## Research Paper

# Simple models in finance: a mathematical analysis of the probabilistic recognition heuristic 

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#### Abstract

It is well known that laypersons and practitioners often resist using complex mathematical models such as those proposed by economics or finance, and instead use fast and frugal strategies to make decisions. We study one such strategy: the recognition heuristic. This states that people infer that an object they recognize has a higher value of a criterion of interest than an object they do not recognize. We extend previous studies by including a general model of the recognition heuristic that considers probabilistic recognition, and carry out a mathematical analysis. We derive general closed-form expressions for all the parameters of this general model and show the similarities and differences between our proposal and the original deterministic model.


We provide a formula for the expected accuracy rate by making decisions according to this heuristic and analyze whether or not its prediction exceeds the expected accuracy rate of random inference. Finally, we discuss whether having less information could be convenient for making more accurate decisions.

Keywords: recognition heuristic; judgment and decision making; fast and frugal; accuracy rate; less-is-more effect (LIME).

## 1 INTRODUCTION

Many scholars argue that individuals are not unboundedly rational and instead are bounded rational. There are many interpretations of bounded rationality (see Katsikopoulos 2014). In one interpretation, individuals do not have complete and stable preferences or have sufficient skills that enable them to achieve the highest attainable point on their preference scale (see Simon 1955). Moreover, rationality is limited by the information-gathering process, the cognitive limitations of the mind and the available amount of time to decide (see Gigerenzer and Selten 2002; Todd and Gigerenzer 2003).

There are many financial decisions where individuals compare (two) objects or items and then must quickly choose one of them. Investors that must choose between two different stocks, creditors that must decide between two different debtors and consumers that must decide between two similar goods with different brands are real-life examples of this type of financial decision process.

To solve this kind of problem, Goldstein and Gigerenzer $(1999,2002)$ claim that decision makers follow a simple heuristic based on object recognition. The heuristic uses the maxim that recognized objects have a higher value of some criterion than unrecognized objects. Furthermore, the recognition heuristic suggests that if neither of the two objects are recognized, then the subject decides randomly (with equal probability); and if both objects are recognized, then the subject decides with the help of some additional information (knowledge cues).

Also, the recognition heuristic challenged the idea that greater accuracy involves greater effort. As some experiments have shown, more information can, instead of increasing the rate of correct choices, decrease it. On the contrary, less information could lead to higher accuracy rates, a feature coined the "less-is-more" effect. The interested reader is referred to Pachur (2010) for an analysis of the experimental evidence for the less-is-more effect (LIME).

Following Katsikopoulos (2010), we shall distinguish two types of effect:
(1) the full less-is-more experience occurs when a person recognizing an intermediate number of objects is more accurate than a person who recognizes all of the objects; and
(2) the "below chance" LIME occurs when a person with no recognition of the objects is more accurate than a person recognizing an intermediate number of objects.

The interested reader is referred to Goldstein and Gigerenzer (2002) for a treatment on the LIME, and to Katsikopoulos (2010) for theoretical predictions by and empirical tests of the effect.

There are many applications of the recognition heuristic to finance. For instance, in a bull market, Borges et al (1999) and Ortmann et al (2008) found evidence that constructing portfolios based solely on the names of recognized companies yields better returns than the market index. They conducted laboratory experiments where participants constructed their portfolios with the most frequently recognized shares. In most cases, the selected portfolios outperformed the market index. These results were surprising, as they contradicted the efficient market hypothesis (see Fama 1970). That is, simple investment strategies cannot consistently beat the market index. A reason for this stunning result is that recognized companies may yield higher average returns than unrecognized ones.

Boyd (2001) replicated the Borges et al (1999) test in a bear market, and reached different conclusions. He found that the recognition heuristic as a strategy for picking stocks does not outperform the market. A possible explanation for these contradictory conclusions can be deduced from the model by Merton (see Andersson and Rakow 2007; Merton 1987), in which it is assumed that investors construct their optimal portfolios only with known securities. This implies that recognized firms will tend to have higher demand and value. Yet, the Merton model predicts a negative correlation between stock returns and recognition, which implies that recognized companies will yield lower returns than average, possibly explaining the results of Boyd (2001).

Andersson and Rakow (2007) study the effectiveness of the recognition heuristic in choosing stocks but find few benefits of using this heuristic to form the portfolio. They conclude that, with respect to changes in value, selecting stocks on the basis of name recognition is a near random method of conforming a portfolio.

Nevertheless, the recognition heuristic has been applied to a wide variety of other situations, for example, comparing cities with respect to their populations (Goldstein and Gigerenzer 2002), predicting sports results (Andersson et al 2005; Scheibehenne and Bröder 2007; Serwe and Frings 2006; Snook and Cullen 2006) and choosing consumer goods (Hauser 2011; Herzog and Hertwig 2011; Oeusoonthornwattana and Shanks 2010; Thoma and Williams 2013). We refer the reader to Gigerenzer and Goldstein (2011) and the references therein for further applications of this heuristic.

The aim of this paper is to develop a theoretical general model of the recognition heuristic. The novelty of our approach is that we assume that objects are recognized according to some probability distribution. The rationale for setting up a probabilistic
model is that memory is imperfect, and therefore people might not always recognize the things they have experienced in the past (see Katsikopoulos 2010; Pleskac 2007; Smithson 2010).

Our model will help us to understand the theoretical structure behind this heuristic and aims to give the following results. First, we derive the explicit formulas for all the parameters of the model and for the accuracy rate. Second, we state the conditions under which making decisions according to the recognition heuristic surpasses the strategy of choosing by simply flipping a fair coin (random inference). Third, we establish the conditions under which the LIME may appear in the decision process. Finally, we provide simulations to study the probability of occurrence of the LIME.

We find that having additional information about the objects has an important role in improving the expected accuracy rate of this heuristic when their recognition is identically distributed. Whereas, if the recognition of objects is nonidentically distributed and there is no additional information about the objects, then a positive correlation between the recognition of objects and the criterion ranking is crucial to improve the expected accuracy rate of the heuristic.

The paper is organized as follows. In Section 2, we present a snapshot of the heuristics used in finance. Section 3 introduces some definitions, provides the explicit formulas for all the parameters and establishes some properties of the accuracy rate of the model. Section 4 develops the general model for the recognition heuristic and presents our main findings. In Section 5, we study the probability of the LIME occurring by simulating the model under a variety of different scenarios. Concluding remarks complete the paper.

## 2 A SNAPSHOT OF HEURISTICS

There are many reasons why individuals may use heuristics. First, decision makers may be unable to obtain all the information necessary to solve (consciously or subconsciously) a given problem. Second, even after obtaining the necessary information, they may be unaware of the optimal method of solving the problem. Third, delay is often not an option and decisions need to be made quickly.

Practitioners in finance and economics have long used simple rules. For instance, the celebrated book by Graham (1973) recommends simple investing rules to obtain abnormally high returns. Benartzi and Thaler (2001) show that successful investors do not use sophisticated models to choose their portfolio, and usually allocate their wealth using a naive strategy that consists in investing equal shares of their initial wealth in each asset. Furthermore, Manasse and Roubini (2009) suggest some simple "rules of thumb" to predict a sovereign debt crisis and Aikman et al (2014) evaluate fast and frugal strategies in determining a bank's capital adequacy and the probability
of bank failure. Other examples of simple rules in finance can be found in Drexler et al (2014) and Neth et al (2014) and the references therein.

Indeed, the use of heuristics has been compared with other decision models in a number of additional applications, such as forecasting the commercial success of patents (Åstebro and Elhedhli 2006), diversifying financial portfolios (DeMiguel et al 2009a,b; Huberman and Jiang 2006; Monti et al 2012), predicting the future purchasing behavior of past customers (Wübben and Wangenheim 2008), prescribing antibiotics to children (Fischer et al 2002), geographical profiling of crimes (Bennell et al 2010; Snook et al 2005), predicting political elections (Gaissmaier and Marewski 2011), predicting the stock market and ranking airline safety (Richter and Späth 2006).

To sum up, these studies conclude the following:
(i) heuristics have greater predictive accuracy than optimization models when information is scarce;
(ii) the opposite appears to be true when information is not scarce; and
(iii) each of the heuristic models can outperform another more complex model, and vice versa.

For a survey of these comparisons and a thorough treatment of heuristics, the interested reader is referred to Gigerenzer et al (1999), Katsikopoulos (2011) and Schwartz (2010).

## 3 DEFINITIONS, NOTATION AND PRELIMINARY RESULTS

In this section, we set out the definitions and notation of the Goldstein and Gigerenzer (1999, 2002) original model of the recognition heuristic. The following mathematical notation is necessary to explain our main results. Suppose that we are dealing with $N$ objects $x_{i}$ for $i=1, \ldots, N$, represented as an $N$-dimensional vector $\boldsymbol{x}=\left(x_{1}, x_{2}, \ldots, x_{N}\right)$, called the recognition vector. The position of each coordinate of $\boldsymbol{x}$, and thus of the underlying object to be ranked, is based on the criterion ranking, denoted by $\boldsymbol{c}=\left(c_{1}, c_{2}, \ldots, c_{N}\right)=(1,2, \ldots, N)$, which is an arrangement of the underlying objects in decreasing order with respect to the topic of interest. For instance, according to the criterion ranking, the $i$ th object is greater in value than the $j$ th object when $i<j$. Each coordinate, $x_{i}$, of the vector is equal to 1 if the $i$ th object is recognizable, and to 0 if the object is unrecognizable.

For a recognition vector $\boldsymbol{x}$, we have

$$
\sum_{i=1}^{N} x_{i}=N_{1} \quad \text { and } \quad \sum_{i=1}^{N}\left(1-x_{i}\right)=N_{0}
$$

where $N_{1}$ is the number of recognized objects and $N_{0}$ is the number of unrecognized objects, with $N=N_{0}+N_{1}$. We define the cue vector or cue ranking as follows: $\boldsymbol{q}=\left(q_{1}, q_{2}, \ldots, q_{N}\right)$. The cue vector is used by the individuals only when both objects are recognized. It indicates the ranking of the underlying objects, which may or may not coincide with the above criterion ranking. We also assume that there are no ties (ie, $q_{i} \neq q_{j}$ when $i \neq j$ ). We shall also use the following permutations of $\boldsymbol{x}$ and $\boldsymbol{q}: \boldsymbol{x}^{-1}=\left(x_{N}, x_{N-1}, \ldots, x_{1}\right)$ and $\boldsymbol{q}^{-1}=\left(q_{N}, q_{N-1}, \ldots, q_{1}\right)$. The following example helps us to clarify the above notation.

Example 3.1 Consider the vector $\boldsymbol{x}=(1,0,1,0,0)$, which means that there are $N=5$ objects, and the (default) criterion ranking is $\boldsymbol{c}=(1,2,3,4,5)$. The objects $x_{1}$ and $x_{3}$ have been recognized, and the other three objects have not been recognized. In addition, suppose the cue ranking is $\boldsymbol{q}=(3,1,2,5,4)$, which gives information about the true object ranking. For instance, the cue ranking says that the first object is ranked third, the second object is ranked first, and so on. We recall that this cue ranking is used only when the two objects have been recognized. Therefore, we would only compare the first and third elements of vector $\boldsymbol{q}$. In this example, it means that when the pairwise comparison is between object $x_{1}$ and object $x_{3}$ the individual would follow the cue ranking and choose $x_{3}$ as the higher value of the pair, as cue ranking indicates this is so.

Finally, following the work by Smithson (2010), we define a parameter that will be used later in this work.

Definition 3.2 Let $v_{c}$ (the knowledge cue validity) be the probability of correct choices between any pair of objects using only the knowledge cue, $\boldsymbol{q}$.

We are interested in finding the probability of correct choices or simply the accuracy rate. Let $A$ be the event "correct guess". Thus, the expected proportion of correct inference is the probability $\boldsymbol{P}(A)$. In other words, $\boldsymbol{P}(A)$ is the proportion of correct answers in all of the pairwise comparisons. To calculate this probability, we first introduce the following mutually exclusive and exhaustive sets or events.

- $E_{00}$ consists of all the pairs of different objects in which both are unrecognized.
- $E_{01}$ consists of all the pairs of different objects in which one is unrecognized and the other is recognized.
- $E_{11}$ consists of all the pairs of different objects in which both are recognized,

Now, using the rule of total probability, we have

$$
\begin{equation*}
\boldsymbol{P}(A)=\boldsymbol{P}\left(A \cap E_{00}\right)+\boldsymbol{P}\left(A \cap E_{01}\right)+\boldsymbol{P}\left(A \cap E_{11}\right) \tag{3.1}
\end{equation*}
$$

Our main goal is equivalent to finding $\boldsymbol{P}(A)$ and calculating the "marginal" probabilities $\boldsymbol{P}\left(A \cap E_{i j}\right)$ on the right-hand side of (3.1). Of course, the probability $\boldsymbol{P}\left(A \cap E_{i j}\right)$ is equal to 0 when $E_{i j}=\emptyset$, the empty set. When, however, $E_{i j} \neq \emptyset$, the probability can be expressed in terms of conditional probabilities by

$$
\begin{equation*}
\boldsymbol{P}\left(A \cap E_{i j}\right)=\boldsymbol{P}\left(A \mid E_{i j}\right) \boldsymbol{P}\left(E_{i j}\right) \tag{3.2}
\end{equation*}
$$

Since computing the probabilities $\boldsymbol{P}\left(E_{i j}\right)$ is straightforward for all $i$ and $j$, our task reduces to the calculation of the conditional probabilities $\boldsymbol{P}\left(A \mid E_{i j}\right)$, which can be classified as follows.

- When $E_{00} \neq \emptyset$, we let $\beta_{0}:=\boldsymbol{P}\left(A \mid E_{00}\right)$. This is called the knowledge validity for unrecognizable objects. Throughout this paper, we set $\beta_{0}=\frac{1}{2}$, because when we face two unrecognizable objects, we choose one of them by flipping a fair coin.
- When $E_{01} \neq \emptyset$, we set $\alpha:=\boldsymbol{P}\left(A \mid E_{01}\right)$. This is called the recognition validity, which is the probability of scoring a correct answer when one object is recognized and the other is not.
- When $E_{11} \neq \emptyset$, we set $\beta:=\boldsymbol{P}\left(A \mid E_{11}\right)$. This is called the knowledge validity for unrecognizable objects, which is the probability of scoring a correct answer when both objects are recognized via an additional cue (knowledge cue).

Remark 3.3 Note that $\alpha$ is a mapping of $\boldsymbol{x}$ into $[0,1]$, while $\beta$ is a mapping of $\boldsymbol{x}$ and $\boldsymbol{q}$ into $[0,1]$.

Goldstein and Gigerenzer (2002) show that the probability of correct guesses, or accuracy rate $f(\boldsymbol{x}, \boldsymbol{q})$, is equal to

$$
\begin{equation*}
\boldsymbol{P}(A)=f(\boldsymbol{x}, \boldsymbol{q})=\beta_{0} \frac{N_{0}\left(N_{0}-1\right)}{N(N-1)}+2 \alpha \frac{N_{1} N_{0}}{N(N-1)}+\beta \frac{N_{1}\left(N_{1}-1\right)}{N(N-1)} \tag{3.3}
\end{equation*}
$$

where $\beta_{0}=\frac{1}{2}, \alpha=\alpha(\boldsymbol{x})$ and $\beta=\beta(\boldsymbol{x}, \boldsymbol{q})$.
So far, we have shown the accuracy rate of deciding between pairs of objects using the recognition heuristic as it appears in the original model by Goldstein and Gigerenzer $(1999,2002)$. Now, we derive closed-form solutions of all the parameters in (3.3). Since we assume $\beta_{0}=\frac{1}{2}$, we need to find the two remaining parameters, $\alpha$ and $\beta$. We start with the explicit formula for the recognition validity, which clearly depends on the recognition vector, denoted by $\alpha(\boldsymbol{x})$.

Proposition 3.4 For any $\boldsymbol{x}$, we have

$$
\begin{equation*}
\alpha(\boldsymbol{x})=\frac{1}{N_{1} N_{0}} \sum_{i=1}^{N}\left(x_{i} \sum_{j=i+1}^{N}\left(1-x_{j}\right)\right) \tag{3.4}
\end{equation*}
$$

Proof The proof is straightforward, since we need to calculate the proportion of correctly guessed pairs among those with one recognized object and one unrecognized object. Since there are $N_{1}$ recognized objects and $N_{0}$ unrecognized objects, using the multiplication rule of counting we obtain $N_{1} N_{0}$ pairs with one recognized and one unrecognized object. According to the heuristic, we shall guess correctly only pairs of the form $(1,0)$, while pairs of the form $(0,1)$ will be guessed incorrectly. We initiate our counting in the vector $\boldsymbol{x}$ containing the pairs $(1,0)$ with the coordinate $x_{1}$; if it is equal to 0 , we discard the case, and continue discarding until we reach the first recognized object, that is, the leftmost " 1 " of the coordinates of $\boldsymbol{x}$, which we denote by $x_{i}=1$. There are $\sum_{j=i+1}^{N}\left(1-x_{j}\right)$ zeros (ie, unrecognized objects) to the right of $x_{i}$. Hence, so far we have correctly guessed $x_{i} \sum_{j=i+1}^{N}\left(1-x_{j}\right)$ pairs. To pick the remaining correctly guessed pairs, we proceed with the next " 1 " and count all the zeros to the right of this, proceeding in the same fashion until no " 1 " remains. In this way, we arrive at $\sum_{i=1}^{N} x_{i} \sum_{j=i+1}^{N}\left(1-x_{j}\right)$ correctly guessed pairs, which are of the form $(1,0)$.

Remark 3.5 Equation (3.4) can be written as

$$
\begin{equation*}
\alpha(\boldsymbol{x})=\frac{1}{2}+\frac{1}{N_{1} N_{0}}\left[\frac{N_{1}\left(N_{1}+N_{0}+1\right)}{2}-\sum_{i=1}^{N} i x_{i}\right] \tag{3.5}
\end{equation*}
$$

which appears in Pachur (2010, p. 598).
Next, we show the explicit formula for the knowledge validity, which will clearly depend on the recognition vector and the cue vector. Hence, we shall denote knowledge validity by $\beta(\boldsymbol{x}, \boldsymbol{q})$.
Proposition 3.6 The knowledge validity can be expressed as

$$
\begin{equation*}
\beta(\boldsymbol{x}, \boldsymbol{q})=\frac{\sum_{i=1}^{N} \sum_{j=i+1}^{N} x_{i} x_{j} \mathbf{1}\left\{q_{i}<q_{j}\right\}}{\sum_{i=1}^{N} \sum_{j=i+1}^{N} x_{i} x_{j}}, \tag{3.6}
\end{equation*}
$$

where the indicator function $\mathbf{1}\left\{q_{i}<q_{j}\right\}$ is equal to 1 when $q_{i}<q_{j}$ and equal to zero otherwise.

Proof This proof is straightforward, noting that here we deal with pairs of two recognized objects, that is, with pairs of the form $(1,1)$. In the numerator, we count those pairs $(1,1)$ that have been correctly recognized by the cue ranking. The denominator is simply the total number of pairs $(1,1)$ that we have to deal with, which is

$$
\sum_{i=1}^{N} \sum_{j=i+1}^{N} x_{i} x_{j}=\frac{N_{1}\left(N_{1}-1\right)}{2}
$$

TABLE $1 \alpha$ and $\beta$ for $N=3$.

| $\beta(x, q)$ depending on $q$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{x}$ | $\boldsymbol{\alpha}(\boldsymbol{x})$ | $(\mathbf{1 , 2 , 3 )}$ | $(\mathbf{1 , 3 , 2 )}$ | $(\mathbf{2 , 1 , 3 )}$ | $(2,3,1)$ | $(\mathbf{3 , 1 , 2 )}$ | $(\mathbf{3 , 2 , 1 )}$ |
| $(1,1,1)$ | - | 1 | $2 / 3$ | $2 / 3$ | $1 / 3$ | $1 / 3$ | 0 |
| $(0,1,1)$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| $(1,0,1)$ | $1 / 2$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $(0,0,1)$ | 0 | - | - | - | - | - | - |
| $(1,1,0)$ | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| $(0,1,0)$ | $1 / 2$ | - | - | - | - | - | - |
| $(1,0,0)$ | 1 | - | - | - | - | - | - |
| $(0,0,0)$ | - | - | - | - | - | - | - |

An example of the calculations of these parameters is shown in Table 1.
We pause here to present the following lemma, which we will use in later proofs.

## Lemma 3.7 For any $\boldsymbol{x}$ and $\boldsymbol{q}$, the following equalities hold:

(1) $\alpha(\boldsymbol{x})+\alpha\left(\boldsymbol{x}^{-1}\right)=1$;
(2) $\beta(\boldsymbol{x}, \boldsymbol{q})+\beta\left(\boldsymbol{x}^{-1}, \boldsymbol{q}^{-1}\right)=1$.

We skip the proof of this lemma, since it is direct, by noting that if $(i, j)$ is a recognized-unrecognized pair and is correctly classified with vector $\boldsymbol{x}$, then if $(N+$ $1-i, N+1-j$ ) is a recognized-unrecognized pair, it will be wrongly classified with vector $\boldsymbol{x}^{-1}$ and vice versa. The same argument applies to vector $\boldsymbol{q}$.

It is worth noting the following relationship between the values of the probability of success and a permutation of $\boldsymbol{x}$ and $\boldsymbol{q}$.

THEOREM 3.8 Given a recognition vector $\boldsymbol{x}$ and a cue ranking $\boldsymbol{q}$, we have that

$$
\begin{equation*}
\boldsymbol{P}(A)+\boldsymbol{P}\left(A^{\mathrm{c}}\right)=f(\boldsymbol{x}, \boldsymbol{q})+f\left(\boldsymbol{x}^{-1}, \boldsymbol{q}^{-1}\right)=1 \tag{3.7}
\end{equation*}
$$

where $A^{\mathrm{c}}$ is the complement of $A$.
The proof is omitted since it is sufficient to note that if $(i, j)$ is a pair correctly guessed by the heuristic corresponding to the setup $(\boldsymbol{x}, \boldsymbol{q})$, then the pair $(N+1-$ $i, N+1, j)$ is wrongly guessed by the heuristic corresponding to the inverse setup $\left(\boldsymbol{x}^{-1}, \boldsymbol{q}^{-1}\right)$, and vice versa.

Remark 3.9 Note that, even though numerically $\boldsymbol{P}\left(A^{\mathrm{c}}\right)=f\left(\boldsymbol{x}^{\mathbf{- 1}}, \boldsymbol{q}^{\mathbf{- 1}}\right)$, the probabilities are not the same, since they have different sample spaces.

## 4 A GENERAL MODEL OF THE RECOGNITION HEURISTIC

Here, we present a general model of the recognition heuristic. Instead of assuming that the individual recognizes (or not) an object with certainty, as Goldstein and Gigerenzer's model supposes, we assume that the recognition of the object is random. As we shall see, this probabilistic approach will make our model more flexible in characterizing the conditions under which the recognition heuristic predicts better than random inference, and will also help us to discuss the occurrence of the LIME. Moreover, the Goldstein and Gigerenzer model is a special case of our probabilistic general model.

We define a random variable $X_{i}$ such that $X_{i}=0$ when the $i$ th object is not recognized and $X_{i}=1$ when the $i$ th object is recognized. We start be analyzing the case when each object recognition probability has the same probability distribution. Later, we study the case when this assumption is relaxed.

### 4.1 Recognition with identical distribution

We begin by assuming that $X_{i}$ are independent random variables with distribution $\boldsymbol{P}\left(X_{i}=1\right)=p$ and $\boldsymbol{P}\left(X_{i}=0\right)=1-p$ for each $i=1,2, \ldots, N$, with $p \in[0,1]$.

More formally, let $X: \Omega \rightarrow \mathbb{R}^{N}$ be the recognition random vector, where $\Omega$ is the sample space. We denote by $\boldsymbol{X}(\omega)=\boldsymbol{x}$ the real vector associated with the sample point $\omega \in \Omega$. For example, assume $N=3$. Then, recall that we have $2^{3}=8$ recognition vectors, ie, $\boldsymbol{X} \in\left\{\boldsymbol{x}_{1}, x_{2}, \ldots, x_{8}\right\}$, with

$$
\boldsymbol{P}\left[\boldsymbol{X}=\boldsymbol{x}_{1}=(0,0,0)\right]=(1-p)^{3}, \quad \boldsymbol{P}\left[\boldsymbol{X}=\boldsymbol{x}_{2}=(1,0,1)\right]=p^{2}(1-p)
$$

and so on. Note that, under the previous assumptions, $\boldsymbol{X}$ and $\boldsymbol{X}^{-1}$ have the same distribution.

We have also assumed that the knowledge cues are random. Let $Q: \Omega^{\prime} \rightarrow \mathbb{R}^{N}$ be a random vector of the knowledge cues, where $\Omega^{\prime}$ is the sample space. Similarly, we denote by $\boldsymbol{Q}(\omega)=\boldsymbol{q}$ the real vector associated with the sample point $\omega \in \Omega^{\prime}$. Likewise, $\boldsymbol{X}^{-1}$ and $\boldsymbol{Q}^{-1}$ are random vectors of, respectively, all possible vectors $\boldsymbol{x}^{-1}$ and $\boldsymbol{q}^{-1}$, as defined earlier.

Our aim is to find the expected probability of correct guesses (expected accuracy rate) of the recognition heuristic for different recognition vectors. This is done by considering all possible combinations of $\boldsymbol{x}$ and $\boldsymbol{q}$ for a given $N$. As a benchmark, we shall consider the simplest case, when $\boldsymbol{X}$ is independent of $\boldsymbol{Q}$.

We denote by $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]$ the expected accuracy rate when $\boldsymbol{X}$ and $\boldsymbol{Q}$ are random. On the other hand, we denote by $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]$ the expected accuracy rate when $\boldsymbol{X}$ is random and $\boldsymbol{Q}=\boldsymbol{q}$.

In Section 3, we derived the explicit formulas of all the parameters in (3.3) for any $\boldsymbol{x}$ and $\boldsymbol{q}$. Now, our goal is to find the expected accuracy rate for the general recognition heuristic model.

To reach this goal, we need the following result.
Theorem 4.1 The following equality holds:

$$
\begin{equation*}
\mathbb{E}[\alpha(\boldsymbol{X}) \mid \boldsymbol{X} \in A]=\frac{1}{2} \tag{4.1}
\end{equation*}
$$

where $A$ is the set of all $\boldsymbol{x}$ with pairs $(0,1)$.
Proof Since $X \in A \Longleftrightarrow X^{-1} \in A$, by invoking the first result of Lemma 3.7, we obtain

$$
\mathbb{E}[\alpha(\boldsymbol{X}) \mid \boldsymbol{X} \in A]+\mathbb{E}\left[\alpha\left(\boldsymbol{X}^{-1}\right) \mid \boldsymbol{X} \in A\right]=1
$$

Since $\boldsymbol{X}$ and $\boldsymbol{X}^{-1}$ have the same distribution, we deduce that

$$
\mathbb{E}[\alpha(\boldsymbol{X}) \mid \boldsymbol{X} \in A]=\mathbb{E}\left[\alpha\left(\boldsymbol{X}^{-1}\right) \mid \boldsymbol{X} \in A\right]
$$

Thus, $\mathbb{E}[\alpha(\boldsymbol{X}) \mid \boldsymbol{X} \in A]=\frac{1}{2}$, which is our desired result.
Theorem 4.1 states that, on average, the recognition validity is equal to $\frac{1}{2}$. In other words, the expected proportion of correct guesses for $(0,1)$ pairs is one-half. This result contradicts a possible correlation between the recognition validity and the number of recognized items, as many works suggest (see, for example, Pachur 2010).

Now, we find the expected value of the knowledge validity.
Theorem 4.2 We have that

$$
\mathbb{E}[\beta(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}, \boldsymbol{X} \in B]=v_{c}(\boldsymbol{q})
$$

where $B$ is the set of all $\boldsymbol{x}$ with pairs $(1,1)$.
We skip the proof of this theorem, since it is easily deduced from Definition 3.2.
Since we have derived the conditional expectation of all the parameters, we are now able to find the expectation of (3.3) given $\boldsymbol{Q}=\boldsymbol{q}$.
Theorem 4.3 The following equality holds:

$$
\begin{equation*}
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]=\frac{1}{2}+\left(v_{c}(\boldsymbol{q})-\frac{1}{2}\right) p^{2} \tag{4.2}
\end{equation*}
$$

Proof We have that

$$
\begin{aligned}
& \mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}] \\
& =\frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p^{2} \mathbf{1}\left\{q_{i}<q_{j}\right\}}{N(N-1)} \\
& \quad+\frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p(1-p)}{N(N-1)}+\frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N}(1-p)^{2}}{N(N-1)} .
\end{aligned}
$$

Since

$$
\sum_{i=1}^{N-1} \sum_{j=i+1}^{N}=\frac{1}{2} N(N-1)
$$

we obtain
$\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]=p^{2} v_{c}(\boldsymbol{q})+\frac{2 p(1-p)+(1-p)^{2}}{2}=\frac{1}{2}+\left(v_{c}(\boldsymbol{q})-\frac{1}{2}\right) p^{2}$.

Equation (4.2) has several implications for the recognition heuristic.
(1) When $p=0$, the vectors have only elements equal to 0 . Thus,

$$
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]=\frac{1}{2}
$$

In addition, when $p=1$, the random vectors have only elements equal to 1 . Thus, in this case, as the heuristic suggests, we decide with the help of the knowledge cue(s), and (4.2) simplifies to

$$
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]=v_{c}(\boldsymbol{q})
$$

(2) $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]$ is an increasing function in $p$ if and only if $v_{c}(\boldsymbol{q})>\frac{1}{2}$, and a decreasing function if and only if $v_{c}(\boldsymbol{q})<\frac{1}{2}$, since larger values of $p$ imply a greater chance of having a recognized object. Thus, there is an expected LIME only when $v_{c}(\boldsymbol{q})<\frac{1}{2}$. This result tells us that, for any knowledge cue whose validity is high enough, ie, is better than guessing $\left(v_{c}(\boldsymbol{q})=1 / 2\right)$, if all objects have an equal probability of being recognized, then there is no expected LIME. Note that we have a new kind of LIME, which we define for the first time. Indeed, it is a natural extension of the deterministic LIME in the probabilistic recognition case, and there is an interesting difference between them: the probabilistic-case condition involves only $v_{c}(\boldsymbol{q})$, not $\mathbb{E}[\alpha(\boldsymbol{X})]$ or even $p$, while in the deterministic case (Goldstein and Gigerenzer's version), the condition involves $\alpha$ and $\beta$.

Finally, taking the expectation with respect to $\boldsymbol{Q}$ on both sides of (4.2), and using the tower property of conditional expectation, yields a simple formula for finding the expectation in (3.3):

$$
\begin{equation*}
\mathbb{E}[\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]]=\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]=\frac{1}{2}+\left(\mathbb{E}\left[v_{c}(\boldsymbol{Q})\right]-\frac{1}{2}\right) p^{2} \tag{4.3}
\end{equation*}
$$

In the next theorem, we show the conditions under which the expected accuracy rate of the recognition heuristic cannot improve on the strategy of deciding by flipping a fair coin.

Theorem 4.4 If $\boldsymbol{Q}$ has the same distribution as $\boldsymbol{Q}^{-1}$, then

$$
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]=\frac{1}{2}
$$

Proof The proof is deduced directly by invoking Lemma 3.7, noting that if $\boldsymbol{Q}$ has the same distribution as $\boldsymbol{Q}^{-1}$, then $\mathbb{E}\left[v_{c}(\boldsymbol{Q})\right]=\frac{1}{2}$. The result follows by simply plugging this equality into (4.3).

A particular case of Theorem 4.4 is when $\boldsymbol{Q}$ is uniformly distributed. In this case, the intuition behind the assertion in this theorem is the following: since any cue vector has the same probability of being available, the cue recognition is, on average, uninformative, and the recognition heuristic accuracy rate cannot improve on random inference.

In conclusion, we find that the knowledge cue is crucial in determining whether or not the expected accuracy rate exceeds the accuracy rate of random inference. A similar conclusion appears in economics: the interested reader is referred to the literature on the value of information and its transparency in decision making (see Blackwell 1953; Broll and Eckwert 2006; Eckwert and Zilcha 2001). In addition, the knowledge cue is fundamental to establish the appearance of the LIME. These findings are consistent with the empirical evidence, where learning additional information plays a fundamental role in increasing the accuracy rate of the recognition heuristic (see, for example, Newell and Shanks 2004; Oeusoonthornwattana and Shanks 2010).

### 4.2 Recognition with nonidentical distribution

So far, we have assumed that each $X_{i}$ is equally distributed with $\boldsymbol{P}\left[X_{i}=1\right]=p$. This is a strong assumption, since it implies that each item has the same probability of being recognized. It is reasonable, however, to assume that objects with a higher value are more likely to be recognized than objects with a lower value. Therefore, here we relax this assumption as follows. Let $X_{i}$ have the following distribution:

$$
\boldsymbol{P}\left[X_{i}=1\right]=p_{i} \quad \text { and } \quad \boldsymbol{P}\left[X_{i}=0\right]=1-p_{i},
$$

with $i=1,2, \ldots, N$ and $p_{i} \in[0,1]$.
The expected accuracy rate given $\boldsymbol{q}$ is easily deduced by the following result.
Theorem 4.5 We have that

$$
\begin{align*}
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]= & \frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{i} p_{j} \mathbf{1}\left\{q_{i}<q_{j}\right\}}{N(N-1)} \\
& +\frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{i}\left(1-p_{j}\right)}{N(N-1)} \\
& +\frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N}\left(1-p_{i}\right)\left(1-p_{j}\right)}{N(N-1)} . \tag{4.4}
\end{align*}
$$

REMARK 4.6 Note that, when $p_{i}=p_{j}=p$, (4.4) collapses to (4.2).
The expected unconditional accuracy rate when the probabilities of recognition are different is presented in the next theorem.

Theorem 4.7 We have that

$$
\begin{align*}
\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]= & \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N}\left(1+p_{i}-p_{j}\right)}{N(N-1)} \\
& +\frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{i} p_{j}\left(2 \mathbb{E}\left[\mathbf{1}\left\{q_{i}<q_{j}\right\}-1\right]\right)}{N(N-1)} \tag{4.5}
\end{align*}
$$

Proof The proof follows from using the expectation tower property:

$$
\begin{align*}
\mathbb{E}[\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q}) \mid \boldsymbol{Q}=\boldsymbol{q}]]= & \mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})] \\
= & \frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{i} p_{j} \mathbb{E}\left[\mathbf{1}\left\{q_{i}<q_{j}\right\}\right]}{N(N-1)} \\
& +\frac{2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{i}\left(1-p_{j}\right)}{N(N-1)} \\
& +\frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N}\left(1-p_{i}\right)\left(1-p_{j}\right)}{N(N-1)} . \tag{4.6}
\end{align*}
$$

After simplifying the terms in (4.6), the assertion follows.
Note that $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]$ now depends on the values of each $p_{i}$ and $\mathbb{E}\left[\mathbf{1}\left\{q_{i}<q_{j}\right\}\right]$. Hence, we can distinguish three cases that can help to elucidate when the expected accuracy rate is greater than the rate of random inference.

THEOREM 4.8 The following assertions hold.
(1) If $p_{i}=p_{j}$ and $\mathbb{E}\left[1\left\{q_{i}<q_{j}\right\}\right]=\frac{1}{2}$ for all $i, j=1,2, \ldots, N$, then $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]=\frac{1}{2}$.
(2) If $p_{i}>p_{j}$ and $\mathbb{E}\left[\mathbf{1}\left\{q_{i}<q_{j}\right\}\right]=\frac{1}{2}$ for all $i \neq j$, with $i, j=1,2, \ldots, N$, then $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]>\frac{1}{2}$.
(3) If $p_{i}>p_{j}$ and $\mathbb{E}\left[\mathbf{1}\left\{q_{i}<q_{j}\right\}\right]>\frac{1}{2}$ for all $i, j=1,2, \ldots, N$, then $\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{Q})]>\frac{1}{2}$.

We omit the proof of Theorem 4.8, since it can easily be derived from (4.5).
There is a big difference between this case and the identically distributed one. Here, the recognition heuristic can have predictions better than random inference, even if the additional information is uninformative. Moreover, in the nonidentically
distributed case, we show that the expected accuracy rate of the heuristic is improved when there is a positive correlation between the probability of recognition and the criterion ranking, an intuitive fact that has been noted by Goldstein and Gigerenzer (1999, 2002).

## 5 THE OCCURRENCE OF THE LESS-IS-MORE EFFECT

One of the characteristic features of the recognition heuristic, supported by experimental data, is the LIME. That is, the accuracy rate might increase as we have less information about the objects (less recognition). To distinguish the different types of this effect, Katsikopoulos (2010) posits the concepts of "full experience" and "below chance" effects as defined below.

## Definition 5.1

(1) The full experience effect appears whenever there is a recognition vector $\boldsymbol{x}$ such that $\sum_{k=1}^{N} x_{k}<N$ and

$$
f(\boldsymbol{x}, \boldsymbol{q})>f(\boldsymbol{z}, \boldsymbol{q})
$$

where $z=(1,1, \ldots, 1)$.
(2) The below chance effect appears when there is a recognition vector $\boldsymbol{x}$ such that

$$
f(\boldsymbol{x}, \boldsymbol{q})<f(\mathbf{0}, \boldsymbol{q})
$$

where $\mathbf{0}=(0,0, \ldots, 0)$.
The full experience effect states that full recognition of the objects does not ensure an individual achieves the highest accuracy rate. In other words, a person with an intermediate amount of experience could be more accurate than a person with all possible experience. The below chance effect, on the other hand, states that an individual with no experience might be more accurate than a person with an intermediate amount of experience.

Next, we study the occurrence of the full experience and below chance effects as defined above. For $N=3$, it is easy to calculate "exactly" the value of these effects, as we show in Table 2.

Note that when we have accurate additional information $(\boldsymbol{q}=(1,2,3))$ the full experience effect appears in one-fifth of the possible recognition vectors, whereas if we have no accurate additional information $(\boldsymbol{q}=(3,2,1)$ ), this effect appears in three-quarters of the possible recognition vectors.

TABLE 2 Less-is-more effect for $N=3$.

|  | $\boldsymbol{c}$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overbrace{\mathbf{1 , 2 , 3})}$ | $\mathbf{( 1 , 3 , 2 )}$ | $\mathbf{( 2 , 1 , 3 )}$ | $(\mathbf{( 2 , 3 , 1 )}$ | $\mathbf{( 3 , 1 , 2 )}$ | $\mathbf{( 3 , 2 , 1 )}$ |
| Experience | 0.20 | 0.25 | 0.13 | 0.50 | 0.50 | 0.75 |
| Below chance | 0.25 | 0.25 | 0.25 | 0.50 | 0.50 | 0.50 |

TABLE 3 Less-is-more effect for $N=10$ and $N=5000$ simulations.

|  |  | $\boldsymbol{q}_{\mathbf{1}}$ | $\boldsymbol{q}_{\mathbf{2}}$ | $\boldsymbol{q}_{\mathbf{3}}$ |
| :--- | :--- | :---: | :---: | :---: |
| Full experience | 0.000 | 0.002 | 0.998 |  |
| Below chance | 0.162 | 0.176 | 0.813 |  |

In the following, we introduce a more complex example for $N=10$ objects, where we consider three knowledge-cue vectors that provide different qualities of information:

- high accuracy, $\boldsymbol{q}_{1}=(1,2,3,4,5,6,7,8,9,10)$;
- medium accuracy, $\boldsymbol{q}_{2}=(2,1,3,4,5,6,7,8,10,9)$;
- low accuracy, $\boldsymbol{q}_{3}=(10,9,8,7,6,5,4,3,2,1)$.

We shall simulate the results depending on whether or not the cue vector is random. If $\boldsymbol{q}$ is not random, we find these effects by choosing randomly (with equal probability) a number of recognition vectors and calculating the effects for the cue vectors $\boldsymbol{q}_{1}, \boldsymbol{q}_{2}$ and $\boldsymbol{q}_{3}$. We show the results in Table 3.

Note that for $N=10$ the probability of finding the LIME with highly accurate additional information is practically zero. This probability increases to near unity as the additional information becomes less accurate. It is interesting to note that the below chance effect has a significant positive probability even when the additional information is very accurate.

The possibility that the relationship between the knowledge cue validity and the full experience effect could be nonlinear merits some discussion. Smithson (2010) shows that when the knowledge cue validity is greater than $\frac{1}{2}$ for pairs of unrecognized objects, the full experience effect occurs only if the recognition heuristic has greater validity than the knowledge cue for choices between pairs where one object is recognized and the other is not. However, when the knowledge cue's validity falls below $\frac{1}{2}$, flipping a fair coin will perform better than the knowledge cue for choosing

TABLE 4 Less-is-more effect with different probabilities of recognition.

|  | $p_{1}=0.95$, <br> $\boldsymbol{a}=-\mathbf{0 . 0 5}$ | $p_{1}=0.45$, <br> $\boldsymbol{a}=\mathbf{0 . 0 5}$ |
| :---: | :---: | :---: |
| Full experience | 0 | 0 |
| Below chance | 0.001 | 0.197 |

between objects, so that the full experience effect has a considerably greater chance of occurring.

Finally, we perform the next simulation assuming that each object has a different probability of recognition, ie, $\boldsymbol{P}\left[X_{i}=1\right]=p_{i}$. In this case, we run simulations assuming that higher-ranked objects have a greater probability of being recognized than lower-ranked objects. We set $N=10$ and $\boldsymbol{P}\left[X_{i}=1\right]=p_{i}$ and $\boldsymbol{P}\left[X_{j}=1\right]=$ $p_{i}+a$, where $a \in \mathbb{R}$ for $i<j, i, j=1,2, \ldots, 10$. We run 1000 simulations with $\boldsymbol{q}$ uniformly distributed over all its possible values. The results are depicted in Table 4.

Here, the magnitude of both effects is small, which is consistent with our assumption that the probability of recognition increases with the objects' ranking.

## 6 CONCLUDING REMARKS

In this paper, we presented a general model of the recognition heuristic that assumes that objects' recognition is random. The recognition heuristic simply states the recognized objects are "better" than unrecognized objects. Despite its simplicity, this model has performed relatively well as a forecast tool in decision making. Our general approach allowed us to theoretically validate, evaluate and compare the recognition heuristic performance with other strategies of financial decision making.

We established the conditions under which the heuristic performs better than choosing randomly in two different settings. First, we studied the case when the recognition of objects is identically distributed. In this case, the additional information plays a fundamental role in increasing the accuracy rate of this heuristic. Otherwise, its performance cannot improve, on average, the accuracy rate of the strategy of random inference. Second, we relaxed the identically distributed assumption and studied the case where the recognition of objects is nonidentically distributed. Here, a positive correlation between the recognition vector and the criterion ranking can make the expected accuracy rate exceed the rate of random inference, even if we have an average knowledge of information. Finally, we performed simulations to analyze the probability of occurrence of the LIME. We found that the probability of its occurrence is strongly correlated with the quality of the information provided by the knowledge cue.

## DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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