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Cosmic accelerated expansion of the universe with phantom fluid

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Abstract

The phantom behavior of the Universe is discussed in an extended version of Gauss-Bonnet (GB) gravity. Following the method proposed by (doi.org/10.1142/S0218271818500785 (2018)), we obtain a viable cosmological model for the phantom phase of the Universe. We find a condition for m in the model $\sim G^m$ which shows a phantom expansion of the Universe. On the other hand, using a phantom source-term $\sim T^{2n}$ in the model we observe that the term $\sim G^n$, with $n > \frac{3}{4}$, gives a phantomic space-time expansion. This form $(G^n + T^{2n})$ obtained for the phantom phase of the Universe exhibits a similar form to the Einstein's gravity theory $(R + L_m)$. However, we are addressed the cosmic coincidence problem for the model.

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I. INTRODUCTION

The observational data [1–3] indicate that the late-time (present) universe has a spatial-flat geometry with a unknown (dark) energy component which causes an accelerating expansion. After it was noticed that an additional term (may be cosmological constant Λ) in the Einstein equations presented a dynamic universe, in theoretical background the modifications of the Einstein's gravity theory have been considered by researchers. Some gravitational theories such as $F(R)$ [4, 5], $F(\boldsymbol{T})$ [6], $f(R,G)$ [7], $f(R,T)$ [8], $F(R,\boldsymbol{T})$ [9], $F(G,T)$ [10] and $F(R,T_{\mu\nu}T^{\mu\nu})$[11, 12] gravity theories, with matter lagrangian L_m, are examples for the modifications of the standard Einstein's gravity $(R+L_m)$, where $\boldsymbol{T}$ and $T_{\mu\nu}T^{\mu\nu}$ are respectively the torsion scalar and a scalar constructed from the square of the energy-momentum tensor $T_{\mu\nu}$. While the solutions of the Einstein's field equations without the additional term produce the dust era and the radiation era of the universe in context of the spatial-flat FriedmannRobertsonWalker (FRW) universe, modifications give a cosmic accelerated expansion in both two eras (for the early-inflationary universe and the late-time universe). In the observational frame these cases can be evaluated by taking into account an important parameter, i.e. the equation-of-state (EoS) parameter, $w=\frac{p}{\rho}$, where p and ρ respectively denote the pressure and the density of ordinary matter obtained from the matter lagrangian, L_m. For the certain values interval of the EoS parameter showing a negative pressure, it can be stated that the universe is in an accelerated phase. For instance, while the range $-1<w<-\frac{1}{3}$ [13] show a quintessential expansion, $w<-1$ [2] expresses a phantomic expansion. However, the case $w=-1$ describes de Sitter expansion (Λ-cold-dark-matter cosmology, ΛCDM model). ΛCDM model is faced with some cosmological problems. One of these is coincide problem related to a specific expansion of the beginning universe (inflation). Since the current ratio of dark energy density to matter energy density are nearly equal to one (why now?), this case requires a specific condition of the beginning inflation. However, in a future cosmic time both dark energy and matter energy densities become different rates as long as the Universe expands[14–17]. In the present study, we are addressed this problem for a phantom phase of the universe.

On the other hand, another approach for modifications is based on scalar fields modifications which are minimal-coupled with gravity. Namely, with helping of a scalar field, ϕ, in the standard Einstein gravity the cosmic acceleration of the universe can be shown in both the early and the late-time universe. In this study, we consider theory of $R + F(G,T) + L_m$ [18] gravity together with the scalar field lagrangian L_ϕ to show the phantom phase of the universe. For phantom phase of the universe we have obtained a viable cosmological model arising from the terms in the consideration gravity. We find a condition for m in the geometric model $\sim G^m$ which shows a phantom expansion of the universe. On the other side, with another geometric term in the system, $\sim G^n$ with $n > \frac{3}{4}$, we obtain a phantomic expansion in which the geometric term is supported by the phantom source-term $\sim T^{2n}$. This gets a difference from the other pure geometric term, $\sim G^m$. Therefore, we have observed that the structure $(G^n + T^{2n})$ for phantom phase exhibits a similar form to Einstein's gravity theory $(R + L_m)$. However, for a long cosmic time we observe that the ratio of dark energy density and matter energy density are nearly equal to one, $\frac{\rho_d}{\rho} \cong 1$, which this can be a solution of the coincide problem for late-time evolution of the universe at least.

II. PHANTOM PHASE SOLUTIONS OF THE GRAVITY THEORY

We consider the following action integral which includes a canonical scalar field lagrangian [18], $L_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$,

$$S = \int [\frac{(R + F(G,T))}{2k^2} + L_m + L_\phi]\sqrt{-g}d^4x, \quad (1)$$

where $k^2 = 8\pi\widetilde{G}$ with Newton constant $\widetilde{G} = 1$. The field equations are derived by varying the action (1) with respect to inverse metric, $g^{\alpha\beta}$. Then we obtain

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R &= k^2T_{\alpha\beta} + k^2T^{(\phi)}_{\alpha\beta} + \frac{1}{2}g_{\alpha\beta}F(G,T) \\ &- (T_{\alpha\beta} + \Theta_{\alpha\beta})F_T - [2RR_{\alpha\beta} - 4R^\mu_\alpha R_{\mu\beta} \\ &+ 2R^{\mu\nu\xi}_\alpha R_{\beta\mu\nu\xi}]F_G - [2Rg_{\alpha\beta} - 4R_{\alpha\beta} \\ &- 2R\nabla_\alpha\nabla_\beta + 4R^\mu_\beta\nabla_\alpha\nabla_\mu + 4R^\mu_\alpha\nabla_\beta\nabla_\mu \\ &- 4g_{\alpha\beta}R^{\mu\nu}\nabla_\mu\nabla_\nu + 4R_{\alpha\mu\beta\nu}\nabla^\mu\nabla^\nu]F_G, \end{aligned} \quad (2)$$

where F_T and F_G show derivatives with respect to the energy-momentum tensor (EMT) and the GB invariant, respectively. Also, $T^{(\phi)}_{\alpha\beta}$ and $\Theta_{\alpha\beta}$ are defined by

$$T^{(\phi)}_{\alpha\beta} = \nabla_\alpha\phi\nabla_\beta\phi + g_{\alpha\beta}L_\phi,\ T_{\alpha\beta} = g_{\alpha\beta}L_m - 2\frac{\partial L_m}{\partial g^{\alpha\beta}},$$
$$\Theta_{\alpha\beta} = -2T_{\alpha\beta} + g_{\alpha\beta}L_m - 2g^{\mu\nu}\frac{\partial^2 L_m}{\partial g^{\alpha\beta}\partial g^{\mu\nu}}. \quad (3)$$

Spatial-flat FRW metric is described by the following metric function,

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]. \quad (4)$$

Herein, a is the scale factor of the universe. After choosing $L_m = p$ [19] in eq. (3) we obtain

$$3H^2 = k^2(\rho + \rho_\phi) + (\rho + p)F_T - \frac{1}{2}F(G,T) + \frac{G}{2}F_G - 12H^3\dot{F}_G, \quad (5)$$

$$-(3H^2 + 2\dot{H}) = k^2(p + p_\phi) + \frac{1}{2}F(G,T)$$
$$-\frac{G}{2}F_G + 8H^3\dot{F}_G + 8H\dot{H}\dot{F}_G + 4H^2\ddot{F}_G, \quad (6)$$

where the trace of the EMT is $g^{\mu\nu}T_{\mu\nu} = T = -\rho + 3p$ due to metric (4). However, the over dot shows derivative with respect to cosmic time and $H = \frac{\dot{a}}{a}$ represents Hubble parameter. According to metric (4), the energy density and pressure of the scalar field, respectively, are as the following

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \quad (7)$$

We assume that the lagrangian function is a sum of the two functions as[18, 20]

$$F(G,T) = f(G) + f(T). \quad (8)$$

Then, the equations (5), (6) can be rewritten as follows,

$$3H^2 = k^2\rho_t, \quad (9)$$

$$-(2\dot{H} + 3H^2) = k^2 p_t, \quad (10)$$

where $\rho_t = (\rho + \rho_e)$ and $p_t = (p + p_e)$. The effective density and the pressure are given by

$$\rho_e = \rho_\phi + \frac{1}{k^2}[(\rho + p)f_T - \frac{f(T)}{2} - \frac{f(G)}{2} + \frac{G}{2}f_G - 12H^3\dot{f}_G], \quad (11)$$

$$p_e = p_\phi + \frac{1}{k^2}[-\frac{G}{2}f_G + \frac{f(G)}{2} + \frac{f(T)}{2} + 8H^3\dot{f}_G + 8H\dot{H}\dot{f}_G + 4H^2\ddot{f}_G]. \tag{12}$$

On the other side, the continuity equations for total energy density and the energy density of the scalar field can be written from the equations (7), (9) and (10)

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0,\ \dot{\rho}_t + 3H(\rho_t + p_t) = 0, \tag{13}$$

and therefore we have

$$\dot{\rho}_e + 3H(\rho_e + p_e) = 0. \tag{14}$$

Also, from the second equation in (13), for ordinary matter, we have

$$\dot{\rho} + 3H(\rho + p) = -\frac{1}{k^2 + f_T}[(\dot{p} - \frac{\dot{T}}{2})f_T + (\rho + p)\dot{f}_T]. \tag{15}$$

Hence, conservation of ordinary matter satisfies the following condition

$$(1 - w)f_T + 2Tf_{TT}(1 + w) = 0, \tag{16}$$

where $w = \frac{p}{\rho}$ is the equation-of-state (EoS) parameter of ordinary matter. However, from first equation in (13) we obtain the equation of motion for the scalar field,

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \tag{17}$$

with $V'(\phi) = \frac{\partial V(\phi)}{\partial \phi}$.

We construct the master equation by using equations (11), (12), (16) and the notation $A = k^2(p_e - \rho_e w_\phi) + 2\dot{H} + 3H^2(1 + w_\phi)$[18, 21] which shows the following equation

$$\begin{aligned} 4H^2\ddot{f}_G + 8H\dot{H}\dot{f}_G - 4H^3\dot{f}_G = -k^2(\rho + \rho_\phi) \\ (1 + w_\phi) + 3H^2(1 + w_\phi) - \frac{2\rho(1 + w)^2}{(w - 1)}Tf_{TT}, \end{aligned} \tag{18}$$

with $w_e \cong w_\phi = -1 - \frac{2\dot{H}}{3H^2}$. In effect, this equation can be easily created by utilizing the equations (5), (6). However the notation A gets an advantage to us. Because, in this notation we have an equality between dark energy density and matter energy density. In order to illustrate this case, we can utilize the continuity equation (14) and first equation in (13),

$$\frac{\rho_e}{\rho_\phi} \cong a^{3(w_\phi - w_e)}. \tag{19}$$

Hence, the case $w_e \cong w_\phi$ in the notation, A, provides that $\rho_e \cong \rho_\phi$. It is observed that the ratio of the densities will not different with cosmic time as it does not depends on the scale factor. For investigation we now proceed to find phantomic lagrangian function. To this purpose, we take into account the phantom solutions of the scale factor given by

$$a(t) = (t_s - t)^{-h} \rightarrow G = 24h^3(h+1)(t_s - t)^{-4} = \gamma(t_s - t)^{-4}, \tag{20}$$

where $h > 0$ and $t_s = t$ shows 'Big-Rip' singularity. Considering a general form of the EMT source, $f(T) = \beta T^\alpha$, the equation (17) can be written as follows,

$$\begin{aligned}\dot{P}(t) - HP(t) = {} & (1+w_\phi)[-k^2\rho_0(t_s - t)^{3h(1+w)} \\ & - k^2\rho_{\phi_0}(t_s - t)^{3h(1+w_\phi)} + 3h^2(t_s - t)^{-2}] - \frac{2(1+w)^2}{(w-1)} \\ & \rho_0^\alpha \beta\alpha(\alpha - 1)(3w - 1)^{\alpha-1}(t_s - t)^{3h\alpha(1+w)},\end{aligned} \tag{21}$$

where $P(t) = 4H^2\dot{f}_G$. Then we find the solution

$$\begin{aligned}f(G) = {} & A'G^{\frac{-3h(1+w)}{4}} + B'G^{\frac{-3h(1+w_\phi)}{4}} \\ & + D'G^{\frac{1}{2}} + F'G^{\frac{-3h\alpha(1+w)}{4}} + K'G^{\frac{h+1}{4}}\end{aligned} \tag{22}$$

where

$$\begin{aligned}A' &= \frac{(1+w_\phi)k^2\rho_0\gamma^{\frac{3h(1+w)+4}{4}}}{h^2[3h(1+w)+h+1][3h(1+w)+4][3h(1+w)]}, \\ B' &= \frac{k^2\rho_{\phi_0}(1+w_\phi)\gamma^{\frac{3h(1+w_\phi)+4}{4}}}{h^2[3h(1+w_\phi)+h+1][3h(1+w_\phi)+4][3h(1+w_\phi)]}, \\ D' &= \frac{3(1+w_\phi)\gamma^{\frac{1}{2}}}{4(h-1)}, \\ F' &= \frac{2(1+w)^2(-1+3w)^{n-1}}{4h^2(w-1)[3h\alpha(1+w)+h+1][3h\alpha(1+w)+4]} \\ & \frac{\rho_0^\alpha\beta\alpha(\alpha-1)\gamma^{\frac{3h\alpha(1+w)+4}{4}}}{[3h\alpha(1+w)]}, \\ K' &= \frac{c_1\gamma^{\frac{-h+3}{4}}}{3h^2(h+1)(3-h)},\end{aligned} \tag{23}$$

where c_1 is an integral constant. For the real value of the function (8) we can write

$$F(G,T) = B'G^m + F'G^n + \beta T^{2n}, \tag{24}$$

with $c_1 = 0$, where the standard FRW solutions $k^2\rho_0 = 3h^2$ and $h = -\frac{2}{3(1+w)}$ [18, 21, 22] are used. Also, $m = \frac{(1+w_\phi)}{2(1+w)}$ and $\alpha = 2n$. On the other hand, from eq. (16) we obtain

$$\alpha = 2n = \frac{(1+3w)}{2(1+w)}. \tag{25}$$

The phantomic system (22) describes a phantom spacetime expansion with the terms $F'G^n + \beta T^{2n}$. However, another geometrical term $B'G^m$ also describes a phantom expansion of the universe under the following condition,

$$m > 0 \tag{26}$$

when $w < -1$. In this case the EoS parameter of the scalar field is $w_\phi < -1$ interval. This interval is provided by the condition $m > 0$.

On the other hand, from the equality (23) for the phantom phase of the universe, $w < -1$, we obtain the following condition,

$$n > \frac{3}{4}. \tag{27}$$

Thus, with the positive value of the power-term of the GB-term a phantomic expansion of the universe is obtained. On the other side, the eq. (19) can be written as follows,

$$\frac{\rho}{\rho_\phi} \cong a^{\frac{6(2m-1)}{3-4n}}, \tag{28}$$

and if we are fixed the $m = \frac{1}{2}$ in the interval given by (26), we obtain equality $\rho \cong \rho_\phi \cong \rho_e$ that is independent from the scale factor of the universe. In other words, under the condition (27) a phantomic Einstein gravity is observed with $F'G^n + \beta T^{2n}$. Even if the current value of the EoS parameter is close to -1, in a future time one expects that $w < -1$, in which matter components of the universe decrease for a long cosmic time. Therefore, the density equality is break down in future. But, in the present study, we show that the density rates can be protected for the conditions $m = \frac{1}{2}$ and $n > \frac{3}{4}$ even if the universe is in a phantom phase. This can be a solution of the coincide problem.

III. CONCLUSION

It is known that while one side of the Einstein equation describes a spacetime geometry (curvature), the other side of it shows the matter which follows inside this curvature geometry. In one respect, this is described by the lagrangian $R + L_m$ in the EH action, with spacetime geometry R and matter L_m. Hence, in our case at hand we have $F'G^n + \beta T^{2n}$ which shows a relation between spacetime geometry $F'G^n$ and the matter βT^{2n}. This form obtained for the phantom phase of the universe is similar to the structure of the field equations of the general relativity theory. However, we state that the geometrical term, $B'G^m$, with the positive values of $m > 0$ or especially the fixed value $m = \frac{1}{2}$ supports the dark energy term $F'G^n + \beta T^{2n}$ which can remove the coincide problem for the phantom phase of the universe. It should be noted that for both the geometric Gauss-Bonnet invariants we obtain positive values of the power-terms, $n > \frac{3}{4}$ and $m = \frac{1}{2}$. The authors [5] show that with the condition $0 < \beta < \frac{1}{2}$ in the term $\sim G^{\beta}$ the universe does not reach the phantom phase. Namely, with the positive values of β in $0 < \beta < \frac{1}{2}$ interval the universe can not be in the phantom phase. But in the present study we have shown that the universe can be in a phantom phase together with the positive values of the power terms of the Gauss-Bonnet invariants. However, even if the energy densities of the matter component of the universe decrease in future time, the rates will not a change. In effect, this case can be originated from a fixed value of the EoS parameter of the extra dimensions in a non-accelerated expansion phase.[23].

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