\Delta\xi_{I-AG}$ ["]	$\Delta\eta_{I-AG}$ ["]
R029	0.76	~~6.34~~	0.24	~~5.54~~
R050	0.11	~~5.45~~	0.75	~~6.65~~
R051	0.59	0.39	0.09	1.11
R060	0.82	~~–9.72~~	0.01	~~–10.63~~
145	–0.69	–0.05	–0.46	–0.45
146	–0.84	0.15	–0.87	–0.39

4. Conclusion

After the conducted analyses, the following can be concluded:

- With the *gravimetric survey,* the improvement of the regional component cannot be expected. It is necessary to initiate the creation of a new gravimetric survey, with appropriate *density* (about 1 point/km^2). Therefore, we must emphasise that the activities on the realisation of a new gravimetric survey of the Republic of Serbia have been initiated within the Republic Geodetic Authority.
- With the *vectorised topographic maps* at the scale of 1:25 000 (TK25) that cover the territory of the Republic of Serbia, a hybrid digital model can be created. The verification shows that in the region of Vojvodina, the 3 second model can be used, and in the regions with pronounced topography, the 2 second resolution is recommended. In order to define functionals with the remove-restore technique in the border region of Serbia as well, it is necessary to start exchanging data related to the shape of topographic masses with the neighbouring countries, especially at the distance up to 50 km out of borders.
- With the *global geopotential models,* equally good trend removal within the Republic of Serbia can be expected only after we submit our terrestric data to relevant international institutions for the creation of a new global geopotential model.

- With the *astronomic determination,* the compatibility with gravimetric determinations cannot be verified for the entire territory of the Republic of Serbia. The conduct of new and verification of the existing astro-geodetic measuring is recommended, on the points of the reference GPS network.
- It is necessary to bring in practice the *model of surface density,* so that the need for introducing hypothesis on the mass density of the Earth crust could be completely eliminated.

References

Bašić, T., Markovinović, D. (2002): Gravimetrijska mreža i kalibracijska baza Republike Hrvatske, Zbornik Geodetskog fakulteta Sveučilišta u Zagrebu, 1962–2002, 81–93.

Bilibajkić, P., Mladenović, M., Mujagić, S., Rimac, I. (1979): Tumač za gravimetrijsku kartu SFR Jugoslavije – Bouguerove anomalije 1:500 000, Savezni geološki zavod, Beograd.

Delčev, S. (2001): Postojeća državna trigonometrijska mreža SRJ u svetlu savremenih zahteva, doktorska disertacija, Građevinski fakultet Univerziteta u Beogradu, Beograd.

Forsberg, R. (1985): Gravity field terrain effect computations by FFT, Bull. Geod., No. 59, 342–360.

Forsberg, R., Sideris, M. G. (1993): Geoid computation by the multi-band spherical FFT approach, Manuscripta geodaetica, No. 18, 82–90.

Forsberg, R., Tscherning, C. C. (1997): Topographic effects in gravity modelling for BVP Geodetic Boundary Value Problems in View of the One Centimeter Geoid, Editor: Fernando Sansó, Reiner Rummel, Lecture Notes in Earth Sciences, Vol. 65, 239–272.

Gučević, J. (2006): Testing the data quality of the shape of topographic masses using different digital terrain models algorithms, Intergeo East, Belgrade 2006, Belgrade.

Haagmans, R., de Min, E., van Gelderen, M. (1993): Fast evaluation of convolution integrals on the sphere using 1D FFT, and comparison with existing methods for Stokes integral, Manuscripta geodaetica, No. 18, 227–241.

Heiskanen, W., Moritz, H. (1967): Physical Geodesy, W. H. Freeman, New York.

Hofmann-Wellenhof, B. (1994): Global Positioning System – Theoty and Practice, Springer-Verlag, Wien – New York.

Milovanović, V., Vasiljev, V. (1984): Određivanje Laplasovih tačaka 145 i 146, tehnička dokumentacija, Institut za geodeziju Građevinskog fakulteta Univerziteta u Beogradu, Beograd.

Odalović, O., Marković, D. Starčević, M. (2004): Digital terrain model of Serbia – gravity effect of topography, Intergeo East, March 3–5, Belgrade.

Ogrizović, V. (2003): Određivanje astrogeodetskih otklona vertikale na području CKN Beograd, Građevinski fakultet Univerziteta u Beogradu, Beograd.

Sideris, M. G., Tziavos, I. N. (1988): FFT-evaluation and applications of gravity-field convolution integrals with mean and point data, Bull. Geod., No. 62, 521–540.

Tscherning, C. C. (1994): Geoid determination by least-squares collocation using GRAVSOFT, International, School for the Determination and Use of the Geoid, International Geoid Service, DIIAR, Milano, Italy.

Tsoulis, D. (1999): Analytical and Numerical Methods in Gravity Field Modelling of Ideal and Real Masses, Deutsche Geodätische Kommission, Munchen.

Vanicek, P., Krakiwisky, E. (1980): Geodesy: The concept, Nort-Holland Publishing company, Amsterdam – New York – Oxford, The Netherlands.

Vasović, J. (2001): Primena brzih Furijeovih transformacija za određivanje lokalnog geoida visoke rezolucije, magistarski rad, Građevinski fakultet Univerziteta u Beogradu, Beograd.

URL 1: International Geoid Service,
http://ipmtf14.topo.polimi.it/~iges/db/model.html, (13. 7. 2010.).

URL 2: KIT – Karlsruher Institut für Technologie – Geodätisches Institut,
http://www.gik.uni-karlsruhe.de/~wenzel/geopmods.htm, (12. 7. 2010.).

URL 3: USGS,
http://edcdaac.usgs.gov/gtopo30/gtopo30.html, (14. 7. 2010.).

Optimalna rezolucija i veličina DTM-a za potrebe modeliranja utjecaja topografije

SAŽETAK. Modeliranje utjecaja topografskih masa, koje nose lokalne promjene gravitacijskog polja, od izuzetne je važnosti u suvremenoj znanstvenoj i praktičnoj primjeni. U ovome radu utjecaj topografije analizira se digitalnim modelom terena Republike Srbije koji je razvijen na osnovi topografske karte mjerila 1:25 000 te uz pretpostavku jednake gustoće površinskih slojeva Zemljine kore. Kratkovalna karakteristika, kao utjecaj lokalne topografije, analizirana je u dijelovima s različitom topografijom za područje Beograda i Paraćina. Parametri gravitacijskog polja određeni su u točkama referentne GPS-mreže Republike Srbije te u točkama baze u Paraćinu, u svrhu usporedbe gravimetrijskih otklona vertikale s astro-geodetskim otklonima vertikale.

Ključne riječi: *utjecaj topografije, korekcija terena, redukcija ubrzanja sile teže, anomalija ubrzanja sile teže, otklon vertikale.*

Prihvaćeno: 2010-08-11