Development of the Drag Coefficient of a Sphere Translating Through a Viscoelastic Fluid

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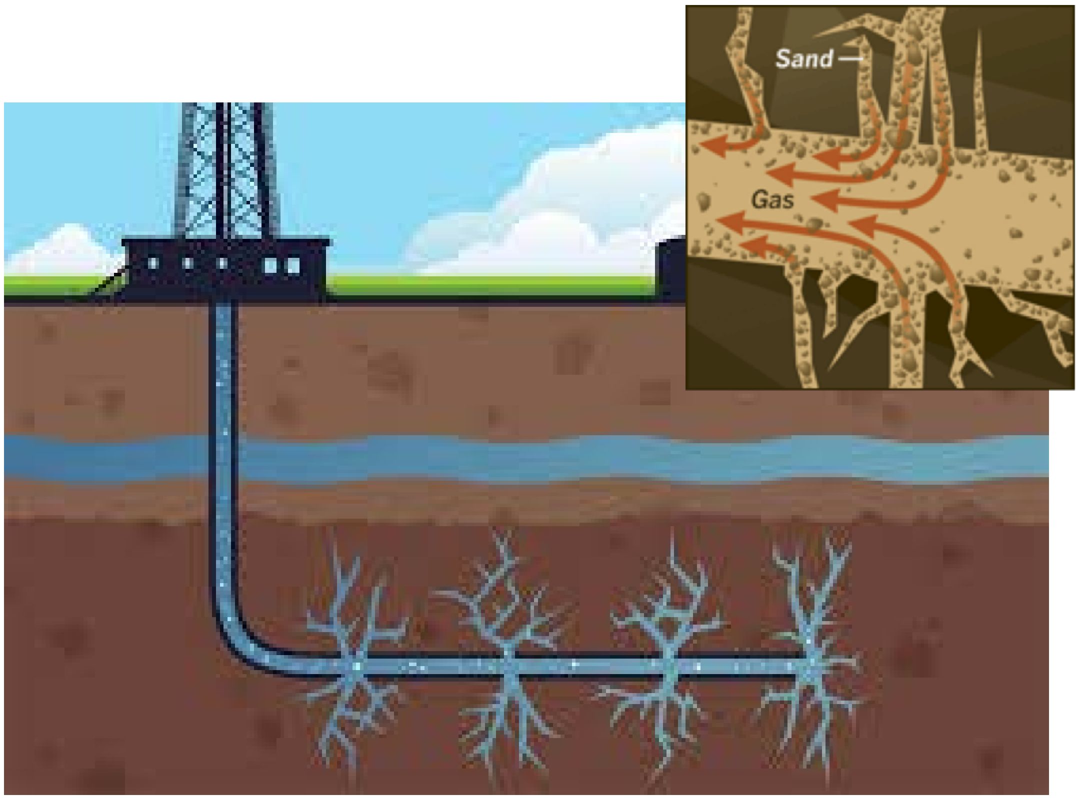
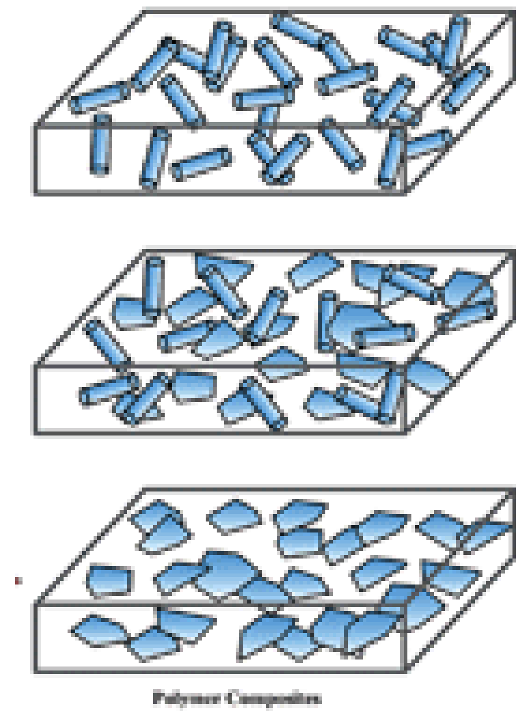
Outline

- Motivation
- Objectives
- Governing Equations and Numerical Method
- Code Verification & Validation
- Drag Model
- Conclusion

The flow of particle-laden complex fluids is an ubiquitous problem…

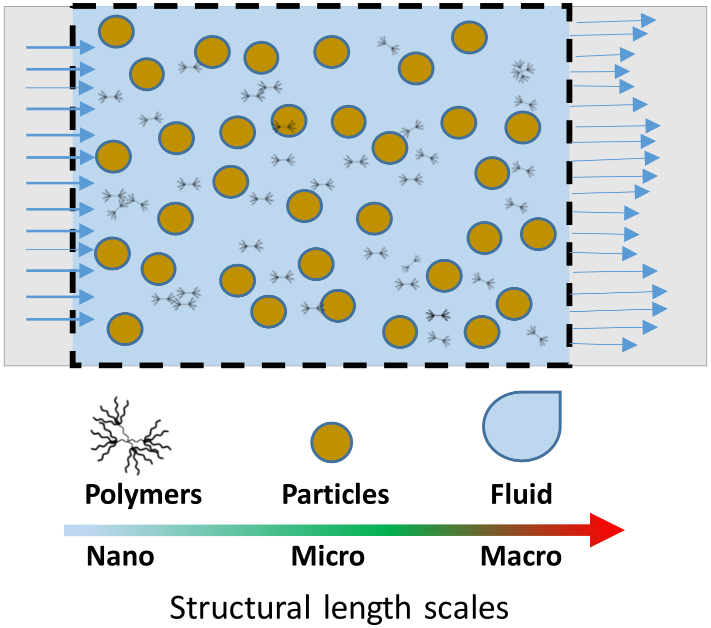


Hydraulic Fracturing



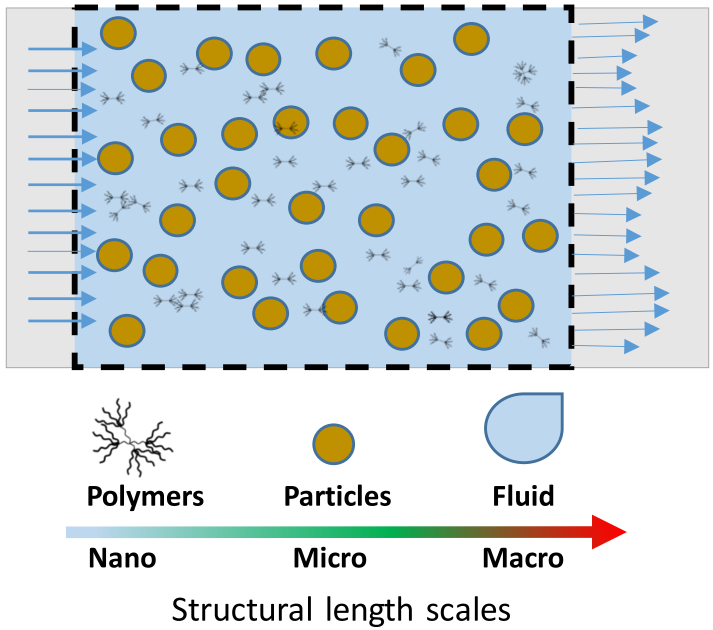
Polymer Composites

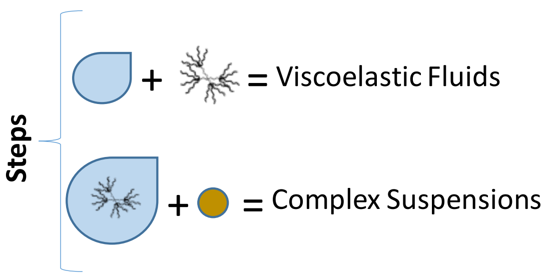
Eulerian-Lagrangian Model



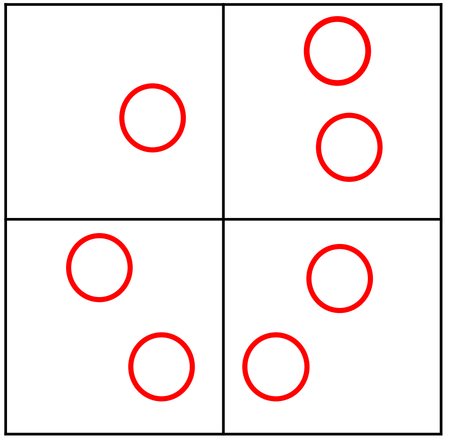
The fluid is described in an Eulerian frame whereas the particles are handled as discrete points which are tracked individually in a Lagrangian frame using Newton's second law.

Eulerian-Lagrangian Model





$$\delta x > d_p$$



But need to know expressions for

1- Drag force
2- Lift force
3- Hindrance effect (due to presence of other particles)

Motivation – Bulk transport model

Eulerian-Lagrangian Model

$$\sum \mathbf{F} = \mathbf{F_a} + \mathbf{F_D} + \mathbf{F}_p + \mathbf{F}_{vol} + \mathbf{F}_{lift} + \mathbf{F}_{buoy} + \mathbf{F}_h,$$

$$\mathbf{F}_a = \frac{1}{2}\rho\frac{m_P}{\rho_P}\left(\frac{D\mathbf{U}}{Dt} - \frac{d\mathbf{U}_P}{dt}\right),$$

$$\mathbf{F}_D = m_p\frac{\mathbf{U}-\mathbf{U}_P}{\tau_P}, \quad \tau_p = \frac{4}{3}\frac{\rho_p D_p}{\rho C_D|\mathbf{U}-\mathbf{U_p}|}$$

$$\mathbf{F}_p = -\frac{m_P}{\rho_P}\nabla p,$$

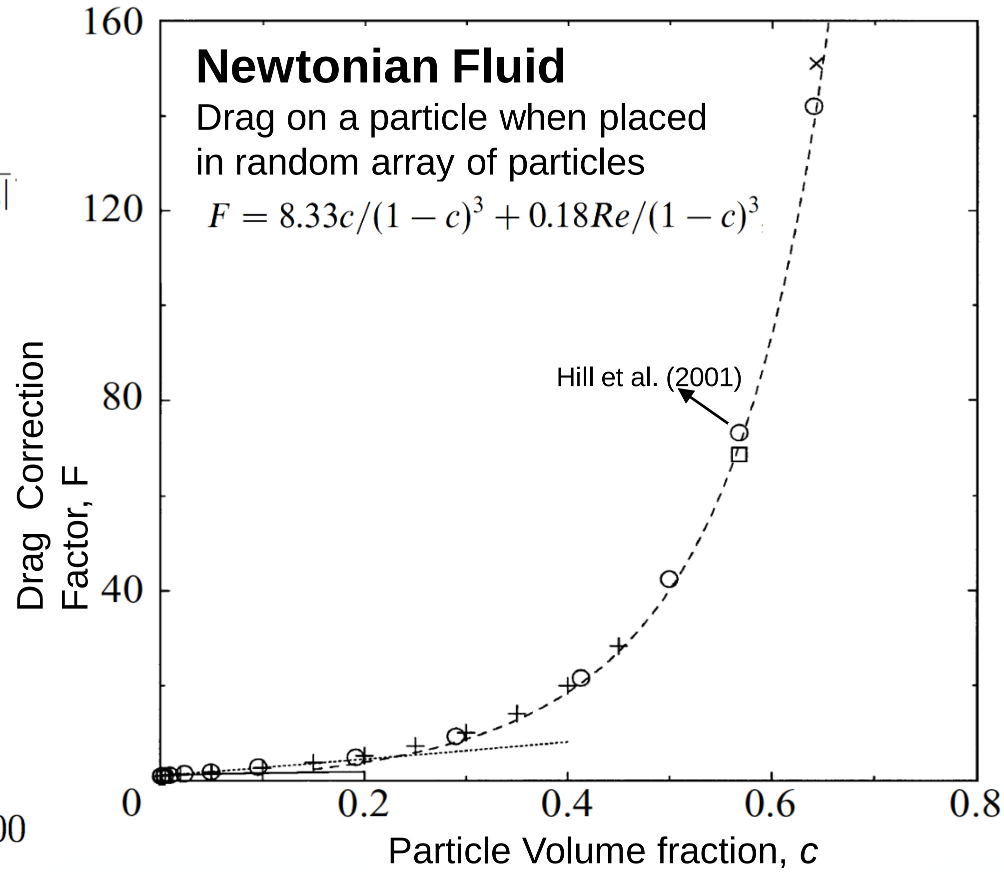
$$\mathbf{F}_{vol} = \frac{1}{2}\rho\frac{dV_P}{dt}(\mathbf{U}-\mathbf{U_p}),$$

$$\mathbf{F}_{lift} = C_L\rho\frac{m_P}{\rho_P}(\mathbf{U}-\mathbf{U_p})\times\omega,$$

$$\mathbf{F}_{buoy} = m_P\left(1-\frac{\rho}{\rho_P}\right)\mathbf{g},$$

$$\mathbf{F}_h = \frac{3}{2}D_P^2\sqrt{\pi\rho\mu}\int_0^t \frac{\frac{D\mathbf{U}}{Dt'} - \frac{d\mathbf{U_P}}{dt'}}{\sqrt{t-t'}}dt',$$

$$C_D = \begin{cases} \frac{24}{Re_p} & \text{if } \mathrm{Re}_p \leq 0.1 \\ \frac{24}{Re_p}\left(1+\frac{1}{6}Re_p^{2/3}\right) & \text{if } 0.1 \leq Re_p \leq 1000 \\ 0.44 & \text{if } \mathrm{Re}_p > 1000 \end{cases}$$



*Faroughi S. A. (2016) Theoretical Developments to Model Microstructural Effects on The Rheology of Complex Fluids, PhD Thesis.
*Subramaniam S., (2013). Progress in Energy and Combustion Science – Elsevier
*Hill, R., Koch, D., & Ladd, A. (2001). *Journal of Fluid Mechanics, 448*, 243-278

Parametrize the effects of fluid elasticity, especially the relaxation and retardation times, as well as inertia on the drag coefficient of a sphere translating through a viscoelastic fluid described by the Oldroyd-B model

(Pseudo) Transient, incompressible laminar flow of an Oldroyd-B fluid

Continuity

$$\nabla \cdot \mathbf{u} = 0$$

Momentum balance

iBSD*

UCM

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - \nabla\cdot\left(\left(\cancel{\eta_S} + \eta_P\right)\nabla\mathbf{u}\right) = -\nabla p - \nabla\cdot\left(\eta_P\nabla\mathbf{u}\right) + \nabla\cdot\boldsymbol{\tau}_P$$

Constitutive model

$$\boldsymbol{\tau}_P + \lambda\left(\frac{\partial \boldsymbol{\tau}_P}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\tau}_P - \boldsymbol{\tau}_P\cdot\nabla\mathbf{u} - \left(\nabla\mathbf{u}\right)^T\cdot\boldsymbol{\tau}_P\right) = \eta_P\left(\nabla\mathbf{u} + \left(\nabla\mathbf{u}\right)^T\right)$$

Retardation ratio $\zeta = \eta_P/(\eta_S + \eta_P)$

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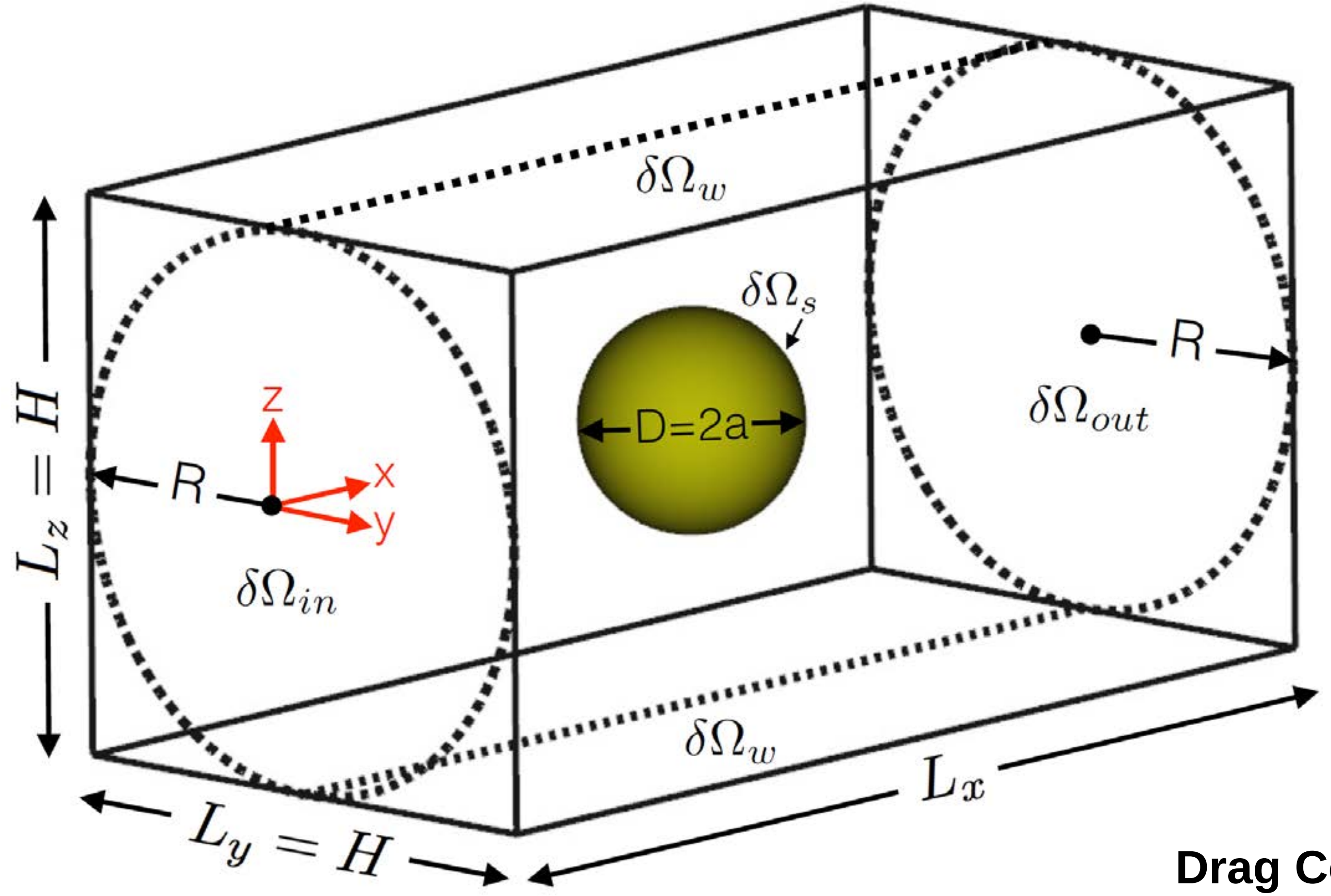
foam-extend 4.0

- Finite Volume Method
- SIMPLE like algorithm
- Second order discretization methods
- Absolute tolerance for $\mathbf{u}$, p, $\boldsymbol{\tau}$ fields 10^{-10}
- Stop criterion 4^{th} decimal place of C_D

* C Fernandes, MSB Araujo, LL Ferrás, JM Nóbrega, Improved both sides diffusion (iBSD): A new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017



$$Re = \frac{2a\rho U}{\eta_0}$$

$$De = \frac{\lambda U}{a}$$

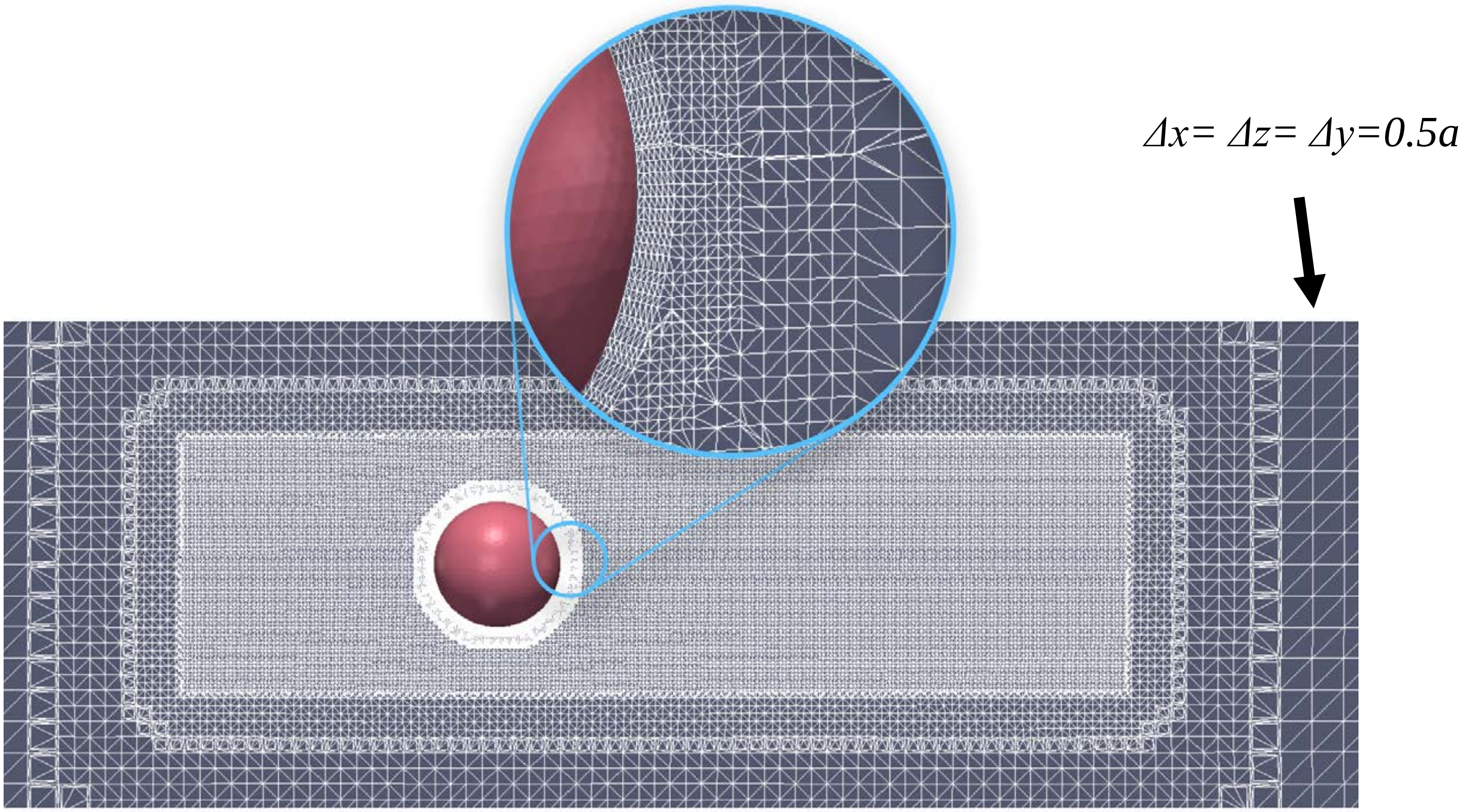
$$El = De/Re$$

Drag Coefficient

Computational domain
(square and circular ducts)

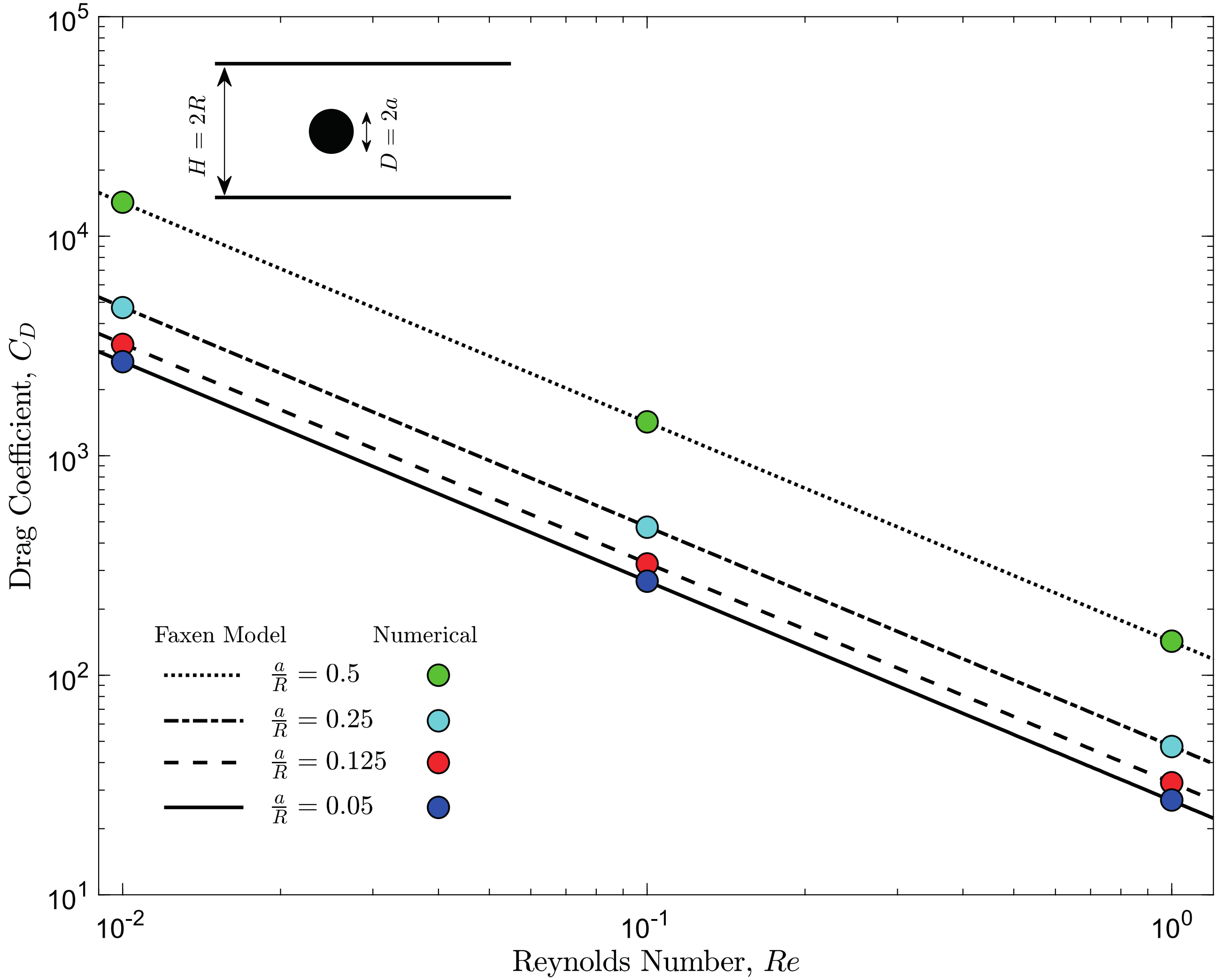
$$C_D = \frac{2F_D}{\rho U^2 A}$$
$$= \frac{2}{\rho U^2 A} \int_{\delta\Omega_s} (\boldsymbol{\tau}_P + \boldsymbol{\tau}_S - p\mathbf{I}) \,.\, \boldsymbol{n} \,.\, \boldsymbol{e}_x dS$$

Case Study – Mesh/geometry independency



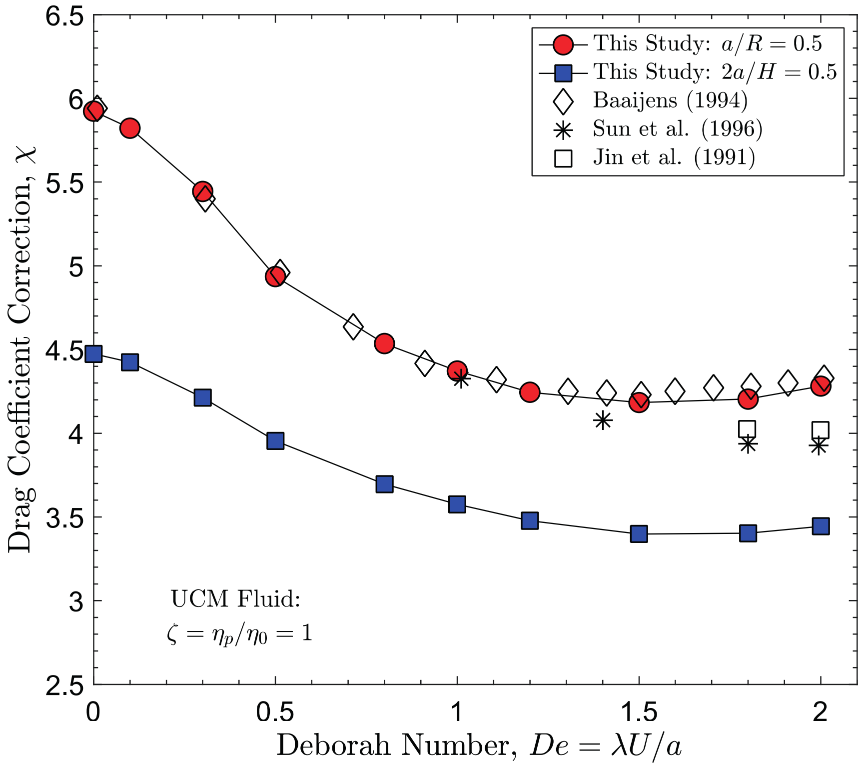
- Mesh generated by *snappyHexMesh* utility
- 7 steps mesh refinement towards the sphere
- 3 steps mesh refinement towards the wall for bounded domain
- $L_x = 180a$
- ~2.5M cells

Newtonian fluids

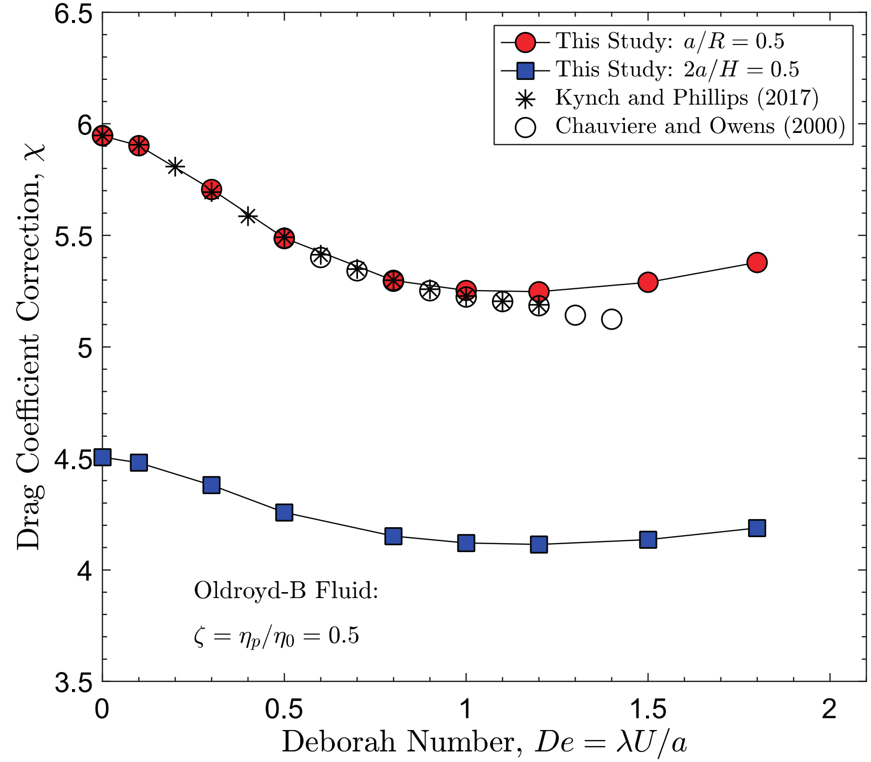


Viscoelastic fluids

$$\chi = \frac{C_D}{(24/Re)}$$



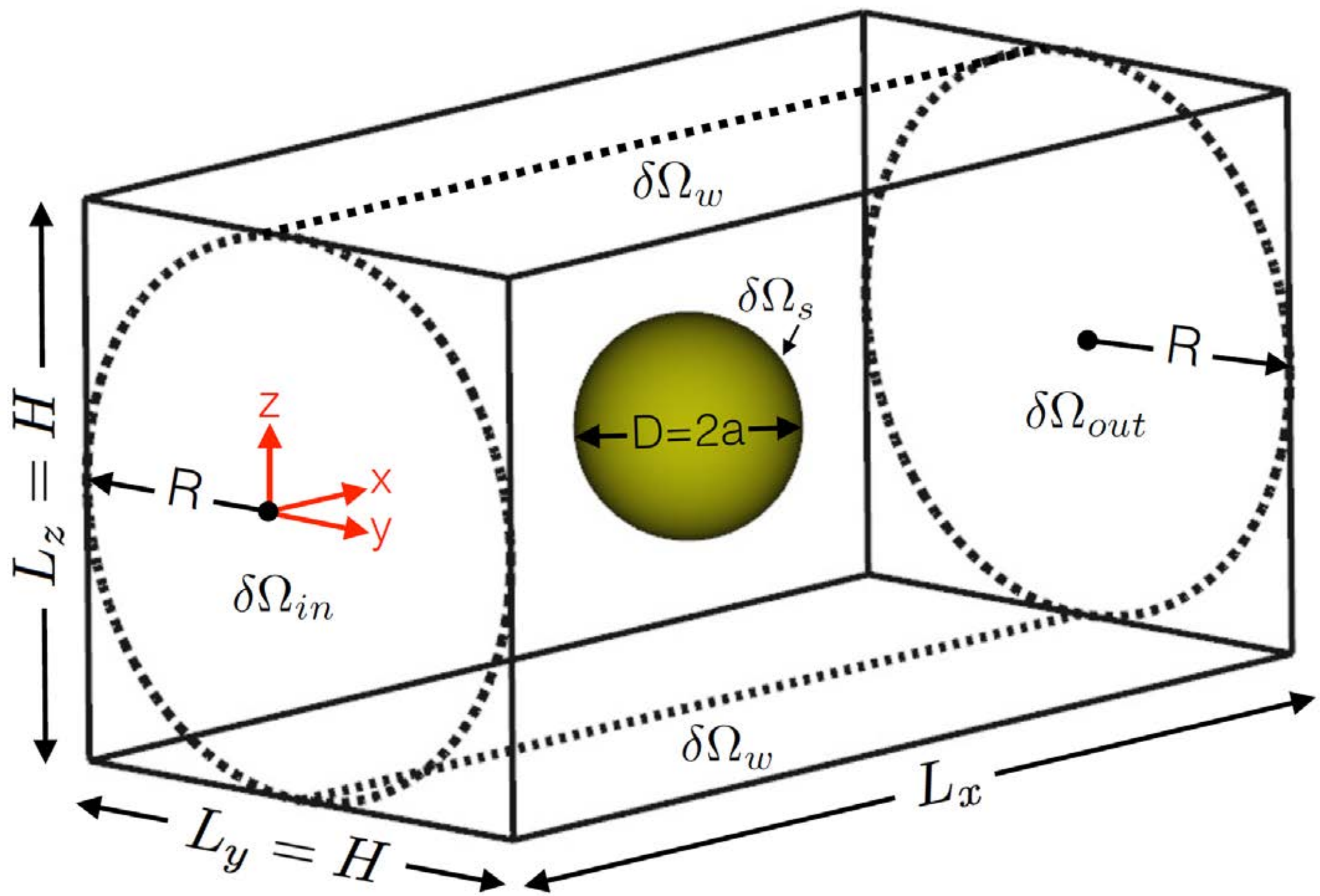
UCM



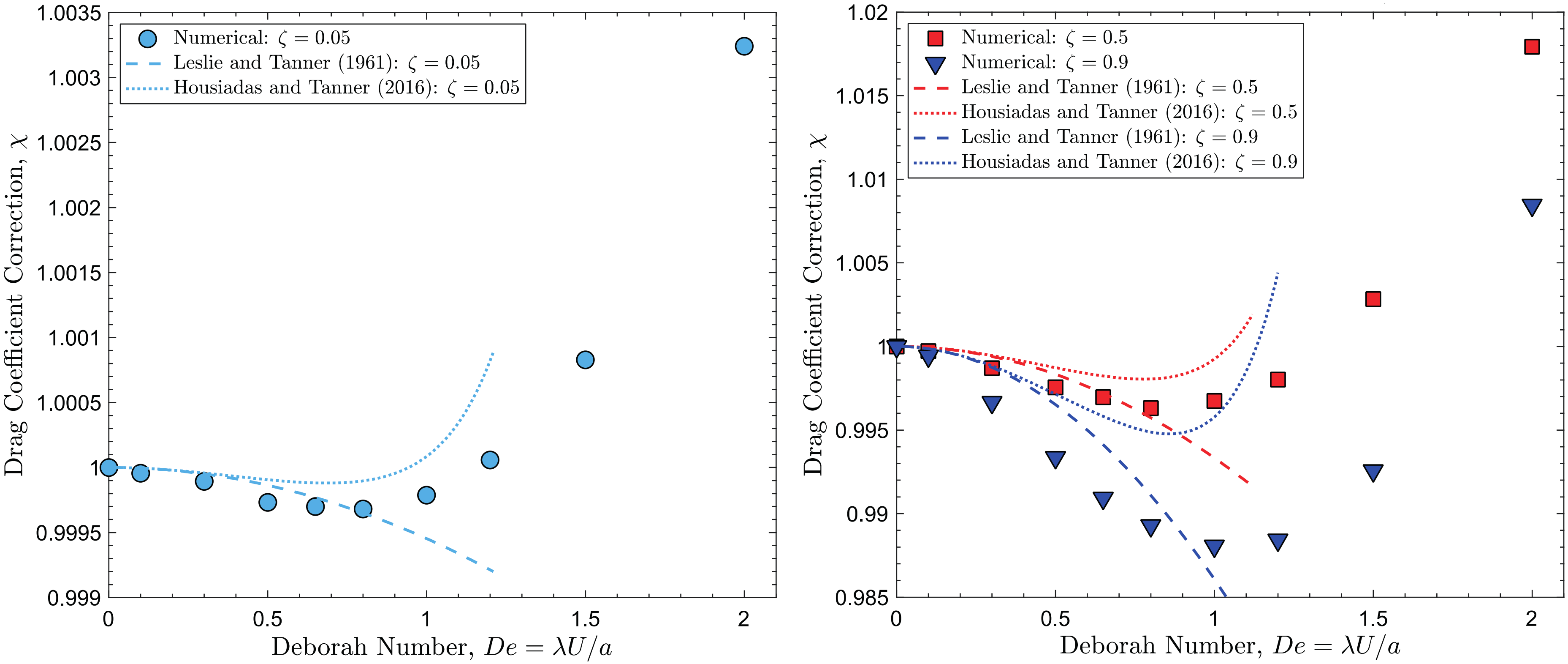
Oldroyd-B

Drag Model

- Square duct
- $L_x = 180a$, $L_y = L_z = 40a$
- Periodic boundary conditions at the lateral walls



Creeping flow conditions



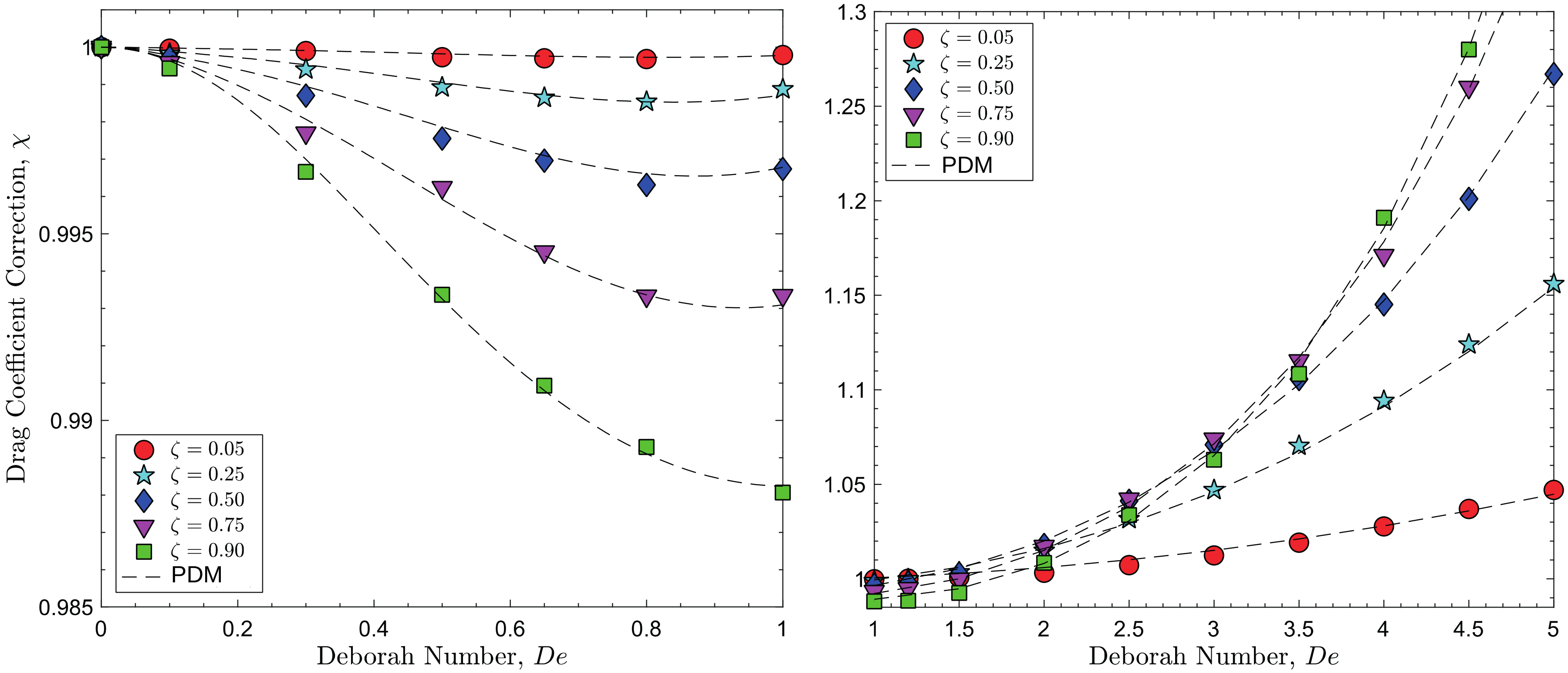
Creeping flow conditions - Proposed drag model (PDM)

$$\chi = \frac{C_D}{(24/Re)} = \begin{cases} 1 + \dfrac{\sum_{i=1}^{3}\left[De^{2i}\left(\sum_{m=1}^{3} a_{im}\zeta^{m-1}\right)\right]}{\sum_{j=1}^{3}\left[De^{2(j-1)}\left(\sum_{n=1}^{3} b_{jn}\zeta^{n-1}\right)\right]} & \text{if } De \leq 1 \\ 1 + \dfrac{\sum_{k=1}^{3}\left[De^{2(k+1)}\left(\sum_{p=1}^{3} c_{kp}\zeta^{p-1}\right)\right]}{\sum_{s=1}^{3}\left[De^{2(s-1)}\left(\sum_{q=1}^{3} d_{sq}\zeta^{q-1}\right)\right]} & \text{if } De > 1 \end{cases}$$

$$a_{im} = \begin{pmatrix} 0 & -0.00443 & 0.002478 \\ 0 & -0.09422 & 0.07025 \\ 0 & 0.06665 & -0.06392 \end{pmatrix} \qquad c_{kp} = \begin{pmatrix} 0 & -0.02511 & 0.0009496 \\ 0.0006047 & 0.02517 & -0.02148 \\ 0 & 0 & 0.0005713 \end{pmatrix}$$

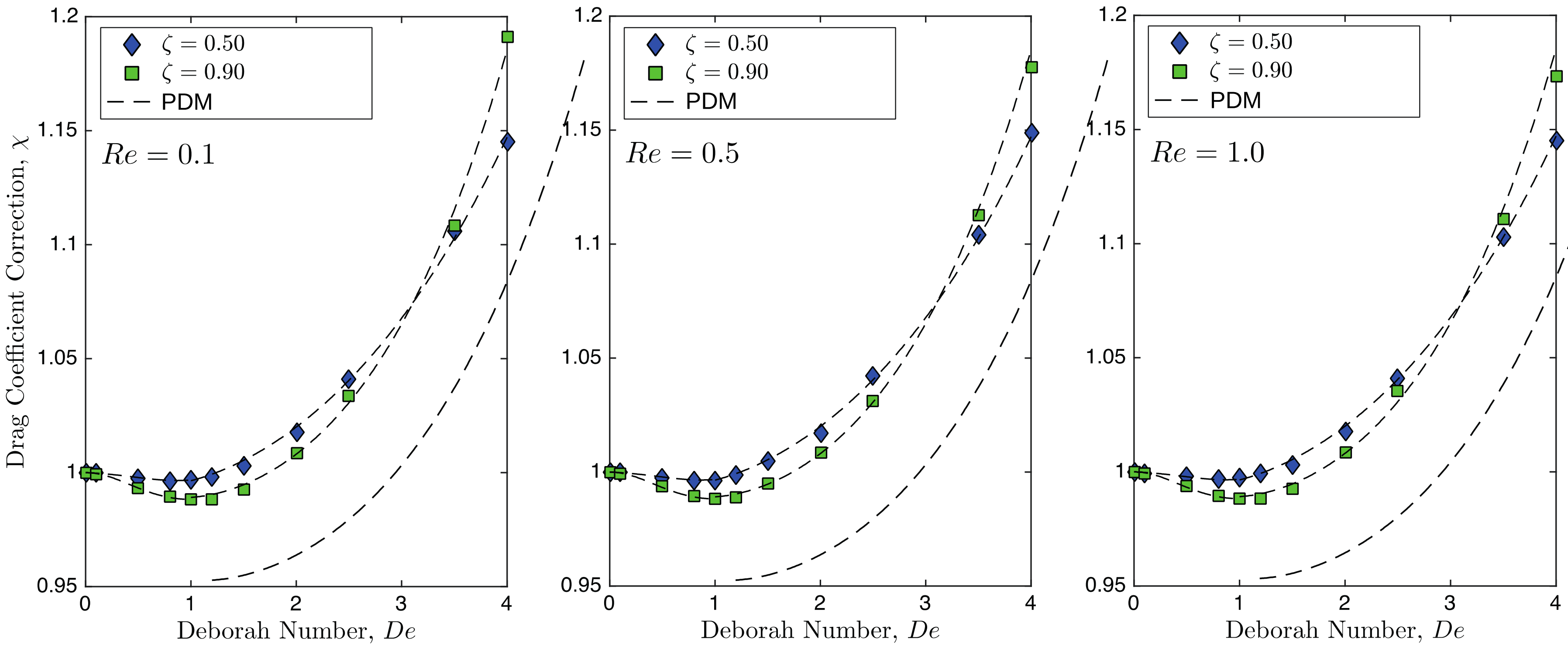
$$b_{jn} = \begin{pmatrix} 0.0534 & 0 & 0 \\ 6.288 & -6.111 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad d_{sq} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.5014 & -0.02511 \\ 1 & 0 & 0 \end{pmatrix}$$

Creeping flow conditions



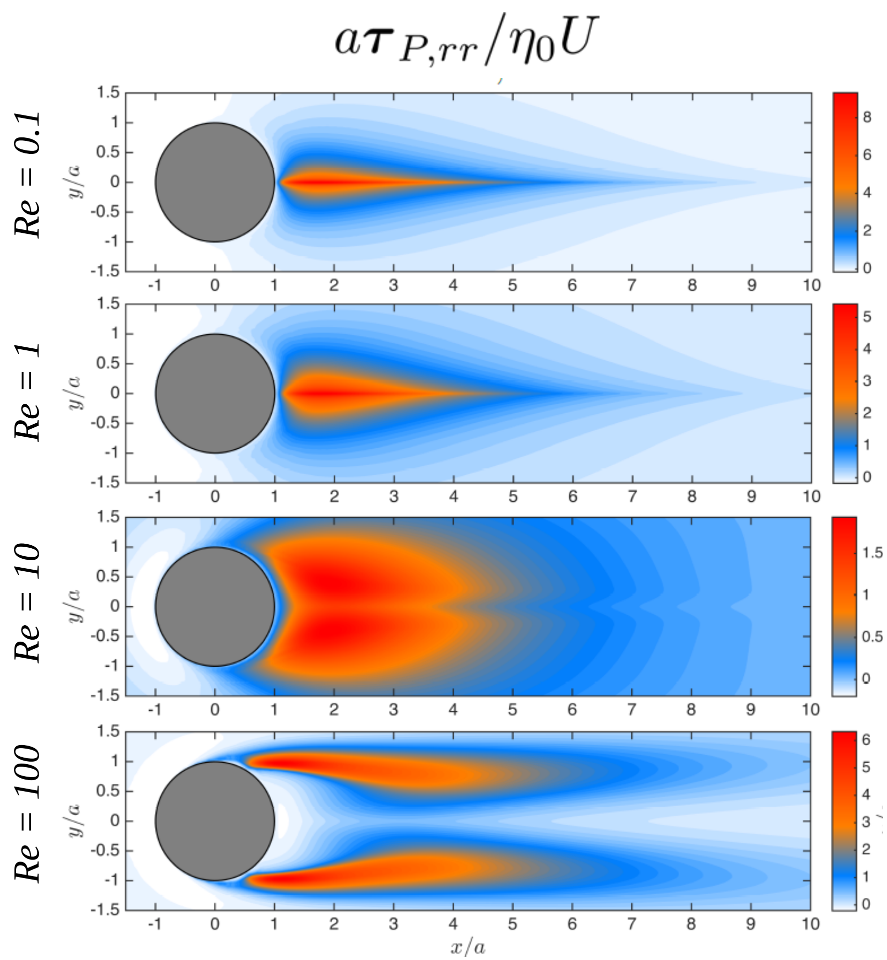
Drag Model

Effect of inertia

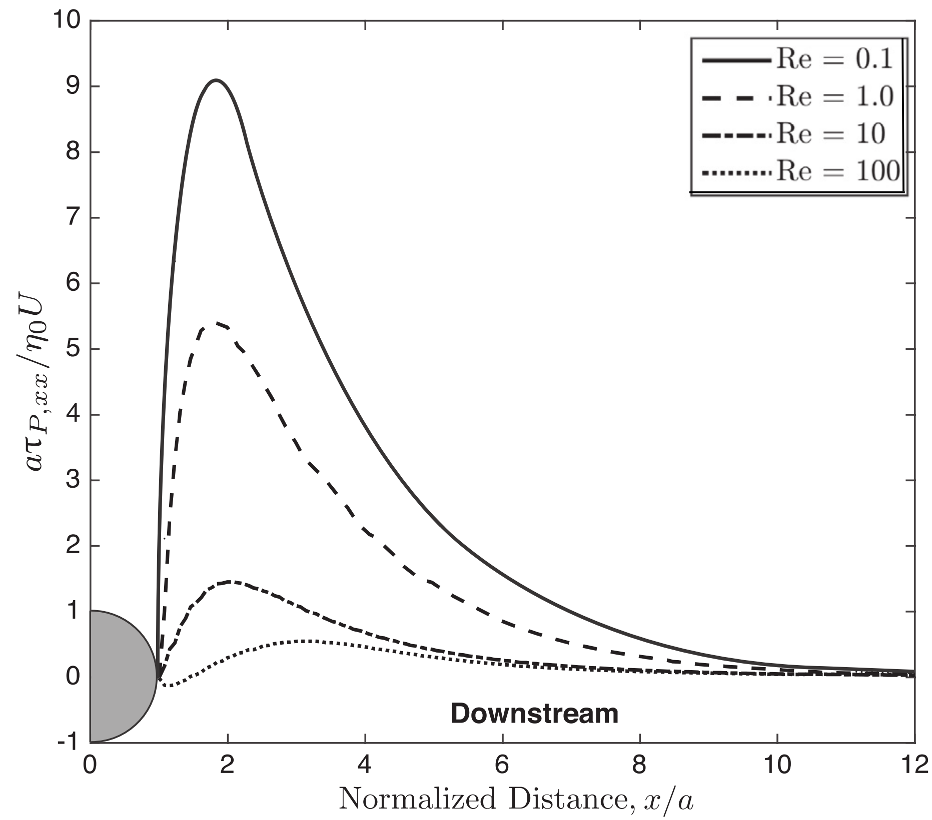


Proposed drag model just valid for $Re \leq 1$!!!

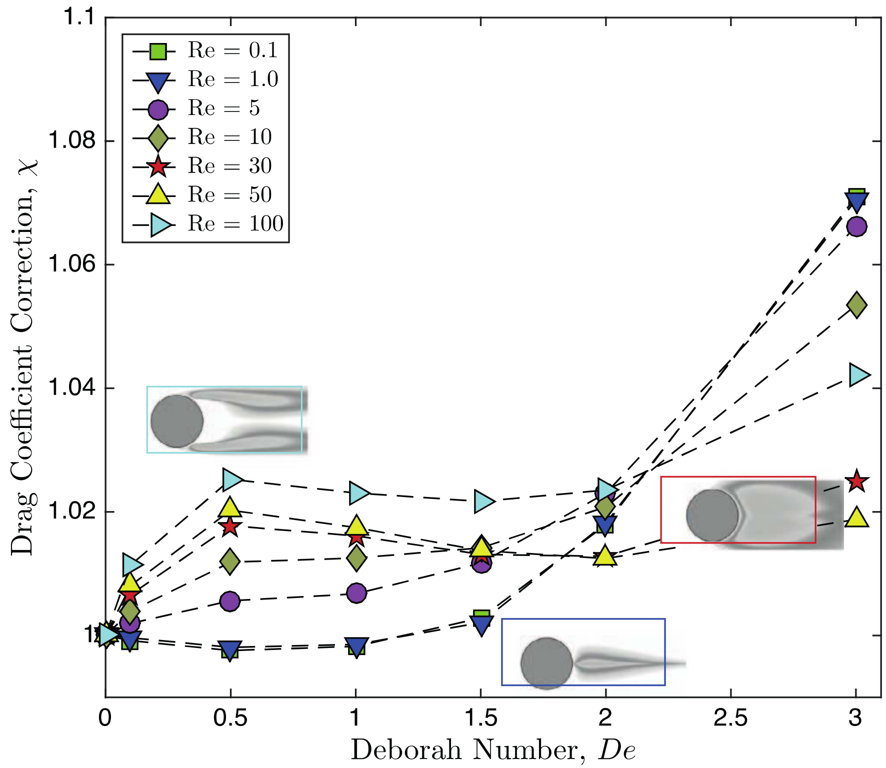
Effect of inertia

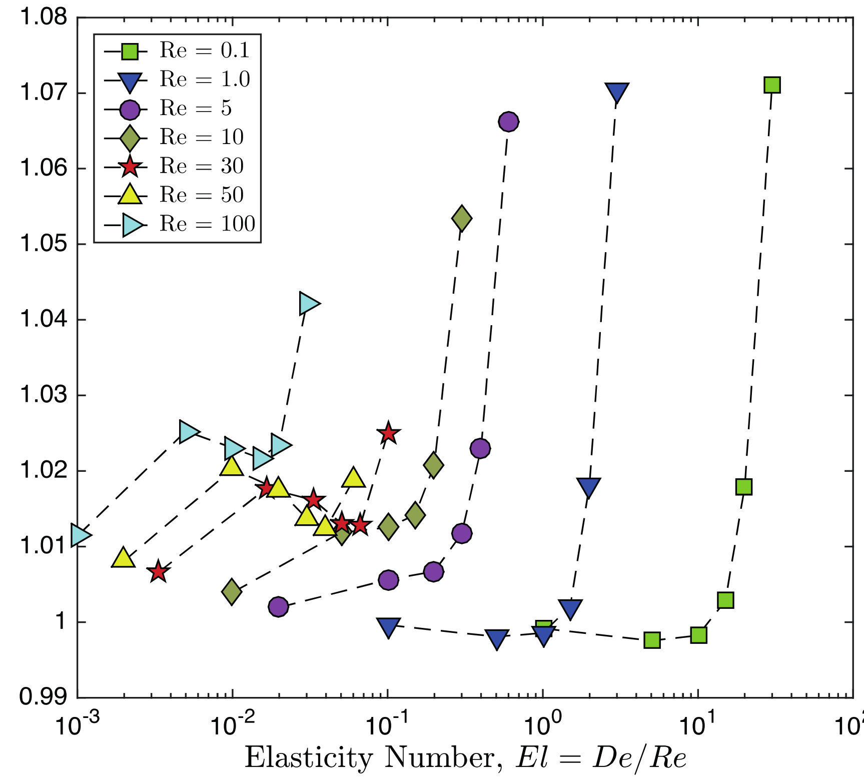


(Spherical polar coordinates)



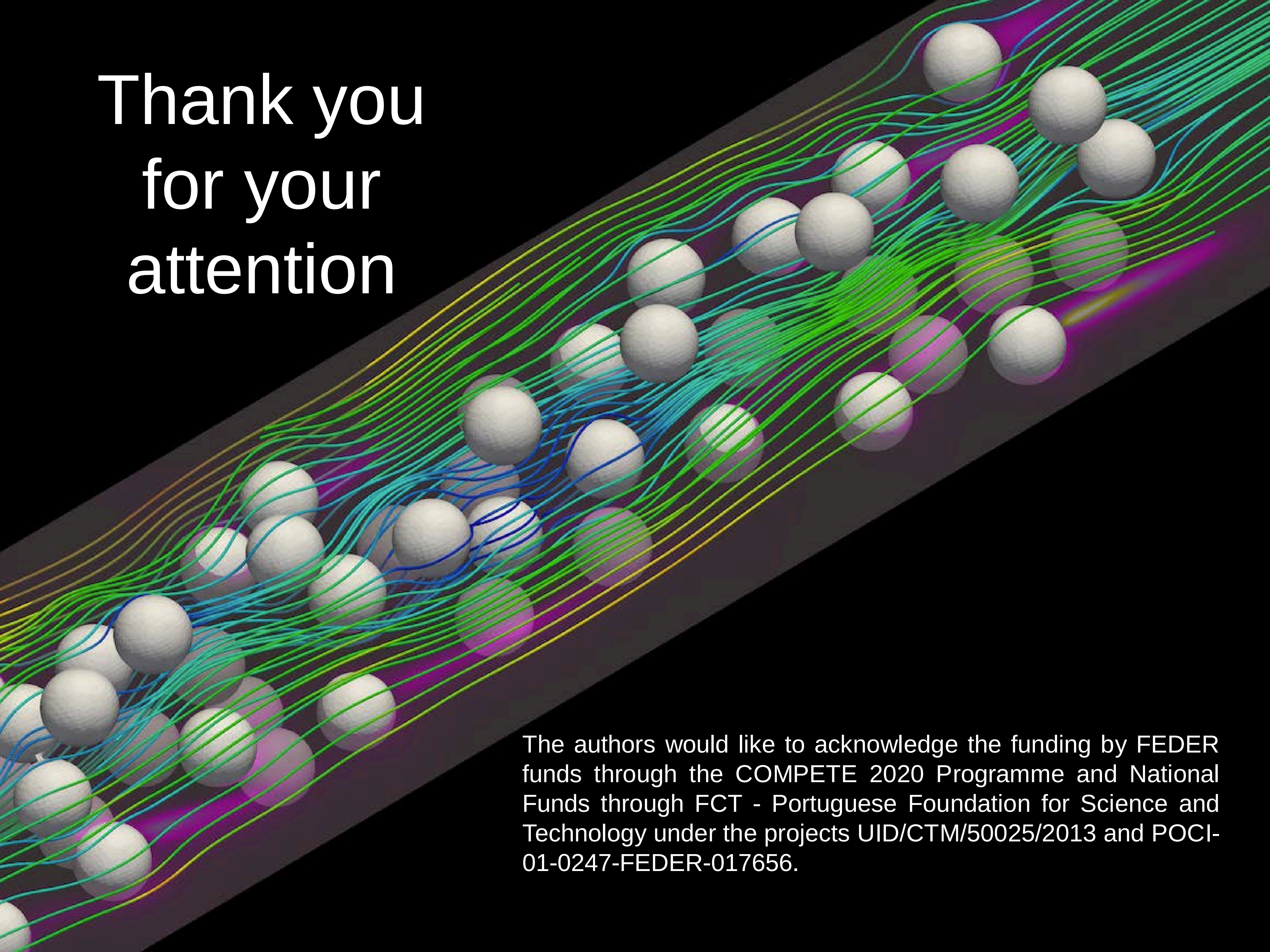
Effect of inertia





- DNS to study the effects of fluid inertia, elasticity and polymer viscosity ratio, on the overall drag coefficient
- A drag model was proposed to fit the data collected on numerical experiments
- Drag model valid (95% accuracy) for $0 < \zeta < 1$, $De < 9$ and $Re \leq 1$

Thank you for your attention



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