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Comments

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In memory of Professor Clas Wihlborg

Abstract. This paper examines asset markets in which the key distinguishing characteristic of the goods is that they can be purchased for resale. Although the distinction between consumption durables and non-durables is clear and universally recognized, less evident is whether asset re-tradability accounts for economic instability. Market instability is strongly associated with goods that can be re-traded; stability with those that are bought for consumptive use. We emphasize the centrality of asset re-tradability in financial theory through a reinterpretation of the fundamental theorem of asset pricing: an arbitrage-free asset market is a market in which there is no advantage to re-trade any asset holdings. This result illustrates the inherent nature of the no-trade problem of neoclassical finance and suggests exploration of a different framework when it comes to dealing with asset re-tradability and speculation. We develop a relatively simple model of speculative asset price dynamics that generates excess, fat-tailed, and clustered volatility, three well-established empirical properties of financial volatility.

Keywords: re-tradable assets, asset experiments, speculation, no-trade theorems, no-arbitrage principle, excess volatility, clustered volatility, equity premium puzzle, trend following, power-law distribution

1 Introduction: Market Instability Arises from Goods That Can Be Purchased for Resale

In this paper we examine asset markets in which the key distinguishing characteristic of the goods is that they can be purchased for resale. Although the distinction between consumption durables and non-durables is clear and universally recognized, less evident is that whether items can be bought for resale, or not, accounts for their contribution to economic instability, or to stability. Market instability in both experiments and the economy is strongly associated with goods that can be re-traded; stability with those that are bought for consumptive use. (Dickhaut et al., 2012; Gjerstad & Smith, 2014, chapters 2 and 3)

The distinction between non-durable and durable re-tradable goods relates directly to Adam Smith's original two concepts of value: "The word VALUE…has two different meanings…sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called 'value in use;' the other, 'value in exchange.'" (A. Smith, 1776 [1904], Vol. I, p. 30)

In markets for non-durables, a buyer's 'value in use,' is expressed as a maximum willingness to pay (WTP) for the good in consumptive use. Because such perishable items are not re-traded by final demand consumers, they convey no "power in purchasing other goods." You buy a hamburger to eat not to resell in exchange for other goods. Similarly, a consumer does not buy a haircut, rent a hotel room, or buy a commercial airline flight ticket for the purpose of reselling it. Any such discrete item satisfies a buyer's demand for its use value if its price does not exceed the buyer's Max WTP limit; moreover, on any given market day many buyers in markets are likely not to buy such an item because the price exceeds their WTP limit.

Consequently, the price found in the market for aggregates of such buyers reflects choices that separate individuals that buy from those that do not and measures the marginal buyer's WTP expression of use value. Hence, for nondurables, value in exchange can only derive from value in use, and the two are inseparable as measures of (marginal individual) market value that are expressed in a good's price. In the above examples we cite, observe that almost none of these buyers feel the urge to buy a second, third, etc., unit until the marginal *individual* dollar spent is equal to the price, although they are entirely free to so choose. Rather, the urge is to buy cheap, for less than Max WTP. Observe that every purchase has a potential "income" (sic, value) effect; the cheaper you buy the more wealth means-of-payment left for buying other goods, including asset savings (not consuming).

For durable re-traded goods, use value is not rigidly bound to its market price. A house has use value as shelter, a function that can be satisfied by purchasing it for occupation and for the consumption of a steam of such use values over its asset life. But it also has value in exchange derived from its being possible, or providing the option, to re-sell it. A house, therefore, may be purchased where the motivation is the possibility, if not the intention, of reselling for a capital gain.

Hence, durable goods assets can involve rather different motivations than non-durables, and therefore may exhibit different forms of market behavior. This interpretation is consistent with observations on aggregates of the two types of goods found in the national accounts.

2 Consumer Components of Domestic Product, Instability and Housing

In final-demand consumer markets (US national accounts) about 75 percent of private product expenditures (GDP less government purchases) are for items that are not purchased for resale, the category of households' non-durable goods and services, *C*. The other 25 percent are households' durable goods (physical assets), *D*, including automobiles, furnishings, appliances, plus new housing construction, *H*. Among durables, however, housing has been both pernicious and episodic, as a source of economic instability: (1) New housing construction expenditures have been identified as a leading indicator of declines in GDP in eleven of the last fourteen recessions (Gjerstad and Smith, 2016, pp. 628-9). (2) Massive declines in the value of all homes in the United States are associated with the Depression, 1929-1933, and the Great Recession, 2008-2009 (Gjerstad and Smith, 2014, pp. 274-5).

The great flywheel of inertial stability in GDP arises from *C*, its largest component and its most stable component. At the other extreme, most of the instability in GDP arising from the choices of households is expressed in D and H, with housing the most volatile. Housing is also the longest-lived: "Its production is easily pushed forward when financing for it is plentiful or postponed when financing is scarce…" (Gjerstad and Smith, 2014, p. 11) Housing is thus a primary source of instability in the GDP and a perpetual target of political economy programs intended to redistribute wealth in the form of houses. Housing is so prominent in the irregular cyclical fluctuations in business, that it would not be an exaggeration to refer to the phenomenon as a consumer housing cycle. If the primary target of central bank monetary policy is interest rates, that policy is certain to impact housing, the most interest sensitive component of private spending.

3 Housing and Household Balance Sheets in Economic Instability; Severe Recessions and Household Balance Sheet Crises.

What accounts for the inordinate contribution of housing to economic instability? Most all of existing and new home purchases are executed by buyers who depend on the availability of what Adam Smith called "other people's money."[2] Home purchases are contingent on the ability of buyers to acquire long term mortgage loans to be paid off gradually over periods of 15-30 years. Consequently, a decline in the market value of all homes, against fixed mortgage debt, disproportionally impacts the owners' equity. Since total household net wealth is composed primarily of home equity, home prices inordinately impact household appending decisions.

Economists and Central Bankers usually think of GDP, and its decline in a recession, as the primary measure of economic distress. But the Great Recession, 2007-9, and the Depression, 1929-1933, are counter examples where loss in GDP as a measure of economic distress paled in comparison with the decline of household wealth in the form of broadly-based home equity. Total US home equity declined massively from $13.25 trillion in early 2006 to $5.54 trillion in early 2009, hovering near that level for the next three years. Levels of home equity this low had not been recorded for the previous 25 years (in 1984). (See Figure 11.1. in Gjerstad and Smith, 2014, p. 274.)

This is not the place, nor is it necessary, to delve more deeply into the collapse of housing markets in the Great Recession. Suffice it to say that a gap grew ever larger between the market price of

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[2] The reference, however, involved a much different context, trading in the stock of the South Sea Company, but is one of those pithy remarks by A. Smith that endures, and reminds us to look for the incentives that are at work in the roots of observations on culture and economy. (A. Smith [176] 1904, Vol. II, pp. 232-3)

a home, and its use value as measured directly by home rental prices, or indirectly by the prices of all other consumer goods and services. Other people's money mattered greatly in the momentum-driven rise in home prices, as measured by the flow of mortgage funds into the housing market: The lowest priced tier of homes rose the most, and subsequently fell the most; negative equity became widespread; GDP and all its components declined during the recession, although true to form, consumer nondurables, *C*, declined the least by far. (See Chapter 3, Gjerstad and Smith, 2014, pp. 52-3, 62-5, 75-8)

4 Asset Market Performance and Experiments

The features of asset markets enumerated above, and that we have illustrated in the market for housing, were discovered by experimentalists in the first asset market experiments.

Asset trading experiments, that were subsequently widely replicated and studied down to the present, were reported in V. L. Smith et al. (1988). These were elementary, quite transparent, markets in which all participants were provided complete common public information on the asset's fundamental value. Subjects each received an endowment of cash and shares. At the end of each trading period, each share received a dividend drawn with replacement from a commonly known distribution announced publicly and paid on each share held in the account of each investor. Therefore, if the expected dividend each period is E(d), the implication is that the fundamental value, FV, of a share during any trading period t, is simply E(d) times the number of periods remaining. For a trading-dividend horizon of periods, $T > 0$, it follows that in period 1, the asset's FV (1) = TE (d); in period 2, FV (2) = (T—1) E(d); and so-on down to FV (T) = E(d). Hence, FV (t) = (T—t+ 1) E (d), t=1, 2, 3,…,T. Asset prices are expected to follow the linear step-function

trend line from H E(d) in the first period down to E(d) in period T. That strong expectation failed and was followed by empirical and theoretical investigations exploring the causes of these surprising results.

Subsequent replications concluded that "common information is not sufficient to induce common rational expectations, but eventually through experience in a stationary environment, the participants come to have common expectations." (Porter and Smith, 1994, p. 16) This conclusion is relevant, however, only for an ideal decision maker who recognizes that exchange price must rationally reflect only dividends as the "use value" source of exchange value.

As an alternative, it has been proposed that stock market bubbles be modeled based on distinct beliefs, where individual actions are rational given the belief motives for profitable trade: (1) momentum in which purchases are positively related to the rate of change in prices, and (2) proportionate deviations from fundamental value, where these respective dynamic components are weighted by constants, F1 and F2. "A key prediction of the momentum model, that a larger initial undervaluation produces a larger positive price movement, is supported by the experiments. The predictions of the momentum model are also more accurate than those of rational expectations." (Caginalp et al., 2000a, pp. 191, 202).

The momentum feature depends on the existence of investors who rationally seek capital gains from trends that they believe they have identified in prices. This feature, however, has been questioned by experiments with inexperienced subjects who cannot profit from capital gains. Purchased shares cannot be resold, and the cash from shares sold cannot be used to repurchase shares. Yet bubbles are observed with individuals violating these elementary forms of rationality. (Lei et al., 2001) These results suggest that there are non-optimizing forces of momentum in the

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behavior of inexperienced subjects. With experience, subjects across replications of all these asset trading environments converge on "rational expectations" prices, but individuals in the process are identify-ably taking actions that are not rational. Hence as a group, and with experience, experimental asset traders achieve efficient price outcomes consistent with the rational expectations' prediction. Our models of the individual fail to capture these rationalizing interactivities, and thus do we fail to model markets as instruments of price discovery.

To summarize: goods in economics have been classified in various ways; but as regards economic instability, the most decisive typology of goods is classification according to their re-tradability. This concept of re-tradability of a good is crucial for understanding market instability at both the macro and micro level (as evidenced by the lab evidence). The next sections will further emphasize that in a deep sense, re-tradability is a fundamental concept in financial theory; thus, owing to the no-trade theorems of neoclassical finance, the concept of asset re-tradability is a core concept of neoclassical finance (Section 6). But before we so engage, we recall two other empirical regularities of asset price dynamics (besides excess volatility, or bubbles) that further emphasize the importance of re-trading and which are hard to reconcile with the neoclassical theory. In the sequel we will focus on financial markets, the one sector of the economy where the importance of re-trading hardly needs documentation: the intense, high-frequency trading occurring all the time, necessarily involves a high-turnover of asset holdings. It generates an intense excess financial volatility that has been carefully studied empirically by various researchers and is now generally regarded as obeying two notably universal empirical regularities.

5 Fat-tailed and Clustered Volatility: Two Universal Empirical Regularities in Financial Markets

Financial volatility obeys two universal empirical regularities: it is fat-tailed (more precisely power-law distributed with a tail exponent often close to 3) and it tends to be clustered in time (Fama, 1963; Mandelbrot, 1963b, 1963a; Engle, 1982a; Engle & Bollerslev, 1986; Ding et al., 1993; Gopikrishnan et al., 1998; Lux, 1998; Plerou et al., 2006; Bouchaud, 2011). The fat tail means that extreme price changes are much more likely than is suggested by the standard assumption of a normal distribution. The volatility clusters reveal a nontrivial predictability in the return process, whose sign is uncorrelated but whose amplitude is long-range correlated. These are fascinating regularities that apply to various financial products (commodities, stocks, indices, exchange rates, CDS, bitcoins) on various markets, on various time scales (ticks, minutes, days), and have also been confirmed in experimental asset market data (Kirchler & Huber, 2007, 2009). Figures 1 to 5 illustrate the two stylized facts, respectively, for the Ford Motor Company common stock; the S&P 500 Index; US-UK Exchange rate; bitcoin price; and experimental asset prices.
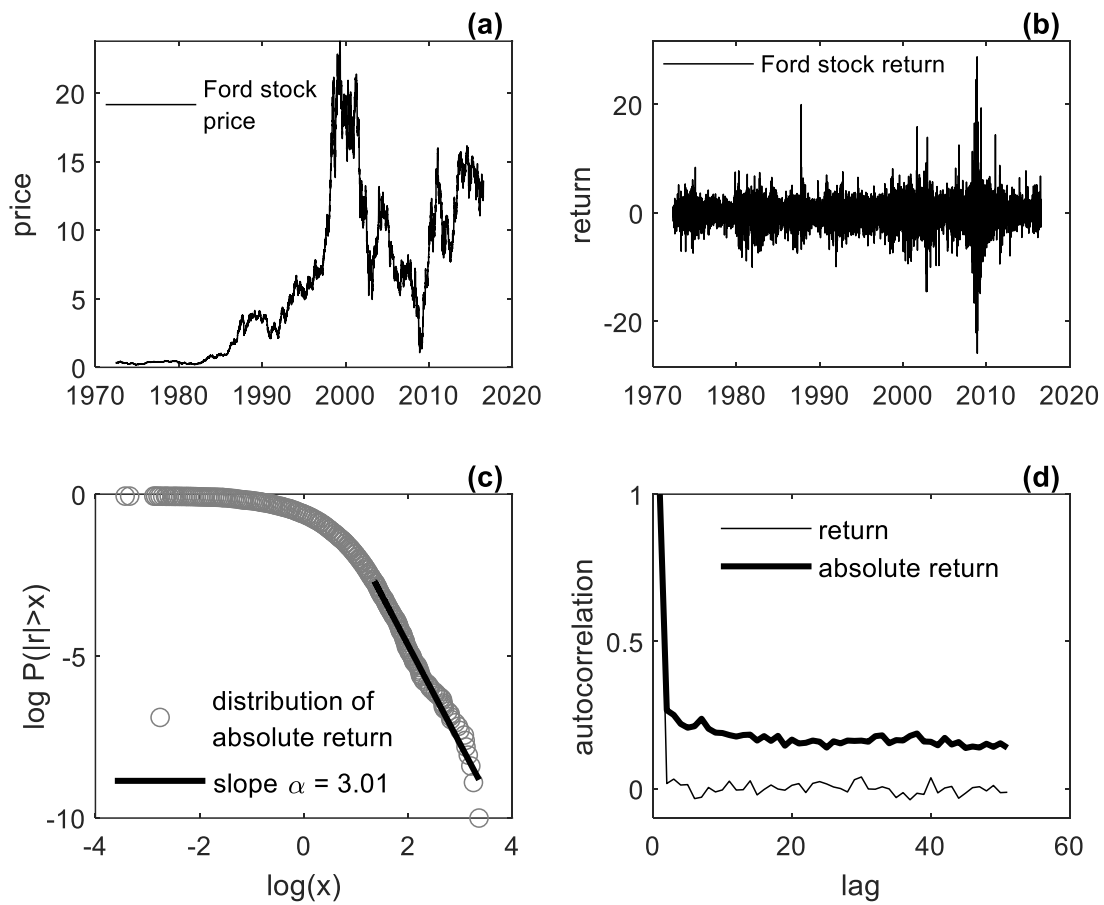
Figure 1. Ford Motor Company stock: (a) Price; (b) Return (in percent); (c) cumulative distribution of volatility in log-log scale, and a linear fit of the tail, with a slope close to 3; (d) Autocorrelation function of return, which is almost zero at all lags, while that of volatility is nonzero over a long range of lags (a phenomenon known as volatility clustering).
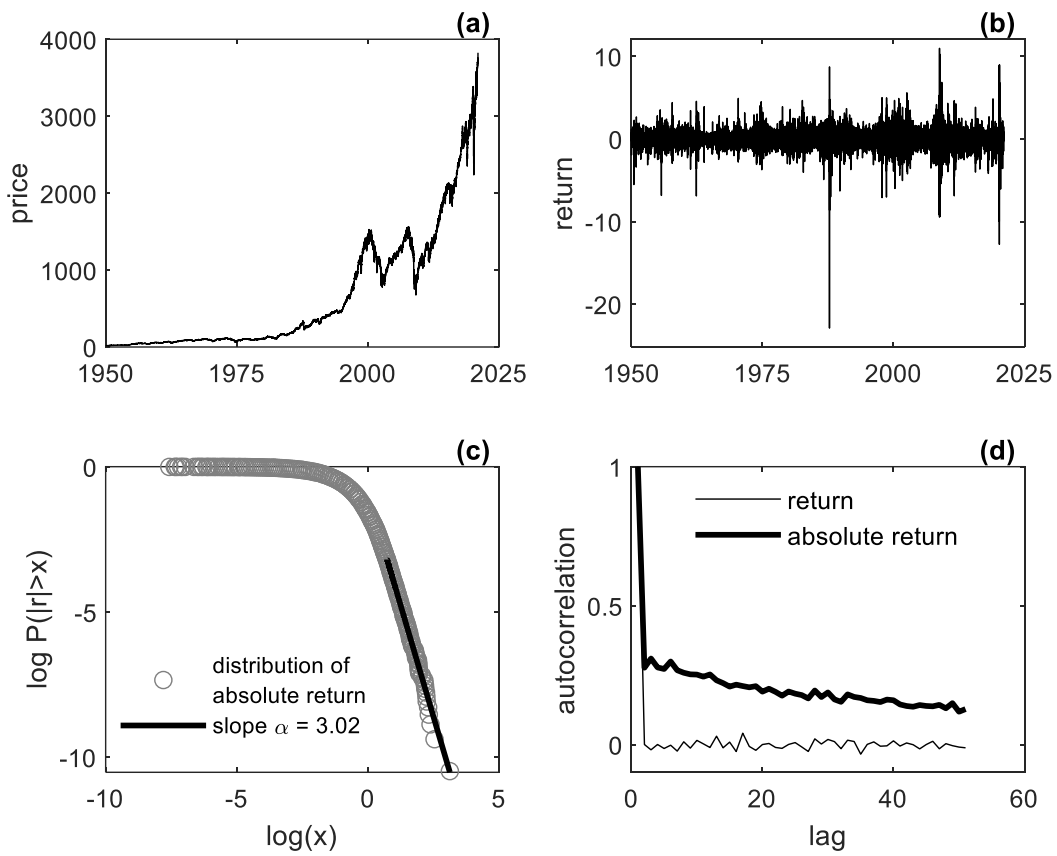
Figure 2. S&P 500 Index: (a) Price; (b) Return (in percent); (c) Cumulative distribution of absolute return; (d) Autocorrelation function of return and absolute return.
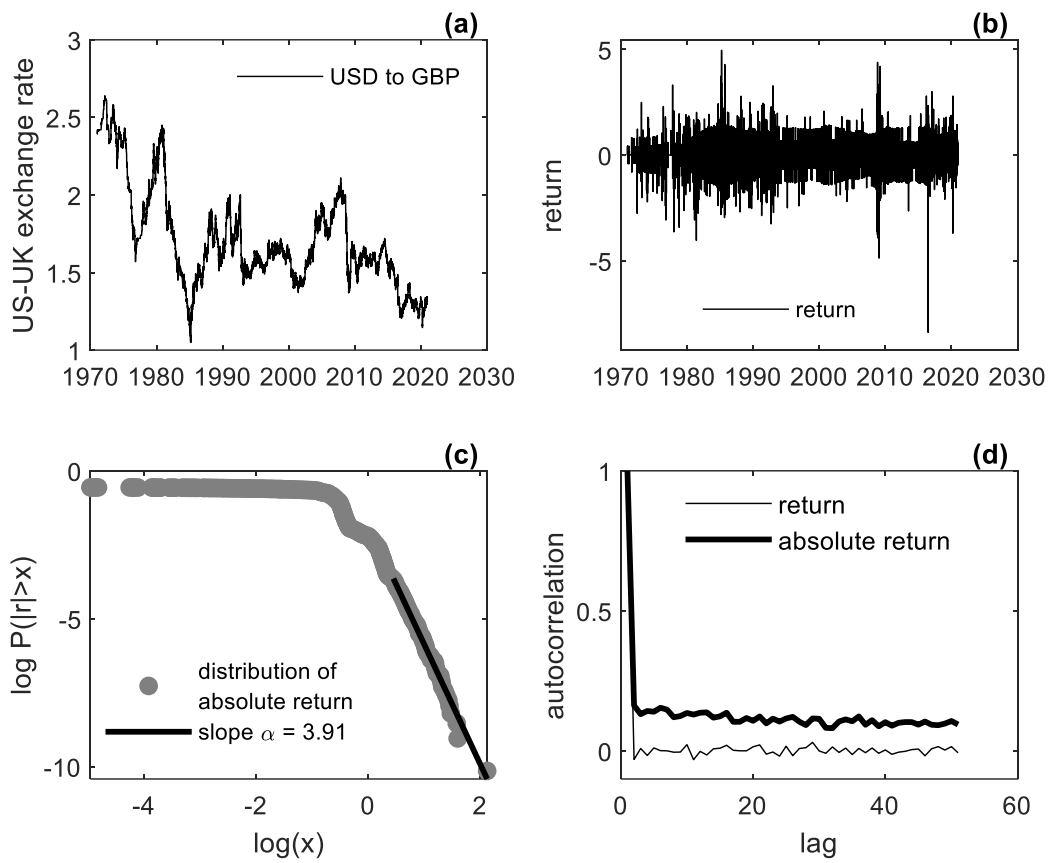
Figure 3. US-UK Exchange rate: (a) Exchange rate; (b) Return (in percent); (c) Cumulative distribution of absolute return; (d) Autocorrelation function of return and absolute return.
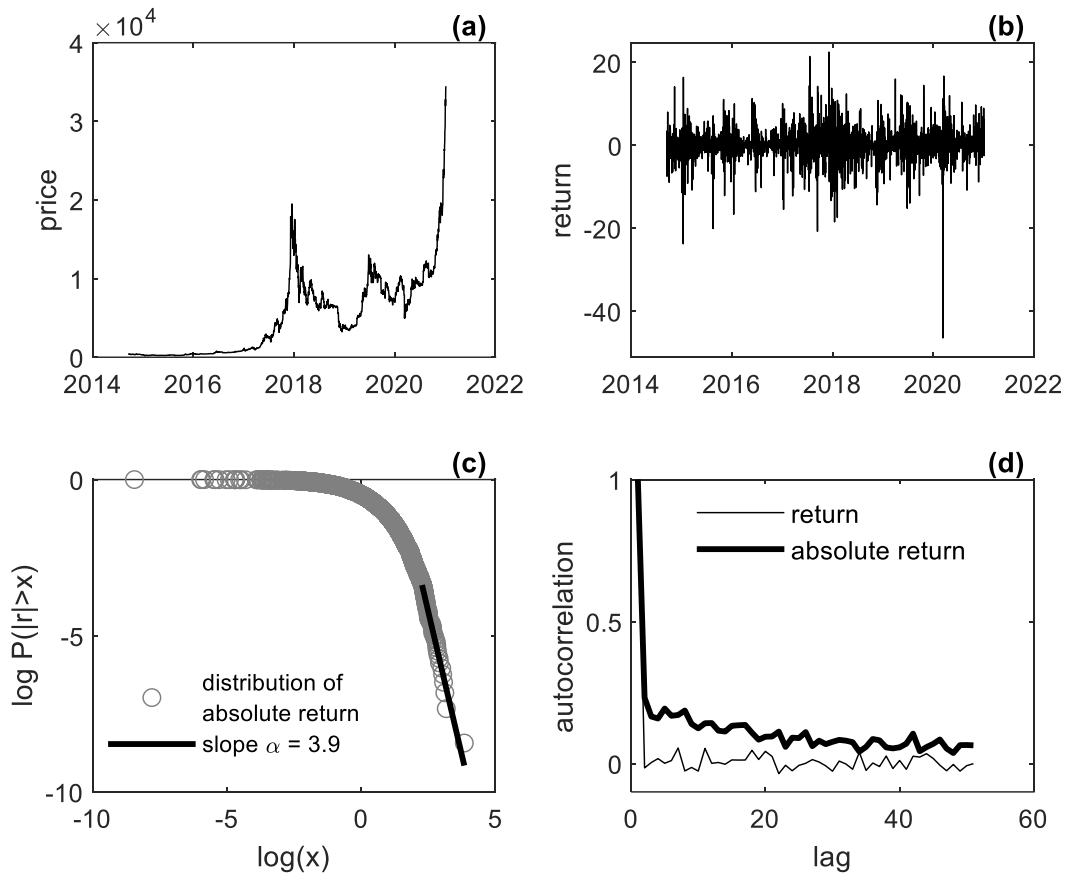
Figure 4. Bitcoin: (a) Price; (b) Return (in percent); (c) Cumulative distribution of absolute return; (d) Autocorrelation function of return and absolute return.
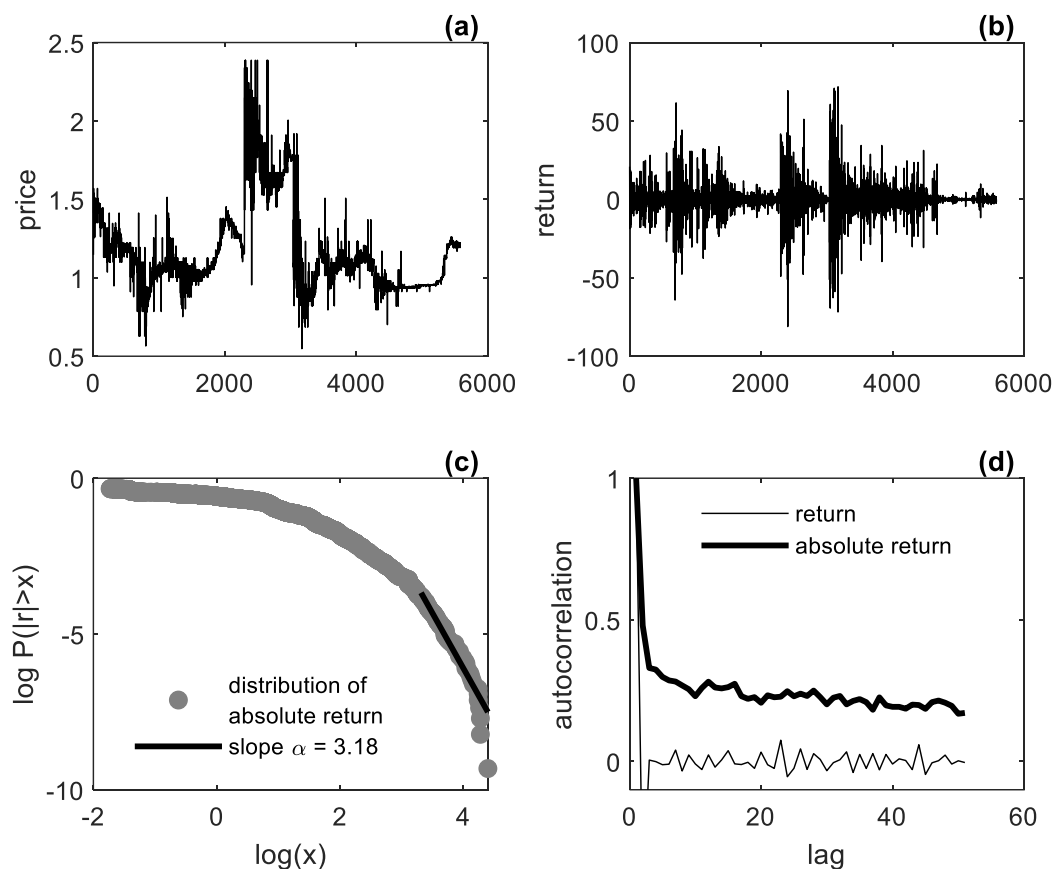
Figure 5. Experimental asset price data: 5 sessions combined data: (a) Price; (b) Return; (c) Cumulative distribution of absolute return; (d) Autocorrelation function of return and absolute return. Data source: Kirchler and Huber (2007, 2009).[3]

The universality and robustness of these empirical laws suggests that there must be some basic and permanent cause intrinsic to the very act of financial trading (supply and demand and price adjustment, or order flow dynamics and price impact) that is causing them; otherwise, data from various asset types, economic activities, and marketplaces cannot lead to uniform patterns that are robust even when the various data are pooled together (as for the S&P index in Figure 1, or

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[3] The authors thank Jürgen Huber for kindly sharing their data. Notice that we pooled 5-market's data with different treatments. This may sound surprising at first sight; but it is not, given the hypothesis of universality of the empirical laws. An index (such as the S&P 500) is also pooling different assets, at different places. But on this see the text.

as in the lab data in Figure 5, which pools various sessions or markets together).[4] To identify these

causes, one should go back to the basics of financial theory and contrast two major paradigms

on financial fluctuations.

The dominant view today, the efficient market hypothesis, treats an asset's price as following a

random walk *exogenously driven by fundamental news* (Bachelier, 1900; Osborne, 1959; Fama,

1963; Cootner, 1964; Fama, 1965a, 1965b; Samuelson, 1965; Fama, 1970). But owing to the

intrinsic no-trade theorems of neoclassical finance (Rubinstein, 1975; Milgrom & Stokey, 1982;

Tirole, 1982), the powerful efficient-market hypothesis is hard to derive theoretically, as an

equilibrium condition of supply and demand, as is known. This is not a minor problem, for what

is at stake in the no-trade concept is the very possibility of a financial market in this theory (since

a market where no trade takes place is not a market). The no-trade problem justified the

prominence of the no-arbitrage approach to neoclassical finance, which in effect renounced an

explicit equilibrium approach, and postulates no-arbitrage pricing as a fundamental principle

without specifying a means for its attainment (Ross, 2005, ch. 3). The no-trade problem remains

unsolved, however, for the elegant no-arbitrage approach to financial economics (taken for

granted in mathematical finance literature) evades the no-trade problem. Rather than solving it,

an arbitrage-free market is precisely a market in which no re-trade takes place (that is, absence

of arbitrage is mathematically equivalent to a no-re-trade), as emphasized in Section 6 (see 6.2).

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[4] It should be insisted that the power law of returns, perhaps even the so-called cubic law (alpha=3), seems to be robust to aggregation across various asset data (see, e.g., Gabaix et al., 2006), making the regularity particularly surprising. At least the (non-Gaussian) fat tail and volatility clustering can be viewed as "universal" regularities of financial volatility. Evidence in favor of a stable and universal relation between order flow history and the direction of price, used to justify pooling various asset data, is also recently suggested by machine learning (Sirignano & Cont, 2019).

Notwithstanding its difficult neoclassical derivation, the notion that asset prices are driven by fundamental news (or exogenous events more generally) is clearly plausible at least in part and relevant for the understanding of at least the second regularity: big volatility clusters tend to correspond to major economic events, such as uncertainty ensuing a crisis for example, although the sense of causality is tricky as usual in economics (Figure 6).
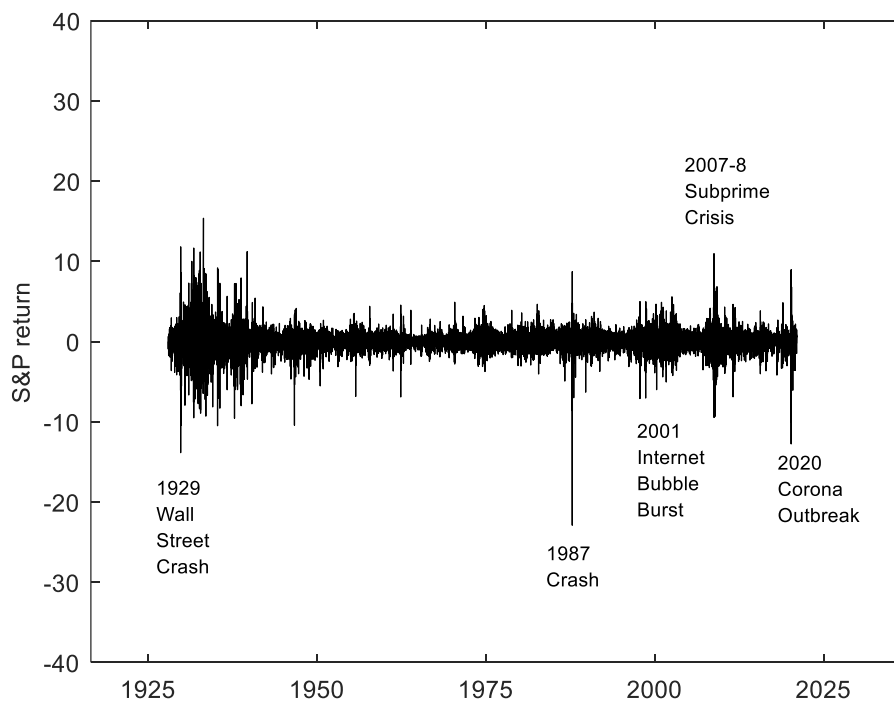
Figure 6. Big Volatility Clusters Triggered by Major Events (Crises).

Nonetheless, the hypothesis that volatility clusters are triggered by economic events that shift traders' expectations of fundamental values seem to be a reasonable one and is also validated experimentally. The arrival of heterogenous dividends news is even suggested to be the main cause of both the fat tail and the clusters of volatility (Kirchler & Huber, 2007, 2009). Yet the purely news-driven view of asset price volatility is not completely satisfactory: the dramatic case often invoked is of course the 1987 crash, which is hardly caused by any apparent perceptible

bad news about economic fundamentals. Moreover, high-frequency asset price moves (some happening at the rhythm of a tick) are hardly compatible with announcements of fundamental news (with occur at much lower frequency); yet the two empirical laws hold even in high-frequency data.[5] So, although important, the purely fundamental news-driven asset price moves hypothesis is not completely satisfactory.

In reaction to the efficient-market hypothesis is the growing resurgence of an old view of financial markets that insists on *endogenous amplifying feedback mechanisms* caused by mimetic or trend-following speculative expectations, fueled by credit, and responsible for bubbles and crashes (Fisher, 1933; Keynes, 1936; Shiller, 1980; V. L. Smith et al., 1988; Cutler et al., 1989; Orléan, 1989; Cutler et al., 1990; Minsky, 1992; Caginalp et al., 2000b; Barberis & Thaler, 2003; Porter & Smith, 2003; Akerlof & Shiller, 2009; Shaikh, 2010; Bouchaud, 2011; Dickhaut et al., 2012; Keen, 2013; Palan, 2013; Soros, 2013; Gjerstad & Smith, 2014; Soros, 2015). The nuance in this diverse literature on endogenous financial instability lies perhaps in the nature of the ultimate destabilizing force that is specifically emphasized in each tradition, notably human psychology (Keynes and behavioral finance) or the easy bank-issued liquidity that backs or fuels the speculative euphoria and without which this latter would be of no significant, macroeconomic, harm (Fisher, Minsky, Kindleberger).

Thus, one view insists on a fundamental news arrival process as the main cause of asset price volatility (where the main and in fact only players are long-run fundamental-value-investors who update continuously with current information), while the other insists on endogenous amplifying

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[5] It is customary to cite the 1987 Crash as a dramatic illustration that price moves can happen absent any apparent fundamental news (Cutler et al., 1990).

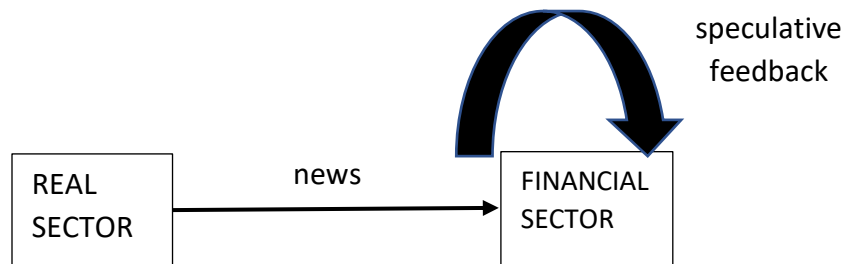feedback intrinsic to speculative asset supply and demand (where the players are short-run speculators).

Figure 7. The two components of asset price dynamics: fundamental news versus amplifying speculative feedback trading.

A faithful picture of financial price formation, one that takes seriously the empirical laws, would most likely involve some synthesis of the two visions, involving both long-run investors and short-run speculators, both fundamental-news-triggered trades but also self-reinforcing endogenous speculative trades. But such a synthesis would necessarily involve a more radical departure from neoclassical than a mere nominal one, owing precisely to the no-trade theorems. For example, the dominant synthesis in terms of noisy rational-expectation asset market equilibrium is theoretically unsatisfactory, in that the very concept of unexplained noise-trading is self-defeating as an explanation of market-level empirical regularities, as is known: if the noise trading is truly based on noise, it should average out on the aggregate; but if it is not averaging out, then it was a misnomer to call it noise-trading.

GARCH offer parsimonious models of the two regularities (Engle, 1982b; Bollerslev, 1986; Bollerslev et al., 1992); yet these statistical models are hardly a theoretical explanation of the empirical laws in terms of explicit economic mechanisms; when fitted to empirical data, moreover, they imply an infinite-variance return process, the integrated GARCH (or IGARCH)

model (Engle & Bollerslev, 1986), which corresponds to a more extreme randomness than the empirical one (Mikosch & Starica, 2000, 2003). Agent-based models, on the other hand, offer a realistic picture of financial markets and mimic the two empirical laws, although through a complex mix of nonlinear mechanisms, such as traders' switching between trading strategies in highly nonlinear way. [For a review of these models, see, e.g., Samanidou et al. (2007) and Lux and Alfarano (2016).] Other types of models offered in the literature explain more specifically one regularity, for example, a model of the extreme (power law) variations of asset return in terms of large trades by major institutional investors (Gabaix et al., 2006).

Section 6.4 will offer a relatively simple model of asset price formation that explains the excess, fat-tailed, and clustered volatility in an intrinsic way, in terms of speculation and reaction to news, and centered on the traditional dichotomy of financial traders (shared by most agent-based models), investors versus speculators. The model reduces their behaviors and expectations to simple, linear, mechanisms (a departure from the agent-based tradition), assuming no switching between trading strategies by traders, an interesting feature of agent-based models suggested in a seminal contribution (Lux & Marchesi, 1999, 2000), but which is not essential to the emergence of the two regularities in our model, nor in the laboratory studies mentioned above (Kirchler & Huber, 2007, 2009).

The power-law tail of volatility can be shown to derive intrinsically from the self-reinforcing amplifications inherent to trend-following speculative trading based on adaptive expectations or moving-averages, a popular financial practice, leading directly to a random-coefficient autoregressive (RCAR) return process. The power-law tail of such processes is rigorously proven in the mathematical literature (Kesten, 1973; Klüppelberg & Pergamenchtchikov, 2004;

Buraczewski et al., 2016). RCAR processes are also known as Kesten processes, named after H. Kesten whose seminal theorem proves their power-law tail behavior. Kesten processes are perhaps the most convincing generic power-law generating processes and are invoked in a few models of financial volatility, notably GARCH processes themselves, although these latter were not originally viewed as such. A first order RCAR process has also been suggested as approximations to complex agent-based mechanisms (Sato & Takayasu, 1998; Aoki, 2002; Carvalho, 2004). 'Rational bubbles' also can be interpreted as first-order RCAR processes assuming a random discount factor (Lux & Sornette, 2002); but this model generates a tail exponent smaller than 1, an infinite-mean bubble and return process, which is much more extreme than justified by the empirical data.[6] The point of the model offered in this paper is to show that a general RCAR return process holds intrinsically, almost by definition, in a competitive market of trend-following speculators.

But it can be proven that the RCAR model cannot explain volatility clustering (Mikosch & Starica, 2000; Basrak et al., 2002; Mikosch & Starica, 2003; Buraczewski et al., 2016). The basic cause of clustered volatility, as this paper suggests, in accordance with a hypothesis already put forward to explain the emergence of the stylized fact in experimental asset data (Kirchler & Huber, 2007, 2009), is none other than the persistent, long-memory, impact that exogenous news impresses upon traders' expectations. Thus, the general model is populated by both speculators and investors: the investors attach a value to an asset and buy it when they think the asset is underpriced, or sell it, otherwise, updating additively their valuations with the advent of a

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[6] In fact, since an infinite-mean process (in absolute value) does not qualify as a martingale process (whose mean should be finite), the presumed neoclassical foundation of this model is uncertain.

fundamental, exogenous news. Speculators' expectations of future price changes are similar but correspond to a shorter memory of fundamental news by definition. The model exhibits the excess, fat-tailed, and clustered volatility in a generic and robust way, although all traders' behavior is simple, linear.

The rest of the paper is organized as follows: Section 6 treats formally the no-trade problems of neoclassical finance, presented in a slightly unconventional way, to emphasize the basic implicit assumption behind the no-trade problem. Section 7 then presents formally the alternative model of asset price formation just sketched.

6 Neoclassical Finance: No Arbitrage is Equivalent to No Re-trade

6.1 Basic Definitions: Information, Asset, Financial Market, Speculation

By neoclassical finance we mean the standard theory of financial markets based on the following axioms: (1) a financial asset is completely specified by the future monetary payoff stream it entitles the holder: it is formally identified with the random process of future payoff stream; (2) the financial traders are price-taking expected-utility maximizers who always prefer more money, and dis-prefer less; (3) the agents' subjective expectations are "rational" in the neoclassical sense: in particular, their subjective expectations behave like mathematical expectations; (4) each trader in a frictionless market can trade any amount of assets without restriction and without affecting the prices; (5) assets' prices are fair prices: a financial market is arbitrage-free.

Since assets in neoclassical finance are identified with the stochastic process of their payoffs, a financial market is completely specified in this approach by a probability space of possible economic events and by the information available to financial traders. Throughout, we assume

given, a finite probability space of economic events $(\Omega, \mathbb{P})$ and we denote by $\mathbb{E}$ the associated expectation operator. The probability space being given, we characterize a financial market by $\{\mathrm{info}_t\}$, the public information available to traders at the end of period t.

Intuitively, information reduces uncertainty due to the realization of an event or collection of events (in the probabilistic sense of the term event): therefore, an information set is identified with a set of events. The events that carry information to financial traders can be a simple event (such as the announcement of a dividend) or the event associated with the realization of a stochastic process (price history for example). We assume the information available to traders is non-decreasing (no information is lost). By a *fundamental news* (or news for short) we shall mean the realization of an event that is relevant for the traders' valuation of an asset's fundamental value, notably a dividend announcement.

This intuitive understanding of the fundamental concept of information is enough for the probabilistic manipulations that follow (while avoiding much intimidating technical jargon from measure theory that would have otherwise cluttered the essence of things, which is simple. These technicalities are hardly needed since we assume a finite number of possible states of nature or the economy).

We assume throughout discrete time periods $t = 0, 1, ..., T$. Defining time as periods is convenient (by simplifying the notation) provided we adopt throughout the following convention: all position or "stock" variables (asset holdings, dividends, information accumulated) are evaluated at the end of each time period; while change or "flow" variables (demand, supply, transactions) apply to a whole period. (This will become clear soon.) We consider a financial market in which there

are K assets (numbered $1,...,K$). At the closing of each period t is announced the (positive) market prices $\mathbf{p} = [p_t^1,...,p_t^K]$ at which any trader is free to buy or sell the assets during the next period, $t+1$. (This convention merely makes the notation more natural.) We express prices in terms of a risk-free asset (a bond), taken as numeraire. Let $\mathbf{d}_t = [d_t^1,...,d_t^K]$ be the payoffs the assets entitle their holders per unit of asset held at the end of each period t. We shall refer to these payments generically as *dividends*, using the term more comprehensively for share dividends, interest coupons, insurance payout. Let $\mathbf{H}_t = [H_t^1,...,H_t^K]$ denote a trader's desired asset holdings of each security at the closing of time period t, counting asset instruments (stocks, bonds, and lending) positively and liability securities (bonds and borrowing) negatively: we refer to the sequence $\mathbf{H} = \{\mathbf{H}_t : t=0,...,T\}$ as the trader's *asset holding strategy*. By definition, $\mathbf{z}_t \equiv \mathbf{H}_t - \mathbf{H}_{t-1}$ is the trader's asset demand (or supply if negative) during period t (where orders to buy are counted positively, and orders to sell, counted negatively): we refer to the sequence $\mathbf{z} = \{\mathbf{z}_t : t=1,...,T\}$ as the trader's *trading strategy*. A trader decides how much to trade based on available information. By construction, the trading strategy for period t is decided based on information available at the beginning of that period, namely info_{t-1}, hence we will say that a trading strategy $\mathbf{z}$ is based on available public information if it is determinate given that information, so that $\mathbb{E}(\mathbf{z}_t \mid \mathrm{info}_{t-1}) = \mathbf{z}_t$.

A trading strategy motivated solely by dividends is called a fundamental *investment* (or investment for short). A trading strategy motivated solely by re-trading gains based on price fluctuations through time will be called a *speculation.* Arbitrage, as broadly used in economics, is a generalization of speculation, in the sense that it allows also for gains based on price fluctuation

through space (the same asset simultaneously traded in two places at different prices). But as mostly used in financial theory (especially in mathematical finance), spatial arbitrage is not considered, making arbitrage and speculation fundamentally equivalent notions, as emphasized below; of course, arbitrage is defined in that literature as a risk-less and costless speculative gain extracted from the market; yet it can be shown that an arbitrage-free market excludes risky speculation as well, so long as traders are risk-averse expected utility maximizers (see Proposition in Subsection 6.2).

Although it is customary in the mathematical finance literature to define arbitrage in terms of portfolio holding strategy $\mathbf{H}$ (there called trading strategy), it is more natural to define arbitrage in terms of trading strategy $\mathbf{z}$ (as defined above).

Finally, in terms of probability theory concepts, we consider that a process, say price, $\{p_t : t = 0,...,T\}$, is fair in view of public information, or formally that it is a martingale with respect to that information, if $\mathbb{E}(p_{t+1} \mid \mathrm{info}_t) = p_t$, for each $t = 0,...,T-1$ (which requires that the process have finite expectation in absolute value, for the conditional expectation to exist). The definition applies to a vector process, which is a martingale if each of its components is a martingale.

The basic concepts being defined, we adopt the following:

Definition 1 (Financial Market). A financial market is specified by $(\Omega,\mathbb{P},\{\mathrm{info}_t : t = 0,1,...,T\})$, where $(\Omega,\mathbb{P})$ is a probability space of states of the economy and info_t is public information available to traders, which includes historical prices and dividends up to period t.

6.2 No Arbitrage = No Re-trade

The bedrock of neoclassical finance is of course the (first) Fundamental Theorem of Arbitrage-free Pricing—or FTAP (Ross, 1976; Rubinstein, 1976; Ross, 1978; Harrison & Kreps, 1979; Harrison & Pliska, 1981; Dalang et al., 1990).[7] Arbitrage, recall, is essentially a costless and riskless speculative strategy informed by price history that guarantees a positive expected gain.

A trader's financial wealth (evaluated at the end of each period t) is by definition the value of the trader's asset holding, minus asset purchase cost, plus resale revenue, plus dividends received:[8]

$$W_t = \mathbf{p}_t \cdot \mathbf{H}_t - \mathbf{z}_t \cdot \mathbf{p}_{t-1} + \mathbf{d}_t \cdot \mathbf{H}_t, \tag{1}$$

for $t \geq 1$, and the initial wealth $W_0 \equiv \mathbf{p}_0 \cdot \mathbf{H}_0$ (the value of the trader's initial asset holdings), and $\mathbf{z}_t \equiv \Delta \mathbf{H}_t$ being the traders' transaction during period t. Had the trader maintained his asset position throughout period t, refraining from re-trading any unit, he would enjoy a wealth of

$$W_t^* = \mathbf{p}_t \cdot \mathbf{H}_{t-1} + \mathbf{d}_t \cdot \mathbf{H}_{t-1}. \tag{2}$$

The relative advantage of re-trading over holding one's position is measured by $R_t = W_t - W_t^*$, namely:

$$R_t = (\mathbf{p}_t - \mathbf{p}_{t-1} + \mathbf{d}_t) \cdot \mathbf{z}_t. \tag{3}$$

In more explicit notation:

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[7] For a simplified and clarified exposition of the mathematics of the finite-market case, see, e.g., Taqqu and Willinger (1987).

[8] Recall that by construction, all trades during period t are to be executed at $\mathbf{p}_{t-1}$, the prices announced at the closing of $t-1$.

$$R_t(\mathbf{z}_t) = W_t(\mathbf{z}_t) - W_t(\mathbf{0}). \tag{4}$$

Let $G_t \equiv \sum_{\tau=0}^{t} R_\tau$ be the accumulated re-trade gains, with $G_0 = R_0 = 0$, a natural convention.

The buying of most financial assets comes with an implicit option of re-trading the asset for speculative gains: then for a single security $(K=1)$ we can naturally define the value of a unit of this implicit *re-trade option* at a given time as obtained by setting $z=1$ in (4). In theory, we can think of this re-trade option itself as a (potential, derivative) asset, identified with its payoff. Thus for any tradable asset, we can have associated to it for each time period t, the unitary

$$Implicit\ Re\text{-}trade\ Option\ = R_t(1) = W_t(1) - W_t(0).$$

As it turns out, this implicit re-trade option is not exercised in an arbitrage-free market: the value of an implicit re-trade option is zero in an arbitrage-free market. In fact, an arbitrage-free market can be characterized as one in which any re-trade option is valueless. Formally, we can define arbitrage as follows:

Definition 2 (Arbitrage). *A market contains an arbitrage opportunity if there exists a trading strategy $\mathbf{z} = \{\mathbf{z}_t : t = 1,..,T\}$ based on available information that yields a terminal re-trade advantage $G_T(\mathbf{z}) \geq 0$ with $\mathbb{E}(G_T(\mathbf{z})) > 0$ starting from $G_0 = 0$.*

This slightly unconventional definition of arbitrage is in fact a natural one, as the following implication suggests. (Recall that the available $info_t$ in the market contains both price and dividend information.) The corresponding formulation of the FTAP reads more simply in terms of the following *discounted conditional expectation operator* process (that will play the role of an

arbitrage-free pricing operator), which for any random variable X dependent on available information, and for each $t = 0, ..., T-1,$ gives its discounted conditional expected value

$$\mathbb{E}_t^*(X) \equiv \mathbb{E}(\delta_{t+1}\delta_t^{-1}X \mid \text{info}_t), \tag{5}$$

where $\{\delta_t\}$ is a (strictly) positive process that depends on info_t and is a fair process given info_t. In other words, (5) defines a weighted averaging operator process, where a discount factor $D_{t+1} \equiv \delta_{t+1}\delta_t^{-1}$ plays the role of weight, since

$$\mathbb{E}_t^*(D_{t+1}) = \mathbb{E}(\delta_{t+1}\delta_t^{-1} \mid \text{info}_t) = \delta_t^{-1}\mathbb{E}(\delta_{t+1} \mid \text{info}_t) = \delta_t^{-1}\delta_t = 1.$$

In particular, if for a given time t, we identify the indicator function 1_Ω, or simply 1, with the asset paying 1 dollar for sure (a riskless bond), then $\mathbb{E}_t^*(1) = 1.$ The operator (5), for t given, is a *(strictly) positive* linear operator: $\mathbb{E}_t^*(X) \geq 0$ if $X \geq 0$; $\mathbb{E}_t^*(X) > 0$ if $X > 0$, since $D_{t+1} > 0.$ Also, for $X \geq 0,$ we have $\mathbb{E}_t^*(X) = 0$ implies $X = 0.$

The following is an interpretation of the Fundamental Theorem of Asset Pricing (FTAP) as a No-Re-trade Theorem (where arbitrage is as reformulated in Definition 2).

Theorem (FTAP as a No-Re-trade Theorem). *A market* $(\Omega, \mathbb{P}, \{\text{info}_t\})$ *is arbitrage-free if and only if there is a positive discounted expectation operator* (5) *such that for each* $t = 0, ..., T-1,$

$$\mathbf{p}_t = \mathbb{E}_t^*(\mathbf{p}_{t+1} + \mathbf{d}_{t+1}). \tag{6}$$

Proof sketch. The proof is the same as in the literature, save for the conceptual simplification due to the formulation of arbitrage in terms of re-trade gains (rather than nominal capital gains of a

portfolio holding).[9] Thus we only prove the easy direction of the equivalence [existence of a positive operator (5) excludes any arbitrage opportunity]. Since $\mathbf{z}_{t+1}$ is constant given info_t, we get $\mathbb{E}_t^*(G_{t+1} - G_t) = \mathbb{E}_t^*(R_{t+1}) = \mathbb{E}_t^*[\mathbf{z}_{t+1} \cdot (\mathbf{p}_{t+1} - \mathbf{p}_t + \mathbf{d}_{t+1})] = \mathbf{z}_{t+1} \cdot \mathbb{E}_t^*(\mathbf{p}_{t+1} - \mathbf{p}_t + \mathbf{d}_{t+1}) = 0$, hence $\mathbb{E}_t^*(G_T) = \mathbb{E}^*(G_0) = \mathbb{E}^*(R_0) = 0$, and since $G_T \geq 0$, $G_T = 0$. The converse [existence of a positive operator (5), given no arbitrage] is omitted: it is involved but standard. ∎

From the viewpoint of the investors, we can state the No-Re-trade Theorem more specifically: [10]

Proposition. *In an arbitrage-free market, no rational risk averse expected-utility maximizing trader would re-trade any asset given information on prices and dividends.*

Proof. A trader's accumulated financial wealth is decomposed into an asset holding wealth and re-trade gains, $W_T(\mathbf{z}) = W_T(\mathbf{0}) + G_T(\mathbf{z})$, without the restriction $G_T \geq 0$, however. Let U be a concave utility function; by Jensen's inequality, $\mathbb{E}_t^*[U(W_T(\mathbf{z}))] \leq U[\mathbb{E}_t^*[W_T(\mathbf{0}) + G_T(\mathbf{z})] = U[\mathbb{E}_t^*(W_T(\mathbf{0}))]$, since $\mathbb{E}_t^*(G_T(\mathbf{z})) = 0$ in an arbitrage-free market. ∎

Remark. The simplest formulation of the no-trade problem follows if we consider a short-time period in which no dividend will be paid (the standard case in the FTAP literature): then the FTAP is a no-speculation result (since, by definition, the only motive for trade would be for speculative reasons in the market). The basic idea in the above no-trade Proposition is a general one, being

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[9] The difference is that we will not need to impose explicitly a "self-financing" portfolio condition. As to the phrasing in terms of discounted conditional operator is equivalent to the usual one in terms of "equivalent martingale measure". The discount ratios defining the conditional operator (5) are in the change of measures.

[10] To avoid a distracting and uninformative qualification, we assume a trader would not trade unless the trader expects a (strictly) positive benefit (which, here, means expected utility), and to exclude the artificial case of a trader who, for no profit, would engage themselves in the toil and trouble of trading. Also, note that under rational expectations, a trader trading in an arbitrage-free knows that the market is arbitrage-free.

a direct consequence of expected-utility theory (by Jensen's inequality); to wit: no risk-averse expected-utility maximizer would trade any asset, or more generally engage in any risky project, knowing that it yields *zero expected return* (independently of initial wealth).

The zero-expected return is theoretically plausible, and not unreasonable in practice. The stock price plotted in the Figures above are daily dividend-and-split-adjusted closing price data, commonly used by financial traders).[11] Such historical stock prices would suggest a zero (or slightly negative) average daily return (Table 1). (Most average returns are slightly negative perhaps due to the 2007-8 crisis. Only the S&P 500 average return over the very long run, 1927-2020, seems to be significantly positive, in accordance with the figures reported in the equity premium literature; but more on that in the next subsection.) So, in the light of neoclassical finance, why is there any trade taking place in these markets, let alone the intense, high-frequency, nature of the trading?

Table 1. Summary Statistics of Empirical Asset Price Volatility (Daily Percent Price Change: Returns are multiplied by 100).

	Fig. 1 Ford	Fig. 2 S&P	Fig. 6 S&P Long	Fig. 3 USD/GBP	Fig. 4 Bitcoin	Fig. 5 Lab Data					
mean(r)	-0.03	-0.03	0.02	-0.4	0.19	-0.08					
std(r)	2.22	0.97	1.2	0.64	3.89	8.5					
alpha($|r|$)	3.01	3.16	2.21	3.91	3.90	3.18					
Ticker	C	BAC	GE	IBM	JNJ	JPN	KO	MMM	PEP	PG	TM	XOM
mean(r)	-0.02	-0.02	-0.04	-0.03	-0.04	-0.04	-0.05	-0.04	-0.05	-0.04	-0.02	-0.04
std(r)	2.59	2.64	1.62	1.59	1.47	2.38	1.60	1.45	1.58	1.43	2.48	1.44
alpha($|r|$)	2.44	2.36	3.17	3.49	3.93	2.68	3.36	4.72	3.58	3.47	3.52	4.71

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[11] The stock price data reported in this paper are from https://finance.yahoo.com/quote/.

Risk-loving trader behavior, as neoclassic-ally defined, is not a theoretically plausible explanation

for asset trading, since (at least in a frictionless market) it would lead to infinite demand or supply

(which is incompatible with any market equilibrium). Explanations appealing to a so-called

irrational behavior such as noise trading is also problematic, as emphasized earlier (Section 5, p.

18). The common ways out of the no-trade problem seem to share the common methodological

premise of letting a theory judge reality, rather than the other way around: the standard theory

predicts no trade; hence one thinks of financial traders as "over-trading". Methodologically, we

should perhaps take the obvious existence of intense trade in finance as a fact that questions a

theory that predicts zero trade. In other words, the problem is in the theory (not in the world it

is supposed to explain). Thus, we ask: what is there in neoclassical finance that is behind its

intrinsic incapacity of dealing with trade itself?

6.3 The Essence of The No-Trade Problem and the Rationality of Speculation

Although we focus on arbitrage-free markets, the no-trade problem of neoclassical finance is a

general problem: consider any asset (or risky project) that yields an expected return $\mathbb{E}(r)$ in a

frictionless market of price-taking traders with rational expectations. If $\mathbb{E}(r) > 0,$ then any trader

would find it beneficial to buy the asset; but then no asset holder would want to resell the asset;

if on the other hand $\mathbb{E}(r) < 0,$ then any trader would want to short-sell the asset, but then no

trader would want to buy the asset. (All these claims follow also from Jensen's inequality and the

basic principle that for trade to take place there should be a buyer for any seller.)

But since we are investigating the no-trade problem within the no-arbitrage approach, we will

assume the martingale (or zero-expected return) hypothesis, and to simplify further, we assume

the standard random walk price model (with independent identically distributed, or i.i.d., price increments, or percent returns, if one thinks in terms of log-price):

$$p_t = p_{t-1} + r_t, \quad \{r_t\} \text{ is i.i.d process with } \mathbb{E}(r_t) = 0. \tag{7}$$

The no-trade problem is known to be related to the rational expectations' hypothesis (or similar assumptions, such as the common-knowledge one, that create homogenous beliefs). More precisely, the basic problem seems to lie in the assumption that an individual trader's subjective expectation behaves like an objective (population) mathematical expectation. An immediate consequence of this underlying premise is the homogeneity of beliefs across traders (by unicity of a mathematical expectation), which excludes the possibility of trade. Thus, it is hard to articulate individual and population expectations. Traders, in practice, form their expectations adaptively, informing their judgement by a relatively small sample of past realizations: they form their beliefs through a moving average of past realizations. In the simplest form, a trader's expected return, for example, can be written as a simple moving average of past returns:

$$r_{Ht}^e = \frac{1}{H} \sum_{h=1}^{H} r_{t-h}. \tag{8}$$

In practice, there is no no-trade puzzle, since only by accident will all traders' trading timing and horizons coincide perfectly: heterogeneity of traders' expectations comes notably from diversity of the moving average window H across traders, which depends on the traders' re-trade horizons. In the random-walk price world (7), rational expectation can be viewed as a model of a very-long-run investor (by the law of large numbers), a special case of the adaptive (moving-average) expectation model (8) with $H \rightarrow \infty$. Alternatively, rational expectation is only

reasonable as a model of traders' average expectations (8); for while $\mathbb{E}(r_{Ht}^e) = \mathbb{E}(r_t) = 0$, only exceptionally would an individual trader's short-run expectation $r_{Ht}^e = 0$.

It can be shown that fundamental tenets of neoclassical finance are not unreasonable as a model of aggregate market behavior, although they are uneasy to derive because of the conflation of the individual and the population perspectives, notably concerning the trading horizon. If we renounce imposing rational expectations, then: (1) absence of market-level arbitrage, in the sense that $\mathbb{E}(r) = 0$, is compatible with many successful arbitrage strategies by individual traders (if we do not interpret an individual trader's expectation of gain as the mathematical expectation of gain); (2) zero serial correlation (or even independence) of basic (one-period) returns (a form of the efficient-market hypothesis) is compatible with strong autocorrelation of a trader's cumulative return process (if we do not confuse the market elementary time scale, which in all rigor is the smallest frequency of price moves, or tick, with a trader's trading horizon).

We elaborate in greater detail on these two points, starting with the practical meaning of the martingale assumption, $\mathbb{E}(r) = 0$. In the objectivist interpretation of probability, a mathematical expectation (or probability for that matter) is a collective concept that applies to a population: it models a class (or ensemble) of individual realizations, as suggests the law of large numbers. Thus, the average height in a country hardly tells you any precise information about the next person you will come across in that country: it is a summary of the country's height population. It was perhaps an unhappy association historically that aggregate concepts such as probability have come to be expressed as if applying to an individual (the misleading notion of representative, or average individual), and that the *integral of a random variable* has been named a mathematical "expectation" (as if someone is expecting that value to come about): clearly if a

fair dice (with faces numbered 1 to 6 as usual) is thrown once in a game, none would rationally

expect it to land showing $3.5 = (1 + \ldots + 6)/6$. The number 3.5 is not relevant to anyone whose

stake or interest in the game does not involve a large number of individual trials (outcomes):

thus, the mathematical expectation (or population average) of re-trade gains in a market is

indeed relevant information only for a long-run investor, who buys and holds for a remote future;

a short-run speculator would hardly care. Hence, a zero average return (arbitrage-free) market

is in practice one that merely excludes the possibility that all traders combined could collectively

earn a positive speculative gain on the aggregate; or, alternatively, a consistent, positive,

speculative gain for an individual trader upon many trials: in other words, an arbitrage-free

market is perfectly consistent in practice with short-run persistent positive gains by many

individual speculators (so long as we do not impose that an individual speculator's expectation

of gains coincide with the mathematical expectation of gains).

In fact, there is an intrinsic incentive to short-run speculation in an arbitrage-free market even in

the standard case of a random walk, which leads us to the second point about serial correlation:

the reason is that a random walk is a *persistent process by construction* (it is in fact the paradigm

of persistent processes). This is a basic yet crucial point that we should emphasize because it is

intuitively well-taken into consideration by practitioners who usually follow trends. What matters

for a speculator is this persistence of price trends, which, by the identity $\sum_{s=1}^{t} r_t = p_t - p_0$, is also

the trend of *cumulative returns*; and the persistence of cumulative returns holds notwithstanding

the serial independence of direct, elementary, price returns $\{r_t\}$. This dependence by

aggregation effect makes it tricky to test the random walk hypothesis based on time-aggregated

data (monthly or yearly), since the random walk is supposed to hold at the elementary time scale: a similar problem has been emphasized concerning variance tests (Brown et al., 1995). All that matters for a trend-following speculator is to be able to buy at some rising price, expecting that the price trend will persist, and to be able to resell the asset at a higher price (or to short-sell when the price is falling, expecting the decline to persist). But this persistence of the cumulative return process is precisely what any random walk process guarantees, as suggest the following autocorrelation formula valid for all random walks:

$$\mathrm{cor}(p_t, p_{t+H}) = \sqrt{\frac{1}{1+H/t}}, t > 0, H \geq 0. \tag{9}$$

In other words, short-run trend-following speculation has a natural justification in the fact that by construction the short run (or small H/t) autocorrelation is close to 1 in any random walk: in fact for a given t, it is clear that only when the horizon (or equivalently lag) $H \to \infty$ do we have $\mathrm{cor}(p_t, p_{t+H}) \to 0$. Thus speculation, re-trading, or arbitrage is a reasonable or *rational practice from the viewpoint of an individual trader* (as long as we do not confuse the individual trader's expectation, or individual gain trajectory, with a population equivalent).
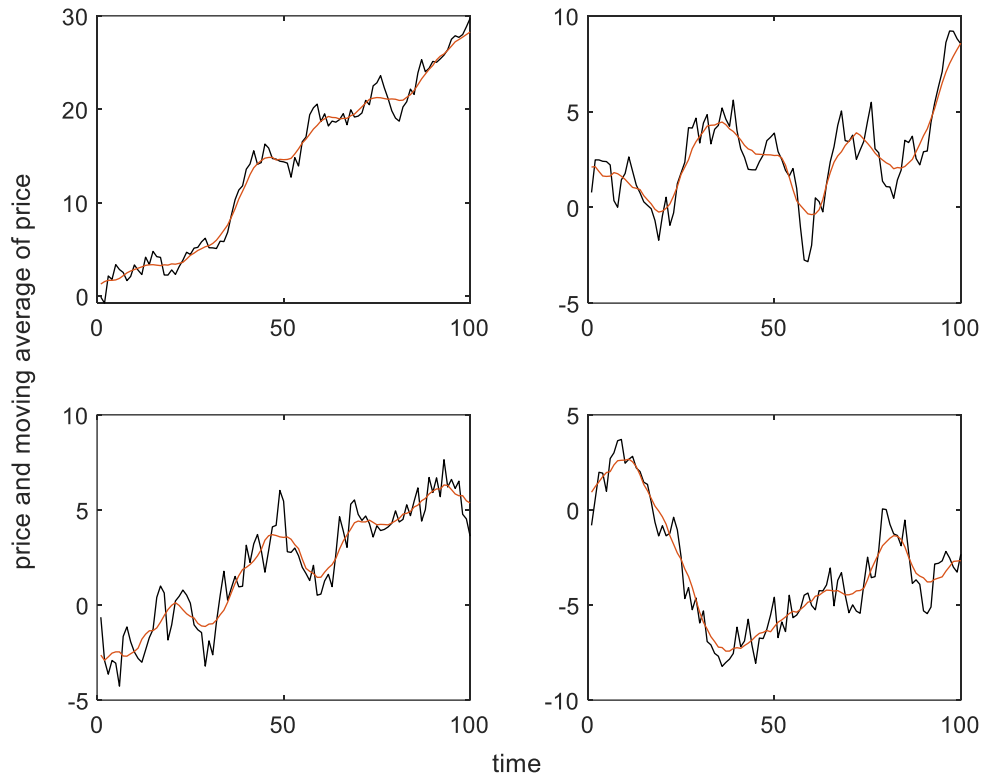
Figure 8. Persistence in time intervals in random walk price model: price and 10-window simple moving average of price. By construction, the price series can also be interpreted as cumulative returns; and the simple moving averages are 10-period average returns.

In summary, the elementary price return $\{p_t - p_{t-1}\}$ is serially independent by assumption, yet the cumulative return process $\{p_{t+H} - p_t\}$ is a persistent process by aggregation, being a random walk. The strong persistence can be stated in terms of the (arithmetic) average return process. Let the average return between t and $t+H$ be

$$r_t^H = \frac{1}{H}(p_{t+H} - p_t) = \frac{1}{H}\sum_{s=0}^{H-1} r_{t+s}.$$

The autocovariance formula for this average return process is

$$\text{cov}(r_t^H, r_{t+h}^H) = \frac{1}{H^2} \text{cov}\left(\sum_{s=0}^{H-1} r_{t+s}, \sum_{s'=0}^{H-1} r_{t+h+s'}\right)$$

$$= \frac{1}{H^2} \text{cov}\left(\sum_{s=0}^{H-1} r_s, \sum_{s=0}^{H-1} r_{h+s'}\right) \quad \text{(by stationarity of the return process)}$$

$$= \frac{1}{H^2} \sum_{s=0}^{H-1}\sum_{s=0}^{H-1} \text{cov}(r_s, r_{h+s'})$$

$$= \frac{1}{H^2} \sum_{s=h+s'}^{H-1}\sum_{s=0}^{H-1} \text{cov}(r_s, r_{h+s'}) \quad \text{(since the return process is i.i.d.)}$$

$$= \frac{H-h}{H^2} \text{var}(r_s) \quad (\#\{(s,s'): 0 \le s, s' \le H-1, s-s'=h\} = H-h).$$

It follows the formula for the autocorrelation function of average cumulative return:

$$\text{cor}(r_t^H, r_{t+h}^H) = \frac{H-h}{H}, H \ge h > 0. \tag{10}$$

Thus, the viewpoint of financial practitioners (who commonly follow trends) is not incompatible with serial independence of 1-period return process.

The emergence of momentum by aggregation is a general effect that holds for martingale price processes and for general re-trading strategies, because the cumulative gain process is a persistent process by aggregation, being an additive process with uncorrelated increments:

$$G_T = \sum_{t=1}^{T} (p_t - p_{t-1}) z_t,$$

For example, consider the simplest trend-following "momentum strategy" in a random-walk price market:

$$z_t = \text{sign}(p_{t-1} - p_{t-2}),$$

This momentum trading yields the arbitrageur an accumulated gain that evolves as

$$G_t = G_{t-1} + (p_t - p_{t-1})\operatorname{sign}(p_{t-1} - p_{t-2}).$$

Assume $G_0 = 0$, so that by construction, $\mathbb{E}(G_T) = 0$. Now, the important point is that $\mathbb{E}(G_T) = 0$ does not apply to an individual trading experience: it characterizes a large population of such individual experiences. Thus, the proposition "an arbitrageur makes on average zero re-trade gains in an arbitrage-free market" is a misleading rendition if read as if it applies to a typical arbitrageur. It is more precisely interpreted in two ways: either that a large number of arbitrageurs collectively make no positive gains on the aggregate or, much less relevant in practice, that an individual arbitrageur who tries a large number of independent strategies will make zero gains overall. In other words, an individual realization of an arbitrage gain is a highly persistent process by construction (being a random walk). It has the same autocorrelation pattern (7) as the price process (or any random walk for that matter). Here is the rationality of trend-following speculation, which many practitioners emphasize: A growing gain will tend to keep growing in the short run: the speculator's major concern, however, is when to stop and leave the game. This is hardly an irrational attitude.

Now of course such easy persistent gains are hardly achievable in practice, because of the competition of speculators for gains (and the resulting price impact of supply and demand): the price will not evolve by exogenous increments (implicit in the price-taking axiom) but rather will react in practice to supply and demand (there is no such thing in practice as costless speculation, or arbitrage, at least because of price impact): this is a second limitation of the expected-utility maximization hypothesis (namely the requirement of price-taking behavior). Notice the irony

here: it is a common critique of the random walk hypothesis (and of neoclassical finance more

generally) to emphasize the empirical evidence of "momentum" as evidence against the random

walk model; yet, if the foregoing discussion is correct, we should think the other way around: the

possibility of short-run momentum in a frictionless random walk price model is true by

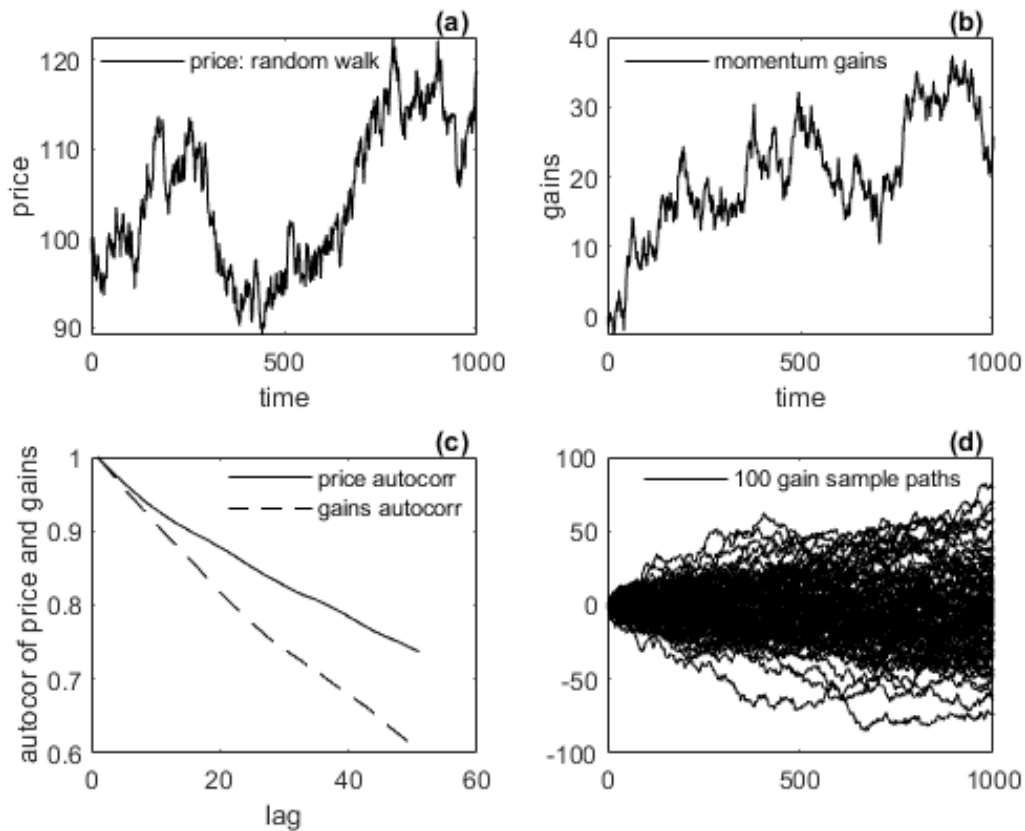construction, and is in fact too high for this standard price model to be realistic.[12]

Figure 9. Simulation of (a) a price random walk (sample path) and (b) an accumulated
momentum arbitrage gain (assuming frictionless market); (c) autocorrelation function
of price and momentum gain. Notice the persistence of the price and gains processes,
which is true of any random walk: the momentum-gain sample path is almost never

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[12] Think of momentum investors as discovering an outlier investment with autocorrelation especially favorable to short-rum trading gains. The process of exploiting that profitable persistence will negatively impact its profitability. Hence, momentum traders must constantly search for stocks that ae outliers in this sense. The process of searching in the tails of distributions, and acting on the findings, will in turn tend to clip these tails. Hence, for complete understanding we cannot ignore, and indeed must ultimately model, the market supply and demand process for momentum stocks.

negative, which is in no way unusual, although persistently negative gains are also possible: (d) 100 sample paths of momentum gains.

6.4 Equity Premium Puzzle and Survivor Bias: A Comment

Granted the previous argument for the rationality of speculation, and the importance of the time scale, there remains no "momentum puzzle" from the viewpoint of neoclassical finance (if we put aside the no-trade problem). A similar reappraisal seems to be in order for the "equity premium puzzle", the excessively high historical average real return of stocks compared to a relatively riskless asset (say Treasury bills) on the very long run (a century or two): about 6 to 7% annual premium for U.S. stocks. Because this section's main object is a discussion of the no-trade problem, which holds even in an arbitrage-prone market, the following brief comment about the equity premium will not pay full justice to the complexity of the issue.

The equity premium puzzle is commonly articulated theoretically in terms of the representative-agent approach to neoclassical finance, in which an asset price is identified with the valuation of an infinitely-lived representative agent (Mehra & Prescott, 1985): the equity premium is a puzzle in this framework because of the unreasonably high degree of risk-aversion required on the part of the representative agent to account for the equity premium. By construction, the more fundamental no-trade puzzle is not directly addressed in this theory (by the representative-agent simplification); but, as emphasized previously, the no-trade problem remains even under the assumption of positive equity expected return if we think in terms of multiple-agent markets. (Under rational expectations, a positive equity premium implies the arbitrage opportunity whereby very-long-run investors would borrow the riskless asset to buy stocks; but then no stockholder would want to resell their stocks in the first place for the same reason.)

Alternatively, one can discuss the equity premium more simply in terms of the no-arbitrage approach, in which the risk-adjusted *expected return* is zero by construction since it characterizes an arbitrage-free market:

$$\mathbb{E}_t^* \left(\frac{\mathbf{p}_{t+1} - \mathbf{p}_t + \mathbf{d}_{t+1}}{\mathbf{p}_t} \right) = 0. \tag{11}$$

The return in (11) is a risk-adjusted expected return (an average excess return above the risk-free rate) because the prices are relative prices, expressed in terms of the risk-free asset's price. Thus, the relatively high historical equity premium reported seems to be a direct challenge to the no-arbitrage assumption.

Brown et al. (1995) has already raised doubt about the equity premium as an accurate estimate of the population expected equity premium: the high historical average equity return, according to these authors, might be due to a sample selection bias, whereby the historical return overstates the population expected return since it is usually estimated from a sample of stocks of firms that survive, thus excluding stocks from markets that did not survive. We have emphasized a basic reason why momentum is to be expected to hold *by construction* under the random walk hypothesis. Here we emphasize a similarly basic reason why Brown et al.'s "survival bias" argument may be well-founded. For the basic intuition behind the survival argument to hold, one needs not invoke the survival or bankruptcy of a whole stock market, but rather that some component firms in a sample reference sample (notably the 500 firms in the S&P selection) will be replaced with positive probability over the very long run by others.

Thus, over the very long run, the average return of an index such the S&P 500 tends to be biased by construction. For example, all the daily individual stock returns used in the previous

illustrations (Table 1), offer no evidence in favor of a positive expected stock return, except the S&P 500 long-run (1927-2020) average return, which is 0.02% per day, consistent with the annual average return reported in the equity premium literature (0.02×365=7.3% annual return).[13] By construction, the S&P average return is a conditional average return, holding for a selection of stocks (500 largest companies), which tend to be a relatively successful group of companies, since over the long run, any relatively small company (by capitalization), let alone failed companies, is replaced by another, relatively stronger, one. An important observation is that over the very long run, bankruptcy is implied for purely probabilistic (model-free) reasons, under the simple random walk model.

Formally, let $\mathbb{E}(r)$ be the unconditional population expectation of the excess return generating process, and let C be the sample selection criterion (such as that of the S&P 500): then

$$\mathbb{E}(r) = \mathbb{E}(r \mid C)\mathbb{P}(C) + \mathbb{E}(r \mid \text{non } C)\mathbb{P}(\text{non } C).$$

Viewed over the very long run (say a century), a selection of companies such as the S&P 500 index is hardly a representative sample of the population of stocks, since companies that do not satisfy the selection criterion are replaced by others; moreover, companies that go bankrupt are excluded by construction. Strictly speaking, the historical expected return reported is an estimate of $\mathbb{E}(r \mid C)$, which can be higher than the unconditional expected return if the companies excluded from the selection include underperforming companies. It is easy to show that if $\mathbb{E}(r_t) = 0$, no large sample of stocks is representative which systematically excludes firms whose

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[13] The fact that we recover an annual S&P 500 (based on nominal price series) consistent with the one reported in the literature suggests that we should not worry too much about inflation in this brief illustration. The exact expected return figure is not anyway at issue here: for, again, what matters in the no-arbitrage approach is that the equity premium is nonzero.

relative stock prices (expressed in terms of the a bond price, say) are small: in fact, if the return generating process $\{r_t\}$ is serially independent with $\mathbb{E}(r_t)=0$, or more generally if $\mathbb{E}(r_t)$ is small enough that $\mathbb{E}(\log(1+r))<0$, then by the law of large numbers, the stock price is asymptotically negligible relatively to the bond price: that is, the price process $\{p_t\}$, with $p_t=(1+r_t)p_{t-1}$, will almost surely converge to zero as $t \to \infty$ (Figure 10). This follows from the identity:

$$p_t = p_0 \exp\left\{t\frac{1}{t}\sum\nolimits_{s=1}^{t}\log(1+r_s)\right\} = p_0 \left\{\exp\left[\frac{1}{t}\sum\nolimits_{s=1}^{t}\log(1+r_s)\right]\right\}^t.$$
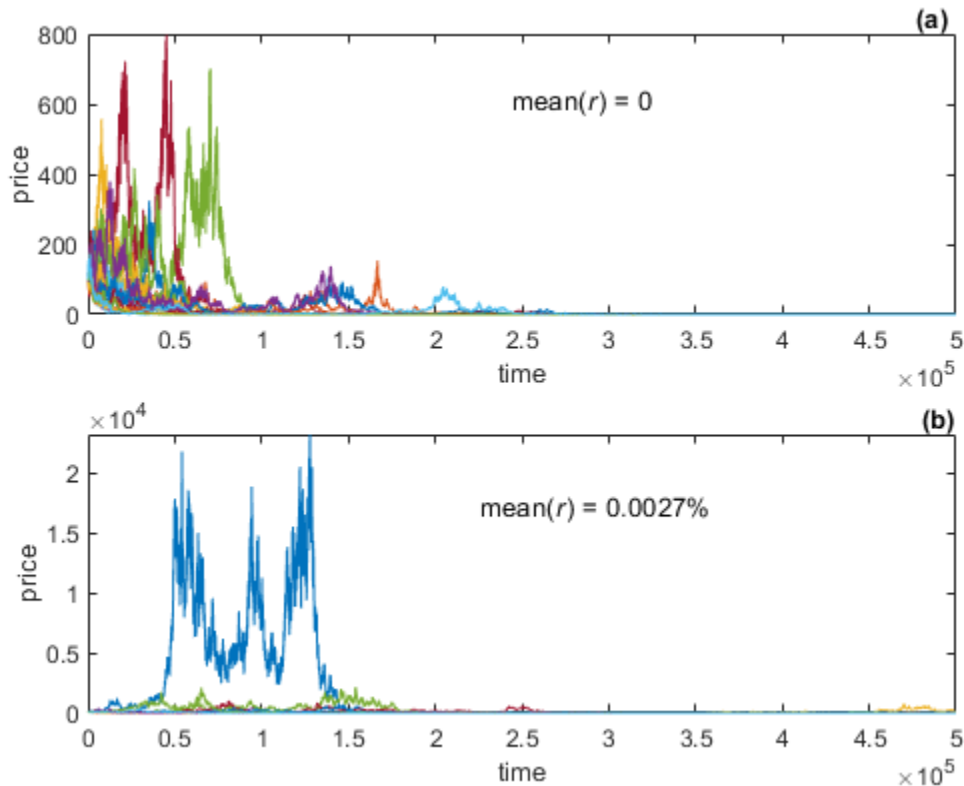

Figure 10. Almost sure asymptotic bankruptcy for a (multiplicative) random walk price process with zero or small average return. Here the Gaussian return process has properties chosen to have a realistic order of magnitude for empirical daily return data; hence 1% standard deviation. (a) zero mean; (b) 0.0027% mean return (roughly the average risk-free rate, 1% per year).

In summary, a fundamental aspect of economic instability, asset re-tradability, is intrinsically problematic to deal with in neoclassical finance. The no-trade problem is the major theoretical limitation of neoclassical finance, and not the famous so-called "anomalies" such as "momentum" which are secondary problems even if they are true puzzles (we indicated reasons to the contrary); and excess, clustered, and fat-tailed volatility are important empirical limitation of neoclassical finance, to the extent that it denies the first and is silent about the two others.

7 A Model of Asset Price Formation with Fat-Tailed and Clustered Volatility

7.1 The General Model: Setup

We adopt a classical approach to financial markets, that is, we realistically describe financial volatility from assumptions that consider the viewpoint of the agents themselves (that is, we do not postulate price-taking utility maximizing agents with rational expectations), and derive emergent patterns, notably the fat-tailed and clustered volatility.

Thus, we assume:

1. The market is populated by investors and speculators.

2. Traders' expectations are adaptive in the general sense that traders form their expectations of future prices or cash flows by means of moving averages of past realizations; yet expectations may be revised with the advent of a newsflash.

3. Prices form competitively, namely are driven by the law of supply and demand.

4. Traders' behaviors are simplified to their first-order linear approximations.

Formally, let the (excess) demands of an investor and a speculator be respectively[14]:

$$z_{it} = \gamma^I \frac{v_{it}^e - p_t}{p_t},$$

(12)

$$z_{st} = \gamma^S \frac{p_{st}^e - p_t}{p_t},$$

(13)

where p_{st}^e is a speculator's estimation of the asset's future price, v_{it}^e is an investor's estimation of the asset's unit present value, with the parameters $\gamma^I, \gamma^S > 0$. [15]Let N_t^I and N_t^S be respectively the numbers of investors and speculators active in period t. The total (market) excess demand is

$$Z_t = \gamma^I N_t^I \frac{v_t^e - p_t}{p_t} + \gamma^S N_t^S \frac{p_t^e - p_t}{p_t},$$

(14)

where $p_t^e = \sum_s p_{st}^e / N_t^S$, the average speculator anticipated future price, and

$$v_t^e = \frac{1}{N_t^I} \sum_{i=1}^{N_t^I} v_{it}^e,$$

(15)

the investors' average assessment of the asset's value, which by an abuse of terminology we call simply *the* asset's *value*: clearly, only if the investors' assessments are overall correct (unbiased) and independent can we identity the average value with the asset's fundamental value. The speculator's relevant variable is more conveniently defined, not so much as the anticipated

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[14] Because nonlinearity adds no further insight to this theory, we assume these standard linear supply and demand functions, which can be viewed as first-order linear approximations of more general functions: one can show that supply and demand are more generally given by the cumulative distribution of traders' reservation prices: anticipated asset valuations (investors) or expected resale prices (speculators' effective asset valuations).

[15] Think of these estimates as being solicited in an experiment measuring the most people would be willing to give to acquire the item of value. The resulting estimates would be predictive of the price of a share in an open outcry auction.

future resale price, but as the anticipated return (percent price change): thus, in the sequel we think in terms of the speculators' average anticipated return denoted as

$$r_t^e = \frac{p_t^e - p_t}{p_t}. \tag{16}$$

We simplify competition through a linear law of supply and demand, assuming the following standard price adjustment, in accordance with the market microstructure literature [16]:

$$r_t = \beta \frac{Z_t}{L_t}, \tag{17}$$

where L_t is the *overall market liquidity* (or market depth) and $\beta > 0$. Let the overall price impact of speculative and investment orders be denoted respectively as

$$n_t^I = \beta \mu \frac{N_t^I}{L_t}, \tag{18}$$

$$n_t^S = \beta \gamma \frac{N_t^S}{L_t}. \tag{19}$$

Combining (14) and (17), we get:

$$r_t = n_t^S r_t^e + n_t^I \frac{v_t^e - p_t}{p_t}. \tag{20}$$

We assume the generalized adaptive news-driven anticipation model for both traders:

$$r_t^e = \mu_t^S r_{t-1}^e + (1 - \mu_t^S) r_{t-1} + \varepsilon_t^S \text{ news}_t, \tag{21}$$

$$v_t^e = \mu_t^I v_{t-1}^e + (1 - \mu_t^I) v_{t-1} + \varepsilon_t^I \text{ news}_t, \tag{22}$$

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[16] The seminal work is Kyle (1985).

where news_t is a dummy variable (indicator function) indicating the announcement of fundamental news in period t; ε_t^S and ε_t^I are the aggregate impacts of news on speculators and investors, respectively, which we assumed Gaussian by aggregation in the simulations below (but other specifications would produce qualitatively similar results).

This generalized adaptive expectations model captures the common practice of technical analysts based on exponential moving averages, which, if anticipative (namely excluding the contemporaneous value to be predicted) is none other than adaptive expectations.[17] The memory parameters μ_t^S, μ_t^I are the averages of the individual traders' memory parameters; hence, they can fluctuate in principle.

7.2 The Purely Speculative Market Model

The dynamics of the asset price is of course decided by the specification of the general expectation model. It is useful to start with the special case of a purely speculative market. Assume

$$n_t^I = 0, \mu_t^S = 0. \tag{23}$$

Then we have the simplest purely speculative (momentum) model:

$$r_t = n_t^S r_{t-1} + \varepsilon_t^S \, \text{news}_t. \tag{24}$$

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[17] As a model of expectations, the standard definition of a moving average needs to be adjusted to apply only to available realizations, which exclude of course the future realization being estimated. Then exponential moving average expectation is just another name for adaptive expectations.

This is a random-coefficient autoregressive process of order 1, in short RCAR(1): it is known in the mathematical literature as a Kesten process, after H. Kesten, who proved a fascinating theorem that even with a relatively mildly fluctuating exogenous input $\{e_t\}$, here $e_t = \varepsilon_t^S \, \text{news}_t$, a RCAR(1) process $r_t = a_t r_{t-1} + e_t$, here $a_t = n_t^S$, converges under mild additional technical condition to a strictly stationary process with a Pareto tail

$$\mathbb{P}(|r_t| > x) \sim C x^{-\alpha}, \tag{25}$$

for big x, where the exponent is the solution to the fundamental equation $\mathbb{E}(|a_t|^\alpha) = 1$.[18]

Kesten's theorem generalizes naturally to a general RCAR(H), or even to a multidimensional (vector) setting. Intuitively, the extreme variability of the output emerges despite a mild input, due to the amplifying feedback component (the autoregressive part). In other words, if the model (23)-(24) is accurate then the Pareto tails of asset price volatility is due to the feedback intrinsic to the trend-following speculation (or more precisely to any speculation informed by past price changes, whether trend-following or mean reverting: this is mathematically inconsequential).

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[18] For example, if each a_t is drawn from an exponential distribution with mean λ (as in Figure 11) then it can be shown that $\mathbb{E}(|a_t|^\alpha) = \Gamma(\alpha)/\lambda^\alpha$ (where the numerator involve the Gamma function); thus $\alpha = 3$ is achieved for roughly $\lambda = 0.55$.
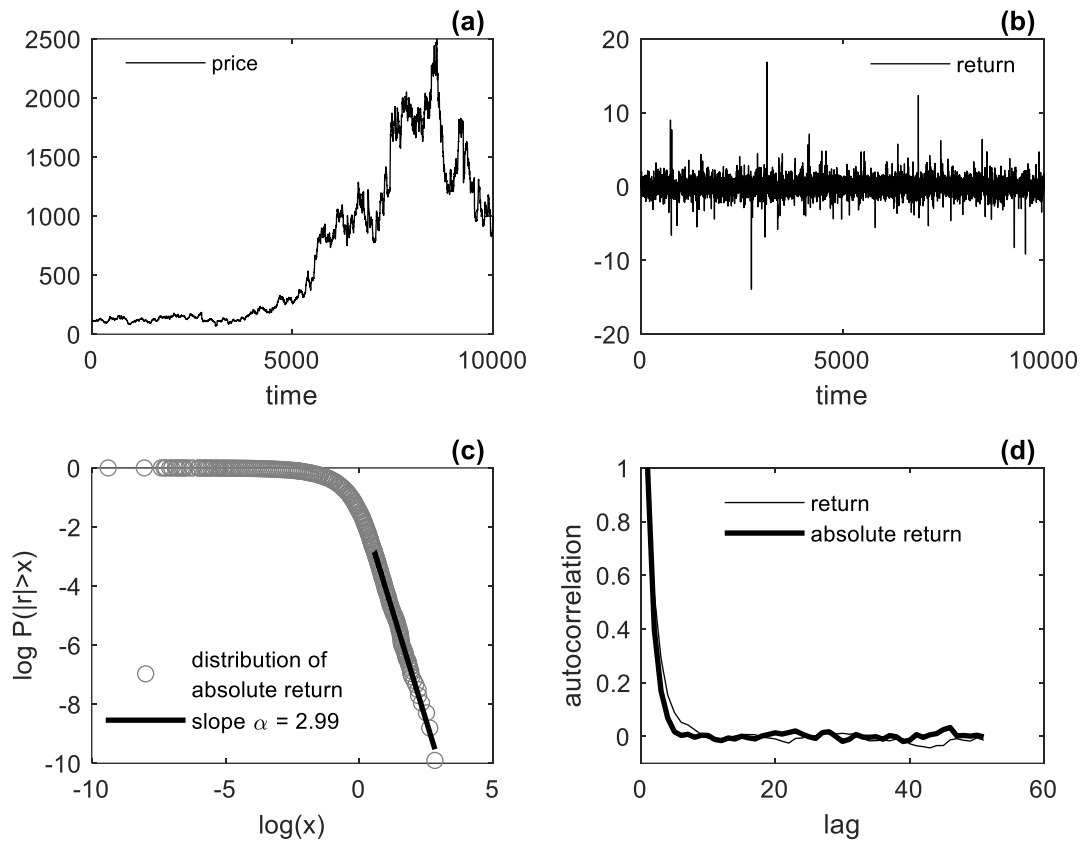
Figure 11. A purely speculative asset market model: the simplest case of momentum speculators or RCAR(1) model, with prob(news at time *t*)=1, in this simulation; feedback term n_t drawn from an exponential distribution with mean 0.55; impact of news is zero-mean Gaussian with standard deviation of 1.

More generally, common moving-average-based trading strategies (not necessarily based on exponential moving averages) implies an effective speculators' average anticipated return of the general form:

$$r_t^e = \sum_{h=1}^{H} \omega_{ht} r_{t-k} + \varepsilon_t^S \text{ news}_t, \qquad (26)$$

where we included the impact of news on speculators' expectations. The effective weighting scheme underlying common moving-average trading strategies used by financial practitioners

can be derived explicitly (Zakamulin, 2014; Levine & Pedersen, 2016; Beekhuizen & Hallerbach, 2017). Thus, asset's price return in a purely speculative trend-following market, is more generally:

$$r_t = n_t^S \sum_{h=1}^{H} \omega_{ht} r_{t-h} + e_t^S, \tag{27}$$

where $e_t^S = n_t^S \varepsilon_t^S \text{news}_t$ captures the total impact of exogenous news on speculators' expectations. The general RCAR(H) model is qualitatively equivalent to simple RCAR (1), as far as the tails of the distribution are concerned. It generates the same strictly stationary Pareto tail (25), where the exponent α depends solely on the distribution of the weights, w_{ht} and n_t^S, but not on the fundamental news (Klüppelberg & Pergamenchtchikov, 2004; Buraczewski et al., 2016, Subsection 4.4.9).

However, the RCAR(H) model cannot account for clustered volatility: for any such autoregressive model, and for any arbitrary function f, $\text{cov}[f(r_{t-m}), f(r_t)]$, when it is well-defined, decays rapidly (at an exponential rate) with the lag m (Mikosch & Starica, 2000; Basrak et al., 2002). So, volatility, whether measured as $|r|$, r^2, or more generally by any function f, cannot be long-range correlated in this purely speculative trend-following model. The key to this model's incapacity for reproducing clustered volatility is its short-memory property: speculators forget fundamental news very quickly, and this short memory forbids any persistent trading behavior capable of explaining a persistent volatility. A more general model is therefore needed in which news play a fundamental role.

7.3 A More General Specification with both Speculators and Investors

To repeat, the general model (with both speculators and investors) reads:

$$p_t = (1 + r_t)p_{t-1},$$

$$r_t = n_t^S r_t^e + n_t^I \frac{v_t^e - p_t}{p_t},$$

$$r_t^e = \mu_t^S r_{t-1}^e + (1 - \mu_t^S)r_{t-1} + \varepsilon_t^S \, \text{news}_t,$$

$$v_t^e = \mu_t^I v_{t-1}^e + (1 - \mu_t^I)v_{t-1} + \varepsilon_t^I \, \text{news}_t.$$

It turns out that both the fat-tiled and the clustered volatility are robustly captured by the specification:

$$\mu_t^I = 1, \mu_t^S \approx 1, \text{but } \mu_t^S < 1. \tag{28}$$

Figures 3-6 are simulations of the model using the parameter specifications in Table 1.
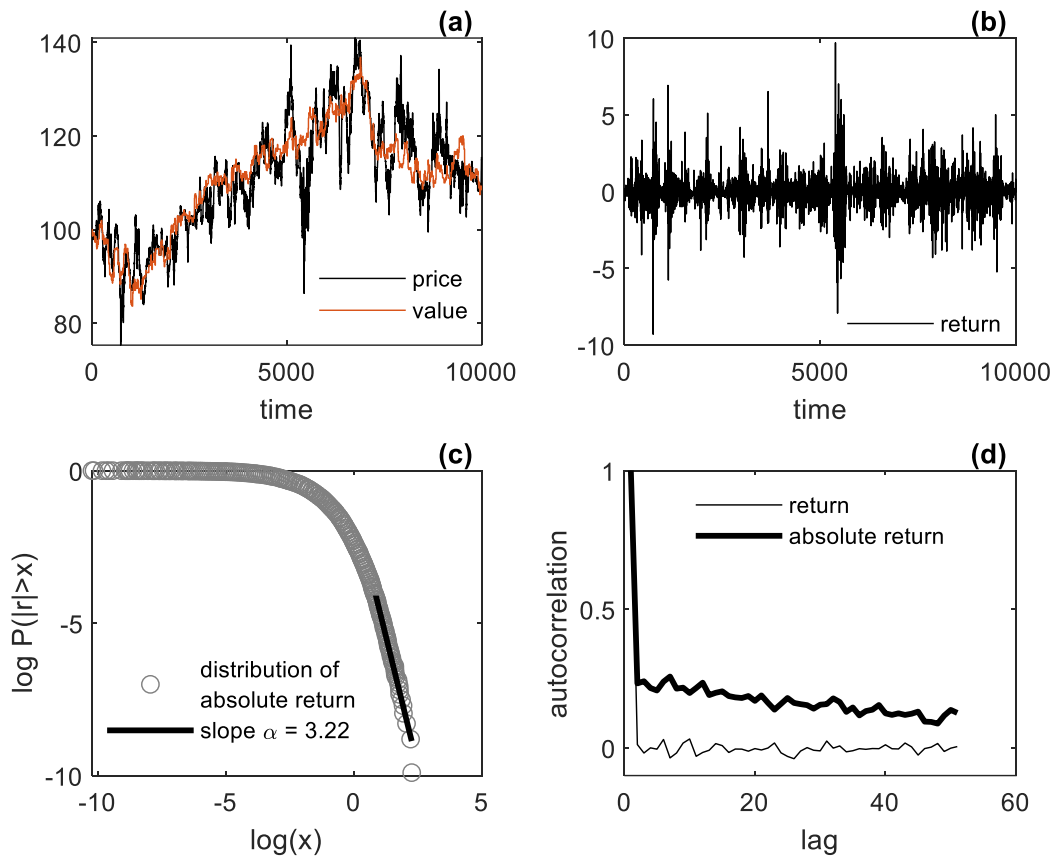
Figure 12. The general model simulated. (a) Price; (b) Return (in percent); (c) cumulative distribution of volatility in log-log scale, and a linear fit of the tail, with a slope close to 3; (d) Autocorrelation function of return and absolute return.

7.4 Discussion

The fat tail and the volatility clustering emerge as generic and robust in the model: they hold for a broad class of distributions and parameters (which we have checked). The specifications in the Figures are merely chosen for illustration, and are reported in Table 2: the key parameters are $\{n_t^S\}$ and $\{n_t^I\}$, referring, respectively, to the price impact triggered by a one percent anticipated price rise, and one percent anticipated mispricing; and their ratio correspond essentially to the ratio of the number of speculators and investors. For illustration, we specify them to be

exponentially distributed i.i.d. processes with means reported in Table 2. The revisions of traders'

beliefs due to news, $\{\varepsilon_t^S\}$ and $\{\varepsilon_t^I\}$, are zero-mean i.i.d. Gaussian processes in all simulations of the

model with standard deviations set merely to have realistic orders of magnitude compared to

empirical data (notably the standard deviation of return which is typically around 1 per cent); all

the simulations involve also $T = 10000$ periods; $P_0 = V_0^e = P^e = 100$, $r_1 = r_1^e = 0$. The differing
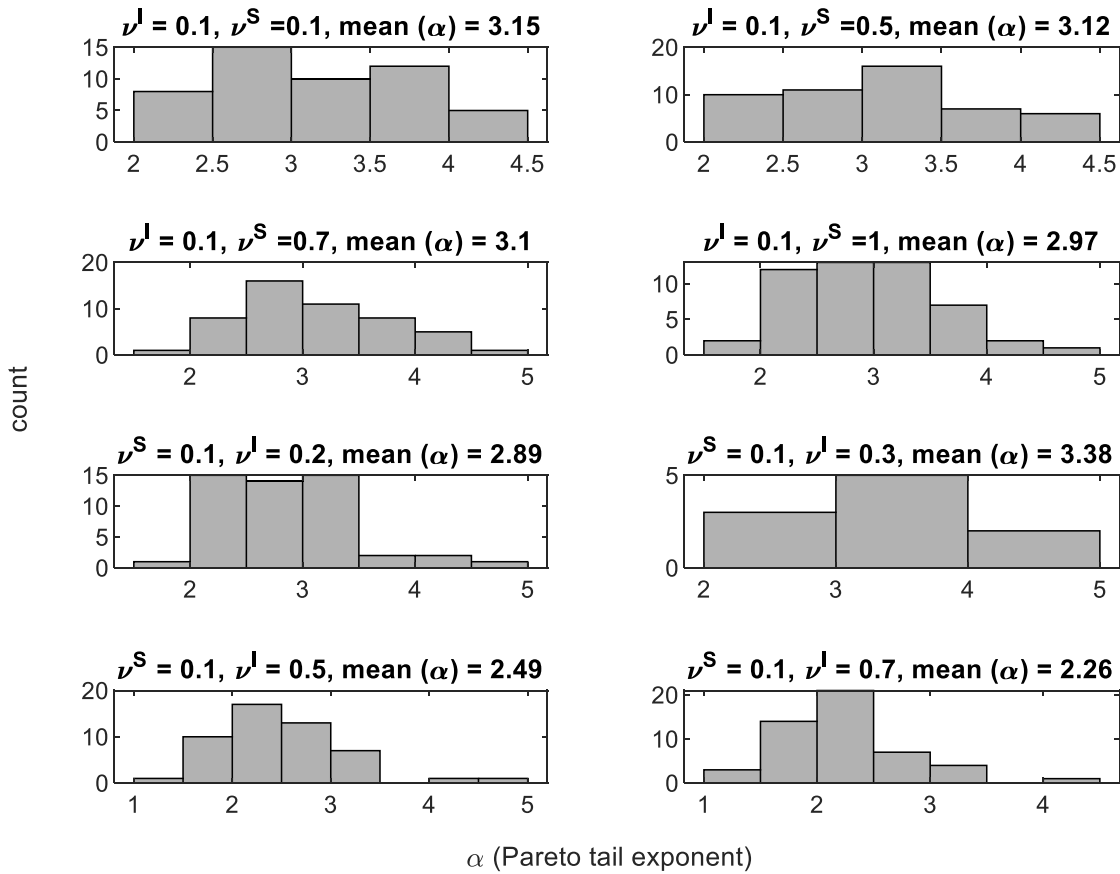
parameter choices are reported in in Table 2.

Figure 13. Sensitivity of Pareto tail index alpha to changes of the key parameters n^S and n^I. Histogram of alpha for 50 realizations of the model. Notice the robustness of the cubic law of returns to changes in the speculators' overall impact n^S, and the sensitivity of alpha to the investor's impact n^I.

Excess volatility of price relatively to fundamental value is also a generic phenomenon in the model. To investigate the bubble phenomenon in view of the experimental evidence discussed in Section 4, we specialize the model's asset value process $\{v_t\}$ to the standard experimental implementation, namely a declining step function, or simply its linear trend approximation $v_t = (T - t + 1)\mathbb{E}(d)$, $t = 1, \ldots, T$. According to this model, the asset bubble size increases ceteris paribus with the dominance of speculators in the market, more precisely the ratio $\mathbb{E}(n^S)/\mathbb{E}(n^I)$, and it decreases with the overall trading horizon T (Figure 14).
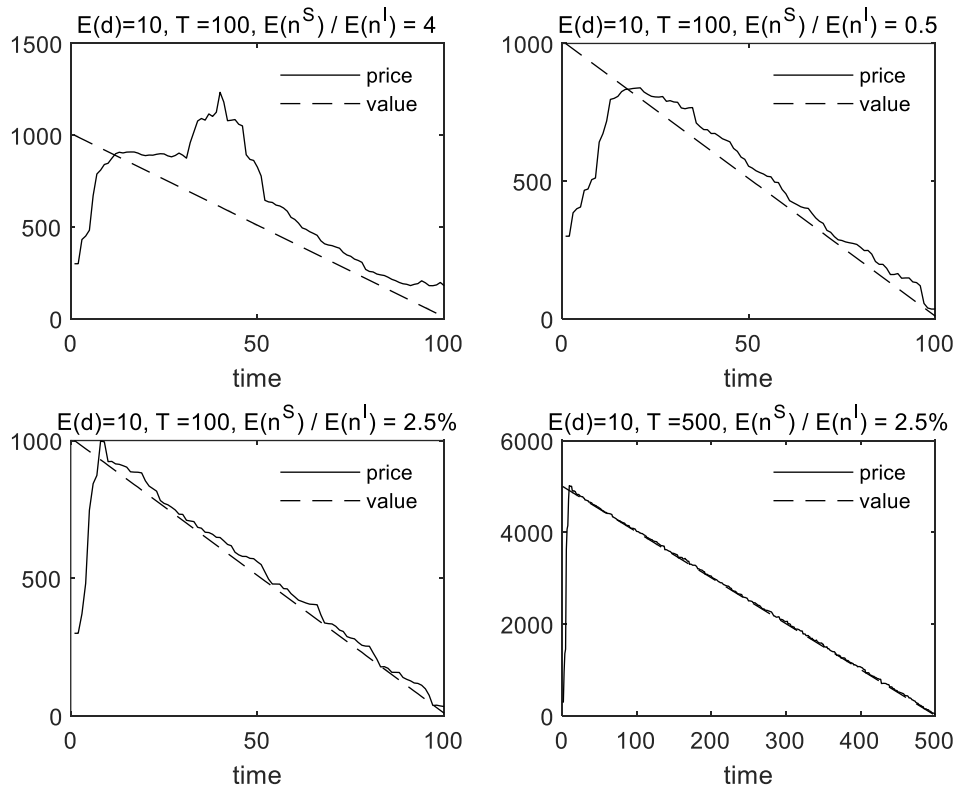

Figure 14. Excess Volatility: ceteris paribus, the asset bubble size increases with the dominance of speculators in the market, namely the ratio $\mathbb{E}(n^S)/\mathbb{E}(n^I)$, and it decreases with the overall trading horizon T.

Table 2. Model's Summary Statistics and Parameter
Specification (Percent Return).

	Figure 11 RCAR (1)	Figure 12 Model
Parameters		
mean (n^S)	0.55	0.2
mean (n^I)	0	0.1
std(ε^S)	0.65	1
std(ε^I)	1	1
prob(News)	0.1	0.1
S-memory μ^S	0.99	0.99
I-memory μ^I	1	1
mean(r)	0.03	0.4
std(r)	1	0.71
alpha($|r|$)	2.99	3.22

8 Conclusion

This paper revisits the old problem of economic and financial instability through the lens of asset re-tradability, a fundamental concept in financial theory, as we emphasize throughout. Market instability is strongly associated with goods' re-tradability and this association, established in the lab, holds at both the micro- and the macro-economic level (Sections 1-4). Owing to the inherent no-trade problem of neoclassical finance (Section 6), the link between economic instability and asset re-tradability is not easy to investigate theoretically within this framework. Also, mounting evidence on relatively high-frequency financial volatility (both in the field and in the lab) suggest at least three robust empirical regularities that are hard to reconcile with the conventional financial theory (Sections 4-5): excess, clustered, and fat-tailed volatility: financial volatility is too high given fundamentals; it is power-law tailed; and it unfolds in clusters (suggesting a nontrivial autocorrelation in asset returns). Both the excess and power-law volatility, as we show, can be

explained intrinsically from trend-following speculation; the volatility clusters are due to traders' long memory of fundamental news (Section 7.3).

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