Relativistic mean field formulation of clustering in heavy nuclei

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Relativistic mean field formulation of clustering in heavy nuclei

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Abstract. Very little is known about clustering in heavy nuclei and in particular the interaction between the correlated cluster nucleons and remaining core nucleons. Currently the phenomenological Saxon-Woods plus cubic Saxon-Woods core-cluster potential successfully predicts the alpha decay half-life and energy band spectra of a number of heavy nuclei. This model, however, lacks a microscopic understanding of clustering phenomenon in these heavy nuclear systems. A fully relativistic microscopic formalism is presented, which generates the core-cluster potential by means of the McNeil, Ray and Wallace based double folding procedure. The core and cluster baryon densities are calculated by using a relativistic mean field approach. The Lorentz covariant IA1 representation of the nucleon-nucleon interaction is folded with the core and cluster densities. Theoretical predictions of the ground-state decay half-life and positive parity energy band of ^{212}Po are obtained with the relativistic mean field formalism and which are compared to the results from the phenomenological Saxon-Woods plus cubic Saxon-Wood core-cluster potential and microscopic M3Y interaction.

1. Introduction

In nuclear dynamics, as seen in light stable nuclei, clustering phenomenon is one of the essential features which has been studied in great detail [1]. In the physics of unstable nuclei, clustering is one of the central areas of study. The cluster-core interaction lies central to the identification of clustering in the nuclear matter and the description of clustering phenomenon in various nuclei. During the last decade the modified phenomenological Saxon-Woods plus Cubic Saxon-Woods cluster potential has successfully described various phenomenon related to alpha clustering in light as well as even-even heavy nuclei. In order to fully describe clustering in nuclear systems one would have to develop a microscopic model of the phenomenon at the nucleon-nucleon scale.

At a more microscopic level the core-cluster interaction may be constructed from a nucleon-nucleon interaction. Prior to the development of the Saxon-Wood plus Saxon-Wood cubed potential form, such a microscopic interaction had been employed in various forms to describe α cluster bound states in light nuclei [2] and the exotic decays in heavy nuclei [3]. In recent years the microscopic M3Y-type potential model has been extended to describe the alpha decay half-lives and the structure of heavy nuclei [4], [5], and [6]. An application of the interaction to ^{94}Mo and ^{212}Po in particular suggests a good amount of α clustering in these nuclei [7].

Relativistic mean field theory (RMFT) [8] has proven to be very successful in describing various properties of nuclear structure [10]. In this work a RMFT description of clustering is presented and a comparison is made between the experimental ground-state decay half-lives and band energy spectral of ^{212}Po and cluster model predictions of these quantities which were obtained from the Saxon-Woods + cubic Saxon-Woods potential, double folded M3Y nucleon-nucleon interaction [9] and the microscopic RMFT based core-cluster interaction.

2. Formulation of Binary cluster model

This model is based on the preformed binary cluster model for which the decay half life is given by

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma}, \tag{1}$$

where Γ represents the cluster decay width. For the breakup of a nucleus into the core and cluster the decay width is defined by the relationship

$$\Gamma = P \frac{\hbar^2}{2\mu} \frac{exp(-2\int_{r_2}^{r_3} k(r)dr)}{\int_{r_1}^{r_2}[k^{-1}(r)]dr} \tag{2}$$

with (P) being the core-cluster preformation probability in the parent nucleus, μ is reduced mass of the core-cluster system and $k(r)$ is cluster wavenumber. The wavenumber depends on both the decay energy (E) and the core-cluster potential $V(r)$, and is given by

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|E - V(r)|}. \tag{3}$$

The energy band structure of the quasi-boundstates can be obtained from a combination of the Bohr-Sommerfeld (BS) quantization integral

$$\int_{r_1}^{r2} \sqrt{\frac{2\mu}{\hbar^2}\left[E_l - V(r)\right] dr} = (2n+1)\frac{\pi}{2} \tag{4}$$

and the Wildermuth condition $G = 2n + l$, where n is the number of nodes of the radial wavefunction and l is the orbital angular momentum of the cluster state. G defines the global quantum number of the core-cluster relative motion. The interaction between the core and

cluster, $V(r)$, is described by the sum of the attractive nuclear cluster-core potential $U(r)$, the Coulomb potential between the two charged centres, and the centrifugal potential.

3. Core cluster potentials

3.1. Phenomenological core-cluster interaction

The recently developed modified Saxon-Wood with an additional cubic Saxon-Woods core-cluster phenomenological potential

$$U(r) = U_0 \left[\frac{x}{1 + exp\left(\frac{r-R}{a}\right)} + \frac{1-x}{1 + exp\left(\frac{r-R}{3a}\right)^3} \right] \tag{5}$$

is found to consistently reproduce not only the alpha and exotic decay half-lives, but also correctly predict the level properties of nuclei in the rare earth and the actinide region. This potential is parameterized in terms of the potential depth (U_0), nuclear radius (R), diffuseness (a), and x is a mixing parameter. Despite its success this potential model tells us very little about the microscopic nature of clustering in closed shell nuclei.

3.2. Relativistic mean field construction of the cluster-core potential

In the IA1 representation of the nucleon-nucleon scattering amplitude [12]

$$F = F^S I^a I_b + F^V \gamma_a^\mu \gamma_{\mu b} + F^{PS} \gamma_a^5 \gamma_{5b} + F^T \sigma_a^{\mu\nu} \sigma_{\mu\nu b} + F^A \gamma_a^5 \gamma_a^\mu \gamma_b^5 \gamma_{\mu b}, \tag{6}$$

Lorentz covariance, parity conservation, isospin invariance, and the constraint that the free nucleons are on the mass shell imply that the invariant NN scattering operator F be written in terms of the five complex functions for pp and five for pn scattering. The quantities $\lambda_i^L = (I, \gamma^\mu, \gamma^5, \sigma^{\mu\nu}, \gamma^5\gamma^\mu)$ represent the five Dirac gamma matrices [13], and the index $(i = a, b)$ labels the two interacting nucleons. The index L labels the scalar, vector, pseudo-scalar, tensor and axial terms.

Out of the Lorentz covariant McNeil, Ray and Wallace (MRW) construction of the optical potential for nucleon-nucleus scattering [11], arises the double folded MRW form which describes the cluster-core potential

$$U^L(r, \epsilon) = -\frac{4\pi i p}{Mc^2} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} F^L(q, \epsilon) \int d^3r' e^{-i\mathbf{q}\cdot\ \mathbf{r}'} \rho_1^L(r') \int d^3r'' e^{-i\mathbf{q}\cdot\ \mathbf{r}''} \rho_2^L(r''), \tag{7}$$

where r represents the separation distance between the cluster (1)and core (2)center, and ϵ is the laboratory energy of the nucleons in the cluster. The momentum of the nucleons in the nucleon-nucleon (NN)center of mass system is given by p while M represents the nucleon mass. Equation (7) contains the Lorentz covariant nucleon-nucleon scattering amplitudes $F^L(q, \epsilon)$, which are functions of the NN centre of mass momentum transfer ($\vec{q}$) and nucleon laboratory energy (ϵ), as well as the respective cluster and core densities ρ_1^L and ρ_2^L.

The Walecka model is based on a relativistic mean field theory with an effective Lagrangian which describes the NN interaction via the electromagnetic interaction and the effective meson fields [8]. The dynamical equation which results from the Lagrangian is given by

$$\hat{H}\psi(\mathbf{r}) = \Big(i\alpha \cdot \nabla - g_v \gamma^0 V^0(r) + \beta[M - g_s \phi(r)])\psi(\mathbf{r}\Big) = E\psi(\mathbf{r}) \tag{8}$$

with the Dirac Hamiltonian operator ($\hat{H} = i\alpha \cdot \nabla - g_v \gamma^0 V^0(r) + \beta[M - g_s \phi(r)]$), vector and scalar fields g_v and g_s respectively, as well as the zeroth component vector field (V_0) and scalar field (ϕ). Equation (8) has both positive and negative solutions $U(\mathbf{r})$ and $V(\mathbf{r})$, and thus the field operator can be expanded as

$$\hat{\psi}(\mathbf{r}) = \sum_{\Lambda} \left[A_{\Lambda} U_{\Lambda}(\mathbf{r}) + B^{\dagger}_{\Lambda} V_{\Lambda}(\mathbf{r}) \right]. \tag{9}$$

The baryon and antibaryon creation operators $A^{\dagger}_{\Lambda}$ and $B^{\dagger}_{\Lambda}$ satify the standard anticommutator relationships and the index Λ specifies the full set of single-particle quantum numbers, which for a spherically symmetric and parity conserving system, are the usual angular momentum and parity quantum numbers, as given by reference [13]. The positive-energy spinor can be written as

$$U_{\Lambda} \equiv U_{njlmt}(\mathbf{r}) = \begin{pmatrix} i\left[G_{njlt}(r)/r\right] \Phi_{jlm} \\ \left[F_{njlt}(r)/r\right] \Phi_{jl+1m} \end{pmatrix} \varsigma_t, \tag{10}$$

where Φ_{jlm} is the angular momentum and spin dependent part of the solution and n and ς_t represents the principal quantum number and two component isospinor which is labeled by the isospin projection t. The functions $G(r)$ and $F(r)$ represent the radial wave functions for the upper and lower components of the positive energy spinor U_{Λ}
Neglecting the negative-energy spinors the local baryon (ρ_B) and scalar (ρ_s) densities can be derived from the positive-energy solutions

$$\left.\begin{matrix} \rho_B(\mathbf{r}) \\ \rho_s(\mathbf{r}) \end{matrix}\right\} = \sum_{\Lambda} \bar{U}_{\Lambda}(\mathbf{r}) \begin{pmatrix} \gamma^0 \\ I \end{pmatrix} U_{\Lambda}(\mathbf{r}). \tag{11}$$

4. Model predictions and conclusions

For the BMP phenomenological form of the cluster-core potential prediction of the positive parity alpha band energy structure of ^{212}Po the parameters $U_0 = 208$ MeV, $a = 0.66$ fm, $x = 0.30$ and $R = 6.784$ were used with $G = 18$ [15].
The Walecka based RMFT prediction uses the experimental masses $M = 939$ MeV, $m_v = m_{\omega} = 738$ MeV, $m_{\rho} = 770$ MeV, $m_s = 520$ MeV, and $\alpha = e^2/4\pi = 1/137.36$ are used. The coupling constants for the scalar, vector, and ρ-meson are $g_s^2 = 109.6$, $g_v^2 = 190.4$, and $g_{\rho}^2 = 65.23$ respectively. We apply the Dirac-Hartree code Timora [14] to calulate the scalar and vector densities for both the protons and neutrons. The densities of the core and cluster systems are inturn used to calculate the core-cluster potential by means of the double folded MRW method. The results of the calculated α-decay half-life of the ground-state as predicted by the phenomenological BMP, the microscopic M3Y with phenomenological core and cluster baryon densities [15], and MRW double folded relativistic mean field nucleon densities with Lorentz covariant NN scattering amplitudes are compared with experimental data [16] in Table I. Table II compares the predicted band energy structure from the BMP, M3Y and RMFT model calculations with available experimental data [16].
From the results in Table (1) and (2) one see that the Saxon-Woods plus cubic Saxon-Woods potential gives a reasonable prediction of the half-life of the 0^+, while the microscopic M3Y and RMFT based models underpredict the ground-state alpha decay half-life of ^{212}Po by a factor of approximately 2. Furturemore the energy spectra of the excited α states are predicted reasonably well by the Saxon-Woods plus cubic Saxon Woods core-cluster potential where as both the microscopic M3Y and RMFT model potentials show a clear inversion of the energy spectra. The energy bands structure as predited by the RMFT model is wider spread at large l values as compared to the M3Y predictions.

Table 1. The experimental ground state decay of ^{212}Po and the corresponding values obtained with the BMP phenomenological potential given by equation (5), double folded M3Y potential with phenomenological Saxon-Woods and Gaussian baryon density distributions (M3Y), and the double folded RMFT potential (RMFT).

$T_{1/2}$(Exp) ns	$T_{1/2}$(BMP) (ns)	$T_{1/2}$(M3Y) (ns)	$T_{1/2}$(RMFT) (ns)
300	348.0	157.4	153.8

Table 2. The experimental energy level scheme of ^{212}Po and the calculated spectra obtained with the BMP phenomenological potential given by equation (5) and the double folded RMFT potential.

J^π	E_{exp} (MeV)	E_{BMP} MeV	E_{M3Y}	E_{RMFT} (MeV)
0^+	0.000	(0.495)	−0.004	−0.007
2^+	0.727	0.659	−0.067	−0.136
4^+	1.132	0.948	−0.229	−0.575
6^+	1.355	1.318	−0.508	−1.584
8^+	1.476	1.730	−0.930	−3.346
10^+	1.834	2.145	−1.538	−5.917
12^+	2.702	2.519	−2.358	−9.407
14^+	2.885	2.805	−3.437	−14.226
16^+	–	2.941	−4.800	−21.000
18^+	2.921	2.841	−6.477	−30.040

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