EPM/RT-93/6

BANG BANG CONTROL OF AN OVERHEAD CARTESIAN CRANE

par

Gratuit

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Mars 1993

Dépôt légal, septembre 1992
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Summary

A motion controller for an over-head Cartesian crane in 3-D space is designed under the constraint that the control action belong to a discrete set of assigned values. The design procedure rests upon a two-step approach: first, one determines a constraint-free motion controller satisfying the required dynamic specifications; second, one replaces this controller with an equivalent controller satisfying the discrete action constraint. The first step is implemented by adopting a heuristic 3-D extension of a wellproven 2-D controller. The second step, by applying recent sliding mode results. Numerical simulations illustrate the properties of the ensuing feedback system under both nominal and perturbed operating conditions.

Keywords: feedback, motion control, overhead crane, sliding mode.

Ce rapport est une version étendue de l'article **THE BANG BANG CONTROL OF AN OVERHEAD CARTESIAN CRANE** accepté pour publication dans les Comptes Rendus de la **1993 American Control Conference**, San Francisco, Juin 1993.

List of Symbols

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x: state (in particular, the state of the crane);

u: control (in particular, a control of the crane);

x_1: displacement of the girder with respect to the inertial frame;

x_2: displacement of the trolley with respect to the girder;

x_3: angle between the z axis of the inertial frame and the z axis of the load frame;

x_4: angle between the z_x plane of the inertial frame and the vertical plane containing the z axis of the load frame;

x_5: torsional twist of the cable;

$x_6 := \dot{x}_1$, $x_7 := \dot{x}_2$, $x_8 = \dot{x}_3$, $x_9 := \dot{x}_4$;

u_1: propulsion force applied to the girder;

u_2: propulsion force applied to the trolley;

m_{gi}: mass of the girder;

m_{tr}: mass of the trolley;

m_{lo}: mass of the load;

l: length of the cable;

g_0: gravitational acceleration;

I_a: inertia of the load with respect to the z axis;

I_t: load inertia with respect to the x axis (and y axis);

p: twist compliance coefficient of the suspended cable;

$a{:}+m_{gi}+m_{tr}+m_{lo}$; $b:=m_{gi}+m_{tr}$; $c:=m_{gi}l$; $d:=m_{gi}l^2+I_t$;

$e:=I_a-I_t-m_{gi}l^2$; $g:=m_{gi}g_0l$; $h:=2m_{gi}l^2+2(I_t-I_a)$

1. Introduction

The operation of an overhead Cartesian crane entails the motion control of the girder, the trolley and the hoisting apparatus, so that the suspended load moves along a prespecified path. Among the techniques successfully explored to automate this operation are open and closed loop optimal control [Ri.1, Sa.1, Ka.1], pole placement and LQG linear state feedback [Hu.1], and fuzzy controllers [Ya.1]. A common feature of these controllers is that they are designed on the assumption that the voltage applied to the crane motors is arbitrarily selectable within a certain continuous range of values. Practical considerations, however, make it convenient or necessary that crane motors be energized with only a finite number of voltage levels.

The natural approach in dealing with this requirement is to have recourse to nonlinear techniques, such as equivalent gain, phase plane and time optimization [Slo.1, Ts.1, Po.1]. The dynamic model of the crane is rather involved, however, and the modalities of application of these techniques are not evident. In addition, difficulties in the practical implementation of the resulting controller as well as in the characterization of its dynamic and sensitivity properties, are to be expected.

A second approach to solving the problem might be to initially design a controller by assuming a continuous range of voltage values, and to then implement this "continuous" controller by means of a "discrete" controller which, though constrained by a finite number of output levels, does, nevertheless, produce an action equivalent to that of the original controller. One way to implement this replacement could be to simply cascade a pulse-width modulator with the continuous controller. However, in addition to the cost of the extra equipment involved, there is the disadvantage that a small delay is generated, which may have a destabilizing influence on the feedback system.

The implementation of the continuous controller by means of an appropriate discrete sliding-mode controller offers an alternative way [Ba.1, De.1, De.2, Ut.1]. As shown in these references, a sliding-mode controller does not require a special equipment, nor does it necessarily introduce an additional delay. Moreover, this controller offers the potential to improve sensitivity to parameter variation and external perturbation. A first application of sliding-mode techniques to the design of "discrete" motion controllers for overhead cranes was explored in [De.3]. Though this development has demonstrated the modalities and the feasibility of the approach, it is somewhat incomplete, however, in that it is confined to a travel motion of the crane's load contained in a vertical plane (a 2-D motion with either the trolley or the girder blocked). In what follows we will consider the more realistic case where the trolley and the girder move concurrently, causing a 3-D motion of the load. In accordance with [De.3] we will pursue this objective by relying on the techniques developed in [De.2]. It should be noted that with the transition from 2-D to 3-D motion, the dynamics of the load becomes intrinsically nonlinear and intercoupled. As a consequence, Lyapunov linearization, which is routinely applied in the 2-D case, is no longer applicable. In addition, a key hypothesis of the basic result in [De.2], which plays an essential role in the planar case [De.3], turns out not to be satisfied by the crane's dynamical model to be considered in the 3-D case. These difficulties will be overcome by modifying the main result in [De.2] so as to make it applicable to the context under consideration.

2. The Mathematical Setting

From a mathematical point of view, the problem under consideration may be formulated as follows: Let a nonlinear time-varying dynamical plant be described by the differential equation

$$\dot{x}(t) = f_0(x,t) + B_0(x,t)(u(t)+p(t)) \qquad x(0) = x_0. \tag{1}$$

where $x(t) \in R^n$ represents the plant state, $u(t) \in R^m$ is the control, $p(t) \in R^m$ represents the influence of parameter variations and external perturbations; $f_0(x,t)$ and $B_0(x,t)$ are appropriately dimensioned real-valued vector and matrix functions. The problem is to design a (discrete) control, $u(t)$, satisfying the following requirements: a) the feedback system dynamics must conform to assigned specifications; b) the components of the control action, $u(t)$, are constrained to belong to an assigned discrete set, $u_i(t) \in U_i := \{u_{i1}, ..., u_{iN}\}$, $i = 1 ... m$.

In many applications (such as the motion control of an overhead crane), various techniques are available which successfully lead to a continuous controller satisfying the first requirement. As mentioned in the introduction, one way to solve the problem is therefore to assume that one such (continuous) controller $u_D(x,t)$, is available, and to develop a controller, $u(x,t)$, satisfying the second requirement, and capable of producing the same feedback system dynamics as $u_D(x,t)$.

References [De.2, De.3] suggest that this latter development may be carried out by means of a sliding-mode controller, obtained by the following procedure: Using the notation

$$\sigma(t) := B_0(x,t)B_0^{+}(x,t)S(t) \tag{2}$$

where $B_0^{+}(x,t)$ denotes the pseudo-inverse of $B_0(x,t)$, and

$$S(t) := \int_0^t \{\dot{x}(t) - f_0(x,t) - B_0(x,t)u_D(x,t)\}dt, \tag{3}$$

the discrete controller is selected so as to provide an output such that

$$u_i^*(t) \in U_i,\ i= 1, .., m \tag{4}$$

and

$$SGN\{u_i^*(t)-u_{Di}(x,t)+p_i(t)\}:= - SGN\{[B_0'\sigma(t)]_i\}. \tag{5}$$

The key property of such a controller is formalized by the following result (a variation of theorem 1 in [De.2]):

Theorem 1. If $B_0(x,t)$ is of full rank, then: the dynamics of system (1) submitted to a discrete control satisfying (2-5) are described by

$$\dot{x}(t) = f_0(x,t) + B_0(x,t)u_D(x,t) \tag{6}$$

Proof:

Consider the positive functional $|\sigma(t)|^2$, and observe that

$$1/2\ d/dt\ |\sigma(t)|^2 = <\dot{\sigma}(t),\sigma(t)> \tag{7}$$

$$=<\sigma(t),B_0B_0^+\{\dot{x}(t)-f_0(x,t)-B_0u_D(x,t)+B_0^+\{d/dt(B_0B_0^+)\}S(t)>$$

$$=<B_0'\sigma(t),u(t)-u_D(x,t)+p(t)+B_0^+\{d/dt(B_0B_0^+)\}S(t)>$$

$$=\Sigma<[B_0'\sigma(t)]_i,u_i(t)-u_{Di}(x,t)+p_i(t)+[B_0^+\{d/dt(B_0B_0^+)\}S(t)]_i>. \tag{8}$$

By selecting $u_i = u^*_i$ with u^*_i such that

$$SIGN\{u_i(t)-u_{Di}(x,t)+p_i(t)+[B_0^+\{d/dt(B_0B_0^+)\}S(t)]_i\}=SIGN\{[B_0'\sigma(t)]_i\} \tag{9}$$

we have

$$d/dt|\sigma(t)|^2=-\Sigma|[B_0'\sigma(t)]_i|*|u_i(t)u_{Di}+p_i(t)+[B_0^+\{d/dt(B_0B_0^+)\}S(t)]_i|$$
$$\leq 0 \tag{10}$$

Setting $\sigma(0)=0$, implies that

$$\sigma(t)=0 \text{ for } t>0, \tag{11}$$

and therefore

$$\dot{\sigma}(t)=0 \text{ for } t>0. \tag{12}$$

This equality implies in turn that the state trajectory of the system is described by

$$\dot{x}(t) = f_0(x,t) + B_0(x,t)u_{equ}(t) + p(t), \tag{13}$$

where u_{equ} (commonly referred to as the equivalent control) must satisfy (12). As this condition gives

$$u_{equ}(t)=u_D(x,t)-p(t)-B_0^+\{d/dt(B_0B_0^+)\}S(t). \tag{14}$$

it follows that

$$S(t) := \int_0^t \{\dot{x}(t) - f_0(x,t) - B_0u_D(x,t)\}dt,$$
$$= \int_0^t \{B_0(u_{equ}(t)-u_D(x,t)+p(t))\}S(t)\} = 0$$
$$= \int_0^t \{B_0B_0^+\{d/dt(B_0B_0^+)\}S(t)\} \tag{15}$$

This implies in turn that $S(0)=0$ also implies $S(t)=0$ for each t, and therefore that

$$\dot{x}(t) = f_0(x,t) + B_0(x,t)u_D(t) \quad x(0) = x_0. \tag{16}$$

Remark 1. Under nominal operating conditions, the discrete controller considered by the theorem generates a state trajectory identical to that of the continuous controller (nominal trajectory). Under perturbed conditions, the state trajectory remains identical to the nominal trajectory provided that parameter variations and external perturbations may be represented in terms of an input equivalent perturbation p(t).

Remark 2. Theorem 1 gives little information about the dynamical behavior of the system in the presence of parameter variations and external perturbations that may not be represented in terms of the vector p(t). This is contrary to theorem 1 in [De.2], which uses the additional hypothesis that $B_0(x,t)B_0^{+}(x,t)$ is a constant matrix. This hypothesis allows a more complete robustness characterization of the dynamics of the plant with respect to more general perturbations. As will be evident from the model developed in the next section (eqns 22, 24), however, such hypothesis is not applicable in the case of an overhead crane operating in 3-D space, hence the necessity of proceeding with a somewhat more restrictive result and leaving open the question of the behavior of the discrete controller under perturbations not representable by p(t).

3. The Overhead Crane Model

With reference to Figure 1, assuming a suspension cable of constant length, the kinematic configuration of a Cartesian overhead crane can be represented in terms of parameters x_1, x_2, x_3 and x_4 where

x_1 gives the position of the girder with respect to the inertial frame;

x_2 gives the position of the trolley with respect to the girder;

x_3 is the angle between the z axis of the inertial frame and the z axis of the load frame;

x_4 is the angle between the z_x plane of the inertial frame and the vertical plane containing the z axis of the load frame.

The state of the crane can in turn be represented in terms of a vector $x \epsilon R^9$,

$$x' := [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9]' \tag{18}$$

where x_1, x_2, x_3, and x_4 are as defined above, x_5 is the torsional twist of the cable, and

$$x_6 := \dot{x}_1,\ x_7 := \dot{x}_2,\ x_8 = \dot{x}_3,\ x_9 := \dot{x}_4. \tag{19}$$

The control can be represented by a 2-D vector $u \epsilon R^2$, $u := [u_1, u_2]'$, where u_1 is the propulsion force developed by the girder motor, and u_2 is the force developed by the trolley motor.

Taking into account the non-holonomic constraint

$$\dot{x}_5 = x_9 \cos x_3, \tag{20}$$

and applying the Euler-Lagrange approach, the dynamics of the crane may be modelled in terms of the differential equation (this model is adapted from [Ka.1])

$$\dot{x} = f(x) + B(x)u \tag{21}$$

where $x \epsilon R^9$, $u \epsilon R^2$, and matrix functions f(x) and B(x) are described by

$$f(x) := \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad B(x) := \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{22}$$

with

$$F_1' := [f_1\ f_2\ f_3\ f_4\ f_5]' \qquad F_2' := [f_6\ f_7\ f_8\ f_9]'$$

$$[f_1\ f_2\ f_3\ f_4\ f_5]' := [x_6\ x_7\ x_8\ x_9\ x_9\cos(x_3)]'$$

$$F_2 := M^{-1}\ \tilde{F}_2 \qquad\qquad \tilde{F}_2' := [\tilde{f}_6\ \tilde{f}_7\ \tilde{f}_8\ \tilde{f}_9]'$$

$$\tilde{f}_6 := c\sin x_3 \cos x_4 (x_8^2 + x_9^2) - 2c\cos x_3 \cos x_4 x_8 x_9$$

$$\tilde{f}_7 := \quad c\sin x_3 \cos x_4 (x_8^2 + x_9^2) + 2c\sin x_4 \cos x_3 x_8 x_9$$

$$\tilde{f}_8 := -e\sin x_3 \cos x_4 x_8^2 - g\sin x_3$$

$$\tilde{f}_9 := -p\cos x_3 x_5 - h\sin x_3 \cos x_3 x_8 x_9 \tag{23}$$

$$B_2 := M^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{24}$$

and

$$M := \begin{bmatrix} a & 0 & c\cos x_3 \sin x_4 & c\sin x_3 \cos x_4 \\ 0 & b & c\cos x_3 \cos x_4 & -c\sin x_3 \sin x_4 \\ c\cos x_3 \sin x_4 & c\cos x_3 \cos x_4 & d & 0 \\ c\sin x_3 \cos x_4 & -c\sin x_3 \sin x_4 & 0 & ml^2 \sin x_3^2 + I_a \cos x_3^2 + I_t \sin x_3^2 \end{bmatrix} \tag{25}$$

The inertia matrix of the load has been assumed to be diagonal and given by

$$J := \begin{bmatrix} I_t & 0 & 0 \\ 0 & I_t & 0 \\ 0 & 0 & I_a \end{bmatrix}$$

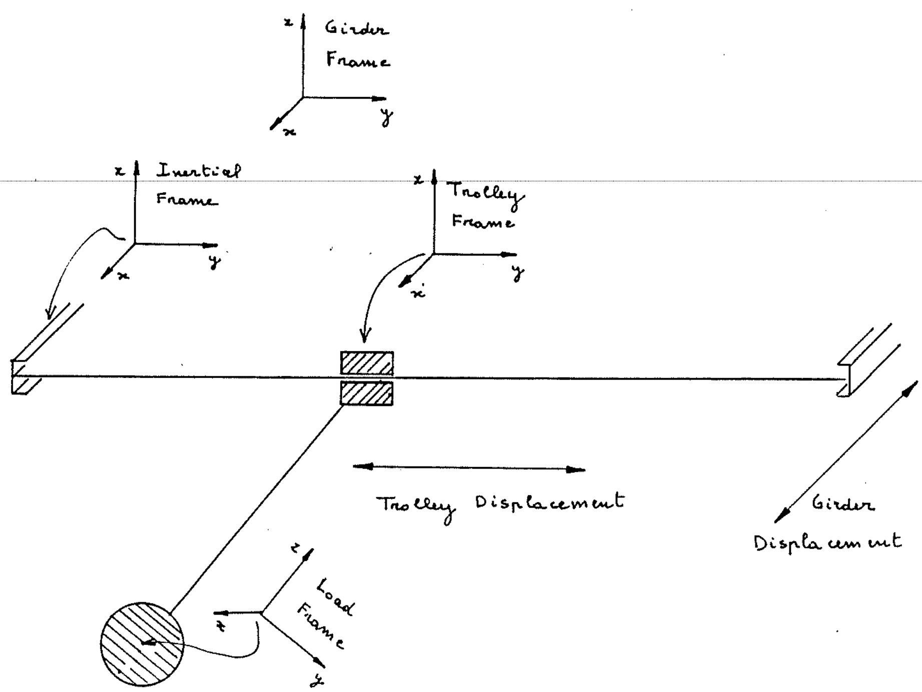

Figure 1: 3-D Euclidean Overhead Crane

4. Motion Controller Design

In accordance with sections 1 and 2, we first design a (continuous) motion controller. As is common practice in similar studies, the controller selected is a member of the state feedback family

$$[u_{D1}\ u_{D2}] := u_D := -K[x - x_D] \tag{26}$$

where x_D represents the desired (equilibrium) state, and K is an appropriate (usually state-dependent) gain matrix.

Letting

$$x_D := [x_{D1}\ x_{D2}\quad 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]', \tag{27}$$

where $(x_{D1}\ x_{D2})$ denotes the desired position of the trolley with respect to the fixed frame, a generalization of the approach in [Hu.1, De.3] suggests that a suitable state feedback controller is given by

$$u_{D1} = -K_A[x_1 - x_{D1}\ x_3\sin(x_4)\ \dot{x}_1\quad \dot{x}_3\sin(x_4)] \tag{28}$$

$$u_{D2} = -K_B[x_2 - x_{D2}\ x_3\cos(x_4)\ \dot{x}_2\quad \dot{x}_3\cos(x_4)] \tag{29}$$

where K_A, (K_B), is a constant matrix computed so as to stabilize the crane under the hypothesis that the trolley (the girder) is blocked and that the crane motion is confined to a vertical plane [De.3, Hu.1, Kr.1].

Adopting the physical parameters in Table 1, and characterizing the linearized model associated with these constrained motions by the eigenvalues $p_1 = p_2 = p_3 = p_4 = -1$ (for the vertical motion with the trolley blocked), and $p_1 = p_2 = p_3 = p_4 = -1.5$ (for the vertical motion with the girder blocked), one obtains

$$K_A = [-3920\ \ -15660\qquad 237470\qquad 28300]$$

$$K_B = [-2478 \ -6608 \ -9940 \ -24063] \tag{30}$$

Next, we consider the implementation of the continuous controller defined by (28-30) in terms of a discrete controller the entries of which will be required to satisfy the constraint

$$u_i^* \in U_i := \{ u_{ij} \}, \quad i=1,2, \, j=1,2, \tag{31}$$

with

$$U_1 = \{ 0, +- 250, +- 600 +- 2500, +- 4500\}$$

$$U_2 = \{ 0, +- 950, +- 1500, +- 3500\}. \tag{32}$$

Following theorem 1, a suitable discrete controller may be obtained by selecting $u_i(t)$ so that $u_i \in U_i$, and

$$SGN\{u_i^*(t) - u_{Di}(x,t) + \mu_i(t)\} := - SGN\{[B_0' \sigma(t)]_i\} \tag{33}$$

where $B_0(x)$ is given by (22, 24, 25) and $\sigma(t)$ is obtained by combining (2, 3) with (22, 23). Observe that

$$S(t) := \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \tag{34}$$

with $S_1=0$, and

$$S_2(t) := q - \int_0^t \{ F_{02}(x) + B_{02}(x) u_D(x)\} dt, \tag{35}$$

where

$$\dot{q} := [x_5 \; x_6 \; x_7 \; x_8 \; x_9]', \tag{36}$$

and $F_{02}(x)$, $B_{02}(x)$ are the nominal values of $F_2(x)$, $B_2(x)$.

It follows that condition (33) becomes

$$SGN\{u_i^*(t)-u_{Di}(x,t)+\max\{|p_i(t)|\}\}:=$$

$$SIGN\{[B_{02}'(x)B_{02}(x)B_{02}^+(x)S_2(t)]_i\} \quad (37)$$

which is met by selecting $u_i^*(t)$ as given by

$$u_i^*(t) = -M(t)_i SIGN\{[B_{02}'(x)B_{02}(x)B_{02}^+(x)S_2(t)]_i\} \quad (38)$$

with $M(t)_i$ the smallest element in $\{u_{ij}\}$ such that

$$M(t)_i > |u_{Di}(x,t)| + \max|p_i(t)|. \quad (39)$$

mass of the girder (m_{gi}) = 5000 Kg

mass of the trolley (m_{tr}) = 4200 Kg

mass of the load (m_{lo}) = 600 Kg

length of the cable (L) = 8 m

gravitational acceleration (g) = 9 m/sec^2

inertia of the load with respect to the z axis (I_a) = .1 Kg m^2

inertia of the load with respect to the x axis (I_t) = .01 Kg m^2

inertia of the load with respect to the y axis (I_t) = .01 Kg m^2

compliance coefficient of the suspended cable (p) = .1 N*m/rad

Table 1: Overhead crane nominal parameter values

5. Simulation Experiments

The comparative behavior of the discrete controller and its continuous counterpart have been investigated by simulation. The basic simulation test procedure adopted was as follows: the crane's dynamic model is defined by (18-24), with the crane's nominal parameters as in Table 1. The continuous controller is described by (28, 29) with K_A, K_B as in (30). The discrete controller is described by (31, 33). Both controllers are required to implement a transfer of the crane from the initial state [0 0 0 0 0 0 0 0 0]' to the final state [1 1 0 0 0 0 0 0 0]'.

Test N.1: STEP RESPONSE UNDER NOMINAL OPERATING CONDITIONS

Objective: To illustrate comparative behavior under nominal operating conditions.

Modalities: As stipulated in the basic test procedure under nominal operating conditions (absence of perturbation, system parameters corresponding to their expected values).

Results and Discussion: As predicted by theorem 1, the results of this test (see Figures 2-4) confirm that under nominal operating conditions the output behavior of the continuous controller and of its discrete counterpart are essentially identical. The slight discrepancy in the coordinate x_4 is due simply to the influence of the integration step used in the simulation.

Test N.2: THE INFLUENCE OF A BIGGER-THAN-EXPECTED LOAD MASS

Objective: To illustrate comparative behavior under a bigger-than-expected load mass.

Modalities: Identical to those of experiment N.1, except that the load mass is now assumed

to be equal to 7000 Kg, rather than to have the nominal value of 5000 Kg.

Results and Discussion: The presence of a bigger-than-expected load mass causes perturbations that cannot be represented in terms of the vector p(t) considered by the theorem. Thus a theoretical result allowing us to predict its influence is not available. The simulation results in Figure 2 indicate, nevertheless, that the dynamics of the continuous and the discrete controllers remain essentially equivalent. They also indicate dynamic behavior only slightly different from the nominal one. The robustness to this type of perturbation appears to be quite acceptable in both the discrete and the continuous controller.

Test N.3: INFLUENCE OF EXTERNAL PERTURBATION

Objective: To illustrate comparative behavior under the application of a sinusoidal perturbation to the trolley.

Modalities: Identical to those in test N.1, except that now a sinusoidal perturbation force is applied to the trolley. The frequency of this perturbation is 0.2 hertz; the amplitude is 100 N.

Results and Discussion: With reference to theorem 1, in this case we have $p(t)= 100B_0 \sin(.4\pi t)$. From theorem 1, we expect this perturbation to have an influence on the dynamics of the continuous controller, and no influence on the dynamics of the discrete controller. The results in Figure 3 confirm that this is in fact the case. It follows that from the perspective of this test the sensitivity performance of the discrete controller is superior to that of the continuous controller it attempts to emulate.

Test N.4: INFLUENCE OF A DELAY IN THE LOOP

Objective: To analyze sensitivity to the presence of a delay in the feedback loop.

Modalities: Identical to those of experiment N.1, except that a delay of 1 msec is now introduced in the application of the control action.

Results and Discussion: As the available theoretical results are not applicable in the presence of a delay, it is once again difficult to predict the outcome of this experiment prior to its implementation. The simulation results (Figure 4), suggest that while the dynamic behavior produced by both the continuous and the discrete controller is influenced by this perturbation, this influence is somewhat more damaging in the case of the latter controller. This in turn suggests that in implementing the continuous controller by means of a discrete action particular attention has to be paid to minimize such a delay.

Conclusions

The design of a motion controller for an over-head Cartesian crane in 3-D space, under the constraint that the control action belong to a set with a finite number of values, may be carried out by means of a two-step approach: first, design a constraint-free (continuous) controller satisfying the required motion specifications; second, replace this continuous controller with an equivalent sliding-mode (discrete) controller satisfying the finite-number-of- values requirement. In implementing this approach, there are three main difficulties: first, the solutions proposed in the literature for carrying out the first step appear to be either excessively complex (as, for instance, in [Ka.1]), or only applicable to the 2-D case (as in [Ri.1, Hu.1, De.3]); second, the presence of nonlinear intercouplings in the 3-D model prevents a Lyapunov- type linearization of the plant model, and hence the adoption of results based on such a linearization (as was the case in [De.3]); third, the dynamics of the crane do not satisfy the hypothesis that the matrix $B_0(x,t)B_0^+(x,t)$ be constant (this hypothesis is required by the main theorem in [De.2]).

An effective way to overcome the first difficulty is to obtain a 3-D controller by the concurrent application of two 2-D controllers of the type proposed in [Hu.1, De.3]. The input to these controllers is represented by the projection of the load motion over the vertical planes associated with the motion of the girder and of the trolley. The second and third difficulties may be overcome by modifying the main result in [De.2] so as to make it applicable to the problem under consideration (theorem 1).

Proceeding in this fashion, a discrete controller is obtained, of which the structure is quite simple and easy to implement. Simulation results confirm theoretical predictions

to the effect that under nominal operating conditions, the dynamics generated by the discrete controller are identical to those characterizing the continuous controller. This remains the case under the presence of input-equivalent parameter variations and external perturbations (such as neglected forces acting on the girder or the trolley). The available theory does enable prediction of the influence of (not input-equivalent) perturbations such as those related to an only approximate knowledge of the crane's physical parameters (e.g. mass load, girder and trolley mass, cable length, etc..). The simulation results do suggest, however, that even in this case the dynamics of the discrete controller compare favorably with those associated with the continuous controller.

On a less optimistic note, it must be added that the simulation results also suggest that the performance of the discrete controller is problematic in the presence of a delay in the control loop. In addition to this, we must expect that practical modifications of the proposed scheme might be required in order to render the high-frequency switching produced by the discrete controller compatible with the physical capabilities of the crane motors.

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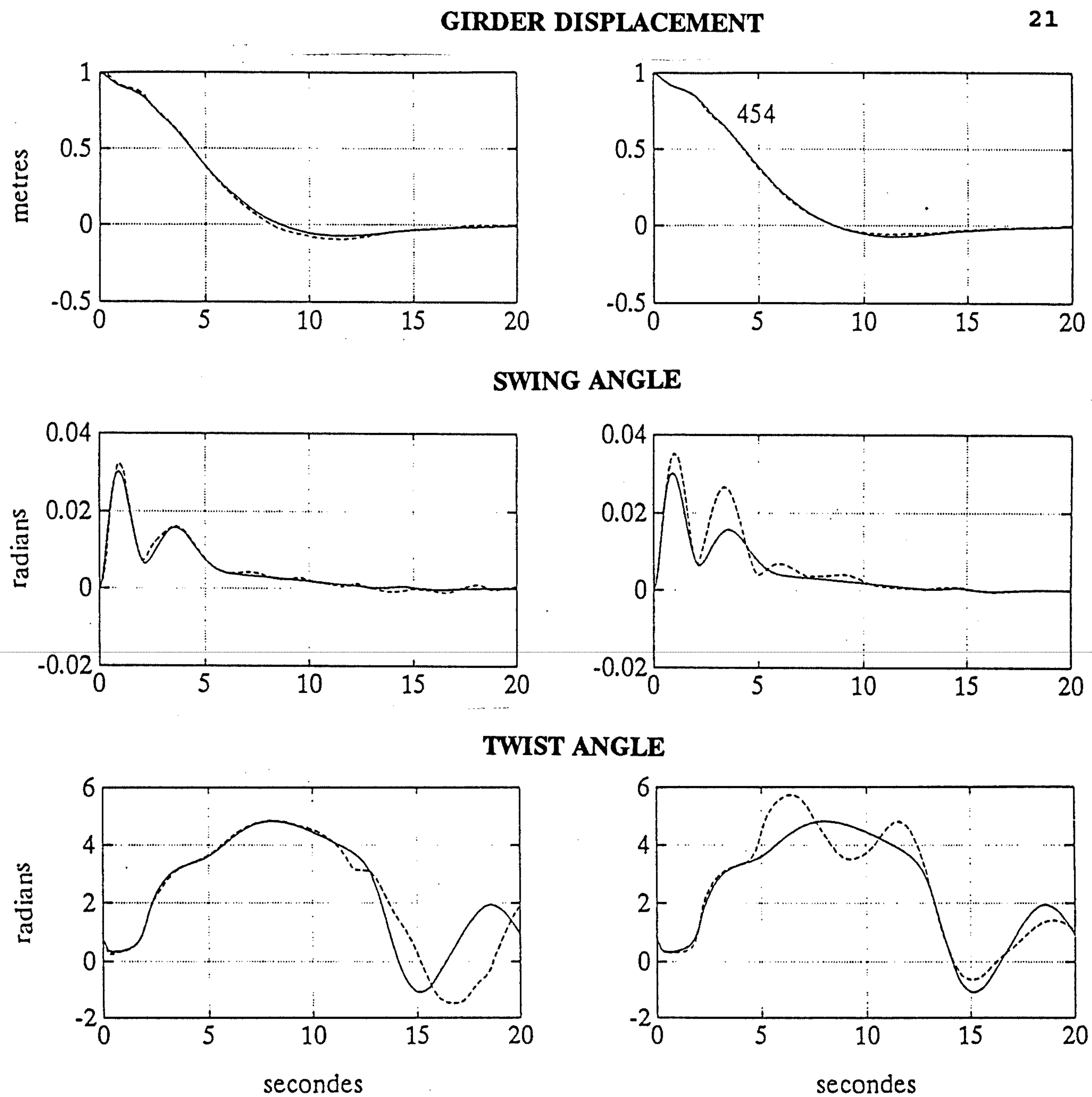

Figure 2: Influence of a bigger-than-expected mass load: a) discrete controller; b) continuous controller

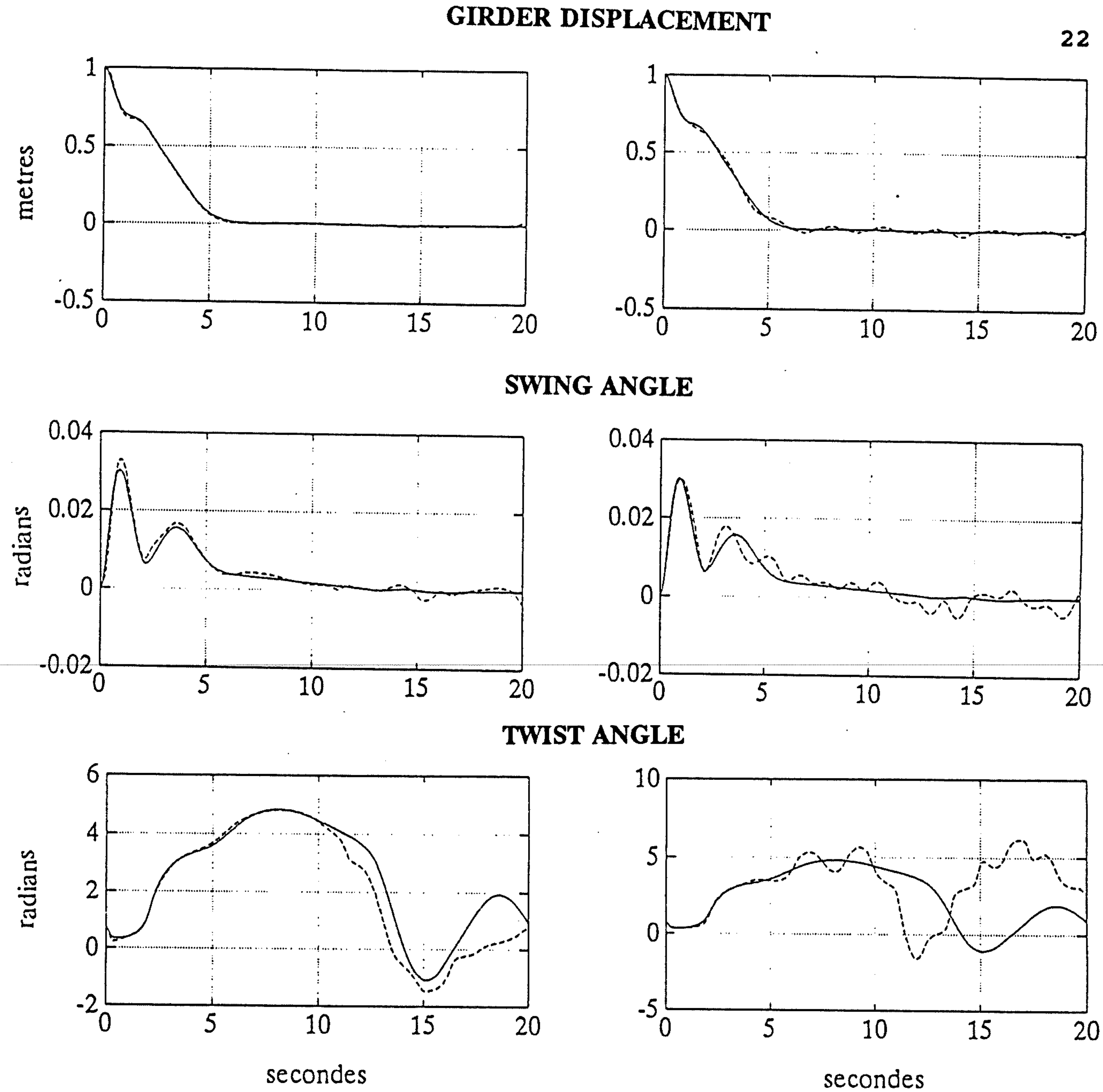

Figure 3: Sensitivity to a sinusoidal perturbation force acting on the trolley: a) discrete controller; b) continuous controller

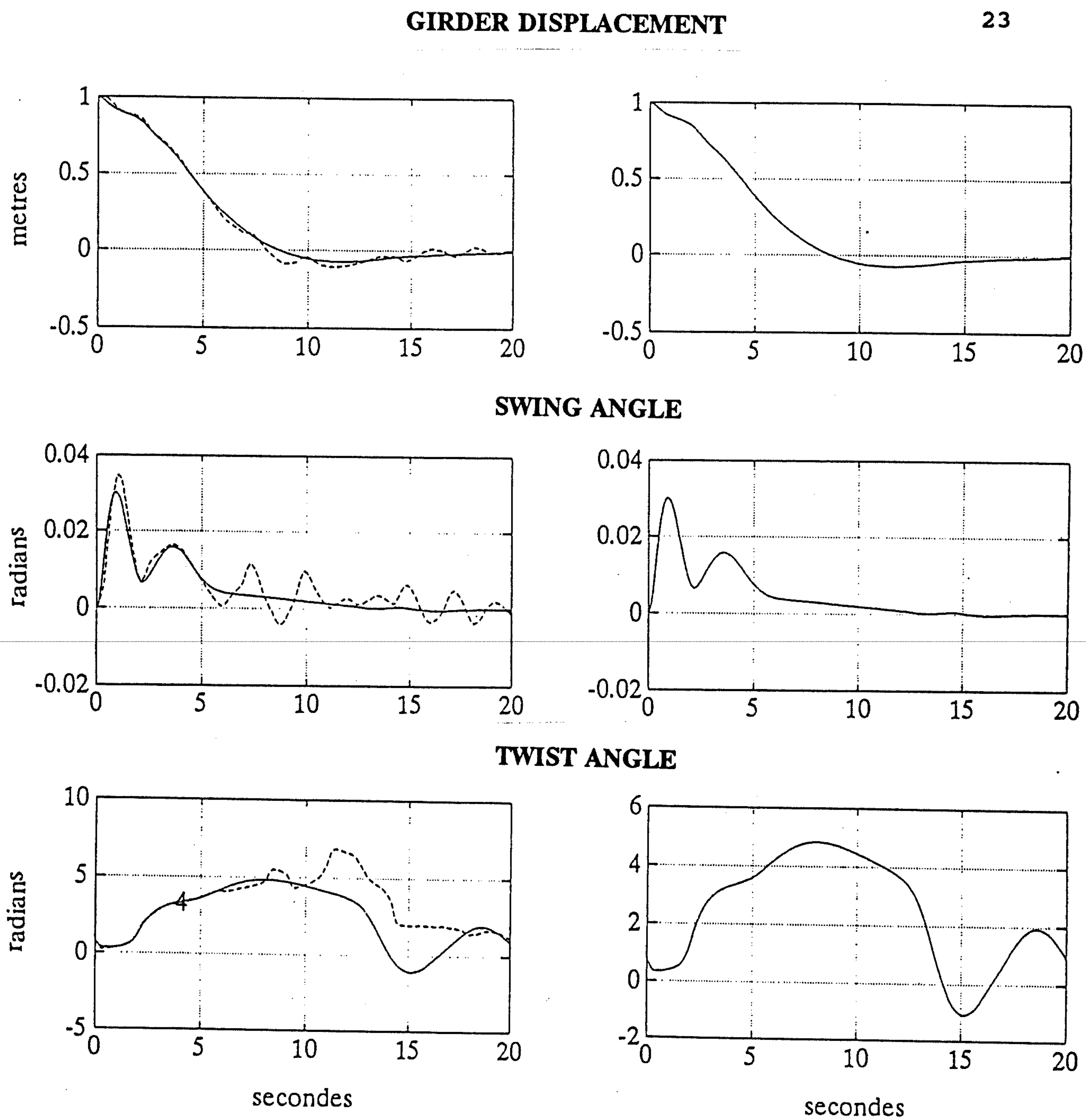

Figure 4: Sensitivity to the presence of a delay in the loop: a) discrete controller; b) continuous controller.