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"A BANG BANG CONTROLLER FOR VIBRATION REDUCTION IN A ROTATING FLEXIBLE BEAM"

par:

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A BANG BANG CONTROLLER FOR VIBRATION REDUCTION IN A ROTATING FLEXIBLE BEAM

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0. Summary

A bang bang controller for vibrations reduction in a flexible beam is designed under the constraint that its action emulate the dynamics which would be generated by a pre-assigned linear controller. This design is carried out by using sliding mode techniques which have been recently developed to implement a continuous controller via a discrete valued controller. The comparative behavior of the ensuing bang bang controller and of its linear counterpart is illustrated via numerical simulation under both nominal and perturbed operating conditions.

Ce rapport est une version étendue de l'article "A Bang Bang Controller for Vibration Reduction in a Rotating Flexible Beam" qui a été accepté pour publication dans les Comptes Rendus du "31st IEEE Conference on Decision and Control", Tucson, Arizona, Décembre 1992.

1. Introduction

A number of real time feedback schemes have been recently proposed to control vibrations in flexible structures such as those characterizing light weight flexible manipulators, large space antennas, space platforms, space stations and similar systems. While the variety of approaches adopted to this end is quite large, typically the flexible structures considered are modelled in terms of a linear dynamical system; the controller schemes are often of the state feedback type with gains computed by either pole placement or LQG techniques [Jo.1, Li.1, Sh.1]. A common feature of these controllers is that they are designed under the assumption that the control action be arbitrarily selectable within a certain continuous range of values. Practical considerations arising from the utilization of on/off electronic hydraulic or air jet components may, on the other hand, make it convenient or necessary to implement these controllers by imposing the control action to be restricted to assume one value out of a discrete set.

One way to solve this problem is to develop the design of the discrete controller from scratch by considering the performance specifications which have led to the linear controller, and by subsequently applying discrete design techniques. This, however, is an inefficient procedure to follow as it does not benefit from the already developed linear controller. An alternative solution is simply to insert in cascade with the linear controller a switching device such as, for example, a pulse width modulator. A disadvantage of this approach, however, is in the introduction of a small delay which on occasion may produce a destabilizing influence on the feedback loop dynamics. Among more advanced alternative avenues, a recently proposed approach, based on sliding mode controllers techniques [De.1], appears to be particularly attractive. In addition to the elimination of the delay, these techniques may also lead to a solution with an improved robustness to parameter variation and external perturbation.

The objective of the present paper is to investigate the possibility to solve our problem by applying this approach, and more specifically, by considering the general sliding mode procedure recently proposed in [De.2]. This objective is pursued by considering, as a prototype of a flexible structure, a planar flexible rotational beam . This beam is clamped to a hub at one hand and free at the other; its rotations are implemented by means of a linear feedback control torque applied at the hub [Da.1, Sh.1, Bha.1]. By applying the procedure proposed in [De.2], we design a bang bang controller which emulates the action of this pre-assigned linear control. A number of simulation experiments illustrate the comparative behavior of the flexible beam submitted to the ensuing bang bang controller and to its linear counterpart. This behavior is investigated under both nominal and perturbed operating conditions.

2. The Mathematical Setting

Consider a linear dynamical plant described by the differential equation

$$\dot{x}(t) = A*x(t) + B*u(t) + p(t), \quad x(0) = x_0 \tag{1}$$

where: $x(t) \in R^n$ represents the plant state, $u(t) \in R^m$ is the control, and $p(t) \in R^n$ a disturbance; the symbols A and B denote appropriately dimensioned real matrices. These matrices are assumed to be given by

$$A := A_0 + \delta A \tag{2}$$

$$B := B_0 + \delta B \tag{3}$$

where A_0 and B_0 characterize the "nominal" behavior of the plant, and δA, δB describe the influence of parameter variations. It is further assumed that a (continuous) control law $u_D(x,t)$ is pre-assigned with values belonging to a certain open set in R^m, (e.g.: an open hypercube).

From a mathematical point of view, the problem of interest is to determine a (discrete) control law $u(x,t)$ with entry values constrained to belong to a given discrete set, $u_i(x,t) \in \{u_{i1}, \ldots, u_{iN}\}$, $i = 1 \ldots m$, and such that its action on the plant is, in some sense, equivalent to that of the (continuous) control $u_D(x,t)$. Under nominal operating conditions, (i.e. with $\delta A=0$, $\delta B=0$ and $p(t)=0$), one would like the discrete controller to produce the same (nominal) state trajectory as the pre-assigned $u_D(x,t)$. Under perturbed operating conditions, (i.e. with $\delta A<>0$, $\delta B<>0$ and $p(t)<>0$), one would like the discrete controller to produce a state trajectory at least as close to the nominal one as the trajectory produced by $u_D(x,t)$.

To solve this problem [De.2] suggests a controller defined according to the following procedure. Using the notation

$$\sigma(t) := B_0 B_0^{+} S(t) \tag{4}$$

where B_0^{+} denotes the pseudo-inverse of B_0 and

$$S(t) := \int_0^t \{\dot{x}(t) - A_0 * x(t) - B_0 * u_D(x,t)\}dt$$

the discrete controller is selected so as to provide an output such that

$$u_i^{*}(x,t) \in \{u_{i1}, \ldots, u_{iNi}\}, \; i = 1, .. , m \tag{6}$$

and

$$SGN\{u_i^{*}(x,t) - u_{Di}(x,t) + \mu_i(t)\} := - SGN\{[B_0'\sigma(t)]_i\} \tag{7}$$

where

$$\mu(t) := B_0^{+}\{\delta A * x(t) + \delta B * u(t) + p(t)\} \tag{8}$$

The properties of such a controller are formalized by the following lemma.

Lemma 1 (A specialization of theorem 1 in [De.2]): If B_0 is of a full rank and the inverse of $B_0'B$ exists, **then:** the dynamics of system (1-3) submitted to a discrete controller satisfying (4-8) has the following properties:

i) the state trajectory of the system is described by

$$\dot{x}(t) = A_0 * x(t) + B_0 * u_D(x,t)$$
$$+ [I - B[B_0^+ B]^{-1} B_0^+]\{- B_0 u_D(x,t) + \delta A * x(t) + p(t)\} \qquad (9)$$

ii) **if** Rank($[\delta A | \delta B | p(t) | B_0]$=Rank($B_0$) **then**

$$\dot{x}(t) = A_0 * x(t) + B_0 * u_D(x,t), \qquad (10)$$

iii) **if** $[I - [B_0 B_0^+]]\delta = \delta$ for $\delta \epsilon \{\delta A, \delta B, p(t)\}$

then

$$\dot{x}(t) = A * x(t) + B * u_D(x,t) + p(t), \qquad (11)$$

iv) **if** Rank($[\delta B | B_0]$) = Rank(B_0)

then

$$\dot{x}(t) = A_0 * x(t) + B_0 * u_D(x,t) + v(t), \qquad (12)$$

where

$$v(t) := [I - [B_0 B_0^+]]\{\delta A * x(t) + p(t)\} \qquad (13)$$

3. The Case of a Flexible Beam

With reference to Figure 1, consider a flexible beam free at one hand and clamped on a rotating rigid hub at the other. It has been demonstrated in a number of recent publications that, under appropriate assumptions, this beam may be adequately controlled by means of a linear feedback controller [Da.1, Sh.1, Li.1]. To further characterize this controller, let the kynematic configuration of the beam be described in terms of the num_ndimensional vector

$$q := [displ' \ \theta]' \tag{14}$$

where: num_n denotes the number of nodes used to develop the finite element model of the beam; displ is a (num_n-1)-dimensional vector representing the displacement of the nodes from the neutral axis of the beam; θ represents the angle of rotation of the hub.

Ignoring rotary inertia and shear deformation effects, the dynamic behavior of the (Euler-Bernoulli) model of the beam is given by

$$\dot{x} = A*x + B*u \tag{15}$$

$$y_1 := C*x$$

where: the (2*num_n)-dimensional vector x represents the state of the beam

$$x := [x_1 \ \ x_2]' \qquad x_1 := q \qquad x_2 := \dot{q}; \tag{16}$$

the scalar u represents the control torque applied to the hub; y_1 is a (num_tp+1)-dimensional output vector: its first num_tp entries represent the displacements measured by the strain gages located at num_tp testing points; the last entry represents the angular rotation of the hub.

Moreover,

$$A := \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix} \qquad B := \begin{matrix} B_1 \\ B_2 \end{matrix} \tag{17}$$

with

$$A_{11} = A_{22} = 0;\ A_{12} := I_{num\_n}\ ;\ A_{21} = -[Mass]^{-1}\ [Rig] \qquad (18)$$

$$B_1 = 0; \qquad B_2 = [Mass]^{-1}\ w; \qquad w' := [0_{num\_n-1} \quad 1]'$$

where the symbols [Mass] and [Rig] represent respectively the beam mass and rigidity matrices. Matrix C extracts from the state vector those entries which correspond to either a testing point or to the hub angular rotation.

The vibrations of the beam are controlled via a linear feedback law

$$u_D(t) = -\ K_1\ y_1(t)\ -\ K_2\ \dot{y}_1(t) \qquad (19)$$

where $K := [K_1 \quad K_2]$ is a gain matrix; K is usually computed by considering a beam model where the number and location of the nodes are selected so as to coincide with the available testing points. The structure of this controller is represented in Figure 2.

In line with section 2, a bang bang controller, subjected to the constraint $u \epsilon \{+- M\}$, and capable of inducing a dynamic behavior equivalent to that produced by the linear feedback control described by eqn (19), may be obtained as follows.

Consider

$$S(t) = \begin{array}{c} 0_{num\_n} \\ \\ y_2(t) - B_{20} * K_2 * (y_1(t) - y_1(0)) - \int_0^t ((B_{20} * K_1 - A_{210}) y_1(t) \end{array}$$

(20)

where $y_2(t) := \dot{y}_1(t)$, and A_{210} and B_{20} represent the nominal values of A_{21} and B_2 which would correspond to a beam model where nodes and testing points are made to coincide.

Following lemma 1, the bang bang controller is given by

$$u := -M*SIGN[\sigma(t)] \tag{21}$$

where M represent the available level of control and

$$\sigma(t) = C_1 [y_1(t) - y_1(0)] + C_2 y_2(t) + y_3(t) \tag{22}$$

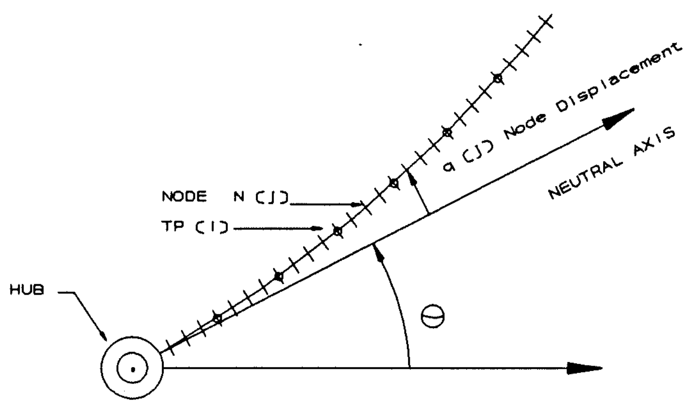
with

$$\dot{y}_3 := C_3 y_1 \tag{23}$$

and

$$C_1 := B_{20}' B_{20} K_2 ; C_2 := B_{20}' ; C_3 := B_{20}' [B_{20} K_1 - A_{210}] \tag{24}$$

The structure of this controller is illustrated in figure 3.



TP_j , j=1,...,num_tp, := testing points

N_j , j=1,...,num_n, := nodes

q_j := displacement of j-th node from neutral axis

theta:= angular rotation of the hub

tau:= control torque

Figure 1: Flexible Rotating Beam [Da.1]

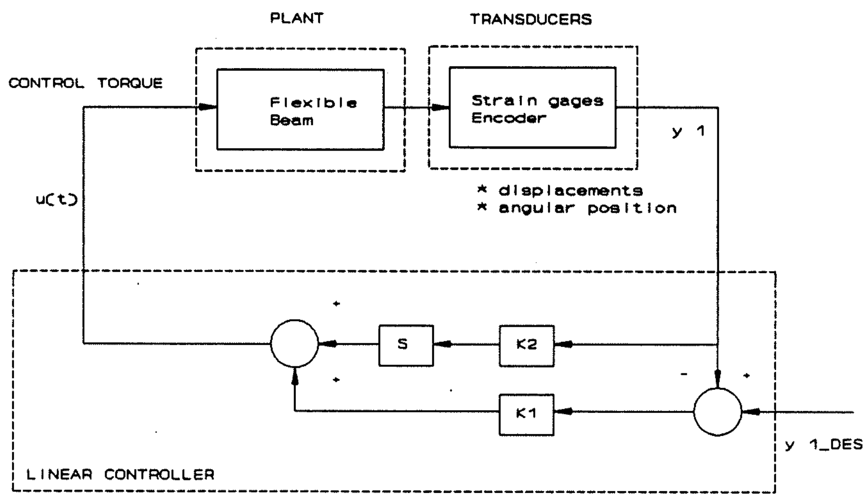


Figure 2: The Structure of the Continuous Controller.

$$C1 := \grave{B}_{20} B_{20} K_{2}$$

$$C2 := \grave{B}_{20}$$

$$C3 := \grave{B}_{20} [B_{20} K_{1} - A_{210}]$$

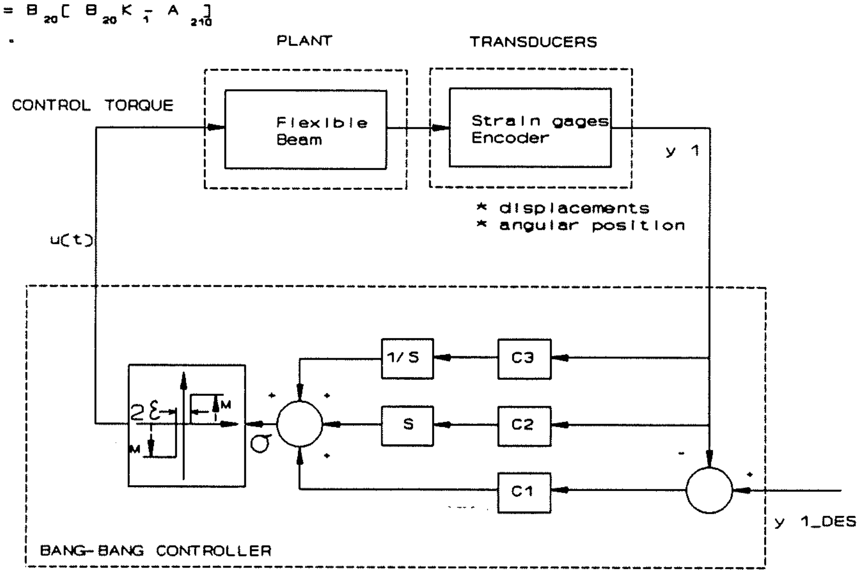


Figure 3: The Structure of the Bang Bang Controller.

4. Simulation Results

The comparative behavior of the bang bang controller and of its continuous counterpart has been investigated by implementing a certain number of simulation tests. These simulations concern a stepped beam with geometrical and physical parameters as specified in the example developed by Angeles and co-workers in [Da.1] (table 1, Figure 4).

Basic Test Procedure: Considering the linear and the bang bang schemes in Figures 2 and 3, the controllers are required to implement a 3.14 rad hub rotation. The initial state of the plant corresponds to a beam configuration characterized by a zero angular speed and a null deformation. The gains of the two controllers are determined using a beam dynamical model based on 8 measuring points. More in particular, the linear gain have been taken as suggested in [Da.1]; the matrices appearing in the description of the bang bang controller have been computed using (24); the value of the control level, M, has been taken equal to 20. In both schemes, the dynamics of the beam is simulated by considering a finite element model with 29 nodes. More details may be found in [De.4].

Graphical Results Presentation : The results for each Test are visualized graphically in Figures 5 through 11. These figures display the evolution of the tip deflection, the mid beam deflection, the hub control and the hub rotation with respect to time. The dashed curves describe the behavior with nominal operating conditions whereas the continuous curves relate to perturbed operating conditions.

a = 0.1 [m]

Hub Inertia = 4.26 E-2 [kg m^ 2]

Section 1 := 0.5 [m]

Section 2 := 0.4 [m]

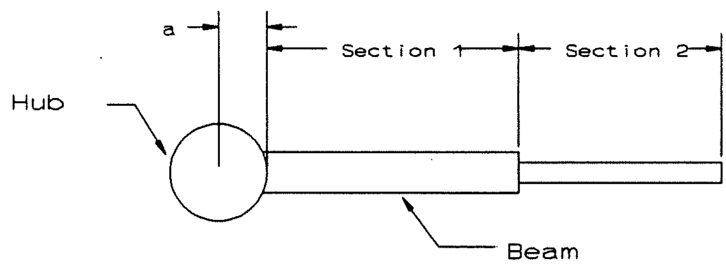


Figure 4: Beam used in simulation experiments [Da.1]

PHYSICAL PROPERTIES	Hub	SECTION 1	SECTION 1
No. Testing Points	0	5	4
Length [m]	a = 0.1	0.5	0.4
Mass Density [kg/m^ 3]	2712	2712	2712
Young Modulus [GPa]	--	71.0	71.0
Vertical Heigth [mm]	--	2.0	1.0
Horizontal Depth [m]	--	0.1	0.1

Table 1: Physical Parameters of the Beam

Test N.1: STEP RESPONSE BEHAVIOR UNDER NOMINAL OPERATING CONDITIONS

Objective: To illustrate comparative behavior under nominal operating conditions.

Modalities: As stipulated in the basic test procedure under nominal operating conditions (absence of perturbation, system parameters correspond to their expected values). The control period is of 0.1 msec.

Results and Discussion: With reference to lemma 1, in this case one has that δA, δB and p(t) are all equal to zero. It follows that, in line with lemma 1, one must expect the dynamics of the linear and bang bang controllers to coincide. The results of the test are represented by the dashed curves in Figures 5-11 and confirm this expectation.

Test N.2: INFLUENCE OF A HUB MOMENT OF INERTIA BIGGER THAN EXPECTED

Objective: To illustrate comparative behavior under operating conditions where the hub moment of inertia is considerably bigger that its nominal value.

Modalities: Identical to those of experiment N.1 with the exception that the actual value of the hub moment of inertia is now taken to be 20 times bigger than the nominal one.

Results and Discussion: The results of this test are reported in Figure 5. The addition of hub mass moment of inertia causes both a δA and a δB different from zero. Contrary to the case of test N.7, the induced perturbation does not satisfy condition ii) of lemma 1. As a consequence, it is more difficult to predict the theoretical outcome of the test. Observe, however, that the component of the perturbation in the range of $\dot{B}_0$ is relatively minor, and that condition iii) of lemma 1 is "almost" satisfied. Moreover, with the implemented selection of the bang bang action, it is still possible to satisfy eqn (7). From this observation, one should expect the dynamics of the linear and bang bang controllers to be similar. This is confirmed by the simulation results in Figure 5 which report a dynamic behavior only slightly different from the nominal one. These results also suggest that the robustness to this type of perturbation is quite acceptable in both the bang bang and the linear controller.

Test N.3: ADDITION OF A MASS LOAD AT THE TIP OF THE BEAM

Objective: To illustrate comparative behavior under operating conditions where a mass load is added at the tip of the beam.

Modalities: Identical to those of experiment N.1 with the exception that a pointwise, unaccounted for, mass load is now assumed to be located at the tip of the beam. The mass of this load corresponds to 10 % of the overall mass of the beam.

Results and Discussion: Figure 6 illustrates the simulation results. The behavior of the two controllers are visibly similar. The discussion of Test N.2 applies once again to this type of perturbation.

Test N.4: INFLUENCE OF A SMALLER THAN ANTICIPATED MODULUS OF ELASTICITY

Objective: To illustrate comparative behavior under perturbed operating conditions corresponding to a smaller than anticipated beam modulus of elasticity.

Modalities: Identical to those of experiment N.1 with the exception that the modulus of elasticity is now smaller (50%) than the anticipated value.

Results and Discussion: Figure 7 illustrates the simulation results. The behavior of the two controllers are visibly similar. This perturbation causes δA to be different from zero. The discussion of Test N.2 applies once again to this type of perturbation.

Test N.5: BEHAVIOR UNDER AN IMPRECISE LOCATION OF THE STRAIN GAGES

Objective: To analyze sensitivity to the location of the strain gages used to measure vibrations.

Modalities: Identical to those of experiment N.1 with the exception that the strain gages at the tip and at the mid point of the beam are now misplaced from their expected location of 5 mm towards the tip of the beam.

Results and Discussion: The influence of this perturbation is difficult to predict on a theoretical basis as no direct result is available to this effect. The simulation results, (Figure 8), suggest that while the dynamic behavior produced by the linear controller is relatively uneffected by this perturbation, its influence is critical to the stability of the bang bang controller. This difficulty reveals a substantial weakness of the bang bang controller which should be dealt with before proceeding to test bench implementation.

Test N.6: INFLUENCE OF A DELAY IN THE CONTROL ACTION

Objective: To analyze sensitivity to the presence of a delay in the feedback loop.

Modalities: Identical to those of experiment N.1 with the exception that a delay of 2.5 msec is now introduced in the application of the control action.

Results and Discussion: It is once again difficult to theoretically predict the outcome of this experiment as the available nonlinear stability and sensitivity results are not directly applicable to this case. The simulation results, (Figure 9), suggest that while the dynamic behavior produced by both the continuous and the bang bang controller is influenced by this perturbation, this influence is somewhat more damaging in the case of the latter controller.

Test N.7: INFLUENCE OF A PERTURBATION TORQUE ACTING ON THE HUB

Objective: To illustrate comparative behavior under the application of a sinusoidal perturbation torque to the hub.

Modalities: Identical to those in Test N.1 with the exception that now a perturbation torque is applied to the hub. This perturbation is sinusoidal with a frequency equal to 1 hertz and an amplitude equal to .7 N*m.

Results and Discussion: With reference to lemma 1, in this case one has that δA and δB are equal to zero and p(t)= .7*B_0 sin(2*pi*t). As this perturbation is trivially in the range of B_0, from lemma 1 one must expect it to have an influence over the dynamics of the linear controller and no influence over the dynamics of the bang bang controller. The results in Figure 10 confirm this expectation. They suggest that the performance of the bang bang controller is , in this test, superior to that of the linear controller it attempts to emulate.

Test N.8: INFLUENCE OF THE CONTROL SAMPLING PERIOD

Objective: To analyze sensitivity to the control sampling period in the feedback loop.

Modalities: Identical to those of experiment N.1 with the exception that the control period is now equal to 1.5 msec.

Results and Discussion: It is once again difficult to theoretically predict the outcome of this experiment as the available nonlinear stability and sensitivity results are not directly applicable to this case. The simulation results, (Figure 11), indicate the presence of beam oscillations when the system is subjected to the bang bang controller. Further simulations demonstrated a stable behavior of the continuous controller with a control period equal to 3 msec.

Closure

The problem to reduce vibrations in a rotating flexible beam by means of a bang bang action may be solved by applying the general procedure proposed in [De.2]. According to this procedure one starts by designing a linear (continuous) controller capable to reduce vibrations. The action of this continuous controller is then emulated by applying the bang bang action of an "equivalent" discrete controller. The results of this application appear to have much in common with the analogous nonlinear study reported in [De.3].

The structure of the overall bang bang controller is quite a simple and easy one to implement. Simulation results confirm theoretical predictions to the effect that the bang bang controller may be designed so that the dynamics it generates under nominal operating conditions be identical to that obtainable with the pre-designed linear state feedback controller. Under certain types of perturbations such as those related to an only approximate knowledge of the beam physical parameters (hub moment of inertia, rigidity modulus, an additional load on the tip of the beam) the dynamics of the bang bang controller is essentially equivalent to that of the linear controller. Furthermore, the discrete controller, unlike its continuous counterpart, is unaffected by a perturbation torque applied at the hub.

These results also indicate, however, that the stability of the bang bang scheme becomes problematic in the face of an only approximate knowledge of the location of the strain gages, or in the presence of a delay or a too long sampling period in the control loop. Improvements to the bang bang controller design should therefore be considered before its test bench performance may be expected to be a satisfactory one. An avenue of potential interest in this direction is the investigation of robustness improvement by means of correctives such as the adoption an adaptive scheme with real time identification, the introduction of thresholds and additional levels in the control action, the use of a state predictor, and similar measures.

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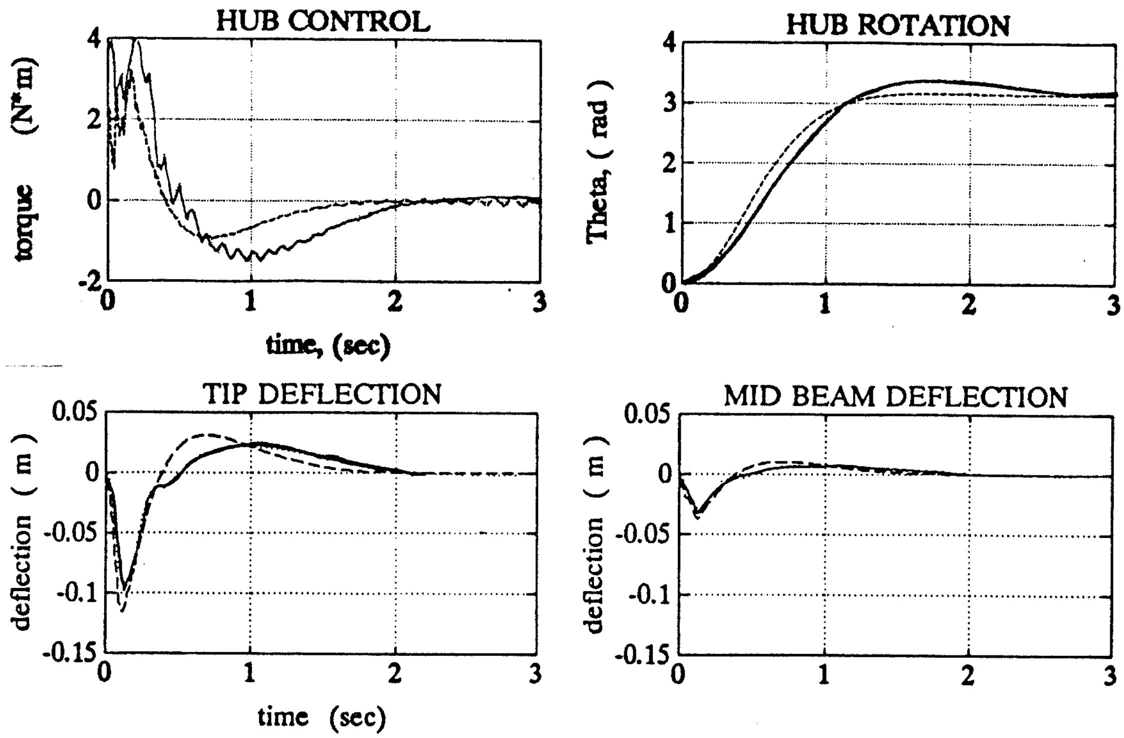
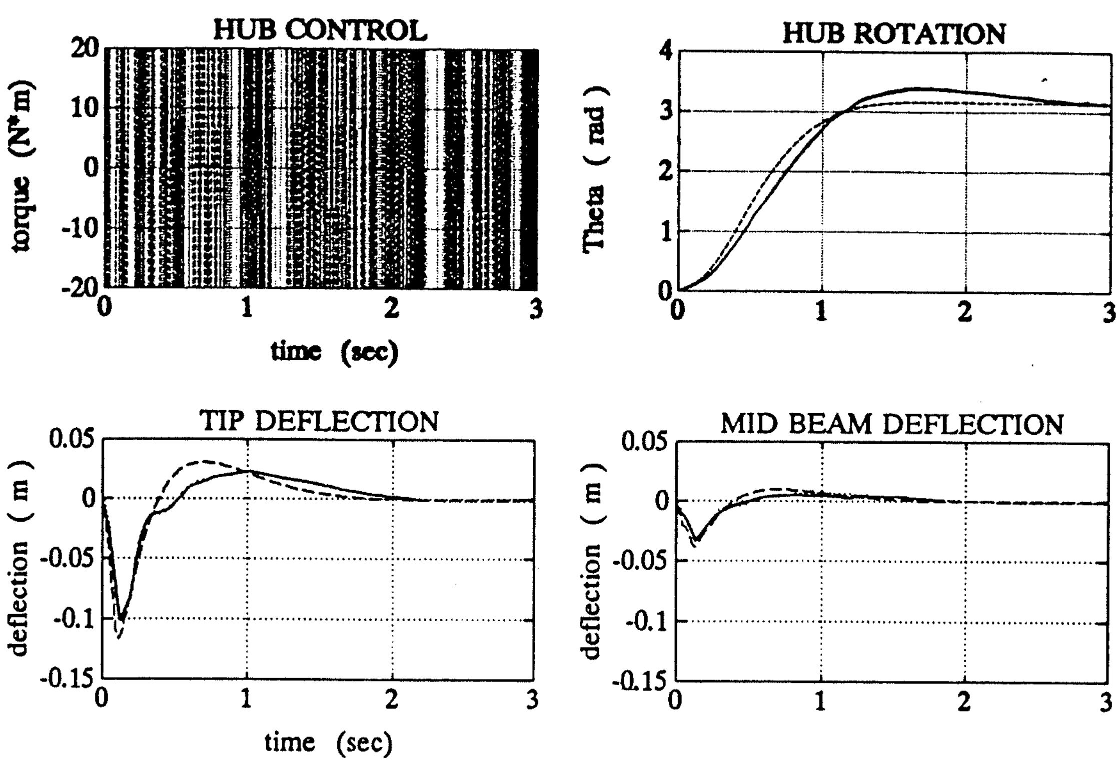
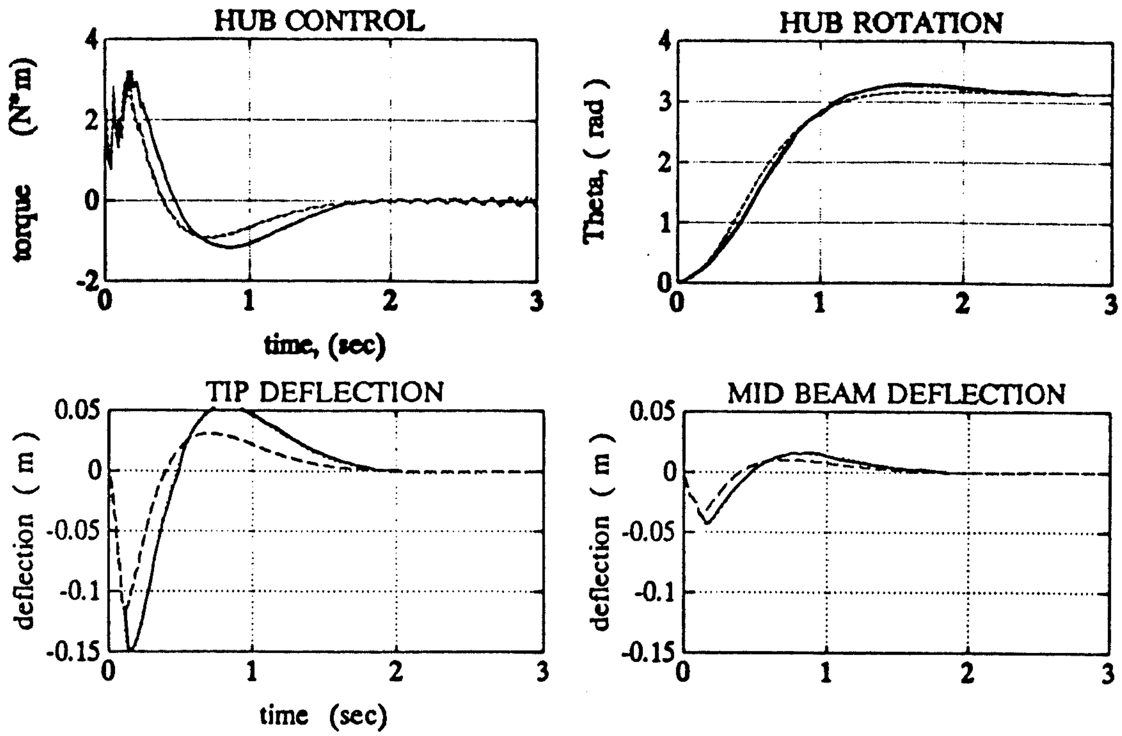


a) Linear Controller

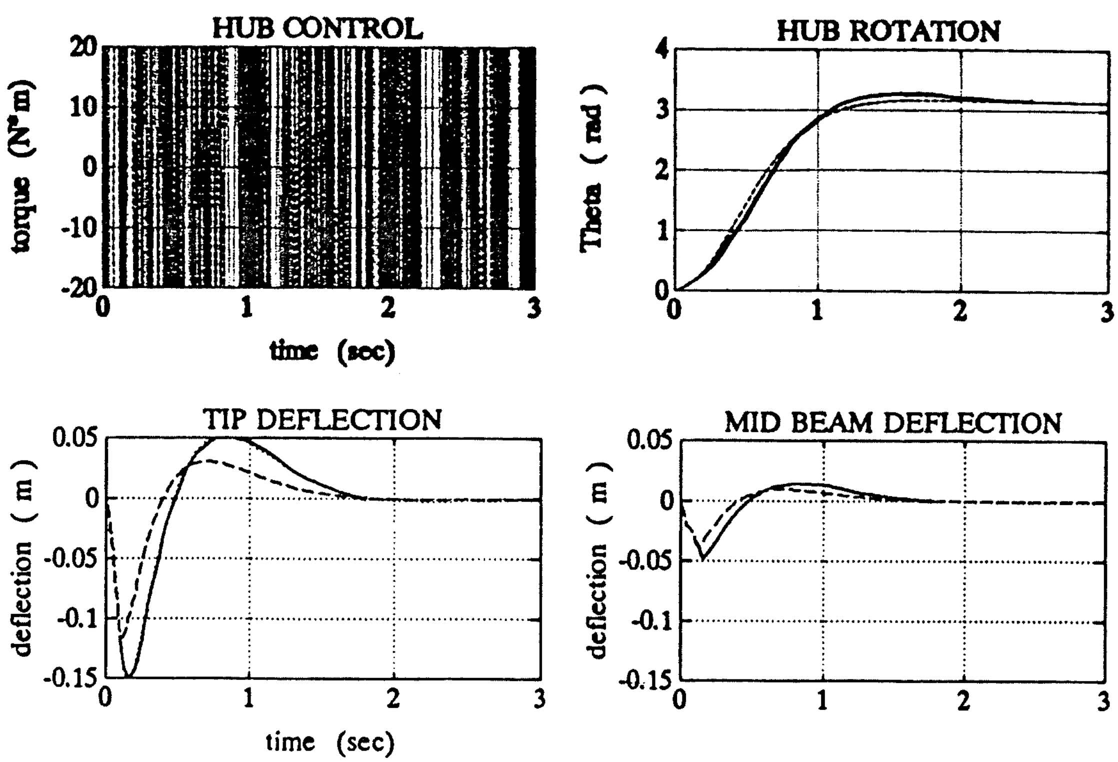


b) Bang Bang Controller

Figure 5: Sensitivity to a larger than expected hub inertia.

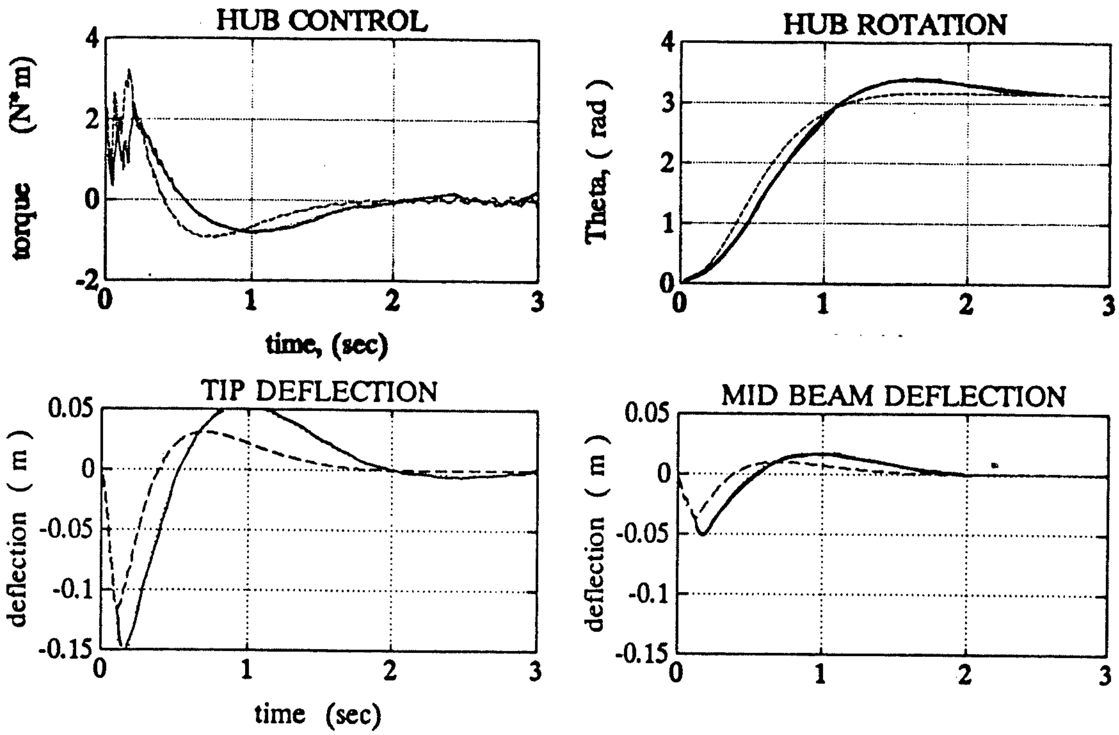


a) Linear Controller

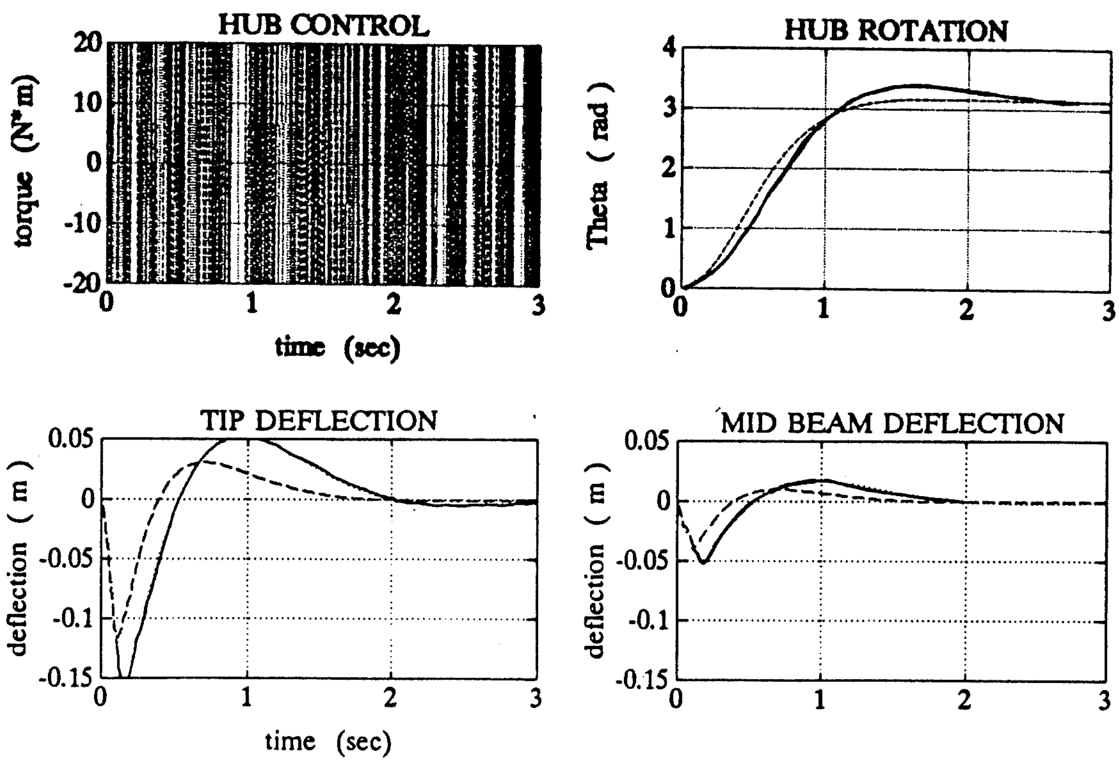


b) Bang Bang Controller

Figure 6: Addition of a load at the tip of the beam.

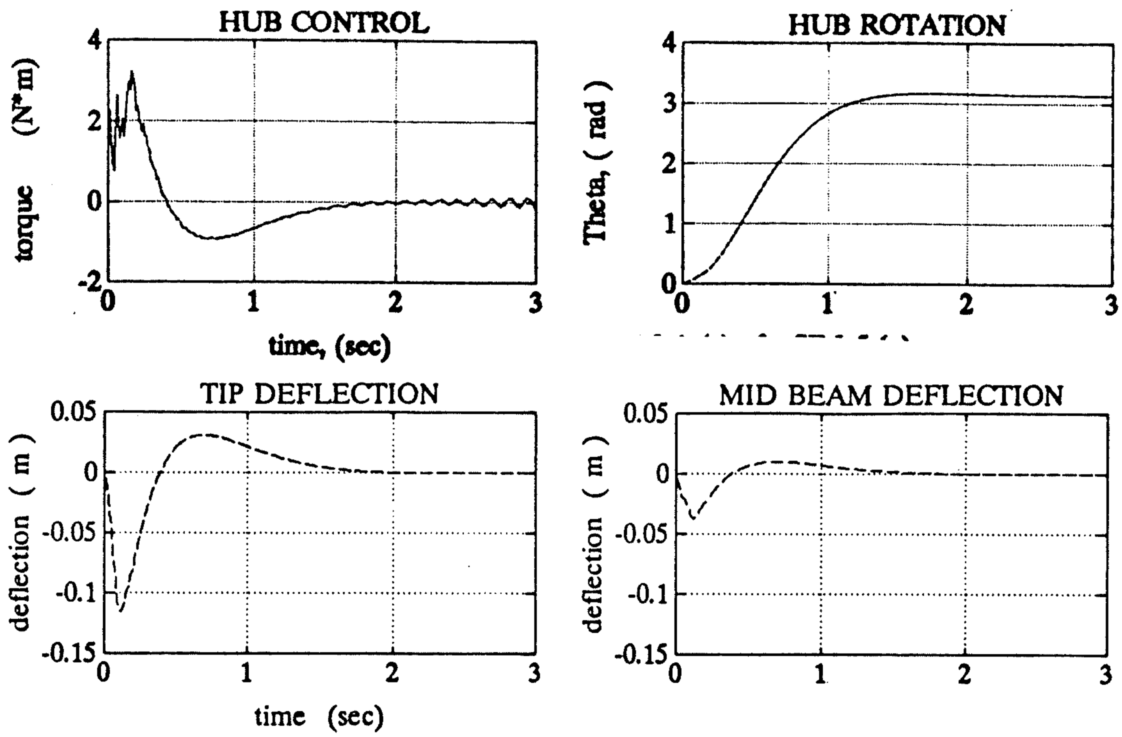


a) Linear Controller

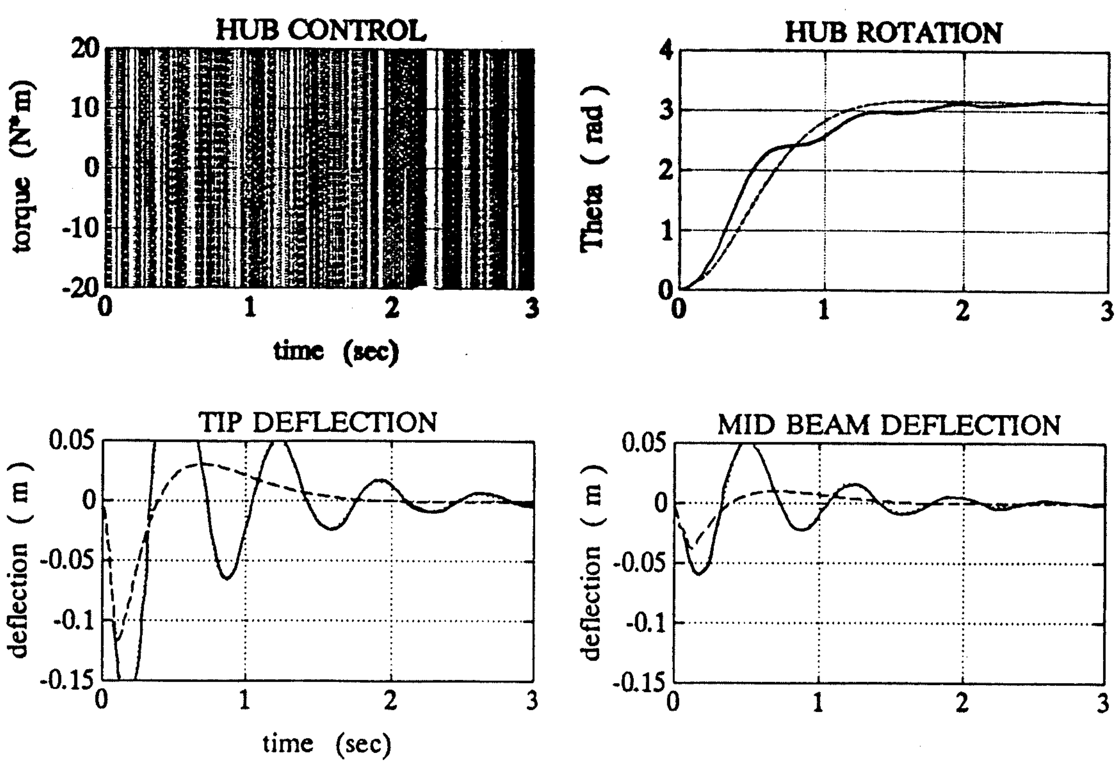


b) Bang Bang Controller

Figure 7: Sensitivity to a diminished modulus of elasticity.

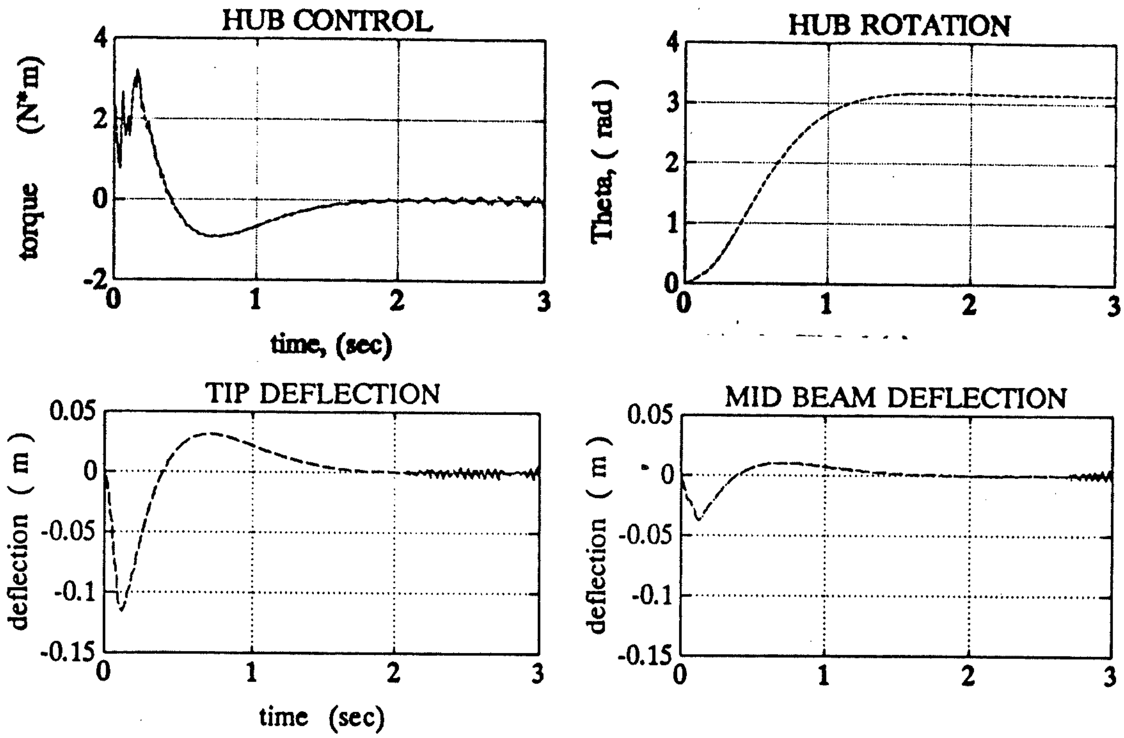


a) Linear Controller

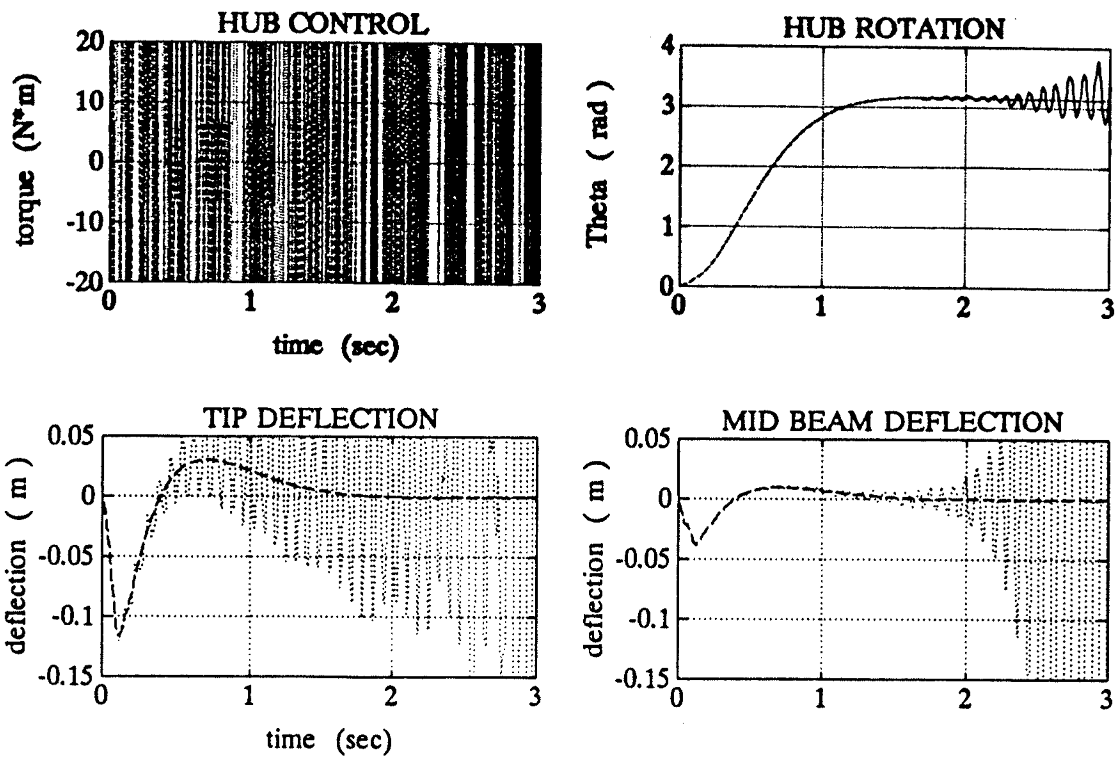


b) Bang Bang Controller

Figure 8: Sensitivity to a misplacement of the strain gages.

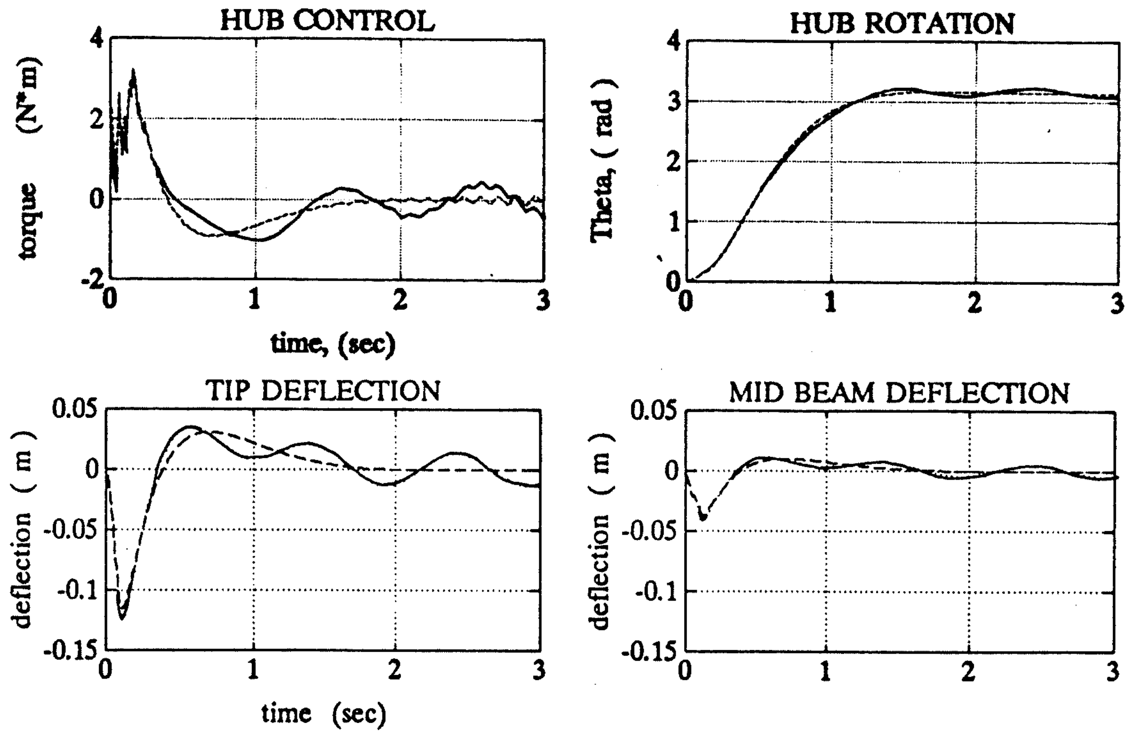


a) Linear Controller

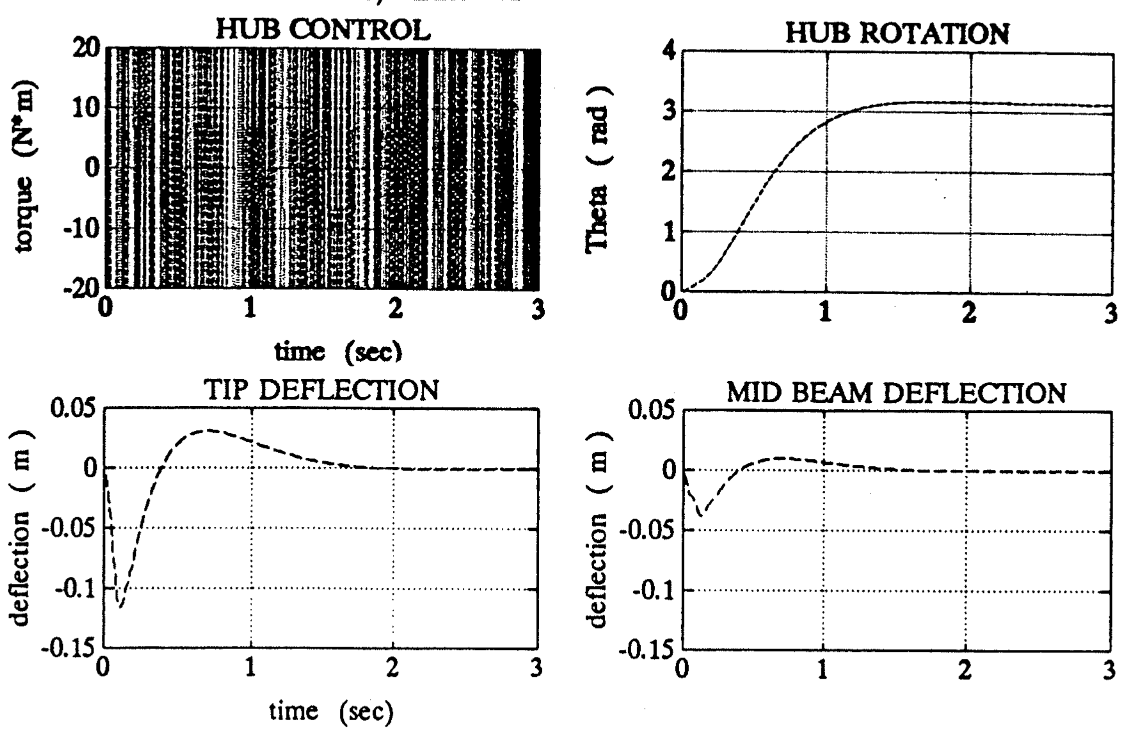


b) Bang Bang Controller

Figure 9: Sensitivity to the presence of a delay in the loop.

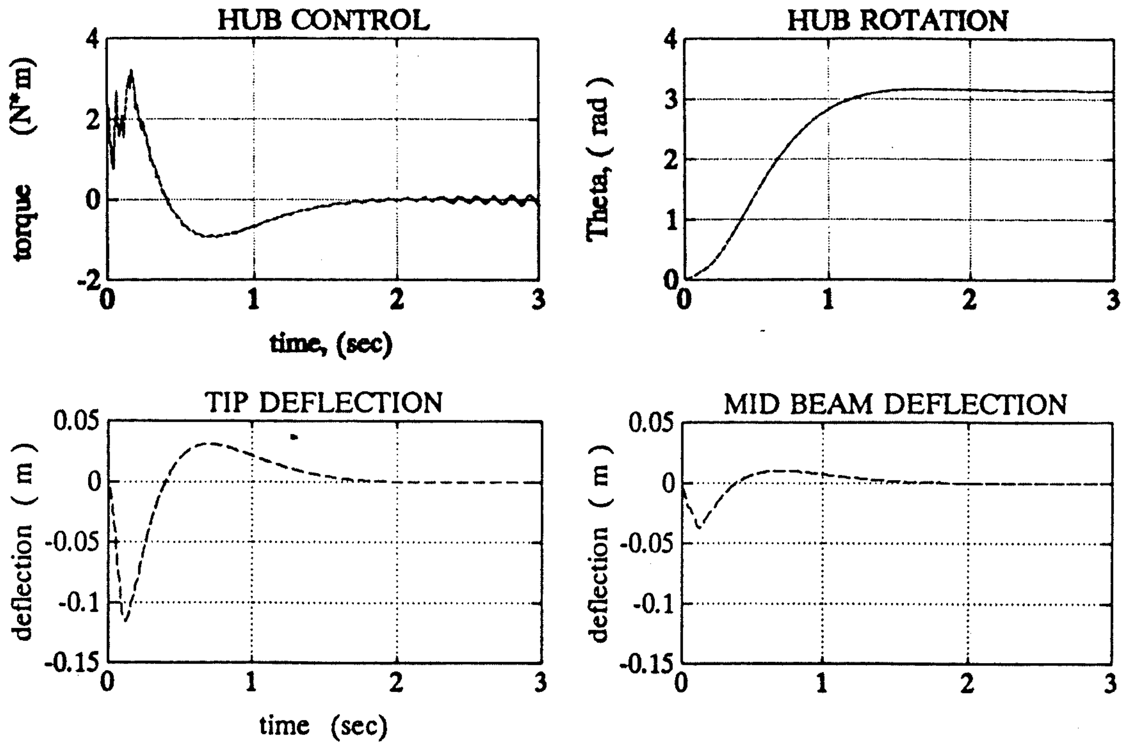


a) Linear Controller

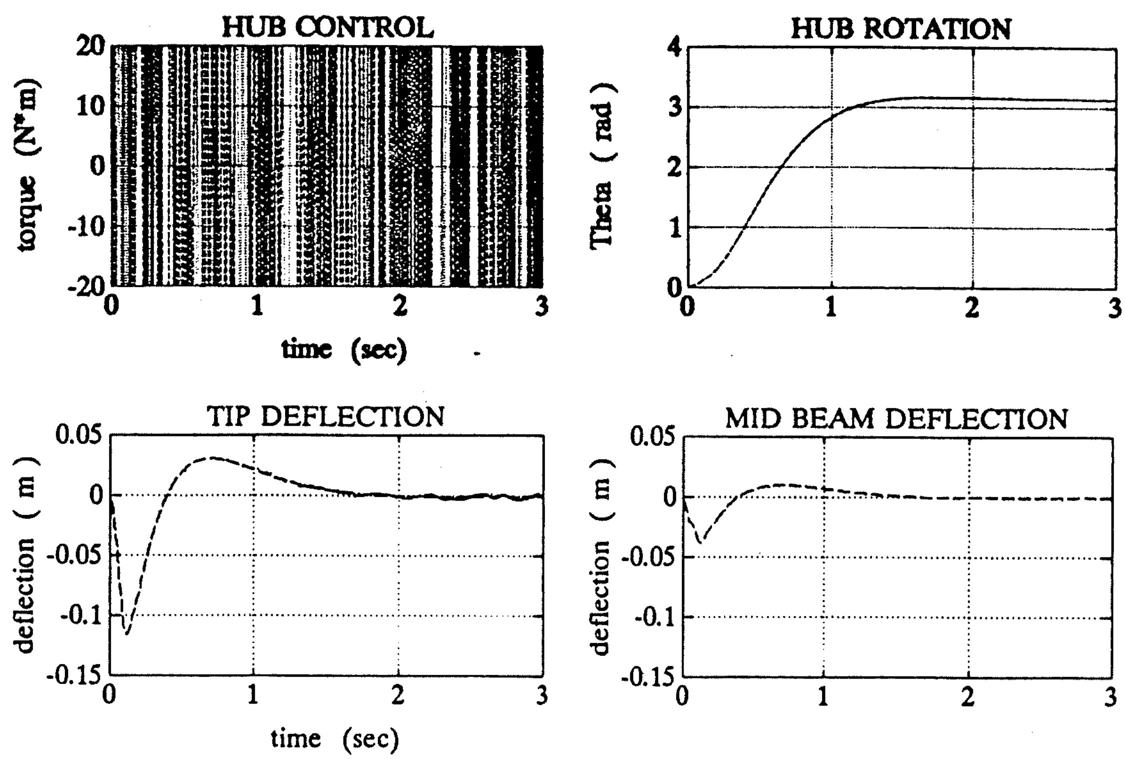


b) Bang Bang Controller

Figure 10: Influence of a sinusoidal perturbation torque at the hub.



a) Linear Controller



b) Bang Bang Controller

Figure 11: Sensitivity to the control period in the loop.

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