Essays on modelling the economics of energy and the climate

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Contents

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Introduction

This thesis builds on key recent developments in modelling climate change and energy through four papers (chapters). Many recent papers have discussed the importance of uncertainty in modelling the economics of climate change. The first paper thesis considers how a risk of tipping points, where there is abrupt and irreversible damage, impacts optimal tax policy for both carbon dioxide and methane.

The ongoing transition to clean energy involves a shift of factors of production and researchers into the clean energy sector. Governments have a role in enabling and incentivising such a shift through various policy options including carbon taxes and research subsidies. The second paper discusses the relative performance of taxes and subsidies when only one of these instruments is available and how different modelling assumptions affect results.

The substitutability between clean and dirty energy is an important factor in determining: the cost of a clean transition; the type, timing and extent of optimal policy; and the additional costs from suboptimal policy. The third paper examines the elasticity of substitution between renewable inputs in electricity and dirty inputs empirically and in a simple theoretical dispatch model, and discusses policy implications of decreasing substitutability as integrating intermittent inputs becomes more difficult as their share rises.

The fourth paper is a shorter comment on multiple equilibria and the innovation framework of a prominent paper in this field (Acemoglu, Aghion, Bursztyn, & Hemous, 2012).

I. Chapter summaries

Using different formulations of climate tipping points that trigger abrupt and irreversible damages, the first paper derives optimal environmental taxes in an analytically tractable model and depend on only a few parameters and a temperature projection. In a stylised approach, optimal taxes are constant as a ratio of income and are the sum of a deterministic damage component and a tipping risk component. If a tipping point may be triggered by temperature crossing a threshold, optimal tax-to-income ratios eventually fall and the price for short-lived methane emissions relative to long-lived carbon dioxide emissions should rise over time.

The second paper considers a hypothetical choice between a carbon tax and a clean research subsidy. This paper argues that the absence of a non-energy sector has led some previous literature to find that subsidies outperform taxes. An integrated assessment model with endogenous technology is described. Numerical exercises find that a permanent global tax-only policy outperforms a permanent subsidy-only policy and this result is robust to many different parameter settings and assumptions. However, in the more optimistic case where optimal policy begins in 2050, the performances of subsidy-only and tax-only policies in the interim are closer.

The third paper argues that a clean transition in electricity generation will likely be driven by variable renewable energy. The elasticity of substitution between wind and solar inputs and dirty inputs in electricity is estimated to be 3 or more by fitting an aggregate production function to OECD panel data. A high elasticity is consistent with detailed electricity models which also predict that the substitutability decreases as the share of clean inputs rises, as integrating intermittent energy supply becomes increasingly difficult. A simple dispatch model of electricity generation demonstrates this characteristic. Decreasing substitutability implies higher costs of a clean transition, greater costs from regions transitioning sequentially rather than together, and a greater role for carbon taxes over research subsidies.

The fourth paper discusses how the framework used to endogenise technology growth by Acemoglu et al. (2012) can exhibit increasing returns to research and hence multiple equilibria, including an unstable interior equilibrium. The paper discusses several methods to determine how a unique equilibrium might be specified. Alternative methods can produce substantially different results when the elasticity of substitution between clean and dirty inputs is high.

1. Optimal environmental taxes for a tipping climate

By ANTHONY WISKICH*

Using different formulations of climate tipping points that trigger abrupt and irreversible damages, optimal environmental taxes are derived in an analytically tractable model and depend on only a few parameters and a temperature projection. In a stylised approach, optimal taxes are constant as a ratio of income and are the sum of a deterministic damage component and a tipping risk component. If a tipping point may be triggered by temperature crossing a threshold, optimal tax-to-income ratios eventually fall and the price for short-lived methane emissions relative to long-lived carbon dioxide emissions should rise over time. (JEL H23, O44, Q40, Q54, Q56, Q58)

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Potential effects from climate change include the risks of abrupt and irreversible events, referred to as tipping points. These events have been shown to have a material impact on optimal policy, and recent reviews have discussed the merits of using models that incorporate such uncertainty (Farmer, Hepburn, Mealy, &

Teytelboym, 2015; Lemoine & Rudik, 2017; Pindyck, 2013a, 2013b). A growing literature has examined tipping points in Integrated Assessment Models (IAMs) using various frameworks of uncertainty and tipping impacts.

This paper adds to this literature by describing optimal environmental taxes under different tipping formulations using an analytically tractable economic model. Tax equations are derived which allow optimal tax paths to be calculated given a choice of formulation, a few parameters and a temperature projection. These parameters involve assumptions about the discount rate, damages from tipping events and how much the probability of tipping rises with each degree of warming. The equations are used to derive optimal taxes for both carbon dioxide and methane, first assuming exogenous temperature projections for illustration and second in a model with endogenous temperature. Two main approaches used in the literature to define the stochastic nature of tipping points are considered and discussed in depth: the threshold approach leads to a fall in the tax-to-income ratio in the long run (assuming temperature eventually starts falling) and a rising price for methane relative to carbon dioxide; and in the non-threshold approach optimal prices simply grow with output. Initial taxes in both frameworks are found to be broadly invariant to temperature projections and the welfare losses from using the wrong framework are discussed.

Many papers have considered the risks of environmental catastrophe, going back to Cropper (1976) who assumes an unknown threshold of pollution which triggers a tipping point. Clarke and Reed (1994) consider a different method where the probability of tipping in a period, referred to as the hazard rate, is a function of pollution. The first formulation implies there is no risk of tipping occurring once temperature stabilises or falls in the long run. The second implies that tipping is certain to occur in the long run for any stabilisation of temperature with a non-zero tipping probability. As tipping risks likely lie somewhere in between these extremes, both methods are considered in this paper and they are referred to as the threshold and non-threshold formulations.[1]

An assessment of whether tipping points discussed in the literature map better to a threshold or non-threshold formulation is out of the scope of this paper, but both have been used extensively in the literature (see Table 1). A threshold formulation is akin to a phase transition in physics, such as a transition from liquid to gas, which occurs at a particular temperature (and pressure). Discussion of the likelihood of tipping events in the literature seems to map to threshold formulations: the collapse of Atlantic thermohaline circulation "probably requires more than 4^{O}C warming"; the disappearance of the Greenland ice sheet "may occur at 0.8^{O}C – 3.2^{O}C (with best estimate 1.6^{O}C)"; and collapse of the West Antarctic ice sheet "may be triggered at >4^{O}C warming" (Lenton, 2013). However, referencing the triggering of events using warming levels is more convenient than doing so in a manner that maps to a non-threshold formulation. Further, when levels of warming are referenced, experts presumably have in mind a projection path such as the ranges Kriegler, Hall, Held, Dawson, and Schellnhuber (2009) provide to elicit views on the likelihood of a tipping event occurring. Lenton (2013) speculates that "Snowball Earth" glaciations that have occurred in the past were due to the combination of steady cooling and a stochastic cooling event such as a volcanic eruption. Such dependence on a stochastic event may map better to a non-threshold approach, likely with non-linear temperature dependence. Ultimately, we may never know the stochastic nature of tipping events that may be triggered by climate change, or at least not until after an event occurs. However, policymakers should be aware that the threshold and non-threshold formulations (and other sensitivities) can lead to different optimal policy paths as described in this paper.

[1] Crépin and Nævdal (2020) discuss an approach which would account for delays between temperature and the hazard rate called inertia risk not considered in this paper.

Optimal climate policy involves pricing the emissions of greenhouse gases and, potentially, actions that can reduce global warming such as solar geoengineering. The timing of effects of these actions differs: while carbon dioxide is long-lived, methane decays relatively quickly. Under the current Intergovernmental Panel on Climate Change (IPCC) policy, the weights (prices relative to carbon dioxide) of greenhouse gases are constant based on a 100-year Global Warming Potential (GWP), independent of temperature outcomes. Such flat weights may be optimal, or close to optimal, in Integrated Assessment Models with a smooth damage function:[2] this paper examines the optimal weights of the short-lived gas methane under a tipping risk.

Section I describes a stylised framework that leads to a simple formula for the optimal carbon tax: tipping is assumed to lead to a fixed proportional damage to output,[3] and future temperatures are restricted.[4] This framework can be thought of as a variation to the model described in Golosov et al. (2014), who find a constant optimal tax-to-income ratio independent of economic growth and climate outcomes. This result occurs because assumptions imply a constant savings rate, so consumption is proportional to output, and damages are exponential-linear so that emissions lead to a linear reduction in log output and thus welfare. Golosov et al. (2014) assume that expected damage combines a <u>fixed probability</u> of severe/moderate damages which are <u>linear in temperature</u>. The tipping framework that I consider reverses these assumptions: the probability of tipping is <u>linear in temperature</u> and damage is a <u>fixed proportion</u> of output. Thus the risk of tipping

[2] For example, the model described by Golosov, Hassler, Krusell, and Tsyvinski (2014) implies constant optimal tax-to-income ratios independent of temperature outcomes, so the weights of short-lived gases would also be constant.

[3] A collapse of major ice sheets leading to severe sea-level rise is an example of a shock that would have long-term and direct economic impacts.

[4] An increasing temperature in the threshold formulation, and the less restrictive constraint in the non-threshold formulation that temperature does not fall far enough to remove the risk of a tipping event.

raises the tax, as found in previous theoretical and numerical papers,[5] by adding a constant component to the optimal tax-to-income ratio (my first proposition). A deterministic damage component using the exponential-linear framework is also included for realism, whilst making identification of the component of the tax due to tipping risks straight forward.

Removing the temperature restrictions means the optimal tax-to-income ratio will be lower in the long run (my second proposition), and dynamics are sensitive to the formulation adopted. Optimal taxes in the non-threshold formulation (hereafter non-threshold taxes) depend on the time when temperature falls sufficiently to remove the risk of tipping: if the risk of tipping persists for centuries the optimal tax-to-income ratios can be considered constant. However, for the threshold formulation, the risk of tipping may be a more temporary phenomenon depending on when peak temperature (the maximum level of projected warming) occurs. The threshold tax rises before peak temperature and following peak temperature the tax drops to the component corresponding to deterministic damages, as there is no risk of tipping. The sensitivity to peak temperature leads to rising weights of short-lived actions like methane.

In section II, the general results are illustrated using numerical examples where temperature outcomes are exogenously set by two IPCC scenarios. This approach is simple while still allowing insights into the dynamics of taxes. While non-threshold taxes are independent of temperature projections, the initial threshold tax is higher the sooner peak temperature occurs, but is independent of the temperature rise. Thus, higher temperature projections where peak temperature occurs in the distant future can lead to a lower initial optimal tax. For a temperature profile consistent with IPCC temperature targets of 1.5^{O}C and 2^{O}C, peak temperature

[5] For example: Lontzek, Cai, Judd, and Lenton (2015); Van der Ploeg (2014); Lemoine and Traeger (2014); and Lemoine and Traeger (2016b).

likely occurs this century and the optimal weights of short-lived actions should be higher today and rise over coming decades.

In section III a model with endogenous temperature is used to examine optimal temperature paths and the timing and interaction of methane abatement. As well as showing results in the tipping formulations, results for the commonly applied framework which imposes an upper limit on warming, referred to as the cost-minimisation formulation, is shown as a comparison for policy-makers.[6] Under cost-minimisation, including methane abatement naturally leads to a lower optimal carbon dioxide tax (and no change in peak temperature by construction), but in the tipping formulations there is little or no effect on the initial carbon dioxide price and peak temperature is lowered. Much of the literature investigating different gases suggests a low optimal weight of methane today based on cost-minimisation approaches, but results in this paper do not support this policy. In a simulation where tipping events have equal probabilities of having a threshold and non-threshold nature, the key qualitative threshold results of lower long-run optimal tax ratios and an increasing methane weight before peak temperature persist. The welfare implications of various suboptimal policies are discussed, including the costs of maintaining a fixed methane weight (as under current policy) as discussed in IPCC (2014).

Section IV discusses various sensitivities. First, I investigate increasing the degree of risk aversion implied by a logarithmic utility, adding to recent literature that applies Epstein-Zin utility to consider climate impacts and policy.[7] I derive an approximate analytical solution to the optimal price and find that a risk aversion coefficient consistent with the literature leads to a small uplift in the tipping tax component. A power utility function with a higher coefficient of relative risk

[6] Most previous papers only show results from one framework, with Goulder and Mathai (2000) an exception.

[7] References include Bretschger and Vinogradova (2018), Cai and Lontzek (2019), Olijslagers and van Wijnbergen (2019) and Traeger (2018).

aversion makes predictable changes to results in line with an increase in the discount rate.

Second, a tipping event could lead to a change in the climate response rather than fixed damages, such as reduced absorption of carbon into the oceans discussed by Lenton et al. (2008) and considered by Lemoine and Traeger (2014). Increased sensitivity to temperature can act as a proxy for a change in the climate response: an exponential damages case examines the implications of both the probability of tipping and impacts being linear in temperature. Deriving optimal taxes is complicated by dependence on expected temperature levels, and a tipping event now changes the level of the tax-to-income ratio. For rising temperatures, the non-threshold tax-to-income ratio grows while the threshold ratio may grow or shrink depending on the concavity of temperature outcomes. Solving for the tax under the possibility of multiple tipping points requires considering multiple future tipping eventualities when temperature is endogenous. However, by assuming that tipping can only occur once, as is often done in the literature, the tax can be derived using only temperature outcomes assuming tipping does not occur *ex-post*. This sensitivity has a differential welfare impact component to the optimal tax (Lemoine & Traeger, 2014), relating to the difference in the marginal welfare effect of the tax before and after tipping events, leading to lower weights of methane than under fixed damages.

Third, while a few other papers consider the possibility of multiple tipping events like this paper,[8] most studies consider the effect of a single tipping event. Unsurprisingly the tax is lower if only one tipping event can occur. Interestingly, in this case higher projected temperatures lead to a lower carbon tax today. Consider that a tipping event occurs at some future point, after which there is no further risk of tipping and hence no corresponding benefits from abatement today

[8] See Bretschger and Vinogradova (2018), Lemoine and Traeger (2016b) and Tsur and Zemel (1998).

from the marginal hazard effect. As higher temperature outcomes increase the risk of tipping, the benefits of abatement today are lower from this “inevitability” effect and there is positive feedback between higher temperature outcomes and a lower carbon tax.

Fourth, as tipping events may take time to become apparent I also investigate the effects of delayed impacts and learning on the optimal carbon price. A delayed tipping impact lowers today's tax due to discounting, but a delay in learning can boost future taxes. For example, in the threshold formulation with exponential damages, the tipping component of the tax persists beyond peak temperature until the risk that tipping has already occurred has gone.

With logarithmic utility, tax-to-income ratios are not constant when the expected damage from tipping is not exponential-linear in temperature. A deterministic model with exponential-quadratic damages or some other exponential-nonlinear function would also lead to a non-constant tax-to-income ratio: Van der Ploeg, 2014 discusses such sensitivity to the functional form of damages.[9] The result that the weight of methane rises in the threshold approach is intuitive and would also apply with convex deterministic damages – if more damage is done at peak temperature and the temperature effect of methane at the peak is greater just before the peak, due to its short-lived nature, the weight will naturally be higher. However, quantitatively the rise will likely be much less in such a deterministic approach as changes in damages are smooth in contrast to the step-change in the marginal hazard effect in the threshold formulation.

This paper adds to two main streams of literature: studies that consider optimal policy for carbon dioxide in IAMs; and studies that consider optimal policy across a range of greenhouse gases and actions such as geoengineering, sometimes using

[9] The threshold formulation is comparable to a deterministic framework where damages depend on the rate of change of temperature.

a cost-minimisation framework. An example of the former which is close to this paper is Engström and Gars (2016), who use a similar framework to consider different types of tipping impacts with a threshold formulation but differ from this paper in at least three respects. First, the current paper considers both threshold and non-threshold formulations and considers methane as well as carbon dioxide. Second, this paper allows temperature to fall which has a large effect on the optimal tax in some instances, while Engström and Gars (2016) use a model where temperature cannot decrease and tipping is sure to occur in the long run. Third, I focus on optimal prices rather than rates of extraction and the green paradox which is the focus of their paper.[10] A list of studies that consider tipping points is shown in Table 1, according to the threshold versus non-threshold formulations and fixed versus temperature-dependent damages.

Differences between optimal policy for methane and carbon dioxide depend entirely on their different temporal effects on temperature. Nævdal (2006) considers the optimal regulation (not prices) of methane and carbon dioxide under a threshold tipping risk and finds a temporary boost in the ratio of methane to carbon dioxide stock above the steady-state, consistent with an increasing optimal methane weight in a decentralised model. Marten and Newbold (2012) find that the social cost of methane relative to carbon rises by up to 50% by 2050 in a deterministic model, due in part to their climate model where the marginal forcing of methane decreases slower than carbon with the increasing atmospheric stock. Other deterministic studies that consider methane include Waldhoff, Anthoff, Rose, and Tol (2011), Hope (2005) and Tol (1999). Finally, other relevant actions considered in the literature with different temporal characteristics include geoengineering (Goes, Tuana, and Keller (2011), Heutel, Moreno-Cruz, and

[10] Scarcity rents have been marginal or non-existent historically (Hart & Spiro, 2011), and scarcity constraints on fossil energy extraction are unlikely to bind under optimal policy.

Shayegh (2018) and Bickel and Agrawal (2013)) and leakage rates and risks from carbon capture and sequestration (van der Zwaan & Gerlagh, 2009).

While this paper considers the risks of tipping in a stochastic framework, the impact of tipping and the formulation of the hazard rate are themselves uncertain. Numerical exercises that incorporate Bayesian learning about climate sensitivity reach different conclusions about the effect of learning on the optimal carbon price. While Gerlagh and Liski (2018) find that the effect of learning on the carbon price is not significant over the next century and Leach (2007) finds that learning may take thousands of years, Hwang, Reynès, and Tol (2017) and Kelly and Tan (2015) find a material impact this century. In the current paper, the agent learns about the location of tipping points in the threshold formulation.[11]

TABLE 1: APPROACHES ADOPTED BY PREVIOUS LITERATURE

Hazard rate formulation	Impacts of tipping cases	
	Fixed impact	Temperature-dependent or climate response impacts
Threshold	(Cropper, 1976; Engström & Gars, 2016; Tsur & Zemel, 1998)	(Engström & Gars, 2016; Keller, Bolker, & Bradford, 2004; Lemoine & Traeger, 2014; Lemoine & Traeger, 2016b)
Non-threshold	(Bretschger & Vinogradova, 2018; Cai, Judd, Lenton, Lontzek, & Narita, 2015; Cai & Lontzek, 2019; Clarke & Reed, 1994; Gerlagh & Liski, 2018; Lontzek et al., 2015; Polasky, De Zeeuw, & Wagener, 2011; Ren & Polasky, 2014; Tsur & Zemel, 1998; van der Ploeg & de Zeeuw, 2017, 2019)	(Polasky et al., 2011; Van der Ploeg, 2014; van der Ploeg & de Zeeuw, 2019)

I. Core model and optimal environmental taxes

The core model uses 5 key assumptions found in Golosov et al. (2014) that lead to analytical tractability: logarithmic utility (i); full one-period depreciation of capital (ii); temperature is a linear function of historical actions (iii); an exponential

[11] Learning that a tipping event has occurred has no effect on the optimal tax-to-income ratio under fixed damages and further possible tipping events, but does change the tax for exponential damages or single tipping event cases.

impact of temperature on output (iv); [12] and Cobb-Douglas production (v). A global representative household maximises the following in discrete time, for consumption C_t and discount rate β:

$$(1) \qquad \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ where } U(C_t) := \log(C_t)\,.$$

Temperature is a linear function of historical non-interacting actions a in period t, E_{at}, such as the use of energy releasing carbon dioxide emission E_{ct},

$$(2) \qquad T_t = \sum_{a} \sum_{i=-\infty}^{\mathrm{t}} T_{t-i}^{a} E_{ai} \text{ where } T_{t-i}^{a} := \frac{\partial T_t}{\partial E_{ai}}.$$

Deterministic damages are set by the parameter $\gamma > 0$ and stochastic damages from tipping by the function f_t which I specify later. A multiplicative exponential damage function of atmospheric temperature T_t above pre-industrial applies, and output is as follows:

$$(3) \qquad Y_t = e^{-(\gamma T_t + f_t)} K_t^{\kappa} F(\boldsymbol{E}_t) \text{ with parameter } 0 < \kappa < 1.$$

Note that no restriction is placed on the function of energy F, with $\boldsymbol{E}_t$ a vector of actions E_{at}. The sensitivity of optimal environmental taxes to these assumptions has been discussed by L Barrage (2014) and Rezai and Van der Ploeg (2015). Assumption (iii) can replicate the more complex climate-economy models well, although tipping impacts on climate feedback, such as a lower rate of decomposition of carbon dioxide, require a more complex framework. For the level

[12] Strictly, Golosov et al. (2014) assume atmospheric carbon concentrations are a linear function of historical emissions, and an exponential impact of carbon concentration on output.

of damages considered in this paper, assumption (iv) leads to an approximately linear relationship between global damages and temperature, consistent with Burke, Hsiang, and Miguel (2015).[13] The effect of higher risk aversion and non-logarithmic utility is considered in section IV.

Optimal taxes

The social cost of carbon is equal to the optimal carbon tax and is derived using a Lagrangian method. The same derivation using Bellman equations for a specific framework is shown in Appendix A. The Lagrangian maximizes (1) subject to production and temperature constraints as follows:

$$(4) \qquad \mathcal{L}(C_t, K_t, \boldsymbol{E}_t, T_t) = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log C_t \right.$$
$$\left. + \sum_{t=0}^{\infty} \lambda_{Yt} \left(e^{-(\gamma T_t + f_t)} K_t^{\kappa} F(\boldsymbol{E}_t) - C_t - K_{t+1} \right) + \sum_{t=0}^{\infty} \lambda_{Tt} \left(T_t - \sum_a \sum_{i=-\infty}^{\mathrm{t}} T_{t-i}^a E_{ai} \right) \right\}.$$

First-order conditions for C, T, K and E_a are

$$(5) \qquad \frac{\beta^t}{C_t} = \lambda_{\mathrm{Y}t}, \qquad \lambda_{Tt} = \lambda_{\mathrm{Y}t} \gamma Y_t + \mathbb{E}_{\mathrm{t}} \left(\sum_{i=0}^{\infty} \lambda_{\mathrm{Y}t+i} \frac{\partial f_{t+i}}{\partial T_t} Y_{t+i} \right),$$

$$\mathbb{E}_{\mathrm{t}} \left(\lambda_{\mathrm{Y}t+1} \kappa \frac{Y_{t+1}}{K_{t+1}} \right) = \lambda_{\mathrm{Y}t} \text{ and } \lambda_{\mathrm{Y}t} e^{-(\gamma T_t + f_t)} K_t^{\kappa} F'(\boldsymbol{E}_t) = \mathbb{E}_{\mathrm{t}} \left(\sum_{i=0}^{\infty} \lambda_{Tt+i} T_i^a \right).$$

[13] For damages up to around 10% of output, an exponential function is approximately linear. Burke, Hsiang, & Miguel, 2015 find non-linear local responses to temperature but approximately linear losses at a global level.

A constant rate of savings is implied by the conditions for C and K. The multiplier for temperature equals marginal deterministic damages and the expected damages from tipping risks. The marginal gain from energy E_a equals the future damages from temperature effects. The social cost of action a in units of the final good Λ_t^a equals the sum of the future effects on temperature T_t^a multiplied by the temperature multiplier:

$$
\begin{aligned}
(6)\quad \Lambda_t^a &= \frac{1}{\lambda_{\mathrm{Y}t}} \mathbb{E}_{\mathrm{t}} \left(\sum_{i=0}^{\infty} \lambda_{Tt+i} \mathrm{T}_i^a \right) \text{ and from (5)} \\
&= \frac{\mathrm{C}_t}{\beta^t} \mathbb{E}_{\mathrm{t}} \left(\sum_{i=0}^{\infty} \beta^{t+i} \mathrm{T}_i^a \left(\frac{\mathrm{Y}_{t+i}}{\mathrm{C}_{t+i}} \gamma + \mathbb{E}_{\mathrm{t}} \left(\sum_{j=0}^{\infty} \beta^j \frac{\mathrm{Y}_{t+i+j}}{\mathrm{C}_{t+i+j}} \frac{\partial f_{t+i+j}}{\partial T_{t+i}} \right) \right) \right).
\end{aligned}
$$

Lemma 1: Given assumptions (i) to (v), the optimal tax-to-income ratio is given by

$$
(7)\qquad \widehat{\Lambda}_t^a := \frac{\Lambda_t^a}{\mathrm{Y}_t} = \widehat{\Lambda}_{det}^a + \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \mathbb{E}_{\mathrm{t}} \left(\sum_{j=0}^{\infty} \beta^j \frac{\partial f_{t+i+j}}{\partial T_{t+i}} \right)
$$

$$
\text{where } \widehat{\Lambda}_{det}^a = \beta \gamma \Gamma^a, \quad \Gamma^a := \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a.
$$

Equation (7) breaks down the optimal tax-to-income ratio into a component due to deterministic damages ($\widehat{\Lambda}_{det}^a$) and a component due to the risk of tipping.

Hazard rate

Tipping occurs in each period with probability p_t, referred to as the hazard rate. An impact variable I_t is zero if tipping does not occur in period *t*, and δ if tipping occurs. I assume that multiple tipping events are possible, implying that the hazard rate is independent of whether events have already occurred, but no more than one event in a period which makes things easier in a discrete-time framework.[14] The function f_t is a function of temperature and previous impacts with parameter μ as follows:

$$(8) \quad f_t = g(T_t) \sum_{i=0}^{t-1} I_i \text{ and } p_t = \begin{cases} \mu(T_t - \bar{T}_t) & \text{if } T_t \geq \bar{T}_t \\ 0 & \text{otherwise.} \end{cases}$$

This paper assumes fixed damages ($g_{FD}(T_t) = 1$) in this section and a sensitivity with exponential damages ($g_{CS}(T_t) = T_t$) is discussed in section IV. Two methods of defining $\bar{T}_t$ are considered which are generally used in the literature: the values $(\bar{T}_t, \mu)$ are $\left(\max_{k<t}(T_k), \mu_T\right)$ for the threshold formulation and (T_{min}, μ_N) for the non-threshold formulation, with T_{min} parameterising the safe temperature below which there is no risk of tipping. Note the discontinuity in the temperature-derivative of the hazard rate at $T_t = \bar{T}_t$. The derivative in (7) that determines the component of the tax due to tipping is

$$(9) \qquad \mathbb{E}_t\left(\frac{\partial f_{t+i+j}}{\partial T_{t+i}}\right) = \delta \mathbb{E}_t \begin{cases} \dfrac{\partial g(T_{t+i})}{\partial T_{t+i}} \displaystyle\sum_{k=0}^{i} I_{t+k} & \text{if } j = 0 \\ g(T_{t+i+j}) \displaystyle\sum_{k=0}^{j} \dfrac{\partial I_{t+i+k}}{\partial T_{t+i}} & \text{if } j > 0. \end{cases}$$

[14] For the threshold approach, one can consider a prior probability function that is flat with temperature (with the caveat that not more than one event can occur each period). Multiple potential tipping points means that the expected number of tipping events increases without bound as temperature rises, and there is no updating the probability function if a tipping event occurs.

The top term is referred to as the differential welfare impact in Lemoine and Traeger (2014) and is proportional to the difference in the marginal welfare effect of the tax before and after tipping events. The bottom term is the marginal hazard effect and captures the marginal reduction in the risk of tipping by the tax.

Fixed damages from tipping

In the core model, tipping induces a fixed proportional impact on output. The first proposition considers a constraint on temperature outcomes such that $T_t > \bar{T}_t$ for all t, implying increasing temperatures for the threshold formulation and future temperatures remaining above T_{min} for the non-threshold formulation.

Proposition 1: Given assumptions (i) to (v), $T > \bar{T}$ and fixed damages from tipping with multiple possible tipping events, the optimal tax-to-income ratio consists of a constant deterministic component and a constant tipping risk component given by

$$(10)\quad \widehat{\Lambda}^a_{T>\bar{T},t} = \widehat{\Lambda}^a_{det} + \widehat{\Lambda}^a_{T>\bar{T}} \text{ where } \widehat{\Lambda}^a_{T>\bar{T}} = \begin{cases} \beta\delta\mu\Gamma^a & \text{for threshold} \\ \dfrac{\beta\delta\mu\Gamma^a}{1-\beta} & \text{for non}-\text{threshold}. \end{cases}$$

Proof: With fixed damages, the differential welfare impact vanishes as $g_{FD}(T_t) = 1$ and the tipping tax-to-income component is

$$(11)\qquad \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \mathbb{E}_\mathrm{t}\left(\sum_{j=0}^{\infty} \beta^j \frac{\partial f_{t+i+j}}{\partial T_{t+i}}\right) = \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \mathbb{E}_\mathrm{t}\left(\sum_{j=0}^{\infty} \beta^j \sum_{k=0}^{\mathrm{j}} \frac{\partial I_{t+i+k}}{\partial T_{t+i}}\right)$$

$$= \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \mathbb{E}_\mathrm{t} \begin{cases} \dfrac{\partial I_{t+i}}{\partial T_{t+i}} + \dfrac{\beta}{1-\beta}\left(\dfrac{\partial I_{t+i}}{\partial T_{t+i}} + \dfrac{\partial I_{t+i+1}}{\partial T_{t+i}}\right) \text{ for threshold as } \mathbb{E}_\mathrm{t}\left(\dfrac{\partial I_{t+i+k}}{\partial T_{t+i}}\right) = 0 \; if \; k > 1 \\ \displaystyle\sum_{j=0}^{\infty} \beta^j \dfrac{\partial I_{t+i+j}}{\partial T_{t+i}} \quad \text{for non} - \text{threshold as } \mathbb{E}_\mathrm{t}\left(\dfrac{\partial I_{t+i+k}}{\partial T_{t+i}}\right) = 0 \; if \; k > 0 \end{cases}$$

$$= \delta \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \begin{cases} \mu_T + \dfrac{\beta}{1-\beta}(\mu_T - \mu_T) \text{ for threshold} \\ \displaystyle\sum_{j=0}^{\infty} \beta^j \mu_N \qquad\qquad \text{for non} - \text{threshold} \end{cases}$$

$$= \delta \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \begin{cases} \mu_T \qquad\quad \text{for threshold} \\ \displaystyle\sum_{j=0}^{\infty} \beta^j \mu_N \quad \text{for non} - \text{threshold} \end{cases}$$

Consider a marginal increase in temperature T_{t+i} which increases the chance of tipping in period $t+i$ by μdT_{t+i}. For the non-threshold formulation there is no effect on the chance of tipping in future periods so $\frac{\partial f_{t+i+j}}{\partial T_{t+i}} = \mu_N$ for all j and the infinite sum leads to the denominator $1-\beta$. However, for the threshold formulation, the chance of tipping in period $t+i+1$ is reduced by $\mu_T dT_{t+i}$, so $\frac{\partial f_{t+i+j}}{\partial T_{t+i}} = 0$ for $j > 0$ explaining the absence of the denominator $1-\beta$. Note that proposition 1 is unaffected by a tipping event occurring in the past. The discount parameter β appears in the numerator as there is a lag of one period between a tipping event and damages. A greater delay of impacts (and learning) can be easily incorporated in this framework and is discussed in section IV.

Now let us consider models that allow temperature to fall. In the very long run, climate models indicate that warming would decline in the absence of man-made emissions. Further, actions such as geoengineering can be used to reduce temperature. Consider the weak assumption that until period τ, $T_t > \bar{T}_t$, and from then on $T_t < \bar{T}_t$. For the threshold formulation, this implies that temperature falls in the future and never rises back above the peak. For the non-threshold formulation, it implies that temperature falls below T_{min} at some point and from then on remains below T_{min}. The optimal tax can be written as

$$(12) \qquad \widehat{\Lambda}_t^a = \widehat{\Lambda}_{det}^a + \beta\delta \begin{cases} \mu_T \left(\sum_{i=1}^{\tau-t-1} \beta^i \mathrm{T}_i^a + \frac{\beta^{\tau-t} \mathrm{T}_{\tau-t}^a}{1-\beta} \right) & \text{for threshold} \\ \mu_N \sum_{i=1}^{\tau-t-1} \beta^i \mathrm{T}_i^a & \text{for non} - \text{threshold} \end{cases} .$$

Proposition 2: Assume $T > \bar{T}$ until period τ and $T < \bar{T}$ thereafter and fixed damages from tipping. The non-threshold tax-to-income ratio is close to flat but will decrease slightly as the risk of tipping eventually disappears. The threshold tax-to-income ratio will rise, provided the temperature effect is falling at peak temperature, and then fall to the deterministic level following the peak.

The proof for non-threshold is straightforward: as time goes on, the number of summands in (12) falls and thus the tax-to-income ratio technically falls but can be considered flat for large τ. For threshold, the change in the tax-to-income ratio implied by (12) is $\Delta\widehat{\Lambda}_{\tau-t-1}^a = \frac{-\beta^{\tau-t}\Delta\mathrm{T}_{\tau-t-1}^a}{1-\beta}$, hence the tax ratio grows provided the temperature effect at peak temperature of action a tomorrow is more than today. Assumed temperature responses are shown in Figure 8 – the temperature response peaks after 10 years for methane and 20 years for carbon dioxide, so threshold tax-to-income ratios will grow provided peak temperature is more than 20 years in the future.

Under both formulations, the proposition leads to an increasing weight of a short-lived action as the sums in (12) are finite and a short-lived action is being compared to long-lived carbon dioxide. The increase in the weight of methane depends on the extent of tipping impacts relative to deterministic damages from warming.

Corollary 1: Assuming $T > \bar{T}$ until period τ and $T < \bar{T}$ thereafter and fixed damages from tipping, the optimal weight of a short-lived action will rise over time until period τ, then fall to the flat deterministic level thereafter.

II. An illustration with exogenous temperature

Most of the insights in this paper are clearly and simply shown using exogenous temperature outcomes. Optimal prices for short-lived methane are considered alongside carbon dioxide given IPCC temperature projections detailed in Stocker et al. (2013) and extrapolated to 2400. Deterministic damages are calibrated to correspond with a loss of 0.48% of GDP from 2.5^{O}C warming (Nordhaus, 2008), leading to a deterministic component of the carbon dioxide tax-to-income ratio of 0.00002. For a global decadal GDP in 2020 of US$1000 trillion, the deterministic tax component is $5.4 per tonne carbon dioxide in 2020 (US$20 per tonne carbon). Fixed damages from a tipping event are 10% of output in addition to deterministic damages, the annual discount rate is 1.5% as used in the DICE 2016R2 model and T_{min} is set to 1 degree as used by Cai and Lontzek (2019). Hazard rate parameters μ_N and μ_T are calibrated so that they both lead to around a 5% chance of tipping over the next 100 years, presuming temperature rises linearly by about 1^{O}C. Values of around $\mu_N = 0.01$ and $\mu_T = 0.05$ deliver this outcome: the chance of tipping each decade rises in the non-threshold formulation and averages $0.5\mu_N$ per decade, while there is a constant 0.5% chance of tipping each decade in the threshold formulation. Lontzek et al. (2015) assume a larger value for μ_N of 0.025.

Panel A in Figure 1 shows temperature projections for each IPCC scenario. Panel D shows the optimal carbon dioxide tax for the RCP2.6 projection. The non-threshold tax increases with output as proposition 1 describes, while the threshold tax drops after peak temperature following proposition 2. Panel E shows the same results as panel D as a ratio of income: this approach is used hereon as it makes identification of the tipping component of the tax easier. Panel F shows tax-to-income results for the RCP4.5 projection: the non-threshold tax ratio is flat and the threshold tax ratio begins slightly lower but persists for longer as peak temperature occurs much later. Panels B and C show the weight of methane relative to carbon dioxide: non-threshold weights are constant while threshold weights rise before peak temperature. Optimal carbon dioxide/methane taxes in 2020 are $29/$50 for threshold and $33/$40 for non-threshold per tonne carbon dioxide equivalent in 2020.[15]

The main messages from this illustration are: non-threshold tax-to-income ratios are flat and independent of temperature projections; threshold carbon dioxide ratios are initially slightly higher the sooner peak temperature occurs, increase slightly and then drop following peak temperature; the calibration method to derive hazard rates leads to roughly similar initial non-threshold and threshold carbon dioxide taxes; and the threshold weight of methane should be higher today and increase if peak temperature occurs in coming decades. Note that the purpose of this section is to show what optimal policy looks like given an exogenous temperature projection. Thus, there is no link between the tax policies shown in Figure 1 and the temperature outcomes. For example, it may seem odd to the reader that the RCP2.6 scenario, which is likely only achievable with high carbon prices, has the same or similar starting optimal carbon price as the RCP4.5 scenario. One could consider that the different temperature projections are due to different assumptions of technology gains in clean energy or substitutability between clean and dirty

[15] For comparison, Nordhaus (2017) finds a social cost of carbon of $44 (converting $31 in 2015 using 2010 $US) per tonne of carbon dioxide using the DICE-2016R2 model.

energy. The next section presents a more complex model that allows a link between policy and temperature outcomes.

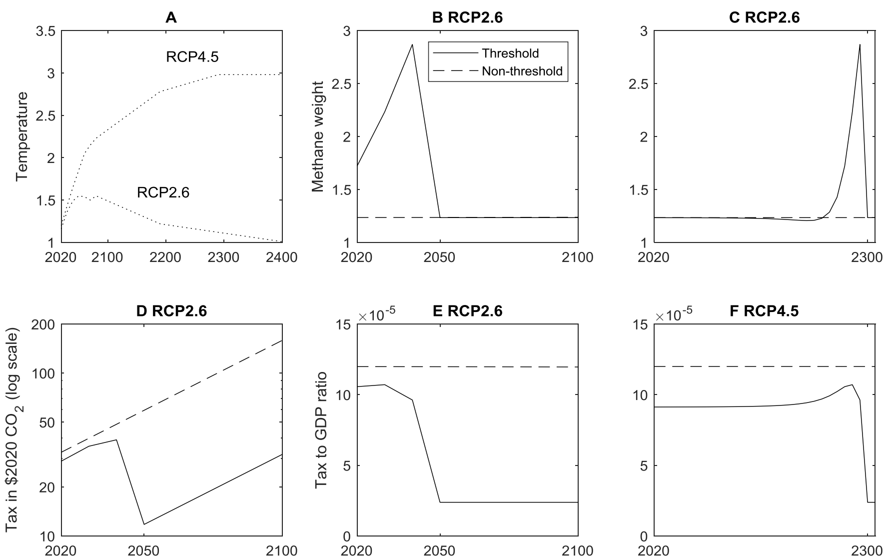

FIGURE 1: IMPLIED TAX, TAX-TO-INCOME RATIOS AND THE WEIGHT OF METHANE FOR IPCC TEMPERATURE PROJECTIONS

CO_2=Carbon dioxide. Tax to GDP ratio in GtC.

III. A model and numerical example with endogenous temperature

This section uses a model with endogenous temperature to consider: the profile of optimal warming; the timing of the different actions of carbon dioxide and methane abatement; and interaction effects between these actions such as the change in the carbon dioxide price and the change in methane weight over time.

Final output is a Cobb-Douglas specification of exogenous technology A_t, capital K_t, final sector labour N_t, energy E_t, and a multiplicative exponential damage function of atmospheric temperature T_t above pre-industrial, as follows:

(13) $$Y_t = e^{-(\gamma T_t + f_t)} A_t K_t{}^{\kappa} N_t{}^{1-\kappa-\nu} E_t{}^{\nu} \text{where } \kappa, \nu > 0 \text{ and } \kappa + \nu < 1.$$

The abatement cost curve for methane abatement may be linear, which would imply a quadratic cost function (IEA, 2020). The costs of the proportion of methane abatement $M_t := 1 - \frac{E_{mt}}{E_{mt\ no\ abate}}$ is parameterised by φ and included in the consumption function:

$$(14) \qquad C_t = Y_t - K_{t+1} - \varphi {M_t}^2 Y_t.$$

Energy E_t is a composite isoelastic function of carbon dioxide-based energy E_{ct} and clean energy, labelled r for renewable E_{rt}, with parameter ρ determining the elasticity of substitution $\sigma = \frac{1}{1-\rho}$:

$$(15) \qquad E_t = \left(E_{\mathrm{ct}}^{\rho} + E_{\mathrm{rt}}^{\rho}\right)^{\frac{1}{\rho}}.$$

Carbon-dioxide energy contributes to carbon dioxide emissions. For simplicity, methane is not included in the output equation (13). Carbon dioxide and renewable sectors require only labour in production[16]

$$(16) \qquad E_{ct} = A_{ct} N_{ct} \text{ and } E_{rt} = A_{rt} N_{rt} \text{ where } N_t + N_{ct} + N_{rt} = 1.$$

Carbon and renewable prices ($p_{\mathrm{i}t}$) are set by wages (w_t) in the final sector, $A_{\mathrm{it}} p_{\mathrm{it}} = w_t$ where $w_t = \frac{Y_t(1-\kappa-\nu)}{N_t}$, leading to:

$$(17) \qquad \frac{p_{\mathrm{ct}}}{Y_t} = A_{ct}\left(\frac{\nu}{E_{ct}^{1-\rho} E_t^{\rho}} - \widehat{\Lambda}_t^c\right) = \frac{w_t}{Y_t} \text{ and } \frac{p_{\mathrm{rt}}}{Y_t} = A_{rt} \frac{\nu}{E_{rt}^{1-\rho} E_t^{\rho}} = \frac{w_t}{Y_t}.$$

[16] This assumption is consistent with Golosov et al. (2014) and assists with model tractability, although energy sectors tend to be capital-intensive.

The optimal levels of methane abatement M_t are set to equate the marginal benefits from reduced temperature with the marginal costs from (14):

$$(18) \qquad \frac{1}{\lambda_{0t}}\sum_{i=0}^{\infty}\lambda_{Tt+i}\mathrm{T}_i^m = -2\varphi M_t Y_t \frac{\partial M_t}{\partial E_{mt}} so \;\; M_t = \frac{E_{mt\ no\ abate}\widehat{\Lambda}_t^m}{2\varphi}.$$

Numerical Example

Results for two simulations are discussed: the first only allows carbon abatement and the second also considers the effect of methane abatement. Projections show a future path where no tipping occurs, but the optimal tax considers uncertainty about the future. Details on how the model is solved are in Appendix D.

Parameters are shown in Table 2. The maximum level of warming under cost-minimisation is set at 2^{O}C. Historical emissions for both carbon dioxide and methane go back a century and together induce warming in 2020 of 1.17^{0}C, in the range of IPCC estimates (IPCC, 2014), with methane contributing 0.29^{0}C. Methane emissions are assumed to be 16% of total emissions historically and remain constant in the future in the absence of a tax on methane, as methane emissions grow only weakly with income (Jorgenson & Birkholz, 2010). Between 2020 and 2030, temperature rises by 0.22^{O}C under Laissez-Faire.

The technology parameter in the final sector grows at 1.3% per annum and renewable energy grows at 2%, following Golosov et al. (2014). Dirty energy is assumed to be mature and hence has no progress, implying a clean transition occurs without climate policy. The renewable input price is assumed to start at 3 times the carbon input price, close to the ratio of 2.7 used recently by Hart (2019). Most empirical estimates of the elasticity of substitution between clean and dirty energy range between 0.5 and 3 (Jo (2020), Papageorgiou, Saam, and Schulte (2017), Lanzi and Sue Wing (2011); Stern (2012) and Pelli (2012)) although higher substitutability has been found in the electricity sector (Wiskich (2021d) and Stöckl and Zerrahn (2020)). Elasticities used in integrated assessment and macroeconomic models have ranged between 10 and 1 (Acemoglu et al. (2012), Hart (2019), Golosov et al. (2014), Greaker, Heggedal, and Rosendahl (2018), and Wiskich (2021b)). I use a value for σ of 2. Parameters κ and ν relating to the shares of capital and energy are set to 0.3 and 0.04 respectively, following Golosov et al. (2014). Complete abatement of methane is calibrated to cost 0.5% of GDP so $\varphi = 0.005$. This implies the marginal cost required for complete methane abatement in 2020 is $\frac{2\varphi_M Y}{(13.92)(3.67)} = \frac{2(0.005)10^{15}}{(13.92)(3.67)10^9}$ = US$196 per tonne carbon-dioxide-equivalent.[17]

TABLE 2: CALIBRATION PARAMETERS

gA_0 (%/year)	gA_c (%/year)	gA_r (%/year)	β (annual)	A_c/A_r	σ	ν	δ	κ
1.3	0	2	0.985	3	2	0.04	0.1	0.3
$E_c(-10:-1)+E_m(-10:-1)$ (GtCe)	$10^{15}Y_{2020}$ ($)	T_{min} (^{0}C)		γ	μ_N	μ_T	φ	
[5,5,10,20,30,40,50,60,80,85]	1	1		0.0016	0.01	0.05	0.005	

The first row of Figure 2 shows the results for the cost-minimisation formulation outlined in Appendix B. The carbon dioxide tax-to-income ratio rises sharply and then falls as temperature peaks and then stabilises for a long period. Weights start low, as methane emissions today contribute less to a distant peak temperature, and rise as peak temperature approaches. Feedback from methane abatement reduces the carbon dioxide tax, reduces variation in the methane weight, and increases temperature marginally in coming decades as warming can be curtailed quickly using methane abatement.

[17] For comparison, a price of US$60 per tonne carbon-dioxide-equivalent is estimated to lead to abatement of 15% in Agriculture(Manure Management), 59% in Coal Mines, 32% in Solid Waste, 47% in Oil and Gas and 8% in Wastewater . https://www.globalmethane.org/documents/gmi-mitigation-factsheet.pdf

Results in the second row show increasing threshold tax-to-income ratios and an increasing methane weight up to peak temperature. Temperature stabilises for a prolonged period. Compared with the cost-minimisation approach: weights start much higher; the carbon dioxide tax is relatively unaffected by methane abatement and peak temperature is reduced. The third row shows a flat non-threshold carbon dioxide tax-to-income ratio, flat methane weights and a temperature that does not stabilise. Optimal carbon dioxide/methane taxes in 2020 are \$11/\$8 for cost-minimisation, \$25/\$31 for threshold and \$33/\$40 for non-threshold per tonne carbon dioxide equivalent in 2020.

Reflecting uncertainty of the stochastic nature of tipping events, Figure 3 shows results when tipping events have an equal probability of being a threshold or non-threshold event: the rise in the weight of methane persists. However, the costs of assuming the wrong tipping formulation are relatively small: around \$0.5 trillion equivalent variation shown in Table 3. Around two-thirds of the cost of using the optimal non-threshold tax in a threshold world is due to a higher-than-necessary tax. The remaining third (\$0.16T) is due to the weight of methane being fixed, as is the case under current policy. The costs of following optimal taxes under a cost-minimisation formulation are much greater, particularly when the true formulation is non-threshold, as are costs when methane abatement is excluded.

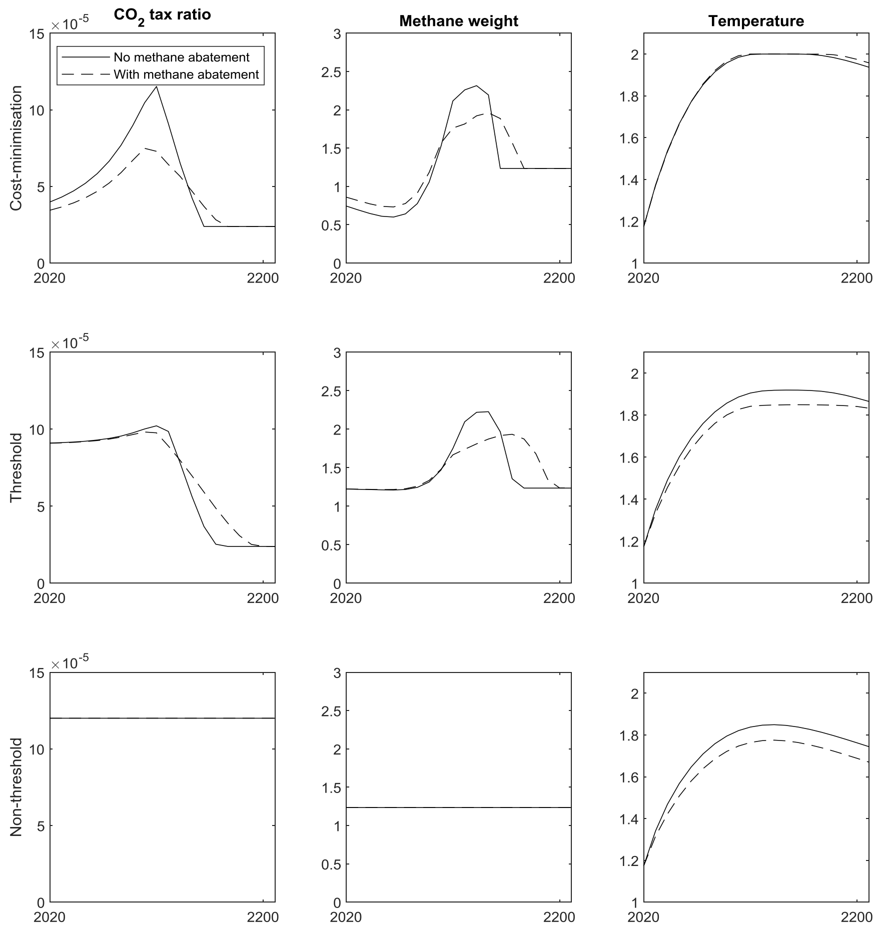

FIGURE 2: OPTIMAL CLIMATE POLICY IN A MODEL WITH ENDOGENOUS TEMPERATURE, WITH AND WITHOUT METHANE ABATEMENT.

CO_2=Carbon dioxide.

TABLE 3: WELFARE LOSS FROM SUBOPTIMAL POLICIES

True Tipping Formulation	With methane abatement		Without methane abatement		
	Wrong formulation	Cost Min	Right formulation	Wrong formulation	Cost Min
Non-threshold	0.0010% ($0.4T)	0.0072% ($2.9T)	0.0035% ($1.4T)	0.0044% ($1.8T)	0.0076% ($3.1T)
Threshold	0.0011% ($0.5T)	0.0030% ($1.2T)	0.0022% ($0.9T)	0.0031% ($1.3T)	0.0037% ($1.5T)

Loss as a % of optimal welfare (Equivalent Variation in 2020$US Trillion). “Wrong formulation” indicates optimal policy for non-threshold is adopted when the true formulation is threshold and vice versa.

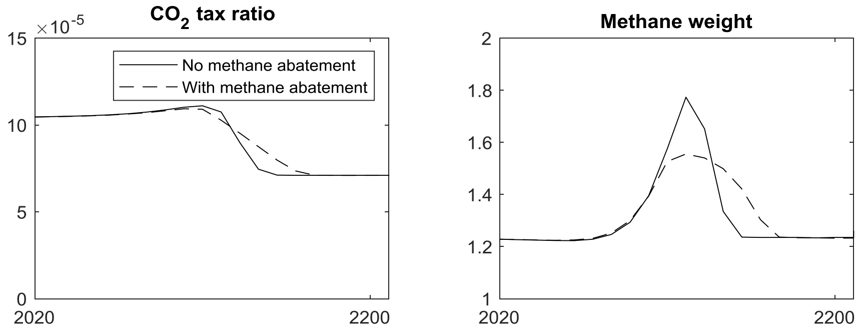

FIGURE 3: OPTIMAL CLIMATE POLICY WITH AN EQUAL CHANCE OF THRESHOLD AND NON-THRESHOLD TIPPING EVENTS, WITH AND WITHOUT METHANE ABATEMENT.

CO_2=Carbon dioxide.

VI. Sensitivities

There are many uncertainties in this field and this section discusses the effect of the following changes to the framework: an increase in risk aversion; exponential damages where the post-tipping impact increases with temperature; limiting the risk of tipping to a single event; and a delay in tipping impacts.

Dependence on risk aversion and the utility function

A logarithmic power utility is commonly used and implies an intertemporal elasticity of substitution of unity. However, some papers disentangle time preferences and risk aversion as described by Epstein and Zin (1990). This approach allows compliance with risk aversion estimates in the literature without leading to excessively high risk-free discount rates. An increase in risk aversion over that implied by a logarithmic utility is achieved by adding an expectation term as shown in the Bellman equation:

$$(19) \quad \mathrm{V}(\mathrm{K}, E, T) = \max_{K,T,E}\left\{U(C) + \frac{\beta}{\alpha}\log\left(\mathbb{E}\left(e^{\alpha V'}\right)\right)\right\} \text{ with parameter } \alpha.$$

The inclusion of additional risk aversion in the value function leads to an increase in the tipping component of the tax, as outlined in the following remark with proof in Appendix A.

Remark 1: For the non-threshold formulation and assuming a simple step function for temperature response, further risk aversion increases the tipping component of the tax-to-income ratio under fixed damages and $T > \bar{T}$ according to the following approximation:

$$(20) \qquad \widehat{\Lambda}^{a}_{EZ\ T>\bar{T}} \xrightarrow{\text{small } \delta\alpha} \widehat{\Lambda}^{a}_{det} + \widehat{\Lambda}^{a}_{T>\bar{T}}\left(1 - \frac{\alpha\delta}{2}\right)$$

Traeger (2018) show that values of $\alpha \in [-1.2, -0.7]$ are consistent with relative risk aversion values between 10 and 6 found in the literature.[18] The uplift approximation in (20) relies on small $\delta\alpha$ which is reasonable for the assumed parameter of $\delta = 0.1$. The range of risk aversion uplift to match the literature then implies an uplift in the tipping tax component of between 3.5% and 6%. Such low values are consistent with some literature including Cai and Lontzek (2019) and Lemoine and Traeger (2016a).

Further, the effect of a power utility function with a coefficient of relative risk aversion (CRRA) of 1.5, close to the value of 1.45 in the DICE 2016R2 model, exhibits predictable differences relating to an increase in the interest rate: optimal prices are lower; the weight of short-lived actions are higher; and the consumption ratio increases and is no longer flat but the dynamics are not material, as discussed in L Barrage (2014). Results for prices are shown in Figure 4.

[18] The standard risk aversion coefficient defined in the Epstein-Zin setting is $1 - \frac{\alpha}{1-\beta}$.

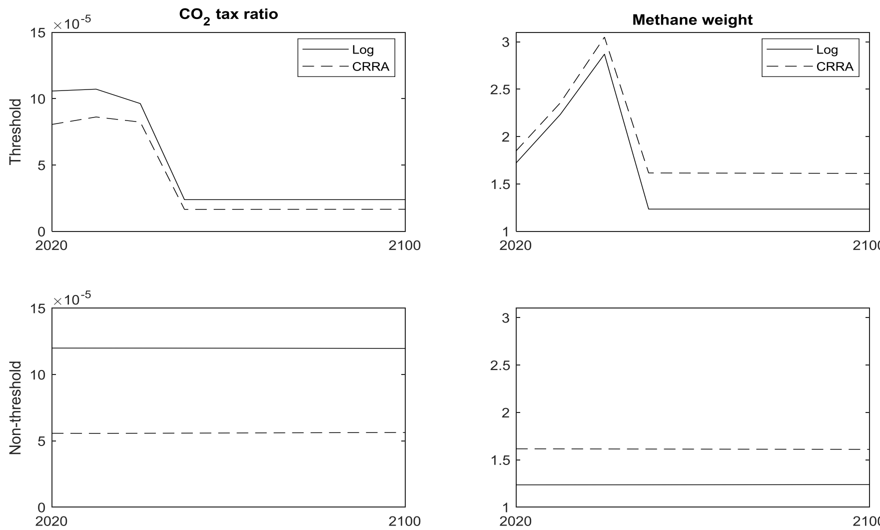

FIGURE 4:COMPARISON OF A UTILITY FUNCTION WITH CRRA PARAMETER OF 1.5 WITH LOG UTILITY. CO_2=Carbon dioxide.

Exponential damages from tipping

Now assume that a tipping event increases the sensitivity to temperature so $g_{CS}(T_t) = T_t$. This "exponential damages" case is calibrated so that 2 degrees of warming post-tipping leads to 10% damages ($\delta_{ED} = \delta/2$), shown in Figure 5.

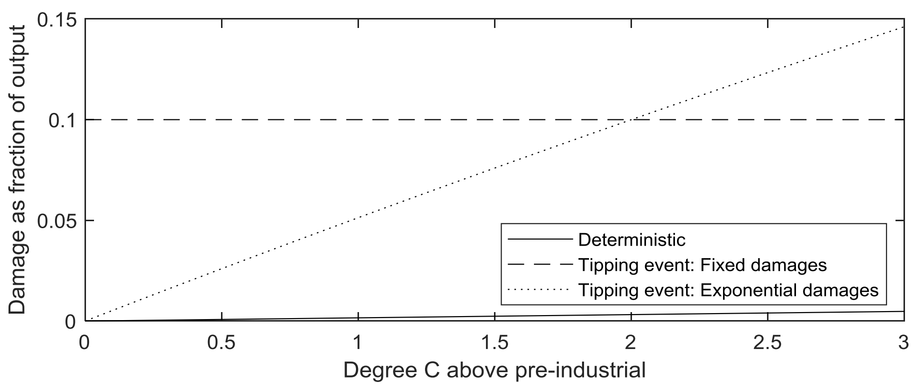

FIGURE 5: DAMAGES AS A FUNCTION OF WARMING

The optimal tax-to-income ratio will now be affected by the occurrence of a tipping event as the marginal damages from temperature are increased: for a single tipping event, the deterministic part of the optimal tax-to-income ratio is boosted as follows:

$$(21) \quad \widehat{\Lambda}^{a}_{det\,ED\,post-tip} = \widehat{\Lambda}^{a}_{det} + \frac{\Gamma^{a}\beta\delta_{ED}}{1-\beta}.$$

The optimal tax will now be a function of expected temperature levels and (9) becomes

$$(22) \quad \mathbb{E}_t\left(\frac{\partial f_{EDt+i+j}}{\partial T_{t+i}}\right) = \mathbb{E}_t \begin{cases} \sum_{k=0}^{i} I_{EDt+k} & \text{if } j = 0 \\ T_{t+i+j}\sum_{k=0}^{j} \frac{\partial I_{EDt+i+k}}{\partial T_{t+i}} & \text{if } j > 0. \end{cases}$$

Consider the restrictive assumption of $T > \bar{T}$ and a concave exogenous temperature projection. The differential welfare impact shown in the top of (22) increases in the non-threshold formulation with rising temperature and decreases in the threshold formulation as the probability of tipping in the future falls (as temperature increases in each period become smaller). The marginal hazard effect in the bottom of (22) increases in both formulations with rising temperatures. Thus, the non-threshold ratio rises while the threshold ratio depends on the concavity of the temperature projection.

Figure 6 extends the numerical example in section II using the optimal tax-to-income formula derived from (9), shown in Appendix A. Taxes now increase with increasing temperature projections for both carbon dioxide and methane. As the chance of tipping increases as time passes (provided a risk is present), the differential welfare tax component tends to increase the optimal tax of carbon dioxide more than short-lived actions. Thus, the weights of methane are lower under exponential damages than under fixed damages. The key result of an increasing threshold methane weight persists and non-threshold taxes are roughly flat initially if temperature rises are constrained.

In a model with multiple possible tipping events and endogenous temperature outcomes dependent on carbon taxes, different future outcomes of the stochastic variable need to be considered. However, by assuming that tipping can only occur once and, as before, it is irreversible, the optimal tax computation is greatly simplified as the tax only has a dependence on temperature outcomes conditional on tipping not occurring. Thus, although tipping can happen in any future period with a non-zero hazard rate *ex-ante*, the optimal tax can be determined by a single future outcome where tipping does not occur *ex-post*.

Remark 2: If tipping only occurs once, the optimal tax can be calculated by temperature outcomes conditional on no tipping *ex-post*.

While this result is not used in simulations in this paper, it could simplify modelling methods in future papers relative to previous papers that resort to complex numerical methods to handle multiple future states.[19] The remark holds as the hazard rate and derivative become zero when tipping occurs and there is no further tipping risk. Thus both the marginal hazard effect and differential welfare impacts in (22) are only dependent on temperature outcomes conditional on no tipping *ex-post*. Engström and Gars (2016) take advantage of this result as they report that their numerical model with a more restrictive climate response is programmed with a simple one-state dynamic programming function.

[19] For example, Lemoine and Traeger (2014) and Lontzek et al. (2015).

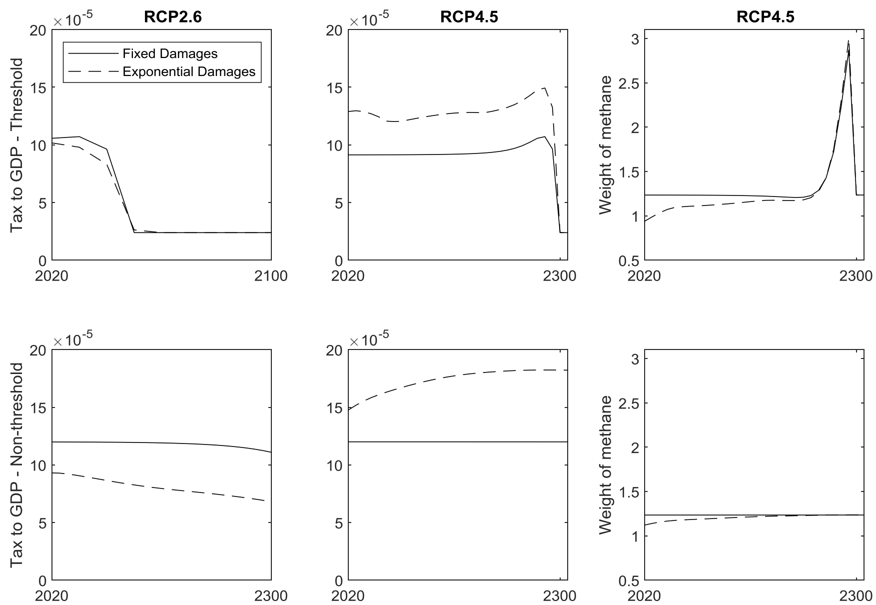

FIGURE 6: EFFECT OF EXPONENTIAL DAMAGES COMPARED WITH FIXED DAMAGES ON THE TAX-TO-INCOME RATIO FOR CARBON DIOXIDE AND THE WEIGHT OF METHANE.

One tipping event only

What if only one tipping event can occur? Consider fixed damages as in section I, exogenous temperature outcomes and tipping has not yet occurred. The expectation at time t of the derivative of the hazard rate (p_{t+i}^{single}) at time $t+i$ is reduced when only one tipping event is possible by the chance that tipping will have occurred between t and $t+i$ as follows: [20]

[20] Papers that use the threshold method typically assume one tipping event where the threshold lies in a given temperature range. As temperature rises and if tipping does not occur, the temperature range which contains the threshold contracts and the marginal risk of tipping rises. Consequently, the hazard rate rises as temperature increases. This paper assumes a constant marginal risk and hence this effect does not arise.

$$(23) \qquad \mathbb{E}_t\left(\frac{\partial p_{t+i}^{single}}{\partial T_{t+i}} \mid f_t = 0\right) = \mu \prod_{k=1}^{i-1}\left(1 - \mathbb{E}_t(p_{t+k})\right) \leq \mu = \mathbb{E}_t\left(\frac{\partial p_{t+i}}{\partial T_{t+i}}\right)$$

A lower tax (given some probability of tipping) in this framework results as $\prod_{k=1}^{i-1}\left(1 - \mathbb{E}_t(p_{t+k})\right) \leq 1$. While this result is intuitive, consider the effect of temperature projections in the case of a single tipping event. As $\mathbb{E}_t\left(\frac{\partial p_{t+i}^{single}}{\partial T_{t+i}}\right)$ in (23) is reduced by the risk of tipping before period $t + i$, the tax today is reduced by higher temperature projections which I call an "inevitability" effect. In a model with endogenous temperature, this effect would create positive feedback from lower taxes to higher temperature projections. As the risk of tipping is very low by assumption in the numerical examples, the effect of this sensitivity is tiny and hence not shown for brevity.

Remark 3: For fixed damages, $T > \bar{T}$ and only one tipping event, the tax is lower the higher the projected temperature outcomes.

Delayed impact

A delay to tipping impacts and learning leads to discounting of the tipping component described in the following corollary to proposition 1.

Corollary 2: Given the assumptions in proposition 1 and assuming a delayed impact from tipping by θ periods, the optimal tax-to-income ratio is

$$(24) \qquad \widehat{\Lambda}_{T>\bar{T}\ delay}^{a} = \widehat{\Lambda}_{det}^{a} + \beta^{\theta-1}\widehat{\Lambda}_{T>\bar{T}}^{a}\ .$$

Figure 7, extending the numerical example in section II, shows a lower tax in the RCP2.6 temperature projection when impacts are delayed due to the discounting term $\beta^{\theta-1}$ in (24).[21] The consequence of this delay is more nuanced in the exponential damages case: the initial tax is reduced due to discounting, but the delay in learning leads to the tipping component of the tax persisting well after peak temperature until the chance that a tipping event was triggered in the past is eliminated.

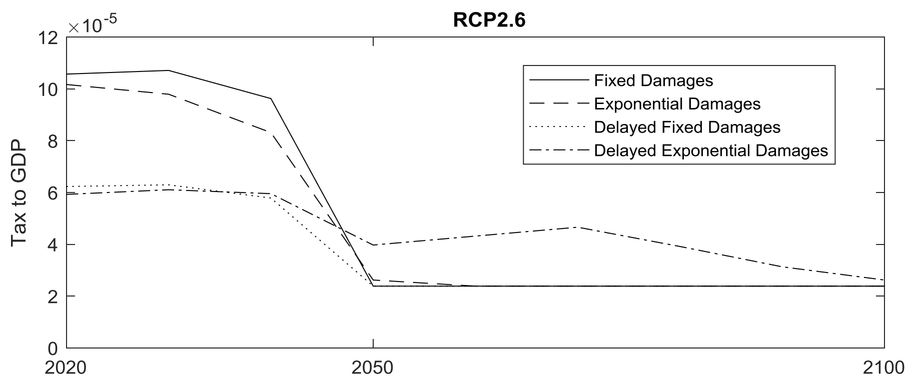

FIGURE 7: EFFECT OF DELAYED IMPACTS ON THE OPTIMAL CARBON DIOXIDE TAX-TO-INCOME RATIO IN THE RCP2.6 PROJECTION

VI. Conclusion

This paper examines the path of optimal environmental taxes under climate tipping risks. As the nature of such risks and the consequences of tipping are highly uncertain, several formulations are considered. The economic framework adopted has restrictive assumptions that allow an easy calculation of the optimal price path given a few parameters and expected temperature outcomes. Results are illustrated

[21] A delay of 5 decades is shown, corresponding to an inital tipping component of around half of the case without delay with the assumed discount rate of 1.5% per annum.

using temperature projections under different IPCC scenarios and in a model with endogenous temperature outcomes.

A key message of this paper is the decline in optimal environmental tax-to-income ratios if there is a risk of tipping from temperature rising above a threshold. A declining carbon price-to-income ratio has been found in other studies: as a consequence of uncertainty in Cai and Lontzek (2019) and Daniel, Litterman, and Wagner (2019) and of directing technical change to clean energy in Acemoglu et al. (2012). Such a decline has implications for temperature and emissions outcomes and potentially on public perceptions of a carbon price. A tax that is much lower after peak temperature has passed may help people appreciate the objective of the tax and its temporary nature may alleviate public resistance.

Another key message is that the weights of short-lived actions should rise given a threshold climate tipping risk. Papers have highlighted the suboptimality of the current policy of constant weights in a cost-minimisation formulation: this paper extends this analysis to a standard economic model maximising a discounted utility function. Reducing the time horizon that determines the weights of greenhouse gases relative to carbon dioxide from 100-years to correspond to the anticipated time of peak temperature may be a reasonable adjustment to current policy. Such an approach maintains (most of) the simplicity and transparency of the existing system.

While this paper considers the risks of tipping in a stochastic framework, the model and associated parameters are assumed to be known *a priori*. The restrictive assumptions used in the economic framework do not allow precautionary capital formation considered in other papers,[22] and exogenous productivity does not allow investigation of the link between climate policy and technical change. Finally, the assumption that temperature is a linear function of previous actions can replicate

[22] For example, van der Ploeg and de Zeeuw (2017).

the more complex climate-economy models well, but tipping impacts on climate feedback, such as a lower rate of decomposition of carbon dioxide, require a more complex framework.

APPENDIX A –DERIVATIONS

BELLMAN DERIVATION OF PROPOSITION 1 FOR NON-THRESHOLD

For simplicity assume a constant temperature effect for carbon $\mathrm{T}_j^c = \mathrm{T}^c \; for \; j \geq 1$, as outlined in Matthews, Gillett, Stott, and Zickfeld (2009) and recently adopted by Dietz and Venmans (2019). Omitting time subscripts and signifying time $t+1$ variables using prime, the value function is

$$(A.1) \quad \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t log(C_t) \text{ where } C = e^{-(\gamma T + f)} K^\kappa F(E) - K',$$

$$T' = T + \mathrm{T}^c E \;\; and \; f' = f + I.$$

$$(\mathrm{A}.2) \qquad \mathrm{V}(\mathrm{K}, E, T) = \max_{K,T,E} \{ log(Y - K') + \beta \, \mathbb{E}\big(\mathrm{V}(K', E', T')\big) \}$$

$$= \max_{K,\mathrm{T},E} \Big(\log\big(e^{-(\gamma T + f)} K^\kappa F(E) - K'\big) + \beta \, \mathbb{E}(\mathrm{V}') \Big).$$

Using a trial solution, we have:

$$(A.3) \qquad \varphi_K \log K + \varphi_T T + \varphi_E E + \varphi_f f$$
$$= \max_{K,T,E} \big(\log(Y - K') + \beta \varphi_K log K' + \beta \varphi_T T' + \beta \varphi_E E' + \beta \varphi_f (f + \mu \delta_{FD} T) \big).$$

The first-order condition for capital leads to $K' = \frac{\beta \varphi_K}{1+\beta \varphi_K} Y$, and substitution into (A.3) leads to

$$(A.4) \quad \varphi_K \log K + \varphi_T T + \varphi_E E + \varphi_f f = \log\left((1-\beta\kappa) e^{-(\gamma T + f)} K^{\kappa} F(E)\right)$$

$$+\beta\varphi_K\left(\kappa log K + log F - (\gamma T + f)\right) + \beta\varphi_T T' + \beta\varphi_E E' + \beta\varphi_f (f + \mu\delta_{FD} T).$$

Equating terms for $logK, f$ and the first-order condition for T and E are

$$(A.5) \qquad logK{:}\ \varphi_K = \kappa + \kappa\beta\varphi_K \ so\ \varphi_K = \frac{\kappa}{(1-\beta\kappa)}.$$

$$(A.6) \quad f{:}\ \varphi_f = -1 - \beta\varphi_K + \beta\varphi_f \ so\ \varphi_f = \frac{-(1+\beta\varphi_K)}{1-\beta} = \frac{-1}{(1-\beta\kappa)(1-\beta)}.$$

$$(A.7) \quad FOC\ T{:}\ \varphi_T = -(1+\beta\varphi_K)\gamma + \beta\varphi_T + \beta\varphi_f \mu\delta \ so$$

$$\varphi_T = \frac{-\left(\frac{\gamma}{(1-\beta\kappa)} + \frac{\beta\mu\delta}{(1-\beta\kappa)(1-\beta)}\right)}{(1-\beta)} = \frac{-\left(\gamma + \frac{\beta\mu\delta}{(1-\beta)}\right)}{(1-\beta)(1-\beta\kappa)}.$$

$$(A.8) \quad FOC\ E{:}\ \varphi_E = \frac{F'(E)}{F(E)}(1+\beta\varphi_K) + \beta\varphi_T T^c.$$

The shadow price of carbon energy φ_E consists of the benefits for production and the negative externality from temperature increase. The latter term is the social cost of carbon expressed in consumption units and the non-threshold result in (10) using this simple temperature effect follows:

$$(A.9) \qquad \frac{\Lambda^c_{T \geq \bar{T}}}{C} = -\beta\varphi_T T^c \text{and C} = (1-\beta\kappa)Y.$$

BELLMAN DERIVATION OF REMARK 1

The value function is

$$(A.10)\ \ \mathrm{V} = \max_{K,T,E}\left(\log\left((1-\beta\kappa)e^{-(\gamma T+f)}K^{\kappa}F(E)\right) + \frac{\beta}{\alpha}\log\left(\mathbb{E}_{\mathrm{t}}\left(e^{\alpha V'}\right)\right)\right).$$

The expected exponential of damages from tipping is

$$(A.11)\ \ \mathbb{E}_{\mathrm{t}}\left(e^{-\alpha\delta}\right) = log\left[pe^{-\alpha\delta}+1-p\right] = log\left[\mu Te^{-\alpha\delta}+1-\mu T\right]$$

$$\sim log\left[\mu T\left(-\alpha\delta+\frac{\alpha^2\delta^2}{2}\right)+1\right]\ as\ e^{-\alpha\delta}\sim 1-\alpha\delta+\frac{\alpha^2\delta^2}{2}$$

$$\sim\mu\delta T\alpha\left(-1+\frac{\alpha\delta}{2}\right).$$

Using the same trial solution as the power utility case leads to a different equation for the temperature coefficient and the optimal tax-to-income ratio is therefore uplifted in (20):

$$(A.12)\quad FOC\ T\!: \varphi_T = -(1+\beta\varphi_K)\gamma+\beta\varphi_T+\beta\varphi_f\mu\delta\left(1-\frac{\alpha\delta}{2}\right).$$

DERIVATION OF OPTIMAL TAXES FOR EXPONENTIAL DAMAGES

$$(\mathrm{A}.13)\quad \sum_{i=0}^{\infty}\beta^i\mathrm{T}_i^{\alpha}\mathbb{E}_{\mathrm{t}}\left(\sum_{j=0}^{\infty}\beta^j\frac{\partial f_{EDt+i+j}}{\partial T_{t+i}}\right)$$

$$=\sum_{i=0}^{\infty}\beta^i\mathrm{T}_i^{\alpha}\mathbb{E}_{\mathrm{t}}\left(I_{EDt+i}+\sum_{j=0}^{\infty}\beta^j\sum_{k=0}^{\mathrm{j}}T_{t+i+k}\frac{\partial I_{EDt+i+k}}{\partial T_{t+i}}\right)$$

$$= \sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \mathbb{E}_\mathrm{t} \begin{cases} I_{EDt+i} + \delta_{ED}\mu_T T_{t+i} & \text{for threshold} \\ I_{EDt+i} + \delta_{ED}\mu_N \sum_{j=0}^{\infty} \beta^j T_{t+i+j} & \text{for non} - \text{threshold} \end{cases}$$

$$= \delta_{ED}\mathbb{E}_\mathrm{t} \begin{cases} \mu_T \left(\sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a (TM_{\mathrm{t}+i-1} - TM_{t-1}) + \sum_{i=0}^{\tau-\mathrm{t}-1} \beta^i \mathrm{T}_i^a T_{t+i} + \beta^{\tau-\mathrm{t}} \mathrm{T}_{\tau-\mathrm{t}}^a \sum_{i=0}^{\infty} \beta^i T_{\tau+i} \right) \\ \mu_N \left(\sum_{i=0}^{\infty} \beta^i \mathrm{T}_i^a \sum_{j=0}^{\min(i,\tau\prime-\mathrm{t}-1)} \left(T_{\mathrm{t}+j} - 1 \right) + \sum_{i=0}^{\tau\prime-\mathrm{t}-1} \beta^i \mathrm{T}_i^a \sum_{j=0}^{\infty} \beta^j T_{t+i+j} \right) \end{cases} .$$

where $TM_t \equiv \max_{i \le t} T_i$

APPENDIX B – COST-MINIMISATION

Optimal weights of greenhouse gases are often considered in a cost-minimisation formulation where an upper bound of temperature or emissions concentration is exogenously imposed.[23] Cost-minimisation can be considered as a peculiar case of a cost-benefit framework: utility is unaffected by temperature up to the maximum temperature level and becomes minus infinity if temperature rises above this level.

The cost-minimisation approach used in section III sets a maximum temperature T_{max} exogenously and the optimal taxes minimise the costs of keeping below this level. While some papers set a maximum temperature at a point in time, this paper allows the model to endogenously determine the onset and end of peak temperature. Deterministic damages are included as in the stochastic tipping frameworks and the optimisation problem is then as described in (4) with the additional constraint that $T_t \le T_{max}$. The optimal tax for action a is simply the deterministic component plus

[23] Cost-minimisation (also called cost-effectiveness) references include Manne and Richels (2001), O'Neill (2003), Aaheim, Fuglestvedt, and Godal (2006), and Johansson, Persson, and Azar (2006). A growing price ratio as a target stock of emissions is approached was perhaps first illustrated by Michaelis (1992). Shine (2009) criticises the current 100-year GWP used to weigh greenhouse gases.

a function of the Lagrange multipliers required to keep temperature at or below the maximum levels in each period, λ_t^{CM}, as follows:

$$(B.1) \qquad \widehat{\Lambda}_t^{aCM} = \widehat{\Lambda}_{det}^{a} + \sum_{i=0}^{\infty} \beta^i \lambda_{t+i}^{CM} T_i^a.$$

Assuming peak temperature occurs at period τ and temperature declines immediately afterwards, from (B.1) the optimal tax-to-income ratio for action j is:

$$(B.2) \qquad \widehat{\Lambda}_t^{aCM} = \widehat{\Lambda}_{det}^{a} + \beta^{\tau-t}\tilde{\lambda}_\tau^{CM} T_{\tau-t}^a.$$

While the benefits of abatement are discounted in the cost-benefit approach due to the damages function, benefits are not in the cost-minimisation formulation and the optimal tax therefore increases much faster before peak temperature.

APPENDIX C –CLIMATE MODEL

The climate model in this paper is taken from Shine, Fuglestvedt, Hailemariam, and Stuber (2005) and described here. Many papers have assumed that the temperature response to a carbon dioxide pulse peaks after several decades which is inconsistent with recent physical science literature. In continuous-time, temperature dynamics are a function of radiative forcing R_t:

$$(C.1) \qquad H\frac{dT_t}{dt} = R_t - \frac{T_t}{\lambda},$$

where H is the heat capacity of the system and λ is a climate sensitivity parameter. For carbon dioxide, radiative forcing and temperature responses at time t after an emissions pulse (in discrete time) are

$$(C.2) \qquad R_t^c := \frac{\partial R_t}{\partial E_{c0}} = a_0 + \sum_{i=1}^{4} a_i e^{-\frac{t}{\alpha_i}} \text{ and}$$

$$(C.3) \qquad T_t^c := \frac{\partial T_t}{\partial E_{c0}} = \frac{B_c}{H}\left\{\zeta a_0 \left(1 - e^{-\frac{t}{\zeta}}\right) + \sum_{i=1}^{4} \frac{a_i \left(e^{-\frac{t}{\alpha_i}} - e^{-\frac{t}{\zeta}}\right)}{(\zeta^{-1} - \alpha_i{}^{-1})}\right\},$$

where a_i are coefficients which sum to 1, α_i reflect gas lifetimes in years, ζ is by definition the constant λH in years, and B_c is the radiative forcing due to a 1-kg change in carbon dioxide. For methane ($a = m$) the equations are simpler:

$$(C.4) \qquad R_t^a = B_a e^{-\frac{t}{\alpha_a}} \text{ and}$$

$$(C.5) \qquad T_t^a = \frac{B_j}{H(\zeta^{-1} - \alpha_a{}^{-1})}\left(e^{-\frac{t}{\alpha_a}} - e^{-\frac{t}{\zeta}}\right).$$

Parameter values are shown in Table 4. The left panel of Figure 8 shows the radiative forcing from emissions pulses of carbon dioxide and methane normalised to the same initial forcing for ease of comparison.

Before examining temperature responses, note that GWPs are used to weigh the climate effects of greenhouse gases. The GWP of a gas is the time-integrated radiative forcing from a pulse emission, relative to an equal mass of carbon dioxide, and thus resulting weights depend on the choice of time horizon. For example, methane has a 100-year GWP of 28 and a 20-year GWP of 84 (IPCC, 2014). The 100-year GWP was adopted by the United Nations Framework Convention on Climate Change and its Kyoto Protocol and is now used widely as the default metric. The clearest recommendation for 100 years is that a significant fraction of carbon dioxide is removed from the atmosphere over this time scale (Fuglestvedt et al., 2003), and this period also roughly corresponds to the anticipated maximum change in temperature (WMO, 1992).

Temperature responses to pulse emissions are shown in the right panel of Figure 8, normalised so that the non-discounted sum over a time horizon of 100 years is the same as for carbon dioxide, approximating the current policy of a 100-year GWP.[24] These impulse functions are central to this paper and highlight the sharp temperature responses to methane relative to the carbon dioxide pulse.[25]

When the discount rate is high, the weight of a short-lived action like methane is high due to the rapid temperature effect of a methane pulse relative to carbon dioxide. As the discount rate decreases, the optimal methane to carbon dioxide weight drops. This sensitivity to the discount rate has been discussed previously in the literature. As the GWP is approximately equal to the area under the temperature impulse responses shown in Figure 8, if the discount rate was zero and $\tau \to \infty$, the weight of a short-lived action would be less than 1 (the approximate value set by current policy). A discount rate of around 1% implies a weight of 1 for methane, and a discount rate of 1.5% as used in this paper leads to a weight above 1 as shown in.

TABLE 4: CLIMATE MODEL PARAMETERS

a_0	a_1	a_2	a_3	a_4	B_c	B_m	B_s
0.1756	0.1375	0.1858	0.2423	0.2589	1.98	$3.95B_c$	$17.97B_c$
H	α_1	α_2	α_3	α_4	ζ	α_m	
4.2	421.09	70.597	21.422	3.4154	10.65	12	

[24] A 100-year GWP is approximately equal to the area under the curve up to 100-years.

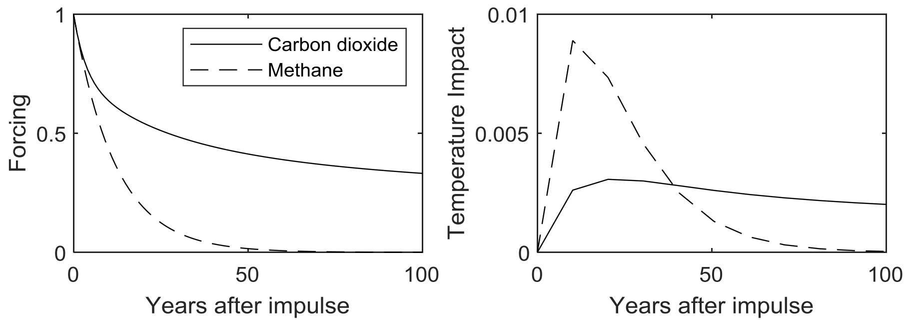

FIGURE 8: FORCING AND TEMPERATURE IMPACT FROM A PULSE EMISSION OF CARBON DIOXIDE AND METHANE.

APPENDIX D – SOLVING THE ENDOGENOUS MODEL

While solving the model without methane abatement use is simplified by a constant savings rate, including methane abatement implies optimal savings rates are no longer constant. Combining (14) and $\frac{1}{\mathrm{C}_t} = \beta \mathbb{E}_t\left(\frac{1}{\mathrm{C}_{t+1}} \kappa \frac{\mathrm{Y}_{t+1}}{\mathrm{K}_{t+1}}\right)$ leads to:

$$(\mathrm{D.1}) \qquad \frac{\mathrm{Y}_t}{\mathrm{C}_t} = 1 + \frac{\varphi_M {M_t}^2 Y_t}{\mathrm{C}_t} + \kappa\beta\mathbb{E}_t\left(\frac{\mathrm{Y}_{t+1}}{\mathrm{C}_{t+1}}\right).$$

However, variation in the savings rate makes no material difference to the variables investigated. Solution steps for the cost-minimisation formulation are as follows.

1. Manually choose a high estimate τ for the period where peak temperature starts.
2. Run the model for 80 periods (800 years) multiple times in a loop assuming tipping does not occur. For each iteration:
    2.1 Estimate the carbon dioxide tax required to ensure the temperature does not rise following the peak temperature period.
    2.2 Calculate multipliers from these prices as described below.
    2.3 Calculate optimal prices from (B.2).

Stop iterating when optimal prices converge and the savings rate converges in the simulation with methane abatement.

3. If temperature rises above T_{max} before τ, reduce τ by one period and repeat. Otherwise stop.

Solving the model for the non-threshold formulation simply involves iterating to solve (12). The threshold formulation is easier to solve by numerically optimising welfare and the derived tax equations are used as a check. The step-change in the marginal hazard effect is smoothed with the following function:

$$(D.2) \qquad \frac{1}{X} log\left(1 + \mathrm{e}^{\mathrm{X}(\mathrm{T}-\overline{\mathrm{T}})}\right) \xrightarrow[X\to\infty]{} \begin{cases} \mathrm{T}-\overline{\mathrm{T}} & \text{if } \mathrm{T}-\overline{\mathrm{T}} > 0 \\ 0 & \text{if } \mathrm{T}-\overline{\mathrm{T}} < 0 \end{cases}$$

Handling temperature stabilisation

Peak temperature multipliers are derived from the taxes needed to stabilize temperature: the carbon tax in the period before the end of peak temp is given by $\widehat{\Lambda}_{n-1}^{cCM} = \widehat{\Lambda}_{det}^{c} + \frac{\tilde{\lambda}_n T_1^c}{\beta^{n-1}}$ which gives $\tilde{\lambda}_n$, the tax in the prior period is $\widehat{\Lambda}_{n-2}^{cCM} = \widehat{\Lambda}_{det}^{c} + \frac{\tilde{\lambda}_{n-1} T_1^c}{\beta^{n-2}} + \frac{\tilde{\lambda}_n T_2^c}{\beta^{n-2}}$ which gives $\tilde{\lambda}_{n-1}$ and so on. The choice of τ is determined through manual iteration as described above: for a high τ, peak temperature occurs before this value, and thus the value of τ is reduced until it corresponds with peak temperature. Any further reduction in τ implies the tax in this period is higher than would be the case without the upper limit constraint applying (holding the tax in all other periods constant), implying suboptimality.

2. Environmental taxes versus research subsidies as suboptimal policy

By ANTHONY WISKICH*

Given a choice between a carbon tax and a clean research subsidy, which one performs better and under what conditions? This paper argues that the absence of a non-energy sector has led some previous literature to find that subsidies outperform taxes. An integrated assessment model with endogenous technology is described. Numerical exercises find that a permanent global tax-only policy outperforms a permanent subsidy-only policy and this result is robust to many different parameter settings and assumptions. However, in the more optimistic case where optimal policy begins in 2050, the performances of subsidy-only and tax-only policies in the interim are closer. (JEL O30, O44, Q54, Q56, Q58)

Keywords: Climate change, directed technical change, optimal policy, energy.

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The ongoing transition to clean energy involves a shift of factors of production and researchers into the clean energy sector. Governments have a role in enabling and incentivising a clean energy transition through various policy options including carbon taxes and research subsidies. However, global policy is difficult to achieve, and we are a long way from optimal policy. Thus, the relative performance of suboptimal policy is important, at least until something approaching optimal policy can be achieved. This paper focuses on a hypothetical choice between taxes and subsidies.

The first contribution is an explanation of why previous literature has reached different conclusions about whether taxes or subsidies alone are preferable. Fischer and Newell (2008) rank policies for reducing carbon dioxide emissions and promoting innovation and diffusion of renewable energy and put an emissions price first and research subsidy last. However, the importance of subsidies are emphasised in Acemoglu et al. (2012) and Acemoglu, Akcigit, Hanley, and Kerr (2016) and subsidies are found to outperform taxes in Greaker et al. (2018) and Lemoine (2017). More recently, Hart (2019) finds that emissions taxes are far more important than research subsidies.

I argue that a key reason for different modelling results is the presence of a non-energy sector, as many of these studies only consider clean and dirty intermediates or technologies in a "two-sector" approach. Such an approach exaggerates the cost of a clean transition. As the transformation of the economy to clean energy is limited to a small proportion of the economy, the costs of abatement in the two-sector approach should be reduced by a factor approximately equal to the inverse of the share of output in the energy sector (25 in this paper) while the effects of damages are unchanged. This straightforward result has important implications for the optimal balance between subsidies and taxes. Taxes can distort production and direct clean research but at a cost.[26] As the cost of distortion is exaggerated under a two-sector approach, subsidy-only policy is favoured.[27]

[26] As found in Gerlagh, Kverndokk, and Rosendahl (2009) and Acemoglu et al. (2012), for example.

[27] Pottier, Hourcade, and Espagne (2014) discuss the high costs of a clean transition in the model described by Acemoglu et al. (2012) and caution against use of a highly aggregated model.

This result would help explain why both Greaker et al. (2018) and Lemoine (2017), who do not include a non-energy sector, find subsidy-only policy can outperform a tax-only policy while Hart (2019) finds the opposite and includes a non-energy sector. Hart argues that the difference in results is due to different model construction: that the model in Greaker et al. (2018) follows Acemoglu et al. (2012) where a corner solution is reached and only clean research occurs. This paper demonstrates that the important difference is the inclusion of non-energy both theoretically and numerically by comparing results in 2-sector and 3-sector models.

The second contribution is an extension of the endogenous growth model described in Acemoglu et al. (2012). A non-energy sector is incorporated in addition to clean and dirty energy sectors, similar to Hart (2019), Hémous (2016) and Fried (2018). The ongoing clean transition has seen an increase in investment and research in both clean and fossil energy (IEA, 2014), and as the model allows movement of labour and researchers such an effect can be considered. Under optimal and tax-only policy, environmental disaster, defined as an ever-increasing use of dirty energy, is avoided. When only subsidies are available, their optimal application only avoids environmental disaster if the substitutability between clean and dirty energy is high enough. However, a novel result is that the movement of researchers between non-energy and energy implies disaster can be avoided suboptimally at the cost of economic growth.

This paper introduces a novel functional form for considering stepping-on-toes which better represents decreasing returns to research due to overlapping research ideas. The commonly used functional form was described in Jones and Williams (1998) who applied it in a macroeconomic context where the research share is contained within reasonably restricted bounds. Some papers such as Hart (2019) have applied this form at a sectoral level where research may fall close to zero, leading to returns to research approaching infinity. While heterogeneous researchers may constitute a basis for such a functional form, I adopt a form more in line with the concept of stepping-on-toes where the returns to research are finite as researchers approach zero. Sensitivity analysis shows that this can make a material difference to results.

Parameter choices are based on a combination of previous literature and recent patent and research data. Parameter values are uncertain and hence this paper undertakes analysis using many different parameter choices for: the elasticity of substitution between clean and dirty energy; the elasticity of substitution between non-energy and energy; the extent of damage from climate change; the coefficient of relative risk aversion (and discount rate); the initial (long-run) clean energy share; the life of patents; the economic cost from applying a research subsidy due to misdirection of resources; spillovers to research from technology in other sectors; and the parameter choice of stepping-on-toes and functional form as discussed above.

There are three mechanisms through which each sensitivity affects the relative performance of taxes and subsidies: changing the marginal costs of a tax as discussed above; changing the marginal benefits of a tax; and changing the initial tax required to direct clean research in the absence of subsidies. For example, an increase in the parameter of relative risk aversion (or equivalently the discount rate) reduces future benefits of abatement today, lowering the marginal benefits of a tax and hence favours subsidies. Thus a higher discount rate both lowers the optimal carbon tax and increases the relative performance of subsidies. In almost all scenarios except when a high coefficient of relative risk aversion is assumed, tax-only policy performs better than subsidy-only policy. In the main scenario, subsidy-only policy involves 3.3 times the utility loss of tax-only policy.

The purpose of considering suboptimal policy options is to recognise that the implementation of optimal global policy is difficult, if not impossible in the short-term, and thus provide policymakers with advice on which policy to focus on. Hassler, Krusell, Olovsson, and Reiter (2020) point out that we are far from optimal policy at present: the average global carbon tax is negative due to coal subsidies. Difficulty in coordination demands research in such multi-region models.[28] But the attainment of something approaching optimal coordinated policy may be possible in coming decades, which begs the question of what policy should be pursued in the interim in a single-region model. The role of subsidies is a short-term one in the sense that once clean technology is sufficiently advanced, clean energy dominates the dirty sector and attracts research without a subsidy. Therefore, the attainment of optimal policy after a delay improves the relative performance of subsidy-only policy in the interim and this is confirmed in every sensitivity conducted in this paper. In the main scenario when optimal policy is achieved in 2050, tax-only policy outperforms subsidy-only policy by a reduced factor of 1.9, and under many sensitivity combinations a temporary subsidy-only policy outperforms a temporary tax-only policy.

The research question is close to that of Hart (2019). The calibration used by Hart (2019) is based on a combination of parameter choices and matching historical data and leads to a small role for subsidies, likely related to an extremely strong stepping-on-toes effect. In addition to explaining why previous literature have reached different conclusions, the current paper differs from Hart (2019) in several respects: a weaker stepping-on-toes parameter is considered; lower substitutability between non-energy and energy rather than Cobb-Douglas; a novel formulation for stepping-on-toes as described above; and the effect of parameter variation on the relative performance of suboptimal policies are more thoroughly examined. This paper supports the finding of Hart (2019) that tax-only policy outperforms subsidy-only policy but is more optimistic about the relative performance of the latter, particularly if optimal policy is eventually reached.

[28] Hémous (2016) is another example.

I. Model

The model builds on Acemoglu et al. (2012)[29]. A representative household maximises

$$(1) \qquad \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(\hat{C}_t) \text{ where } u(\hat{C}_t) = \frac{\hat{C}_t^{\,1-\kappa}}{1-\kappa} \text{ and } \hat{C}_t = D(T_t)C_t$$

in discrete time, where C_t is consumption, D is a damage function of temperature T_t above pre-industrial levels, κ is a constant coefficient of relative risk aversion and ρ is the discount rate. In the main scenario the damage function is an exponential function of temperature, $D(T_t) = e^{-\theta T_t}$ where θ is a damage parameter, and the quadratic form assumed in the DICE 2016R2 model is considered as a sensitivity. Aggregate output at time t is an isoelastic function of energy inputs Y_{et} and non-energy inputs Y_{0t}, with elasticity of substitution ε and share parameter δ_e:

$$(2) \qquad Y_t = \left(\delta_e^{\frac{1}{\varepsilon}} Y_{et}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\delta_e)^{\frac{1}{\varepsilon}} Y_{0t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Total energy production Y_{et} is produced competitively using clean and dirty inputs, Y_{ct} and Y_{dt}, according to an isoelastic function with an elasticity of substitution σ:

[29] Many other papers also build on Acemoglu et al. (2012) including Greaker and Heggedal (2012), Greaker et al. (2018), Acemoglu, Aghion, and Hémous (2014), Durmaz and Schroyen (2013), Van den Bijgaart (2017), Lemoine (2017) and Hémous (2016).

(3) $$Y_{et} = \left(Y_{ct}^{\frac{\sigma-1}{\sigma}} + Y_{dt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

The environmental externality is caused by the production of the dirty input so that temperature evolves as follows:

(4) $$T_t = \sum_{u=-\infty}^{\mathrm{t}} T_{t-u}^c Y_{du} \text{ where } T_{t-u}^c := \frac{\partial T_t}{\partial Y_{du}}.$$

Details of the temperature response to dirty energy use are discussed in Appendix B. The inputs Y_{jt} are produced using labour L_{jt} and a continuum of sector-specific intermediates:

(5) $$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha}\, di,$$

for parameter $0 < \alpha < 1$, A_{jit} is the quality of intermediate of type i used in sector j at time t and x_{jit} is the quantity of this intermediate. Total labour supply is normalised to 1:

(6) $$L_{0t} + L_{et} = 1 \text{ where } L_{et} := L_{ct} + L_{dt}.$$

Intermediates are supplied by monopolistically competitive firms and cost ψ units of the final good which is normalised to $\psi := \alpha$. Market clearing for the final good implies that

(7) $$C_t = Y_t - \psi \int_0^1 (x_{cit} + x_{dit} + x_{0it})\, di - \chi sub_t.$$

where an economic cost of applying research subsidies sub_t due to misdirection of resources is specified by parameter χ, following Acemoglu et al. (2016). For shadow prices of input *j* equal to the ratio of Lagrange multipliers for (5) and (7), $p_{jt} := \frac{\lambda_{jt}}{\lambda_{Ct}}$, the first-order condition with respect to x_{jit} leads to

(8) $$x_{jit} = \left(\frac{\alpha p_{jt}}{\psi}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt}.$$

Combining (8) with (5) implies

(9) $$\mathrm{Y}_{jt} = \left(\frac{\alpha}{\psi} p_{jt}\right)^{\frac{\alpha}{1-\alpha}} L_{jt} A_{jt}$$

where the average productivity in sector *j* is

(10) $$A_{jt} := \int_0^1 A_{jit}\, di.$$

The equalisation of wages sets the price of the non-energy sector and the relative prices of clean and dirty inputs as follows:

(11) $$p_{0t}{}^{\frac{1}{1-\alpha}} A_{0t} = p_{ct}{}^{\frac{1}{1-\alpha}} A_{ct} = p_{dt}{}^{\frac{1}{1-\alpha}} A_{dt}.$$

Sector *j* maximises pretax profit

(12) $$p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha}\, di - w_{jt} L_{jt} - \int_0^1 \psi x_{jit}\, di$$

where a subsidy of $1-\alpha$ is applied to the use of all machines to account for monopoly distortions so that the post-subsidy price equals marginal cost ψ. This leads to the isoelastic inverse demand curve (8) and the profit-maximising price is a constant markup over marginal cost, $p_{jit} = \frac{\psi}{\alpha}$. Pretax profit $\pi_{jit} = (p_{jit} - \psi)x_{jit} = \left(\frac{\psi}{\alpha} - \psi\right) x_{jit}$ and (8) imply

$$(13) \qquad \pi_{jit} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \quad p_{jt}^{\frac{1}{1-\alpha}} L_{jt} A_{jit}.$$

Technology advances due to the research of scientists, and each scientist decides at the start of each period to direct their research. Scientists are successful in innovation in sector j with probability η_j, where innovation increases the quality of intermediates by a factor $1+\gamma$. The total number of scientists is normalised to 1:

$$(14) \qquad s_{0t} + s_{ct} + s_{dt} \leq 1.$$

I apply a stepping-on-toes effect which captures a duplication externality from research, leading to decreasing returns (Jones & Williams, 1998). The standard functional form for researchers s_{jt} and a sector diversity parameter s_j^{LR}, which in part determines the long-run research share, is $R(s_{jt}) = \left(\frac{s_{jt}}{s_j^{LR}}\right)^{\omega}$ where $0 < \omega \leq 1$.[30] The denominator s_j^{LR} is present to account for sector diversity: a single successful scientist has a lower impact on non-energy productivity as this sector is larger and more diverse than the clean energy sector. This standard functional form has the advantage of simplicity and can avoid corner solutions where no research occurs in a sector.

[30] For example see Fried (2018).

However, the implied infinite returns to research as researchers approach zero (if $\omega < 1$) is not consistent with a duplication externality and may make a material difference to results under a complete transition between highly substitutable sectors as this paper discusses.[31] Indeed, numerical exercises in this paper show that the standard form leads to lower taxes required to direct clean research and hence a greater disadvantage of subsidy-only policy relative to tax-only policy. Instead, I consider this standard form as a sensitivity and adopt the more complex form $R(s_{jt}) = aln\left(1 + b\frac{s_{jt}}{s_j^{LR}}\right)$ which is finite as research approaches zero and rises without bound but with first derivative approaching zero. Parameters a and b are calibrated so that $R(s_j^{LR}) = 1$ and first derivative $R'(s_j^{LR}) = \omega$ for comparability with the standard form. Kruse-Andersen (2019) use a similar stepping-on-toes form of $(1 + s_{jt})^{\chi}$ where $0 < \chi \leq 1$.

In addition, spillovers into sector j from other sectors are included in the form $\left(\frac{A_t}{A_{jt}}\right)^{\varphi}$ for $0 < \varphi \leq 1$ and $A_t := \delta_e(A_{ct-1} + A_{dt-1}) + \delta_0 A_{0t-1}$ following Fried (2018). For long-run shares s_j^{LR}, with $s_0^{LR} = 1 - s_e^{LR}$, productivity evolves according to:

$$(15) \qquad A_{jt} = \left(1 + \gamma\eta_j R(s_{jt})\left(\frac{A_t}{A_{jt}}\right)^{\varphi}\right)A_{jt-1}.$$

There is no doubt that spillovers occur between sectors. Hart (2019) motivates this effect by imagining wind power in the year 1900 with windmills made of wood with cloth sails which are then not developed further until the year 2000: the knowledge developed in the intervening period would be a major help to

[31] While a similar concern could be made for the Cobb-Douglas production form, the motivation for decreasing returns in this case is not a duplication externality.

researchers in clean energy. However, researchers would not be able to extract rents from all advances, such as simply using more recent material, and so I assume that researchers only extract a part of the gain from spillovers. A successful scientist obtains a one-period patent in the main scenario, following Acemoglu et al. (2012) and expected contemporaneous profits for a single scientist are

$$(16) \qquad \Pi'_{jt} = (1+q_{jt})\eta R(s_{jt})\left(\frac{A}{A_j}\right)^{\varphi'}(1+\gamma)(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} L_{jt}A_{jt-1}$$

where $0 < \varphi' \leq \varphi$ and q_{jt} is a proportional research subsidy in sector j.[32] Total pre-tax profits in a sector are then $s_{jt}\Pi'_{jt}$. As shown in (7), a cost to consumption of applying a research subsidy is assumed in proportion to the value of research subsidies.

I also consider extended patent lifetimes (of life *n* periods) as a sensitivity following Greaker et al. (2018). Profits are discounted by a replacement rate $z_{jt} := \eta_j s'_{jt}$ which is the chance that an innovation will be superseded and hence profits become zero, and scientists' discount rate which I approximate as $\rho + \gamma\eta$ where η is the long-run growth in the economy:

$$(17) \qquad \Pi_{jt} = \sum_{k=0}^{n} \Pi'_{jt+k} \prod_{\pi=1}^{k}\left(\frac{1-z_{jt+\pi}}{1+\rho+\gamma\eta}\right).$$

When a subsidy is used, the total subsidy expenditure $sub_t = \mathrm{q}_{ct}\frac{\Pi'_{ct}}{1+\mathrm{q}_{ct}}$ required to direct research is such that the critical profit ratio $\frac{\Pi_{ct}}{\Pi_{0t}}$ is 1.

32 Note that multiple equilibria may apply, and the derived tax or subsidy can be considered a lower bound for the required policy to direct a clean transition (Wiskich, 2021a).

To demonstrate differences in results between previous studies, a two-sector model is also used where research can occur in clean or dirty energy, described in Appendix C.

II. Model characteristics

There are three market failures in the economy: (i) the underutilisation of machines due to monopoly pricing; (ii) the environmental externality; and (iii) the knowledge externality in the technology frontier. Each can be corrected as follows:

LEMMA 1: Excluding an economic cost of using research subsidies from the misdirection of resources, the socially optimal allocation can be implemented using: (i) a subsidy for the use of all machines (all proceeds from taxes/subsidies being redistributed/financed lump sum); (ii) a tax on dirty input (a "carbon" tax); and (iii) an innovation subsidy (or tax) to each of the energy sectors.

Consider the carbon tax first. Let λ_{Tt} be the Lagrange multiplier for (4). If utility is logarithmic in consumption, the first-order condition with respect to T implies $\lambda_{Tt} = \beta^t \theta$. Excluding misdirection costs from subsidies, the socially optimal tax Λ_t is equal to the social cost of carbon (in units of the consumption price), leading to a flat tax as a ratio of consumption.

$$(18) \qquad \Lambda_t = \frac{-1}{\lambda_{Ct}} \sum_{u=0}^{\infty} \lambda_{Tt+u} \mathrm{T}_u^c = \frac{C_t}{\beta^t} \theta \sum_{u=0}^{\infty} \beta^{t+u} \mathrm{T}_u^c \ \ so\ \frac{\Lambda_t}{C} = \theta \sum_{u=0}^{\infty} \beta^u \mathrm{T}_u^c.$$

A subsidy for the use of all machines is required to boost the supply of machines so that the price is equal to the marginal cost.[33] The knowledge externality can be corrected with a profits subsidy/tax to the clean energy sector (and dirty energy sector) following Acemoglu et al. (2012). Let $\lambda_{A_j t}$ be the Lagrange multiplier for (15) and the first-order condition for technology gives:

$$(19) \qquad \lambda_{A_j t} = \lambda_{Ct} \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) p_{jt}{}^{\frac{1}{1-\alpha}} L_{jt} + \left(1 + \gamma \eta_j R(s_{jt}) \left(\frac{A_t}{A_{jt}}\right)^{\varphi}\right) \lambda_{A_j t+1}$$

The optimal research allocation is determined by the social gain from innovation $\gamma \eta_j \lambda_{A_j t} A_{jt-1}$. The social planner will assign researchers to energy sector j when the following ratio exceeds 1 (until the ratio equals 1):

$$(20) \qquad \frac{\eta_j \left(s_j^{LR} \left(1 + \gamma \eta_j R(s_{jt}) \left(\frac{A_t}{A_{jt}}\right)^{\varphi}\right)\right)^{-1} \sum_{u \geq t} \lambda_{Cu} p_{ju}{}^{\frac{1}{1-\alpha}} L_{ju} A_{ju}}{\eta_0 \left(s_0^{LR} \left(1 + \gamma \eta_0 R(s_{0t}) \left(\frac{A_t}{A_{0t}}\right)^{\varphi}\right)\right)^{-1} \sum_{u \geq t} \lambda_{Cu} p_{0u}{}^{\frac{1}{1-\alpha}} L_{0u} A_{0u}}$$

The focus of this paper is on suboptimal tax-only and subsidy-only policy. As well as distort production to account for the environmental externality, a tax alone can direct clean research but is less efficient at doing so than a subsidy.

LEMMA 2: Directing technical change using a tax will distort labour allocation in the energy sector more than a subsidy for the same research allocation. The labour allocation is

[33] This subsidy of $(1-\alpha)$ is not a focus of this paper. For further details see Acemoglu et al. (2012).

$$\text{(21)} \qquad \frac{L_{ct}}{L_{dt}} = (1+\tau_t)^{\sigma}\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\sigma)(1-\alpha)} \text{ where } \tau_t := \frac{\Lambda_t}{p_{dt}}.$$

This result follows from combining the first-order conditions for wage-equalisation (3) and prices (11) and applies to both optimal and constrained (second-best) approaches. The next proposition may help explain the different findings of previous studies on whether tax-only or subsidy-only policy is preferable.

PROPOSITION 1: Removing substitutability options between non-energy and energy has the following effects.
(1) The cost of a distortion in the energy sector to the macroeconomy is increased.
(2) The performance of tax-only policy relative to subsidy-only policy is reduced if: (i) the tax required to direct clean research in the absence of subsidies is more than the optimal tax by a constant increment; and (ii) misdirection costs from the research subsidy are excluded.

Part (1) results from the envelope theorem and holds for any positive elasticity of substitution between non-energy and energy. Consider a degree, however small, of substitutability between non-energy and energy inputs into final production. Then an infinitesimal reduction in energy input of dx leads to a $shr_e dx$ fall in total output where shr_e is the share of energy in total output.[34]

[34] Even if there is no substitutability between non-energy and energy in final output, the movement of labour inputs leads to the same result in both the short and long-run, as energy output is linear in labour so any reduction in energy sector output can be mitigated by a shift in labour from the non-energy sector. The result for the movement of researchers holds in the long-run, as in the limit that clean technology dominates and $\sigma > 1$, energy output is also linear in clean technology.

The dynamic nature of the model excludes an analytical proof of part (2), so strictly it is a conjecture. However, the reason for the result can be understood in a static approach. Assume that optimal policy involves clean research, which is typically the case in the literature, and that this research is binary so that it either occurs or does not. The top panel of Figure 9 shows the marginal costs and benefits of applying a tax. Marginal costs rise as the tax grows as is typically the case, while the marginal reduction in dirty energy output, considered a benefit due to the climate externality, falls. Consider optimal policy involving a tax and research subsidy: the tax is set as shown, leading to a surplus of area A, and the research subsidy ensures clean research occurs. Under tax-only policy, a higher tax is needed to direct clean research, leading to a welfare cost of area B relative to optimal policy. Under research-only policy, the welfare cost relative to optimal policy is area A as this surplus is not accessible without a carbon tax.

However, before clean sources have dominated, output is less than linear in either technology so the aggregate costs are higher.

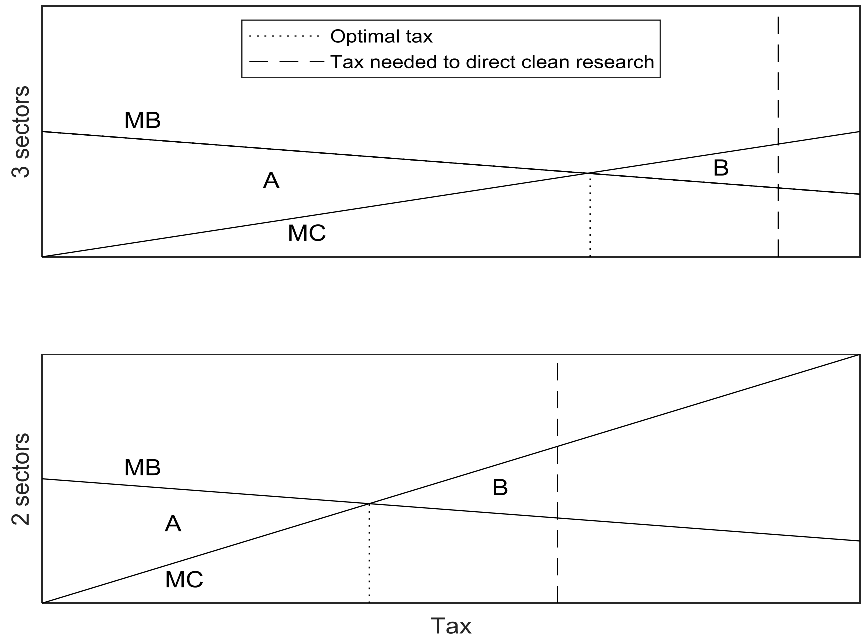

FIGURE 9: MARGINAL COSTS (MC) AND BENEFITS (MC) OF A CARBON TAX IN 2 AND 3 SECTOR MODELS

Now consider a model with only 2 sectors (clean and dirty), shown in the bottom panel of Figure 9. As Proposition 1 outlines, the marginal cost is exaggerated in this case and thus the marginal cost line is steeper. As the substitutability between clean and dirty energy is high relative to the substitutability between non-energy and energy, the tax primarily affects the ratio of clean and dirty energy in the three-sector model and so the marginal benefits line is similar under both models.[35] Under tax-only policy, the additional tax increment required to direct clean research is the same by assumption. This leads to a larger area B representing the welfare cost of tax-only policy relative to optimal policy, and a smaller area A representing the welfare cost of research-only policy. Thus, in the 2-sector model, the optimal tax is

[35] As energy demand is inelastic, a tax leads to a large shift in demand from dirty to clean energy and a small shift from energy to non-energy.

lower and subsidy-only policy is favoured for two reasons: the same tax increment needed to direct clean research comes at a higher cost; and the benefits of the optimal tax are lower, so the cost of subsidy-only policy are lower.

The result in part (2) comes from the exaggerated macroeconomic cost as described in part (1), leading to steepening of the marginal cost curve while the marginal benefit curve is relatively unaffected. This holds when no misdirection costs from subsidies are considered, as assumed by Acemoglu et al. (2012). Including such a cost would raise the marginal benefit curve for the tax (as increasing the tax lowers the subsidy required to direct clean research) and hence the relative effect on suboptimal policy may differ.

Due to the assumed low substitutability between non-energy and energy, the movement of researchers between sectors leads to the equalisation of productivities.

LEMMA 3: Assume a clean transition where there is an equal chance of innovation in clean energy and non-energy $\eta := \eta_c = \eta_0$, long-run shares s_0^{LR} and s_c^{LR} equal share parameters δ_0 and δ_e respectively and variables stabilise. Both non-energy and energy sectors asymptotically grow at the rate $\gamma\eta$ and productivities asymptote to each other.

A proof is shown in Appendix D. Even under a clean transition where the clean share approaches 100%, dirty inputs may continue to grow in the long run and the following definition is useful for intuition.

DEFINITION 1: An environmental disaster occurs if dirty energy inputs continue to increase in the long run.

From (9), (11) and (22), long-run dirty inputs, with $A_{dt} = 1$ for simplicity, are:

$$(22) \qquad Y_{dt} \xrightarrow{t\to\infty} \left(\frac{\alpha p_{c\infty}}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{{A_{c\infty}}^{\alpha+(1-\sigma)(1-\alpha)} L_{c\infty}}{(1+\tau_{\infty})^{\sigma}}.$$

As (22) shows, increasing the carbon tax to infinity drives dirty input Y_{dt} to zero and thus environmental disaster can always be avoided with a tax alone. But while possible, is disaster avoided under optimal policy settings? The following proposition describes the conditions that avoid disaster both under optimal and suboptimal policy.

PROPOSITION 2: Assume an elasticity of substitution between clean and dirty energy $\sigma > 1$ and between non-energy and energy $\varepsilon < 1$. Then (i) environmental disaster is certainly avoided under optimal policy and tax-only policy, (ii) disaster is avoided under subsidy-only policy if $\sigma > \frac{1}{1-\alpha}$; and (iii) if $1 < \sigma < \frac{1}{1-\alpha}$, disaster can be avoided under subsidy-only policy (although it is not optimal to do so) at the cost of long-run growth, which is $\left(1 + \frac{\frac{1}{1-\alpha}-\sigma}{\sigma-\varepsilon} s_e^{LR}\right)^{-1} \gamma\eta$.

The proof is in Appendix D. The movement of researchers between sectors increases the power of directed technical change alone to avert environmental disaster. The intuition is that while ${A_{ct}}^{\alpha+(1-\sigma)(1-\alpha)}$ rises when growth in A_{ct} is boosted and growth in A_{0t} is reduced, labour in clean energy and the clean price are reduced which dominate in (22) and dirty energy inputs are reduced.

Note that profits are proportional to $p_{jt}^{\frac{1}{1-\alpha}} \frac{L_{jt}}{s_j^{LR}} A_{jt-1}$ from (16), while wage equalisation implies $p_{jt}^{\frac{1}{1-\alpha}} A_{jt}$ are equal between sectors from (11). As labour allocations approach their long-run shares and productivities grow at the same rate from lemma 3, clean energy and non-energy profits equalise and therefore the long-run subsidy approaches zero.

REMARK 1: The profits subsidy to clean energy is positive if $\frac{L_{ct}}{s_e^{LR}}\frac{A_{ct-1}}{A_{ct}} > \frac{L_{0t}}{s_0^{LR}}\frac{A_{0t-1}}{A_{0t}}$ and negative if $\frac{L_{ct}}{s_e^{LR}}\frac{A_{ct-1}}{A_{ct}} < \frac{L_{0t}}{s_0^{LR}}\frac{A_{0t-1}}{A_{0t}}$. In the long run, the subsidy approaches zero.

III. Numerical results

This section first discusses parameter choices shown in Table 5. Results for optimal policy as well as second-best policy when only a tax or subsidy is used are presented. Then I consider a more optimistic case where optimal policy is achieved in 2050, and only one instrument applies until then. Of course, there is an infinite number of possible suboptimal frameworks that could be considered, but this approach would seem to be of interest to policymakers and leads to a result that is quite different to the case where only one instrument is available permanently.

Parameterisation

My calibration approach is to set parameters based upon the literature, with some appeal to the data for stepping-on-toes, and check robustness using sensitivities. First, consider the size of the energy sector as a proportion of output. As indicated by proposition 1, this parameter is important in determining the relative performance of tax-only and subsidy-only policies. Golosov et al. (2014) assume a value of 0.04 while the DICE 2016R2 model uses a value of 0.05. A value of 0.04 is in line with US values from the Energy Information Administration (Administration, 2012) which also projects this share to decline. The share should reflect the components of the economy that need to transform under a clean transition: for example, electricity grid costs should largely be excluded as a grid will be required with clean or dirty inputs. The main scenario uses a value of 0.04

and 0.05 is considered as a sensitivity, with quite different results despite the small change in value.

Results are also highly sensitive to the chosen elasticities of substitution: for example, the climate policy needed to avoid environmental disaster depends on the substitutability between clean and dirty energy. Most empirical estimates of the elasticity of substitution between clean and dirty energy range between 0.5 and 3 (Papageorgiou et al. (2017), Lanzi and Sue Wing (2011); Stern (2012) and Pelli (2012)) although higher substitutability has been found in the electricity sector (Wiskich (2021d) and Stöckl and Zerrahn (2020)). Elasticities used in integrated assessment and macroeconomic models have ranged between 10 and 1.[36] I use a value of 3 in the main part of this paper and consider a lower value of 1.5 as a sensitivity.

Substitutability between non-energy and energy allows for reductions in energy demand as the energy price rises. Kaufmann, Karadeloglou, and Di Mauro (2008) report an implied energy own-price elasticity in EIA scenarios of about -0.13. Hassler, Krusell, and Olovsson (2012) advocate a low elasticity of substitution between non-energy and energy in a model with endogenous technology, and this is my main approach. I choose a low elasticity of 0.1 for a discrete period of 5 years but consider a higher elasticity of 0.5 as a sensitivity.

Regarding the climate model, historical emissions go back a century and induce warming at 2020 of 1.18^{0}C, within the range of IPCC projections (IPCC, 2014). The damage parameter is set so that every degree of warming leads to roughly 1% of output reduction. A linear relationship between global damages and temperature is consistent with Burke, Hsiang, & Miguel, 2015,[37] although the magnitude of damage assumed is much less in the current paper. But the level of damage is

[36] 3 and 10 in Acemoglu et al. (2012), 4 in Hart (2019), about 1 in Golosov et al. (2014), 3 and 1.5 in Greaker et al. (2018).

[37] For damages up to around 10% of output, an exponential function is approximately linear.

greater than typically assumed in economic models for a moderate temperature increase, and the quadratic form assumed in the DICE 2016R2 model is considered as a sensitivity where damages of 2.1% and 8.5% result from warming of 3°C and 6°C respectively.

Acemoglu et al. (2012) assume clean energy initially makes up 18% of total energy. On one hand, this is a high estimate as it includes hydro and nuclear power, while renewable energy that will likely drive the clean transition (wind and solar) constitutes a much smaller share. For example, Hart (2019) assumes an initial clean share of 5%. On the other hand, the long-run share of clean energy should arguably be used which, due to long-lived capital and recent advances in clean technology, would likely be materially higher than the current renewable share. Therefore I consider an initial share of 10% and a sensitivity where the initial clean share is 18%.

Each period is 5 years and the time discount rate is 1.5% per annum, consistent with Acemoglu et al. (2012) and the DICE 2016R2 model. The main utility function is logarithmic, consistent with Golosov et al. (2014), while a function with relative risk aversion parameter κ of 1.5 is considered as a sensitivity, close to the value of 1.45 assumed in the DICE 2016R2 model. The share of machines in production is about equal to the share of capital at 1/3.

Patents last one period (5 years) in the main scenario, consistent with Acemoglu et al. (2012), and 10 years in an extended patent lifetime sensitivity. As it is difficult for the government to identify which research projects should be supported, a distortionary cost is assumed with parameter χ set so a consumption cost of 10% of subsidy expenditure is applied, consistent with Acemoglu et al. (2016). A sensitivity excludes such misdirection costs.

The diminishing returns parameter is set to 0.8, much greater than the value of 0.19 used in Hart (2019) which would seem to make rapid advancement of clean energy extremely costly, but similar to values of 0.7 and 0.79 used in Greaker et al.

(2018) and Fried (2018). A high value seems appropriate to apply at a sectoral level and allows a rapid advancement of clean technology as seen over the past two decades. For example, the number of renewable energy patents has increased from around 36,000 per annum between 2000 and 2004 to over 400,000 per annum between 2010 and 2014 (IRENA, 2017). Such a measure is of course a rough proxy for technological advance, but it is consistent with tremendous advances in technology. Holding the effects of research spillovers constant, a stepping-on-toes parameter of 0.19 would require over a 400-fold increase in researchers to generate a 10-fold increase in research output over two 5-year periods. In contrast, a parameter close to 1 is consistent with around a 3-fold increase in researchers for such a gain, roughly consistent with the increase in the total OECD public renewable energy research budget over this period (IEA, 2014). A stepping-on-toes parameter of 0.9 is considered as a sensitivity, along with the standard functional form for stepping-on-toes as described in the previous section.

The spillover parameter φ is assumed to be 0.5, consistent with Fried (2018) who uses the same functional form, and a sensitivity considers a higher value of 1. A difference in this paper is that the benefits of spillovers are not able to be captured fully by researchers. I consider that half of the benefits are captured (for small gains) so that $\varphi' = 0.5\varphi$.

The initial price for dirty energy is set equal to the non-energy price. Due to limits in the amount of energy extractable from a given unit of fossil fuel, Hart (2019) imposes a limit of technical progress in dirty energy, noting that the best modern coal-fired power stations are at about 75% of the thermodynamic limit. This paper recognises this limit and the maturity of dirty energy by assuming a lower value for the chance of innovation success in dirty energy. For non-energy and clean energy the chance of success is 2% per annum, implying a long-run annual growth rate of $\gamma\eta_0 = \gamma\eta_c = 2\%$ in a clean transition, but only 1% for dirty energy. Thus, this paper does not attempt to provide a structure that is consistent with the historical development of clean and dirty energy: Hart (2019) uses a structure where historical advances in dirty energy were more fruitful as the frontier was further away, and Lemoine (2017) explores transitions between dirty inputs by assuming different qualities of resources which determine long-run input shares. However, projections of the clean share are broadly consistent with Hart (2019). Without climate policy, there is no material energy research for a few decades before clean research occurs – the clean share reaches 50% in around 2100, about the same time as in the climate-change denial scenario in Hart (2019). Under optimal policy, the transition occurs quickly: after 50 years the clean share is around 90% in the main scenario although this takes a century in sensitivity s3 with low substitutability between clean and dirty energy: for comparison Hart (2019) finds it takes around 75 years under optimal policy.

TABLE 5 — PARAMETER AND FUNCTIONAL ASSUMPTIONS.

Parameter		Main Value (sensitivity)
Number of years in a period		5
Discount rate	ρ	0.015 (0.025) per annum
Elasticity of substitution between non-energy and energy	ε	0.1 (0.5)
Elasticity of substitution between clean and dirty	σ	3 (1.5)
Share of machines in production	α	1/3
Size of innovation	γ	1
Probability of success in clean and non-energy research	η_c, η_0	0.02 per annum
Probability of success in dirty research	η_d	0.01 per annum
Patent lifetime	n	5 (10) years
Historical dirty energy use (GtC,1925,...,2015)	Y_{dt}	[5,5,7.5,7.5,10,12.5,15,17.5,20,22.5,25,25,27.5,30,35,40,45,50,50]
Initial production of clean energy (2015)	Y_{c0}	5.6 (8.8)
Damage function	D	$e^{-0.01T_t}\,(1-0.00236{T_t}^2)$
Subsidy misdirection cost parameter	χ	0.1 (0)
Long-run share of researchers in energy and the share parameter for energy in final production	$s_e^{LR}=\delta_e$	4% (5%)
Diminishing returns to research parameter	ω	0.8 (0.9)
Sectoral spillovers to research	φ	0.5 (1)
Utility function - coefficient of relative risk aversion	κ	1 (1.5)

Numerical results

Figure 10 shows results for the main scenario under optimal policy and suboptimal policy where only a tax or a subsidy is possible. Optimal policy involves a subsidy to clean energy in the first few decades and a carbon tax which is almost flat as a ratio of income. The starting tax in 2020$US is $60 per tonne carbon dioxide (panel A).[38] This corresponds to a tax-to-consumption ratio of 0.000333 per GtC which is higher than the ratio derived from (18) of 0.000316 due to the extra benefit of taxes from reducing subsidy misdirection costs (as lower subsidies are needed). Under tax-only policy, the tax is initially higher to direct clean research, and then drops below the optimal tax level because the combination of the optimal tax level and research spillovers lead to more clean research than is optimal after the first few decades. Panel B also demonstrates the potential for excessive clean research (when a carbon tax and research spillovers exist) as the clean subsidy under optimal policy falls below zero, similar to the effect described by Gerlagh, Kverndokk, and Rosendahl (2014).[39] Without a tax available, a much higher subsidy is needed to direct clean research early on and the subsidy persists for longer.

Both the clean research share and clean labour share exhibit a hump-shaped profile (panels C and D). There is a period of clean technology catch-up with the non-energy sector before clean energy and non-energy research shares asymptote to long-run shares.[40] Panel E shows the high cost of tax-only policy initially, through lower consumption growth, and a long-run cost of subsidy-only policy due to greater damages from warming. As the effects of subsidies take time, the clean

[38] For comparison, Nordhaus (2017) finds a social cost of carbon of $44 (converting $31 in 2015 using 2010 $US) per tonne of carbon dioxide using the DICE-2016R2 model.

[39] A negative subsidy (a profits tax) applied to clean energy is not generally considered as a policy option. However, restricting the subsidy-only case to positive subsidies does not change the welfare result materially.

[40] This stabilization of research shares is similar to that described in Lemoine (2017). Fried (2018) and Hémous (2016) also consider three-sector models but focus on different insights to this paper.

transition is slower under subsidy-only policy and hence warming is considerably higher (panel F).

Consider the performance of suboptimal policy. Figure 11 shows how subsidy-only policy compares with tax-only policy using the ratio of utility loss: a number over one means that tax-only policy is preferred. Reflecting the uncertainty in parameter values and at the risk of showing too much information, I include not only the main scenario and 9 sensitivities, but also combinations of 2 sensitivities to make 46 scenarios in total: the main scenario is shown circled in the sensitivity 1 column; scenarios with only one sensitivity are circled in columns 2 to 10; and each combination of 2 sensitivities are shown by numbered data points (number *y* in column *x* combines sensitivities *x* and *y*).

The top panel shows results for permanent suboptimal policy: subsidy-only policy leads to 3.3 times the utility loss of tax-only policy relative to optimal policy, and tax-only policy outperforms in most scenarios. Subsidies perform as well as taxes with the high coefficient of relative risk aversion (reflecting the higher effective discount rate) alone, and performs better than taxes when this sensitivity (s6) is combined with many others.

There are three main considerations regarding different parameter choices corresponding to three effects to Figure 9: changing the marginal cost line; changing the marginal benefits line; and changing the tax required to direct clean research. First, the bigger the energy sector as a proportion of the economy the more subsidy-only policy is favoured and this is a key message of this paper. If the energy sector is assumed to be 5% of the economy (sensitivity s2) rather than 4%, tax-only policy outperforms by a factor of 1.9 instead of 3.3. Proposition 1 explains why a 2-sector approach can exaggerate the performance of subsidy-only policy relative to tax-only policy (the marginal cost line is raised in Figure 9). For a comparison between 2 and 3-sector results, I consider the sensitivity without subsidy misdirection costs for comparability with previous literature. In the 3-sector model in this sensitivity, subsidy-only policy leads to a utility loss that is 286% of the tax-only policy, while in the 2-sector model this ratio is only 5% and so subsidy-only policy is preferred.

Second, parameter choices that increase the (future) impact of climate change tend to favour tax-only policy, as the benefits of distorting production tend to increase (raising the marginal benefit line in Figure 9). Thus, low substitutability between clean and dirty energy (s3) improves the relative performance of tax-only policy while high substitutability between non-energy and energy (s4), quadratic (lower at moderate temperatures) damages (s5), a high coefficient of relative risk aversion with high interest rate (s6),[41] and removing subsidy misdirection costs (s7) tend to improve the performance of subsidy-only policy. The dependence of the optimal carbon tax on the discount rate is well understood: this paper finds that the relative performance of taxes and subsidies is also highly dependent on the discount rate.

Third, parameter choices implying a higher tax is required to direct clean research (in the absence of subsidies) improve the relative performance of subsidy-only policy (the dashed vertical line in Figure 9 is shifted to the right). Thus, while future temperatures are lower with a high initial clean share (s8), a lower tax is required to direct clean research and hence the relative performance of tax-only policy is improved. High research spillovers (s9) and the standard functional form for stepping-on-toes (s10) also lower the tax (and subsidy) required to direct clean research, favouring tax-only policy.[42] A lower stepping-on-toes effect (not shown) reduces the loss of the “hump” of clean catch-up shown in the third panel of Figure 10: this tends to increase the size of this hump and reduce optimal clean research

[41] Increasing the time discount rate is not shown as it has a similar effect to the high coefficient of relative risk aversion sensitivity.

[42] Returns to research are higher for low clean research shares in s10, so the effect is similar to higher spillovers for research.

initially, lowering the required tax hence favouring tax-only policy but only marginally. The initial tax required under extended patent life (not shown) is slightly higher: while a future shift to clean energy lowers the subsidy required to direct clean research, the initial tax needed is higher as the future tax falls which marginally favours subsidy-only policy.[43]

As an example, the starting optimal tax in the main scenario is $60 per tonne carbon dioxide and the clean research share is 3.5%. With only a tax available, the starting tax rises to $143 which induces a clean share of 2.8% and tax-only policy outperforms subsidy-only policy. The combination of sensitivities using the DICE damage function (s5) and with a coefficient of relative risk aversion of 1.5 (s6), close to that of the DICE 2016R2 model, leads to a starting optimal tax of only $33 per tonne and clean share of 2.7%. With only a tax available, the tax starts at only $29 and rises to $122 after a delay of 5 years, inducing a clean research share of 2.0%. As the tax rises further as a proportion of the optimal level, subsidy-only policy is preferred in this scenario.

I have shown combinations of two sensitivities as the effect of sensitivities are sometimes complex. For example, the quadratic damage function (s5) has lower damages for moderate temperature increases and hence this sensitivity tends to favour subsidy-only policy. However, when combined with low substitutability between clean and dirty energy (s3) with resulting higher future temperatures, tax-only policy is favoured due to increasing marginal damages as temperature rises.

Considering a more optimistic scenario where one instrument is used until 2050 and then both are available, the relative performance of subsidy-only policy is improved in all sensitivity combinations: in the main scenario the utility cost

[43] Extended patent lifetimes lead to a diminished role for subsidies, as found by Greaker and Heggedal (2012). This result is unsurprising as the knowledge externality that is corrected by the subsidy involves long-run gains from knowledge that are not captured in returns to investors over the patent life. Note that this paper assumes a constant number of researchers in aggregate, and thus the effect of changing patent life is through changing relative profits between sectors and not through changing aggregate research.

reduces from 3.3 to 1.9 times the cost of tax-only policy, and subsidy-only policy outperforms in many scenarios. As the cost of subsidy-only policy is spread over time while the cost of tax-only policy occurs early, the attainment of optimal policy in the long run improves the relative performance of subsidy-only policy.

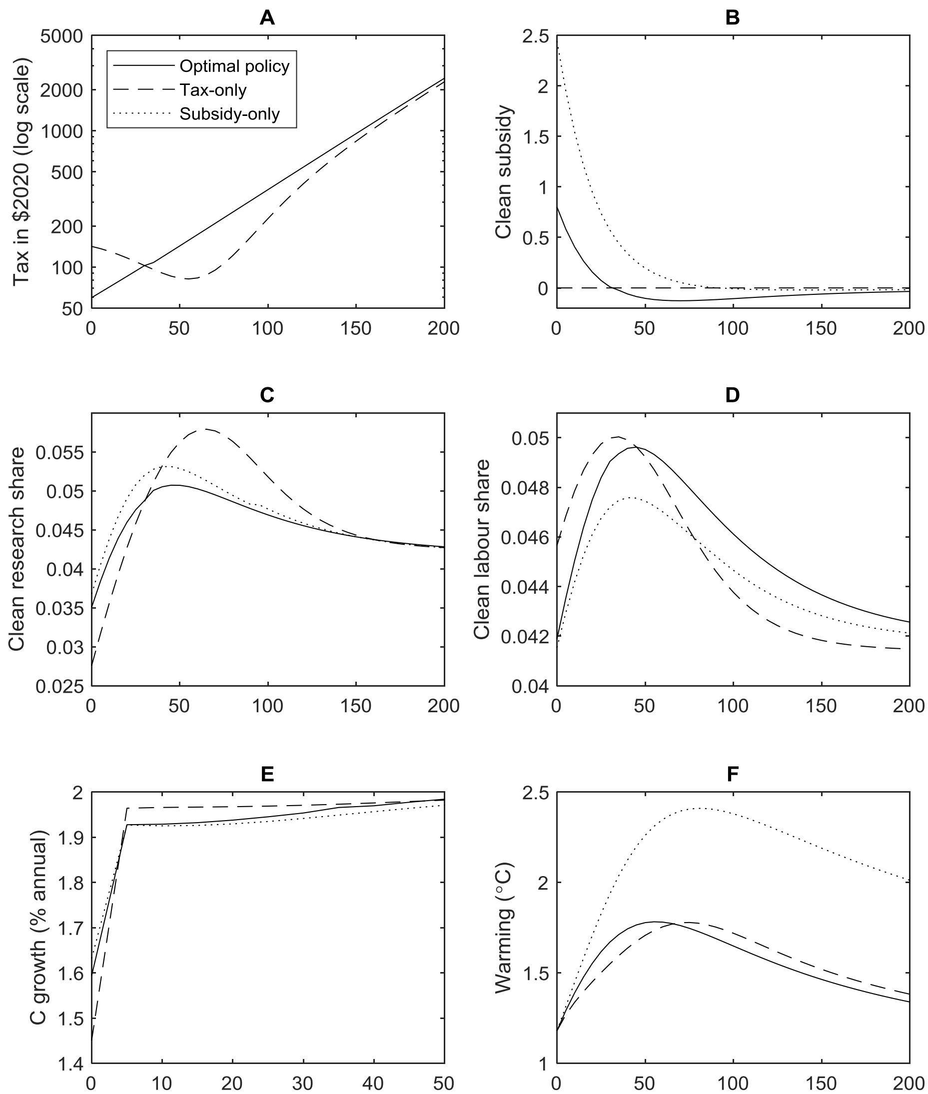

FIGURE 10: OPTIMAL AND PERMANENT SUBOPTIMAL POLICY RESULTS

Tax level in \$2020 per tonne carbon dioxide. Clean subsidy is a proportion of profits.

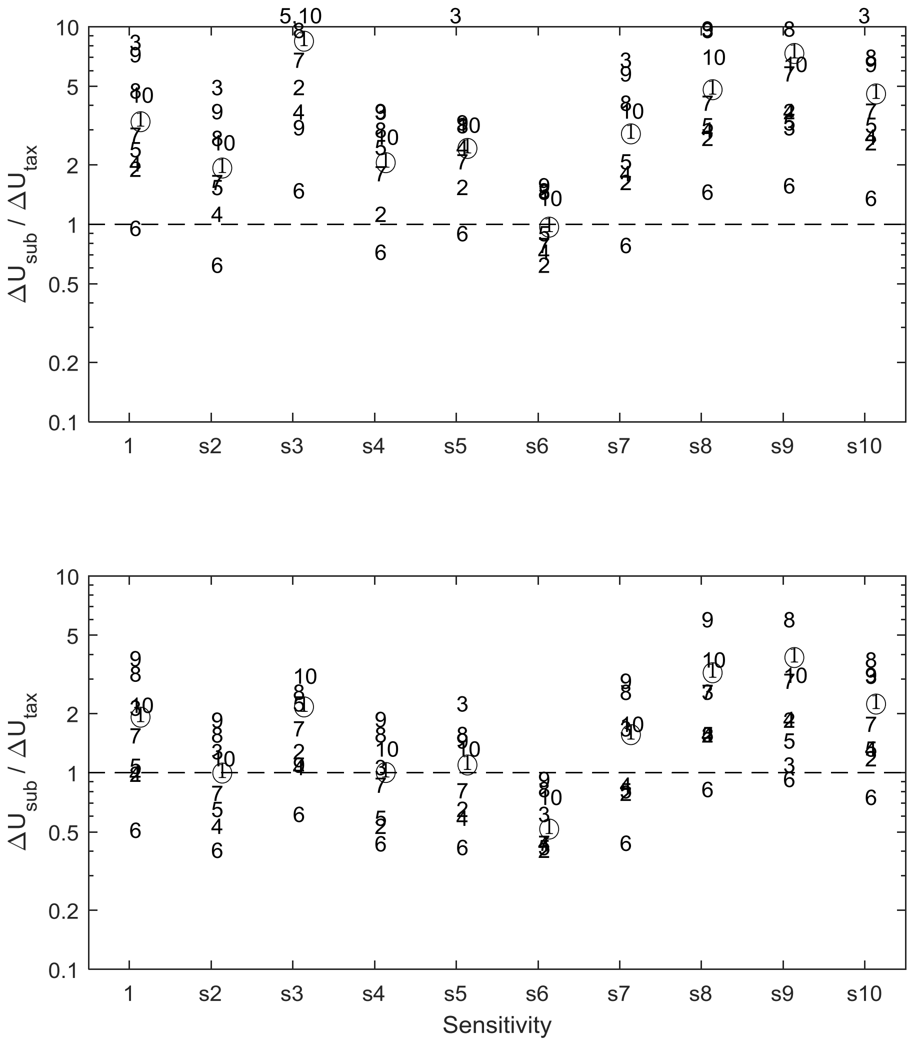

FIGURE 11: RATIO OF UTILITY COSTS (COMPARED WITH OPTIMAL POLICY, LOG SCALE) FOR SUBOPTIMAL SUBSIDY-ONLY POLICY AND TAX-ONLY POLICY.

A number over one means tax-only policy outperforms subsidy-only policy. Results for every combination involving two sensitivities are shown (in duplicate). Sensitivities: 1=Main; s2=High energy share of output; s3=Low substitutability between clean and dirty energy; s4=High substitutability between non-energy and energy; s5=Quadratic climate damage; s6= High coefficient of relative risk aversion; s7=No subsidy misdirection costs; s8=High initial clean energy share; s9=High research spillovers; s10=Commonly-used stepping-on-toes functional form.

IV. Conclusion

The model outlined in this paper incorporates endogenous technology and the free movement of workers and researchers between energy and non-energy sectors under a clean transition. The movement of researchers increases the power of policy to avert environmental disaster and leads to a period of intense research in the clean sector above the long-run share, as productivity in the clean sector catches up to the non-energy sector. This paper helps explain the differing results found in previous literature on the relative performance of tax-only and subsidy-only policies, and some drivers of relative performance are discussed using numerical examples. Permanent tax-only policy outperforms subsidy-only policy across a broad range of parameter assumptions, while a high discount rate favours subsidies. If optimal policy is eventually reached and suboptimal policy is only temporary, the relative performance of subsidy-only policy is closer to tax-only and performs better in some scenarios.

Regarding limitations, the model is deterministic and hence the uncertainty of climate impacts, such as the risk of tipping points, are not considered. If tipping points are triggered by temperature rising above an unknown threshold, optimal policy may be stronger in the near term to limit the maximum level of warming (Wiskich, 2021c) – this would likely favour tax-only policy as a lower peak temperature is achievable than with subsidy-only policy. Further, physical capital is not included and hence precautionary capital formation considered in other papers is not present.

Distortionary fiscal costs from pre-existing taxes are not considered. A recent paper finds that considering this effect in the US reduces the optimal tax below the Pigouvian level (Lint Barrage, 2020), which would probably improve the relative performance of subsidy-only policy. However, considering existing production subsidies to dirty energy including coal would improve the relative performance of tax-only policy, as would raising other distortionary taxes to finance the research subsidies, so the net effect is unclear and could be investigated.

An interesting extension would include research in less dirty technology such as natural gas. Although the model considers substitutability between non-energy and energy, energy savings through demand-side capital investment (such as making houses more energy efficient) or supply-side investment (such as making fossil power stations more efficient) are not explicitly included in the model and may also make an interesting extension. The lifetime of energy assets, which would reduce the potential for transition in the short term, could be considered. Finally, other suboptimal policy experiments might be considered. While this paper considers an optimistic future where optimal policy is eventually achieved, global policy coordination may deteriorate in the future instead which would further favour a carbon tax in the interim. The suboptimal nature of policy in this paper is the restriction of available instruments: it may be useful to weigh feasible magnitudes of tax-only and subsidy-only policies against each other.

APPENDIX A – SOLVING THE NUMERICAL MODEL

From (11) and the price equations

$$(A.1) \qquad {p_{ct}}^{1-\sigma} + \left(p_{dt}(1+\tau_t)\right)^{1-\sigma} = {p_{et}}^{1-\sigma}$$

$$(A.2) \qquad \delta_0 {p_{0t}}^{1-\varepsilon} + \delta_e {p_{et}}^{1-\varepsilon} = 1$$

each price can be derived given technologies, such as:

$$(A.3)\ p_{0t} = \left(\delta_0 + \delta_e\left[\left(\frac{A_{0t}}{A_{ct}}\right)^{(1-\sigma)(1-\alpha)} + \left(\frac{A_{0t}}{A_{dt}}\right)^{(1-\sigma)(1-\alpha)}(1+\tau_t)^{1-\sigma}\right]^{\frac{1-\varepsilon}{1-\sigma}}\right)^{\frac{1}{\varepsilon-1}}.$$

First-order conditions for (2) lead to

$$(A.4) \qquad \frac{p_{0t}}{p_{et}} = \left(\frac{\delta_0}{\delta_e}\right)^{\frac{1}{\varepsilon}} \left(\frac{Y_{0t}}{Y_{et}}\right)^{\frac{-1}{\varepsilon}} \quad \text{and (9) leads to}$$

$$(A.5) \qquad \frac{Y_{it}}{Y_{jt}} = \left(\frac{p_{it}}{p_{jt}}\right)^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{L}_{it}\mathrm{A}_{it}}{\mathrm{L}_{jt}\mathrm{A}_{jt}}.$$

Labour input ratios can then be determined, such as

$$(A.6) \qquad \frac{\mathrm{L}_{0\mathrm{t}}}{\mathrm{L}_{ct}} = \left(\frac{A_{0\mathrm{t}}}{A_{ct}}\right)^{(\varepsilon-1)(1-\alpha)} \frac{\delta_0}{\delta_e} \left((1+\tau_t)^{1-\sigma} A_{ct}{}^{(1-\sigma)(1-\alpha)} + 1\right)^{\frac{\sigma}{\sigma-1}}$$

and labour inputs are then derived from (6) and (21). Optimal taxes and subsidies are determined numerically to optimise welfare.

APPENDIX B – CLIMATE MODEL

Figure 12 shows the assumed temperature impact from a unit of carbon emissions from Shine et al. (2005). Many papers have assumed that the temperature response to a carbon pulse peaks after several decades (Gerlagh and Liski (2018) and the DICE 2013 model) which is inconsistent with recent physical science literature. In continuous-time, temperature dynamics are a function of radiative forcing R_t:

$$(\mathrm{B.1}) \qquad H\frac{dT_t}{dt} = R_t - \frac{T_t}{\lambda},$$

where H is the heat capacity of the system and λ is a climate sensitivity parameter. For carbon, radiative forcing and temperature responses at time t after an emissions pulse (in discrete time) are

$$(B.2) \qquad R_t^c := \frac{\partial R_t}{\partial Y_{d0}} = a_0 + \sum_{i=1}^{4} a_i e^{-\frac{t}{\alpha_i}} \text{ and}$$

$$(B.3) \qquad T_t^c := \frac{\partial T_t}{\partial Y_{d0}} = \frac{B_c}{H}\left\{\zeta a_0\left(1 - e^{-\frac{t}{\zeta}}\right) + \sum_{i=1}^{4} \frac{a_i\left(e^{-\frac{t}{\alpha_i}} - e^{-\frac{t}{\zeta}}\right)}{(\zeta^{-1} - \alpha_i{}^{-1})}\right\},$$

where a_i are coefficients which sum to 1, α_i reflect gas lifetimes in years, ζ is by definition the constant λH in years, and B_c is the radiative forcing due to a 1-kg change in carbon dioxide. Parameter values are shown in Table 6.

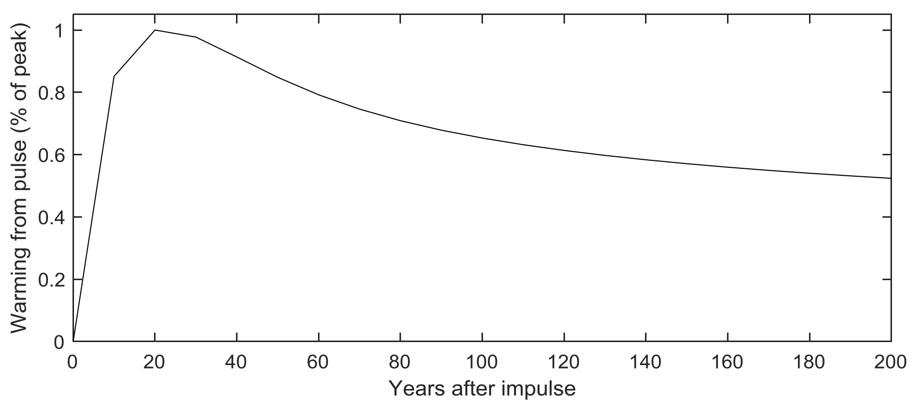

FIGURE 12: TEMPERATURE RESPONSES TO AN EMISSIONS PULSE

TABLE 6: DETAILED CLIMATE MODEL PARAMETERS

B_c	a_0	a_1	a_2	a_3	a_4
1.98	0.1756	0.1375	0.1858	0.2423	0.2589
H	ζ	α_1	α_2	α_3	α_4
4.2	10.65	421.09	70.597	21.422	3.4154

Appendix C – Two-sector model

Consider energy output Y_{et} defined in (3) and consumption given by

$$(C.1) \qquad C_t = Y_{et} - \psi \int_0^1 (x_{cit} + x_{dit})\, di.$$

Other equations are as given in section 1, with $L_{ct} + L_{dt} = 1$ for (6) and $s_{ct} + s_{dt} \leq 1$ for (14).

Appendix D – Proofs

Lemma 3

The elasticity of substitution between non-energy and energy is less than one ($\varepsilon > 1$) and if long-run prices, labour and research allocations stabilise, from (20) $\frac{p_{ct}^{\frac{1}{1-\alpha}} \mathrm{L}_{ct} \mathrm{A}_{ct}}{s_c^{LR}} = \frac{p_{0t}^{\frac{1}{1-\alpha}} \mathrm{L}_{0t} \mathrm{A}_{0t}}{s_0^{LR}}$. From (11), this implies that $\frac{\mathrm{L}_{0t}}{\mathrm{L}_{ct}} = \frac{s_0^{LR}}{s_c^{LR}}$, and (A.6) becomes

$$(D.1) \qquad \frac{s_0^{LR}}{s_c^{LR}} = \left(\frac{A_{0t}}{A_{ct}}\right)^{(\varepsilon-1)(1-\alpha)} \frac{\delta_0}{\delta_e} \text{ as } \sigma > 1$$

Since $\frac{\delta_0}{\delta_e} = \frac{s_0^{LR}}{s_c^{LR}}$ by assumption, we must have $\frac{A_{0t}}{A_{ct}} = 1$, $\frac{\mathrm{L}_{0t}}{\mathrm{L}_{ct}} = \frac{\delta_0}{\delta_e}$ and $p_{\mathrm{ct}} = p_{0t} = 1$ from (11) and (B.2).

Proposition 2

From (22) and if prices and labour and research allocations stabilise, the long-run growth in dirty inputs $\mathrm{g}_{\mathrm{Y}_d} = [\,\alpha + (1-\sigma)(1-\alpha)]\mathrm{g}_{\mathrm{A}_c} - \sigma \mathrm{g}_\tau$. As $\tau_t = \frac{\Lambda_t}{p_{dt}} =$

$\frac{\Lambda_t}{p_{ct}A_{ct}{}^{1-\alpha}}$ (from (11) and setting $A_d = 1$) and as the carbon tax grows with consumption $\mathrm{g}_{\Lambda} = \mathrm{g}_{\mathrm{C}} = \mathrm{g}_{\mathrm{A}_c}$, then $\mathrm{g}_{\mathrm{Y}_d} < 0$ if $\alpha + (1-\sigma)(1-\alpha) - \sigma(1-(1-\alpha)) < 0$ which reduces to $\sigma > 1$ which holds by assumption. Without a carbon tax, $\mathrm{g}_{\mathrm{Y}_d} \xrightarrow{t\to\infty} [\,\alpha + (1-\sigma)(1-\alpha)]\mathrm{g}_{\mathrm{A}_c}$.

Considering the suboptimal application of subsidies without a carbon tax, environmental disaster can still be avoided if $1 < \sigma < \frac{1}{1-\alpha}$. Labour in clean energy and the clean price are

$$(D.2) \qquad \mathrm{L}_{ct} = \frac{1}{1 + \left(\frac{A_{0\mathrm{t}}}{A_{ct}}\right)^{(\varepsilon-1)(1-\alpha)} \frac{\delta_0}{\delta_e}} \xrightarrow{t\to\infty} \frac{\delta_\mathrm{e}}{\delta_0}\left(\frac{A_{0\mathrm{t}}}{A_{ct}}\right)^{(1-\varepsilon)(1-\alpha)} \text{ and}$$

$$(D.3) \qquad p_{ct}{}^{1-\varepsilon} = \frac{1}{\left(\frac{A_{\mathrm{ct}}}{A_{0t}}\right)^{(1-\varepsilon)(1-\alpha)} \delta_0 + \delta_e} \xrightarrow{t\to\infty} \frac{1}{\delta_0}\left(\frac{A_{0\mathrm{t}}}{A_{ct}}\right)^{(1-\varepsilon)(1-\alpha)} \text{ so}$$

$$(D.4) \qquad \mathrm{g}_{\mathrm{L}_c} \xrightarrow{t\to\infty} (1-\varepsilon)(1-\alpha)\left(\mathrm{g}_{\mathrm{A}_0} - \mathrm{g}_{\mathrm{A}_c}\right) \text{ and } \mathrm{g}_{\mathrm{p}_c} \xrightarrow{t\to\infty} (1-\alpha)(\mathrm{g}_{\mathrm{A}_0} - \mathrm{g}_{\mathrm{A}_c})$$

Thus from (22)

$$(D.5) \qquad \mathrm{g}_{\mathrm{Y}_d} \xrightarrow{t\to\infty} \frac{\alpha}{1-\alpha}\mathrm{g}_{\mathrm{p}_c} + \ [\alpha + (1-\sigma)(1-\alpha)]\mathrm{g}_{\mathrm{A}_c} + \mathrm{g}_{\mathrm{L}_c} \text{ leading to}$$

$$(D.6) \qquad \mathrm{g}_{\mathrm{Y}_d} < 0 \ if \ \mathrm{g}_{\mathrm{A}_c} > \left(1 + \frac{\frac{1}{1-\alpha} - \sigma}{\sigma - \varepsilon}\right)\mathrm{g}_{\mathrm{A}_0}.$$

Now $s_e^{LR} g_{A_c} + (1 - s_e^{LR}) g_{A_0} = \gamma\eta$ so the long-run growth consistent with avoiding environmental disaster is

$$(D.7) \quad g_Y = g_C = g_{A_0} \leq \left(1 + \frac{\frac{1}{1-\alpha} - \sigma}{\sigma - \varepsilon} s_e^{LR}\right)^{-1} \gamma\eta.$$

APPENDIX E – SENSITIVITY RESULTS

TABLE 7 — RATIO OF UTILITY COSTS (COMPARED WITH OPTIMAL POLICY) FOR SUBOPTIMAL SUBSIDY-ONLY POLICY AND TAX-ONLY POLICY

	(1)	(s2)	(s3)	(s4)	(s5)	(s6)	(s7)	(s8)	(s9)	(s10)
Main (1)	3.3 (1.9)	1.9 (1.0)	8.4 (2.2)	2.0 (1.0)	2.4 (1.1)	1.0 (0.5)	2.9 (1.6)	4.8 (3.2)	7.3 (3.8)	4.6 (2.2)
High energy share of output $\delta_e = 0.05$ (s2)			5.0 (1.3)	1.1 (0.5)	1.6 (0.7)	0.6 (0.4)	1.6 (0.8)	2.8 (1.6)	3.8 (1.9)	2.6 (1.2)
Low elasticity between clean and dirty $\sigma = 1.5$ (s3)		5.0 (1.3)		3.7 (1.1)	16.9 (2.3)	1.5 (0.6)	6.8 (1.7)	9.7 (2.6)	3.1 (1.1)	12.9 (3.1)
High elasticity between non-energy and energy $\varepsilon = 0.5$ (s4)		1.1 (0.5)	3.7 (1.1)		2.5 (0.6)	0.7 (0.4)	1.8 (0.9)	3.0 (1.6)	3.8 (1.9)	2.8 (1.3)
Quadratic climate damage from DICE-2016R2 (s5)		1.6 (0.7)	16.9 (2.3)	2.5 (0.6)		0.9 (0.4)	2.1 (0.8)	3.2 (1.6)	3.3 (1.5)	3.2 (1.3)
High coefficient of relative risk aversion $\kappa = 1.5$ (s6)		0.6 (0.4)	1.5 (0.6)	0.7 (0.4)	0.9 (0.4)		0.8 (0.4)	1.5 (0.8)	1.6 (0.9)	1.4 (0.8)
No- subsidy misdirection costs $\chi = 0$ (s7)		1.6 (0.8)	6.8 (1.7)	1.8 (0.9)	2.1 (0.8)	0.8 (0.4)		4.1 (2.6)	5.8 (3.0)	3.8 (1.8)
High initial clean energy share $\delta_e = 0.05$ (s8)		2.8 (1.6)	9.7 (2.6)	3.0 (1.6)	3.2 (1.6)	1.5 (0.8)	4.1 (2.6)		9.8 (6.1)	7.1 (3.8)
High research spillovers $\varphi = 1$ (s9)		3.8 (1.9)	3.1 (1.1)	3.8 (1.9)	3.3 (1.5)	1.6 (0.9)	5.8 (3.0)	9.8 (6.1)		6.5 (3.2)
Commonly used stepping-on-toes functional form (s10)		2.6 (1.2)	12.9 (3.1)	2.8 (1.3)	3.2 (1.3)	1.4 (0.8)	3.8 (1.8)	7.1 (3.8)	6.5 (3.2)	

Results for permanent suboptimal policy are shown without brackets, and results for temporary suboptimal policy where optimal policy is achieved in 2050 are bracketed. A number greater than 1 means tax-only policy outperforms subsidy-only policy. Results for every combination involving two sensitivities are shown (in duplicate)

3. Substitutability between clean and dirty electricity generation under a clean transition

By ANTHONY WISKICH*

A clean transition in electricity generation will likely be driven by variable renewable energy. The elasticity of substitution between wind and solar inputs and dirty inputs in electricity is estimated to be 3 or more by fitting an aggregate production function to OECD panel data. Such a high elasticity is consistent with detailed electricity models which also predict that the substitutability decreases as the share of clean inputs rises, as integrating intermittent energy supply becomes increasingly difficult. A simple dispatch model of electricity generation demonstrates this characteristic. Decreasing substitutability implies higher costs of a clean transition, greater costs from regions transitioning sequentially rather than together, and perhaps a greater role for carbon taxes over research subsidies. (JEL O33, Q40, Q41, Q42)

Keywords: Elasticity of substitution; climate change; energy; electricity; production function.

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Responding to climate change almost certainly involves transitioning the global economy from dirty to clean energy. The substitutability between these inputs is an important factor in determining: the cost of this transition; the type, timing and extent of optimal policy; and the additional costs from suboptimal policy. The electricity generation sector is particularly important as it has the highest levels of greenhouse gas emissions and electrification may allow other sectors to transition away from dirty fuels, such as in transport. Variable renewable energy (VRE) wind and solar will likely drive the clean transition in electricity. This paper: (i) empirically finds an elasticity of substitution of 3 or more between VRE and dirty energy using panel OECD data; (ii) finds a high but decreasing elasticity is derived from electricity dispatch models and presents a stylised version of such a model that provides a micro foundation for decreasing substitutability; and (iii) discusses some policy implications of a decreasing elasticity.

The first contribution builds on the empirical approach described in Papageorgiou et al. (2017) and fits an isoelastic production function of electricity output to OECD panel data. Wind and solar, which are enabling most of the clean transition in electricity, are found to have a high elasticity of 3 or above with dirty electricity generation. Such a high elasticity is robust to different specifications and exceeds previous empirical estimates of 2 between clean and dirty electricity inputs (Papageorgiou et al., 2017)[44]; 1.6 between fossil-fuel and renewable energy (Lanzi & Sue Wing, 2011); and around 0.5 for the electricity sector (Pelli, 2012). However, recently Stöckl and Zerrahn (2020) derive some elasticity estimates of around 10 and such a high elasticity has been assumed in integrated assessment models that use an isoelastic function of clean and dirty inputs (Acemoglu et al., 2012; Greaker & Heggedal, 2012).[45]

[44] In an extension using non-parametric estimation methods Malikov, Sun, and Kumbhakar (2018) find that the substitutability may not be that strong

There are many reasons for less than perfect substitutability between VRE and dirty energy, such as geography, market distortions and the costs of integrating intermittent supply. The empirical analysis allows all these factors to drive results in some form but comes with many limitations. Regressions use capital costs as independent variables: as electricity assets are long-lived and capital costs are sunk, dirty capital will persist provided it remains profitable operationally which likely bias the empirical estimates. Further, the clean transition is still at an early stage so the estimation of substitutability at high clean shares is challenging.[46] Electricity dispatch models arguably give a better prediction of substitutability at high clean shares as they anticipate the impact of the variable nature of VRE.

The second contribution applies electricity dispatch models to the question of substitutability: a high but decreasing elasticity is derived from electricity papers describing regional models of electricity. The fall in elasticity relates to the increasing difficulty of integrating intermittent sources as the clean share rises, with the extent and timing dependent on storage and flexibility in demand.[47] While this phenomenon is well understood, this paper is the first to my knowledge which derives estimates of the elasticity of substitution from integration costs reported in such models.

Stöckl and Zerrahn (2020) also derive elasticity estimates from a dispatch electricity model for Germany and find similarly high substitutability which decreases with the VRE share. Some studies impose a changing elasticity between clean and dirty energy aggregates in model exercises. Mattauch, Creutzig, and

[46] Unfortunately, few economies have high shares of clean energy that can help determine how the elasticity between clean and dirty inputs might change as the share of clean energy rises. Regions that do have high clean shares generally take advantage of endowments that may not be transferable to other countries, such as hydro resources.

[47] Electricity is not easily storable and demand varies hour by hour and day by day, which increases the total system costs and means the optimal supply consists of a mix of technologies with different fixed and variable cost ratios. VRE sources wind and solar increase the variation in demand that must be met by dispatchable generation: one part of the integration costs associated with VRE. At low clean shares the cost is relatively low, implying a high elasticity between clean and dirty generation. As the clean share increases, the utilisation rates of dispatchable generation decrease and curtailment of intermittent generation occurs, further increasing costs, so substitutability will likely fall.

Edenhofer (2015) investigate an increasing elasticity of substitution as a proxy for the temporal consideration of a gradual increase in energy infrastructure. In contrast, the decreasing substitutability described in this paper is linked with the share of clean energy and reflects long-run considerations of supply and demand. Gerlagh and Lise (2005) consider a "hump-shaped" symmetric elasticity of substitution between clean and dirty inputs that decreases towards 1 as either input dominates. Other models include greater sectoral detail which implies a changing effective elasticity between clean and dirty aggregates: Golosov et al. (2014) differentiate between fuel inputs coal, gas/oil and renewables; McKibbin and Wilcoxen (1999) and models included in the EMF 27 and EMF 22 international comparison exercises have even more sectoral detail.

A simple cost-minimising[48] electricity dispatch (supply-side) model is described to provide a micro-foundation and help identify the determinants of substitutability in electricity, building on Wiskich (2014). This dispatch model reflects the range of integration costs reported in the electricity literature. Ambec and Crampes (2019) describe a similar model consisting of intermittent and reliable energy sources and storage, assuming one type of reliable energy and no variability in demand for electricity: in contrast, variable electricity demand is central to my model. The dispatch model suggests a high elasticity (over 4) for clean shares below about 50 per cent, with an elasticity of around 1 beyond this share.

The third contribution is a discussion of the policy implications of decreasing substitutability. The most obvious implication is that the costs of a clean transition will be higher if substitutability falls, so a model that assumes a constant elasticity calibrated from empirical estimates using data points at low clean shares will tend to understate the transition cost. Therefore, climate policies in general need to be

[48] The effects of market power discussed by Acemoglu, Kakhbod, and Ozdaglar (2017), Genc and Reynolds (2019) and Samano, Bahn, and Sarkis (2019) are not included for simplicity.

bigger given the same level of warming, or warming will be higher given the same level of policy intervention. Higher temperature outcomes have been found to lead to a greater optimal carbon tax-to-income ratio, which would likely result in smaller optimal research subsidies (Wiskich, 2021c).

Suboptimal policy considerations and the role of carbon taxes and research subsidies are receiving growing interest in the literature (Acemoglu et al. (2012), Greaker et al. (2018), Lemoine (2017), Hart (2019), Wiskich (2021b) and Hassler et al. (2020)). Coordinated international action is needed to deliver optimal climate policy – a common carbon price between regions helps ensure that global abatement occurs at the lowest cost - and two identical regions should undertake the clean transition at the same rate according to a stylised model. However, some regions have far more stringent climate policies than others and are further down the clean transition path. Decreasing substitutability can magnify the cost of this suboptimal action as differences in the marginal costs of abatement become higher if regions act sequentially, making it more important for regions to act in a coordinated way. Further, subsidy-only policy cannot distort production and this limitation becomes more important the greater the rise in future temperature: thus decreasing substitutability increases the performance of tax-only policy relative to subsidy-only policy.

I. An empirical investigation of substitutability

This section stands on the shoulders of Papageorgiou et al. (2017) who estimate the elasticity of substitution between clean and dirty inputs in electricity generation and other sectors directly from aggregate production functions. The current paper focuses on the substitutability between dirty energy and VRE, rather than a clean energy aggregate dominated by hydro and nuclear. The tremendous growth of VRE over recent years allows insights into this question. This paper also diverges in methodology and questions the robustness of the estimated elasticity of 2 by Papageorgiou et al. (2017) in electricity.

Figure 13 shows that the clean transition over the past decade has been driven by wind and solar generation. In 2020, wind and solar reached 9% of global generation, twice as high as in 2015, displacing generation share from dirty generation (and nuclear generation). Further, the potential for hydro uptake is limited by geography. Consequently, this paper focuses on the substitutability between VRE and dirty energy which is presumed to be the critical factor in the clean transition, rather than consider a clean energy aggregate including hydro and nuclear.

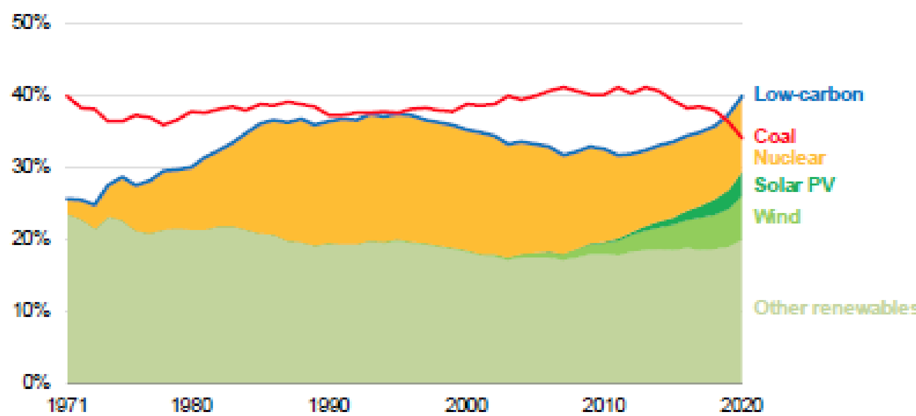

FIGURE 13: GLOBAL GENERATION SHARES FROM COAL AND LOW-CARBON SOURCES, 1971-2020

Source: Global Energy Review 2020.

Rather than identify the elasticity of substitution through responses in energy inputs to price changes, the estimation is based on the aggregate production function combining an input measure for VRE and a measure for dirty inputs. Such an approach does not require consideration of prices or regional policies which may induce VRE investment through price or quantity mechanisms. However, the approach assumes input-augmenting technological change to be neutral. For the main specification used by Papageorgiou et al. (2017) where input measures are generation capacity, this assumption means equal trends in the capacity factors (output in gigawatt-hours per unit of capacity) of capital, absent the effect of integrating VRE itself. This paper uses a cost-adjusted measure of dirty capacity and the data indicates that there has been no significant trend in capacity factors over the time range. However, the capacity factor of VRE has increased significantly due to technology gains, which could bias the estimate of the elasticity. The main specification uses the VRE output in gigawatt hours which should mitigate this bias. Labour costs are excluded as they are a minor part of generation costs and likely not substitutable.

Using output in VRE rather than capacity might raise a concern of endogeneity through simultaneity. However, wind and solar generation depends on the weather and hence cannot respond to demand. Further, given inelastic demand for electricity, any variation in annual generation per unit of VRE will be balanced by a change in dirty dispatchable generation, provided output is not traded, which should limit correlation with the error term. A final concern with using output is the potential effect that VRE has on itself: for high VRE shares curtailment may occur and the quality of additional sites may fall materially. However, for most of the data, it is unlikely curtailment is an issue. For dirty capacity, electricity generation assets are long-lived and take years to build, so there is little risk of endogeneity between these regressors and the error term. For example, capital stocks are unable to adjust to a demand shock in a year. Papageorgiou et al. (2017) consider a Cobb Douglas function of capital and fuel for dirty energy as a robustness check. As dirty energy is dispatchable, there is likely a strong correlation between fuel inputs and the error term as dispatchable dirty generation is worked harder (greater fuel input) under a demand shock. For this reason, in addition to simplicity, fuel is excluded in my main specification.[49]

A challenge with focussing on VRE is how to treat other clean generation technologies: hydro, nuclear and to a lesser extent geothermal and tidal. The main regressions in this paper subtract generation from these sources from the independent output variable, allowing identification of the substitutability between VRE and dirty energy. This method excludes effects from substitution between VRE and other clean sources, which may bias results.

For parameters ψ and $0 < \omega < 1$, country *i*, year *t*, output Y_{it}, VRE output Y_{Cit} and dirty capital K_{Dit}, consider the levels equation:

$$(1) \qquad lnY_{it} = a_i + b_t + \frac{1}{\psi} ln\left(\omega_i {Y_{Cit}}^{\psi} + (1 - \omega_i){K_{Dit}}^{\psi}\right) + \varepsilon_{it}.$$

Note that total output is simply the sum of clean and dirty outputs by definition ($Y_{it} = Y_{Cit} + Y_{Dit}$). If dirty output and capital were to approach zero, then clean and total output must equalise which creates a restriction that can be imposed as shown in (2). Country and time dummies ($a_i$and b_t) are brought into the logarithm and multiply the dirty measure only and the omega terms (ω_i) are removed, so that as $K_{Dit} \to 0$ we have $lnY_{it} \to lnY_{Cit} + \varepsilon_{it}$. I leave the error term outside of the production function, rather than enforce the strict equality of $Y_{it} = Y_{Cit}$ when $Y_{Dit} = 0$, so that the error continues to represent aggregate shocks.

$$(2) \qquad lnY_{it} = \frac{1}{\psi} ln\left({Y_{Cit}}^{\psi} + \mathrm{e}^{a_i + b_t}{K_{Dit}}^{\psi}\right) + \varepsilon_{it}.$$

[49] It is unclear how big a problem endogeneity is in this case, as our parameter of interest is the elasticity in the CES production function.

The elasticity of substitution is derived as $\sigma = 1/(1-\psi)$. The dependent variable of electricity output is measured as generation in gigawatt-hours (excluding hydro, nuclear, geothermal and tidal). As electricity demand is inelastic, this measure should accurately reflect output in the sector although it excludes reliability and electricity grid considerations. Capital is measured by generation capacity in megawatts (MW) adjusted for cost differences, as described in the data section below.

Six alternative specifications are used for robustness and to provide further insight. A limitation of the approach is that the long-lived and lumpy nature of capital may bias the elasticity estimate. Early adopters of VRE such as Denmark have had more time for dirty capital stocks to adjust and may therefore provide a better estimate. Thus, the first alternative specification weights each country by the number of years that VRE generation exceeds 1% of total generation. Second, fuel is combined with capital for the dirty input ($K_{Dit} \rightarrow 0.7K_{Dit} + 0.3F_{Dit}$), assuming fuel ($F_D$) makes up 30% of total costs. Third, nuclear is combined with dirty energy as nuclear may perform a similar baseload role to coal. Fourth, results are shown for countries with low hydro shares, as countries with a high hydro share might have lower costs of integration of VRE. Fifth, the VRE measure becomes solar and wind capital rather than output and so regression equation (3) is used. Finally, all clean capital is included which is comparable with results from Papageorgiou et al. (2017).

$$(3) \qquad lnY_{it} = a_i + b_t + \frac{1}{\psi} ln\left(\omega_i {K_{Cit}}^{\psi} + (1-\omega_i){K_{Dit}}^{\psi}\right) + \varepsilon_{it}.$$

Data

Data is taken from the International Energy Agency (IEA) Electricity Information Statistics. The data has 36 countries and includes the years 1995 to 2018. As noted by Papageorgiou et al. (2017), Luxembourg is excluded due to the high amount of traded electricity in this market. The key reason for less than perfect substitutability between VRE and dirty energy is a reduction in dirty capital utilisation rates resulting from meeting inelastic demand: a high level of electricity trade breaks this link. Also, Iceland is excluded as generation is almost entirely hydro and geothermal (>99.9%) which are omitted from the main regression. The share of solar and wind generation in each region is shown in Figure 19 in Appendix A.

To reflect differences in the cost of capital, capacity is adjusted according to overnight costs listed in the EIA Annual Energy Outlook (AEO) 2021. Coal capacity is valued at 3 times the cost of all other dirty capacities: most of the non-coal dirty capital is gas generation, and this factor is roughly consistent with the overnight costs of Ultra-supercritical coal and Combined Cycle gas generation. The IEA data has some gaps in the capital stock breakdown for different technologies. Data from Global Coal Plant Tracker is used to fill in gaps for Belgium, Canada, Germany, Spain, Netherlands and Slovakia.[50] In the specifications using VRE capacity as an input, raw capacity is used for wind and solar as their overnight costs are similar, while nuclear, hydro and geothermal are uplifted by factors of 5,2 and 2 respectively for the specification using the clean aggregate. For the specification that includes fuel use, the fuel input is simply the sum of all fuel sources in terajoules. Final capacity and fuel inputs are scaled so that average generation per unit of capacity/fuel across the entire dataset equals one: this scaling does not affect

[50] Global Energy Monitor, January 2021
https://globalenergymonitor.org/projects/global-coal-plant-tracker/summary-data/

elasticity estimates but means that the value of ω should be close to 0.5 for strong substitutes which provides a useful check. The data is comprised of main electricity generators which sell output to third parties and a small component (7% of total) for autoproducers that produce electricity for their own use.

Results

Results for the main specification, for both nonlinear least squares in levels and first differences, are shown in Table 8. The levels regressions all indicate an elasticity of 10 or more, while there is greater variation for the difference regressions with the elasticity varying between 1.6 and 5.7. The difference regression in column 8 seems more unstable when regions are removed or the time range of the regression is altered: the ψ estimate changes from 0.492 to 0.955 when Sweden is omitted. Consequently, the preferred regressions correspond to columns 4 and 6 which are also the preferred models according to Akaike Information Criterion (AIC), with elasticity estimates of 20 and 5.7.

Standard errors are robust and clustered at the country level.[51] Papageorgiou et al. (2017) use bootstrapped errors and discard generated data which lead to estimates of ψ greater than 1. While this approach ensures consistency with the isoelastic functional form in the generated data, it does not seem to be a conservative method to derive the standard error and not applying this restriction can lead to very high errors for ψ when bootstrapping.[52] This paper tests the robustness of the central estimates in the preferred regressions in three ways, shown in Figure 14.[53] First, each region is excluded, making 34 estimates: all are

[51] As the elasticity is a nonlinear function of ψ, note that confidence intervals are not symmetric around the implied estimate of σ. For example, for the central estimate of σ of 3.4, the 95% confidence interval is the range $(1.4, \infty)$

[52] Consider an estimate for ψ just under 1. Rejecting generated data which lead to an estimate of ψ greater than 1 means the derived standard error is likely very small by construction and so ψ is significantly different from 0 (corresponding to an elasticity of 1).

[53] The ψ estimate displayed is restricted to be a maximum of 1, consistent with an isoelastic function.

reasonably tightly distributed around the central estimate. Second, 34 estimates are shown when the regression is undertaken on each region individually, leading to a much wider distribution. This is to be expected due to the limited number of data points, with some regions having little adoption of solar and wind, and the true elasticity will likely differ substantially between regions. Third, 8 estimates show results when the starting year is changed from 1995 to 2000 inclusive or the ending year is changed from 2018 to 2015 inclusive (labelled >95,>96,>97,>98,>99,<18,<17,<16): this makes little difference to results.

The levels method is more efficient when the errors are serially uncorrelated while the difference method is more efficient when the residuals follow a random walk (Wooldridge, 2010). The lumpy nature of capital investment and slow dynamics due to long-lived capital would lead to dependence of the error on historical values of independent variables. Indeed, there is strong serial correlation in the residuals in the levels but not in the difference method. However, statistical tests reject the presence of unit roots in levels residuals. As there is no clear reason to prefer one method over the other, results for both are reported.

Table 9 shows results for alternative specifications which also use VRE output as the clean measure. Estimates when countries are weighted by the number of years where VRE generation exceeds 1% of total generation (columns 1 and 5), when fuel is included in the dirty cost measure (columns 2 and 6), and when countries with a high hydro share are omitted (columns 4 and 8) are all within one standard error of the main estimates. Including nuclear capital with dirty capital leads to greater variation in the estimates, with the elasticity estimate in the levels regression reduced to 3.6 and the first difference regression implying perfect substitutability by restricting ψ to a maximum of 1.

Table 10 shows results for other specifications which use clean capital rather than output as a cost measure. The levels specifications including all country and time dummies (a_i, b_t and ω_i) outperforms according to the AIC criterion and are therefore shown. Columns 1 and 3 show that using VRE capital rather than output leads to elasticity estimates of around 4, perhaps indicating that any bias from an increasing capacity factor for VRE is minor. Using a clean aggregate leads to very different elasticity estimates of 1.6 and 11.9. Papageorgiou et al. (2017) report a robust elasticity estimate of 2 undertaking similar regressions of clean and dirty energy substitutability in electricity. However, while Papageorgiou et al. (2017) note that Luxembourg is excluded, their results are determined from regressions which include Luxembourg: excluding Luxembourg using their data changes the estimates for ψ from 0.46 to 2.05 for the levels regression and from 0.49 to 1.80 for the first difference regression.

Considering all specifications, the following conclusion is drawn: VRE has a high elasticity with dirty inputs – an elasticity of 3 or above seems appropriate. Such a high value is consistent with the dispatch models discussed in the next section, with Stöckl and Zerrahn (2020) and with the intended regressions in Papageorgiou et al. (2017). However, it exceeds estimates of 1.6 from Lanzi and Sue Wing (2011) and around 0.5 from Pelli (2012). I see four reasons that could explain the differences in estimates between the supply approach adopted in this paper and estimates from changes in input shares induced by price changes (the price approach), used by Pelli (2012).

First, the supply approach imposes an isoelastic production function across all input ratios: an elasticity less than 1 means that total output goes to zero if one input approaches zero. As many data points have zero or very low VRE input shares, it is unlikely that the supply approach will lead to an estimate less than 1. The price approach does not have this property as the next example demonstrates.

Consider a hyperthetical: VRE and dirty inputs (and total output) do not change in a region despite price falls in VRE. The price approach sees no change in input shares with the falling VRE price and finds an elasticity of zero, while the supply approach cannot determine an elasticity as there is no change in supply measures.

If price falls had no effect until VRE became "competitive", then estimates using the price approach might increase over time. Thus, the second reason is that a more recent estimate using the price approach may be higher due to the rapid gains in VRE technology and adoption in recent years.

Now extend the hyperthetical to another region which increases its share of VRE as the VRE price falls, say with an elasticity of 2. The price approach finds an elasticity estimate of about 1 (between 0 and 2), but the supply approach finds an elasticity estimate of 2 as the region which experiences no VRE adoption has only one point and thus errors are zero and independent of the estimated elasticity. Thus, the third reason is that the supply approach puts a higher weight on regions which have undergone a greater change in VRE inputs, which probably have a higher elasticity.

The fourth reason relates to the long-lived nature of generation assets: reductions in dirty capital to long-run levels (due to the uptake of VRE) may take many years as plants may continue to operate until their end-of-life or until refurbishment costs are required. In the price approach this adjustment delay likely biases the elasticity downwards, as the change in input shares are lower than the long-run change. But in the supply approach the direction of bias is less clear and is likely upwards. As the elasticity parameter is determined by the curvature (second derivative) of data points rather than the slope (first derivative) as in the price approach, delays in adjustment may reduce the convexity of data points and hence the estimated elasticity. To illustrate, consider constant output as VRE inputs increase from 1 to 2 to 3 and long-run dirty inputs change from 10 to 9 to 9. The fact that 9 dirty inputs are still needed despite the increase in VRE input from 2 to 3 leads to convexity and hence a finite long-run elasticity. But if dirty inputs change from 10 to 9.5 to 9 due to adjustment delays, there is no convexity and the supply method will find perfect substitutability between VRE and dirty inputs. The electricity dispatch models discussed in the next section consider the implied long-run substitutability between VRE and dispatchable generation.

TABLE 8 —MAIN NONLINEAR ESTIMATIONS

	Levels				First difference			
	1	2	3	4	5	6	7	8
ψ	0.924***	0.899***	0.932***	0.949***	0.743***	0.802***	0.380***	0.492***
	(0.0921)	(0.166)	(0.0721)	(0.114)	(0.113)	(0.149)	(0.0806)	(0.170)
Time dummies	No	Yes	No	Yes	No	Yes	No	Yes
Country dummies	No	No	Yes	Yes	No	No	Yes	Yes
Regressors	2	25	35	58	2	25	35	58
σ	13	10	15	20	3.9	5.1	1.6	2.0
Log-likelihood	-273.8	-262.2	348.8	423.7	570.0	606.3	587.8	620.0
Obs	806	806	806	806	772	772	772	772
Regions	34	34	34	34	34	34	34	34

***, ** and * indicates significance at the 1%, 5% and 10% levels. Standard errors in parentheses. Implied elasticity of substitution σ from the estimate of ψ.

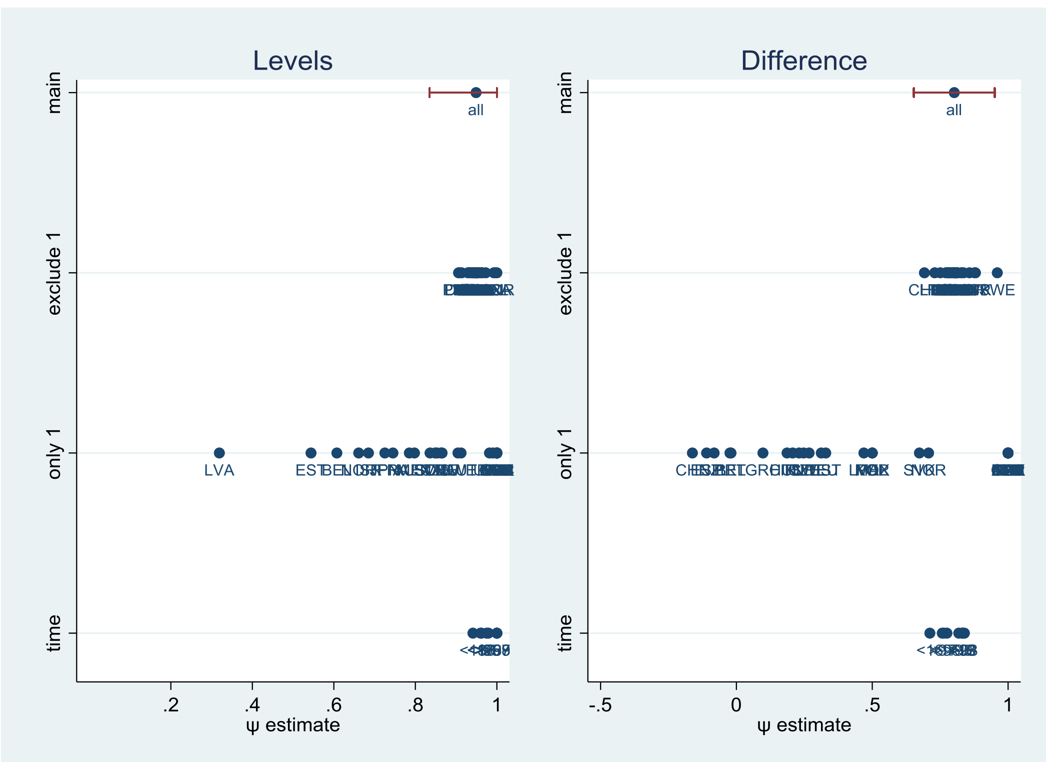

FIGURE 14: ROBUSTNESS CHECKS FOR THE PREFERRED REGRESSIONS

"Main" shows the central estimates for levels and difference regressions and standard error, with $\psi \leq 1$. "Exclude 1" shows ψ estimates when one region is excluded. "Only 1" shows ψ estimates for each country individually. "Time" shows results when the starting year is changed to 1996 to 2000 inclusive or ending year is 2015 to 2017 inclusive.

TABLE 9 —NONLINEAR ESTIMATIONS FOR ALTERNATIVE SPECIFICATIONS USING VRE GENERATION

	Levels				First difference			
	Weighted	Fuel	Nuclear + Dirty	Low Hydro	Weighted	Fuel	Nuclear + Dirty	Low Hydro
	1	2	3	4	5	6	7	8
$\boldsymbol{\psi}$	1.063***	0.990***	0.721***	0.986***	0.678***	0.858***	1.423***	0.799**
	-	(0.121)	(0.154)	(0.176)	-	(0.210)	(0.249)	(0.325)
Regressors	58	58	58	58	25	25	25	25
$\boldsymbol{\sigma}$	∞	100	3.6	71	3.1	7.0	∞	5.0
Obs	806	806	806	542	772	772	772	519
Regions	34	34	34	23	34	34	34	23

***, ** and * indicates significance at the 1%, 5% and 10% levels. Standard errors in parentheses. Implied elasticity of substitution σ from the estimate of ψ. Time and country dummies included in levels regression. Time dummies included in First difference regressions.

TABLE 10 — NONLINEAR ESTIMATIONS FOR ALTERNATIVE SPECIFICATIONS USING CLEAN CAPITAL

	Levels		First difference	
	VRE	Clean	VRE	Clean
	1	2	4	5
$\boldsymbol{\psi}$	0.745***	0.386*	0.780***	0.916***
	(0.0736)	(0.221)	(0.171)	(0.147)
Time dummies	Yes	Yes	Yes	Yes
Country dummies	Yes	Yes	No	No
Country $\boldsymbol{\omega}$ dummies	Yes	Yes	No	No
Regressors	90	90	25	25
$\boldsymbol{\sigma}$	3.9	1.6	4.5	11.9
Obs	806	806	772	772
Regions	34	34	34	34

***, ** and * indicates significance at the 1%, 5% and 10% levels. Standard errors in parentheses. LL is Log Likelihood.

II. Substitutability according to electricity dispatch models

This section considers substitutability between VRE and dirty generation in two parts. The first considers some literature on regional electricity models with detailed generation types that can extrapolate the costs of integrating intermittent sources at high VRE shares. The second develops a simple electricity dispatch model broadly consistent with this literature that helps our understanding of substitutability dynamics and determinants.

For VRE (clean) inputs Y_c and dirty inputs Y_d and the marginal rate of technical substitution between VRE and dirty inputs $MRTS_{cd}$, the elasticity of substitution is given by:

$$(3) \qquad elasticity = \frac{\mathrm{d}\ln\left(\frac{Y_c}{Y_d}\right)}{\mathrm{dln}(MRTS_{cd})} = \frac{\mathrm{d}\ln\left(\frac{Y_c}{Y_d}\right)}{\mathrm{dln}\left(\frac{p_c}{p_d}\right)} = \frac{\mathrm{d}\ln\left(\frac{Y_c}{Y_d}\right)}{\mathrm{dln}\left(\frac{p_c}{p_d}\right)} = \frac{\mathrm{d}\left(\frac{Y_c}{Y_d}\right)\Big/\frac{Y_c}{Y_d}}{\mathrm{d}\left(\frac{p_c}{p_d}\right)\Big/\frac{p_c}{p_d}}$$

where the last two equalities result if the ratio of prices reflects the ratio of the marginal increases in output from a change in input. This section seeks to identify potential changes in elasticity as the VRE share increases, requiring sources to show the marginal changes in inputs with changes in input prices at <u>different input shares</u>.

Results from regional electricity models

Several papers use electricity dispatch models, typically calibrated to a region, to discuss the costs of integrating VRE inputs into the residual (dirty) system. Such integration costs vary with the share of VRE and can be used to infer an indicative elasticity of substitution. For the ratio of VRE to dirty energy, $\lambda := \frac{Y_c}{Y_d}$, integration costs ($p_{int}$) can be added to the price of VRE (p_c) to create a metric called the system price of VRE (p_{sc}):

$$(4) \qquad p_{sc}(\lambda) := p_c(\lambda) + \mathrm{p}_{int}(\lambda)$$

where all the prices are marginal long-run costs so that effects on utilisation, and thus the proportion of fixed and variable costs, are considered.[54] An optimal

[54] Such long-run marginal costs are often referred to as levelised costs of electricity.

quantity of VRE generation occurs when the system price of VRE equals the price of the conventional (dirty without VRE) system:

$$(5) \qquad p_{sc}(\lambda^*) = p_d(0).$$

A definition of integration costs and proof of (5) is in Appendix B. As an example, if the cost of dirty energy was \$60 per megawatt-hour (MWh) and the cost of VRE was \$40/MWh, the optimal level of VRE would be such that integration costs were \$20/MWh. As the price of VRE changes relative to dirty energy, the degree to which VRE inputs change depends on how integration costs vary with the VRE input share. Consider a fixed price for dirty and a shift in VRE price Δp_c, induced by technological change or policy such as a renewable subsidy.[55] For (4) to hold, the VRE share adjusts until the change in integration cost (Δp_{int}) balances Δp_c and the elasticity is then derived from (3).[56]

I extract integration costs from three studies. Hirth, Ueckerdt, and Edenhofer (2015), hereafter HUE, report long-run wind profile costs from a survey of 30 publications.[57] The data is limited to wind generation shares up to 40 per cent and, as they show the line of best fit, is naturally linear. Ueckerdt et al. (2013), hereafter UHLE, report system costs for wind and profile costs of solar. As optimal policy could include both technologies, I construct a combined integration cost described in Appendix B. Elliston, Riesz, and MacGill (2016), hereafter ERM, report average energy cost profiles for different shares of renewable energy up until a 100 per cent

[55] The important point is not whether the clean or dirty price changes, but the change in relative prices.

[56] I assume that changing the clean share does not in itself influence the price of clean energy supply, therefore ignoring the fact that $p_c(w)$ will likely be a decreasing function of w as the most productive VRE sites are used first, implying the inferred elasticity is an overestimate.

[57] Profile costs are the dominant integration cost (HUE, Ueckerdt, Hirth, Luderer, and Edenhofer (2013)) and relate to the impact of timing of generation: for example, the utilisation rates of dispatchable generation decrease as VRE increases.

share, for a low and high gas price of $3 and $9 per GJ. I use the low gas price and convert these average costs to integration costs, described in Appendix B.

All integration costs are combined in the first panel of Figure 15. While there is a significant range of estimates, integration costs increase as the VRE share rises and gradients tend to increase at high (above 50 per cent) VRE shares, where this data exists. Inferred elasticities are shown in the second panel of Figure 15. Although there is a considerable variation between papers, elasticities tend to decrease as the VRE share increases.[58]

A key reason for the decreasing elasticity discussed in the literature is the increasing rate of curtailment with the share of VRE. Curtailment can be reduced using storage technology, demand-side measures and integration between regions with different temporal VRE characteristics. The studies include assumptions of storage options that help balance supply and demand. For example, ERM find that pumped storage hydro and concentrating solar thermal help reduce costs at high clean shares. Therefore, consideration of storage at high clean shares is important.

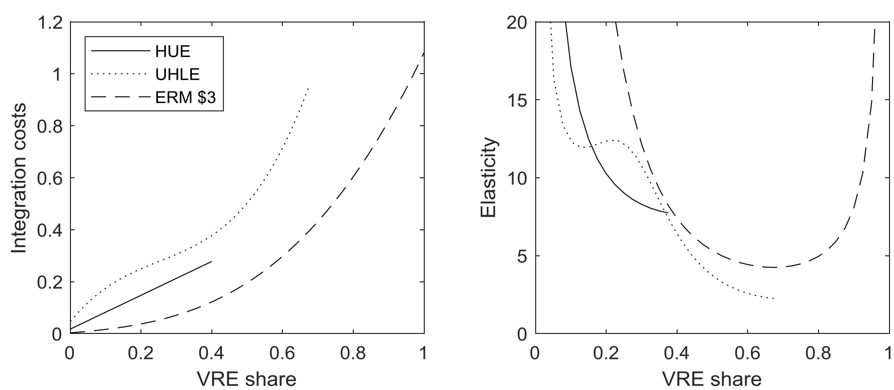

FIGURE 15: COMBINED INTEGRATION COST ESTIMATES AND INFERRED ELASTICITIES

Notes: Integration costs are in units of the average cost of the conventional system ($p_d(0)$).

[58] High elasticities for very small or high VRE shares relate to large changes in the ratio of inputs as one input becomes very small.

A simple electricity dispatch model

This section develops a simple electricity dispatch model that forms a micro foundation for substitutability by considering the profile costs of VRE. The market is structured to minimise the total cost of meeting demand. Two high-level types of generation exist: dirty which is dispatchable and VRE (clean). The profile of demand is assumed to be fixed: it is common to consider the load duration curve (LDC) which shows the (hourly) demand for a year in descending order. Figure 16 gives an example for Denmark (which has a high level of wind generation) from the IEA: demand is approximately linear and this profile is common across regions. Residual demand (the demand that is not met by wind power) is shown in the lower line.

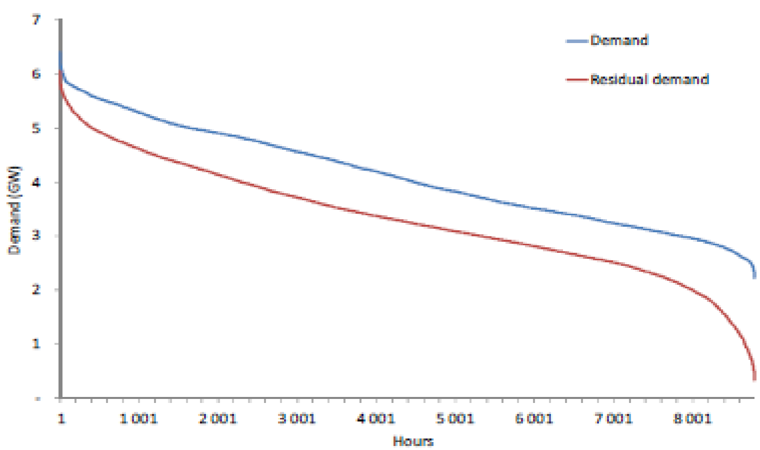

FIGURE 16: LOAD DURATION CURVE AND RESIDUAL DEMAND IN DENMARK, 2008

Source: IEA analysis, Figure 21 of (Vos, 2012).

A stylised linear LDC is a common assumption and is a good approximation to the LDC in Figure 16: for duration x such that $0 \leq x \leq 1$, the LDC is assumed to be $\mathrm{LDC}(x) = 2 - x$ in Figure 17.

Dirty dispatchable generation—Production consists of three types of dirty dispatchable generation – base (B), intermediate (I) and peak (P) - as used in

Wiskich (2014) and Ueckerdt et al. (2015). Each technology is characterised by a fixed and variable cost. Given fixed costs $F_B > F_I > F_P$ and variable costs $V_B < V_I < V_P$, the cost of production given capacity factor X is the sum of fixed costs and variable cost: $F + XV$. The generation of each type is shown in the shaded areas in the top of Figure 17.

Peak capacity is only used a small proportion of the time; when the capacity factor is low ($X < X_1$) and the fixed cost dominates the total cost. Intermediate capacity is used around half of the time on average and base capacity is used almost all the time when the capacity factor is high ($X > X_2$).[59] Capacity factors X_1 and X_2 depend on fixed and variables costs as follows:

$$
\begin{aligned}
(6) \qquad & F_P + X_1 V_P = F_I + X_1 V_I \quad \text{(Peak to Intermediate)} \\
& F_B + X_2 V_B = F_I + X_2 V_I \quad \text{(Intermediate to Base).}
\end{aligned}
$$

Clean variable generation—Intermittency of clean generation lowers and changes the shape of the residual LDC (RLDC) faced by dispatchable generation, as shown empirically in Figure 16 and in the model at the bottom of Figure 17. The key effects of intermittent generation include reduced utilisation of base and peak generation, and increased/decreased capacity of peak/base generation. The RLDC derivation generalises an approach that I have previously used (Wiskich, 2014).[60] The key assumption is a uniform distribution of variable generation with a minimum supply proportion of m.[61] The RLDC (7) describes the effect of

[59] Total fixed costs are $X_1 F_P + (X_2 - X_1) F_I + (2 - X_2) F_1$ and total variable costs are $\frac{{X_1}^2}{2} V_P + \left(\frac{(X_2 - X_1)^2}{2} + (X_2 - X_1) X_1\right) V_I + \left(1 + (1 - X_2) X_2 + \frac{(1 - X_2)^2}{2}\right) V_B$.

[60] Ueckerdt et al. (2015) also use a RLDC approach but assume the quadrilateral shape shrinks and distorts in such a way to match the variable generation supply and demand data.

[61] While this assumption is not a good fit for all regions, the general shape of the RLDC resembles many regions, is simple, and different values of parameter m can be considered.

intermittent generation which varies uniformly between mW ($0 \leq m \leq 1$) and W. Let $W' := (1-m)W$, then we have

$$(7)\ \ RLDC = \max(0, Y) \ \text{ where } Y = \begin{cases} Y_1 & x < X_T \\ Y_2 & X_T < x < 1 - X_T \\ Y_3 & x > 1 - X_T \end{cases}$$

$$Y_1 = 2 - mW - \sqrt{2W'x}$$

$$Y_2 = \begin{cases} 2 - mW - \dfrac{W'}{2} - x & W' < 1 \\ 1.5 - mW - W'x & W' > 1 \end{cases} \qquad \text{and } X_T = \begin{cases} \dfrac{W'}{2} & W' < 1 \\ \dfrac{1}{2W'} & W' > 1. \end{cases}$$

$$Y_3 = 1 - W + \sqrt{2W'(1-x)}$$

Consider the case where $m = 1$ so clean generation consistently generates W units of electricity. Thus $X_T = 0$ and the RLDC is simply the LDC straight line lowered by W. For $m < 1$, the variability of generation between mW and W implies different functions Y_1 and Y_3 for the peak load and minimum load areas of the curve, with $Y_1(0) = 2 - mW$ corresponding to $LDC(0) = 2$ lowered by the minimum intermittent generation mW and $Y_3(1) = 1 - W$ corresponding to $LDC(1) = 1$ lowered by the maximum intermittent generation W. The quadratic forms of Y_1 and Y_3 follow from the assumption of a uniform distribution between mW and W which is uncorrelated with aggregate demand. That is, intermittent supply can be considered as a random number between mW and W for every point in time. When clean energy is high enough such that $W > 1$, curtailment occurs as $Y_3(1) < 0$. In other words, when maximum intermittent generation occurs at minimum demand, excess supply occurs and the excess is assumed to be wasted.

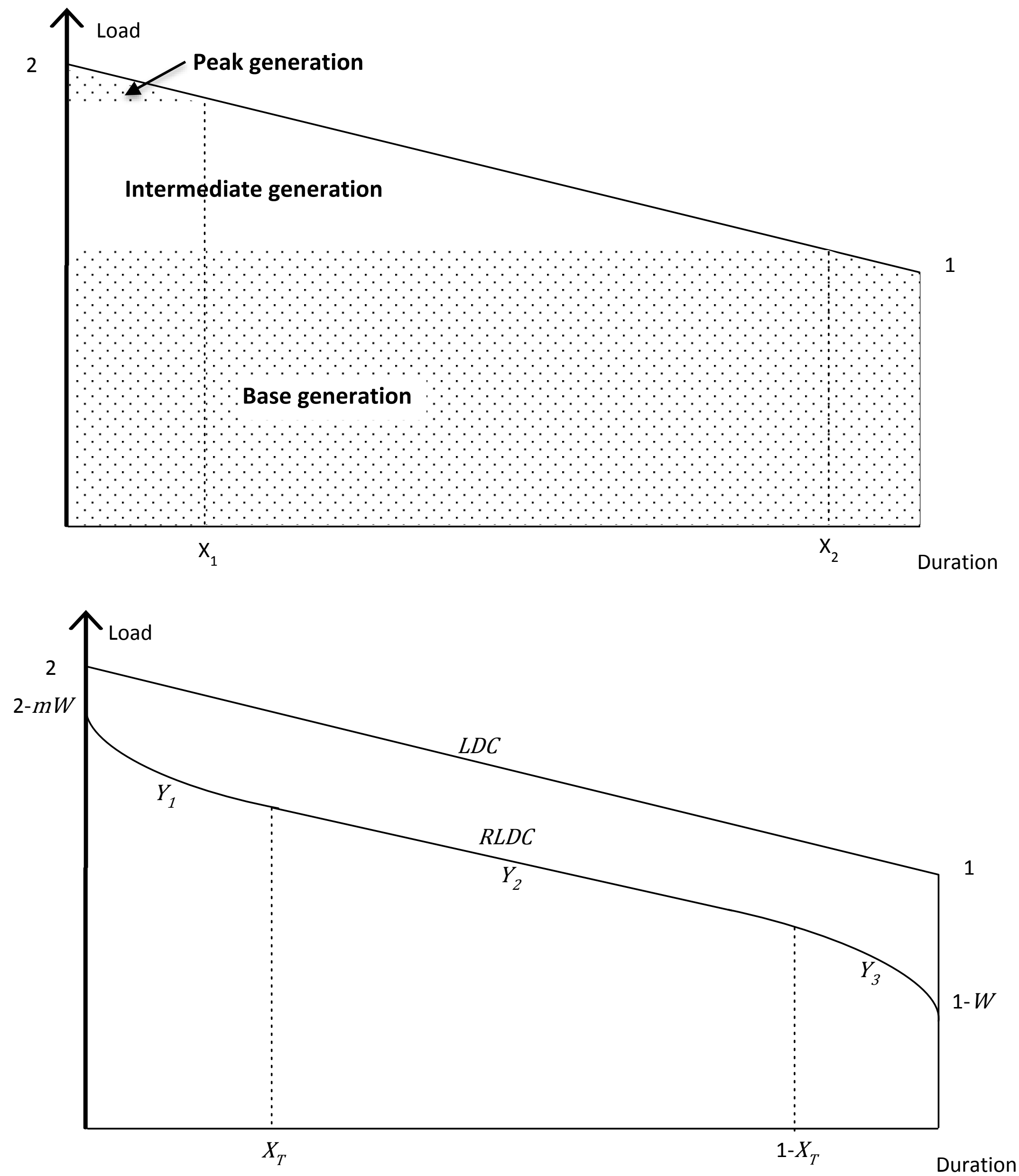

FIGURE 17: GENERATION SUPPLY (TOP) AND THE EFFECT OF VRE ON RESIDUAL DEMAND (BOTTOM)

Calibration of the model is discussed in Appendix B. Figure 18 shows clean shares for a typical dispatch model simulation and a fitted "trimodal" isoelastic function with three elasticity regimes (see (8) below for the functional form of a similar "bimodal" function). The x-axis depicts the ratio of VRE and dirty input prices: as the price of VRE falls, or a carbon tax increases the dirty price, the share of VRE rises. The value of m is assumed to be zero and the availability of storage, which is found to be an important consideration in the literature reviewed above, is omitted. However, the key profile of a decreasing elasticity when clean shares become very high is consistent when a higher value of m or storage are considered.[62]

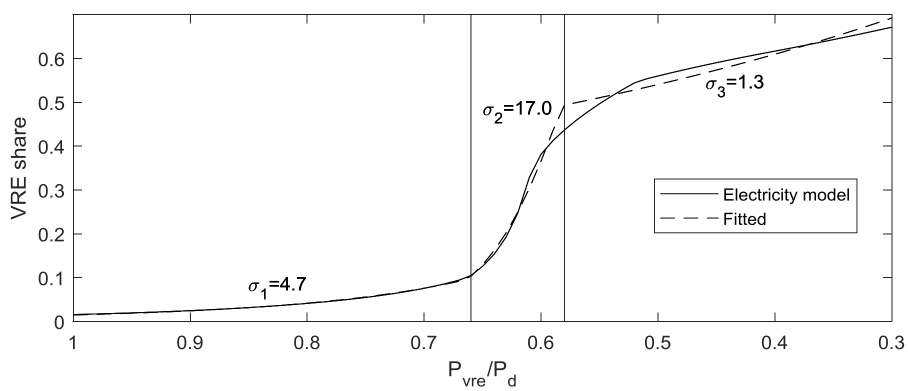

FIGURE 18: COMPARISON OF THE ESTIMATED TRIMODAL FUNCTION (DASHED LINES) WITH DISPATCH MODEL RESULTS

The sharp fall in elasticity is largely due to the curtailment of intermittent supply. The switch point of around 50 per cent suggested by the model is increased by storage (not shown) and as m increases (Appendix C). Also, considering different carbon emission intensities of peak, intermediate and base generation would likely imply a higher switch share in terms of emissions as coal typically fits into the base generation category which is displaced first by VRE.

While the "trimodal" function matches the output from the dispatch model well, a bimodal function would also do a reasonable job and is an easy way of

[62] A simple representation of storage was simulated but is omitted for brevity. As one would expect, storage is complementary to VRE generation and increases the elasticity with dirty energy and raises the switch point.

incorporating decreasing substitutability in a model with an isoelastic function between clean and dirty energy. While other production functions exhibit a decreasing elasticity (for example, Sato and Hoffman (1968) and Revankar (1971)), the bimodal approach benefits from conceptually clarity and simplicity: the three key parameters - low and high share elasticities and the switch point where the elasticity changes – all have a clear conceptual interpretation.

Finally, it is worth noting that most countries have a low share of VRE in generation and hence a panel regression like the one undertaken in the previous section is mostly analysing data points in the leftmost section of Figure 18, with $\sigma_1 = 4.7$. This value is comparable with the estimates in the previous section. A natural question is how well the regression specifications used in section I perform when applied to output from this dispatch model where the "true" elasticity is known. It turns out that including fuel costs is important to identify the elasticity from the dispatch model output. While the dispatch model is stylised, this result indicates a potential limitation in the main empirical specification in section I which considers capital costs but misses the effects of fuel use. Perhaps the weight assigned to the specification that includes fuel costs in section I, which happens to estimate an elasticity of 7 or more, should increase. In any case, results in both sections are consistent in indicating a high elasticity at low clean shares, and the main message of this section is that the substitutability will decrease as the clean transition progresses.

III. Consequences of decreasing substitutability

This section discusses some consequences of decreasing substitutability, including higher costs of a clean transition, greater costs from regions transitioning sequentially rather than together, a potentially higher optimal carbon tax and lower clean research subsidy, and an increase in the performance of tax-only policy versus subsidy-only policy.

Higher costs of a clean transition

The most obvious consequence of a decreasing elasticity is that the transition to clean energy becomes more difficult as the clean share rises, increasing costs. As empirical estimates of the elasticity tend to be based on data points with low clean shares, analysis using an isoelastic function based on these estimates may underestimate the costs of transition.

Regions should transition in parallel

A decreasing elasticity implies a greater cost of regions transitioning to clean energy sequentially rather than in parallel. Consider two identical regions: it is optimal for them to transition together when the elasticity of substitution between clean and dirty energy is constant, or equivalently a carbon price should be uniform between them. Transitioning sequentially has a higher cost due to cheaper abatement options not being used. A decreasing elasticity tends to magnify this effect and is demonstrated using the following model.

For dirty energy in region i of D_i and elasticity of substitution σ, output in two regions Y_1 and Y_2 is as follows:

$$(8)\quad Y_i = A\left(\beta {D_i}^{1-1/\sigma} + 1\right)^{\frac{\sigma}{\sigma-1}}$$

$$\text{where } \sigma, A, \beta = \begin{cases} \sigma_1, 1, 1 & if\ D_i > \overline{D} \\ \sigma_2, \left(\overline{D_i}^{\frac{\sigma_1 - 1}{\sigma_1}} + 1\right)^{\frac{\sigma_1}{\sigma_1 - 1} - \frac{\sigma_2}{\sigma_2 - 1}}, \overline{D_i}^{\frac{1}{\sigma_2} - \frac{1}{\sigma_1}} & if\ D_i < \overline{D}. \end{cases}$$

A similar production function is described in Antony (2009) and builds on Jones (2003): I refer to this function as a bimodal isoelastic function as it allows for a shift in the elasticity of substitution at a particular switch point $\bar{D}$. Assume that pre-transition dirty energy use is $D_1 = D_2 = 0.05$, high and low isoelastic cases with elasticities 3 and 1.5, and a bimodal case where $\sigma_1 = 3$, $\sigma_2 = 1.5$ and $\bar{D} = 0.025$ corresponding to a fall in elasticity when 50% abatement is achieved in a region. Consider the costs of 25% and 50% global abatement, where abatement is either uniform in both regions (the parallel case) or skewed such that only one region undertakes abatement (the sequential case).

The costs of the transition are shown in Table 11. The cost of abatement falls as the elasticity of substitution increases, and the additional cost from a skewed approach also falls as the elasticity increases.[63] Key results are in columns 5 and 6 showing the extra cost from an uneven abatement across the two regions. For the bimodal case, results for a 25% reduction correspond with the high isoelastic case as the region abating 50% still has high substitutability. However, for deeper abatement, the additional cost of a sequential approach is boosted above both isoelastic cases as regional abatement is more difficult above 50%.

TABLE 11: COSTS OF SUBOPTIMAL SEQUENTIAL APPROACH TO ABATEMENT

	25% abatement		50% abatement		Increase in cost from sequential transition	
(D_1, D_2)	Parallel (0.0375,0.0375)	Sequential (0.025,0.05)	Parallel (0.025,0.025)	Sequential (0,0.05)	25% abatement	50% abatement
	1	2	3	4	5	6
CES 1.5	6.8%	7.5%	15.0%	29.3%	9.5%	96.0%
CES 3	2.7%	2.9%	5.8%	7.6%	5.5%	32.8%
Bimodal 3 & 1.5	2.7%	2.9%	5.8%	12.5%	5.5%	118.0%

The optimal carbon tax may be higher, reducing clean research subsidies

[63] While absolute costs are exaggerated, the purpose of this simple model is to demonstrate that there can be additional costs in a sequential approach with a decreasing elasticity.

What might a decreasing elasticity mean for the profile and relative magnitudes of carbon taxes and clean research subsidies? An integrated assessment model with endogenous technical change could investigate this question but is beyond the scope of this paper. Instead, the likely consequence is discussed qualitatively. While Golosov et al. (2014) find that the optimal carbon tax-to-income ratio is constant and independent of temperature outcomes, it is arguable that the optimal carbon price would increase with higher projected temperatures. For example, the optimal tax would rise with temperature outcomes in the model used in (Acemoglu et al., 2012) due to the convexity in the damage function and in another paper, I show that higher temperature outcomes can boost the optimal tax when the risk of tipping points (triggering abrupt and irreversible damage) are considered (Wiskich, 2021b). With a higher tax, a lower subsidy would probably be required as the tax itself incentivises clean research (Fried, 2018).

For suboptimal policy, tax-only is favoured over subsidy-only

While climate change has been known for some time, optimal policy is difficult to achieve so it is useful to weigh second-best policy options. In the model framework with a carbon tax and clean research subsidy, an obvious consideration is when only one instrument is available. Under these models, a clean research subsidy is often temporary as clean research eventually dominates dirty research with technology advances. The limitation of subsidy-only policy is that it cannot distort production and this limitation becomes more important the greater the rise in future temperature, so a fall in substitutability tends to lower the effectiveness of this policy. As the cost of tax-only policy derives from the higher-than-optimal tax needed to direct clean research in the short term, an increase in the optimal tax lowers the additional tax increment required to direct clean research. Thus, the performance of a tax-only policy is likely improved relative to a subsidy-only policy.

III. Conclusion

The elasticity of substitution between clean and dirty electricity generation is a central parameter in determining the costs and optimal policy settings in a clean transition. This transition is being driven by the adoption of the variable renewable energy technologies wind and solar, and this paper produces the first empirical evidence for the substitutability between these technologies and dirty generation. A high elasticity is found, consistent with micro dispatch models of electricity which predict that the elasticity will decrease as the clean share rises. A dispatch model is described to demonstrate this characteristic, mostly resulting from the difficulty in incorporating intermittent sources in meeting inelastic demand. Finally, the policy implications of decreasing substitutability are discussed.

The econometric and dispatch model approaches have limitations. The substitutability at different clean shares is not investigated, largely due to the absence of data points with high VRE shares. As many data points have very low VRE shares, an elasticity estimate greater than one is a natural consequence of using an isoelastic production function: an elasticity less than one would imply output would be zero if VRE input was zero. Further, the approach likely overestimates the long-run elasticity of substitution, whereas an estimation approach using prices likely underestimates the elasticity, perhaps explaining low previous estimates in the literature. Detailed electricity dispatch models capture the well-understood nature of electricity markets at a micro level, but it is difficult to understand how generalisable results are across regions and how complex the dispatch model needs to be. For example, the simple dispatch model assumes that supply is met at the lowest cost – distortions such as market power are not considered.

Future empirical research could investigate substitutability in sub-national markets and the emissions intensities of dirty technologies could be considered.

The effects of decreasing substitutability could be quantitatively investigated: models with an isoelastic function between clean and dirty energy inputs could easily be modified to include a bimodal function for example. Studies could examine the interaction between decreasing substitutability and uncertainty in environmental damages, including tipping points, and the elasticity dynamics itself could be modelled under uncertainty.

APPENDIX A – FURTHER EMPIRICAL DATA

Figure 19 shows the solar and wind generation for each region as a share of total generation excluding Nuclear, Hydro, Geothermal and Tidal.

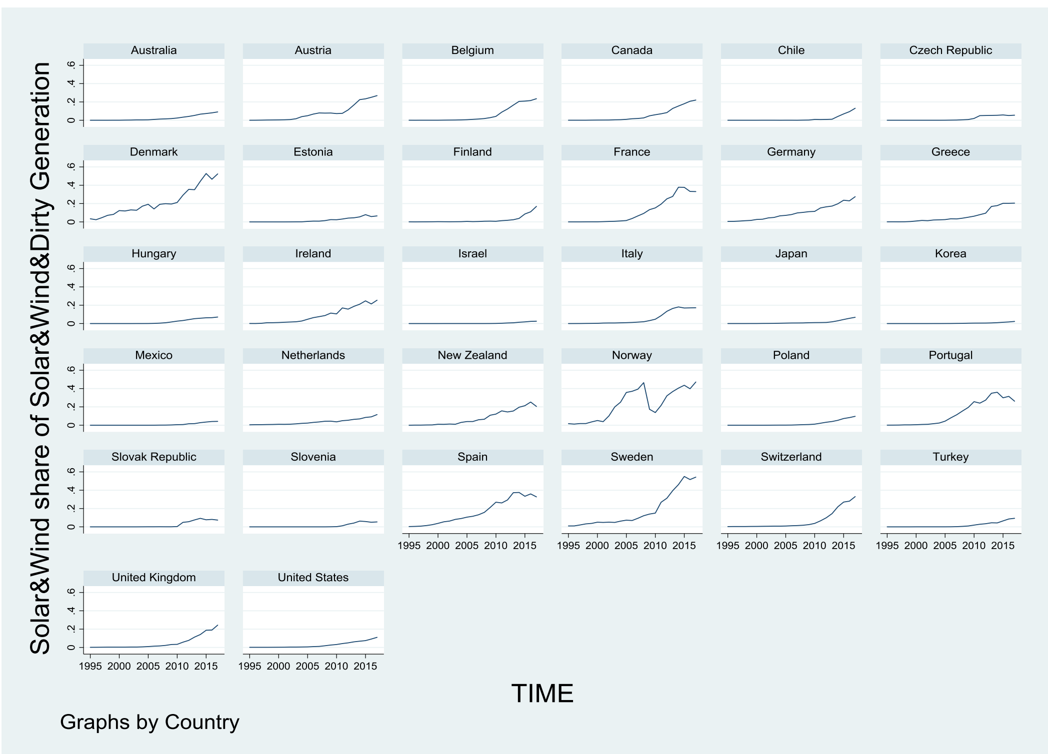

FIGURE 19: VRE SHARES EXCLUDING NUCLEAR, HYDRO, TIDAL AND GEOTHERMAL

APPENDIX B – INTEGRATION COSTS EXTRACTION AND CONVERSION

This section discusses integration costs and the proof of (5), and discusses further details of extracting integration costs from the literature.

Integration costs and proof of (5)

UHLE define total integration costs (C_{int}) as the extra cost in the residual system imposed by VRE. However, this definition excludes curtailment costs[64], defined as excess VRE generation that is wasted times the VRE price (p_c). To understand this exclusion, note that if a unit of VRE is added and (in the extreme case) all its output is curtailed, the residual system is unaffected**.** Like UHLE and other papers, I wish to include the costs of curtailment as an integration cost, and therefore I add curtailment costs (C_{curt}) to the definition of level integration costs. For constant total supply ($\overline{\mathrm{Y}}$) made up of VRE (Y_c) and dirty (residual) supply (Y_d) and marginal integration costs (p_{int}), level integration costs are:

$$(\mathrm{A}.1) \qquad C_{int}(\lambda) = C_d(\lambda) - \frac{Y_d(\lambda)}{\overline{\mathrm{Y}}}\, C_d(0) + C_{curt}(\lambda, p_c)$$

$$\text{where } p_{int} := \frac{\partial C_{int}}{\partial Y_c} \text{ and } \bar{Y} = Y_c + Y_d.$$

Thus, integration costs are the extra costs of the residual system over a conventional one (without VRE), plus curtailment costs. Curtailment costs vary not only with the VRE share but also with the price of VRE.[65] Total costs (C_{tot}) are the

[64] Although curtailment costs are excluded in the definition in UHLE's methodology section, they are included as an integration cost in their results.

[65] This complicates the conceptual framework, as ideally we would be able to define integration costs relative to the price of a conventional system and independently of the VRE price. When curtailment applies, the inferred elasticity only applies for the assumed VRE price and thus should only be taken as indicative.

sum of non-curtailed VRE costs (C_c), curtailment costs and residual costs, and using (A.1) can be written as the sum of VRE costs, integration costs and conventional system costs:

$$(\mathrm{A}.2) \qquad C_{tot} = C_c + C_{int} + \frac{Y_d}{\overline{\mathrm{Y}}}\, C_d(0).$$

Optimality implies that the quantity of VRE generation minimises total costs:

$$(\mathrm{A}.3) \qquad \frac{\partial C_c}{\partial Y_c} + \frac{\partial C_{int}}{\partial Y_c} + \frac{\partial}{\partial Y_c}\left(\frac{Y_d}{\overline{\mathrm{Y}}}\, C_d(0)\right) = 0.$$

As $\overline{\mathrm{Y}}$ is constant we have $\frac{\partial Y_d}{\partial Y_c} = -1$ and marginal integration costs are the difference between the average price of a conventional system minus the price of VRE, consistent with (3):

$$(\mathrm{A}.4) \quad \mathrm{p}_{int}(\lambda^*) = p_d(0) - p_c(\lambda^*) \ \text{ where } p_c := \frac{\partial C_c}{\partial Y_c} \text{ and } p_d(0) := \frac{C_d(0)}{\overline{\mathrm{Y}}}.$$

For a positive price $p_c > 0$, (A.4) indicates that the integration cost is bounded above by the conventional price under optimal conditions when no climate externality is considered. However, the integration price $p_{int}(\lambda) := \frac{\partial C_{int}}{\partial Y_c}$ is not bounded from above under suboptimal conditions, such as setting a predetermined VRE share, as used in the literature showing integration costs.

Extracting integration costs from the literature

UHLE report system costs for wind up to a 40 per cent generation share and profile costs for solar up to a 25 per cent share.[66] By assuming profile costs are independent of each other and using the AEO solar/wind cost ratio, a combined integration cost of both wind and solar is derived.[67]

ERM report average energy cost profiles for a low and high gas price of $3 and $9 per gigajoule. I use the low gas price (a similar profile is derived from the high gas price) and derive (marginal) integration costs, then fit a cubic polynomial to smooth this line and ensure it is non-decreasing with the clean share.[68] Average costs are based on constant prices, and the increase in the average price with renewable share may be due to both a higher price for clean energy over dirty energy, and to integration costs: $\frac{\partial C_{tot}}{\partial E_{vre}} = p_c - p_d(0) + p_{int}$. Thus, if integration costs are zero, the slope of the average cost simply reflects a higher price of clean energy. I assume integration costs are close to zero when the clean share is zero and hence approximate the difference in prices ($p_c - p_d(0)$) by the slope of the average price at a zero clean share.

Appendix C – Dispatch model additional details

Generation shares of technologies that could be considered as base, intermediate and peak vary between regions. These shares can also vary within regions over time as prices change: a lower gas price might expand the generation share of intermediate, for example. Further, the availability of wind and solar resources, and the correlation of these sources with peak demand, varies between regions.

66 UHLE report that solar system costs start for zero penetration at double the cost of wind. Capital costs have changed significantly in recent years and costs are projected to continue to decline. Projected capacity-weighted system costs for new generation resources entering service in 2022 are 48 $/MWh and 59.1 $/MWh for wind (onshore) and solar respectively (Annual Energy Outlook 2018). These costs imply solar costs 23 per cent higher than wind.

67 The combined costs have been smoothed with a cubic polynomial.

68 Non-decreasing integration costs ensure that the VRE share increases as the renewable price falls and the inferred elasticity is positive.

Rather than model one region or set of fuel prices, results for a range of different configurations are considered. Dispatch model simulations consider multiple values $X_1 \in (0.05, 0.1, 0.15)$ and $X_2 \in (0.75, 0.85, 0.95)$ which, in the absence of intermittent generation, correspond to peak generation shares of 0.08 per cent, 0.3 per cent and 0.8 per cent and base generation shares of 70 per cent, 76 per cent and 81 per cent. X_1 and X_2 are derived in two ways: the first by altering fixed costs and the second by altering variable costs, leading to 18 simulations.[69] The base capital share is around 50 per cent greater than the combined intermediate and peak capital, consistent with the OECD data used in section I.

For high VRE shares, residual costs tend to be dominated by fixed capacity costs. As the maximum load is $Y_1(0) = 2 - mW$, each additional unit of W lowers the maximum load by mW and so residual capacity is reduced by m. Thus, integration costs at high VRE shares are highly sensitive to m. A small value of m close to zero would be consistent with the RLDC profile in UHLE, but values above 0.2 might be appropriate for some regions based on Ueckerdt et al. (2017). I consider the results for the extreme values of $m = 0$ and $m = 0.3$, making 36 simulations in total. While no amount of intermittent generation can obviate residual generation when m is zero, for $m = 0.3$ no residual generation is required if $\mathrm{W} > \frac{2}{m} = 6.67$.[70]

Consider generation costs[71] for coal, combined cycle gas turbines and renewables of \$70, \$55 and \$50/MWh (IEA 2015). Thus, a reasonable cost estimate for dirty (fossil fuel) is \$60/MWh, implying a clean to dirty price ratio of $\frac{\mathrm{P_c}}{\mathrm{P_d}} = \frac{50}{60} = 0.83$. I show clean generation shares for a wide range of price ratios from one to 0.3 in Figure 20 for all simulations. In dollar terms with a fixed dirty cost of \$60/MWh,

69 The simulation shown in Figure 18 corresponds to $X_1 = 0.1$ and $X_2 = 0.85$ with fixed costs altered.

70 This result is in the absence of storage which can reduce the amount of VRE required to obviate residual generation substantially.

71 Levelised costs without carbon costs.

the clean price range is \$60/MWh to \$18/MWh, or with a fixed clean price of \$50/MWh, the range of dirty prices (which would include a carbon price) is \$50/MWh to \$167/MWh. The simulations are split into two halves: calibrating by varying fixed costs results in a wider spread than varying variable costs, but the general profiles of results are similar.

Integration costs are derived using (A.4) and are shown in the first panel of Figure 21, along with the upper and lower bounds from the literature taken from panel 1 of Figure 15.[72] The dispatch model can replicate the range of integration costs for different clean shares and leads to a decreasing elasticity shown in the second panel.

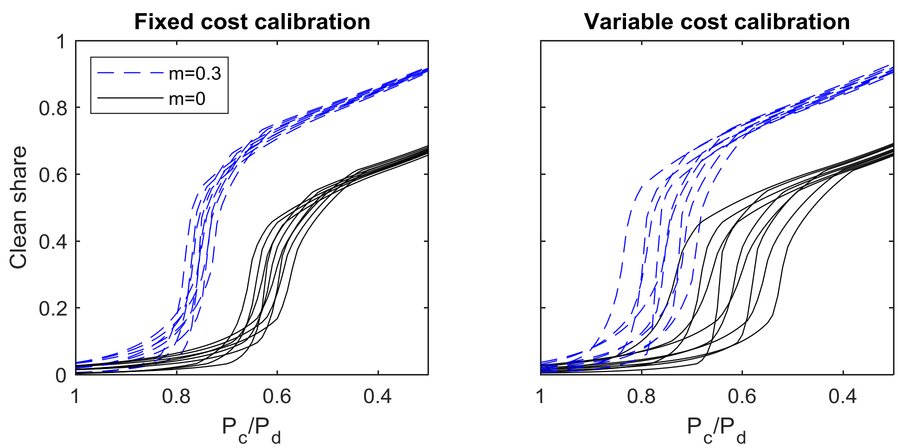

FIGURE 20: SIMULATION RESULTS FOR RATES OF VRE SHARES AS THE PRICE OF VRE INPUTS DECREASE

[72] As discussed above, integration costs are a function of the clean price due to curtailment. Consistent with the electricity model literature discussed in section I, the integration costs are calculated based on a fixed clean price for comparison: I use a value of 0.65 midway between the bounds of the simulations of 1 and 0.3.

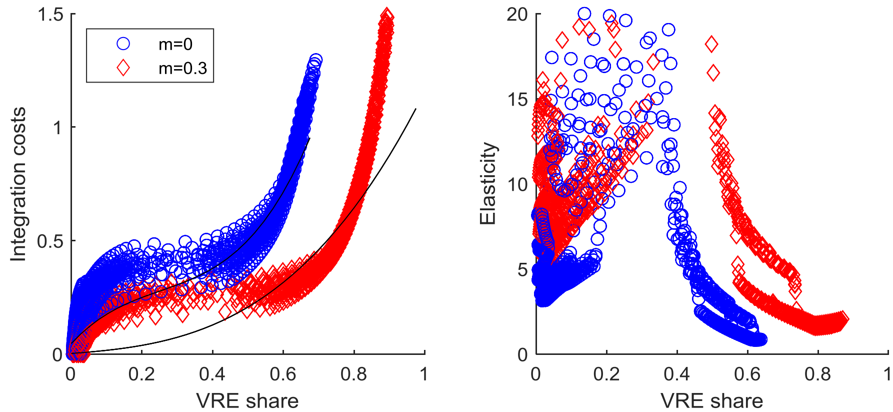

FIGURE 21: MARGINAL INTEGRATION COSTS AND INFERRED ELASTICITY FOR ALL SIMULATIONS

Notes: Integration costs are in units of the average cost of the conventional system ($p_d(0)$), and are calculated based on a fixed clean to dirty price ratio of 0.65.

4. A comment on innovation with multiple equilibria and "The Environment and Directed Technical Change"

By ANTHONY WISKICH*

The framework used to endogenise technology growth by Acemoglu et al. (2012) can exhibit increasing returns to research and hence multiple equilibria, including an unstable interior equilibrium. This paper discusses several methods to determine how a unique equilibrium might be specified. Alternative methods can produce substantially different results when the elasticity of substitution between clean and dirty inputs is high. (JEL O33, O44, Q54, Q56, Q58)

Keywords: Climate change, directed technical change, innovation policy.

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The paper by Acemoglu et al. (2012), hereafter AABH, is prominent in the literature and many subsequent papers have built on or analysed their work.[73] The model considers just two sectors (clean and dirty) and optimal policy relies on both a carbon tax and a research subsidy. Increasing returns to research can arise due to

[73] For example, Greaker and Heggedal (2012), Greaker et al. (2018) , Pottier et al. (2014) , Acemoglu et al. (2014), Wiskich (2021b), Durmaz and Schroyen (2013), Van den Bijgaart (2017), (Hémous, 2016), Lemoine (2017) and (Hart, 2019).

market size effects, which encourages innovation towards the larger input sector (Acemoglu, 2002), leading to multiple equilibria.[74]

The focus of this paper is the specification of a unique equilibrium and thus the extent of climate policy required to direct clean research. Under optimal policy in AABH, clean research is directed through a research subsidy with no cost of funds (subsidies can be financed through lump-sum taxation which does not involve any distortion). Thus, although the optimal subsidy depends on how equilibria are specified, this has no economic consequence. However, an economic effect would arise if a cost of funds was assumed as in (Acemoglu et al., 2016). AABH also discuss the welfare costs of using only a carbon tax. In this case, the specification of equilibria does have an economic effect which I explore in section 2. The discussion in this paper is relevant for any analysis with increasing returns to research, including papers examining the relative performance of tax-only and subsidy-only policy in a clean transition.[75]

Section 1: Equilibria and optimal policy with increasing returns to research

Consider profits Π in two sectors clean (c) and dirty (d) which exhibit increasing returns to research, with clean research of s and dirty research of $1-s$ as shown in Figure 22. At first glance, a unique equilibrium would seem to be at the intersection of the profit lines. But this interior equilibrium is dynamically unstable as a change in the research share leads to a positive feedback on profits which would then drive research allocation towards one of the corner solutions, where all research is undertaken in one sector.

[74] As discussed in (Pottier, 2014).

[75] Multiple equilibria can be avoided with a stepping-on-toes effect, which introduces decreasing returns to research.

Of course this internal equilibrium could be implemented by a social planner and could also be an outcome in laissez-faire if, for example, there was uncertainty in future market shares and scientists have different guesses of what the share will be. This equilibrium could be decentralised using complex market mechanisms which create a variation in profits, such as giving different research subsidies to different scientists. However, the results would be peculiar: for example, as a research subsidy increases, clean profits Π_c increase which shifts the intersection to the left and so the share of clean research decreases. Thus, it seems reasonable to assume that this unstable equilibrium will not arise in a decentralised approach.

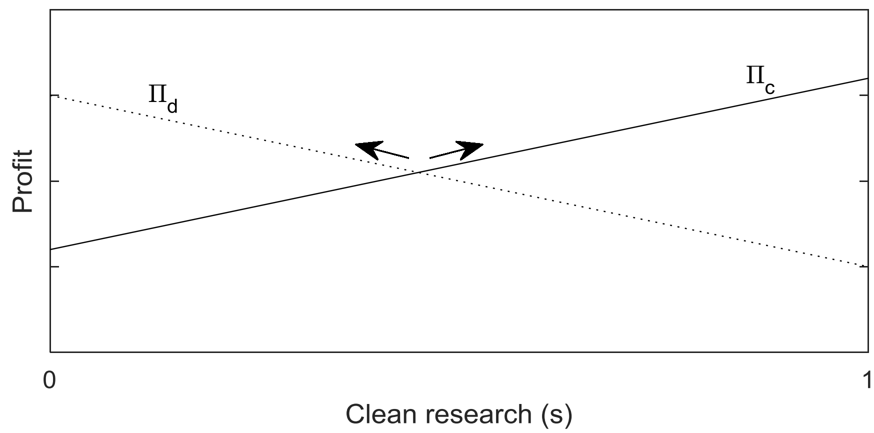

FIGURE 22: INCREASING RETURNS FROM RESEARCH

Alternatively, consider the conditions under which the corner solutions arise. One method is to assume that scientists (or their employers) coordinate, recognise the externality from the research conducted by other scientists on their expected profits and thus allocate themselves to achieve maximum expected profits with perfect foresight. This calculation involves the ratio $\frac{\Pi_c(1)}{\Pi_d(0)}$: if $\frac{\Pi_c(1)}{\Pi_d(0)} > 1$ (as shown in Figure 22) all research is in clean energy, and if $\frac{\Pi_c(1)}{\Pi_d(0)} < 1$ all research is in dirty. If $\frac{\Pi_c(1)}{\Pi_d(0)} = 1$ the equilibrium could be determined to be the same as the previous period, introducing path-dependence.

Such path-dependence could be taken further: consider all research is in dirty initially and researchers face profit outcomes shown in Figure 22. A plausible assumption is that, while researchers have an incentive to shift to clean research en masse, all research remains in dirty as the marginal researcher is not incentivized to switch to clean research as $\Pi_d(0) > \Pi_c(0)$.

Now consider the extent of subsidy (or carbon tax) required to direct clean research. Figure 23 shows pre-subsidy profits for clean Π'_c and dirty Π_d such that all research is in dirty without policy ($\Pi_d > \Pi'_c$) , and outlines three methods of determining the subsidy required to direct clean research completely, lifting clean profits to Π_c.[76]

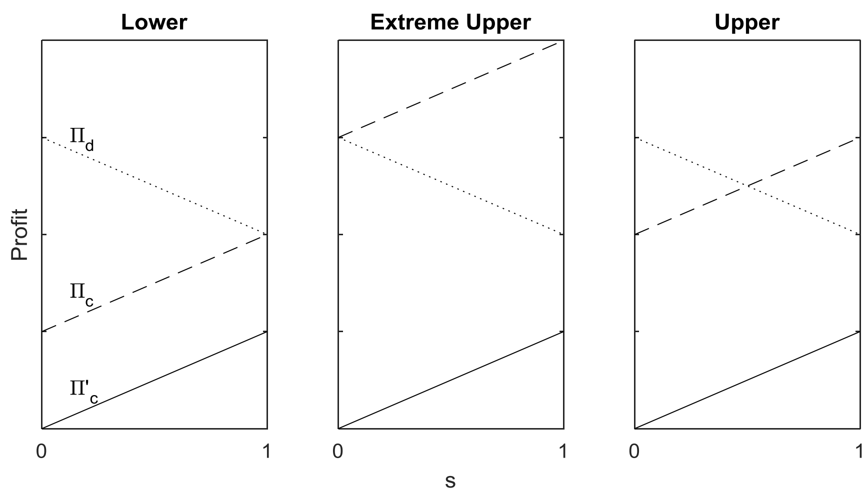

FIGURE 23. THREE METHODS OF DETERMINING THE SUBSIDY REQUIRED TO DIRECT CLEAN RESEARCH

The *Lower* method in the first panel corresponds to $\frac{\Pi_c(1)}{\Pi_d(1)} = 1$. A slightly higher subsidy would imply that, when $s = 1$, the marginal researcher is not incentivized to switch to dirty research. However, researchers have an incentive to shift to dirty

[76] A single profit line for dirty research is shown for clarity and is consistent with a clean subsidy – dirty profits may change when a carbon tax is applied.

research *en masse*, or remain there if the initial research allocation is all dirty. Thus, this equilibrium seems reasonable in providing a lower bound to the subsidy required to direct clean research.

The second '*Extreme Upper*' panel shows a symmetric solution where the critical ratio $\frac{\Pi_c(0)}{\Pi_d(0)} = 1$ applies to direct clean research. A larger subsidy is required and, even if the subsidy is reduced marginally, clean research is still more profitable than dirty when $s > 0$: the subsidy is the minimum required to induce the marginal researcher to switch to clean research when $s = 0$. Such a high subsidy could conceivably be required to induce a switch to clean research from a prior state of dirty research, but should not be needed once clean research dominates dirty research. Further, if subsidies are costly or have even a small administrative burden, the government would have an incentive to keep subsidies as low as possible. Thus, this equilibrium may be appropriate to use for a period if path-dependence is considered and dirty research initially is dominant. While the critical ratio in the *Lower* method consists of functions where $s = 1$, the critical ratio in the *Extreme Upper* method consists of a counterfactual where $s = 0$. Thus the way policy is fixed between the counterfactuals becomes important – in AABH the subsidy q is a proportion of profits and the tax τ is ad valorem, so these are assumed to be fixed in the numerical results in section 2.

The third panel assumes an equilibrium where scientists (or their employers coordinate and allocate themselves to achieve maximum expected profits. This calculation involves the ratio $\frac{\Pi_c(1)}{\Pi_d(0)} = 1$. As $\Pi_d(0) > \Pi_d(1)$ this equilibrium is Nash. Any reduction in subsidy from this level would mean that researchers are all better off undertaking dirty research than clean research. I label this method as '*Upper*' as I consider it a reasonable upper bound in a model without path-dependence. The downside of this method is greater complexity in the profit calculation because the critical ratio now involves different values of s in the numerator and denominator.

The next section examines how important these different methods are for numerical results in the AABH framework.

Section 2: Implications for results in the AABH innovation framework

AABH do not specify which equilibrium should apply when multiple equilibria exist and make different assumptions in their numerical simulations. For first-best policy simulations, AABH use the critical ratio $\frac{\Pi_c(s)}{\Pi_d(s)} = 1$ when $s > 0$ (the *Lower* case for $s = 1$). As discussed above, different specification methods imply different subsidies are required to direct technical change but have no economic impact as there is no costs of funds. However, for tax-only scenarios, the welfare costs differ between the methods as the tax is used to direct technical change as well as shift production. AABH tax-only results are consistent with the *Extreme Upper* corner equilibrium, implying a large tax needed to direct clean research.

Table 12 shows the welfare costs of tax-only policy for the different elasticities of substitution and discount rates as used by AABH. The welfare loss is reduced under the *Upper* method and even more so under the Lower method. A path-dependent method is also shown where a large tax (*Extreme Upper*) is required to first shift researchers from dirty to clean energy and then a lower tax (*Lower*) is required from then on. The most important determinant of the difference in results is the elasticity: the higher the elasticity, the greater the slope of the profit lines shown in Figure 23 and hence the greater separation between required policies to direct clean research. No matter which method is used, the welfare loss is smaller when the elasticity is high as a smaller tax is required to direct technical change. The effect of the discount rate depends on the timing of clean research. For the high elasticity case where clean research occurs immediately, a high discount rate increases welfare costs under second best as greater weight is placed on earlier periods where a higher tax is imposed. For the low elasticity case, clean research is delayed when the discount rate is high and the associated loss at this time is therefore reduced, leading to a lower welfare loss.[77]

TABLE 12— WELFARE COSTS OF RELYING SOLELY ON A CARBON TAX

Elasticity of substitution	10		3	
Discount rate	0.001	0.015	0.001	0.015
Extreme Upper (AABH)	1.02	1.66	1.92	1.48*
Upper	0.54	0.91	1.84	1.48**
Lower	0.22	0.37	1.65	1.30
Extreme Upper if $s_{t-1} = 0$, *Lower* if $s_{t-1} = 1$	0.65	1.09	1.71	1.36

Notes: Percentage reductions in utility relative to first-best policy. Utility is a discounted CRRA function of consumption with separable preferences in environmental quality as described in AABH. *AABH report a value of 3.15 due to an apparent programming error. **Extreme Upper critical ratio applies – see Appendix for details.

In summary, this paper discusses different rationales for choosing conditions for equilibria when multiple equilibria exist, as found in the framework used by AABH. The alternative methods can produce substantially different results when a high elasticity of substitution between clean and dirty inputs is assumed.

APPENDIX – SPECIFICATIONS USING THE AABH INNOVATION FRAMEWORK

For subsidy q_t, carbon tax τ_t, probability of innovation success η where innovation increases the quality of a machine by a factor $1+\gamma$, average productivity A_{jt} with $\overline{A}_t := \frac{A_{ct}}{A_{dt}}$ and $\varphi := (1-\alpha)(1-\sigma)$ where σ is the elasticity of substitution between the two sectors and α is the share of income spent on machines, sectoral profits follow from AABH (B.3), (A.16), (7) and (16) with policy:

[77] A programming error mean that AABH miss this finding and they conclude that a high discount rate increases the welfare loss under both elasticities.

$$(A.1)\ \ \Pi_{ct}(s) = \mathrm{Z}_t(1+q_t)\eta(1+\tau_t)^{\sigma}A_{ct-1},\ \Pi_{dt}(s) = \mathrm{Z}_t\eta(X_t\bar{A}_{t-1})^{1+\varphi}A_{dt-1}$$

$$\text{with } \mathrm{Z}_t := \frac{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}(1+\gamma)(1-\alpha)}{\left(1+(1+\tau_t)^{1-\sigma}\bar{A}_t{}^{\varphi}\right)^{\frac{1}{\varphi}}\left(\bar{A}_t{}^{\varphi}+(1+\tau_t)^{\sigma}\right)} \text{ and } \bar{A}_t = \frac{(1+\gamma\eta s)\bar{A}_{t-1}}{1+\gamma\eta(1-s)}.$$

Critical ratios for the three methods for tax-only policy are as follows:

$$(A.2)\ \ Lower \qquad \frac{\Pi_{ct}(1)}{\Pi_{dt}(1)} = \frac{(1+\tau_t)^{\sigma}}{1+\gamma\eta}\left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi}\Bigg|_{s=1}.$$

$$(A.3)\ \ Extreme\ Upper \quad \frac{\Pi_{ct}(0)}{\Pi_{dt}(0)} = (1+\tau_t)^{\sigma}(1+\gamma\eta)^{1+\varphi}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}.$$

$$(A.4)\ \ Upper \qquad \frac{\Pi_{ct}(1)}{\Pi_{dt}(0)} = (1+\tau_t)^{\sigma}M\bar{A}_{t-1}^{-\phi} \text{ where}$$

$$M = \left(\frac{(1+\gamma\eta)^{\phi}(1+\tau_t)^{\sigma}+\bar{A}_{t-1}^{\phi}}{(1+\tau_t)^{\sigma}+(1+\gamma\eta)^{\phi}\bar{A}_{t-1}^{\phi}}\right)\left(\frac{(1+\gamma\eta)^{\phi}+(1+\tau_t)^{1-\sigma}\bar{A}_{t-1}^{\phi}}{1+(1+\tau_t)^{1-\sigma}(1+\gamma\eta)^{\phi}\bar{A}_{t-1}^{\phi}}\right)^{\frac{1}{\phi}}.$$

Note that for a high tax τ, clean profits tend to exhibit decreasing returns to research as the price effect dominates the market size effect. In the case where $\sigma = 3$ and the discount rate is 1.5% (final column of Table 12), clean research is delayed and a large tax is needed to direct clean research. As such $\Pi_{ct}(1) < \Pi_{ct}(0)$ and the tax required to direct clean research using the *Upper* critical ratio is more than under the *Extreme Upper* method, so the *Extreme Upper* tax is selected for both methods.

Conclusion

Uncertainty of the impacts of global warming can have a large effect on optimal policy. The risk of tipping points that trigger abrupt and irreversible damages is an important example, and effects on the optimal carbon dioxide tax depend upon the probability of tipping, impacts and temperature projections. Methane is short-lived relative to carbon dioxide, and the risk of tipping may be important for determining optimal weights. If the probability of tipping is a smooth linear function of temperature, the optimal weights are flat, consistent with current policy. But if tipping events are triggered when temperature rises above unknown thresholds, the weights of short-lived actions should rise over time.

Permanent tax-only policy outperforms subsidy-only policy across a broad range of parameter assumptions, while a high discount rate favours subsidies. If optimal policy is eventually reached and suboptimal policy is only temporary, the relative performance of subsidy-only policy is closer to tax-only and performs better in some scenarios.

The elasticity of substitution between solar,wind and dirty inputs in electricity generation is a central parameter in determining the costs and optimal policy settings in a clean transition. A high elasticity of 3 or more is found empirically, broadly consistent with micro dispatch models of electricity which predict that the elasticity will decrease as the clean share rises.

Increasing returns to research can lead to multiple equilibria, and several methods can determine how a unique equilibrium might be specified. Alternative methods can produce substantially different results when the elasticity of substitution between clean and dirty inputs is high.

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